Module 6 : PHYSICS OF SEMICONDUCTOR DEVICES

Lecture 34 : Intrinsic Semiconductors

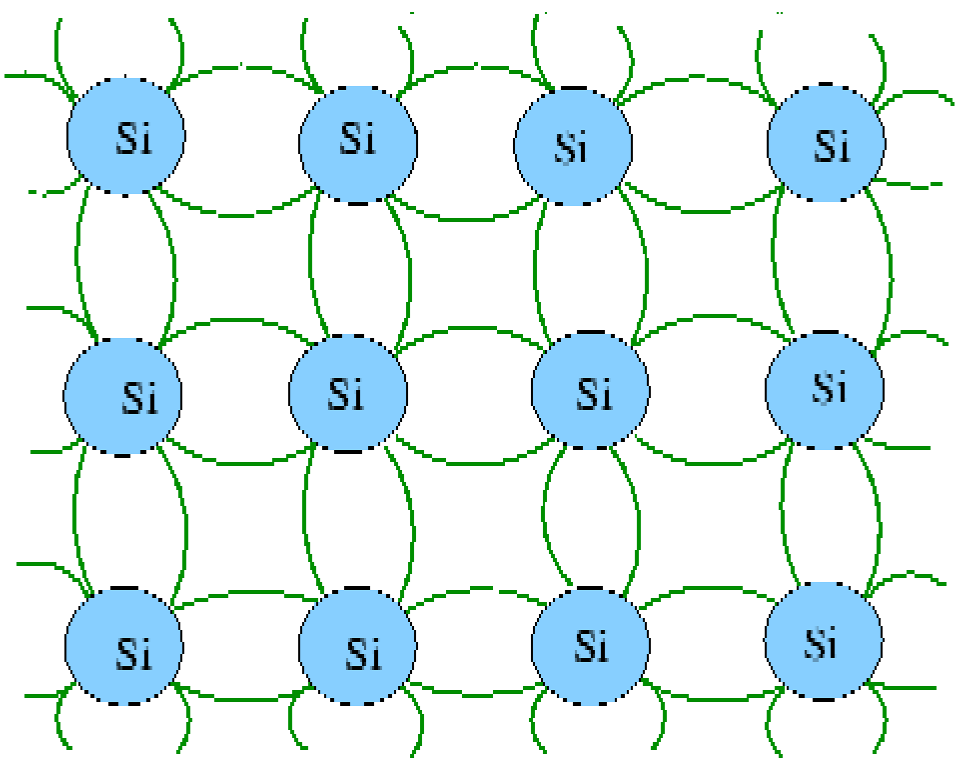
Objectives

In this course you will learn the following

- Intrinsic and extrinsic semiconductors.
- Fermi level in a semiconductor.
- p-type and n-type semiconductors.
- Compensated semiconductors.
- Charge neutrality and law of mass action.

Intrinsic Semiconductors

An intrinsic semiconductor is a pure semiconductor, i.e., a sample without any impurity. At absolute zero it is essentially an insulator, though with a much smaller band gap. However, at any finite temperature there are some charge carriers are thermally excited, contributing to conductivity. Semiconductors such as silicon and germanium, which belong to Group IV of the periodic table are covalently bonded with each atom of Si(or Ge) sharing an electron with four neighbours of the same specis. A bond picture of silicon is shown in the figure where a silicon atom and its neighbour share a pair of electrons in covalent bonding.



Gallium belongs to Group III and bonds with arsenic which belongs to Group V to give a III-V semiconductor. In GaAs, the bonding is partly covalent and partly ionic. Other commonly known III-V semiconductors are GaN, GaP, InSb etc. Like the III-V compounds, Group II elements combine with Group VI elements to give semiconductors like CdTe, CdS, ZnS etc. Several industrially useful semiconductors are alloys such as Al $_x$GA $_{1-x}$As.

The number of carriers in a band at finite temperatures is given by $\int n(E)f(E)dE$, where $N_c = \frac{1}{4}\left(\frac{2m_e kT}{\pi\hbar^2}\right)^{3/2}$ is the density of state and $f(E)$ is the Fermi function which gives the thermal probability. If $E - E_F \gg kT$, we may ignore the term 1 in the denominator of the Fermi function and approximate it as

$$f_e(E) \simeq e^{(E-E_F)/kT}$$

Using this the density of electrons in the conduction band (n) may be written as follows.

$$n = \int_{E_c}^{\infty} e^{(E_F-E)/kT} \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} (E-E_c)^{1/2} dE$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} \int_{E_c}^{\infty} e^{(E_F-E)/kT} (E-E_c)^{1/2} dE$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e}{\hbar^2}\right)^{3/2} \int_{0}^{\infty} e^{(E_F-xkT-E_c)/kT} (kT)^{3/2} x^{1/2} dx$$

$$= \frac{1}{2\pi^2}\left(\frac{2m_e kT}{\hbar^2}\right)^{3/2} e^{(E_F-E_c)/kT} \int_{0}^{\infty} e^{-x} x^{1/2} dx$$

where we have substituted

$$x = \frac{E-E_c}{kT}$$

The integral $\int_0^\infty e^{-x}x^{1/2}dx$ is a gamma function $\Gamma(3/2)$ whose value is $\sqrt{\pi}/2$. Substituting this value, we get for the density of electrons in the conduction band

$$\boxed{n = \frac{1}{4}\left(\frac{2m_e kT}{\pi\hbar^2}\right)^{3/2} e^{(E_F-E_c)/kT} = N_c e^{(E_F-E_c)/kT}} (A)$$

where

$$N_c = \frac{1}{4}\left(\frac{2m_e kT}{\pi\hbar^2}\right)^{3/2}$$

One can in a similar fashion one can calculate the number density of holes, p, by evaluating the expression

$$p = \int_{E_v}^{-\infty} n_h(E) f_h(E) dE$$

where $f_h(E) = 1 - f_e(E)$ is the Fermi function for the occupancy of holes which is the same as the probability that an electron state at energy E is unoccupied. For $E_F - E \gg kT$, the density of holes is given by

$$\boxed{p = \frac{1}{4}\left(\frac{2m_h kT}{\pi\hbar^2}\right)^{3/2} e^{(E_v - E_F)/kT} = N_v e^{(E_v - E_F)/kT}} (B)$$

where

$$N_v = \frac{1}{4}\left(\frac{2m_h kT}{\pi\hbar^2}\right)^{3/2}$$

The following table gives generally accepted values of some of the quantities associated with the three most common semiconductors at room temperature (300 K).

	Δ	m_e/m_0	m_h/m_0	N_c	N_v	n_i
	in eV			/m ^{3}	/m ^{3}	/m ^{3}
Si	1.12	1.08	0.56	2.8×10^{25}	1.0×10^{25}	1.5×10^{16}
Ge	0.66	0.55	0.37	1.0×10^{25}	6.0×10^{24}	2.4×10^{19}
GaAs	1.4	0.04	0.48	4.7×10^{24}	7.0×10^{24}	1.8×10^{12}

Exercise 1

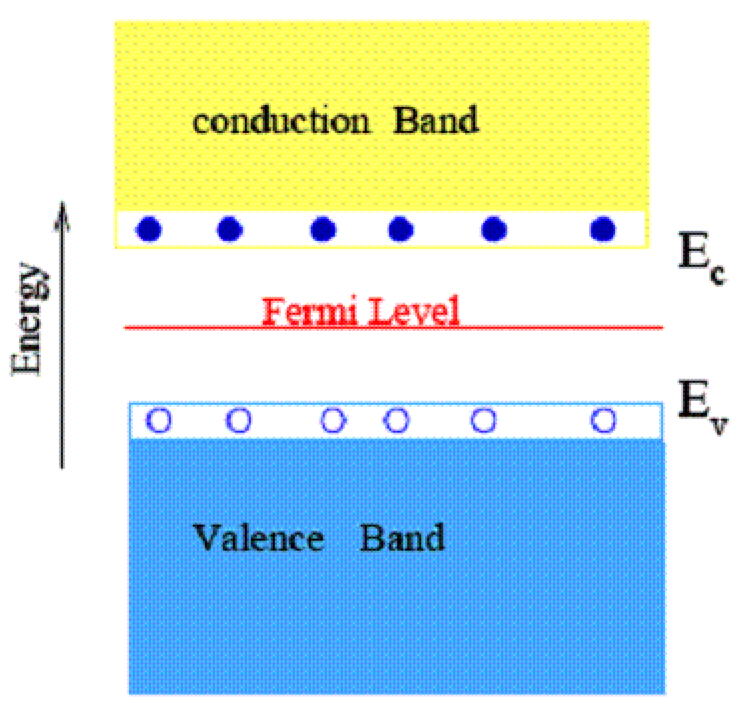
Derive expression (B).

For an intrinsic semiconductor the number of electrons in the conduction band is equal to the number of holes in the valence band since a hole is left in the valence band only when an electron makes a transition to the conduction band,

$$n = p$$

Using this and assuming that the effective masses of the electrons and holes are the same one gets

$$e^{(E_F - E_c)/kT} = e^{(E_v - E_F)/kT}$$



giving

$$\boxed{E_F = \frac{E_c + E_v}{2}} (C)$$

i.e. **the Fermi level lies in the middle of the forbidden gap** . Note that there is no contradiction with the fact that no state exists in the gap as E_F is only an energy level and not a state.

By substituting the above expression for Fermi energy in (A) or (B), we obtain an expression for the number density of electrons or holes ($n = p = n_i$)

$$\boxed{n_i = \frac{1}{4}\left(\frac{2kT}{\pi\hbar^2}\right)^{3/2} (m_e m_h)^{3/4} e^{-\Delta/2kT}} (D)$$

where Δ is the width of the gap.

Exercise 2

Derive the expression (D).

Exercise 3

For a two band model of silicon, the band gap is 1.11 eV. Taking the effective masses of electrons and holes as $m_e = 1.08 m_0$ and $m_h = 0.81 m_0$, calculate the intrinsic carrier concentration in silicon at 300 K.

(Ans. 1.2×10^{16} m $^{-3}$.)

Exercise 4

Show that, if the effective masses of electrons and holes are not equal, the position of the Fermi energy for an intrinsic semiconductor is given by

$$E_F = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln \frac{m_e}{m_h}$$

Current in an intrinsic semiconductor

For semiconductors both electrons and holes contribute to electric current. Because of their opposite charge, their contribution to the current add up. For an intrinsic semiconductor with a single valence band and a conduction band, the current density is given by

$$J = q(nv_e + pv_h)$$

where n and v_e are respectively the electron density and speed while p and v_h are the hole density and speed. Using $v_e = \mu_e \mathcal{E}$ and $v_h = \mu_h \mathcal{E}$ and the fact that $n = p = n_i$, we have

$$J = q\mathcal{E} n_i(\mu_e + \mu_h)$$

which gives the conductivity as

$$\sigma = qn_i(\mu_e + \mu_h)$$

Example 7

Estimate the electrical conductivity of intrinsic silicon at 300 K, given that the electron and hole mobilities are $\mu_e = 0.15$ m ^{2}/V-s and $\mu_h = 0.05$ m ^{2}/V-s.

Solution

The conductivity arises due to both electrons and holes

$$\sigma = qn_i(\mu_e + \mu_h)$$

The intrinsic carrier concentration n was calculated to be $1.2 \times 10^{16}/\mathrm{m}^3$ at 300 K. Thus

$$\sigma = 1.6 \times 10^{-19} \times 1.2 \times 10^{16} \times 0.2 = 3.84 \times 10^{-4}\ \mathrm{ohm} - \mathrm{m}^{-1}$$

Exercise 5

A sample of an intrinsic semiconductor has a band gap of 0.7 eV, assumed independent of temperature.

Taking $\mu_h = 0.5\mu_e$ and $m_h = 2m_e$, find the relationship between the conductivity at 200 K and 300 K.

(Ans. ratio of conductivity = 2014.6, $E_F(300K) - E_F(200K) = 4.33 \times 10^{-3}$ eV)

Extrinsic Semiconductors

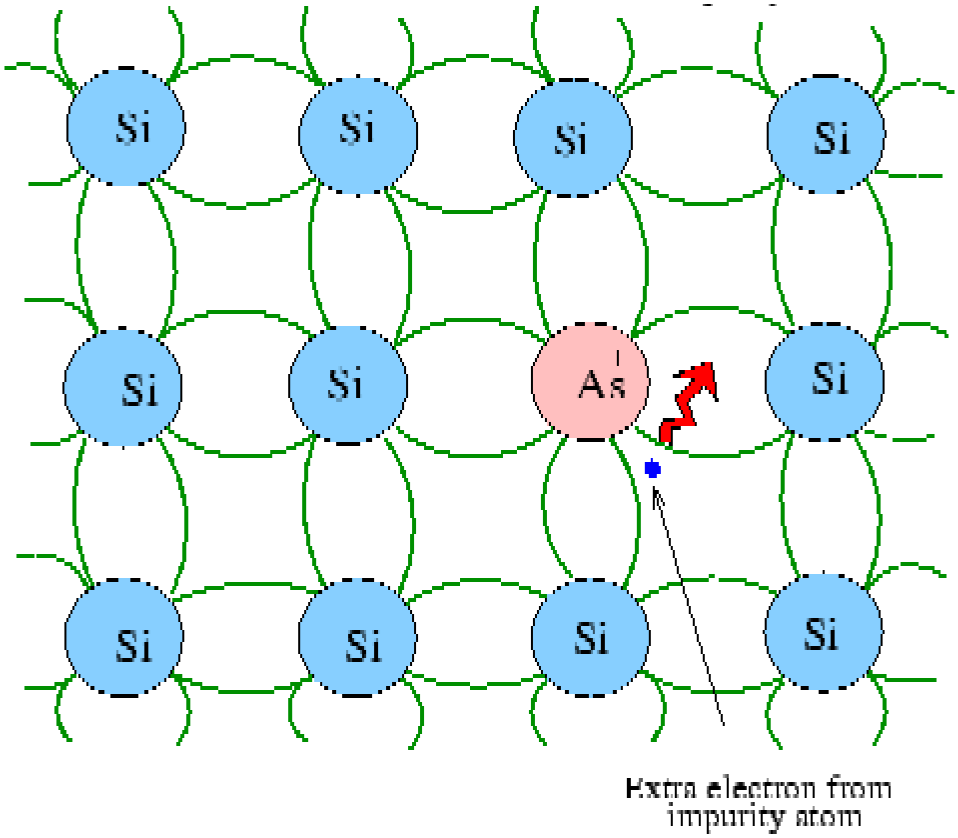
An extrinsic semiconductor is formed by adding impurities, called **dopants** to an intrinsic semiconductor to modify the former's electrical properties. There are two types of such impurities - those which provide electrons as majority carriers are known as **n-type** and those which provide holes as majority carriers are known as **p-type** .

Using this and assuming that the effective masses of the electrons and holes are the same one gets

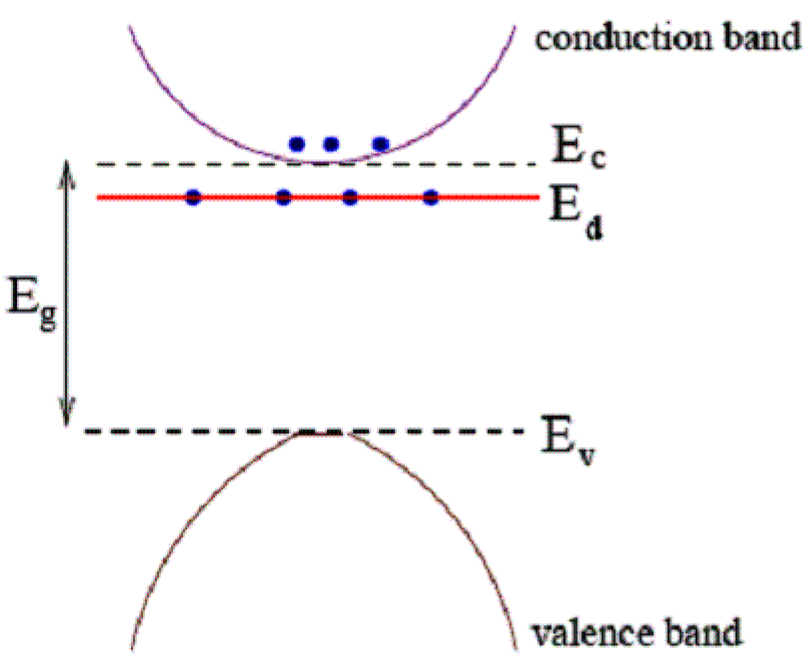
n- type Semiconductors

Consider a matrix of silicon where the atoms are covalently bonded.

If we add a pentavalent atom (As, P etc.) as an impurity, the dopant atom replaces a silicon atom substitutionally. As the dopant has five electrons, only four of these can be used in forming covalent bonds while the fifth electron is loosely bonded to the parent atom. This electron can become detached from the dopant atom by absorbing thermal energy.



In the band picture, the energy level of the additional electron lies close to the bottom of the conduction band. Such an energy level E_d is called a *donor level* as it can donate an electron to the empty conduction band by thermal excitation. We may see this by assuming that the fifth electron of the donor is orbiting around a hydrogen-like nucleus consisting of the core of the donor atom with the following modifications made into the formula for the energy of an electron in the hydrogen atom.



- permittivity of the free space ϵ_0 is replaced by ϵ, the permittivity of the medium (silicon).

- free electron mass being replaced by the effective mass of the donor electron.

Recalling that the energy of an electron in the the hydrogen atom is given by

$$-\frac{me^4}{8\epsilon_0^2 h^2 n^2} = -\frac{13.6}{n^2}\ \mathrm{eV}$$

where $n = 1, 2, 3, \ldots$, we need to replace m by m^* and e^2/ϵ_0 by $e^2/\kappa\epsilon_0$, where κ is the relative dielectric constant of the medium. Using $\kappa = 12$ for Si and $m^* \simeq m$, the free electron mass, the ionization energy of the electron bound to the donor atom is $13.6/(12)^2 \simeq 0.094$ eV, if the electron is in the ground state. Thus the donor energy level lies close to the bottom of the conduction band. In case of semiconductors, the donor **ionization energy** is defined as the energy required to elevate the donor electron to the conduction band.

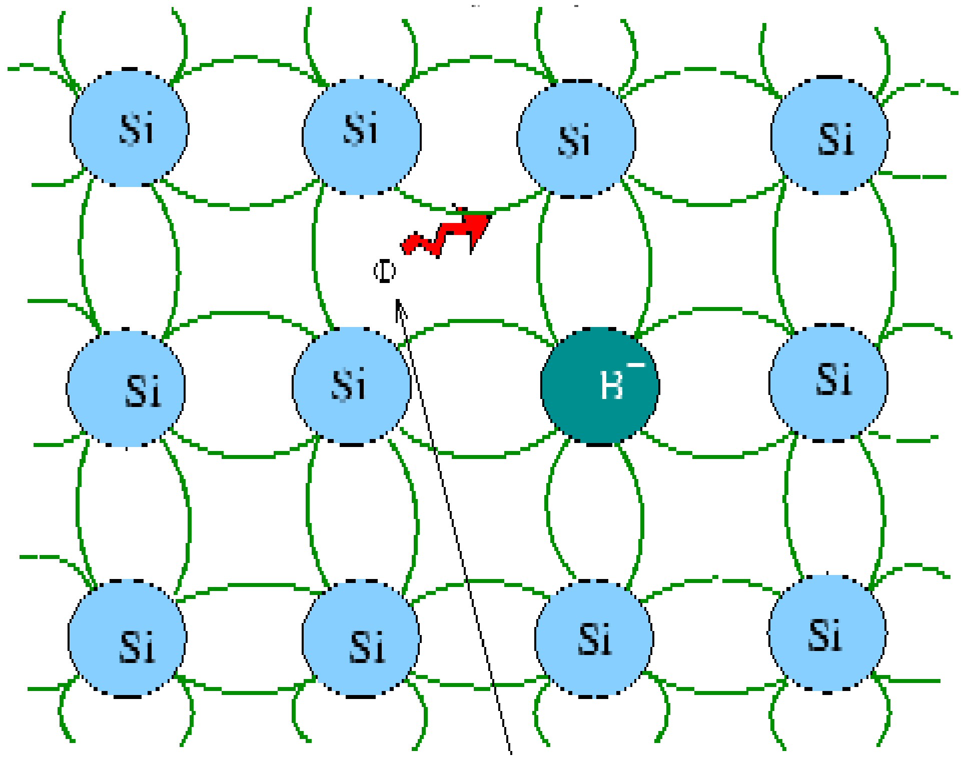
Exercise 6

Calculate the ionization energy of a donor impurity in Ge. The effective mass of electrons is $0.12m_0$ and the dielectric constant is 16.

(6.4 meV)

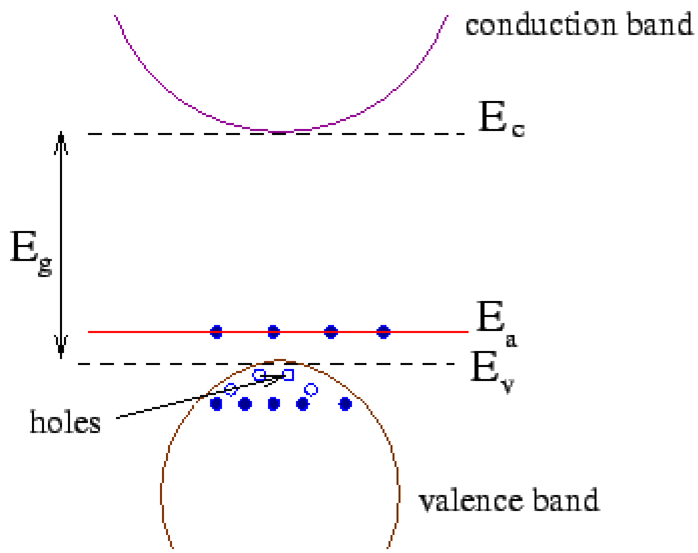
p- type Semiconductors

If the Si matrix is doped with Group III impurities like boron or aluminium, it cannot provide electrons to complete the covalent bonds. However, the impurity readily accepts an electron from a nearby Si-Si bond to complete its own bonding scheme. A hole is thereby created which can freely propagate in the lattice.



deficiency in a bond
(a hole)

In the band picture, the acceptor energy level E_a lies close to the top of the valence band. Electrons near the top of the valence band can be thermally excited to the acceptor level leaving holes near the top of the valence band. In these semiconductors, known as P-type semiconductors, the primary current is due to majority carriers which are holes.



Example 8

In an n-type semiconductor 25% of the donor atoms are ionized at 300 K. Determine the location of the Fermi level with respect to the donor level.

Solution

As 25% of donor atoms are ionized, the occupation probability of donor level is 0.75. Thus

$$F(E_D) = \frac{1}{1+\exp(\frac{E_D-E_F}{kT})} = 0.75$$

Solving, $E_D - E_F = -0.028$ eV.

Exercise 7

In a p-type semiconductor 40% of atoms are ionized at 300 K. Find the location of the Fermi level with respect to the acceptor level.

($E_a - E_F = 0.016$ eV)

A **compensated semiconductor** contains both donor and acceptor impurities. The compensation is said to be complete if $N_d = N_a$ in which case the semiconductor behaves like an intrinsic semiconductor.

Example 9

Solution

Given

$$\frac{1}{1+e^{(E_a-E_F)}/kT} = 0.05$$

we get $\exp((E_a - E_F)/kT) = 0.053$. Using $kT = 0.026$ eV corresponding to room temperature, we get $E_F - E_a = 0.076$ eV. Rewriting this as

$$E_F - E_d + (E_d - E_a) = E_F - E_d + 0.1 = 0.076$$

which gives $E_d - E_F = 0.0236$. The occupation probability of the donor level is

$$\frac{1}{1+e^{(E_d-E_F)}/kT} = 0.29$$

Thus 71% of donor atoms are ionized. The Fermi level is situated 0.0236 eV below the donor level.

Condition of Charge Neutrality

In the absence of an electric field, a bulk material is charge neutral. Let

$n =$ number density of electrons

$p =$ number density of holes

For an intrinsic semiconductor $n = p = n_i$ so that the number density of electrons may be written as

$$n_i = \sqrt{np}$$

Let the density of donor atoms be denoted by N_d and that of acceptor atoms by N_a. If the corresponding densities of ionized donors and acceptors are N_d^+ and N_a^- respectively, the charge neutrality condition for the bulk sample becomes

$$q(N_d^+ + p) = q(N_a^- + n)$$

If all the donors and acceptors are ionized, then,

$$q(N_d + p) = q(N_a + n)$$

Using $np = n_i^2$, we get

$$n = (N_d - N_a) + p$$
$$= (N_d - N_a) + \frac{n_i^2}{n}$$

Thus we get a quadratic equation for the electron density

$$n^2 - (N_d - N_a)n - n_i^2 = 0$$

with solution

$$\boxed{n = \frac{N_d - N_a}{2} + \sqrt{\frac{(N_d - N_a)^2 + 4n_i^2}{4}}}$$

Example 10

Pure germanium has a band gap of 0.67 eV. It is doped with 3×10^{21} per m ^{3} of donor atoms. Find the densities of electrons and holes at 300 K. (effective masses $m_e = 0.55m_0,\ m_h = 0.37m_0$)

Solution

For Ge, the intrinsic concentration is

$$n_i = \sqrt{N_c N_v} e^{-\Delta/2kT}$$

Substituting given numerical values, $n_i = 2.4 \times 10^{19}\ /\mathrm{m}^3$. The density of donor atoms is $N_d = 3 \times 10^{21}$ /m ^{3}. Thus the electron density n is given by

$$n = \frac{N_d}{2} + \sqrt{\frac{(N_d)^2 + 4n_i^2}{4}} \simeq N_d \quad \mathrm{as} N_d \gg n_i$$

Thus $n = 3 \times 10^{21}\ /m^3$. Using $n_i^2 = np$, we get, for the density of holes $p = 1.92 \times 10^{17}\ /\mathrm{m}^3$.

Exercise 8

A sample of Ge at 300 K is doped with $3 \times 10^{21}\ /\mathrm{m}^3$ of donor atoms and $4 \times 10^{21}\ /\mathrm{m}^3$ acceptor atoms. Find the densities of electrons and holes at 300 K.

(Ans. $n = 5.76 \times 10^{17}\ /\mathrm{m}^3$, $p = 10^{21}\ /\mathrm{m}^3$)

Fermi Energy

Let be the Fermi level for a n-type semiconductor. The electron density is given by

E_F^n

$$n = N_c e^{(E_F^n - E_c)/kT}$$

where

$$N_c = \frac{1}{4}\left(\frac{2m_n kT}{\pi\hbar^2}\right)^{3/2}$$

We may rewrite the above equation as follows. Denoting the **intrinsic Fermi energy** as E_F^i,

$$\begin{aligned} n &= = N_c e^{[(E_F^i - E_c)+(E_F^n - E_F^i)]/kT} \\ &= n_i e^{(E_F^n - E_F^i)/kT} \end{aligned}$$

where n_i is the intrinsic electron density. In a similar way one can show that for p type impurities, the concentration of holes is given by

$$p = p_i e^{(E_F^i - E_F^p)/kT}$$

where $p_i\ (= n_i)$ is intrinsic hole density. Thus

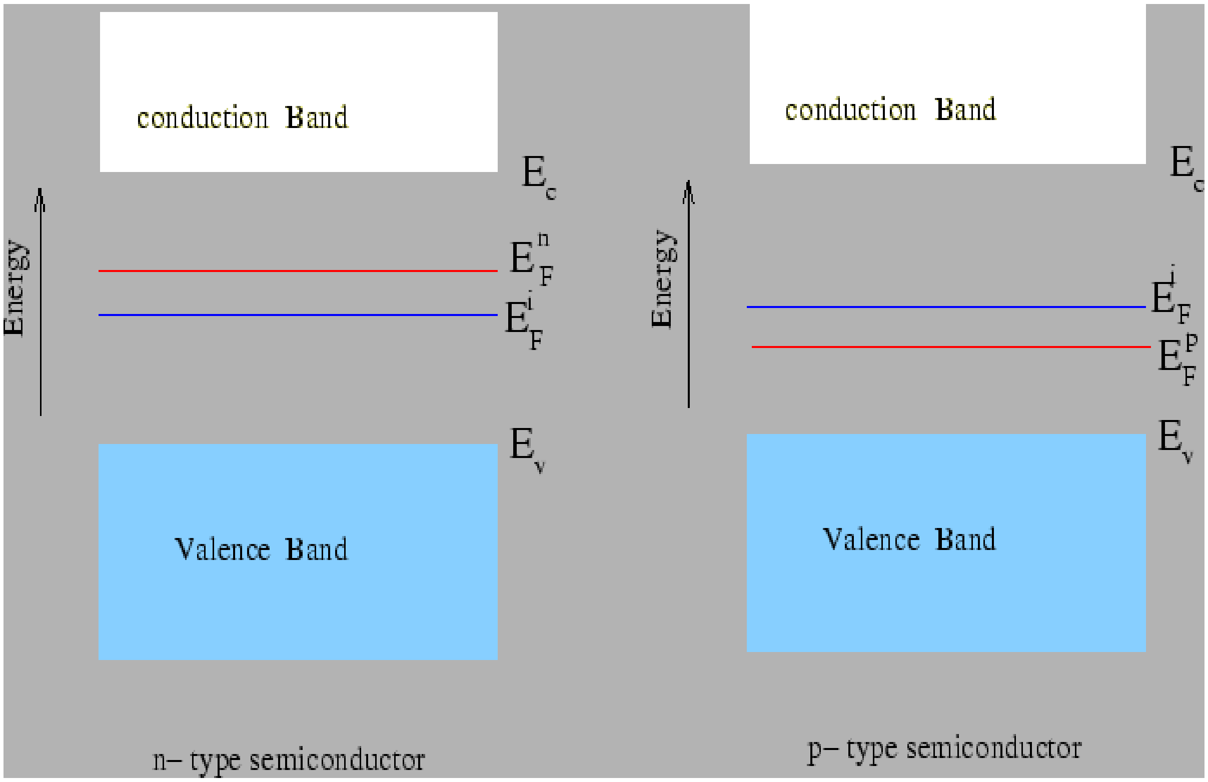
$$\boxed{np = n_i p_i = n_i^2}$$

This relationship is known as the **Law of Mass Action** .

Taking the logarithm of the equations for n and p, the shift in the Fermi energies due to doping for n-type and p-type semiconductors are given by

$$E_F^n - E_F^i = kT \ln \frac{n}{n_i}$$

$$E_F^i - E_F^p = kT \ln \frac{p}{p_i}$$



Since $n > n_i$ for an n-type material, E_F lies above the intrinsic Fermi level, whereas, for a p-type material, $p > p_i$ and the Fermi level lies below the intrinsic Fermi level.

Example 11

Silicon crystal is doped with 5×10^{20} atoms per m ^{3}. The donor level is 0.05 eV from the edge of the conduction band. Taking the band gap to be 1.12 eV, calculate the position of the Fermi level at 200 K.

Solution

The intrinsic carrier concentration can be obtained from the known carrier concentration in Si at 300 K. As the carrier concentration at 300 K is $1.5 \times 10^{16}\ /\mathrm{m}^3$, the carrier concentration at 200 K is $(200/300)^{3/2} \times 1.5 \times 10^{16} = 0.82 \times 10^{16}\ /\mathrm{m}^3$. As the doping concentration is much larger than n_i, we can take $n \simeq N_d = 5 \times 10^{20}\ /\mathrm{m}^3$. Thus

$$E_F^n - E_F^i = kT \ln \frac{n}{n_i} = 0.183\ \text{eV}$$

Exercise 9

Germanium has ionized acceptor density of 4×10^{21} /m ^{3} and donor density of 6×10^{21} /m ^{3}. Taking the band gap to be 0.67 eV, calculate the equilibrium density of majority and minority carriers at 450 K and also the Fermi energy. [Hint : Using the intrinsic concentration at 300 K, find n_i at 450 K and use the expression for n.]

(Ans. $n = 2.02 \times 10^{21}\ /\mathrm{m}^3$ $p = 9.62 \times 10^{17}\ /\mathrm{m}^3$ $E_F^n - E_F^i = 0.143$ eV)

Recap

In this course you have learnt the following

- At very low temperatures, semiconductors are like insulators as there are no free carriers in their conduction band. As temperature is raised, thermal excitation of carriers takes place to the conduction band leading to non-zero conductivity. Such semiconductors are called intrinsic.
- The band gap in semiconductors is much smaller than that in insulators, which facilitates thermal excitation in the former.
- For an intrinsic semiconductor, the Fermi level lies in the middle of the forbidden gap.
- Electrical properties of semiconductors can be modified by introducing impurities (known as dopants). Dopants can have excess valence electrons per atom over that of the host material. Such semiconductors are called n-type semiconductors for which the majority carriers are electrons.
- In a p-type semiconductor, the dopant atoms have a deficit of valence electrons and in such cases the majority charge carriers are holes.
- A compensated semiconductor is one in which both types of dopants exist.