Module 15 : Vector fields, Gradient, Divergence and Curl

Lecture 45 : Curves in space [Section 45.1]

Objectives

In this section you will learn the following :

- Concept of curve in space.
- Parametrization of a curve.
- Motion of tangent and normal to a curve.

45 .1 Curves in Space

Mathematically, the path or the trajectory of a particle moving in space in described by a function of time.

45.1.1 Definition:

Let $I \subset \mathbb{R}$ be an interval. A vector field

$$\mathbf{r} : I \to \mathbb{R}^3, n = 2, 3$$

is called a curve in $\mathbb{R}^2/\mathbb{R}^3$. For $n = 2$ it is called a **plane curve** , and for $n = 3$ it is called a **space curve** . If $I = [a, b]$, then $\mathbf{r}(a)$ in called the **initial point** and $\mathbf{r}(b)$ in called the **final point** of the curve. The set

$$C = \{\mathbf{r}(t) \mid t \in I\}$$

is called the **path** traced by the curve. The function

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}), t \in I,$$

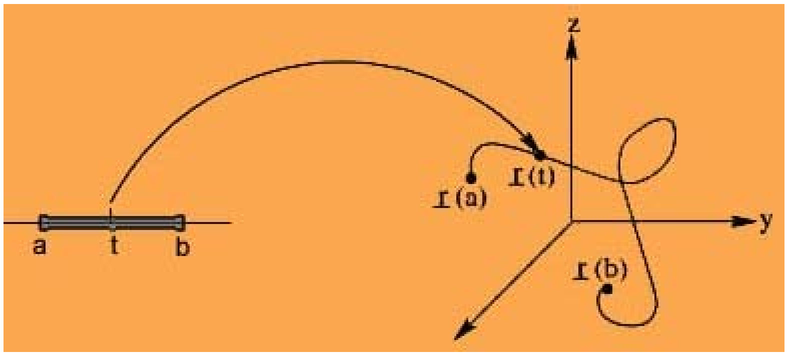

Figure 1. Curve in space

is called a **parametrization** of the curve, and the functions $x(t), y(t), z(t)$ are called the **components** of the curve $\mathbf{r}$. One also writes this as

$$\mathbf{r}(t) = (x(t), y(t), z(t)), t \in I.$$

45.1.2 Example:

(i) Consider the curve

$$\mathbf{r} : [0,1] \to \mathbb{R}^3, \mathbf{r}(t) = (t,t,t), t \in [0,1].$$

The path traced by this curve is the line segment OP, joining the origin with the point $P(1,1,1)$. A parametrization of this curve is given by

$$\mathbf{r}(t) = t\,\mathbf{i} + t\,\mathbf{j} + t\,\mathbf{k}, t \in [0,1].$$

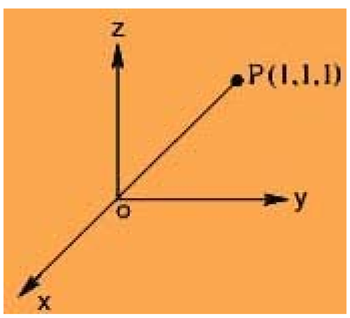

Figure 2. Line segment

In fact

$$\tilde{\mathbf{r}} : [0,1] \to \mathbb{R}^3, \tilde{\mathbf{r}}(t) = t^2\,\mathbf{i} + t^2\mathbf{j} + t^2\,\mathbf{k}, t \in [0,1],$$

is another curve with the same path as that of $\mathbf{r}$ described above. Thus, two different curves can have the same path

(ii) Let

$$f : [a,b] \to \mathbb{R}$$

be a function. Then its graph is a plane curve given by

$$\mathbf{r}(t) := t\,\mathbf{i} + f(t)\,\mathbf{j}, t \in [a,b],$$

(iii) Consider a curve with parameterizations

$$\mathbf{r}: \mathbb{R} \to \mathbb{R}^3, \mathbf{r}(t) = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j} + bt\,\mathbf{k}, t \in \mathbb{R},$$

where $a, b \in \mathbb{R}$ with $a > 0$. Since,

$$(x(t))^2 + (y(t))^2 = a^2,$$

for every $t \in \mathbb{R}, \mathbf{r}(t)$ is a point on the circle of radius a at a hight $z(t) = bt$. Thus, $\mathbf{r}(t)$ lies on the cylinder

$$x^2 + y^2 = a^2.$$

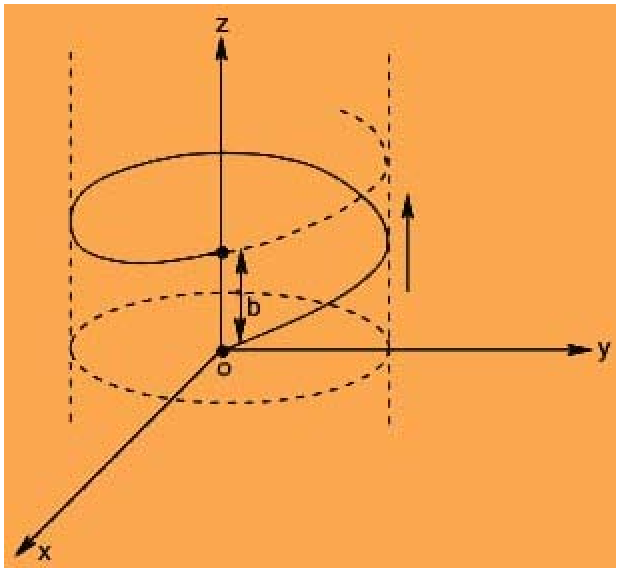

Figure 3. Right-handed helix

As t increases, $\mathbf{r}(t)$ winds around the cylinder upwards. As t change from t_0 to $t_0 + 2\pi$, the point $(x(t), y(t))$ return to their original values, $z(t)$ having increased by 2π. Thus, the curve looks like the threads on a screw, with pitch 2π. For $b > 0$, the curve $\mathbf{r}(t)$ is called the **right handed helix** and for $b < 0$, it's called the **left handed helix** .

(iv) Geometrically, curves in space also arise as intersection of surfaces. We give an example. Consider the

intersection of the cylinders

$z = x^3$ and $y = x^2$.

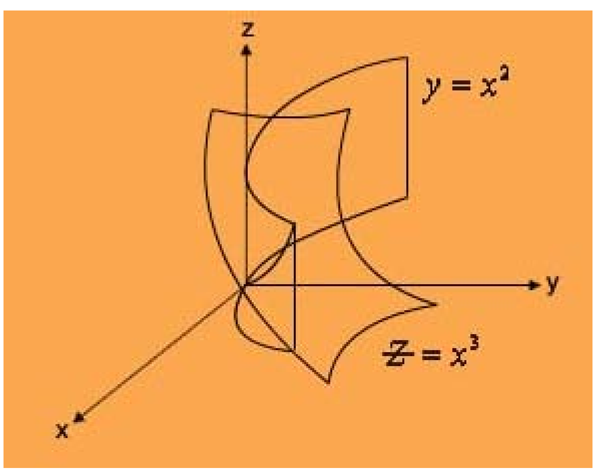

Figure 4. Twisted cuic

To find a parametric representation, we can choose $x = t$, then $y = x^2 = t^2$ and $z = x^3 = t^3$. Then, a parametric representation of the curve of intersection, is given by

$x = t, y = t^2, z = t^3, t \in \mathbb{R}, \mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j} + z(t)\,\mathbf{k}.$

This curve is called the **twisted-cubic** .

(v) Consider the curves

$$\mathbf{r}_1(t) = \cos t\ \mathbf{i} + \sin t\mathbf{j}, t \in [0, 2\pi] \text{ and } \mathbf{r}_2(t) = \cos t\ \mathbf{i} + \sin t\ \mathbf{j}, t \in [2\pi, 4\pi].$$

Then, both the curves $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ have the same path, namely the unit circle.

(vi) Consider the curves

$$\mathbf{r}_1(t) = t\,\mathbf{i} + t^2\,\mathbf{j}, t \in \mathbb{R} \quad \text{and} \quad \mathbf{r}_2(t) = t^2\,\mathbf{i} + t^4\,\mathbf{j}, t \in \mathbb{R}.$$

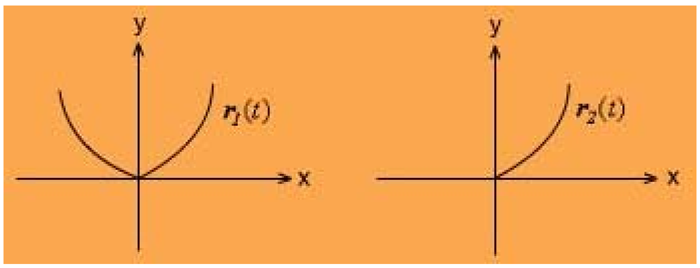

Figure 5. Parabola and half parabola

Then, $\mathbf{r}_1(t)$ represents the well-known curve in the plane, namely the parabola $y = x^2$. The curve $\mathbf{r}_2(t)$ represents only half of the parabola, namely $y = x^2, x \geq 0$.

45.1.3 Definition:

Let $\mathbf{r} : I \longrightarrow \mathbb{R}^3$ be a curve.

(i) We say $\mathbf{r}$ is a **simple curve** if $\mathbf{r}$ is a one-one map.

(ii) We say that the curve $\mathbf{r}$ is a **smooth curve** if $\mathbf{r}$ is continuously differentiable on I. We say $\mathbf{r}$ in **piecewise-**

smooth if $\mathbf{r}$ is continuously differentiable on all I, except a finite number of points.

(iii) A smooth curve $\mathbf{r}(t)$ is said to be **regular** on I if

$\mathbf{r}'(t) \neq \mathbf{0}$ for $t \in I$.

45.1.4 Example:

(i) Consider the curve

$$\mathbf{r}(t) = (t^2 - 1, t^3 - t, 0), t \in \mathbb{R}.$$

Since

$$\mathbf{r}(1) = (0, 0, 0) = \mathbf{r}(-1)$$

it is not a simple curve. However it is smooth and regular.

(ii) Let

$$\mathbf{r}(t) = (|\,t\,|, t, t), t \in \mathbb{R}$$

Then, is not a smooth curve, since does not exist for

$\mathbf{r}(t)$ $\mathbf{r}'(t)$ $t = 0.$

(iii) Let

$$\mathbf{r}(t) = (t^3, t^2, 0), t \in [-1, 1].$$

It is a simple, smooth curve. Since,
$\mathbf{r}'(t) = \mathbf{0}$ for $t = 0,$
it is not a regular curve.

45.1.5 Remark:

We saw in earlier examples that a path can be the image of more than one smooth curves. For example, consider the curves, for $t \in \mathbb{R}$

$$\mathbf{r}_1(t) = (t, 0, 0), \mathbf{r}_2(t) = (t^3, 0, 0), \text{ and } \mathbf{r}_3(t) = (t^3 + t, 0, 0), t \in \mathbb{R}.$$

These are there different curves having the same path, namely the x-axis. Both curves $\mathbf{r}_1$ and $\mathbf{r}_3$ are regular, smooth and simple, while $\mathbf{r}_2$ in smooth and simple, but not regular.

The above remark raises the natural question:

When can two curves having same path be treated as same?

To analyze such questions, we make the following definition.

45.1.6 Definition:

Two curves

$$\mathbf{r}_1 : J = [a, b] \text{ and } \mathbf{r}_2 : I = [c, d] \to \mathbb{R}$$

are said to be **equivalent** if there exist a one to one onto map

$$\phi : (a, b) \to (c, d)$$

such that ϕ and its inverse, $\psi := \phi^{-1}$, are both continuously differentiable and

$$\mathbf{r}_2 = \mathbf{r}_1 \, o \, \phi.$$

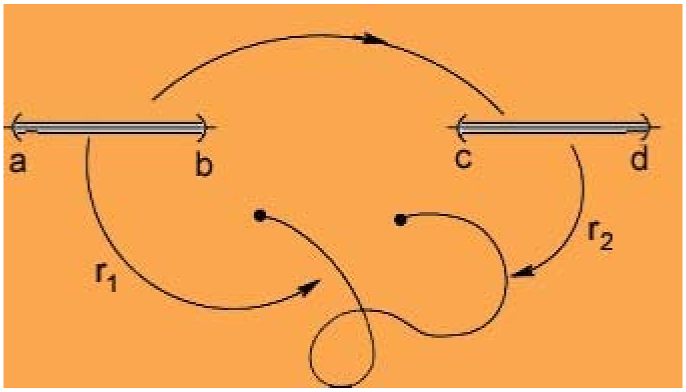

Figure 6. Equivalent curves

45.1.7 Example:

Consider the curves

$\mathbf{r}_1 : [0,2] \to \mathbb{R}^3$ and $\mathbf{r}_2 : [0,1] \to \mathbb{R}^3$,
defined by
$\mathbf{r}_1(t) = (t, 0, 0), \mathbf{r}_2(t) = (t + t^3, 0, 0), t \in \mathbb{R}.$

Let
$\phi : [0,1] \to [0,2]$ be defined by $\phi(t) = t + t^3, t \in [0,1]$.
Then, ϕ in one-one and onto. Further, ϕ in differentiable with
$\phi'(t) = 1 + 3t^2 \neq 0$ for all $t \in (0,1)$.
Thus, ϕ^{-1} in also differentiable in $(0,1)$ and
$\mathbf{r}_2(t) = (\mathbf{r}_1 o \phi)(t)$ for all $t \in (0,1)$.

Hence, $\mathbf{r}_1$ is equivalent to $\mathbf{r}_2$.

Practice Exercises

Prove the following :

(1) (i) The curve

$$\mathbf{r}(t) = t \sin t \ \mathbf{i} + t \cos t \ \mathbf{j} + t^2 k$$

lies on a paraboloid

$$z = x^2 + y^2$$

(ii) The curve.

$$\mathbf{r}(t) = t\mathbf{i} + \frac{1+t}{t}\mathbf{j} + \frac{1-\mathbf{t}^2}{t}\mathbf{k}, t > 0,$$

lies in the plane $x - y + z = -1$.

(2) Find a parametrization of the curve of intersection of the sphere $x^2 + y^2 + z^2 = 16$ and the cylinder $x^2 + (4-2)^2 + 4$ that lies is the first octant.

Answer

(3) Consider the curve in $\mathbb{R}^2$ going from $A(1,2)$ to $B(-1,-2)$ to $C(4,0)$. Find a parameterizations $r(\mathbf{t}), 0 \leq t \leq 3$ of this piecewise smooth curve.

Answer

Recap

In this section you have learnt the following

- Concept of curve in space.
- Parametrization of a curve.
- The motion of tangent normal to a curve.

[Section 45.2]

Objectives

In this section you will learn the following :

- The notion of velocity vector.
- The notion of tangent vector to a curve.

45 .2 Velocity Vector, Tangent Vector

Let $\mathbf{r} : I \longrightarrow \mathbb{R}^3$ be a smooth curve. Physically, $\mathbf{r}(t)$ represents the position of a particle moving in space. If $\mathbf{r}$ is differentiable then, the velocity of the particle at time t_0 is given by

$$\mathbf{r}'(t_0) = \lim_{\Delta t \to 0}\left(\frac{\mathbf{r}(t_0 + \Delta t) - \mathbf{r}(t_0)}{\Delta t}\right).$$

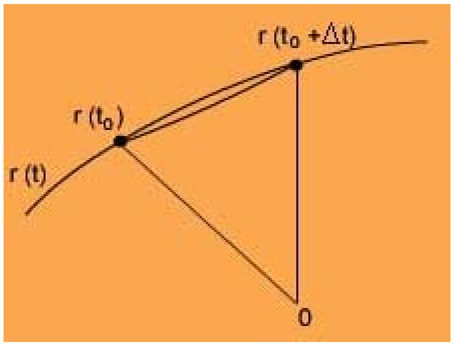

Figure 1. Tangent vector to a curve

45.2.1 Definition:

Let $\mathbf{r} : I \longrightarrow \mathbb{R}^3$ be a regular curve and $t_0 \in I$. Then, $\mathbf{r}'(t_0)$ is called the **tangent-vector** or the **velocity-vector** of the curve at t_0. The vector

$$\mathbf{T}_{\mathbf{r}}(t_0) = \frac{\mathbf{r}'(t_0)}{\|\mathbf{r}'(t_0)\|}$$

is called the **unit tangent vector** or the **unit velocity** to $\mathbf{r}$ at t_0 . The line

$$\phi(t) = \mathbf{r}_0 + t\,\mathbf{r}'(t_0), t \in \mathbb{R},$$

is called **tangent-line** to $\mathbf{r}(t)$ at $t = t_0$, where $\mathbf{r}_0 = (t_0)$.

We describe next the change in unit tangent-vector under re-parameterizations.

45.2.2 Theorem:

Let

$$\mathbf{r}_1 : [a,b] \to \mathbb{R}^3 \; and \; \mathbf{r}_2 : [c,d] \to \mathbb{R}^3$$

be regular curves which are equivalent. Then,

$$\mathbf{T}_{\mathbf{r}_2}(t) = \pm\, \mathbf{T}_{\mathbf{r}_1}\ (g(t)), \text{for all } t \in (c,d),$$

where

$$g : (c,d) \to (a,b)$$

is the re-parametrization of $\mathbf{r}_1$ *to* $\mathbf{r}_2$ *.*

PROOF

45.2.2 Theorem:

Let

$$\mathbf{r}_1 : [a,b] \to \mathbb{R}^3 \; and \; \mathbf{r}_2 : [c,d] \to \mathbb{R}^3$$

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where

$$g : (c,d) \to (a,b)$$

is the re-parametrization of $\mathbf{r}_1$ *to* $\mathbf{r}_2$ *.*

Proof

The map $g : (c,d) \to (a,b)$ is a bijective map with continuous derivative such that

$$\mathbf{r}_2(t) = (\mathbf{r}_1 o g)\,(t) \quad \text{for all } t \in (c,d).$$

Thus,

$$\mathbf{r}'_2(t) = \mathbf{r}'_1(g(t))\; g'(t), t \in (c,d).$$

Hence,

$$\mathbf{T}_{\mathbf{r}_2}(t) = \frac{\mathbf{r}'_2(t)}{\|(t)\|} = \left(\frac{\mathbf{r}'_1(g(t))}{\|\mathbf{r}'_1(g(t))\|}\right)\left(\frac{g'(t)}{|g'(t)|}\right) = \left(\frac{g'(t)}{|g(t)|}\right)\mathbf{T}_{\mathbf{r}_1}(t), t \in (c,d).$$

Note that since g in bijective and $g'(t) \neq 0$, the function $g(t)$ is a monotone function and hence the scalar

$$\frac{g'(t)}{|g'(t)|} = \begin{cases} +1 & \text{if} \quad g \text{ is monotonically increasing} \\ -1 & \text{if} \quad g \text{ is monotonically decreasing.} \end{cases}$$

45.2.3 Examples :

(i) Consider the circular-helix

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + t\,\mathbf{k},\ t \in \mathbb{R}.$$

Then,

$$\mathbf{r}\,(\pi) = -\mathbf{i} + \pi\mathbf{k}, \mathbf{r}'(t)\mathbf{r}'\,(\pi) = -\mathbf{j} + \mathbf{k},\ \text{and}\ \mathbf{r}'(t) = -\sin t\,\mathbf{i} + \cos t\,\mathbf{j} + \mathbf{k}.$$

Hence, the equation of the tangent line to the circular-helix at $\mathbf{t} = \pi$ is given by

$$\begin{aligned} \phi(t) &= (-\mathbf{i} + \pi\mathbf{k}) + t\,(-\mathbf{j} + \mathbf{k}) \\ &= -\mathbf{i} - t\,\mathbf{j} + (t + \pi)\mathbf{k},\ t \in \mathbb{R}. \end{aligned}$$

(ii) In the plane , the tangent line to any point on a circle is perpendicular to the redius vector. The same holds for any curve on the sphere in 3-space. Let $\mathbf{r}(t)$ be a curve on a sphere of radius k .

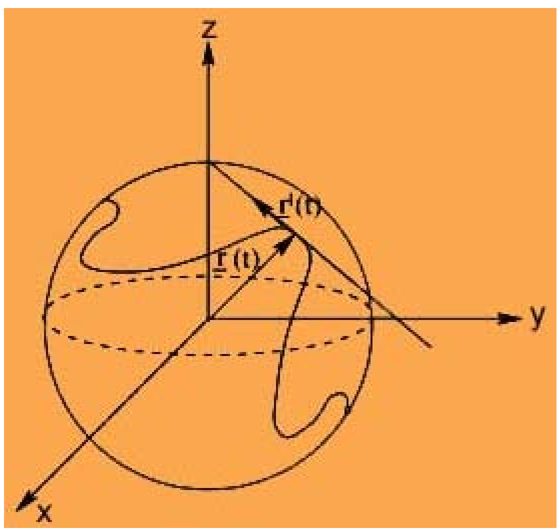

Figure 2. Curve on a sphere .

Since ,

$$||\mathbf{r}(t)|| = k \text{ for all } t, \text{ we have } \mathbf{r}(t) \cdot \mathbf{r}(t) = k^2 \text{ for all } t.$$

Differentiating the above with respect to t, we have,

$$2\mathbf{r}'(t) \cdot \mathbf{r}(t) = 0.$$

Hence,

$\mathbf{r}'(t)$ is perpendicular to $\mathbf{r}(t)$ for every t

45.2.4 Definition:

Analogous to a plane curve, for a regular smooth curve $\mathbf{r}(t), t\in[a,b]$, in space, we define its length to be

$$L=\int_a^b\left(\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2+\left(\frac{d3}{dt}\right)^2}\right)dz$$

$$=\int_a^b\left\|\frac{d\mathbf{r}}{dt}\right\|dt.$$

45.2.5 Example:

Consider the circular-helix

$\mathbf{r}(t)=a\cos t\,\mathbf{i}+a\sin t\,\mathbf{j}+bt\,\mathbf{k}, t\in\mathbb{R}.$

Then

$\mathbf{r}'(t)=-a\sin t+\mathbf{i}+a\cos t\,\mathbf{j}+b\,\mathbf{k}$ and hence $\|\mathbf{r}'(t)\|=\sqrt{a^2+b^2}$.

Thus, the length of the helix from $t=0$ to a general point $t\in\mathbb{R}$ in given by

$$s(t)=\int_0^t\left|\left|\mathbf{r}'(u)\right|\right|du=t\sqrt{a^2+b^2}.$$

45.2.6 Arc-length parameterizations :

Let $\mathbf{r}(t), t\in[a,b]$ be a smooth regular curve C. Then the arc length of C from $t=a$ to a point P on C, with $t\in[a,b]$, is given by

$$s(t)=\int_a^t\left|\left|\mathbf{r}'(t)\right|\right|dt.$$

If we treat $s(t):=s$ as a variable, then every point P on the curve determine a unique value $s\in[0,\ell]$, where

$$\ell=\int_a^b\|\mathbf{r}'(t)\|\,dt,$$

and conversely, every value $s\in[0,\ell]$ is determined by unique point P an C since $||\mathbf{r}'(t)||\neq 0$ for all t.

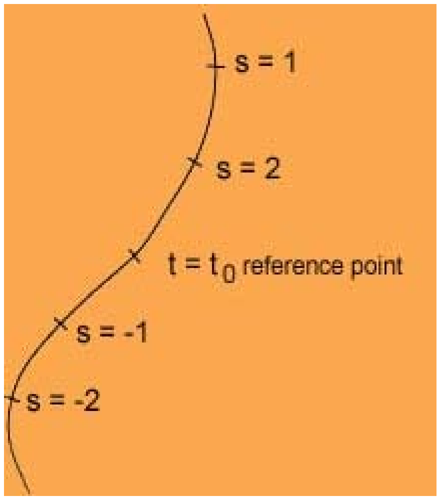

Figure 3. Arc length parameterization

Hence, s can be treated as a parameter for the curve and this new parameterizations $s \mapsto s(t), s \in [0, \ell]$, is called the **arc-length parameterization** or the **unit speed parameterization** of the curve. In case the curve $\mathbf{r}(t)$ in defined on $\mathbb{R}$, we can choose any point t_0 as a reference point(initial point) and define

$$s := s(t) = \int_{t_0}^{t} ||\mathbf{r}'(t)||\, dt, \text{ for all } t \in \mathbb{R}.$$

as the arc-length parameter. Note that $s > 0$ for $t > t_0$ and $s < 0$ for $t < t_0$.

45.2.7 Examples:

(i) Consider the circle in the plane

$$\mathbf{r}(t) = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j},\ 0 \le t \le 2\pi.$$

Then

$$\mathbf{r}'(t) = -a\sin t\,\mathbf{i} + a\cos t\,\mathbf{j}. \text{ and } \|\mathbf{r}'(t)\| = a \text{ for every } t \in [0, 2\pi].$$

Hence,

$$s = s(t) = \int_0^t \|\mathbf{r}'(t)\|\, dt = a\,t \in [0, 2\pi].$$

This gives $t = s/a$, and we have the arc-length parameterization of the circle

$$\mathbf{r}(s) := a\cos t\,(s/a)\,\mathbf{i} + a\sin t\,(s/a)\,\mathbf{j},\ s \in [0, 2\pi].$$

Consider the circular helix

$$\mathbf{r}(t) = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j} + bt\,\mathbf{k},\ t \in \mathbb{R}$$

If we choose $t = 0$ as our reference point, then

$$s = s(t) = \int_0^t \|\mathbf{r}'(u)\|\, du = t\sqrt{a^2 + b^2}, \text{ for every } t \in \mathbb{R}.$$

Thus, the arc-length parameterizations for the helix is

$$s(s) = a\cos\left(\frac{s}{\sqrt{a^2+b^2}}\right)\mathbf{i} + a\sin\left(\frac{s}{\sqrt{a^2+b^2}}\right)\mathbf{j} + b\,\frac{s}{\sqrt{a^2+b^2}}\,\mathbf{k}.$$

The advantage of selecting the arc-length parameterizations of a curve C is described in the following theorem.

45.2.8 Theorem:

(i) *Let C be the path of a smooth regular curve $\mathbf{r}(t)$, with $\mathbf{r}(s)$ as the arc length parameterization. Then,*

$$\left\| \frac{d\mathbf{r}}{dt} \right\| = \frac{ds}{dt},$$

i.e., $\mathbf{T}$, the tangent vector at any point t has length ds/dt. Further,

$$\left\| \frac{d\mathbf{r}}{ds} \right\| = 1 \text{ for every } s.$$

(ii) *If for a smooth regular curve C with parameterization $\mathbf{r}(t)$,*

$$\left\| \frac{d\mathbf{r}}{dt}(t) \right\| = 1, \text{ for every } t,$$

then for any value t_0 in the domain of $\mathbf{r}$, the parameter $s = t - t_0$ gives an arc-length parameterizations of $\mathbf{r}(s)$ for C with the reference point t_0.

45.2.8 Theorem:

(i) *Let C be the path of a smooth regular curve $\mathbf{r}(t)$, with $\mathbf{r}(s)$ as the arc length parameterization. Then,*

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then for any value t_0 in the domain of , the parameter $s = t - t_0$ gives an arc-length parameterizations of $\mathbf{r}(s)$ for C with the reference point t_0.

Proof

(i) Since

$$s(r) = \int_0^t \| \mathbf{r}'(u) \| \, du, \text{ for all } t.$$

By the Fundamental Theorem of calculus,

$$\frac{ds}{dt} = \| \mathbf{r}'(t) \|.$$

Further, by chain-rule

$$\frac{ds}{dt} \frac{d\mathbf{r}}{ds} = \frac{d\mathbf{r}}{dt}.$$

Since, the curve is regular, $\mathbf{r}'(t) \neq 0$ for all t, and hence

$$\frac{d\mathbf{r}}{ds} = \frac{1}{\|\mathbf{r}'(t)\|}\frac{d\mathbf{r}}{dt}, \text{i.e.,} \left\|\frac{d\mathbf{r}}{ds}\right\| = 1.$$

(ii)

Let t_0 be any reference point for the curve $\mathbf{r}(t)$. Then,

$$s = s(t) = \int_{t_0}^{t} \|\mathbf{r}'(t)\|\, du$$

gives the arc-length parameter. Since $\|\mathbf{r}'(t)\| = 1$ for every t,

$$s = s(t) = \int_{t_0}^{t} 1 du = t - t_0.$$

Hence, the arc length parameterizations in $r(s) = \mathbf{r}(t - t_0)$.

Practice Exercises

(1) For the following curve $\mathbf{r}(t)$, find the equation of the tangent line to it at t_0 :

(i) $\mathbf{r}\,(t) = t^2\,\mathbf{i} + (2 - \ln t)\,\mathbf{j};\ \ t_0 = 1$.

(ii) $\mathbf{r}\,(t) = 2\cos t\,\mathbf{i} + 2\sin \pi t\,\mathbf{j} + 3t\,\mathbf{k};\ \ t_0 = \frac{1}{3}$.

(iii) $\mathbf{r}\,(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k};\ \ t = 0$.

Answer

(2) Find the length of the following :

(i) $\mathbf{r}\,(t) = t\,\mathbf{i} + \frac{4}{3}\,(t^3/2)\,\mathbf{j} + \frac{t^2}{2}\,\mathbf{k}, t \in [0, 2]$.

(ii) $\mathbf{r} = (2\cos h\, 3t)\,\mathbf{i} - (2\sin h\, t)\,\mathbf{j} + 6t\,\mathbf{k}, t \in [0, 5]$.

Answer

(3) Show that the following parameterizations are unit speed parameterizations :

(i) $\mathbf{r}\,(s) = \dfrac{(1+s)^{3/2}}{3}\,\mathbf{i} + \dfrac{(1-s)^{3/2}}{3} + \dfrac{s}{\sqrt{2}}\,\mathbf{k}, -1 \le s \le +1.$

(ii) $\mathbf{r}\,(s) = \frac{1}{2}\left(\cos^{-1} - s\sqrt{1-s^2}\right)\mathbf{i} + (1 - s^2)\,\mathbf{j}, 0 \le s \le 1$

(4) Find the relation between the arc length parameter s and the given parameter t :

(i) $\mathbf{r}\,(t) = e^t\,\cos\,\mathbf{i} + e^t\,\sin t\,\mathbf{j} + e^t\,\mathbf{k}, t \in [a, b]$.

(ii) $\mathbf{r}\,(t) = (\cos h\, t)\,\mathbf{i} + (\sin h\, t)\,\mathbf{j} + t\,\mathbf{k}, t \in [a, b]$.

Answer

Recap

In this section you have learnt the following

- The notion of velocity vector.
- The notion of tangent vector to a curve.