

Module 3 : MAGNETIC FIELD

Lecture 14 : Force on a Current Carrying Conductor

Objectives

In this lecture you will learn the following

- Calculate the force on a current carrying conductor in a magnetic field.
- Find the torque on a current loop.
- Define magnetic dipole and its magnetic moment.
- Determine potential energy of a magnetic dipole

Force on a Current Carrying Conductor

A conductor has free electrons which can move in the presence of a field. Since a magnetic field exerts a force $q\vec{v} \times \vec{B}$ on a charge moving with a velocity \vec{v} , it also exerts a force on a conductor carrying a current.

Consider a conducting wire carrying a current I . The current density at any point in the wire is given by'

$$\vec{J} = ne\vec{v}$$

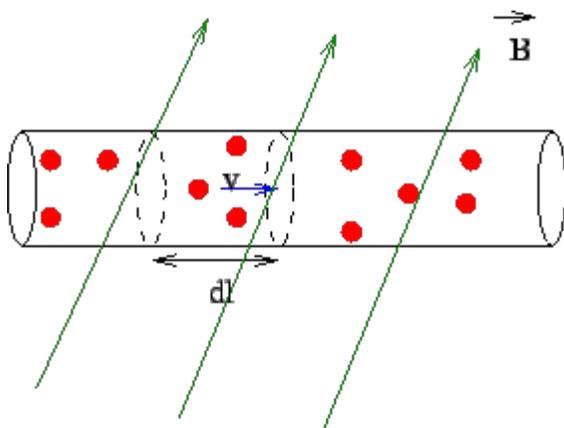
where n is the number density of electrons having a charge e each and \vec{v} is the average drift velocity at that point.

Consider a section of length dl of the wire. If A is the cross sectional area of the section oriented perpendicular to the direction of \vec{J} , the force on the electrons in this section is

$$d\vec{F} = dq\vec{v} \times \vec{B}$$

where dq is the amount of charge in the section

$$dq = neAdl$$



Thus the force on the conductor in this section is

$$d\vec{F} = neA d\vec{l} \vec{v} \times \vec{B}$$

If $d\vec{l}$ represents a vector whose magnitude is the length of the segment and whose direction is along the direction of \vec{v} , we may rewrite the above as

$$\begin{aligned} d\vec{F} &= neAv d\vec{l} \times \vec{B} \\ &= JA d\vec{l} \times \vec{B} \\ &= I d\vec{l} \times \vec{B} \end{aligned}$$

The net force on the conductor is given by summing over all the length elements. If \hat{u}_l denotes a unit vector in the direction of the current, then $\vec{F} = I \int \hat{u}_l \times \vec{B} dl$

Example 1

Force on a Straight Wire carrying current I

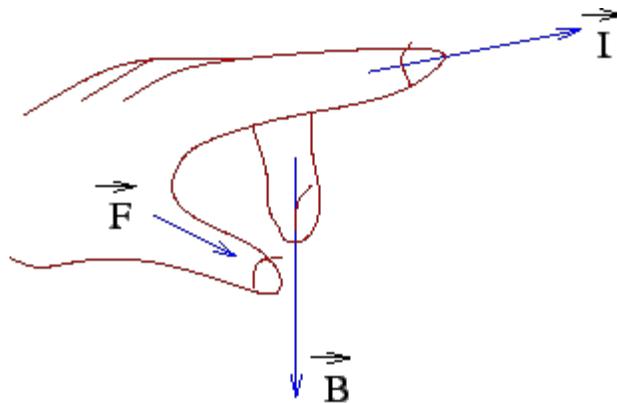
If the wire is straight \hat{u}_l is a constant vector. For a uniform magnetic field \vec{B}

$$\vec{F} = I(\hat{u}_l \times \vec{B}) \int dl = IL(\hat{u}_l \times \vec{B})$$

The wire experiences a sidewise force. The direction of the force is given by Fleming's Left Hand Rule.

Fleming's Left Hand Rule

If one stretches out the forefinger, the middle finger and the thumb of one's left hand so that they are mutually perpendicular to one another, then, if the middle finger points in the direction of current and the forefinger points in the direction of the magnetic field, the thumb will point in the direction of the force.

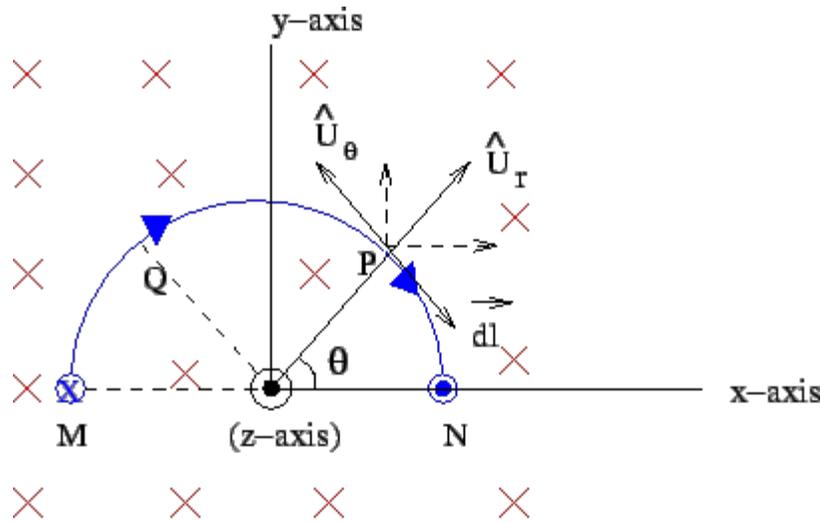


Fleming's Left Hand Rule

Example 2

Force on a semi-circular current element

Consider a semicircular element in the plane of the paper (the current enters the page at M along a direction parallel to the negative z-axis and leaves at N in a direction parallel to the positive z-axis). These incoming and outgoing sections of the conductor do not experience any force as \vec{B} is parallel (or anti-parallel) to the direction of the current in these.



Magnetic Field into the plane of the paper. \times

The length element $dl = R d\theta$. Since the current direction opposite to the direction of the tangential unit vector \hat{u}_θ , the vector \vec{dl} is

$$\vec{dl} = -R d\theta \hat{u}_\theta$$

Thus the force is

$$\begin{aligned} d\vec{F} &= I \vec{dl} \times \vec{B} \\ &= -IR d\theta \hat{u}_\theta \times (-B \hat{k}) \\ &= IRB \hat{u}_r d\theta \end{aligned}$$

The force on the semicircular arc is obtained by integrating the above. By symmetry, the force on a current element situated at a point Q on the second quadrant, situated symmetrically with P, has the same magnitude and is directed along the radial direction at Q. On resolving the forces along the x and y directions, the components parallel to the x-axis cancel for such symmetric pairs while the component parallel to the y-axis add up. Thus

$$\vec{F} = 2IRB \int_0^{\pi/2} \hat{j} \sin \theta = 2IRB \hat{j}$$

The magnitude of the force is seen to be equal to the product of IB with the diameter of the semi-circle.

Example 3

Force on a conductor of arbitrary shape in a uniform field

For an wire of arbitrary shape, $\vec{F} = I \hat{u}_l \times \vec{B} dl = I \left(\int \hat{u}_l dl \right) \times \vec{B}$

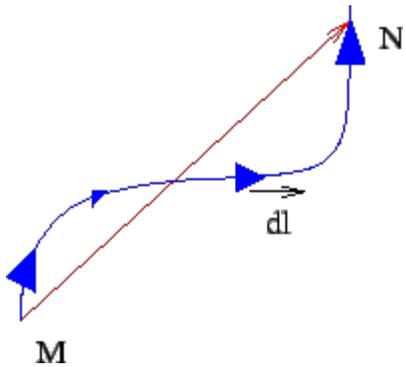
Now, $\int \hat{u}_l dl$ is the sum of vectors along the curve from the initial point M to the final point N. By law of

vector addition, the sum is equal to the vector \vec{MN} connecting the two end points by a straightline segment. Thus

$$\vec{F} = I (\vec{L}' \times \vec{B})$$

where \vec{L}' is the vector connecting the two end points. (Using this, the result of the previous example trivially

follows).

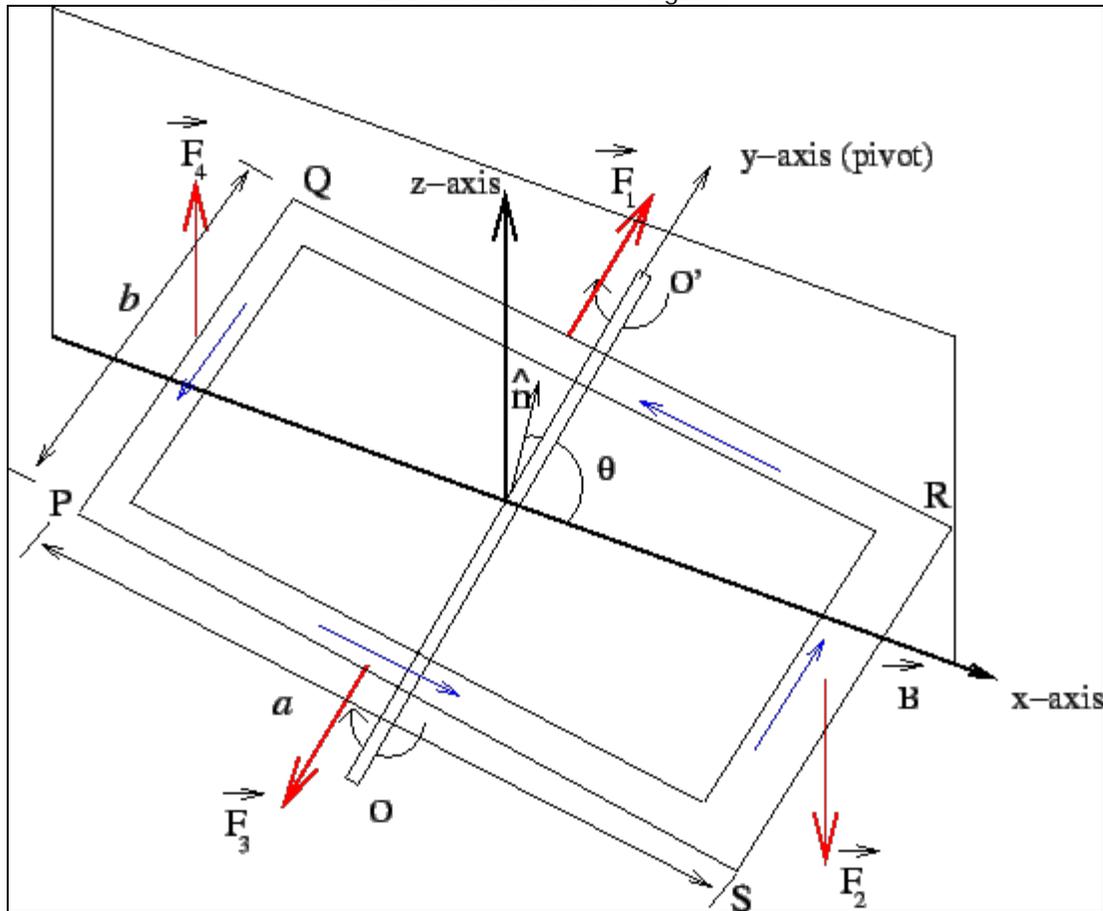


Corollary : The force on a closed current loop in a constant magnetic field is zero.

Torque on a Current Loop in a Uniform Magnetic Field

Though the net force on a closed current loop in a uniform magnetic field is zero, it experiences a torque.

Consider a rectangular current loop PQRS of length a and width b . The loop is in a uniform magnetic field which acts parallel to the x-axis. The loop, which is pivoted about an axis OO' , carries a current I along the direction shown in the figure.



The plane of the loop (i.e. the normal \hat{n} to the loop) makes an angle θ to the direction of the field. We take the shorter sides PQ and RS (as well as the pivot axis OO') to be perpendicular to the field direction, OO' being taken as the y-axis. The longer sides QR and QS make an angle ϕ with the field direction. Since the force on

a current segment is $I \vec{dl} \times \vec{B}$, the force is directed perpendicular to both \vec{B} and to the direction of the current in these segments. The force has a magnitude $IaB \sin \phi$ and is directed oppositely on the two

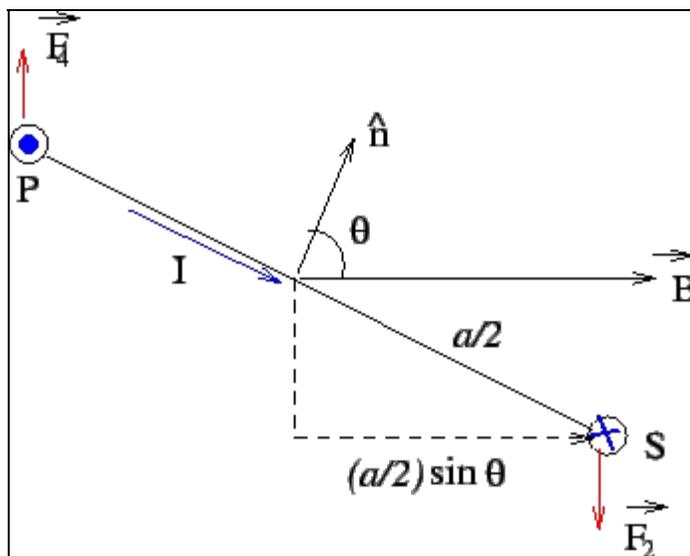
sides. These forces are labelled \vec{F}_1 and \vec{F}_3 in the figure. (The forces are actually distributed along the lengths and the cancellation occurs for the forces acting on symmetrically placed elements on these two arms.) Further, since the lines of action of the forces acting on corresponding elements on these two sides are the same, there is no torque.

The forces acting on the sides PQ and RS (labelled \vec{F}_4 and \vec{F}_2 respectively) are also equal and opposite and have magnitude IbB . However, these forces do not act along the same line. The force on PQ acts parallel to $+\hat{z}$ axis while that on RS acts parallel to $-\hat{z}$ axis. Note that \hat{z} axis is not in the plane of the loop. The situation can be better visualized by redrawing the figure in the plane containing one of the longer sides and \vec{B} .

The current enters the branch SR at S marked with \odot and reenters from QP at P, marked with \otimes . The directions of the forces on the branches SR and QP are shown. The magnitude of the torque about the pivot is

$$\tau = 2(IbB) \times \frac{a \sin \theta}{2} = IabB \sin \theta = IAB \sin \theta$$

where A is the area of the loop.



The result above is independent of the shape of the loop. The following example for a circular loop gives an identical result.

Example 4

A circular loop of radius R carrying a current I is in the x - y plane. The loop is pivoted about its diameter along the y -axis. A uniform magnetic field \vec{B} acts parallel to the x -axis. Calculate the torque on the loop.

Solution :

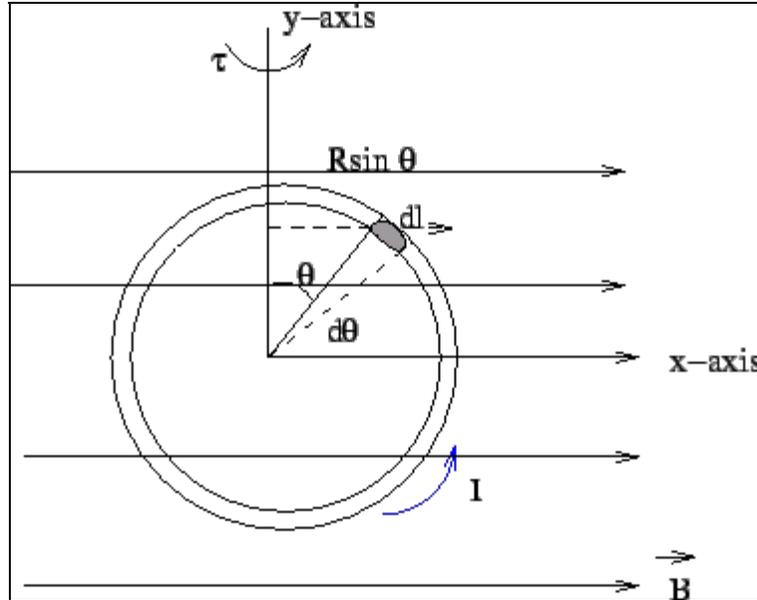
Consider a length element $d\vec{l} = R d\theta$ at an angle θ to the y -axis. The force on the element is $I d\vec{l} \times \vec{B}$ is into the plane of the figure.

The force on the element is

$$d\vec{F} = -IRB \sin \theta d\theta \hat{k}$$

The torque due to this force about the y-axis is

$$\begin{aligned} d\vec{\tau} &= \vec{r} \times d\vec{F} \\ &= (R \sin \theta) \hat{i} \times (-IRB \sin \theta d\theta) \hat{k} \\ &= IR^2 B \sin^2 \theta d\theta \hat{j} \end{aligned}$$



The net force on the loop is zero because of equal and opposite forces acting on diametrically opposite elements of the loop. However, the torques due to all the elements are parallel to the y-direction and add up. The net torque is obtained by integrating over all the elements

$$\vec{\tau} = IR^2 B \int_0^{2\pi} \sin^2 \theta d\theta \hat{j} = IR^2 B \pi \hat{j} = IBA \hat{j}$$

where $A = \pi R^2$ is the area of the loop.

Exercise 1

An coil of 10 turns is in the shape of an equilateral triangle of side 4 cm. The coil is in a magnetic field of strength 0.01 T parallel to the plane of the loop. The current in the coil is 0.1 A. Find the torque acting on the coil.

[Ans. $4\sqrt{3} \times 10^{-6}$ N-m]

Potential Energy of a Magnetic Dipole

A current loop does not experience a net force in a magnetic field. It however, experiences a torque. This is very similar to the behaviour of an electric dipole in an electric field. A current loop, therefore, behaves like a magnetic dipole.

We define the magnetic dipole moment $\vec{\mu}$ of a current loop to be a vector of magnitude

$$|\mu| = IA$$

and direction perpendicular to the plane of the loop (as determined by right hand rule). If the loop has N turns, $|\mu| = NIA$. In a magnetic field, the dipole experiences a torque

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The form of torque suggests that in a magnetic field the dipole tends to align parallel to the field. If the orientation of the dipole is at some angle θ to the field, there must be some potential energy stored in the dipole. This is because, if we wish to bring the dipole from $\theta = 0$ to some arbitrary angle θ , we have to oppose the torque due to the field and do work in the process.

The work done is given by

$$\begin{aligned} W &= \int_0^\theta \tau d\theta \\ &= \int_0^\theta \mu B \sin \theta d\theta \\ &= -\mu B \cos \theta \Big|_0^\theta = (1 - \cos \theta) \mu B \end{aligned}$$

This amount of work is stored as the additional potential energy of the dipole. In analogy with the case of electric dipole in an electric field, the potential energy of the magnetic dipole in a magnetic field is given by

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \theta$$

The energy is lowest when $\vec{\mu}$ and \vec{B} are along the same direction and is maximum when they are anti-parallel.

Recap

In this lecture you have learnt the following

- A conductor carrying current experiences a force in a magnetic field.
- A current loop is equivalent to a magnetic dipole with a magnetic moment equal to the product of the current and the area of the loop.
- A current loop does not experience a force in a magnetic field but experiences a torque which tends to align the dipole moment parallel to the direction of the field.
- A magnetic dipole has a potential energy in a magnetic field.