## Linear Algebra - Video course

## COURSE OUTLINE

Systems of linear equations, Matrices, Elementary row operations, Row-reduced echelon matrices. Vector spaces, Subspaces, Bases and dimension, Ordered bases and coordinates.

Linear transformations, Rank-nullity theorem, Algebra of linear transformations, Isomorphism, Matrix representation, Linear functionals, Annihilator, Double dual, Transpose of a linear transformation.

Characteristic values and characteristic vectors of linear transformations, Diagonalizability, Minimal polynomial of a linear transformation, Cayley-Hamilton theorem, Invariant subspaces, Direct-sum decompositions, Invariant direct sums, The primary decomposition theorem, Cyclic subspaces and annihilators, Cyclic decomposition, Rational, Jordan forms.

Inner product spaces, Orthonormal bases, Gram-Schmidt process.

COURSE DETAIL

| Lectures | Topic |
| :---: | :--- |
| 1 | Introduction to the Course Contents. |
| 2 | Linear Equations |
| $3 a$ | Equivalent Systems of Linear Equations I: Inverses of <br> Elementary Row-operations, <br> matrices |
| $3 b$ | Equivalent $\quad$Rowstems <br> Homogeneous Equations, Examples <br> 4 |
| Row-reduced Echelon Matrices |  |


http://nptel.ac.in

## Mathematics

## Additional Reading:

1. S. Axler, Linear Algebra Done Right, 2nd Edition, John-Wiley, 1999.
2. S. Lang, Linear Algebra, Springer UTM, 1997.
3. S. Kumaresan, Linear Algebra: A Geometric Approach, Prentice-Hall of India, 2004.

## Coordinators:

Dr. K.C. Sivakumar
Associate ProfessorDepartment of MathematicsIIT Madras

| 6 | Elementary Matrices, Homogeneous Equations and Non-homogeneous Equations |
| :---: | :---: |
| 7 | Invertible matrices, Homogeneous Equations Nonhomogeneous Equations |
| 8 | Vector spaces |
| 9 | Elementary Properties in Vector Spaces. Subspaces |
| 10 | Subspaces (continued), Spanning Sets, Linear Independence, Dependence |
| 11 | Basis for a vector space |
| 12 | Dimension of a vector space |
| 13 | Dimensions of Sums of Subspaces |
| 14 | Linear Transformations |
| 15 | The Null Space and the Range Space of a Linear Transformation |
| 16 | The Rank-Nullity-Dimension Theorem. Isomorphisms Between Vector Spaces |
| 17 | Isomorphic Vector Spaces, Equality of the Row-rank and the Column-rank I. |
| 18 | Equality of the Row-rank and the Column-rank II |
| 19 | The Matrix of a Linear Transformation |
| 20 | Matrix for the Composition and the Inverse. Similarity Transformation |
| 21 | Linear Functionals. The Dual Space. Dual Basis I |
| 22 | Dual Basis II. Subspace Annihilators I |
| 23 | Subspace Annihilators II |
| 24 | The Double Dual. The Double Annihilator |


| 25 | The Transpose of a Linear Transformation. Matrices of a Linear Transformation and its Transpose |
| :---: | :---: |
| 26 | Eigenvalues and Eigenvectors of Linear Operators |
| 27 | Diagonalization of Linear Operators. A Characterization |
| 28 | The Minimal Polynomial |
| 29 | The Cayley-Hamilton Theorem |
| 30 | Invariant Subspaces |
| 31 | Triangulability, Diagonalization in Terms of the Minimal Polynomial |
| 32 | Independent Subspaces and Projection Operators |
| 33 | Direct Sum Decompositions and Projection Operators I |
| 34 | Direct Sum Decomposition and Projection Operators II |
| 35 | The Primary Decomposition Theorem and Jordan Decomposition |
| 36 | Cyclic Subspaces and Annihilators |
| 37 | The Cyclic Decomposition Theorem I |
| 38 | The Cyclic Decomposition Theorem II. The Rational Form |
| 39 | Inner Product Spaces |
| 40 | Norms on Vector spaces. The Gram-Schmidt Procedure I |
| 41 | The Gram-Schmidt Procedure II. The QR Decomposition |
| 42 | Bessel's Inequality, Parseval's Indentity, Best Approximation |
| 43 | Best Approximation: Least Squares Solutions |
| 11 | Orthogonal Complementary Subspaces, Orthogonal |


| 44 | Projections |
| :---: | :--- |
| 45 | Projection Theorem. Linear Functionals |
| 46 | The Adjoint Operator |
| 47 | Properties of the Adjoint Operation. Inner Product <br> Space Isomorphism |
| 48 | Unitary Operators |
| 49 | Unitary operators II. Self-Adjoint Operators I |
| 50 | Self-Adjoint Operators II- Spectral Theorem |
| 51 | Normal Operators - Spectral Theorem |

## References:

1. K.Hoffman and R. Kunze, Linear Algebra, 2nd Edition, Prentice- Hall of India, 2005.
2. M. Artin, Algebra, Prentice-Hall of India, 2005.
