

# Nonlinear Vibration - Web course

## COURSE OUTLINE

**Introduction:** linear and nonlinear systems, conservative and non-conservative systems; potential well, Phase planes, types of forces and responses, fixed points, periodic, quasi-periodic and chaotic responses; Local and global stability; commonly observed nonlinear phenomena: multiple response, bifurcations, jump phenomena.

**Development of nonlinear governing equation of motion** of Mechanical systems, linearization techniques, ordering techniques; commonly used nonlinear equations: Duffing equation, Van der Pol's oscillator, Mathieu's and Hill's equations.

**Analytical solution methods:** Harmonic balance, perturbation techniques (Linstedt-Poincare', method of Multiple Scales, Averaging – Krylov-Bogoliubov-Mitropolsky), incremental harmonic balance, modified Lindstedt Poincare' techniques.

**Stability and bifurcation analysis:** static and dynamic bifurcations of fixed point and periodic response, different routes to chaotic response (period doubling, torus break down, attractor merging etc.), crisis.

**Numerical techniques:** time response, phase portrait, FFT, Poincare' maps, point attractors, limit cycles and their numerical computation, strange attractors and chaos; Lyapunov exponents and their determination, basin of attraction: point to point mapping and cell to cell mapping, fractal dimension.

**Application:** Single degree of freedom systems: Free vibration-Duffing's oscillator; primary-, secondary-and multiple- resonances; Forced oscillations: Van der Pol's oscillator; parametric excitation: Mathieu's and Hill's equations, Floquet theory; effects of damping and nonlinearity. Multi degree of freedom and continuous systems.

## COURSE DETAIL

Lecture No.	Topic
<b>Module 1: Introduction</b>	
1.	Mechanical vibration: Linear nonlinear systems, types of forces and responses
2.	Conservative and non conservative systems, equilibrium points, qualitative analysis, potential well, centre, focus, saddle-point, cusp point
3.	Commonly observed nonlinear phenomena: multiple response, bifurcations, and jump phenomena.
<b>Module 2: Derivation of nonlinear equation of motion</b>	
4.	Force and moment based approach



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<http://nptel.iitm.ac.in>

## Mechanical Engineering

### Pre-requisites:

- Mechanical Vibration
- Engineering Mechanics

### Additional Reading:

- International Journal of Nonlinear Mechanics
- Nonlinear Dynamics
- Journal of Sound and Vibration
- Journal of Vibration and Acoustics (ASME)

### Coordinators:

**Prof. S.K. Dwivedy**  
Department of Mechanical Engineering IIT Guwahati

5.	Lagrange Principle
6.	Extended Hamilton's principle
7.	Multi body approach
8.	Linearization techniques
9.	Development of temporal equation using Galerkin's method for continuous system
10.	Ordering techniques, scaling parameters, book-keeping parameter. Commonly used nonlinear equations: Duffing equation, Van der Pol's oscillator, Mathieu's and Hill's equations.
<b>Module 3: Approximate solution method</b>	
11.	Straight forward expansions and sources of nonuniformity
12.	Harmonic Balancing method
13.	Linstedt-Poincare' method
14.	Method of Averaging
<b>Module 4: Perturbation analysis method</b>	
15.	Method of Averaging
16.	Method of multiple scales
17.	Method of multiple scales
18.	Method of normal form
19.	Incremental Harmonic Balance method
20.	Modified Lindstedt-Poincare' method
<b>Module 5: Stability and Bifurcation Analysis</b>	
21.	Lyapunov stability criteria

22.	Stability analysis from perturbed equation
23.	Stability analysis from reduced equations obtained from perturbation analysis
24.	Bifurcation of fixed point response, static bifurcation: pitch fork, saddle-node and trans-critical bifurcation
25.	Bifurcation of fixed point response, dynamic bifurcation: Hopf bifurcation
26.	Stability and Bifurcation of periodic response, monodromy matrix, poincare' section
<b>Module 6: Numerical techniques</b>	
27.	Time response, Runge-Kutta method, Wilson- Beta method
28.	Frequency response curves: solution of polynomial equations, solution of set of algebraic equations,
29.	Basin of attraction: point to point mapping and cell-to-cell mapping
30.	Poincare' section of fixed-point, periodic, quasi-periodic and chaotic responses.
31.	Lyapunov exponents
32.	FFT analysis, Fractal Dimensions
<b>Module 7: Applications</b>	
33.	SDOF Free-Vibration: Duffing Equation
34.	SDOF Free-Vibration: Duffing Equation
35.	SDOF Forced-Vibration: Van der pol's Equation
36.	SDOF Forced-Vibration: Van der pol's Equation
37.	Parametrically excited system- Mathieu-Hill's equation, Floquet Theory
38.	Parametrically excited system- Instability regions

39.	Multi-DOF nonlinear systems
40.	Continuous system: Micro-cantilever beam analysis

**References:**

1. Nayfeh, A. H., and Mook, D. T., Nonlinear Oscillations, Wiley-Interscience, 1979.
2. Hayashi, C. Nonlinear Oscillations in Physical Systems, McGraw-Hill, 1964.
3. Evan-Ivanowski, R. M., Resonance Oscillations in Mechanical Systems, Elsevier, 1976.
4. Nayfeh, A. H., and Balachandran, B., Applied Nonlinear Dynamics, Wiley, 1995.
5. Seydel, R., From Equilibrium to Chaos: Practical Bifurcation and Stability Analysis, Elsevier, 1988.
6. Moon, F. C., Chaotic & Fractal Dynamics: An Introduction for Applied Scientists and Engineers, Wiley, 1992.
7. Rao, J. S., Advanced Theory of Vibration: Nonlinear Vibration and One-dimensional Structures, New Age International, 1992.