Module 3 : MAGNETIC FIELD

Lecture 20 : Magnetism in Matter

Objectives

In this lecture you will learn the following

- Study magnetic properties of matter.
- Express Ampere's law in the presence of magnetic matter.
- Define magnetization and H-vector.
- Understand displacement current.
- Assemble all the Maxwell's equations together.
- Study properties and propagation of electromagnetic waves in vacuum.

Magnetism in Matter

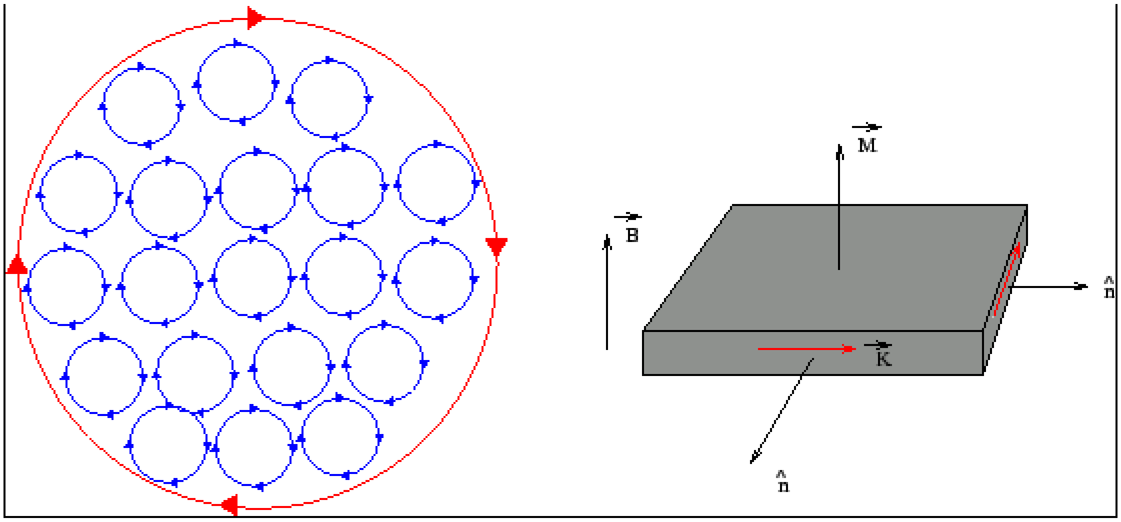
In our discussion on electrostatics, we have seen that in the presence of an electric field, a dielectric gets polarized, leading to bound charges. The polarization vector is, in general, in the direction of the applied electric field.

A similar phenomenon occurs when a material medium is subjected to an external magnetic field. However, unlike the behaviour of dielectrics in electric field, different types of material behave in different ways when an external magnetic field is applied.

We have seen that the source of magnetic field is electric current. The circulating electrons in an atom, being tiny current loops, constitute a magnetic dipole with a magnetic moment whose direction depends on the direction in which the electron is moving. An atom as a whole, may or may not have a net magnetic moment depending on the way the moments due to different elecronic orbits add up. (The situation gets further complicated because of electron spin, which is a purely quantum concept, that provides an intrinsic magnetic moment to an electron.)

In the absence of a magnetic field, the atomic moments in a material are randomly oriented and consequently the net magnetic moment of the material is zero. However, in the presence of a magnetic field, the substance may acquire a net magnetic moment either in the direction of the applied field or in a direction opposite to it. The former class of material is known as **paramagnetic** material while the latter is called **diamagnetic.**

When a paramagenetic material is subjected to a magnetic field, the atomic moments are oriented in the direction of the magnetic field. However, the current directions in adjacent current loops being in opposite directions, there is no current within the volume of the material. However, the currents do not cancel at the surface of the substance on which there is a net current which gives rise to its own magnetic field.



We define magnetization $\vec{M}$ of a sample as the net magnetic moment per unit volume. The unit of magnetization is Ampere/meter. From the figure it can be seen that the surface current flows along the side faces. Denoting the normal to the face by $\hat{n}$, the surface current density $\vec{K}$ (in A/m) is given by

$$\vec{K} = \vec{M} \times \hat{n}$$

The magnetization current is called **bound current** because the electron is not free to move through the material as they would in a conductor, but are attached to a particular atom or molecule.

If, however, the magnetization is not uniform within the sample, the internal currents do not cancel and a magnetization current exists even in the bulk. It can be shown that the bound current density $\vec{J}_b$ is given by

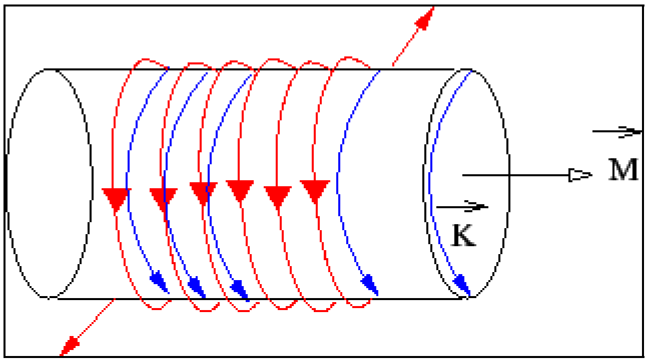
$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

Ampere's Law in Presence of Magnetization

Since magnetization of a material produces bound current, it would modify Ampere's law for magnetic field. Consider a solenoid wound around a hollow cylinder with n turns per unit length carrying a current I. The magnetic field in the solenoid is uniform and is given by $B = \mu_0 nI$. If now, a magnetic material is inserted in the hollow of the cylinder, the material gets magnetized with a magnetization $\vec{M}$. The surface current density $\vec{K}$ has a magnitude M and has the dimension of current per unit length.

A unit length of the solenoid has an effective current given by the sum of free current nI and the magnetization current M. Ampere's law would then give the magnetic field in the solenoid as

$$B = \mu_0(nI + M)$$



In analogy with the way we introduced displacement vector $\vec{D}$ to account for the effect of a dielectric in electrostatics, we introduce a new field, which we will call **H-field**

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

Dimensions of $\vec{H}$ is the same as that of magnetization $\vec{M}$, i.e. those of A/m. Using $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$,

and splitting $I = I_f + I_m$, where I_f denotes the free current which arises due to mobile charges and

I_m is the magnetization (bound) current, we get

$$\oint \vec{H} \cdot d\vec{l} = I_f$$

Note that just as the displacement field in a dielectric is determined by free charges, the H-field is determined only by free currents.

Define **magnetic susceptibility** χ through the relation

$$\vec{M} = \chi \vec{H}$$

Using this, we get

$$\vec{B} = \mu_0(1 + \chi)\vec{H}$$

The **permeability** of the medium is defined as

$$\mu = \mu_0(1 + \chi)$$

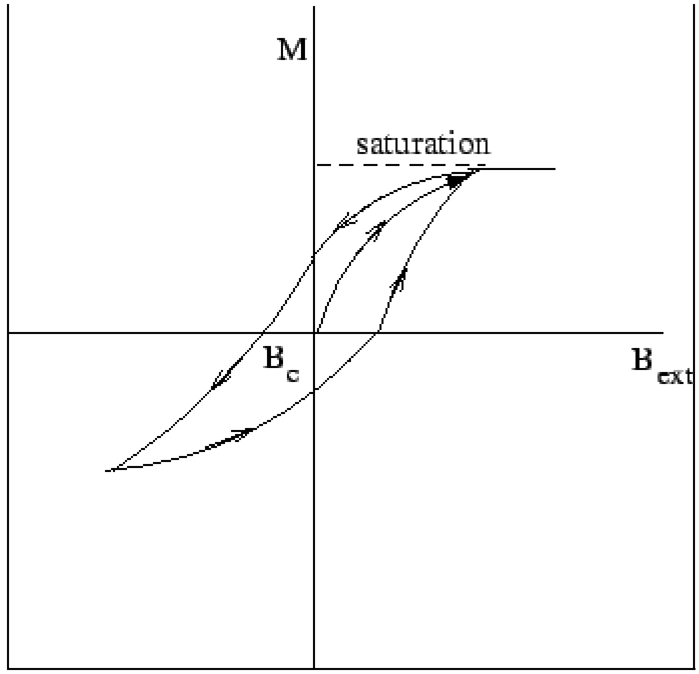
Exercise 1

Show that Ampere's law may also be expressed as $\oint \vec{B} \cdot d\vec{l} = \mu I_f$.

Ferromagnets

In certain substances like Fe, Co, Ni etc. magnetization not only depends on the applied magnetic field but it also depends on the history of the material. In a phenomenon known as **hysteresis** , a sample of a ferromagnet may exhibit magnetization even when no magnetic field is present exhibiting a memory effect.

When a sample of such a material which has no initial magnetic moment is subjected to a magnetic field, the magnetization increases with increasing strength of magnetic field and soon saturates when all the atomic moments have got aligned in the direction of the magnetic field. If field strength is now decreased gradually, the magnetization decreases. However, the magnetization curve does not retrace its path. When the field strength has been reduced to zero, the sample still has some magnetization left. In order to bring the sample to a state of zero magnetization, a coercive field B_c must be applied in the reverse direction.



Diamagnetism

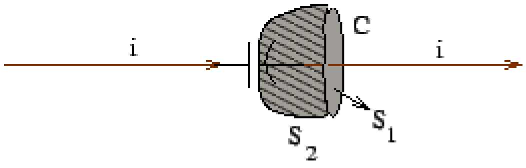
Diamagnetism, being a consequence of Lenz's law is present in all substance though its effect may be masked because of other strong magnetic effects. However, certain substances like Bismuth are strongly diamagnetic. The effect arises because when an atom is placed in a magnetic field, the flux through the atomic orbit changes. This results in an induced current being generated which opposes the changing flux. The effect is equivalent to the atom developing an induced magnetic moment opposite to the direction of the applied field.

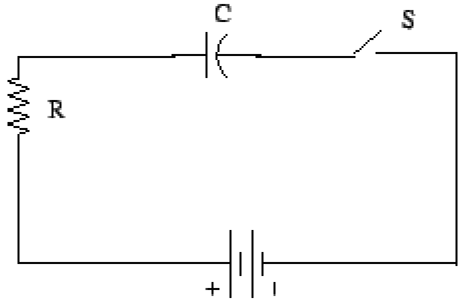
Displacement Current

We have seen that the magnetic field due to a current is given by Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

where the current passes through the surface of the boundary over which the above integral is taken. Maxwell pointed out that the equation is logically inconsistent.
Consider a parallel plate capacitor being charged by a battery. During the process of charging a current passes through the terminals of the battery. The current produces a magnetic field around it. Consider an Amperian loop C located just outside one of the plates. Let the plane of the loop S_1 be parallel to the plates. On applying Ampere's law to such a loop, we get the value of the above integral to be non-zero as there is a current passing through the wires. However, consider a second surface S_2 which is located just inside the plates of the capacitor and does not intersect the wire at all. The flux of the current (the the surface integral of the current density) through this surface is zero. However, as both S_1 and S_2 are bounded by the same loop C, the flux through both must be the same. This is clearly inconsistent.





To remove the apparent inconsistency, we need to revise our notion of what constitutes a current. We generally believe that current being a flow of charge must be the same at all cross sections of a series circuit. This is obviously not true for circuits containing capacitors. Consider an RC circuit. When the switch S is closed, the capacitor charges and the current is given by

$$i = \frac{V}{R}\left[1 - \exp(-t/RC)\right]$$

This does not hold inside the capacitor as the dielectric between the plates is a dielectric (could be air) and does not conduct electricity. In order to preserve the continuity of current inside the dielectric, we need to reconcile the apparent inconsistency. Since the current through the wire $i = dQ/dt$ is the rate of flow of charge through the wires, we may express it in terms of the electric field inside the capacitor plates. Since the electric field inside the capacitor is given by $E = Q/A\epsilon_0$, an equivalent expression for current is

$$i = A\epsilon_0\frac{dE}{dt} = \epsilon_0\frac{d\phi_E}{dt}$$

where ϕ_E is the electric flx through the capacitor.

Maxwell suggested that a term $\epsilon_0 d\phi_E/dt$ should be considered as *current* through the dielectric. He termed this current as **displacement current.** The Ampere's law is modified to read

$$\oint \vec{B}\cdot d\vec{l} = \mu_0\left(i + \epsilon_0\frac{d\phi_E}{dt}\right)$$

Equation above is now logically consistent because inside the dielectric $i = 0$ and the value of the integral is given by the second term. The current exists whenever the electric field, and hence the electric flux, is changing with time. Using Stoke's theorem, we may convert the above to a differential form

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{dE}{dt}$$

Exercise-2

Show the above relationship.

Example-25

A parallel plate capacitor with circular plates of radii 10 cm separated by 5 mm is being charged by an external source. The charging current is 0.2A. Find (a) the rate of change of potential difference between the plates and (b) the displacement current.

Solution :

The capacitance is $C = \epsilon_0 A/d = 55.6\times 10^{-12}$ F. Given $dQ/dt = CdV/dt = 0.2$, $dV/dt = 3.6\times 10^{11}$ V/s. The displacement current is

$$i_d = \epsilon_0\frac{d\phi_E}{dt} = \epsilon_0 A\frac{dE}{dt} = \frac{dQ}{dt} = i$$

Thus the displacement current is 0.2 A.

Exercise-3

A 50 pF parallel plate capacitor is being charged so that the voltage is increasing at a rate 300 V/s. The capacitor plates are circular with radii 10 cm each. Calculate (i) the displacement current density and (ii) the magnetic field strength at a distance of 5 cm from the axis of the capacitor.

(Ans. (i) 4.8×10^{-7} A/m ^{2} (ii) 1.4×10^{-14} T.)

Maxwell's Equations

We are now in a position to collect together all the laws governing the dynamic of electromagnetic field. These equations are collectively known as **Maxwell's equations.**

- Gauss's Law of Electrostatics :

$$\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}, \;\longleftrightarrow\; \oint_S \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0}}$$

- Gauss's Law of Magnetism :

$$\boxed{\vec{\nabla}\cdot\vec{B} = 0, \;\longleftrightarrow\; \oint_S \vec{B}\cdot d\vec{S} = 0}$$

- Faraday's Law

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}, \;\longleftrightarrow\; \oint_C \vec{E}\cdot d\vec{S} = -\frac{d}{dt}\oint_S \vec{B}\cdot d\vec{S}}$$

- Ampere-Maxwell Law

$$\boxed{\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t}, \;\longleftrightarrow\; \oint_C \vec{B}\cdot d\vec{S} = \mu_0 i + \epsilon_0\mu_0\frac{d}{dt}\oint_S \vec{E}\cdot d\vec{S}}$$

These equations are to be supplemented by Lorentz force equation.

In terms of free charges and currents, the first and the fourth of Maxwell's equations are generally expressed in terms of the vectors $\vec{D}$ and $\vec{H}$:

$$\begin{aligned} \vec{\nabla}\cdot\vec{D} &= \rho_f \\ \vec{\nabla}\times\vec{H} &= \vec{J}_f + \frac{\partial\vec{D}}{\partial t} \end{aligned}$$

with the displacement vector $\vec{D}$ and the H-vector defined as

$$\begin{aligned} \vec{D} &= \epsilon_0\vec{E} + \vec{P} \\ \vec{H} &= \frac{\vec{B}}{\mu_0} - \vec{M} \end{aligned}$$

The polarization vector $\vec{P}$ and the magnetization vector $\vec{M}$ are related to the vectors $\vec{E}$ and $\vec{B}$ respectively by constitutive relations

$$\vec{P} = \epsilon\vec{E} \qquad \vec{M} = \chi\vec{H}$$

Electromagnetic Waves

In the absence of any source of charge or current, Maxwell's equations in free space are as follows :

$$
\begin{aligned}
\vec{\nabla}\cdot\vec{E} &= 0 && (1)\\
\vec{\nabla}\cdot\vec{B} &= 0 && (2)\\
\vec{\nabla}\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} && (3)\\
\vec{\nabla}\times\vec{B} &= \epsilon_0\mu_0\frac{\partial\vec{E}}{\partial t} && (4)
\end{aligned}
$$

The last two equations couple the electric and the magnetic fields. If $\vec{B}$ is time dependent, $\vec{\nabla}\times\vec{E}$ is non-zero. This implies that $\vec{E}$ is a function of position. Further, if $\partial\vec{B}/\partial t$ itself changes with time, so does $\vec{\nabla}\times\vec{E}$. In such a case $\vec{E}$ also varies with time since the $\vec{\nabla}$ operator cannot cause time variation. Thus, in general, a time varying magnetic field gives rise to an electric field which varies both in space and time. It will be seen that these coupled fields propagate in space.

We will first examine whether the equations lead to transverse waves. For simplicity, assume that the electric field has only x-component and the magnetic field only y-component. Note that we are only making an assumption regarding their directions - the fields could still depend on all the space coordinates x, y, z, in addition to time t.

Gauss's law gives

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0$$

Since only $E_x \neq 0$, this implies

$$\frac{\partial E_x}{\partial x} = 0$$

Thus E_x is independent of x coordinate and can be written as $E_x(y,z,t)$. A similar analysis shows that B_y is independent of y coordinate and can be written explicitly as $B_y(x,z,t)$.

Consider now the time dependent equations eqns. (3) and (4). The curl equation for $\vec{B}$ gives, taking z-component

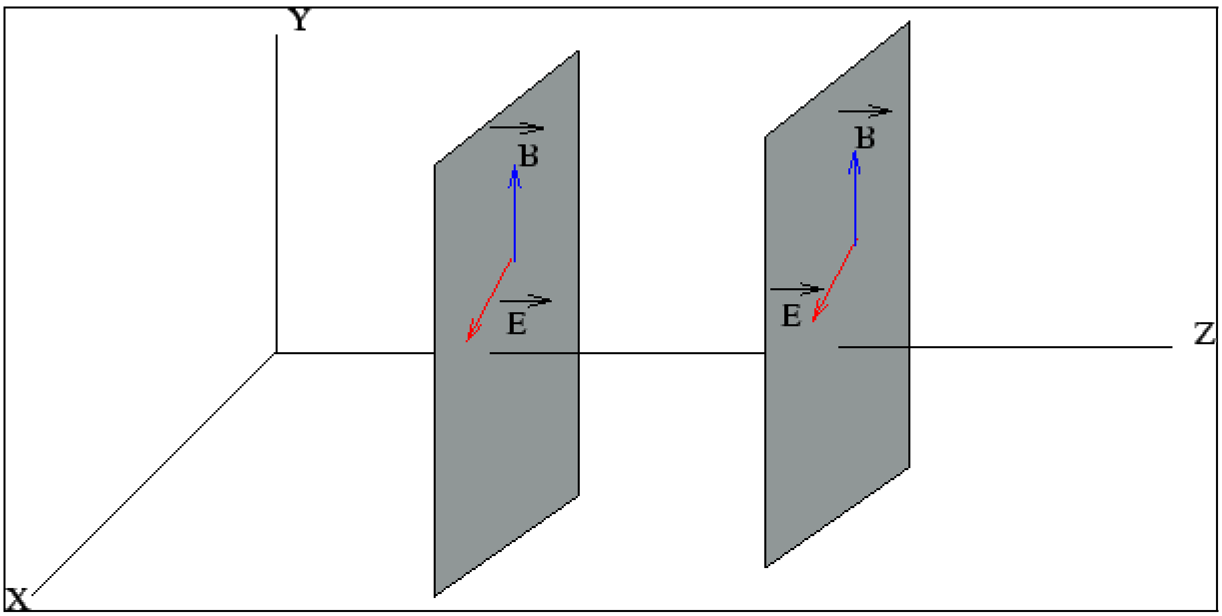
$$\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} = \frac{\partial E_z}{\partial t} = 0$$

Since $B_x = 0$, this gives

$$\frac{\partial B_y}{\partial x} = 0$$

showing that B_y is independent of x and hence depends only on z and t. In a similar manner we can show that E_x also depends only on z and t. Thus the fields $\vec{E}$ and $\vec{B}$ do not vary in the plane containing them. Their only variation takes place along the z-axis which is perpendicular to both $\vec{E}$ and $\vec{B}$. The direction of propagation is thus direction.

$z-$



To see that propagation is really a wave disturbance, take y-component of Eqn. (3) and x-component of Eqn. (4)

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t} \quad (5)$$

$$-\frac{\partial B_y}{\partial z} = \mu_0\epsilon_0\frac{\partial E_x}{\partial t} \quad (6)$$

To get the wave equation for E_x, take the derivative of eqn. (5) with respect to z and substitute in eqn. (6) and interchange the space and time derivatives,

$$\frac{\partial^2 E_x}{\partial z^2} = \mu_0\epsilon_0\frac{-\partial^2 B_y}{\partial z\partial t} = -\frac{\partial}{\partial t}\left(\frac{\partial B_y}{\partial z}\right) = \mu_0\epsilon_0\frac{\partial^2 E_x}{\partial t^2}$$

Similarly, we can show, We get

$$\frac{\partial^2 B_y}{\partial z^2} = \mu_0\epsilon_0\frac{\partial^2 B_y}{\partial t^2}$$

Each of the above equations represents a wave disturbance propagating in the z-direction with a speed

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

On substituting numerical values, the speed of electromagnetic waves in vacuum is 3×10^8 m/sec.

Consider plane harmonic waves of angular frequency ω and wavlength $\lambda = 2\pi/k$. We can express the waves as

$$E_x = E_0 \sin(kz - \omega t)$$
$$B_y = B_0 \sin(kz - \omega t)$$

The amplitudes E_0 an B_0 are not independent as they must satisfy eqns. (5) and (6) :

$$\begin{aligned}\frac{\partial E_x}{\partial z} &= E_0 k\cos(kz-\omega t)\\ \frac{\partial B_y}{\partial t} &= -B_0\omega\cos(kz-\omega t)\end{aligned}$$

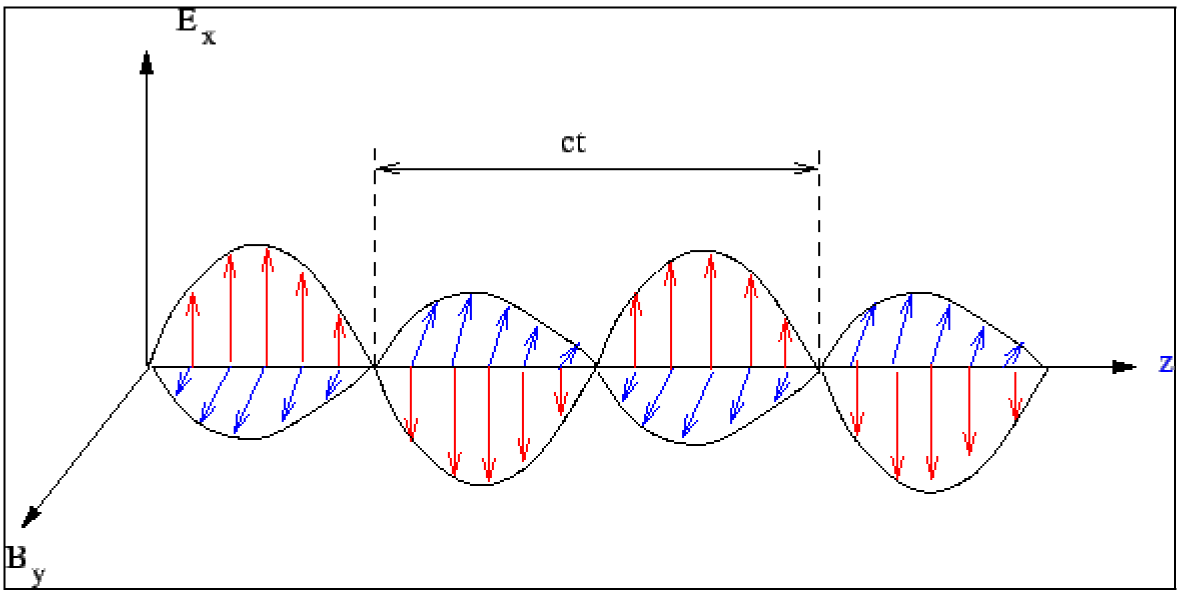
Using Eqn. (5) we get

$$E_0 k = B_0\omega$$

The ratio of the electric field amplitude to the magnetic field amplitude is given by

$$\frac{E_0}{B_0} = \frac{\omega}{k} = c$$

Fields $\vec{E}$ and $\vec{B}$ are in phase, reaching their maximum and minimum values at the same time. The electric field oscillates in the x-z plane and the magnetic field oscillates in the y-z plane. This corresponds to a **polarized wave .** Conventionally, the plane in which the electric field oscillates is defined as the plane of polarization. In this case it is x-z plane. The figure shows a harmonic plane wave propagating in the z-direction. Note that $\vec{E}, \vec{B}$ and the direction of propagation $\hat{k}$ form a right handed triad.



Example-26

The electric field of a plane electromagnetic wave in vacuum is $E_y = 0.5\cos[2\pi\times 10^8(t - x/c)]$ V/m, $E_x = E_z = 0$. Determine the state of polarization and the direction of propagation of the wave. Determine the magnetic field.

Solution :

Comparing with the standard form for a harmonic wave

$$\begin{aligned}\omega &= 2\pi\times 10^8 \ \text{rad/s}\\ k &= 2\pi\times 10^8/c\end{aligned}$$

so that $\lambda = c/10^8 = 3$ m. the direction of propagation is x-direction. Since the electric field oscillates in x-y plane, this is the plane of polarization. Since $\vec{B}$ must be perpendicular to both the electric field direction and the direction of propagation, $\vec{B}$ has only z-component with an amplitude

T. Thus

$B_0 = E_0/c \simeq 1.66 \times 10^{-9}$

$$B_z = 1.66 \times 10^{-9} \cos[2\pi \times 10^8 (t - x/c)] \text{ T}$$

Exercise-4

The magnetic field of a plane electromagnetic wave is given by

$$B_y = B_z = 10^{-8} \sin[\frac{2\pi}{3} x - 2\pi \times 10^8 t] \text{ T}$$

Determine the electric field and the plane of polarization.

(Ans. Strength of electric field is $3\sqrt{2}$ V/m)

Wave Equation in Three Dimensions

We can obtain the wave equation in three dimensions by using eqns. (1) to (4). On taking the curl of both sides of eqn. (3), we get

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

Using the operator identity

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \vec{\nabla}^2 \vec{E} = -\vec{\nabla}^2 \vec{E}$$

wherein we have used $\vec{\nabla} \cdot \vec{E} = 0$, and substituting eqn. (4) we get

$$\vec{\nabla}^2 \vec{E} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

A three dimensional harmonic wave has the form $\sin(\vec{k} \cdot \vec{r} - \omega t)$ or $\cos(\vec{k} \cdot \vec{r} - \omega t)$ Instead of using the trigonometric form, it is convenient to use the complex exponential form

$$f(\vec{r}, t) = \exp(i\vec{k} \cdot \vec{r} - \omega t)$$

and later take the real or imaginary part of the function as the case may be. The derivative of $f(\vec{r}, t)$ is given as follows :

$$\frac{\partial}{\partial x} f(\vec{r}, t) = \frac{\partial}{\partial x} \exp(ik_x x + ik_y y + ik_z z - i\omega t) = ik_x f(\vec{r}, t)$$

Since

$$\nabla = \hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

we have,

$$\nabla f(\vec{r}, t) = i\vec{k} f(\vec{r}, t)$$

In a similar way,

$$\frac{\partial}{\partial t} f(\vec{r}, t) = -i\omega f(\vec{r}, t)$$

Thus for our purpose, the differential operators ∇ and may be equivalently replaced by

$$\partial/\partial t$$

$$\begin{aligned}\frac{\partial}{\partial t} &\rightarrow -i\omega \\ \vec{\nabla} &\rightarrow i\vec{k}\end{aligned}$$

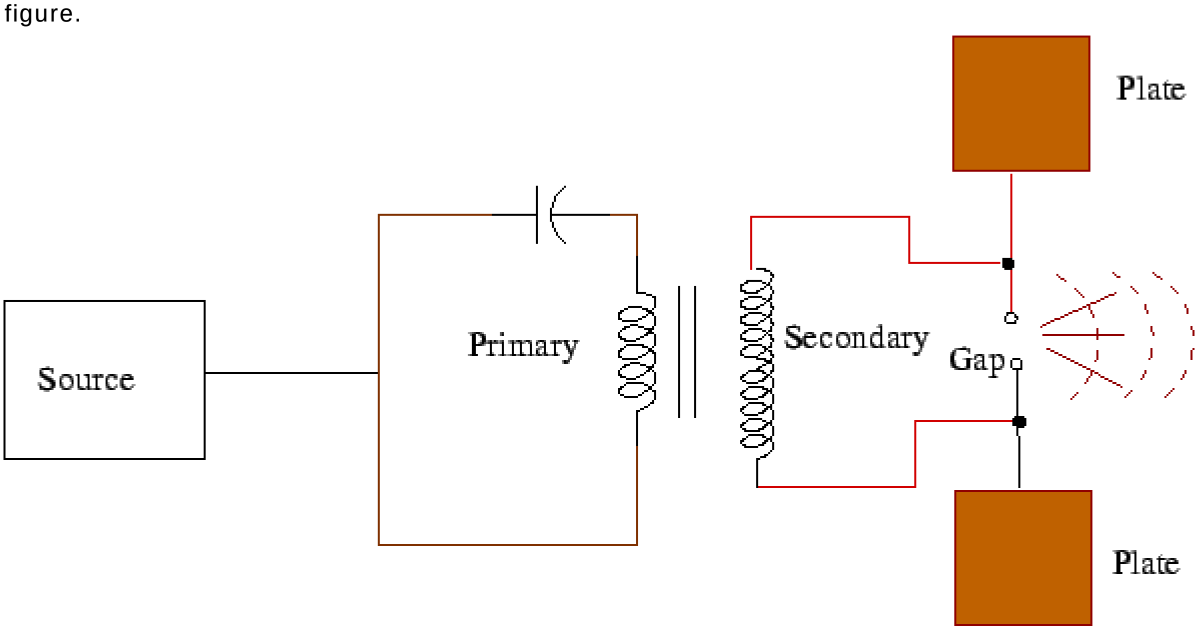
Using these, the Maxwell's equations in free space become

$$\begin{aligned}\vec{k}\cdot\vec{E} &= 0 &(7)\\ \vec{k}\cdot\vec{B} &= 0 &(8)\\ \vec{k}\times\vec{B} &= -\mu_0\epsilon_0\vec{E} &(9)\\ \vec{k}\times\vec{E} &= B &(10)\end{aligned}$$

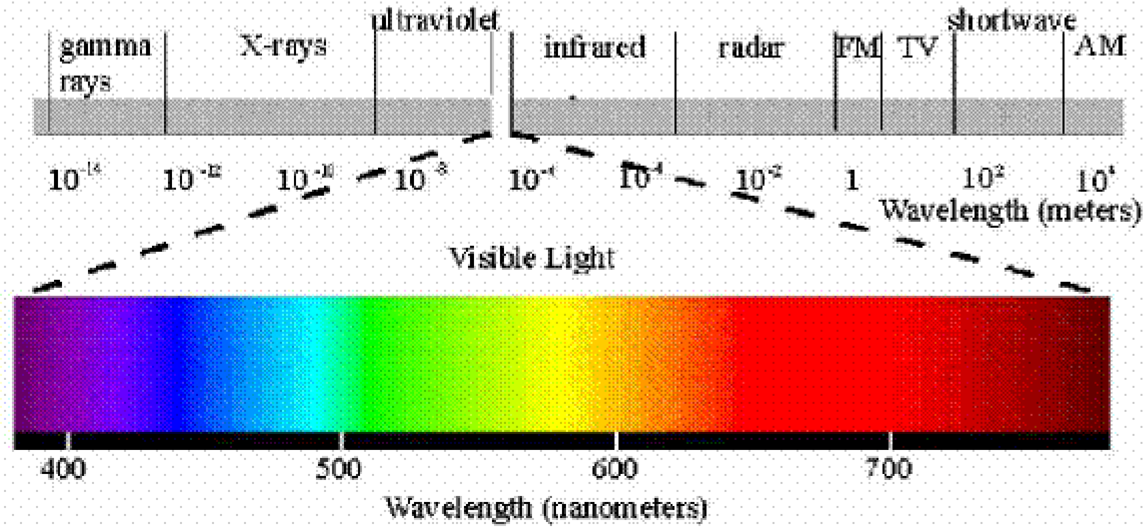
We can see that $\vec{E}, \vec{B}$ and $\vec{k}$ form a mutually orthogonal triad. The electric field and the magnetic field are perpendicular to each other and they are both perpendicular to the direction of propagation.

Generation of Electromagnetic Waves

We have looked for solutions to Maxwell's equations in free space which does not have any charge or current source. In the presence of sources, the solutions become complicated. If ρ = constant, i.e. if $\vec{J} = 0$, we only have a steady electric field. If ρ varies uniformly with time, we have steady currents which gives us both a steady electric field as well as a magnetic field. Clearly, time varying electric and magnetic fields may be generated if the current varies with time, i.e., if the charges accelerate. Hertz confirmed the existence of electromagnetic waves in 1888 using these principles. A schematic diagram of Hertz's set up is shown in the figure.



The radiation will be appreciable only if the amplitude of oscillation of charge is comparable to the wavelength of radiation that it emits. This rules out mechanical vibration, for assuming a vibrational frequency of 1000 cycles per second, the wavelength work out to be 300 km. Hertz, therefore, made the oscillating charges vibrate with a very high frequency. The apparatus consists of two brass plates connected to the terminals of a secondary of a transformer. The primary consists of an LC oscillator circuit, which establishes charge oscillations at a frequency of $\omega = 1/\sqrt{LC}$. As the primary circuit oscillates, oscillations are set up in the secondary circuit. As a result, rapidly varying alternating potential difference is developed across the gap and electromagnetic waves are generated. Hertz was able to produce waves having wavelength of 6m. It was soon realized that irrespective of their wavelength, all electromagnetic waves travel through empty space with the same speed, viz., the speed of light.



Depending on their wavelength range, electromagnetic waves are given different names. The figure shows the electromagnetic spectrum. What is known as visible light is the narrow band of wavelength from 400 nm (blue) to 700 nm (red). To its either side are the infrared from 700 nm to 0.3 mm and the ultraviolet from 30 nm to 400 nm. Microwaves have longer wavelength than the infrared (0.3 mm to 300 mm) and the radio waves have wavelengths longer than 300 mm. The television broadcast takes place in a small range at the end of the microwave spectrum. Those with wavelengths shorter than ultraviolet are generally called *rays* . Prominent among them are x-rays with wavelengths 0.03 nm t0 30 nm and γ-rays with wavelengths shorter than 0.03 nm.

Poynting Vector

Electromagnetic waves, like any other wave, can transport energy. The power through a unit area in a direction normal to the area is given by **Poynting vector** , given by

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

As $\vec{E}, \vec{B}$ and $\vec{k}$ form a right handed triad, the direction of $\vec{S}$ is along the direction of propagation. In SI units $\vec{S}$ ismeasured in watt/m ^{2}.

The magnitude of $\vec{S}$ for the electromagnetic wave travelling in vacuum is given by

$$S = \frac{EB}{\mu_0} = \frac{E^2}{c\mu_0}$$

where we have used the relationship between E and B in free space. For harmonic waves, we have

$$S = \frac{E_0^2}{c\mu_0}\sin^2(kx - \omega t)$$

The average power transmitted per unit area, defined as the **intensity** is given by substituting the value 1/2 for the average of the square of sine or cosine function

$$I = \frac{E_0^2}{2c\mu_0}$$

Example-27

Earth receives 1300 watts per squar meter of solar energy. assuming the energy to be in the form of plane electromagnetic waves, compute the magnitude of the electric and magnetic vectors in the sunlight.

Solution :

From the expression for the average Poynting vector

$$\frac{E_0^2}{2c\mu_0} = 1300$$

which gives $E_0 = 989$ V/m. The corresponding rms value is obtained by dividing by $\sqrt{2}$, $E_{rms} = 700$ V/m. The magnetic field strength is $B_{rms} = E_{rms}/c = 2.33 \times 10^{-6}$ T.

Exercise-5

A 40 watt lamp radiates all its energy isotropically. Compute the electric field at a distance of 2m from the lamp.

(Ans. 30 V peak)

Radiation Pressure

We have seen that electric field, as well as magnetic field, store energy. The energy density for the electric field was seen to be $(1/2)\epsilon_0 E^2$ and that for the magnetic field was found to be $(1/2)B^2/2\mu_0$. For the electromagnetic waves, where $E/B = c$, the total energy density is

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{B^2}{2\mu_0} = \epsilon_0 E^2$$

where we have used $c^2 = 1/\mu_0\epsilon_0$.

In addition to carrying energy, electromagnetic waves carry momentum as well. The relationship between energy (U) and momentum (p) is given by relatistic relation for a massless photons as $p = U/c$. Since the energy density of the electromagnetic waves is given by $\epsilon_0 E^2$, the momentum density, i.e. momentum per unit volume is

$$\mid p \mid = \frac{\epsilon_0 E^2}{c} = \epsilon_0 \mid \vec{E} \times \vec{B} \mid$$

Since the direction of momentum must be along the direction of propagation of the wave, the above can be converted to a vector equation

$$\vec{p} = \epsilon_0 \vec{E} \times \vec{B}$$

If an electromagnetic wave strikes a surface, it will thus exert a pressure. Consider the case of a beam falling normally on a surface of area A which absorbs the wave. The force exerted on the surface is equal to the rate of change of momentum of the wave. The momentum change per unit time is given by the momentum contained within a volume cA. The pressure, obtained by dividing the force by A is thus given by

$$P = cp = c\epsilon_0 EB = \epsilon_0 E^2$$

which is exactly equal to the energy density u.

If on the other hand, the surface reflects the wave, the pressure would be twice the above value.

The above is true for waves at normal incidence. If the radiation is diffuse, i.e., if it strikes the wall from all directions, it essentially consists of plane waves travelling in all directions. If the radiation is isotropic, the intensity of the wave is the same in all directions. The contribution to the pressure comes from those waves which are travelling in a direction which has a component along the normal to the surface. Thus on an average a third of the radiation is responsible for pressure. The pressure for an absorbing surface is $u/3$ while that for a reflecting surface is $2u/3$.

The existence of radiaton pressure can be verified experimentally. The curvature of a comet's tail is attributed to the radiation pressure exerted on the comet by solar radiation.

Exercise-6

Assuming that the earth absorbs all the radiation that it receives from the sun, calculate he radiation pressure exerted on the earth by solar radiation.

(Ans. Assuming diffuse radiation 1.33×10^{-6} N/m ^{2})

Recap

In this lecture you have learnt the following

- The response of different substances to magnetic field are different. In paramagnetic material the magnetization in parallel to the direction of the external field. Diamagnetism aroses due to Lenz's law.
- A ferromagnetic material shows memory effect, i.e., hysteresis.
- Ampere's law is modified in the presence of a magnetic material. The ampere's law for the H-field is determined by free currents.
- When a current passes through circuit containing a capacitor, the continuity of current is established by defining a displacement current through the capacitor.
- By solving the Maxwell's equations in free space, propagating electromagnetic waves was studied. Electromagnetic waves can carry both energy and momentum.