

NPTEL ONLINE CERTIFICATION COURSES

Course Name: Fuzzy logic and Neural Networks

Faculty Name: Prof. D. K. Pratihar

Department : Mechanical Engineering

Week 2

NPTEL



NPTEL ONLINE CERTIFICATION COURSES

Course Name: Fuzzy logic and Neural Networks

Faculty Name: Prof. D. K. Pratihar

Department : Mechanical Engineering

Topic

Lecture 03: Applications of Fuzzy Sets

NPTEL

CONCEPTS COVERED

Concepts Covered:

- Applications of Fuzzy Sets
- Fuzzy Logic Controller/Fuzzy Reasoning Tool
- Fuzzy Clustering

NPTEL

Applications of Fuzzy Sets

- Reasoning tool like Fuzzy Logic Controller
- Fuzzy Clustering
- Fuzzy Mathematical Programming
- Fuzzy Graph theory
- Others

NPTEL

Fuzzy Logic Controller (FLC)

- Developed by Mamdani and Assilian around 1975

- Performance depends on Knowledge Base (KB), which consists of Data Base (DB) and Rule Base (RB)

- The performance of an FLC largely depends on the RB and optimizing the DB is a fine tuning process

NPTEL

Two Forms of FLC

- **Linguistic Fuzzy Modeling:** High interpretability and Low accuracy.

  Ex: Mamdani Approach

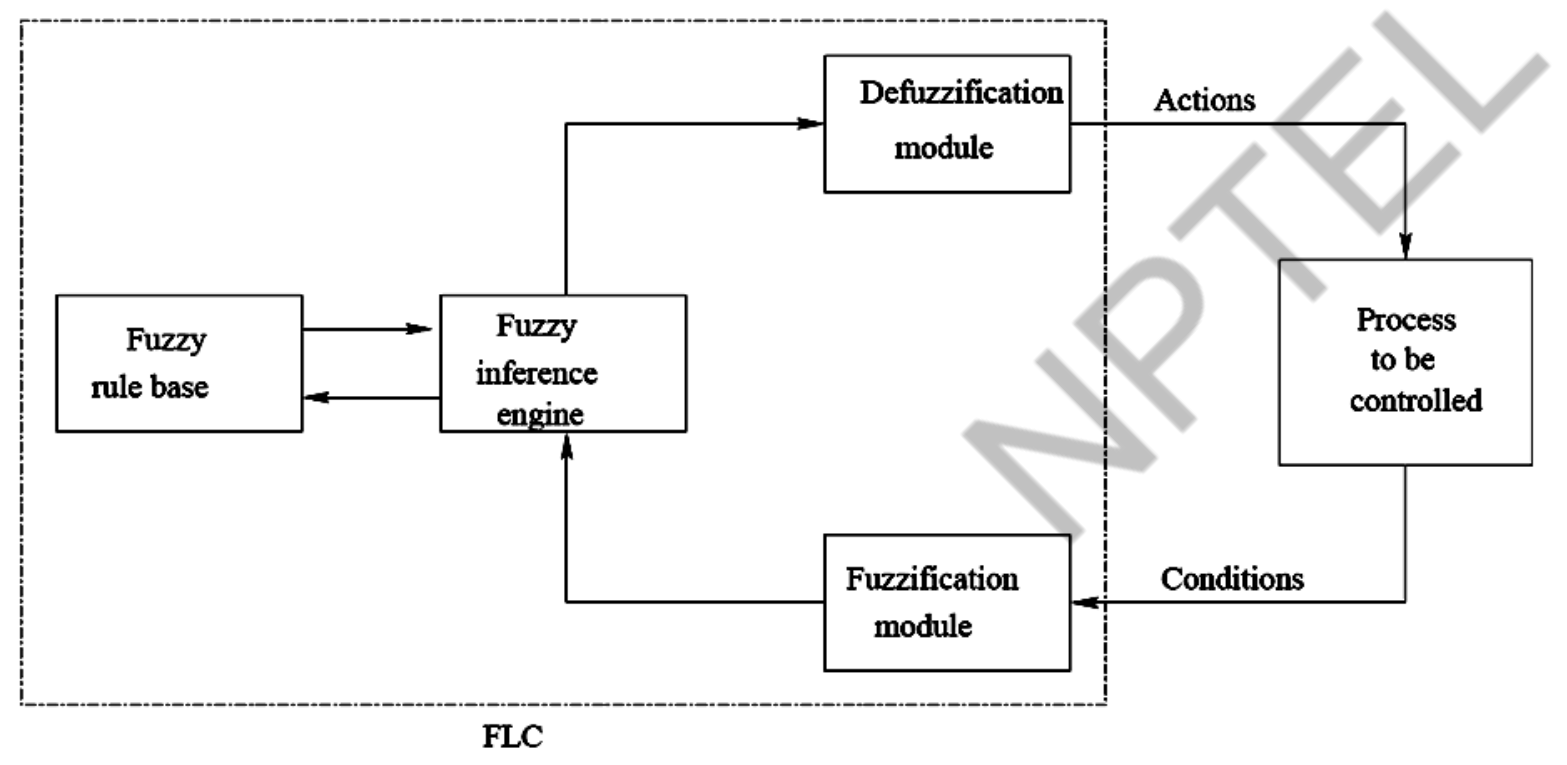
- **Precise Fuzzy Modeling:** Low interpretability and High accuracy.

  Ex: Takagi and Sugeno's Approach

NPTEL

Mamdani Approach

•An FLC consists of four modules: Rule Base; Inference Engine; Fuzzification; De-fuzzification



NPTEL

Steps Involved in the Working Cycle of FLC

- Identify the condition (also known as antecedent) and action (also called consequent) variables of the process

- **Fuzzification:** Measurements of input variables are converted into appropriate fuzzy sets to express measurement uncertainties

NPTEL

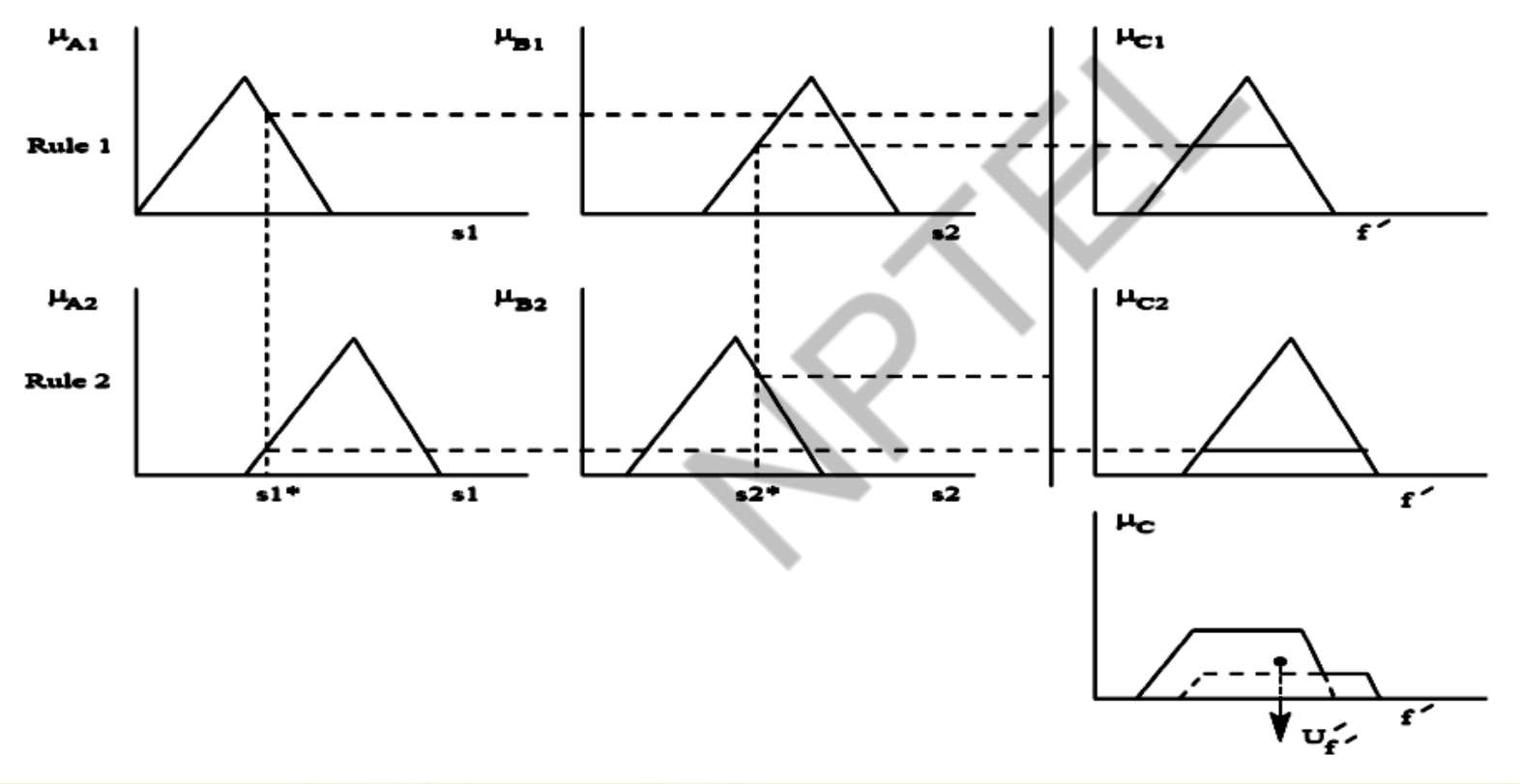
- **Inference Engine** uses fuzzified measurements to evaluate the control rules stored in the fuzzy rule base

- **De-fuzzification:** Fuzzified output is then converted into a single crisp value

NPTEL

Fuzzy Reasoning Process

Rule1: IF S_1 is A_1 AND S_2 is B_1 THEN f' is C_1
Rule2: IF S_1 is A_2 AND S_2 is B_2 THEN f' is C_2



NPTEL

Let S_1^* and S_2^* are the inputs for fuzzy variables S_1 and S_2

Firing strengths of the first and second rules can be calculated as follows:

$$\alpha_1 = \min\left(\mu_{A_1}\left(S_1^*\right), \mu_{B_1}\left(S_2^*\right)\right)$$

$$\alpha_2 = \min\left(\mu_{A_2}\left(S_1^*\right), \mu_{B_2}\left(S_2^*\right)\right)$$

NPTEL

Membership value of the combined control action C is given by

$$\mu_C(f') = \max\left(\mu_{C_1}^*(f'), \mu_{C_2}^*(f')\right)$$

NPTEL

De-fuzzification Methods

- **Center of Sums Method**

$$U'_{f'} = \frac{\sum_{j=1}^{p} A(\alpha_j) \times f_j}{\sum_{j=1}^{p} A(\alpha_j)},$$

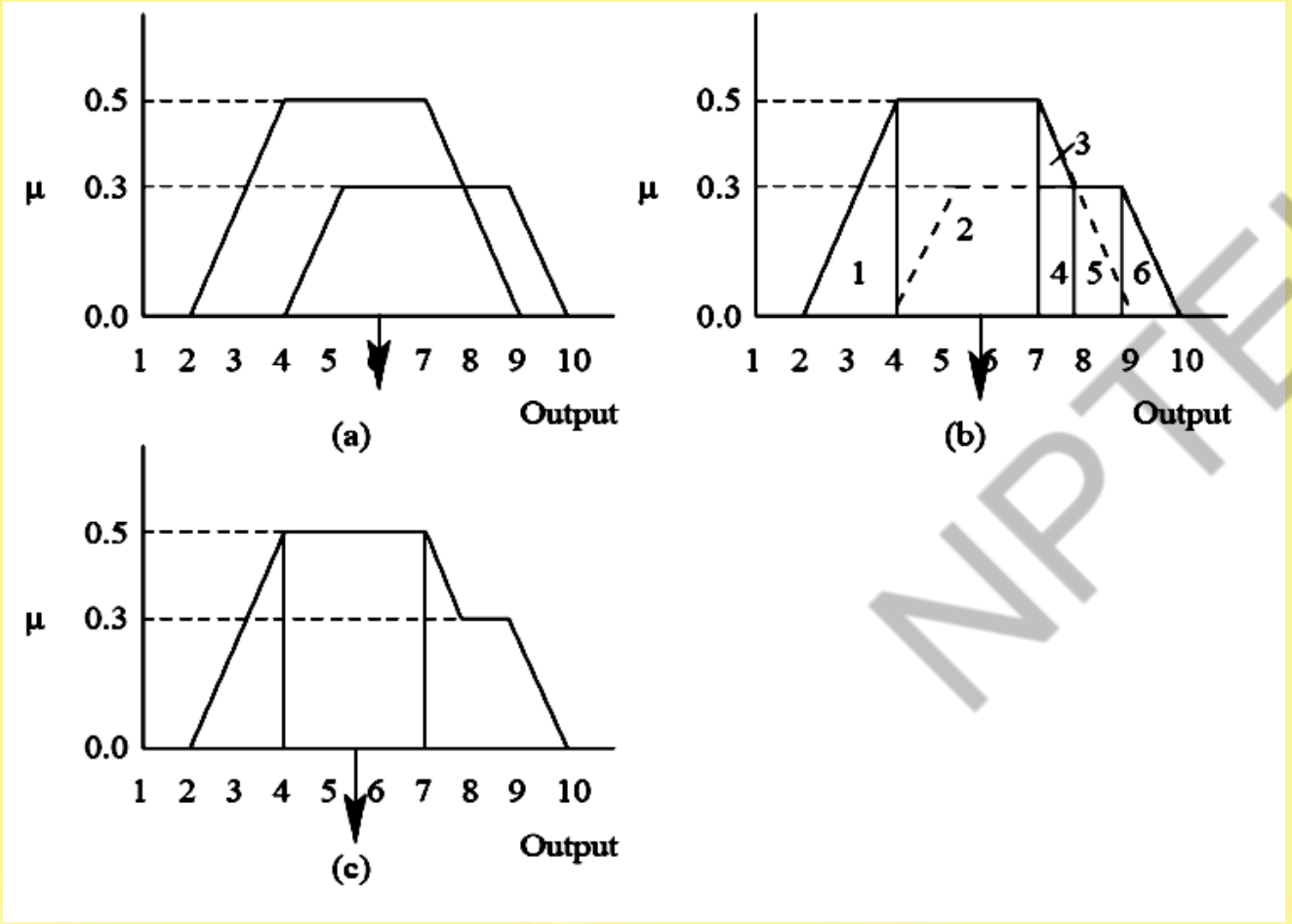
where $A(\alpha_j)$: firing area of j-th rule

f_j: center of the area

p: No. of fired rules

NPTEL

Methods of De-fuzzificztion



NPTEL

- **Centroid Method**

$$U'_{f'} = \frac{\sum_{i=1}^{N} A_i \times f_j}{\sum_{i=1}^{N} A_i},$$

where N: No. of small areas or regions

A_i: Area of i-th small region

f_i: Center of area of i-th small region

NPTEL

• Mean of Maxima Method

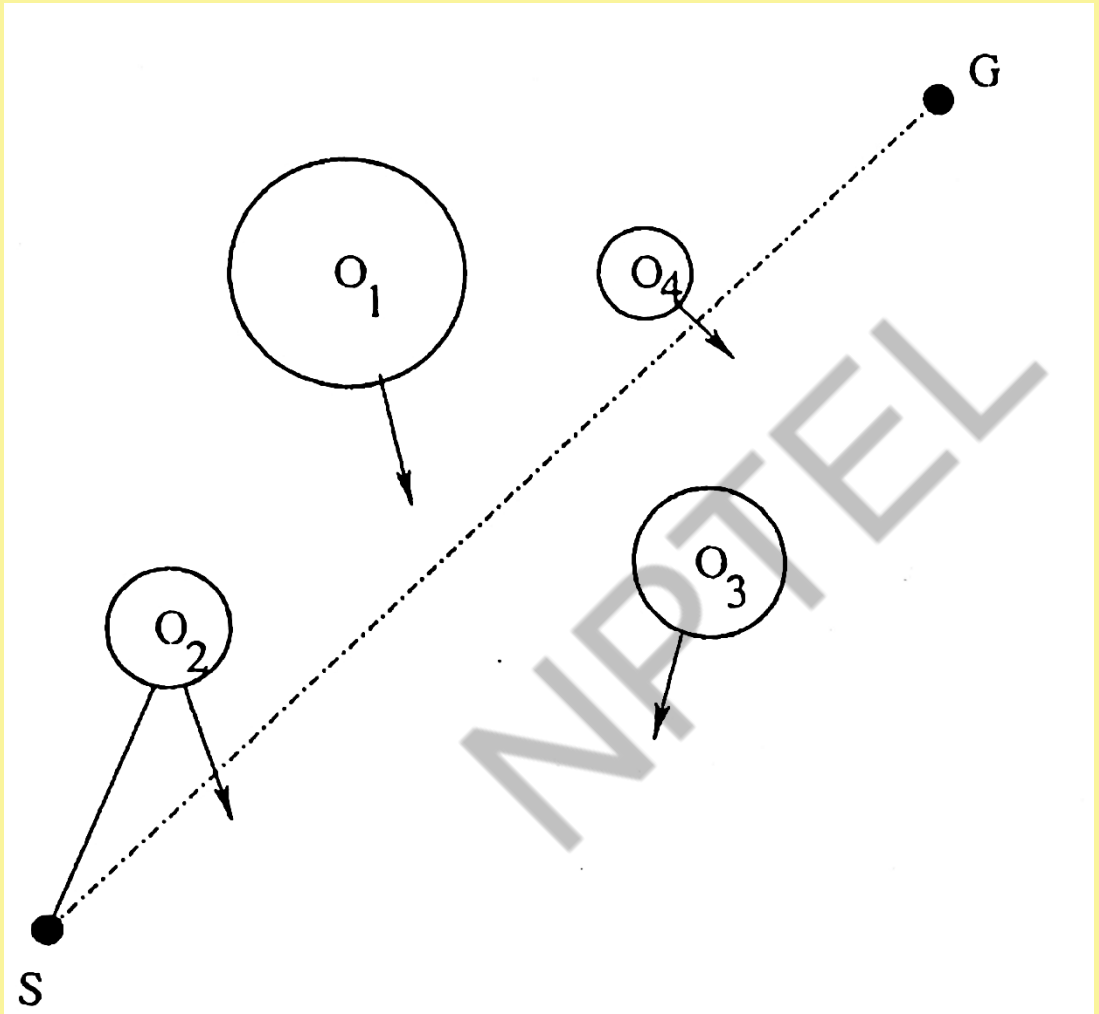
From the membership function distribution of combined control action, the range of the output variable corresponding to the maximum value of membership is identified. The mid-value of this range is considered to be the crisp output of the controller.

NPTEL

Numerical example (Mamdani Approach)

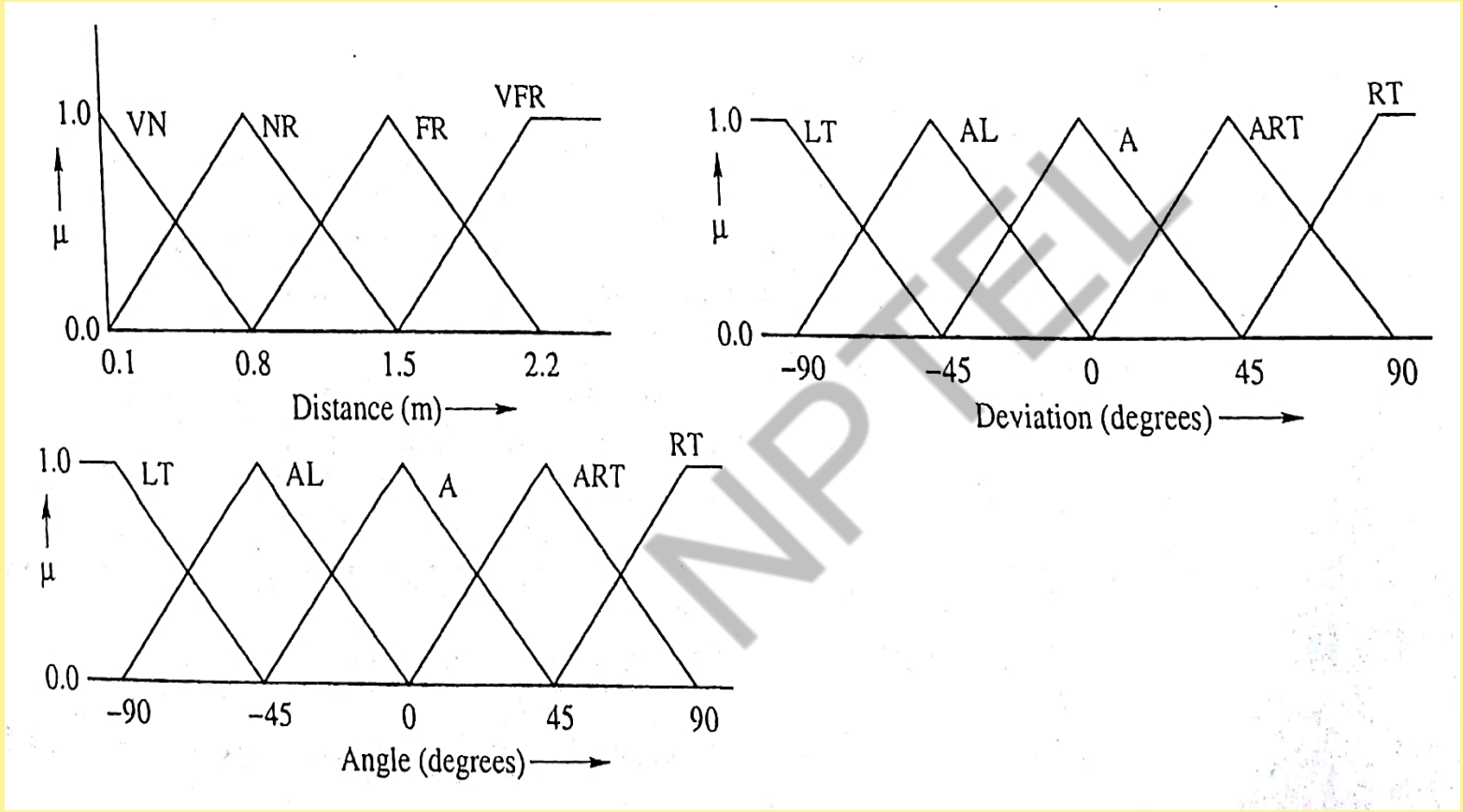
Let us consider a problem scenario related to navigation of a mobile robot in the presence of four moving obstacles. Obstacle 2 is found to be the most critical one. Our aim is to develop a fuzzy logic-based motion planner that will be able to generate the collision-free path for the robot. There are two inputs, namely the distance between the robot and the obstacle (D) and angle ($\angle GSO_2$) for the motion planner and it will generate one output, that is, deviation. Distance is represented using four linguistic terms, namely Very Near (VN), Near(NR), Far(FR) and Very Far (VFR), whereas the input: angle and output: deviation are expressed with the help of five linguistic terms, such as Ahead (A), Ahead Left (AL), Left (LT) Ahead Right(ART) and Right (RT).

NPTEL



NPTEL

The data base (DB) and rule base (RB) of the FLC are shown below.



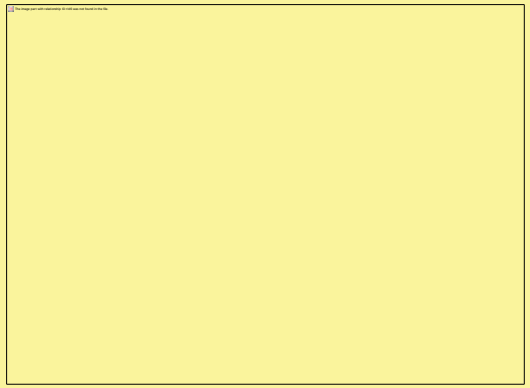
NPTEL

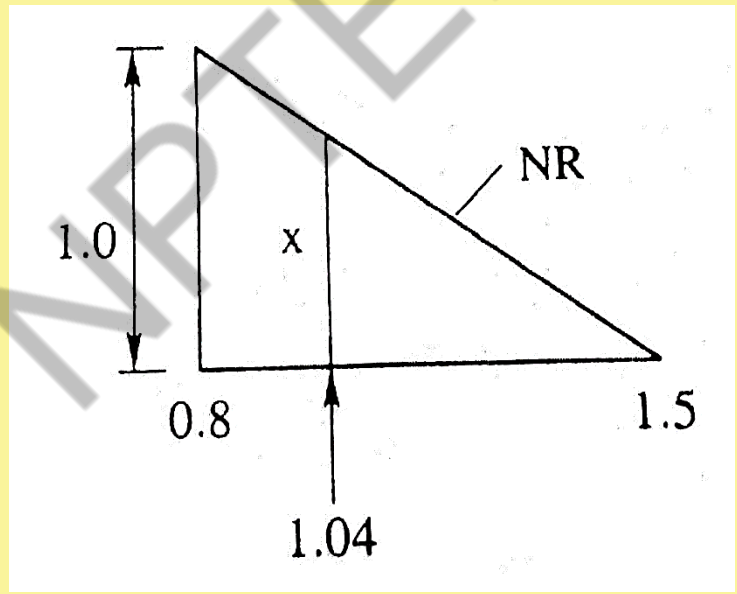
		Angle				
		LT	AL	A	ART	RT
Distance	VN	A	ART	AL	AL	A
	NR	A	A	RT	A	A
	FR	A	A	ART	A	A
	VFR	A	A	A	A	A

NPTEL

Solution:







NPTEL



NPTEL

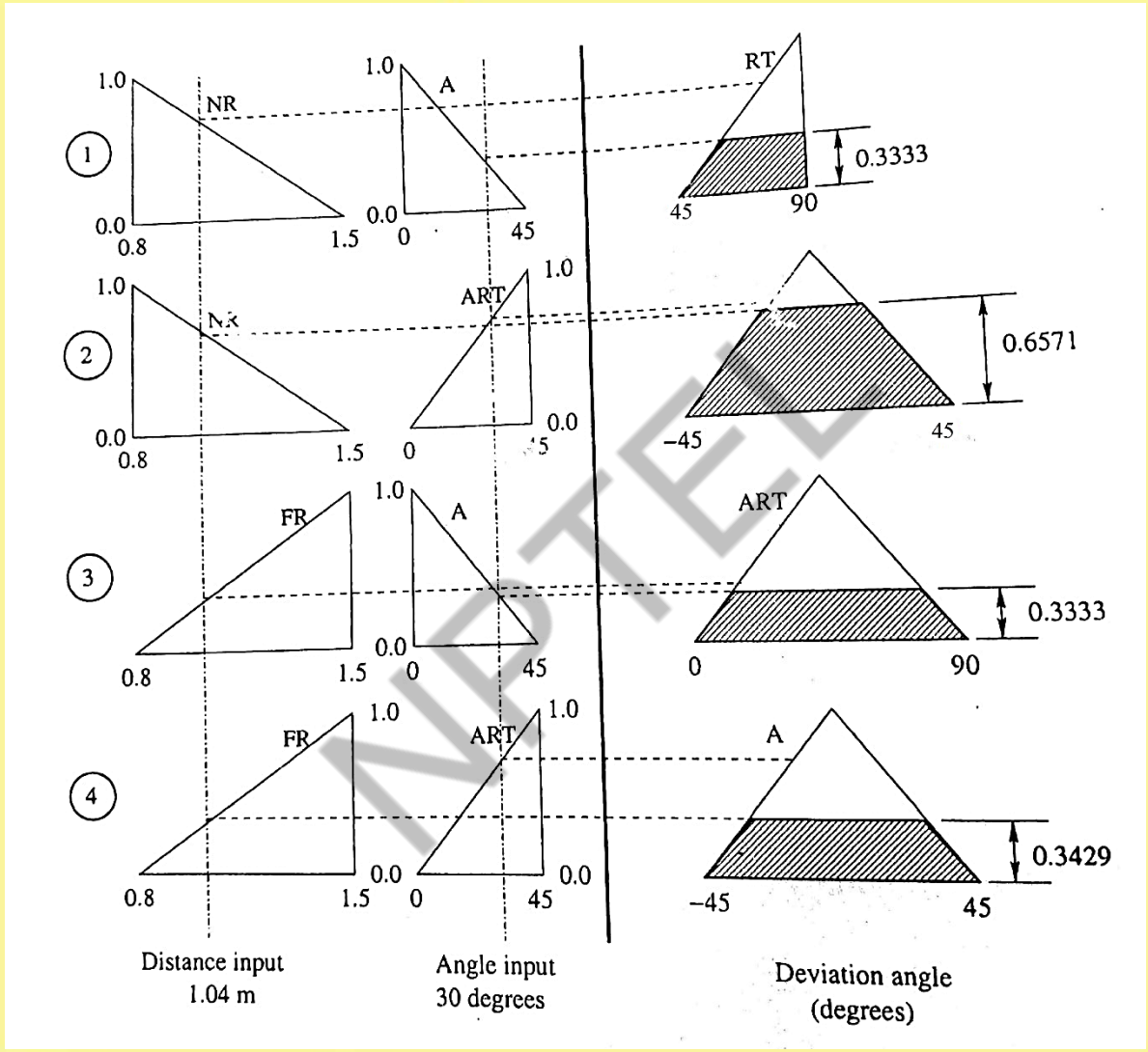
FREE ONLINE EDUCATION
swayam



NPTEL

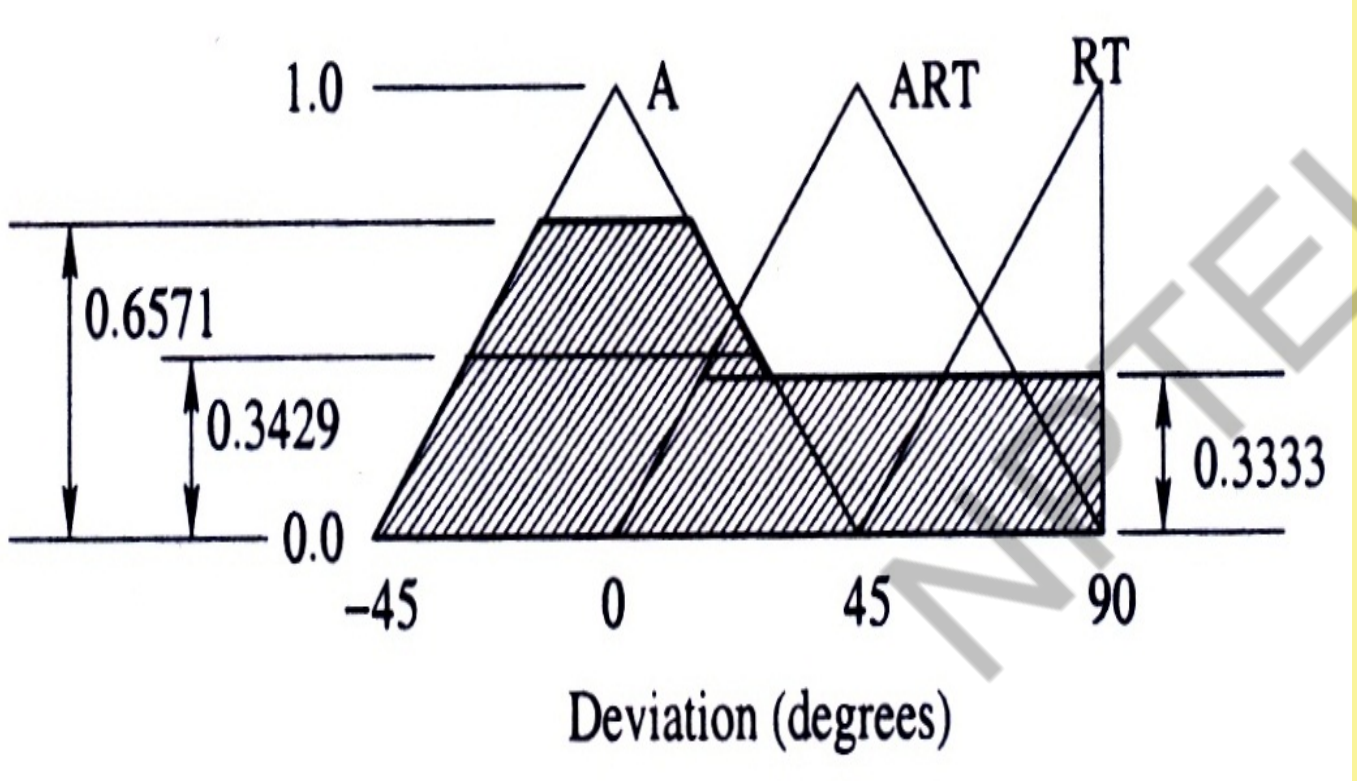
FREE ONLINE EDUCATION
swayam

Fuzzified outputs:



NPTEL

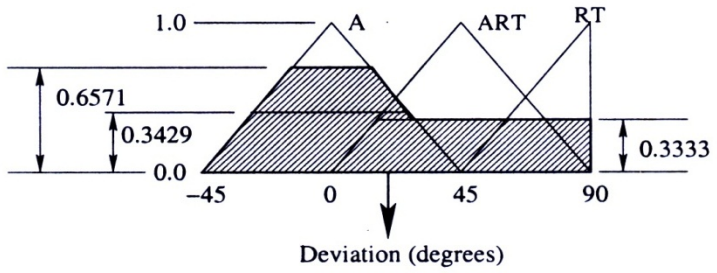
- **Union of the fuzzified outputs:**

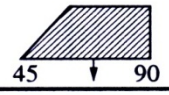
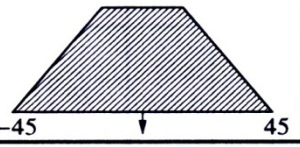
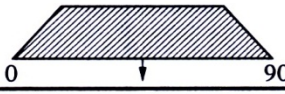
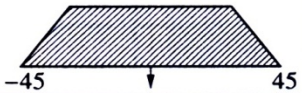


NPTEL

Methods of Defuzzification

1. Center of Sums Method



Fired rule	Shaded region	Area	Center of area
1		12.5	71
2		39.7089	0
3		25	45
4		25.5699	0

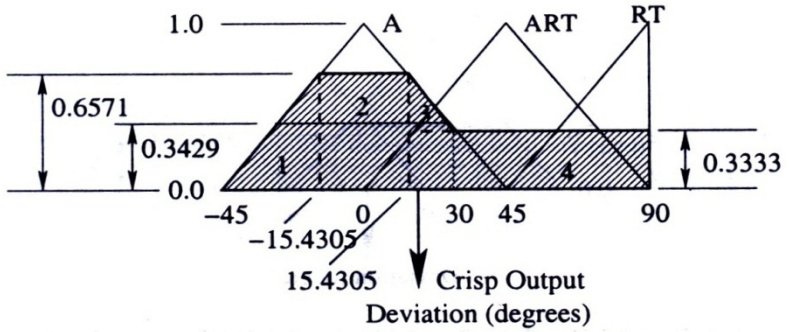
NPTEL

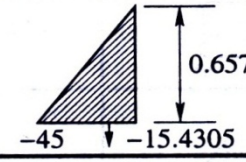
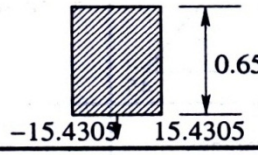
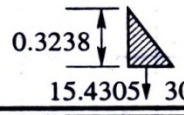
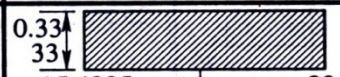
$$U = \frac{12.5 \times 71.0 + 39.7089 \times 0.0 + 25.0 \times 45.0 + 25.5699 \times 0.0}{12.5 + 39.7089 + 25.0 + 25.5699}$$

$$= 19.5809$$

NPTEL

2. Centroid Method



Marked regions	Shaded regions	Area	Center of area
1		9.7151	–25.2860
2		20.2788	0
3		2.3588	20.2870
4		24.8540	52.7153

NPTEL

$$U = \frac{A}{B}$$

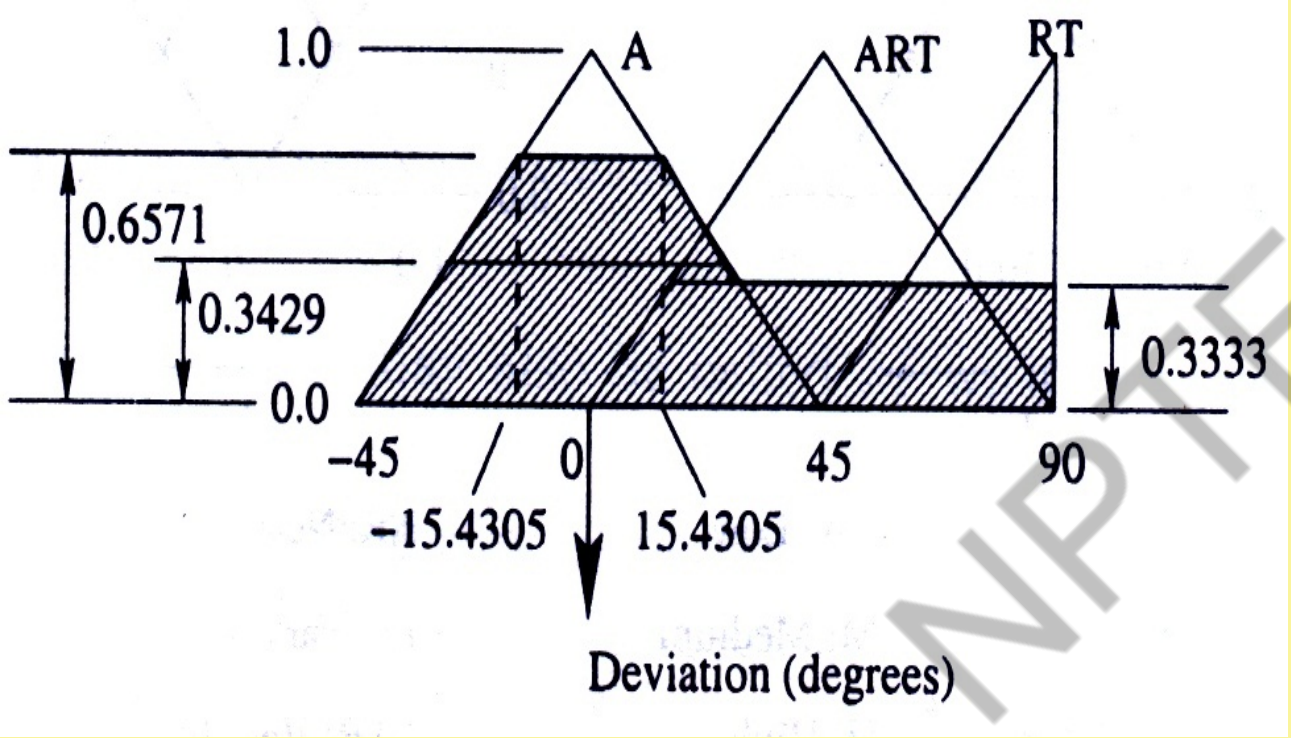
$$A = 9.7151 \times (-25.2860) + 20.2788 \times 0.0 + 2.3588 \times 20.2870 + 24.8540 \times 52.7153$$

$$B = 9.7151 + 20.2788 + 2.3588 + 24.8540$$

$$\therefore U = 19.4450$$

NPTEL

3. Mean of Maxima



$$U = 0$$

Takagi and Sugeno's Approach

- A rule is composed of fuzzy antecedent and functional consequent parts

- i-th rule can be represented as follows:

If x_1 is A^i_1 and x_2 is A^i_2 ..... and x_n is A^i_n then

$$y^i = a^i_0 + a^i_1 x_1 + \ldots\ldots\ldots + a^i_n x_n$$

NPTEL

FREE ONLINE EDUCATION swayam

where $a_0, a_1, \ldots\ldots, a_n$ are the coefficients

- A non-linear system is considered to be a combination of several linear systems

•Weight of i-th rule can be determined as follows:

$$w^i = \mu^i_{A_1}(x_1)\ \mu^i_{A_2}(x_2)\ldots\ldots\ \mu^i_{A_n}(x_n)$$

NPTEL

• Combined control action can be determined as follows:

$$y = \frac{\sum_{i=1}^{k} w^i y^i}{\sum_{i=1}^{k} w^i}$$

where k indicates the total number of rules.

NPTEL

Numerical example (Takagi and Sugeno's Approach

•A fuzzy logic-based expert system is to be developed that will work based on Takagi and Sugeno's approach to predict the output of a process. The DB of the FLC is shown below. As there are two inputs: I_1 and I_2 and each input is represented using three linguistic terms (for example, LW, M, H for I_1 and NR, FR, VFR for I_2) there is a maximum of 3 x 3 = 9 feasible solutions. The output of the *i*-th rule, that is, y^i $(i = 1,2,\ldots,9)$ is expressed as follows:

NPTEL

$$y^i = f(I_1, I_2) = a_j^i I_1 + b_k^i I_2,$$

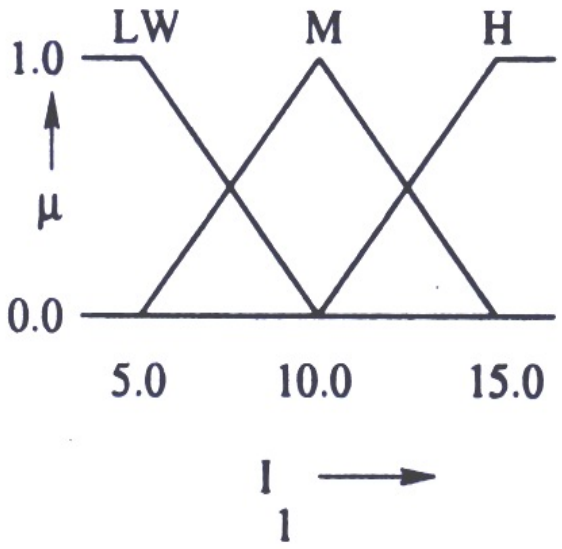
- Where $j, k = 1,2,3$; $a_1^i = 1, a_2^i = 2,$ and $a_3^i = 3,$ If I_1 is found to be LW, M and H, respectively; $b_1^i = 1, b_2^i = 2,$ and, $b_3^i = 3,$ if I_2 is seen to be NR, FR and VFR, respectively. Calculate the output of the FLC for the inputs: I_1=6.0, I_2=2.2

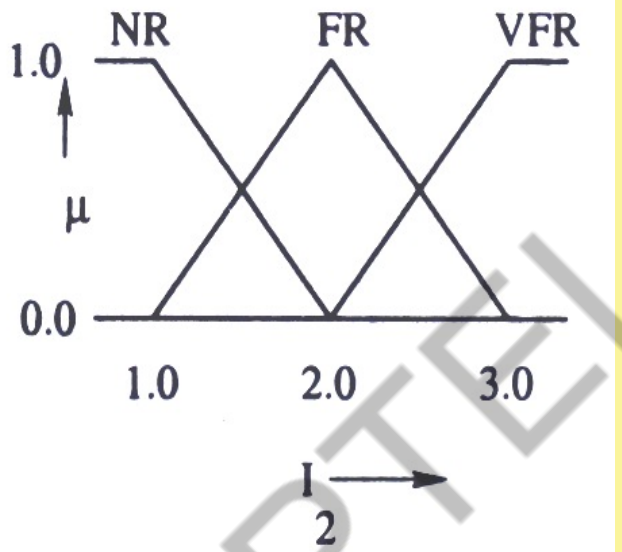
NPTEL











LW: Low

M: Medium

H: High

NR: Near

FR: Far

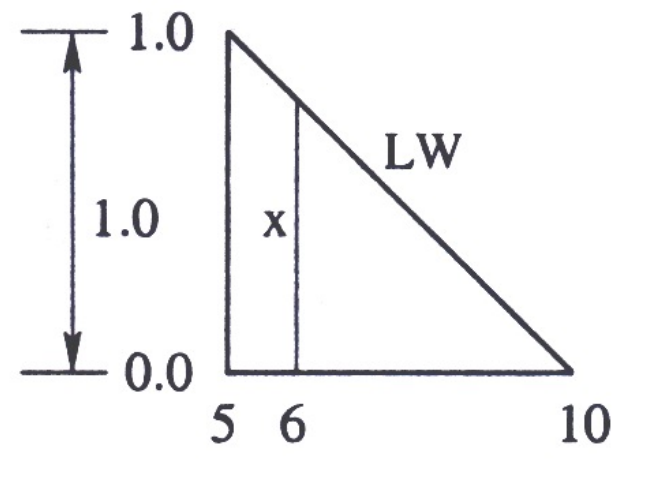
VFR: Very Far

NPTEL

Solution:

- Inputs: I_1=6.0, I_1= 2.2
- Inputs: I_1 of 6.0 unit may be called LW or M.
- Inputs: I_2 of 2.2 unit may be called FR or VFR.
- Determination of membership value:

NPTEL



- **Using the principle of similar triangle**

$$\frac{x}{1.0} = \frac{10-6}{10-5}$$

$$x = 0.8$$

NPTEL

Input I_1 of 6.0 may be called LW with μ_{LW} =0.8

Input I_1 of 6.0 may be called M with μ_M =0.2

Similarly, Input I_2 of 2.2 may be called FR with μ_{FR} =0.8

Input I_2 of 2.2 may be called VFR with μ_{VFR} =0.2

NPTEL

FREE ONLINE EDUCATION swayam

Fired Set of Inputs

I_1 is LW and I_2 is FR

I_1 is LW and I_2 is VFR

I_1 is M and I_2 is FR

I_1 is M and I_2 is VFR

NPTEL

Weights

$$w^1 = \mu_{LW} \times \mu_{FR} = 0.8 \times 0.8 = 0.64$$

$$w^2 = \mu_{LW} \times \mu_{VFR} = 0.8 \times 0.2 = 0.16$$

$$w^3 = \mu_{M} \times \mu_{FR} = 0.2 \times 0.8 = 0.16$$

$$w^4 = \mu_{M} \times \mu_{VFR} = 0.2 \times 0.2 = 0.04$$

NPTEL

Functional consequent values

$$y^1 = I_1 + 2I_2 = 6.0 + 2 \times 2.2 = 10.4$$

$$y^2 = I_1 + 3I_2 = 6.0 + 3 \times 2.2 = 12.6$$

$$y^3 = 2I_1 + 2I_2 = 2 \times 6.0 + 2 \times 2.2 = 16.4$$

$$y^4 = 2I_1 + 3I_2 = 2 \times 6.0 + 3 \times 2.2 = 18.6$$

NPTEL

Output,

$$y = \frac{w^1 y^1 + w^2 y^2 + w^3 y^3 + w^4 y^4}{w^1 + w^2 + w^3 + w^4}$$

$$= \frac{0.64 \times 10.4 + 0.16 \times 12.6 + 0.16 \times 16.4 + 0.04 \times 18.6}{0.64 + 0.16 + 0.16 + 0.04} = 12.04$$

NPTEL



FREE ONLINE EDUCATION
swayam
शिक्षित भारत, उन्नत भारत

NPTEL ONLINE CERTIFICATION COURSES

Thank you!

NPTEL