## Probability and Random Processes -Video course

### **1. Introduction to Probability**

- Definitions, scope and history; limitation of classical and relative-frequency-based definitions
- · Sets, fields, sample space and events; axiomatic definition of probability
- Combinatorics: Probability on finite sample spaces
- Joint and conditional probabilities, independence, total probability; Bayes' rule and applications

### 2. Random variables

- Definition of random variables, continuous and discrete random variables, cumulative distribution function (cdf) for discrete and continuous random variables; probability mass function (pmf); probability density functions (pdf) and properties
- Jointly distributed random variables, conditional and joint density and distribution functions, independence; Bayes' rule for continuous and mixed random variables
- Function of random a variable, pdf of the function of a random variable; Function of two random variables; Sum of two independent random variables
- Expectation: mean, variance and moments of a random variable
- Joint moments, conditional expectation; covariance and correlation; independent, uncorrelated and orthogonal random variables
- Random vector: mean vector, covariance matrix and properties
- Some special distributions: Uniform, Gaussian and Rayleigh distributions; Binomial, and Poisson distributions; Multivariate Gaussian distribution
- Vector-space representation of random variables, linear independence, inner product, Schwarz Inequality
- Elements of estimation theory: linear minimum mean-square error and orthogonality principle in estimation;
- Moment-generating and characteristic functions and their applications
- Bounds and approximations: Chebysev inequality and Chernoff Bound
- 3. Sequence of random variables and convergence:

• Almost sure (a.s.) convergence and strong law of large numbers; convergence in mean square sense with examples from parameter estimation; convergence in probability with examples; convergence in distribution

Central limit theorem and its significance

## 4. Random process

• Random process: realizations, sample paths, discrete and continuous time processes, examples

• Probabilistic structure of a random process; mean, autocorrelation and autocovariance functions

- Stationarity: strict-sense stationary (SSS) and wide-sense stationary (WSS) processes
- Autocorrelation function of a real WSS process and its properties, cross-correlation

function • Ergodicity and its importance

 Spectral representation of a real WSS process: power spectral density, properties of power spectral density; cross-power spectral density and properties; auto-correlation function and power spectral density of a WSS random sequence

• Linear time-invariant system with a WSS process as an input: sationarity of the output, auto-correlation and power-spectral density of the output; examples with white-noise as input; linear shift-invariant discrete-time system with a WSS sequence as input.

Spectral factorization theorem

• Examples of random processes: white noise process and white noise sequence; Gaussian process; Poisson process, Markov Process

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