### Objectives

In this course you will learn the following

- Concept of holes in a semiconductor.
- Drift velocity and mobility.
- Hall effect in semiconductors.

### Electrons and Holes

We have seen that the charge carriers in metals are electrons which are in partially occupied conduction band. Electrons in fully occupied valence bands do not contribute to net current. We may understand this by a consideration of the effective mass of electrons near the top of the valence band. It may be seen that near the top of a band, the effective mass is negative. This means that in an electric field, the electrons which are near the top of the valence band are accelerated in the direction of the field while those near the bottom are accelerated in the opposite direction. The overall current in a filled band $-q\sum_i v_i$ is zero. Physically, this happens because there being no empty state in a filled band, the best that electrons can do is to trade places.

In semiconductors, the width of the forbidden band is small enough so that electrons near the top of a valence band can easily jump to the unoccupied conduction band.

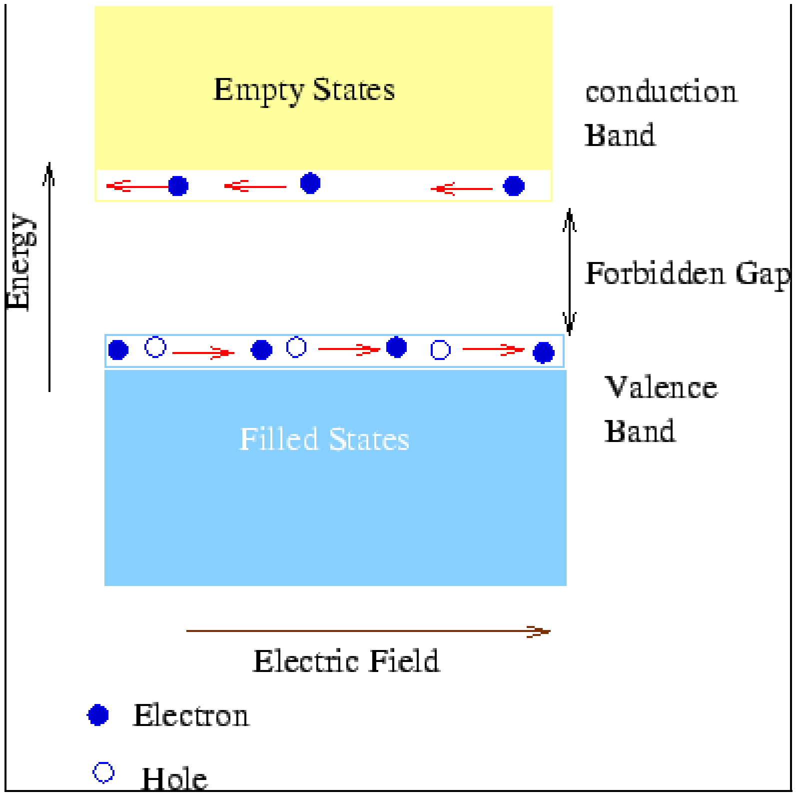
Consider what happens if one electron moves from near the top of valence band to the bottom of conduction band. The net effect of such a transition is to give a negative contribution to the current because had the electron been there at the top of the valence band it would have moved in the direction of the current (because of negative effective mass). Equivalently, the effect is to have one more electron moving opposite to the direction of the field. Mathematically, the current in a nearly filled band is given as follows.

$$I = -q \sum_{i \in \text{filled states}} v_i = \boxed{-q \sum_{i \in \text{all states}} v_i + q \sum_{i \in \text{vacant states}} v_i}$$

$$= 0 + q \sum_{i \in \text{vacant states}} v_i$$

$$= q \sum_{i \in \text{vacant states}} v_i$$

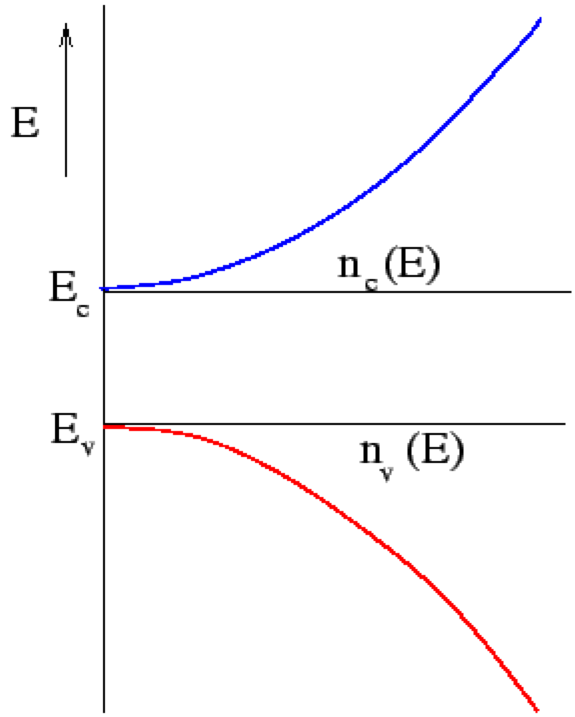
Thus, the vacancy in the valence band behaves like a particle of positive effective mass and positive charge as far as its behaviour in an electric field is concerned. This vacancy is known as a **hole** . In a semiconductor, the conduction takes place by motion of electrons in the conduction band and that of holes in the valence band.




If $E_c$ is the energy of the bottom of the conduction band and $E_v$ is that of the top of the valence band, the density of states of electrons and holes in these two bands are respectively given by

$$n_c(E) = \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2}(E - E_c)^{1/2}$$

$$n_v(E) = \frac{1}{2\pi^2}\left(\frac{2m_h}{\hbar^2}\right)^{3/2}(E_v - E)^{1/2}$$




**Mobility of Electrons and Holes**

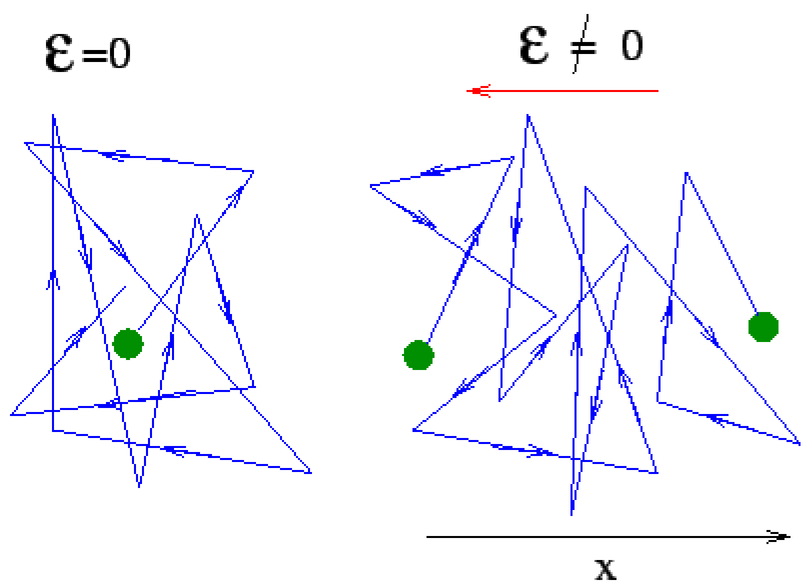
When an electric field $\mathcal{E}$ is applied to a metal or a semiconductor, the carriers are accelerated; the equation of motion being given by Newton's law

$$\frac{d^2x}{dt^2} = \frac{q\mathcal{E}}{m}$$

Since the carriers continuously collide with atoms, the above equation is valid only during the time between two successive collisions. The change in velocity between two successive collisions with an interval $t$ is given by

$$\Delta v = \frac{q\mathcal{E}}{m}t$$

In the absence of a field the carriers exhibit random motion due to chaotic changes in their thermal velocities so that the average change in velocity over a long period of time is zero.




However, if , the carriers have a net motion in the direction of the field (for ) over which the

$\mathcal{E} \neq 0$ $q > 0$

random motion is superimposed. This is known as the **drift velocity** . The figure shows drift of an electron in the field, the abrupt changes in the direction is due to a collision with an atom. The drift velocity is proportional to the strength of the electric field, the constant of proportionality being known as mobility. Taking the average time between collisions (called the relaxation time) to be $\tau$, the average increase in velocity between collisions is given by $v_{ave}$ is given by

$$v_{ave} = \frac{q\mathcal{E}}{m}\tau = \mu\mathcal{E}$$

where $\mu = q\tau/m$ is the mobility of the carrier.

For a semiconductor we need to replace $m$ by $m^*$, the effective mass of the carrier. Mobility depends on the relaxation time - the longer the interval between successive scattering, greater is the increase in incremental velocity. Likewise a smaller effective mass means larger acceleration and consequently a higher velocity.

The unit of mobility is m $^2$/V-s.

**Example-5**

The average distance travelled by a carrier between collisions is called the **mean free path** . The electron mobility in GaAs at 77 K (the temperature at which nitrogen becomes a liquid) is 30 m $^2$/V-s. If the effective mass of electron is 0.067 $m_0$, determine the mean free path.

**Solution**

The relaxation time can be calculated from mobility

$$\tau = \frac{m\mu}{q} = \frac{0.067 \times 9.1 \times 10^{-31} \times 30}{1.6 \times 10^{-19}} = 11.375 \times 10^{-12} \ \text{s}$$

The thermal velocity is given from kinetic theory to be

$$v_{th} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 77}{0.067 \times 9.1 \times 10^{-31}}} = 22.92 \times 10^4 \ \text{m/s}$$

Thus the mean free path is $v_{th}\tau = 2.6 \times 10^{-6}$ m.

**Exercise 1**

Electrons in the conduction band of silicon have effective mass 0.25 $m_0$ and mobility 0.14 m $^2$/V while the holes in one of the valence bands have effectives mass 0.54 $m_0$ and mobility 0.048 m $^2$/V-s, $m_0$ being free electron mass. Determine the relaxation times for the carriers.( $\tau_e = 2 \times 10^{-13}$ s $\tau_h = 1.47 \times 10^{-13}$ s.)

**Exercise 2**

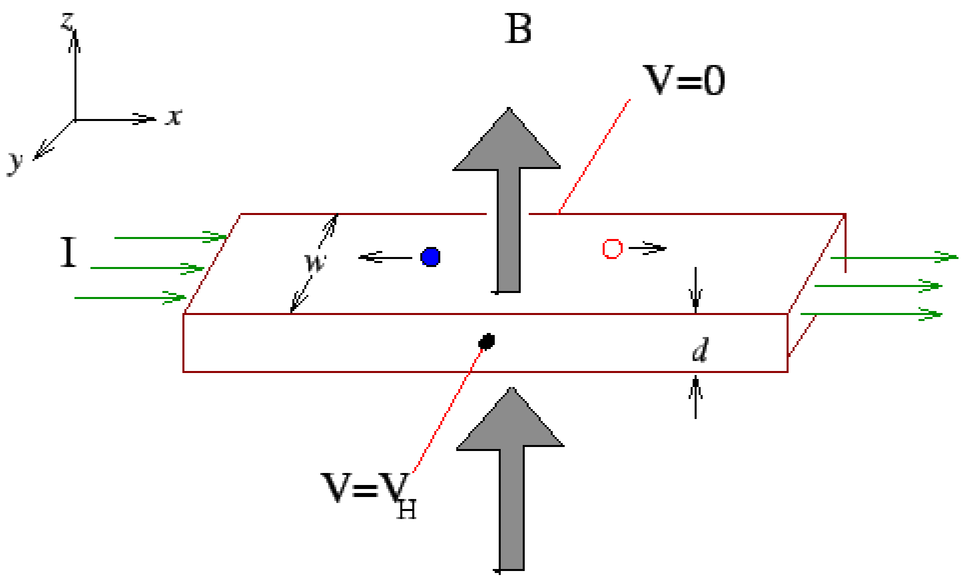
A sample of copper has an electron drift velocity of 2.5 m/s in an electric field of 500 V/m. Determine (i) electron mobility and (ii) relaxation time.

(Ans. (i) m $^2$/V-s (ii) s.)

$5 \times 10^{-3}$ $2.84 \times 10^{-14}$

**Hall Effect**

Hall effect provides a direct evidence of the existence of holes in semiconductors. In a typical Hall effect experiment, a magnetic field is applied in a direction perpendicular to the direction of current in a flat strip of a semiconductor. In the figure shown, the current flows along the length (positive x-direction) of the strip.




When a magnetic field is applied in a direction transverse to the current direction, a potential difference appears across the direction perpendicular to both current and the magnetic field. This transverse voltage is called the **Hall voltage** .

The origin of Hall voltage is Lorentz force that acts on charge carriers. The magnetic force acting on the charges $q\vec{v} \times \vec{B}$ deflects the charge carriers in a direction perpendicular to the direction of the velocity.

This leads to charge separation because of which an electric field $\vec{\mathcal{E}}$ appears across the strip. The bending of the trajectory stops when the force due to the electric field balances that due to the magnetic field, i.e. $\vec{\mathcal{E}} = -\vec{v} \times \vec{B}$. The forces being in the opposite directions, their magnitudes are equal, i.e $\mathcal{E} = vB$.

The current density $J$ is given by $J = nqv$, where $n$ is the density of carriers. Thus we have, on equating the forces,

$$n = \frac{JB}{q\mathcal{E}}$$

- By measuring the current density $J$, the magnetic field strength $B$ and the electric field $\mathcal{E}$, we may determine the carrier density. Since $J = I/wd$ and $\mathcal{E} = V_H/w$, we get

$$n = \frac{IB}{qdV_H}$$

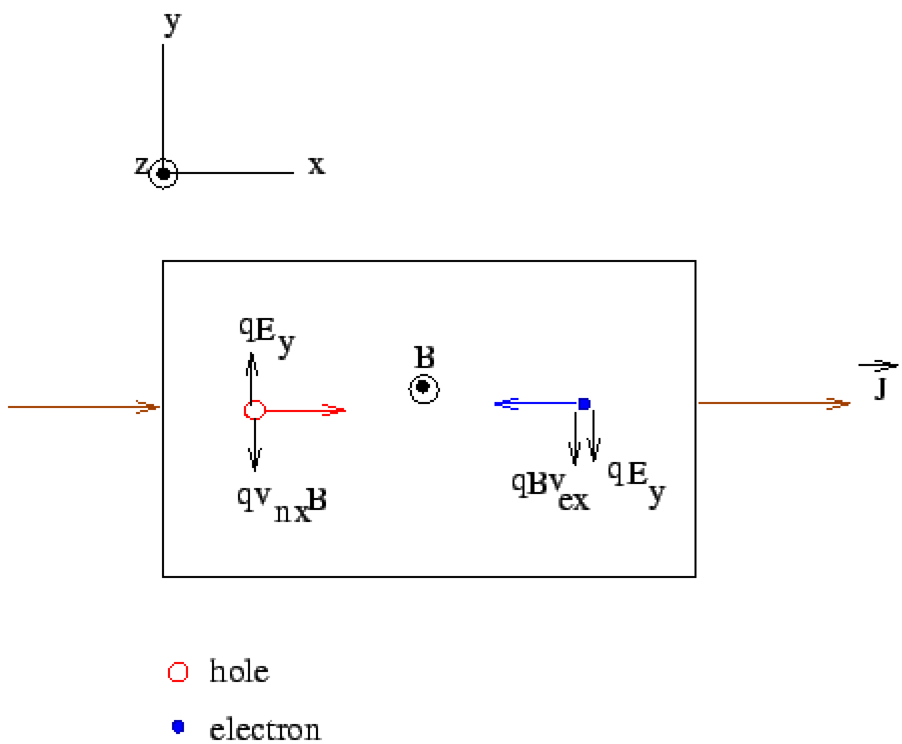
- The direction of Hall voltage depends on the sign of the charge of the carrier. Hall effect, therefore, provides a direct way of establishing the existence of holes. We define **Hall constant** $R_H$ through

$$R_H = \frac{1}{nq} = \frac{V_H d}{IB}$$

If $\rho$ is the resistivity of the material of the strip, the mobility of the carrier is given by

$$\mu = \frac{1}{\mid \rho n q \mid} = \frac{\mid R_H \mid}{\rho}$$

When both electrons and holes are present, they drift in opposite directions. with holes moving in the direction of current and electrons in the opposite direction. As the sign of their charges are opposite, the Lorentz force acting on them will be in the same direction. Thus both the carriers are deflected towards the same side and they pile up along one of the edges. As there is no closed path for current to flow in the y-direction, the sum of the electron and the hole current in the y-direction must be zero




$$qpv_{hy} + qnv_{ey} = 0$$

where $p$ and $n$ are respectively the electron and the hole densities and $v_{hy}$ and $v_{ey}$ the y-component of the respective drift velocities.

**Example-6**

A long strip of material of width $w = 1$ cm and thickness $d = 1$ mm is subjected to a magnetic field along a direction parallel to the edge $d$. A current of 1 A flows in the strip parallel to its length. The mobility of the carriers (of density $2.5 \times 10^{22}$ per m $^3$) is 5 m $^2$/V-s.

(1) Determine the magnitude and direction of the net electric field.

(2) If $B = 0.1$ T, calculate the potential difference $V_H$ between the opposite points of the strip parallel to the width $w$.

**Solution**

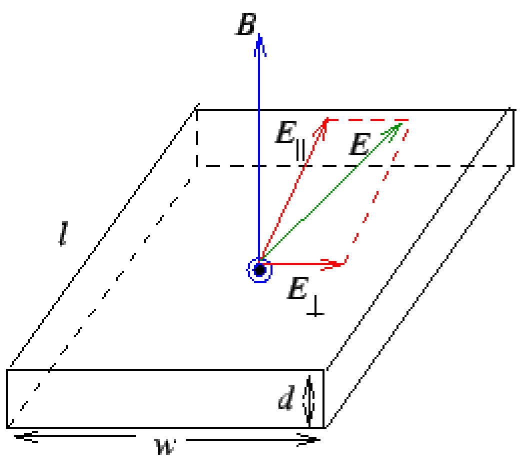
$$I = JA = nevwd$$

The current which gives the drift velocity as

$$v = \frac{I}{nedw} = 25 \text{ m/s}$$

Thus the electric field parallel to the length is

$$E_{\parallel} = \frac{v}{\mu} = \frac{25}{5} = 5 \text{ V/m}$$




The magnitude of the electric field perpendicular to the direction of drift velocity and the magnetic field is

$$E_{\perp} = vB = 25 \times 0.1 = 2.5 \text{ V/m}$$

Thus the net electric field is

$$E = \sqrt{E_{\parallel}^2 + E_{\perp}^2} = 5.59 \text{ V/m}$$

The transverse potential difference is

$$V_H = E_{\perp} w = 25 \text{ mV}$$

**Exercise 3**

Hinall effect experiment is made in a sample of a flat semiconductor of length 1 cm and width 0.3 cm. The mobility of carriers in the sample is 4500 cm $^2$/V-s. If the voltage along the length of the conductor is 1 volt, determe the Hall voltage across the width when a magnetic field of 0.02 T is applied.

(Ans. 2.7 mV)

**Recap**

In this course you have learnt the following

- Charge carriers in semiconductors are negatively charged electrons and positively charged holes. Holes are essentially vacancies in an otherwise filled band so that when an electron moves to such a vacancy, the effect is equivalent to movement of a hole in reverse direction.
- In the absence of an electric field, charge carriers move randomly so that their average velocity is zero. When an electric field is applied, the positve charge carriers move in the direction of the field and the negative charge carriers move against the field direction. This directed motion is superimposed over the random direction and is called drift.
- Drift velocity is proportional to the direction of the field, the constant of proportionality is the mobility of the carrier.
- If a magnetic field is applied on a flat strip of semiconductor in a direction perpendicular to the direction of current flow, a voltage develops in a direction which is perpendicular to both the direction of the field and

the magnetic field. This is known as Hall voltage.

- Hall voltage arises due to Lorentz force that acts on charge carriers and provides a direct means of verifying existence of holes.