Basic Algebraic Geometry : Varieties, Morphisms, Local Rings, Function Fields and Nonsingularity - Video course

COURSE OUTLINE

This course is an introduction to Algebraic Geometry, whose aim is to study the geometry underlying the set of common zeros of a collection of polynomial equations. It sets up the language of varieties and of morphisms between them, and studies their topological and manifold-theoretic properties. Commutative Algebra is the "calculus" that Algebraic Geometry uses. Therefore a prerequisite for this course would be a course in Algebra covering basic aspects of commutative rings and some field theory, as also a course on elementary Topology. However, the necessary results from Commutative Algebra and Field Theory would be recalled as and when required during the course for the benefit of the students.

Algebraic Geometry in its generality is connected to various areas of Mathematics such as Complex Analysis, PDE, Complex Manifolds, Homological Algebra, Field and Galois Theory, Sheaf Theory and Cohomology, Algebraic Topology, Number Theory, QuadraticForms, Representation Theory, Combinatorics, Commutative Ring Theory etc and also to areas of Physics like String Theory and Cosmology. Many of the Fields Medals awarded till date are for research in areas connected in a non-trivial way to Algebraic Geometry directly or indirectly. The Taylor-Wiles proof of Fermat's Last Theorem used the full machinery and power of the language of Schemes, the most sophisticated language of Algebraic Geometry developed over a couple of decades from the 1960s by Alexander Grothendieck in his voluminous expositions running to several thousand pages. The foundations laid in this course will help in a further study of the language of schemes.

SYLLABUS

Affine Varieties, Hilbert's Basis Theorem and the Hilbert Nullstellensatz, projective and quasi-projective varieties, morphisms, rational maps and function fields, nonsingularity, smooth varieties. The course will try to stress the nexus between Commutative Algebra and Algebraic Geometry. It begins the attempt to justify the philosophy that "Commutative Algebra is the Calculus for Algebraic Geometry" and illustrate the translation back and forth between concepts in Commutative Algebra and in Algebraic Geometry, in the spirit of Sophie Germaine's statement that "Algebra is none other than Geometry written down (in Mathematical language), and Geometry is none other than Algebra drawn out (as a beautiful picture)". For more details, please look at the lecture-wise titles, goals and keywords given below.

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COURSE DETAIL

Unit Number / Title	**Lecture Number / Title**
Unit 1: The Zariski Topology **Goals & keywords**	Lecture 1: What is Algebraic Geometry?
	Lecture 2: The Zariski Topology and Affine Space
	Lecture 3: Going back and forth between subsets and ideals
Unit 2: Irreducibility in the Zariski Topology **Goals & keywords**	Lecture 4: Irreducibility in the Zariski Topology
	Lecture 5: Irreducible Closed Subsets Correspond to Ideals Whose Radicals are Prime

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Mathematics

Pre-requisites:

A course in Algebra covering basic aspects of commutative rings and some field theory, and a course on elementary Topology

Additional Reading:

1) Algebra by Serge Lang

2) Basic Algebraic Geometry by I. R. Shafarevich

Hyperlinks:

https://en.wikipedia.org/wiki/Alexander_Grothendieck

http://www.grothendieckcircle.org/

On topics related to Algebraic Geometry:

https://en.wikipedia.org/wiki/Algebraic_geometry

http://www.jmilne.org/math/index.html

Coordinators:

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Unit 3: Noetherianness in the Zariski Topology **Goals & keywords**	Lecture 6: Understanding the Zariski Topology on the Affine Line; The Noetherian property in Topology and in Algebra	
	Lecture 7: The Noetherian Decomposition of Affine Algebraic Subsets Into Affine Varieties	
Unit 4: Dimension and Rings of Polynomial Functions **Goals & keywords**	Lecture 8: Topological Dimension, Krull Dimension and Heights of Prime Ideals	
	Lecture 9: The Ring of Polynomial Functions on an Affine Variety	
	Lecture 10: Geometric Hypersurfaces are Precisely Algebraic Hypersurfaces	
Unit 5: The Affine Coordinate Ring of an Affine Variety **Goals & keywords**	Lecture 11: Why Should We Study Affine Coordinate Rings of Functions on Affine Varieties ?	
	Lecture 12: Capturing an Affine Variety Topologically From the Maximal Spectrum of its Ring of Functions	
Unit 6: Open sets in the Zariski Topology and Functions on such sets **Goals & keywords**	Lecture 13: Analyzing Open Sets and Basic Open Sets for the Zariski Topology	
	Lecture 14: The Ring of Functions on a Basic Open Set in the Zariski Topology	
Unit 7: Regular Functions in Affine Geometry **Goals & keywords**	Lecture 15: Quasi-Compactness in the Zariski Topology; Regularity of a Function at a point of an Affine Variety	
	Lecture 16: What is a Global Regular Function on a Quasi-Affine Variety?	
Unit 8: Morphisms in Affine Geometry **Goals & keywords**	Lecture 17: Characterizing Affine Varieties; Defining Morphisms between Affine or Quasi-Affine Varieties	
	Lecture 18: Translating Morphisms into Affines as k-Algebra maps and the Grand Hilbert Nullstellensatz	

	Lecture 19: Morphisms into an Affine Correspond to k-Algebra Homomorphisms from its Coordinate Ring of Functions
	Lecture 20: The Coordinate Ring of an Affine Variety Determines the Affine Variety and is Intrinsic to it
	Lecture 21: Automorphisms of Affine Spaces and of Polynomial Rings - The Jacobian Conjecture; The Punctured Plane is Not Affine
Unit 9: The Zariski Topology on Projective Space and Projective Varieties **Goals & keywords**	Lecture 22: The Various Avatars of Projective n-space
	Lecture 23: Gluing (n+1) copies of Affine n-Space to Produce Projective n-space in Topology, Manifold Theory and Algebraic Geometry; The Key to the Definition of a Homogeneous Ideal
Unit 10: Graded Rings, Homogeneous Ideals and Homogeneous Localisation **Goals & keywords**	Lecture 24: Translating Projective Geometry into Graded Rings and Homogeneous Ideals
	Lecture 25: Expanding the Category of Varieties to Include Projective and Quasi-Projective Varieties
	Lecture 26: Translating Homogeneous Localisation into Geometry and Back

References:

1) Algebraic Geometry by Robin Hartshorne, Graduate Texts in Mathematics GTM 52, Springer

2) The Red Book of Varieties and Schemes by David Mumford

3) An Introduction to Commutative Algebra by M. F. Atiyah and I. G. Macdonald, Addison-Wesley