Module 2 : Electrostatics

Lecture 8 : Properties Of Conductors

Objectives

In this lecture you will learn the following

- Properties of Conductors in equilibrium

- Induced charges in a conductor

- Electrostatic Shielding

- Concept of electric potential and its calculation

- Determination of electric field from a knowledge of potential

Properties of Conductors

A conductor (typically,a metal or ionic conductors like HCl or NaCl dissolved in water) allows free movement of charges,
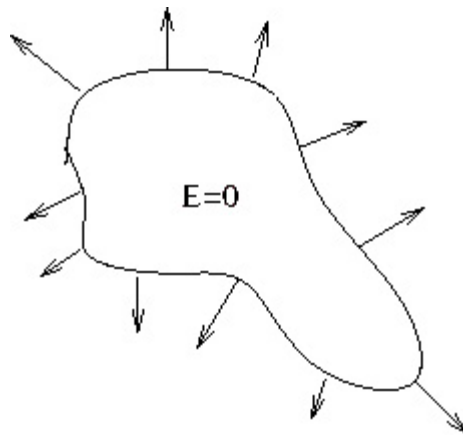
- **The electric field inside a conductor is zero.** In an equilibrium situation, there cannot be an electric field inside a conductor as this would cause charges (electrons or ions) to move around.

- **Charge density inside a conductor is zero.** This follows from Gauss's law

$$\nabla \cdot E = \rho / \epsilon_0$$

As $\vec{E} = 0$, the charge density $\rho = 0$.

(This does not suggest that there is no charge inside, only that the positive and negative charges cancel inside a conductor.)
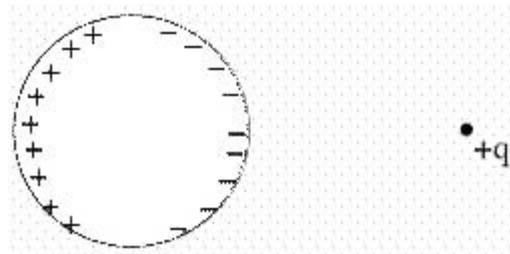
- **Free charges exist only on the surface of a conductor.** Since there is no net charge inside, free charges, if any, have to be on the surface.

- **At the surface of a conductor, the electric field is normal to the surface.** If this were not so, the charges on the surface would move along the surface because of the tangential component of the field, disturbing equilibrium.
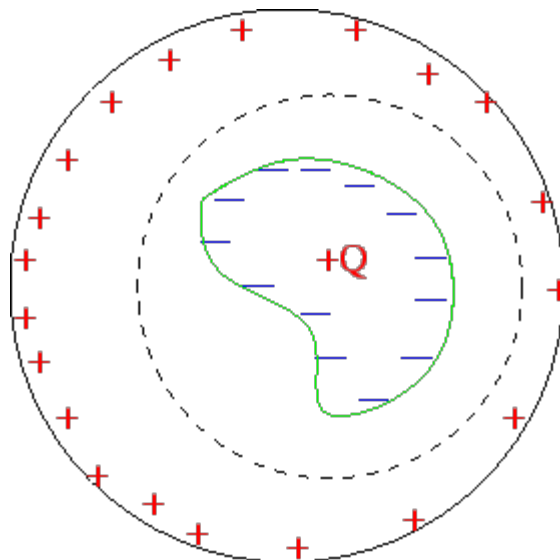
E=0

Induced Charges in a conductor:

The above properties of a conductor influence the behaviour of a conductor placed in an electric field. Consider, for instance, what happens when a charge $+q$ is brought near an uncharged conductor. The conductor is placed in the electric field of the point charge. The field inside the conductor should, however, be zero. his is achieved by a charge separation within the conductor which creates its own electric field which will exactly compensate the field due to the charge $+q$. The separated charges must necessarily reside on the surface.

Another way of looking at what is happening is to think of the free charges in the conductor being attracted towards (or repelled from) the external charge. Thus the surface of the conductor towards the external charge is oppositely charged. To keep the charge neutrality, the surface away from the external charge is similarly charged.

Example 9

A charge Q is located in the cavity inside a conducting shell. In addition, a charge $2Q$ is distributed in the conducting shell. Find the distribution of charge in the shell.

Take a gaussian surface entirely within the conducting shell, completely enclosing the cavity. Everywhere on the gaussian surface $\vec{E} = 0$. The flux and therefore, the charge enclosed is zero within the gaussian surface. As the cavity contains a charge Q, the inner surface of the cavity must have charge $-Q$. As the conductor has distributed charge $2Q$, the charge on the outside surface is $3Q$.

The electric field outside the shell is same as that due to a point charge $+3Q$ located at the centre of the sphere. The reason is that the charge in the cavity is compensated exactly by the induced charges in the surface of the cavity. Thus the effect of the charge Q 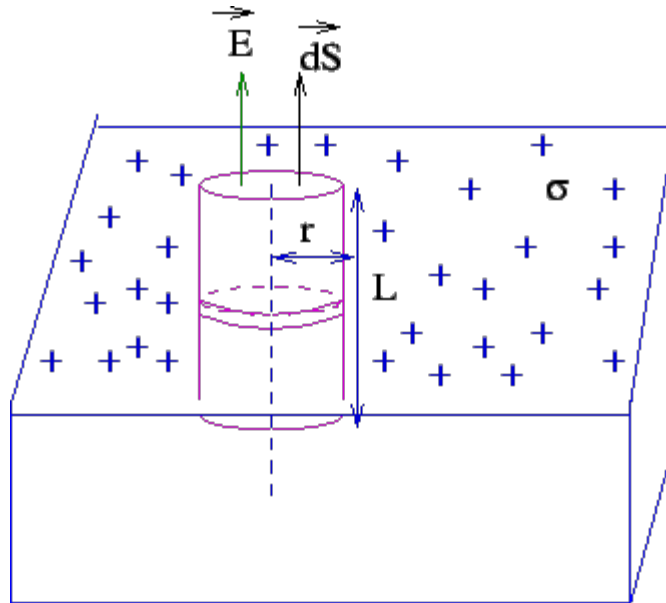is not felt at points outside the sphere. The uniforml distributed charge of $3Q$ over the surface of the sphere gives the same field as that of a point charge located at the centre.

Exercise

Consider the system of conductors shown with two cavities. A charge $+Q$ is kept at the center. (i) Determine the charge distributions on the surfaces marked 1,2,3 and 4, (ii) Is the potential of surface 1 lower, higher or same as that of surface 2 ? (iii) Is the potential of surface 4 lower, higher or same as that of surface 1 ? (Answer : (i) $-Q$ for 1 and 3, (ii) $+Q$ for 2 and 4 (ii) equal (iii) lower.)

Example 10

Calculate the electric field outside a conductor carrying a surface charge density σ.

Take a gaussian pillbox in the shape of a cylinder of height h with $h/2$ inside and $h/2$ outside the conductor.

Lat the cross sectional area be dS normal to the surface. The electric field is normal to the surface. As the field inside is zero and there is no tangential component of the field at the surface, the flux goes out only through the outer cap of the cylider. The charge enclosed is σdS and the flux is $E dS$. The electric field is normal to the surface. applyinng Gauss's law

$$E = \frac{\sigma}{\epsilon_0}\hat{n}$$

Exercise 1

Two parallel, infinite plates made of material of perfect conductor, carry charges   and   . The plates have

Q_1 Q_2

finite thickness. Show that the charge densities on the two adjecent inside surfaces are equal and opposite while that on the two outside

surfaces are equal.

(Hint : Field inside the plates due to four charged surfaces must be zero.)

ELECTROSTATIC POTENTIAL

Electrostatic force is a conservative force, i.e., the work done by the force in moving a test charge from one point to another is independent of the path connecting the two points.

Example 11

Calculate the work done by the electric field due to a charge Q located at the origin to take a unit test charge
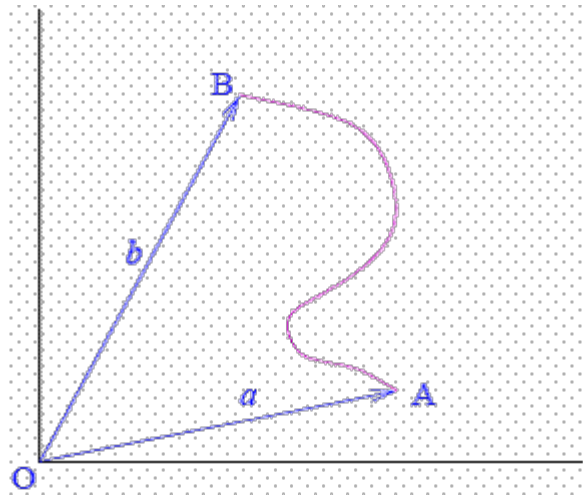
from the point A to the point B, along the path shown.

Solution :

The force on the unit test charge due to the charge Q is, by definition, the electric field due to the charge Q,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$. The work done by the field on the test charge is

$$W = \int_a^b \vec{E} \cdot \vec{dl}$$

where $\vec{dl}$ is along the path connecting A and B.

We use the spherical polar coordinates to express the length element $\vec{dl}$

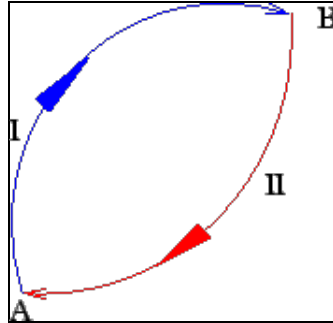$$\vec{dl} = dr\hat{r} + rd\theta\hat{\theta} + r\sin\theta\hat{\phi}$$

As the force is central, only the radial component of $\vec{dl}$ contributes to the integral.

$$W = \frac{Q}{4\pi\epsilon_0}\int_a^b \frac{1}{r^2}dr$$

$$= \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right]$$

which depends only on the end points. **Line integral of the electric field around any closed path is zero.**

Consider the line integral $\int_a^b \vec{E}\cdot\vec{dl}$. Since the integral is independent of the path of integration, we have

$$\int_a^b \vec{E}\cdot\vec{dl} \text{ path } 1 = \int_a^b \vec{E}\cdot\vec{dl} \text{ path } 2$$

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Since $\int_a^b \vec{E}\cdot\vec{dl} = -\int_b^a \vec{E}\cdot\vec{dl}$ along any particular path,

$$\int_a^b \vec{E}\cdot\vec{dl} \text{ alongpath I} + \int_a^b \vec{E}\cdot\vec{dl} \text{ alongpath II} = 0$$

L.H.S. is the line integral of the electric field along the closed loop,

$$\oint \vec{E}\cdot\vec{dl} = 0$$

Since the work done by the electric field is independent of path, we may write

$$\int_A^B \vec{E}\cdot\vec{dl} = U(A,B)$$

where $U(A,B)$ is a scalar function which depends only on the end points A and B of integration. By property of integrals

$$\int_A^B \vec{E}\cdot\vec{dl} = \int_A^C \vec{E}\cdot\vec{dl} + \int_C^B \vec{E}\cdot\vec{dl}$$

where C is any arbitrary point. Thus

$$U(A,B) = U(A,C) + U(C,A)$$

An intutive form for $U(A,B)$ is

$$U(A, B) = \phi(A) - \phi(B)$$

where $\phi(x)$ is a scalar function which depends only on the position x. $\phi(x)$ is called the **electrostatic**

potential of the position x. $\phi(A) - \phi(B)$ is the *potential difference* between the positions A and B. The

absolute value of the potential at a position is meaningless unless we define a reference position at which the potential is zero. Since Coulomb force vanishes only at infinite distances from a source, it is convenient to take infinity to be such a reference position. Thus, the potential at a position x is

$$\phi(\vec{x}) = -\int_{\infty}^{\vec{x}} \vec{E} \cdot \vec{dl}$$

It may be noted that such a reference point is an inappropriate choice for some infinite distribution of charges (e.g. a line charge) where the field does not fall off fast enough to make the integral above vanish.
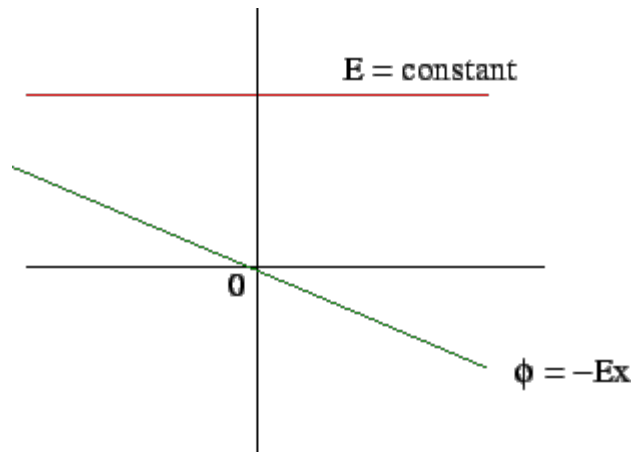
Example 12

Potential due to a uniform electric field $\vec{E}$.

If the electric field is $\vec{E}$ and is directed along the x-direction,

$$\phi(x) = -\int_{x_0}^{x} E dx = -E(x - x_0)$$

choosing the reference point x_0 to be at the origin,

$$\phi(x) = -Ex$$

E = constant

0

φ = −Ex

Example 13

Obtain an expression for the potential at a distance x from a line charge distribution with a linear charge density λ.

Solution

The electric field due to the charge distribution at a point P located at a distance x is
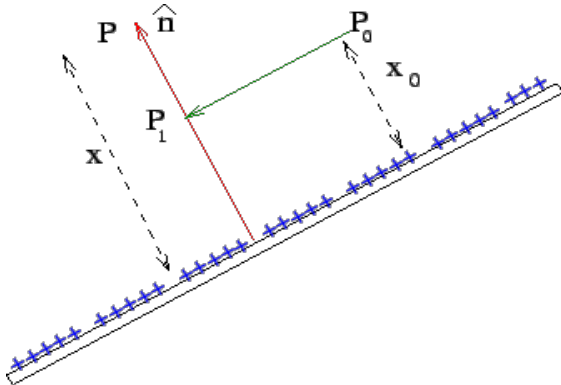
$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0}\frac{1}{x}\hat{n}$$

where $\hat{n}$ is along the perpendicular from the point P to the line charge, as shown (the direction is opposite if line charge density is negative). The potential difference between the point P and a reference point P_0 is

obtained by calculating the value of the integral $\int \vec{E} \cdot \vec{dl}$ from the point P_0 to the point P. As the integral is

independent of path, we calculate it along the path $P_0 \rightarrow P_1 \rightarrow P$, as shown. The contribution to the

integral from the path $P_0 \rightarrow P_1$ is zero as along this path $\vec{dl}$ is perpendicular to $\vec{E}$. Along the path $P_1 P$,

the directions of $\vec{dl}$ and $\vec{E}$ are parallel.

Hence, if the perpendicular distance of P from the line charge is denoted by x

$$
\begin{aligned}
\phi(P) - \phi(P_0) &= -\frac{\lambda}{2\pi\epsilon_0}\int_{x_o}^{x}\frac{dx}{x} \\
&= -\frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{x}{x_o}\right)
\end{aligned}
$$

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One can see that taking x_0 to be infinite will make the integral diverge. In this case, it it is convenient to take

the zero of the potential to be at unit distance from the line ($x = 1$). With this choice the potential at a distance x from the line is

$$\phi(x) = -\frac{\lambda}{2\pi\epsilon_0}\ln x$$

Exercise 1

Find the potential at a height h above a uniformly charged infinite plane having a charge density σ. What is a good reference point for the zero of the potential ?

[Ans. $-\sigma z/2\epsilon_0$, with $\phi(0) = 0$]

Unit of Potential

Since potential is the energy per unit charge, the unit of potential is Joule/Coulomb, which is called a **volt** . The unit of the electric field which we have so far been using as Newton/Coulomb is more commonly referred as volt/meter.

Potential Function satisfies Superposition Principle

Consider a collection of charges $Q_1, Q_2 \ldots$ The electric field at a point due to the distrbution of charges obeys superposition principle. If $\vec{E_i}$ is the electric field at a point P due to the charge Q_i, the net electric field at P is

$$\vec{E} = \sum_i \vec{E_i}$$

The potential ϕ at the point P (with respect to the reference point P $_0$) is

$$\phi(P) = \int_{P_o}^{P} \vec{E} \cdot d\vec{l} = \int_{P_0}^{P} \sum_i \vec{E_i} \cdot d\vec{l} = \sum_i \int_{P_0}^{P} \vec{E_i} \cdot d\vec{l} = \sum_i \phi_i(P)$$

where $\phi_i(P)$ is the potential at P due to the charge Q_i.

Determining Electric Field from a knowledge of Potential

The potential at the position $\vec{x}$ is given by the expression

$$\phi(\vec{x}) = -\int_{\vec{x_0}}^{\vec{x}} \vec{E} \cdot d\vec{l} \qquad\qquad (A)$$

where $\vec{x_0}$ is a reference point such that $\phi(\vec{x_0}) = 0$.

In one dimension,

$$\phi(x) = \int_{x_0}^{x} E(x) dx$$

Differentiate both sides with respect to the upper limit of integration, i.e. x

$$\frac{\partial \phi}{\partial x} = -E(x)$$

In three dimension, we use the fundamental theorem on gradients

$$\phi(\vec{x}) - \phi(\vec{x_0}) = \int_{\vec{x_0}}^{\vec{x}} (\vec{\nabla}\phi) \cdot d\vec{l}$$

which gives

$$\phi(\vec{x}) = \int_{\vec{x_0}}^{\vec{x}} (\vec{\nabla}\phi) \cdot d\vec{l}$$

Comparing the above with eqn. (A) above,

$$\vec{E} = -\nabla\phi$$

In cartesian coordinates,

$$\vec{E} = -\hat{i}\frac{\partial\phi}{\partial x} - \hat{j}\frac{\partial\phi}{\partial y} - \hat{k}\frac{\partial\phi}{\partial z}$$

Example 14

In a certain region of space, the potential function is given by the expression

$$\phi(x, y, z) = x^2 + xy$$

where the potential is measured in volts and the distances in meters. Determine the electric field at the point $(2,1)$.

Solution :

Using

$$\vec{E} = -\nabla\phi = -\hat{i}\frac{\partial\phi}{\partial x} - \hat{j}\frac{\partial\phi}{\partial y} = (-2x - y)\hat{i} - x\hat{j}$$

Substituting for x and y

$$\vec{E} = -5\hat{i} - 2\hat{j}$$

Exercise 2

The potential in a certain region of space is given by the function xy^2z^3 with respect to some reference point.

Find the y-component of the electric field at $(1, -3, 2)$.

(Ans. $-48\hat{j}$)

Exercise 3

Find the potential at a distance h from the mid-point of a charged line of length L carrying a total charge Q.

Using this determine the electric field at the point. (Compare your result for the electric field with the field calculated in Example 2.)
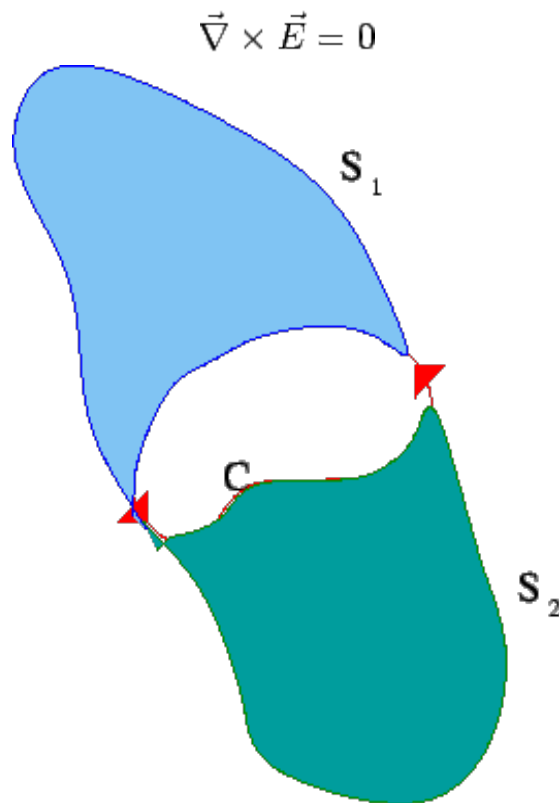
$$\left[\text{Ans, } \frac{1}{4\epsilon_0}\frac{Q}{L}\ln\left(\frac{\frac{L}{2} + \sqrt{\frac{L^2}{4} + h^2}}{-\frac{L}{2} + \sqrt{\frac{L^2}{4} + h^2}}\right)\right]$$

Electric Field is Irrotational, i.e. Curl $\vec{E} = 0$

This follows from Stoke's theorem.

$$\oint \vec{E} \cdot \vec{dl} = \int_S (\nabla \times \vec{E}) \cdot \vec{dS}$$

where the surface integral is over *any* surface bounded by the closed curve. As the surface S is arbitrary (as long as it is bounded by the same curve) , the integrand must vanish. Hence,

$$\vec{\nabla} \times \vec{E} = 0$$

S_1

C

S_2

Two open surface bounded
by the same closed curve C

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Potential and Potential Energy

Potential and Potential energy are different, though they are related. Electric potential at a point is the potential energy of a unit test charge kept at that point.

Potential is the property of the field and is defined at every point, whether or not a charge is located at the point. It is the potential energy that the unit test charge would have if it happens to be located at that point.

The potential energy of a positive charge q increases if it is taken to a region of higher potential. If electric force is the only force acting on the particle, its kinetic energy would decrease by a similar amount.

Let the charge have a velocity v_1 at the position P $_1$ where the electrostatic potential is ϕ_1. If it moves to a position

P $_2$, where the potential is ϕ_2, then, the velocity v_2 of the particle at this point is given by the energy conservation

$$\frac{1}{2}mv_1^2 + q\phi_1 = \frac{1}{2}mv_2^2 + q\phi_2$$

The work done *on* the particle in moving from a potential ϕ_1 to the potential ϕ_2 is given by the ``work-energy theorem''

$$\text{Work Done} = \text{Change in K.E.}$$
$$= \frac{1}{2}m(v_2^2 - v_1^2)$$
$$= q(\phi_1 - \phi_2)$$

Volt,the unit of potential difference,may be interpreted as follows.If a charge of one coulomb moves through a

potential difference such that in the new position the potential is lower by 1 Volt, the kinetic energy of the charge increases by 1Joule.

Electron Volt

In atomic and nuclear physics, a commonly used unit of energy is *electron volt*. An electron volt is the change in the kinetic energy of *an electron* when it is taken through a potential difference of one volt. Thus,
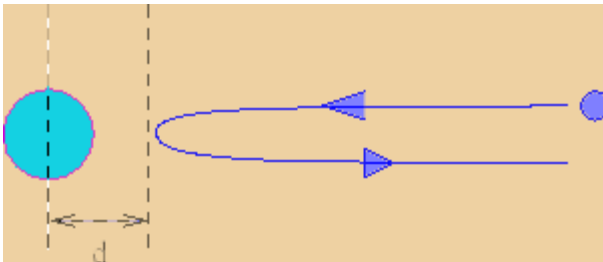
$$
\begin{aligned}
1\text{eV} &= \mid \text{charge of an electron} \mid \times 1\text{volt} \\
&= 1.6 \times 10^{-19}\text{Coulomb} \times 1 \text{ volt} \\
&= 1.6 \times 10^{-19} \text{ Joule}
\end{aligned}
$$

Example 15

An α- particle with a kinetic energy of 1 MeV is projected towards a stationary nucleus with a charge $75 \mid e \mid$

. Neglecting the motion of the nucleus, determine the distance of closest approach of the α- particle.

Solution :

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Initial energy of α-particle = 1 Mev = $10^6 \times (1.6 \times 10^{-19}) = 1.6 \times 10^{-12}$ eV.

At the distance of closes approach, the velocity (and the kinetic energy) of the α- particle is zero. Hence, all its kinetic energy has been converted into potential energy. The potential energy (reference at infinity) at a distance d is

$$
\frac{1}{4\pi\epsilon_0} \frac{75 \times 2 \times (1.6 \times 10^{-19})^2}{d} = \frac{3.456 \times 10^{-26}}{d}
$$

Equating this to the initial kinetic energy, $d = 2.16 \times 10^{-14}$ m.

Recap
In this lecture you have learnt the following

- Conductors are those in which there are free carriers which conduct electricity. In an equilibrium situation, electric field cannot exist inside a conductor.

- Electric field may exist on the surface of a conductor which must then be directed normal to the surface of the conductor.

- When a conductor is placed in an electric field, charges may be induced on the surface of a conductor.

- Electric field being a conservative field of force, the work done only depends on the end points and not on the path connecting them. This enables us to define a potential for the field whose negative gradient gives the electric field.

- As the electric potential is a scalar, it is convenient to compute the potential for a charge distribution and use this to calculate the electric field using superposition principle.