# Nonlinear Dynamical Systems - Video course

#### COURSE OUTLINE

This course "Nonlinear Dynamical Systems" covers basics of nonlinear differential equations that are encountered when dealing with practical dynamical systems in the context of their control.

Classification of stable and unstable equilibrium points using phase portraits forms the initial focus, after which we study various features of dynamical systems that one encounters only in nonlinear systems: robust sustained oscillations, finite escape time, finite time to reach equilibrium position, amongst others.

The course then covers necessary and sufficient conditions for existence and uniqueness of a solution and proof of this theorem (the Lipschitz condition). Study of periodic orbits forms an important aspect due to their practical utility.

Lyapunov stability concepts followed by Lyapunov's theorems for stability, and some converse theorems, are introduced next. The Lure problem is studied next to address absolute stability considerations for sector-nonlinearity type of characteristics.

A review of Nyquist stability criteria (of linear systems) is done here so that understanding the small gain theorem, passivity results and the circle/Popov criteria is easier and more thorough. Finally, we cover the describing function method and relate this topic to earlier stability conditions.

#### Contents:

Introduction to nonlinear dynamical systems' features, existence and uniqueness of solutions, Lipschitz condition, Linearization and local analysis, classification of equilibrium points (planar case), periodic orbits, Lyapunov theory, Lure problem, sector nonlinearity, Nyquist criterion, Lp stability, Small gain theorem, passivity result, circle/Popov criteria, Describing function method.

#### **COURSE DETAIL**

SI. No	Торіс	No. of Hours
1	Introduction and outline of course, notation, some features of nonlinear dynamical systems.	01
2	Scalar differential equations, examples of population dynamics, definition of equilibrium point.	03
3	Vector differential equations, solution for the linear case. Vector fields and phase portraits.	03
4	Planar case: classification of equilibrium points, relation with eigenvalues and eigenvectors, phase portraits and vector fields.	04



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## Electrical Engineering

#### **Pre-requisites:**

1. UG ordinary differential equations, UG Control Systems course.

#### Additional Reading:

1. V.I. Arnold, Ordinary Differential Equations (translated from Russian edition).

#### **Coordinators:**

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5	Oscillators: van der Pol oscillators, necessary conditions for periodic orbits, sufficient conditions for such orbits.	04
6	Conditions for existence and uniqueness of solutions, proof using fixed point theorem, examples of non- uniqueness and finite-escape time features.	06
7	Lyapunov stability definition and theorem/proof, converse theorems, linearization about an equilibrium point.	06
8	Nyquist criterion, Feedback interconnections, Absolute stability, Lp stability, Small-gain theorem, Passivity results.	05
9	Loop transformations, circle/Popov criteria, derivations.	05
10	Describing function method.	06
	Total	43

#### **References:**

- 1. Hassan Khalil, "Nonlinear Systems", 3<sup>rd</sup> Edition, Prentice-Hall, 2002.
- 2. M. Vidyasagar, "Nonlinear Systems Analysis", Prentice-Hall, 1993.
- 3. Shankar Sastry, "Nonlinear Systems: Analysis, Stability and Control", Springer, New-York, 1999.

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