Module 4 : THERMOELECTRICITY

Lecture 22 : Temperature Measurement Using Thermocouple

Objectives

In this lecture you will learn the following

- Use of thermocouples for measurement of temperatures particularly in higher range of temperatures.

- Empirical laws of thermoelectricity, viz., law of homogeneous circuits, law of successive temperatures and aw of intermediate metals.

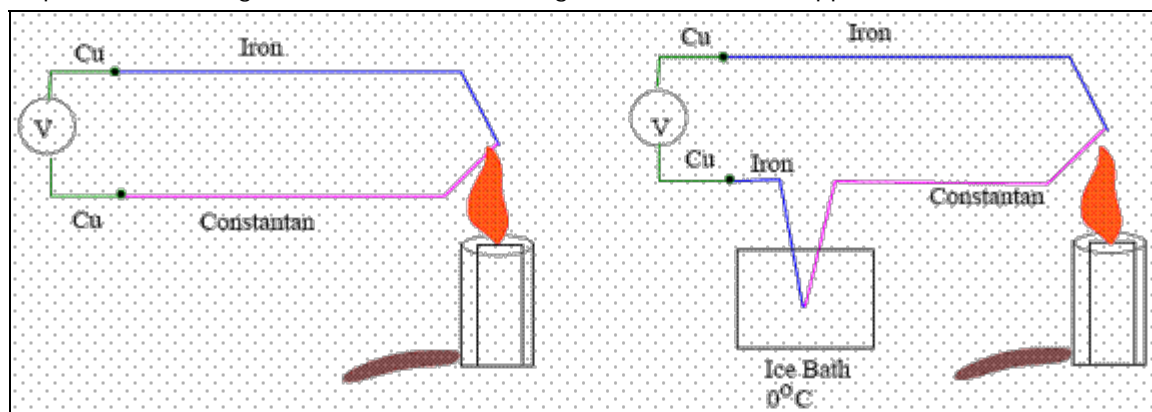- Application of laws of thermodynamics to thermoelectric phenomena.

Temperature Measurement Using Thermocouple

The emf of thermocouple is rather too small for it to be used as a device for converting heat energy to electrical energy. However, thermocouples are of great practical use in measurement of temperatures, particularly in high temperature region, where the tolerable sensitivity is about 1° C. The principle of measurement is to use the Seebeck equation

$$\Delta V = S \Delta T$$

where ΔV is the change in voltage, S is the Seebeck coefficient for the thermocouple pair and ΔT is the change in temperature. (The relationship is not quite linear and one can use a nonlinear relationship with tabulated values of coefficients.) We can measure the emf of the thermocouple with one of the junctions at a known temperature (called the reference temperature) and use the above equation to determine the temperature of the second junction. The system whose temperature is to be measured can easily be brought into thermal equilibrium with the second junction because of the small mass of the thermocouple. In scientific work, it is standard to use a thermocouple with Pt as one of the materials and an alloy of Pt-Rh as the other, to measure temperatures in the range of 600° C to 1000° C.

Seebeck voltage cannot be directly measured because of the need to connect a voltmeter in the circuit, whose terminals introduce additional junctions. Consider, for instance an iron-constantan thermocouple whose open circuit voltage is to be measured using a voltmeter with copper leads.

In doing so we introduce two additional junctions, one of Cu-Fe and the other of Cu-Cn whose effects must be compensated. An alternative is to introduce a reference junction, as shown in the figure to the right. With this arrangement both the junctions with the copper leads are with the same material with the voltages across the pair of junctions are equal and opposite so that their effects cancel. It is common to keep the compensating junction at a reference temperature of 0° C so that the reading of the voltmeter can be directly calibrated to give the temperature of the junction whose temperature is being measured.

Empirical Laws of Thermoelectricity

Experiments with thermocouples have led to formulation of three empirical laws. They are as follows :

Law of Homogeneous Circuits

It is observed that electric current cannot be sustained in a circuit with a single homogeneous metal by application of heat alone. A consequence of this law is that given that junctions between two dissimilar metals are maintained at some temperatures, the thermal emf is not affected by the lead wires even when a thermal gradient exists along the lead wires.

Law of Successive temperatures

If the thermal emf given by a thermocouple is $\mathcal{E}_{12}$ when the junctions are maintained at temperatures

T_1 and $T_2 (< T_1)$ and is $\mathcal{E}_{23}$ when the junction temperatures are T_2 and $T_3 (< T_2)$, then the

emf given by the thermocouple when the junction temperatures are T_1 and T_3 is given by

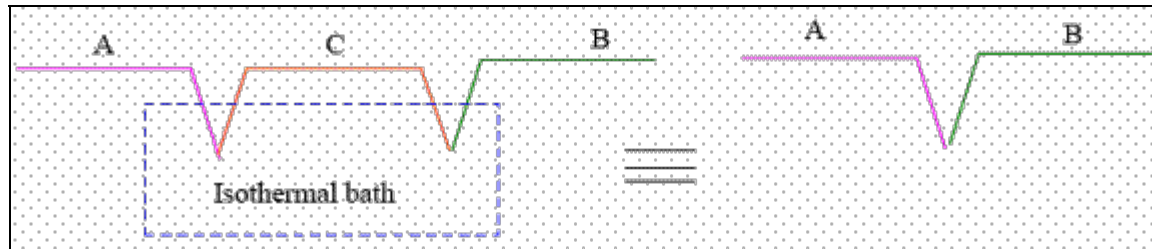$$\mathcal{E}_{13} = \mathcal{E}_{12} + \mathcal{E}_{23}$$

The conclusion is obvious from Seebeck's expression for emf as

$$\mathcal{E}_{13} = \int_{T_1}^{T_3} S_{AB} dT = \int_{T_1}^{T_2} S_{AB} dT + \int_{T_2}^{T_3} S_{AB} dT$$

The law is useful in relating the value of emf found using some arbitrary but known reference junction temperature to that which one would obtain if a standard reference junction temperature (like freezing point of water) were used.

Law of Intermediate Metals

According to this law, inserting an wire of arbitrary material into a thermocouple circuit has no effect on the thermal emf of the original circuit, if the additional junctions introduced in the circuit are at the same temperature.

The law is useful in using thermocouples with elements which are both different from the material of the lead wires of a voltmeter.
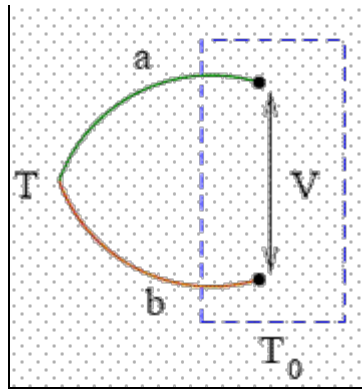
The most convenient way to analyze a thermocouple circuit is to use Kirchhoff's voltage law, which states that the algebraic sum of voltages around a closed loop is zero.

Thus when the junction of two metals a and b are maintained at a temperature T and the open circuit voltage V is measured under isothermal condition at a temperature T_0, we may write
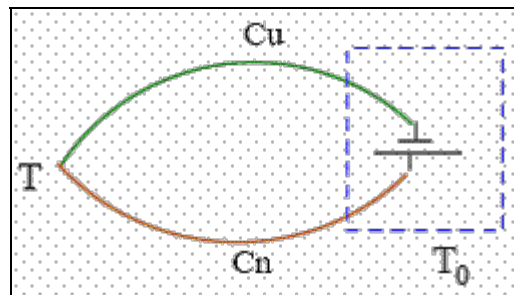
$$S_a(T - T_0) + V + S_b(T_0 - T) = 0$$

so that

$$V = (S_b - S_a)(T - T_0)$$

Values of S_a and S_b determine the polarity of the voltmeter. For instance, in the Cu-Cn junction shown,

$$V = (S_{Cn} - S_{Cu})(T - T_0)$$
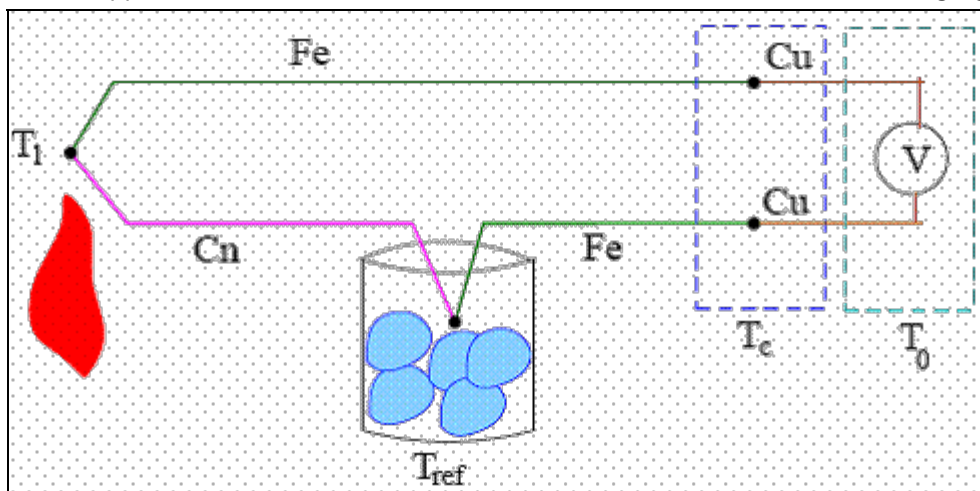$$= -40.5(T - T_0)$$

If T is the temperature of the hot junction, V is negative and therefore, if the circuit were closed, the current would flow from Cu to Cn at the cold junction.

Example-1

Consider an Iron-Constantan thermocouple. If the lead wires of the voltmeter are made of copper, suggest a method for measuring the temperature of the junction.

Solution

One of the ways to handle this is to introduce an intermediate metal in such a way that both the junctions with the copper terminals are with the same material, as shown in the following figure.

Applying Kirchhoff's loop law, we have

$$S_{Fe}(T_1 - T_c) + S_{Cu}(T_c - T_0) + V$$
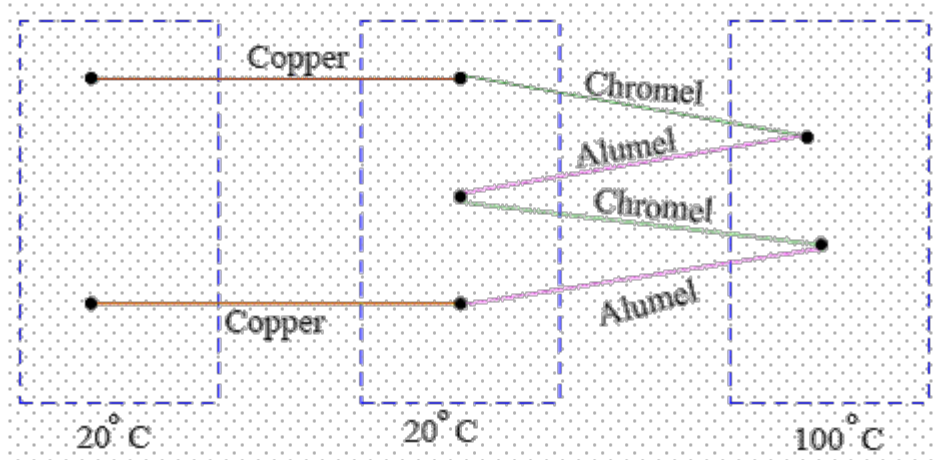$$+ S_{Cu}(T_0 - T_c) + S_{Cn}(T_{ref} - T_1) = 0$$

giving

$$V = (S_{Fe} - S_{Cn})(T_{ref} - T_1)$$

Exercise-2

Consider an Iron - copper thermocouple. Suggest a method of measuring the temperature of a junction using a voltmeter with copper lead wires.

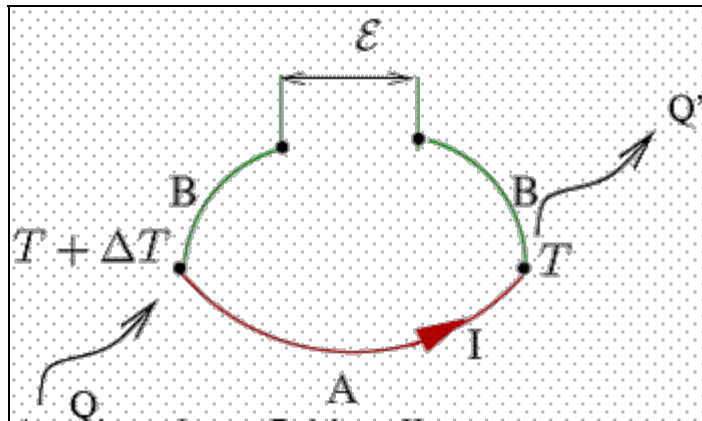<div align="center">Exercise</div>

For the circuit shown below, calculate the voltage measured at the copper terminals under isothermal conditions. Use the table for thermo-emf. (Hint. you do not need thermo emf data at $20°$ C.)(4.1 mV)

Application of Thermodynamics to Thermocouple

Consider a junction of two dissimilar metals A and B, with the cold junction at a temperature T and the hot junction at a temperature $T + \Delta T$. Let $\mathcal{E}$ be the thermo-emf generated in the circuit.

If a small charge δq is made to circulate around the loop in time t, it gives rise to a current I. We consider the system to be adiabatic. In order to conserve energy, the work done on the charge carriers by $\mathcal{E}$ must be equal to the sum of the following :

- Heat absorbed and liberated at the junctions due to Peltier effect.

- Heat exchange within the individual conductors due to Thomson effect.

We have neglected the Joule heat. Applying Kirchhoff's loop law, we have, for the power delivered,

$$\mathcal{E}I = \Pi_{AB}(T+\Delta T)I - \Pi_{AB}(T)I + I\int_T^{T+\Delta T}\sigma_B dT - I\int_T^{T+\Delta T}\sigma_A dT$$

$$= (\Pi_{AB}(T+\Delta T) - \Pi_{AB}(T))I + I\int_T^{T+\Delta T}(\sigma_B - \sigma_A)dT$$

Here $\Pi_{AB}(T+\Delta T)I$ is the rate at which Peltier heat is absorbed at the hot junction while $\Pi_{AB}(T)I$ is that given out at the cold junction. Likewise, the term with is the Thomson heat absorbed by the metal B the term is the heat evolved by the metal A.

If the temperature difference ΔT is small, the emf is written and one can approximate the right hand side of the above by an expansion keeping only the linear term in ΔT

$$(\Pi_{AB}(T+\Delta T) - \Pi_{AB}(T)) \approx \frac{d\Pi_{AB}}{dT}\Delta T$$

$$\int_T^{T+\Delta T}(\sigma_B - \sigma_A)dT \approx (\sigma_B - \sigma_A)\Delta T$$

so that in the limit of $\Delta T \rightarrow 0$, one has

$$\frac{d\mathcal{E}}{dT} = \frac{d\Pi_{AB}}{dT} + (\sigma_B - \sigma_A) \qquad (1)$$

which shows that Seebeck effect is due to the combined effect of Peltier effect and Thomson effect. Equation (1) is known as **Kelvin's First Relation** and is a consequence of the first law of thermodynamics.

If the Joule heat is ignored, the exchange of heat is reversible. According to the second law of thermodynamics the change in entropy around the closed loop is zero,

$$\oint \frac{dQ}{T} = 0$$

The Peltier heat is evolved or absorbed only at the junctions where as Thomson heat is exchanged throughout the wire. Thus

$$\frac{\Pi_{AB}(T+\Delta T)}{T+\Delta T} - \frac{\Pi_{AB}(T)}{T} + \int_T^{T+\Delta T}\frac{(\sigma_B - \sigma_A)}{T}dT = 0$$

In the limit of $\Delta T \rightarrow 0$, we get

$$\frac{d}{dT}\frac{\Pi_{AB}}{T} + \frac{(\sigma_B - \sigma_A)}{T} = 0 \qquad (2)$$

Equation (2) is known as **Kelvin's Second Relation** . Carrying out the differentiation above explicitly, we get

$$\frac{d\Pi_{AB}}{dT} - \frac{\Pi_{AB}}{T} + (\sigma_B - \sigma_A) = 0$$

Using Eqn. (1) in the above, we get a relation between the Thermoelectric Power and the Peltier coefficient

$$\frac{d\mathcal{E}}{dT} = \frac{\Pi_{AB}}{T} \qquad (3)$$

A second differentiation of Eqn. (3) gives

$$\frac{d^2\mathcal{E}}{dT^2} = \frac{1}{T}\frac{d\Pi_{AB}}{dT} - \frac{1}{T^2}\Pi_{AB}$$

$$= \frac{1}{T}[\frac{d\mathcal{E}}{dT} - (\sigma_B - \sigma_A)] - \frac{1}{T^2}\Pi_{AB}$$

$$= \frac{(\sigma_A - \sigma_B)}{T} \qquad\qquad (4)$$

Since Π_{AB}/T represents the entropy of the carriers, according to the third law of thermodynamics,

$$\lim_{T\to 0}\frac{\Pi_{AB}}{T} = 0$$

Using this we may integrate Eqn. (2) to get

$$\frac{\Pi_{AB}}{T} = \int_0^T \frac{\sigma_A - \sigma_B}{T}dT$$

The integral above should be convergent, which requires

$$\lim_{T\to 0}(\sigma_A - \sigma_B) = 0$$

Since σ_A and σ_B are material dependent, it is necessary that not just their difference σ_A, and σ_B must individually vanish in the limit of absolute zero temperature.

Exercise

From the table of Seebeck coefficients, calculate the Peltier coefficient of an Iron-copper thermocouple at 0° C. (Ans. 3.4 V)

Example

The thermoelectric power of a thermocouple is given by the expression (in μV /K)

$$\alpha(T) = 125 + 0.3T - 3 \times 10^{-4}T^2$$

where T is in ^{o}K. Determine (i) the temperature at which the Peltier coefficient is maximum and the value of the maxiumum Peltier coefficient, (ii) the temperature at which the Thomson coefficients of both the components of the thermocouple are equal.

Solution

Peltier coefficient is given by

$$\Pi = \alpha T = 125T + 0.3T^2 - 3 \times 10^{-4}T^3$$

The maximum of occurs when

$$\frac{d\Pi}{dT} = 125 + 0.6T - 9 \times 10^{-4}T^2 = 0$$

Solving for the quadratic equation, we get the maximum Peltier voltage to occur at $T = 833.3$ K. Substituting in the expression for Π, the value of the maximum Peltier voltage is 0.14 volt.

From Eqn. (4) above, the Thomson coefficients of the two components are equal at temperature at which $d\alpha/dT$ vanishes, i.e. when $6 \times 10^{-4}T - 0.3 = 0.$ Solving, $T = 500$ K.

Exercise

A chromel-constantan thermocouple with the cold junction at 0° C has a linear variation of the open circuit voltage with the temperature of the hot junction. Using the thermo-emf values at 100° C from the table, calculate (i) the thermo-emf when the hot junction is maintained at 150° C, (ii) Peltier coefficient at 150° C and (iii) the difference between Thomson emf at this temperature. [Hint : Emf is zero when the hot junction is at 0° C]

(Ans. (i) 9.5 mV (ii) 26.7 mV (iii) 0)

Recap

In this course you have learnt the following

- The Seebeck equation is useful in using thermocouples for measurement of temperature by maintaining one of the thermocouple junctions at a known temperature and calibrating the thermocouple voltage by maintaining the other junction at different temperatures. Usually, thermocouples are used for measurement of high temperatures.

- A set of three empirical laws have been established for the behaviour of thermocouples. Thermo-emf cannot be sustained in a circuit with a single homogeneos metal by application of heat.

- Laws of thermodynamics can be applied to a circuit of dissimilar metals to obtain expressions for variation of thermo-emf with temperature. The Thompson heat evolved for all material vanish at absolute zero as a consequence of third law of thermodynamics.