Module 5 : MODERN PHYSICS
Lecture 24 : PHOTONS

Objectives

In this lecture you will learn the following

- Radiation itself is quantized and consists of a collection of particles called photons.
- The phenomenon of photoelectric effect and its characteristics.
- How classical wave theory of radiation fails in explaining photoelectric effect.
- Einstein's theory of photoelectric effect.

PHOTONS

Planck's explanation of black body radiation was revolutionary as it suggested that atoms could exchange energy only in multiples of quantum of energy. Five years later, in 1905, Einstein put forward a theory of photoelectric effect which suggested that the quantum of energy was not a property associated with the radiation emitted by atoms but is **a property of radiation itself.** Radiation, according to Einstein's theory consists of discrete bundles of energy, called **photons.** Thus, electromagnetic energy is seen as a collection of photons. A photon is characterized by an energy E, related to the frequency by the relationship

$$E = h\nu$$

Further, each photon carries a momentum p given by

$$p = \frac{E}{c} = \frac{h\nu}{c}$$

In Einstein's special theory of relativity, the energy of a particle of rest mass m and momentum p is given by

$$E^2 = p^2c^2 + m^2c^4$$

which implies that photons have zero rest mass.

Exercise 1

Calculate the wavelength of a 2eV photon. ($1eV = 1.6 \times 10^{-19}$ J.)

(Ans. 620 nm)

Example-6

Earth receives 1.4 kW of energy from the sun. If it is assumed that the sunlight consists of monochromatic radiation of wavelength 600 nm, how many photons arrive at the earth every second ?

Solution

If $\lambda = 600$ nm, the corresponding frequency is $\nu = c/\lambda = 5 \times 10^{14}$ Hz. Thus the number of photons per second is

$$N = \frac{P}{h\nu} = \frac{1400}{6.63 \times 10^{-34} \times 5 \times 10^{14}} = 4.21 \times 10^{21}$$

Example-7

Assuming the formula for black body radiation to the valid for the universe, calculate the number density of photons in the universe due to cosmic microwave background.

Solution

Taking the expression for energy density in the interval ν and $\nu + d\nu$

$$N = \frac{P}{h\nu} = \frac{1400}{6.63 \times 10^{-34} \times 5 \times 10^{14}} = 4.21 \times 10^{21}$$

$$u(\nu)d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu\beta} - 1}d\nu$$

the number density of photons with energy in this frequency interval is obtained by dividing the above expression by $h\nu$. The total number density of photons is obtained by integrating the above expression over all frequencies

$$n = \int_0^\infty \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu\beta} - 1}d\nu$$

Substitute $h\nu\beta = x$,

$$n = \frac{8\pi}{(ch\beta)^3}\int_0^\infty \frac{x^2}{e^x - 1}dx$$

$$= 1.65 \times 10^8 \int_0^\infty \frac{x^2}{e^x - 1}$$

The integral has to be done numerically, say by using Simpson's rule. The value of the integral is 2.4, which gives the number density of photons of the cosmic radiation to be 3.96×10^8 per m ^{3}.

Exercise 2

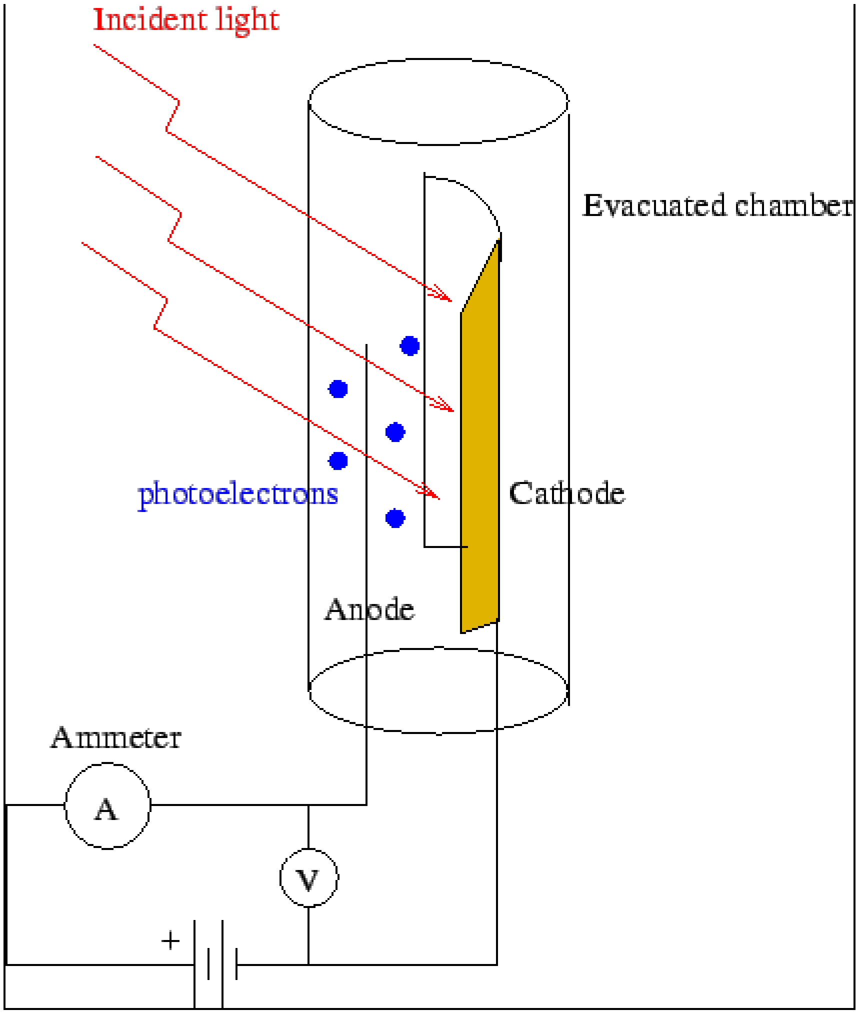
Assuming the sun to be a black body, calculate the number of photons emitted by the sun every second.

(Ans. 2.1×10^{45})

Photoelectric Effect

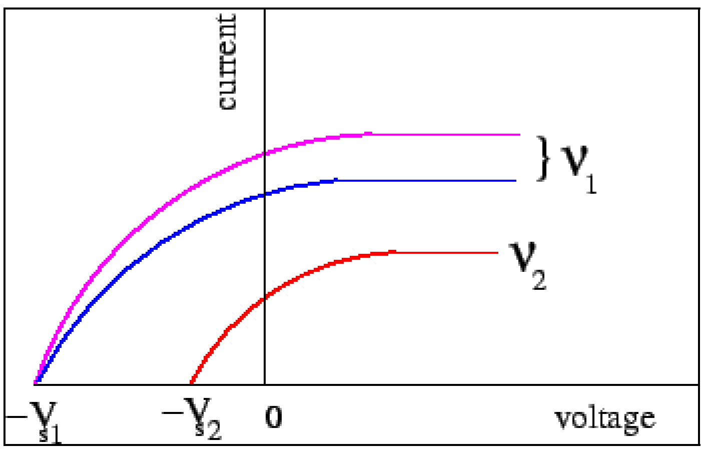
When light falls on certain metals, electrons are ejected from the surface of the metal. In the arrangement shown in the figure, the wire marked anode is held at positive potential with respect to the curved plate marked **cathode**

When light of certain minimum frequency falls on the cathode, electrons are emitted in all directions. These electrons are called **photoelectrons** . Some of these electrons reach the anode wire which provides a path to the electrons to give rise to a mesurable **photo-current** . By making the anode more positive with respect to the cathode, more electrons are attracted towards the anode and the photo-current increases. When the anode potential is such that all the emitted electrons reach the anode, any further increase in the anode voltage does not increase current any further.



See the animation

If the voltage is reversed making the anode negative with respect to the cathode, the electrons get decelerated and only the more energetic of the electrons can reach the anode.



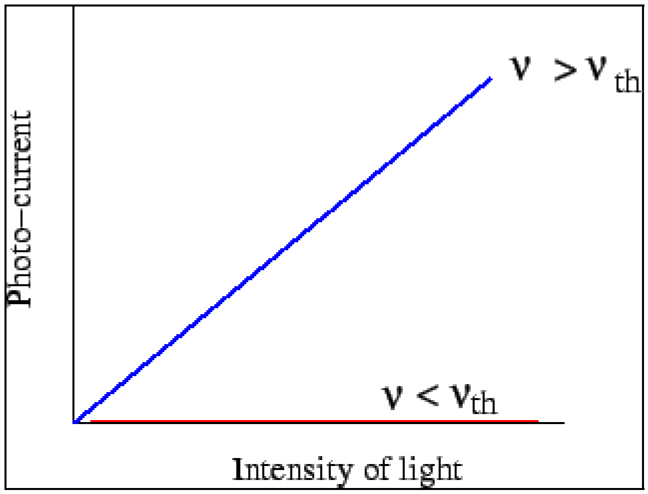
If the reverse voltage is such that even the electrons which are ejected with the maximum kinetic energy cannot overcome the potential, the photo-current becomes zero. The reverse voltage which is just enough to stop the most energetic photoelectrons is called the stopping potential. If K_{max} is the maximum kinetic energy of the electrons, the stopping potential is V_s defined by

$$eV_s = K_{max}$$

The photoelectric effect exhibits the following features :

- Photoelectrons are not ejected unless the frequency of incident light is above a certain threshold value ν_{th}. The value of ν_{th} depends on the material of the cathode.

- If the frequency of incident radiation ν is greater than ν_{th}, even a light of very weak intensity will cause photoelectrons to be emitted. If $\nu < \nu_{th}$, even the most intense light will not cause photoelectrons to be emitted. For $\nu > \nu_{th}$, the photo-current increases linearly with the intensity of light.



- The maximum kinetic energy of the photoelectrons depend on the frequency of incident radiation and not on its intensity.

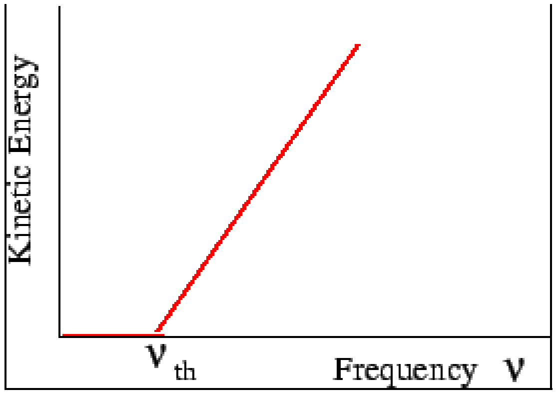
- The emission of photoelectrons is almost instantaneous. i.e. there is no time lag between the emission of electrons and switching on of the light source.

Failure of Classical Wave Theory

According to the classical wave theory, when electromagnetic wave falls on the surface of a metal, an atom on the surface will absorb energy from the electric field of the wave. The rate at which the energy is absorbed depends on the surface area of the atom. An electron can be dislodged from an atom once it absorbes sufficient amount of energy. By increasing the intensity of light (irrespective of its frequency) more energy can be transferred to the atom causing electrons to be ejected.

What is observed is that **unless $\nu > \nu_{th}$ photoelectrons are not emitted, no matter how intense the radiation is.**

Further, according to wave theory, the kinetic energy of emitted electrons would increase with the intensity of light as it would impart more energy to an electron. However, the kinetic energy of photoelectrons is found to depend only on the frequency of radiation and not on the intensity.



Another problem with the classical theory is that it would predict a time lag between the time light falls on a surface and the instant photoelectrons are emitted. The reason why one would expect such a time lag is that the surface are of an atom is very small, as a result of which an atom can only absorb a small fraction of energy that falls on the surface. The following example gives a rough estimate of the expected time lag. However, it is observed that the **emission of electrons is practically instantaneous,** with time lag, if any, being less than 10^{-9} seconds.

Example-8

Consider a light source such as a laser with a power output of 1mW spread over a narrow beam of cross section 0.1 cm ^{2} falling on a surface of a metal. Estimate the time lag of photoelectron emission as per wave theory.

Solution

Taking atomic diameter to be of the order of 10^{-8} cm, the area exposed to the beam is 10^{-16} cm ^{2}. The fraction of light energy absorbed is $10^{-16}/0.1 = 10^{-15}$. Thus the energy absorbed from the beam every second is $10^{-15} \times 10^{-3} = 10^{-18}$ J, which is approximately 6 eV (1 eV = 1.6×10^{-19} J). The amount of energy required to ionize an atom by dislodging an electron is typically 10 eV. Thus it takes about 1.6 seconds to absorb the required energy which is rough estimate of the time lag.

Einstein's Photoelectric Equation :

According to Einstein's explanation, photoelectric effect occurs due to absorption of a single photon by an electron in the atom. When radiation falls on a metal surface, an electron may absorb one quantum of energy and increase its energy by $h\nu$. Some of the absorbed energy, W, will be used to separate the electron from the metal surface. The surplus energy appears as the kinetic energy of the emitted electron

$$K.E. = h\nu - W$$

The electrons which are more tightly bound to the metal (e.g. electrons which lie two or three atomic layers below the surface) require more energy to be removed. We define **Work Function** φ of a metal as the minimu energy that must be supplied to an electron at the metal surface to dislodge it from the metal. Such electrons are emitted with maximum possible kinetic energy. Thus Einstein's equation becomes

$$K_{max} = \frac{1}{2}mv_{max}^2 = h\nu - \varphi$$

Since kinetic energy cannot be negative, the above equation implies the existence of a minimum frequency ν_{th} for photoemission to take place

$$h\nu_{th} = \varphi \quad \text{i.e.} \nu_{th} = \frac{\varphi}{h}$$

Using this, we can reqrite rewrite Einstein's equation as

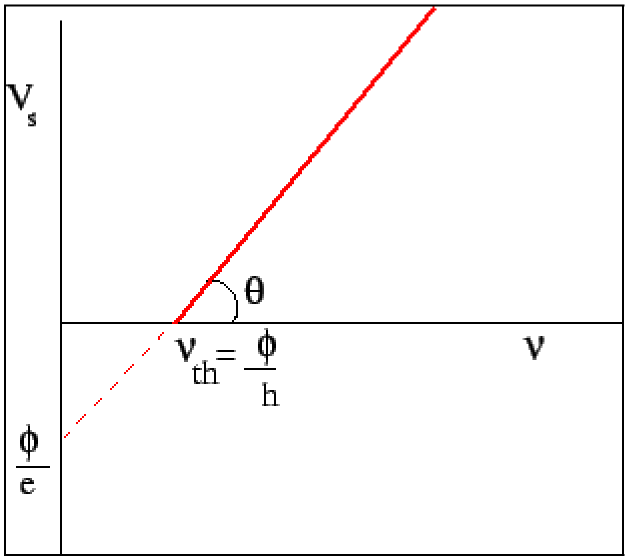
$$K_{max} = h(\nu - \nu_{th})$$

To stop such maximum energy electrons from reaching the anode, we must apply a reverse potential V_s, given by $eV_s = K_{max}$. Thus

$$eV_s = h\nu - \varphi$$

Photoelectric Effect

We can experimentally determine the stopping potential corresponding to various values of incident light frequency ν. The linear plot of V_s versus ν enables us to determine the work function. The slope of the curve is given by $\tan\theta = h/e$, which is used to determine the Planck's constant.



Work Function for some metals in eV

Metal	Work Function
Cs	1.9
Na	2.3
Co	3.9
Al	4.1
Cu	4.7
Ag	4.7
Pt	6.4

Exercise 3

The minimum energy required to remove an electron from a metal is 2.5 eV. What is the longest wavelength of radiation that can cause photoelectrons to be emitted from such a metal ?

(Ans. 495 nm)

Exercise-4

The work function of Potassium is 2 eV. If the surface of the metal is illuminated by a radiation of 360 nm, what will be (i) stopping potential, (ii) energy of the fastest photoelectron ? (Ans. (i) 1.45 eV (ii) 1.45 V)

Exercise 5

The maximum kinetic energy emitted from the surface of a metal has a value equal to twice its work function. By what factor should the frequency of incident radiation be increased so that the kinetic energy is doubled ?

(Ans. 5/3)

Example 9

A monochromatic source of light with a wavelength 200 nm and power output of 2 watts is held at a distance of 0.1m from the surface of an aluminium foil. Aluminium has a work function of 4.2 eV and an atomic radius of 0.15 nm. Take the photo-emission efficiency to be 2.5%. Calculate

- the kinetic energy of the fastest and the slowest photoelectron emitted,
- the average number of photons falling on an atom of Al per second,
- the number of photoelectrons emitted per unit area per second.

Solution

- 200 nm corresponds to a photon energy of 9.95×10^{-19} J, which is equal to 6.2 eV. Thus the kinetic energy of the fastest electron is $h\nu - \varphi = 6.2 - 4.2 = 2$ eV. The kinetic energy of the slowest electron is zero.
- The light falling on an unit area at a distance D from the source has an intensity of

$I/4\pi D^2 = 2/4\pi(0.1)^2 = 15.92$ W/m ^{2}. The amount of radiation captured by an atom of radius r is

$$\frac{I}{4\pi D^2}\pi r^2 = 4.5 \times 10^{-18} \ \ \mathrm{Js}^{-1}$$

The number of photons is obtained by dividing this by the energy of a single photon. Thus the number of photons captured by an atom on the foil is $4.5 \times 10^{-18}/9.95 \times 10^{-19} = 4.5$ per second.

- The **photo-emission efficiency** is the ratio of the number of photoelectrons emitted from a surface to the number

of photons falling on the surface in a given time. The number of electrons falling on unit area of the foil is $15.92/9.95 \times 10^{-19} = 1.6 \times 10^{19}$ per second. With 2.5% efficiency, the number of electrons emitted is 4×10^{17} /m ^{2}-s.

Example 10

Radiation from a black body at 6000 K strikes the surface of a metal with work function 2 eV. What fraction of the black body's total radiant intensity is effective in producing photoelectrons ?

Solution

The work function 2 eV corresponds to a threshold wavelength $\lambda_{th} = 621$ nm. Thus irradiance due to very small wavelengths up to 621 nm will cause photoelectrons to be emitted. Using the expression for the radiant intensity, the fraction of the total intensity is

$$f = \frac{\int_{x_0}^{\infty} \frac{x^3 dx}{e^x - 1}}{\int_0^{\infty} \frac{x^3 dx}{e^x - 1}}$$

where $x_0 = hc/\lambda_{th} kT = 3.87$. The integral in the numerator has to be done numerically while the value of the denominator is known to be $\pi^4/15 = 6.494$. The value of the numerator is found to be 2.775, which gives the fraction to be 0.43.

Exercise-6

Radiation from a black body at a temperature of 500 K falls on a metal with a work function of 0.2 eV. Find the longest wavelength of the spectrum capable of releasing photoelectrons from the surface. What percentage of the total radiant energy of the black body contributes to the process ?

(Ans. 6.19 μm, 29%)

Recap

In this lecture you have learnt the following

- Planck had suggested that atoms can emit or absorb radiation in bursts called quanta. Einstein proposed that quantum nature is an inherent property of radiation itself.
- When light falls on some metals, electrons are emitted from their surfaces. This is known as the photoelectric effect.
- Classical theory is inadequate in explaining several features of photoelectric effect. For instance, the emission of photoelectrons is practically instantaneous without any measurable time lag between shing of light and emission of photoelectrons.
- The photoelectrons are not emitted, no matter how intense is the radiation, unless the frequency of incident radiation exceeeds a critical value.
- Einstein's was able to explain photoelectric effect on the basis of photon theory of light. Einstein's euation states that the maximum kinetic energy of emitted photoelectrons is the difference between the energy of the incident quantum and the work-function for the material.