Structural Analysis-1

Mock Assignment - 1

January 25, 2022

Question 1

The T – beam shown in the figure below is made of concrete of density 2400 kg/m³. The three reinforcing bars are of diameter 20 mm each and has a density of 7850 kg/m³. Determine the dead load of the beam per meter length. All the dimensions in the figure are in mm. (Consider g = 9.81 m/sec²)

Solution :

Dead weight of concrete

\[ W_c = 2400 \times [(0.15 \times 0.9) + (0.3 \times 0.4) - (3\pi0.01^2)] \times 9.81 = 5.98\text{kN/m} \]

Dead weight of steel

\[ W_s = 7850 \times [(3\pi0.01^2)] \times 9.81 = 0.07\text{kN/m} \]

Total dead weight

\[ W = W_c + W_s = 6.05\text{kN/m} \]
Question 2

Consider the beam in the figure below. Calculate the bending moment at A and the vertical reaction at C.

Solution:
Moment equilibrium at internal hinge B

\[ V_c \times 15 - 600 = 0 \]

\[ V_c = 40 \text{kN (upwards)} \]

From force equilibrium, vertical force at B for the segment BC

\[ V_B = -40 \text{kN (downwards)} \]

This reaction is transferred as upward force for (cantilever) segment AB. Now consider AB alone. Moment equilibrium at A demands

\[ M_A = -40 \times 20 + 40 \times 20 \times 20 / 2 \]

\[ = 7200 \text{kNm} \]
Question 3

Consider the structure below. Joints A and C are pinned supported and B has an internal hinge. Calculate the horizontal and vertical reactions at B.

Solution:

Approach A:
Consider segment AB. Vertical equilibrium demands

\[ B_y = 10 \times \frac{4}{2} = 20 \text{kN} \]

In segment BC, moment equilibrium about C needs

\[ B_x \times 2 \cos 45^\circ - B_y \times 2 \sin 45^\circ = 0 \]
\[ B_x = 20 \text{kN} \]

Approach B:
Note that BC is a truss member. Therefore it carries only axial force \( R_B \) (directed along the span BC). Consider (simply supported) segment AB alone.

\[ B_y = R_B \sin 45^\circ = 10 \times \frac{4}{2} \]
\[ R_B = 20\sqrt{2} \]

Therefore,

\[ B_x = R_B \cos 45^\circ = 20 \text{kN} \]
\[ B_y = R_B \sin 45^\circ = 20 \text{kN} \]
Question 4

Determine the shear force and the bending moment in the beam below at a distance of \( x \) from the fixed end.

**Solution:**

At a distance \( x \) from the fixed end, the intensity of load \( w \) is (from similarity of triangles),

\[
w = \frac{2(30 - x)}{30} \text{ kN/m}
\]

Shear force

\[
S_x = \frac{1}{2} \times \frac{2(30 - x)^2}{30}
\]

\[
= 0.0333x^2 - 60x + 30
\]

Bending moment

\[
M_x = -(0.0333x^2 - 60x + 30) \times \frac{30 - x}{3}
\]

\[
= 0.0111x^3 - x^2 + 30x - 300
\]
Question 5

Out of the options below, which one is the correct shear force diagram? There is an internal hinge at B.

Solution:

5
Shear force in CD

\[ S_{BC} = 50\text{kN} \]

Consider segment BCD. Bending moment at internal hinge B must be zero.

\[
50 \times 4 - R_C \times 2 + 50 \times 1 = 0
\]

\[ R_C = 125\text{kN} \text{ (upwards)} \]

Therefore shear force in CB is initially \(50 - 125 = -75\text{ kN}\) and then increases by 50 because of the concentrated load and gets to \(-75 + 50 = -25\text{ kN}\). There are no other forces in the span AB and hence the shear force stays at \(-25\text{ kN}\). This variation of shear force is correctly shown in (a).
Consider free body AB. From moment equilibrium

\[ M_{BC} = 5 \text{kNm (hogging)} \]

In span BC, from moment equilibrium about C demands,

\[ R_C \times 5 - 0.5 \times 20 \times 5 \times 5/3 + 5 = 0 \]

\[ R_C = 15.667 \text{kN} \]

The load intensity at \( x \) distance from C is \( 4x \text{kN/m} \). Bending moment at this section is

\[ M_x = 15.667x - \frac{1}{2} \times 4x^2 \times \frac{x}{3} \]

which attains maximum when \( dM_x/dx = 0 \). This happens for \( x = 2.799 \text{ m} \). \( M_x \)

at this locations is

\[ M^*_x = 15.667(3.917) - 4(3.917)^3/6 \]

\[ = 29.2 \text{kNm (sagging)} \]
Question 7

Determine the moment and the vertical reaction at A.

Shear force

\[ V_A = \frac{(10 + 5)}{2} \times 12 \]
\[ = 90 \text{kN} \]

To determine bending moment, first find centroid \( x^* \) of the load from fixed end

\[ x^* = \left( \frac{12}{3} \right) \left( \frac{10 + 2 \times 5}{10 + 5} \right) \]
\[ = 5.33 \text{m} \]

Bending moment

\[ M_A = 90 \times 5.33 \approx 480 \text{ kNm} \]
Question 8

Determine the vertical reaction at A and the bending moment at C in the beam shown in figure below. B is an internal hinge.

Solution:
From force and moment equilibrium of span AB, one can find that $R_A$ and $R_B$ are one third and two third of the total load from the triangular variation.

$$R_A = \frac{1}{3} \times \frac{1}{2} \times 2 \times 6 = 2 \text{kN} // R_B = 2R_A = 4 \text{kN}$$

Consider span BC. From moment equilibrium at C,

$$M_c = 4 \times 4 + 2 \times 4 \times 4/2 = 32 \text{kNm}$$
Let the left and right ends be denoted by points A and B respectively. Then from moment equilibrium about point B,

\[ R_A (3L) - wL^2/2 = 0 \]

\[ R_A = \frac{wL}{6} \]

For a section at distance \( x \) from A, shear force is

\[ S_x = \frac{wL}{6}; \quad x < 2L \]

\[ S_x = \frac{wL}{6} - w(x - 2L); \quad 2L \leq x \leq 3L \]

Maximum (magnitude) of shear force occurs at \( x = 3L \)

\[ S^*_x = \frac{wL}{6} - wL = \frac{5wL}{6} \]

For maximum bending moment, find where shear force becomes zero

\[ S_x = 0 = \frac{wL}{6} - w(x^* - 2L) \]

\[ x^* = \frac{13}{6}L \]

Bending moment at this location is

\[ M^*_x = \frac{wL}{6} \times \frac{13}{6} - \frac{1}{2}w\left(\frac{13}{6} - 2\right)^2 \]

\[ = \frac{25wL^2}{72} \]
Question 10

The steel structure in the figure below is used to support the 100 mm thick concrete slab. For the design of the beams and columns, it is important to know the amount of dead load (due to the weight of the slab) acting on the members. To estimate the loading on each of the beams, lines are drawn at 45° from the corners. The self weight corresponding to the resulting triangles and trapeziums is the loading on the respective beams. The load on beam AB is the self weight of the lightly shaded trapezium with the hashed lines and similarly the load on the beam BC is the darkly shaded triangular load. Assume the beams to be simply supported. (i) First calculate the reaction at support B of beam AB due to the self weight of the slab. (ii) Then calculate the reaction at support B of beam BC due to the self weight of the slab. (iii) Finally, what will be the force on the column at B due to the self weight of the slab? (density of concrete = 2400 kg/m³, g = 9.81 m/sec²).

Solution:
To find the dead weight of the trapezoidal portion of concrete, first find the area
\[ A = \frac{12 + 4}{2} \times 4 \]
\[ = 32 \text{m}^2 \]

Dead weight of that portion is

\[ W_{AB} = A \times 0.1 \times 2400 \times 9.81 \]
\[ = 75.35 \text{kN} \]

This load is equally transferred to beam supports A and B. Therefore reaction at B is

\[ R_{B1} = \frac{75.35}{2} = 37.68 \text{kN} \]

Doing similar calculations from BC,

\[ R_{B2} = \frac{1}{2} \times 8 \times 4 \times 0.1 \times 2400 \times 9.81 \times \frac{1}{2} \]
\[ = 18.84 \text{kN} \]

Total reaction in the column

\[ R_B = R_{B1} + R_{B2} = 56.52 \text{kN} \]