# Advanced Matrix Theory and Linear Algebra for Engineers - Video course

### COURSE OUTLINE

Introduction, Vector Spaces, Solutions of Linear Systems, Important Subspaces associated with a matrix, Orthogonality, Eigenvalues and Eigenvectors, Diagonalizable Matrices, Hermitian Matrices, General Matrices, Jordan Canonical form (Optional)\*, Selected Topics in Applications (Optional)\*

### COURSE DETAIL

Module No.	Topic/s	Hours
1	<ol> <li>Introduction:</li> <li>First Basic Problem – Systems of Linear equations - Matrix Notation – The various questions that arise with a system of linear eqautions</li> <li>Second Basic Problem – Diagonalization of a square matrix – The various questions that arise with diagonalization</li> </ol>	3
2	<ul> <li>Vector Spaces</li> <li>1. Vector spaces</li> <li>2. Subspaces</li> <li>3. Linear combinations and subspaces spanned by a set of vectors</li> <li>4. Linear dependence and Linear independence</li> <li>5. Spanning Set and Basis</li> <li>6. Finite dimensional spaces</li> <li>7. Dimension</li> </ul>	6
3	<ol> <li>Solutions of Linear Systems         <ol> <li>Simple systems</li> <li>Homogeneous and Nonhomogeneous systems</li> <li>Gaussian elimination</li> <li>Null Space and Range</li> <li>Rank and nullity</li> <li>Consistency conditions in terms of rank</li> <li>General Solution of a linear system</li> <li>Elementary Row and Column operations</li> <li>Row Reduced Form</li> <li>Triangular Matrix Factorization</li> </ol> </li> </ol>	6
4	Important Subspaces associsted with a matrix 1. Range and Null space 2. Rank and Nullity 3. Rank Nullity theorem 4. Four Fundamental subspaces 5. Orientation of the four subspaces	4



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### **Mathematics**

### **Coordinators:**

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5	Orthogonality	5
	<ol> <li>Inner product</li> <li>Inner product Spaces</li> <li>Cauchy – Schwarz inequality</li> </ol>	
	4. Norm 5. Orthogonality	
	<ol> <li>6. Gram – Schmidt orthonormalization</li> <li>7. Orthonormal basis</li> <li>2. Expansion in terms of orthonormal basis</li> </ol>	
	<ol> <li>8. Expansion in terms of orthonormal basis – Fourier series</li> <li>9. Orthogonal complement</li> <li>10. Decomposition of a vector with respect to a subspace</li> </ol>	
	and its orthogonal complement – Pythagorus Theorem	
6	Eigenvalues and Eigenvectors	5
	<ol> <li>What are the ingredients required for diagonalization?</li> <li>Eigenvalue – Eigenvector pairs</li> <li>Where do we look for eigenvalues? – characteristic equation</li> </ol>	
	<ol> <li>Algebraic multiplicity</li> <li>Eigenvectors, Eigenspaces and geometric multiplicity</li> </ol>	
7	Diagonalizable Matrices	5
	<ol> <li>Diagonalization criterion</li> <li>The diagonalizing matrix</li> <li>Cayley-Hamilton theorem, Annihilating polynomials, Minimal Polynomial</li> </ol>	
	<ol> <li>Diagonalizability and Minimal polynomial</li> <li>Projections</li> <li>Decomposition of the matrix in terms of projections</li> </ol>	
8	Hermitian Matrices	5
	<ol> <li>Real symmetric and Hermitian Matrices</li> <li>Properties of eigenvalues and eigenvectors</li> <li>Unitary/Orthoginal Diagonalizbility of Complex Hermitian/Real Symmetric matrices</li> <li>Spectral Theorem</li> <li>Positive and Negative Definite and Semi definite matrices</li> </ol>	
9	<u>General Matrices</u>	5
	<ol> <li>The matrices AA<sup>T</sup> and A<sup>TA</sup></li> <li>Rank, Nullity, Range and Null Space of AA<sup>T</sup> and A<sup>T</sup>A</li> <li>Strategy for choosing the basis for the four fundamental subspaces</li> <li>Singular Values</li> </ol>	
	<ol> <li>Singular Value Decomposition</li> <li>Pseudoinverse and Optimal solution of a linear system of equations</li> <li>The Geometry of Pseudoinverse</li> </ol>	
10	Jordan Cnonical form*	5
	<ol> <li>Primary Decomposition Theorem</li> <li>Nilpotent matrices</li> <li>Canonical form for a nilpotent matrix</li> </ol>	

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### **Selected Topics in Applications\***

- 1. Optimization and Linear Programming
- 2. Network models
- 3. Game Theory 4. Control Theory
- 5. Image Compression

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