



NPTEL ONLINE CERTIFICATION COURSES

COMMUNICATIONS NETWORK

Dr. Goutam Das

G. S. Sanyal School of Telecommunications, IIT KHARAGPUR

Module 04: Circuit Switched Networks

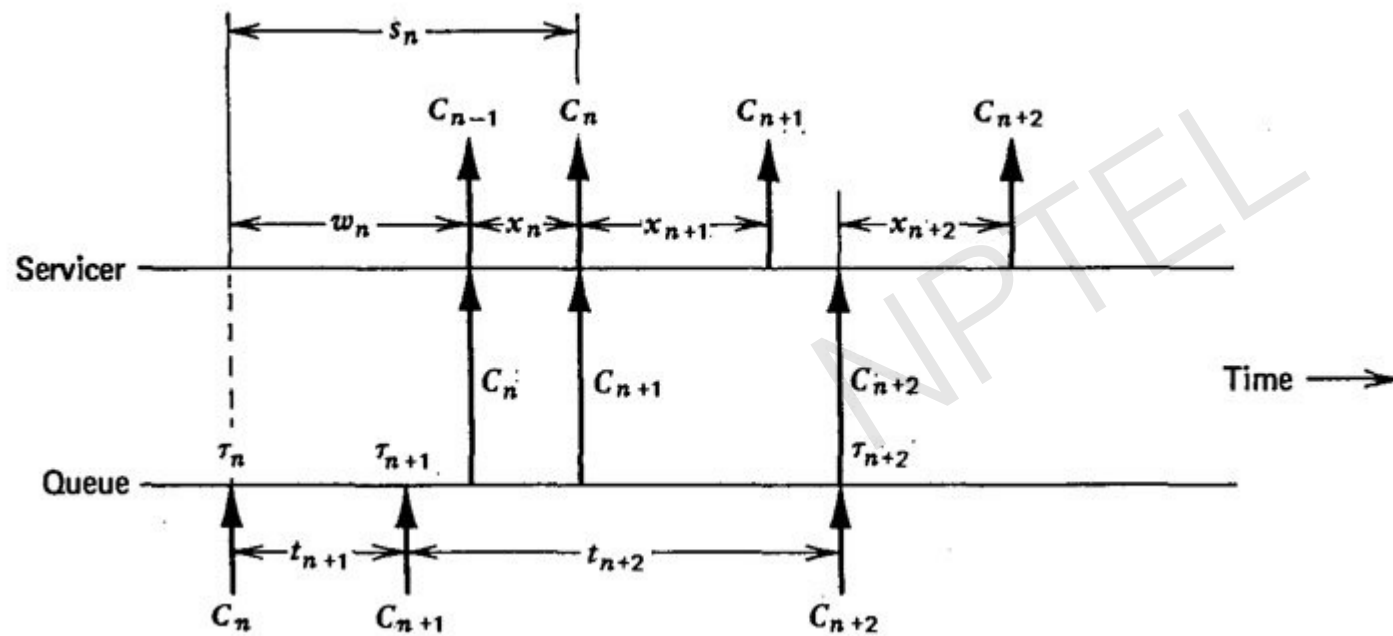
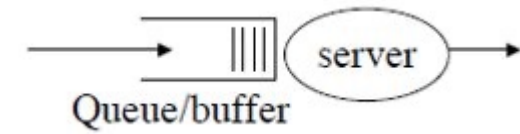
Lecture 15: Introduction to Queuing Theory

Concepts Covered

- Space and Time Switching
- Synchronization
- Multiplexing

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Typical Arrival Departure – time bookkeeping



Trunk Switch Blocking

Queuing Theory

Markovian queues with Multiple servers

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Conclusion

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References

Book:
Queuing Systems Vol. 1 – L. Kleinrock

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Module 04: Queuing Theory

Lecture 16: Arrival and Service Process

Concepts Covered

- Introduction
- Arrival Process
- Service Process

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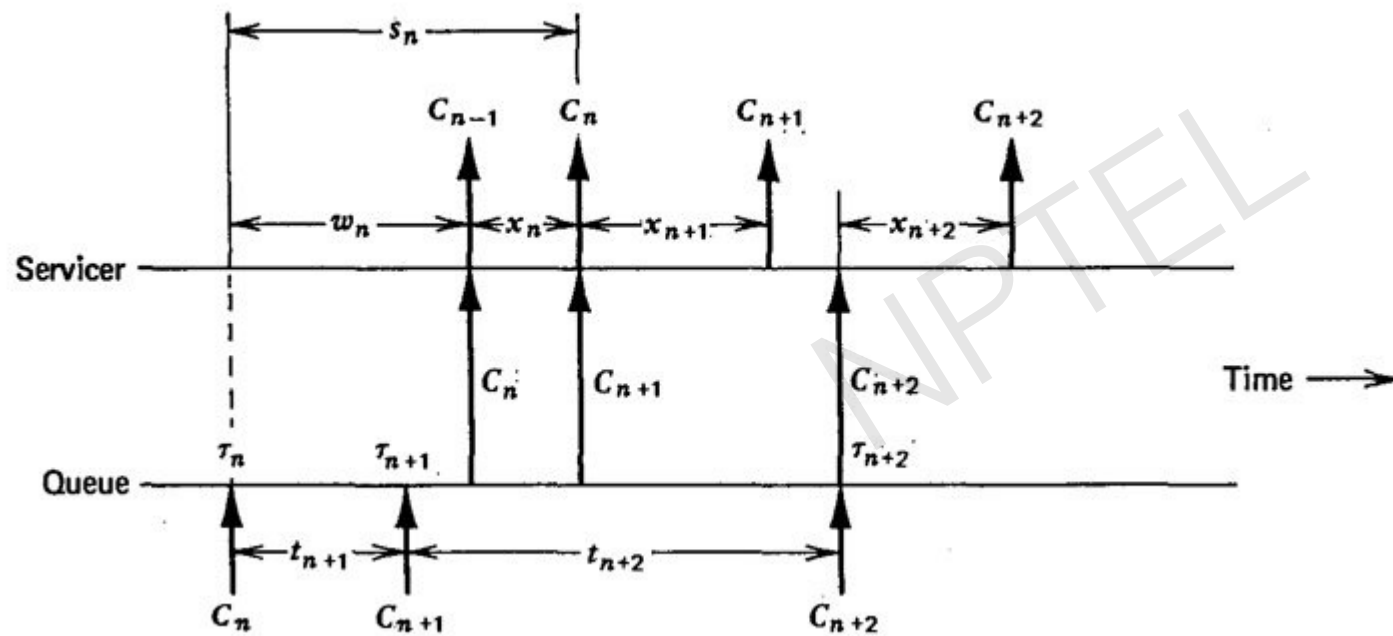
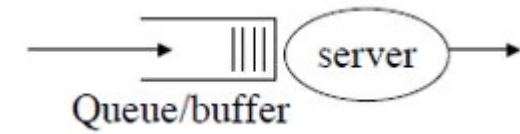
Trunk Switch Blocking

Queuing Theory

Markovian queues with Multiple servers

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Typical Arrival Departure – time bookkeeping



Conclusion

Markovian Process – Poisson and Exponential

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References

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Module 04: Queuing Theory

Lecture 16: Poisson Process

Concepts Covered

- Introduction
- Arrival Process
- Service Process

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Trunk Switch Blocking

Queuing Theory

Markovian queues with Multiple servers

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Conclusion

Markovian Process – Poisson and Exponential

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Module 04: Queuing Theory

Lecture 18: Poisson Process (cont'd)

Concepts Covered

- Introduction
- Arrival Process
- Service Process

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Trunk Switch Blocking

Queuing Theory

Markovian queues with Multiple servers

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Module 04: Queuing Theory

Lecture 19: Memorylessness

Concepts Covered

- Introduction
- Arrival Process
- Service Process

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Trunk Switch Blocking

Queuing Theory

Markovian queues with Multiple servers

NPTTEL

Conclusion

Markovian Process – Poisson and Exponential

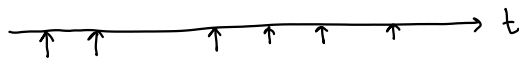
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References

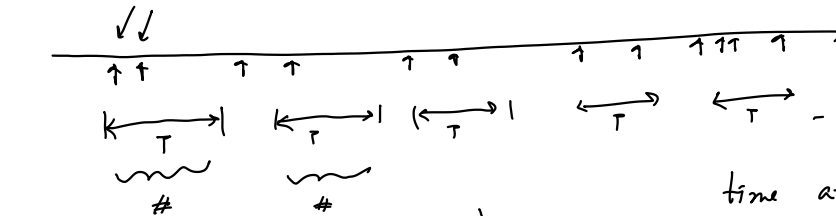
Book:
Queuing Systems Vol. 1 – L. Kleinrock

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Arrival Process

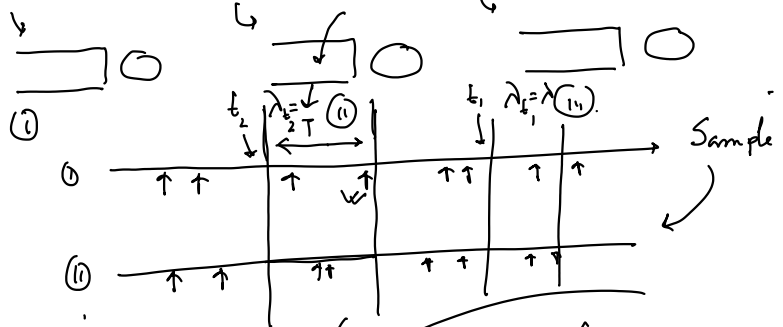


Stationary RP

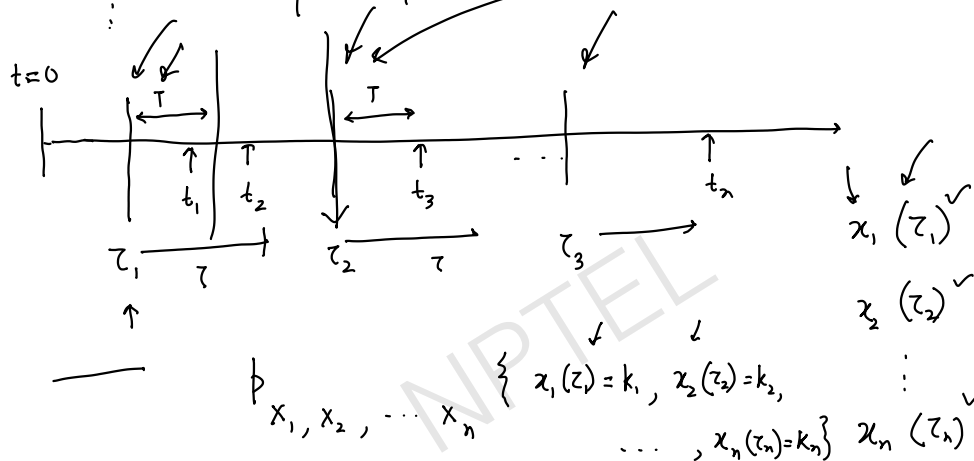


no of arrival / unit time.
= λ

time average.



no of arrival / unit time
= λ



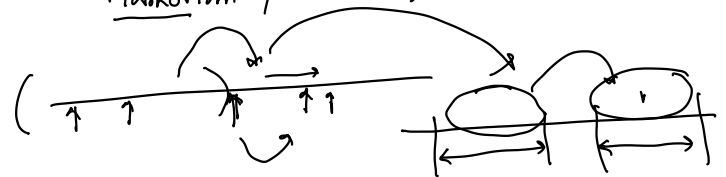
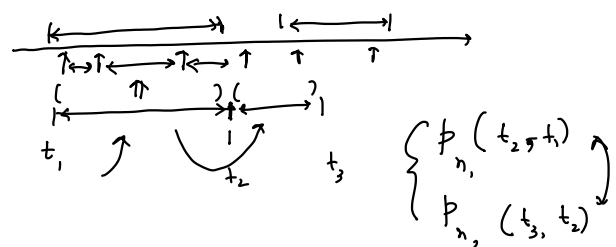
$$= P_{X_1, X_2, \dots, X_n} \{ X_1(z_1+z) = k_1, X_2(z_2+z) = k_2, \dots, X_n(z_n+z) = k_n \}$$

(ii) Stationary

⇒ (i) Independent Increments

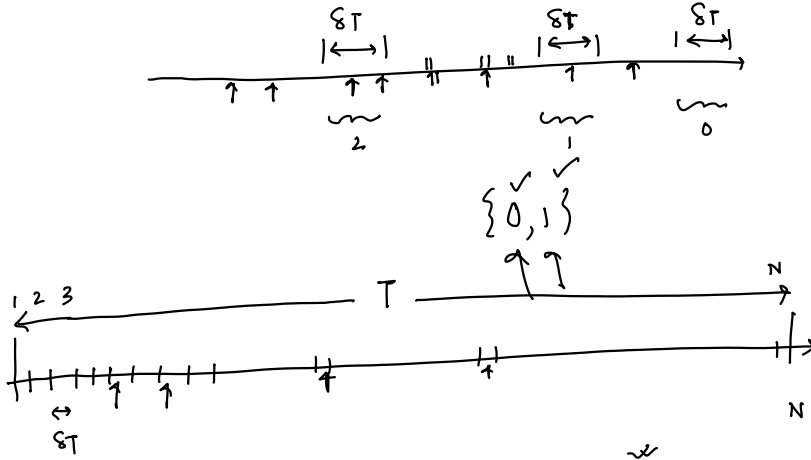
⇒ (ii) Non-Simult. princ.

Markovian / Memoryless



$$\begin{cases} P_{n_1}(t_2, t_1) \\ P_{n_2}(t_3, t_2) \end{cases}$$

T



$$\delta T \rightarrow 0$$

$$\text{arrival rate} = \lambda$$

$$\# \text{ of arrivals} = \lambda T$$

$$p_n(\delta t) = \frac{\lambda T}{N} = \frac{\lambda \delta t}{1} = \lambda \delta t$$

$$\delta T \rightarrow 0 \Rightarrow N \rightarrow \infty$$

$$N = \frac{T}{\delta T}$$

$$p_1(\delta t) = \lambda \delta t$$

$$p_1(\delta t) = c_1 \delta t + c_2 (\delta t)^2 + c_3 (\delta t)^3 + \dots$$

$$p_n(\delta t) = \lambda \delta t + o(\delta t)$$

$$\lim_{\delta t \rightarrow 0} \frac{o(\delta t)}{\delta t} = 0$$

$$\lim_{\delta t \rightarrow 0}$$

$$p_n(\delta t) = 0 \quad n > 1$$

$$p_n(\delta t) = o(\delta t)$$

$$p_0(\delta t) = 1 - p_1(\delta t) - \sum_n p_n(\delta t)$$

$$= 1 - \lambda \delta t + o(\delta t)$$

$$p_1(\delta t) + p_2(\delta t) + \dots = c_1(\delta t) + c_2(\delta t)^2 + \dots + c'_1(\delta t)^4 + c'_2(\delta t)^5 + \dots$$

$$= (c_1 + c'_1)(\delta t)^2 + (c_2 + c'_2)(\delta t)^3 + \dots$$

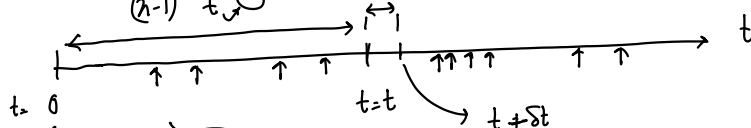
$$= o(\delta t)$$

$$p_0(\delta t) = 1 - \lambda \delta t + o(\delta t)$$

$$p_1(\delta t) = \lambda \delta t + o(\delta t)$$

$$p_n(\delta t) = o(\delta t) \quad \forall n > 1$$

$$\lim_{\delta t \rightarrow 0} \frac{o(\delta t)}{\delta t} = 0$$



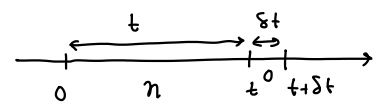
$$p_n(t) \rightarrow p_n(t + \delta t)$$

$$p_n(t + \delta t) = \text{prob.} [n \text{ arrival in } 0 \text{ to } (t + \delta t)]$$

$$= \text{prob.} [n \text{ Ar. in } t \text{ and } 0 \text{ in } \delta t] + \text{prob.} [(n-1) \text{ in } t \text{ and } 1 \text{ in } \delta t] + \dots$$

$$\delta t \rightarrow 0$$

$$= p_n(t) \cdot \underline{p_0(\delta t)}$$



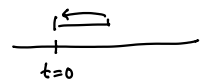
$$+ p_{n-1}(t) p_1(\delta t) + p_{n-2}(t) p_2(\delta t) + \dots$$

$$= p_n(t) [1 - \lambda \delta t + o(\delta t)] + p_{n-1}(t) [\lambda \delta t + o(\delta t)] + o(\delta t)$$

$$p_n(t + \delta t) = p_n(t) - \lambda p_n(t) \delta t + p_{n-1}(t) \lambda \delta t + o(\delta t)$$

$$\lim_{\delta t \rightarrow 0} \frac{p_n(t + \delta t) - p_n(t)}{\delta t} = \lim_{\delta t \rightarrow 0} \left[-\lambda p_n(t) + \lambda p_{n-1}(t) + \frac{o(\delta t)}{\delta t} \right]$$

$$\textcircled{n} \quad \frac{\partial p_n(t)}{\partial t} = -\lambda p_n(t) + \lambda p_{n-1}(t)$$



$n=0$

$$\Rightarrow \frac{\partial p_0(t)}{\partial t} = -\lambda p_0(t)$$

$$\begin{cases} p_n(0) = 1 & n=0 \\ = 0 & n > 0 \end{cases}$$

$$p_n(t) = 0 \quad n < 0$$

$$\frac{\partial p_1(t)}{\partial t} = -\lambda p_1(t) + \lambda p_0(t) \quad n=1$$

$$\int \frac{\partial p_0(t)}{p_0(t)} = -\int_0^t \lambda dt$$

$$\ln p_0(t) \Big|_{p_0(0)}^{p_0(t)} = -\lambda t$$

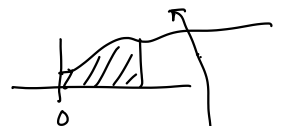
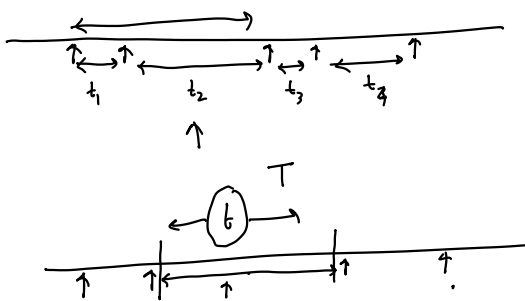
$$p_0(0) = 1$$

$$p_0(t) = e^{-\lambda t}$$

$$\left\{ \frac{\partial p_1(t)}{\partial t} = -\lambda p_1(t) + \lambda e^{-\lambda t} \right\} \quad p_1(t) = (\lambda t) e^{-\lambda t}$$

$$p_n(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}$$

Poisson Dis.



$$\begin{aligned} \text{CDF} &= \text{prob.} [\tau \leq t] \\ &= 1 - \text{prob.} [\tau > t] \\ &= 1 - p_0(t) \\ &= 1 - e^{-\lambda t} \end{aligned}$$

$$pdf = \frac{\partial}{\partial t} \text{CDF} = \lambda e^{-\lambda t} \quad \text{exponential Dis.}$$