



# LINEAR ALGEBRA

**PROF. ARBIND KUMAR LAL**

Department of Mathematics and Statistics  
IIT Kanpur

**TYPE OF COURSE** : New | Core | UG/PG

**COURSE DURATION** : 12 weeks (20 Jul' 20 - 9 Oct' 20)

**EXAM DATE** : 17 Oct 2020

**INTENDED AUDIENCE** : Mathematics Honours

## **COURSE OUTLINE :**

The course will assume basic knowledge of class XII algebra and a familiarity with calculus. Even though, the course will start with defining matrices and operations associated with it. This will lead to the study of system of linear equations, elementary matrices, invertible matrices, the row-reduced echelon form of a matrix and a few equivalent conditions for a square matrix to be invertible. From here, we will go into the axiomatic definition of vector spaces over real and complex numbers, try to understand linear combination, linear span, linear independence and linear dependence and hopefully understand the basis of a finite dimensional vector space.

## **ABOUT INSTRUCTOR :**

Prof. Arbind Kumar Lal interests are in algebraic graph theory.

## **COURSE PLAN :**

- Week 1:** Introduction to matrices, matrix operations, such as addition of matrices, transpose of a matrix, scalar multiplication and matrix multiplication etc. Invertible matrices, examples and submatrix of a matrix
- Week 2:** System of linear equations, elementary row operations, elementary matrices, Gauss elimination and Gauss - Jordan methods, LU decomposition, Row-reduced echelon form of a matrix, Rank of a matrix and the solution set of a linear system
- Week 3:** Application of the solution set of a linear system to linear systems where the coefficient matrix is a square matrix. Determinant of a matrix, Inverse using the classical adjoint method and the Cramer's rule
- Week 4:** Axiomatic definition of a vector space, examples, subspaces and linear combination and linear span, finite dimensional vector space, fundamental subspaces associated with a matrix
- Week 5:** Linear independence and dependence, linear independence and the rank of a matrix, basis of a vector space, constructing a basis of a finite dimensional vector space
- Week 6:** Linear transformations, rank-nullity theorem and its application to maps between finite dimensional vector spaces
- Week 7:** Ordered bases, matrix of a linear transformation and similarity of matrices
- Week 8:** Dot/Inner product in a vector space, Cauchy Schwartz inequality, angle between two vectors, Projection of a vector onto another vector
- Week 9:** Gram-Schmidt orthogonalization process and the QR- decomposition, least square solution of a non-consistent linear system and the orthogonal projections,
- Week 10:** Motivation, definition and examples for eigenvalues and eigenvectors, Schur's unitary triangularization,
- Week 11:** Diagonalizable matrices, Criteria for diagonalizability, diagonalizability of Normal matrices, Spectral theorem for Hermitian matrices
- Week 12:** Quadratic forms, Sylvester's law of inertia, Classification of quadrics