# Module 2

# Lossless Image Compression Systems

# Lesson
# 2

# Image Compression Systems

## Instructional Objectives

At the end of this lesson, the students should be able to:

1. Identify and classify the redundancies present in an image.
2. Distinguish between lossless and lossy image compression.
3. Measure the quality of reconstructed images.
4. State the elements of an image compression system.
5. State the objectives of each of the elements of image compression system.

## 2.0 Introduction

In the last lesson, we have seen that one of the major challenges in multimedia communication is to transmit the multimedia signals, especially the image and the video signals through limited bandwidth channels. As we had outlined, it is possible to achieve compressions through significant reduction of bit-rate requirements by exploiting the redundancies that is typically present in image and video signals.

In this lesson, we shall first discuss the types of redundancies present in an image. Redundancies are exploited at the transmission end, where the compressed image and video signals are represented in an encoded form. A reversal of this process is needed at the receiver end before the signal can be reconstructed and presented. All image and video compression techniques are not exactly reversible. Some of the compression techniques are lossless in the sense that exact reconstruction is possible, whereas others achieve a greater amount of compression by incurring some loss of quality. After discussing lossless and lossy image compression schemes, we shall show the basic blocks of an image compression and discuss the roles of each of those elements.

The compression schemes adopted for images and videos are same in nature, except for the fact that video signals being spatio-temporal in nature, exploitation of temporal redundancies is possible and hence significant compression can be achieved. However, in this lesson, we only discuss image compression and defer the discussions related to temporal redundancy exploitation and video compression for later lessons.

## 2.1 Redundancies and how to exploit those?

As already discussed in lesson-1 (Section-1.4.1), image compression is largely possible by exploiting various kinds of redundancies which are typically present in an image. The extent of redundancies may vary from image to image.

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***Fig 2.1 Image at the left has less details and more Redundancy than the image at the right***

It may be observed that Fig.2.2 has much higher degree of details and hence less redundancies, as compared to Fig.2.1. Example of two synthetic images will exaggerate the situation. Click at the next link.

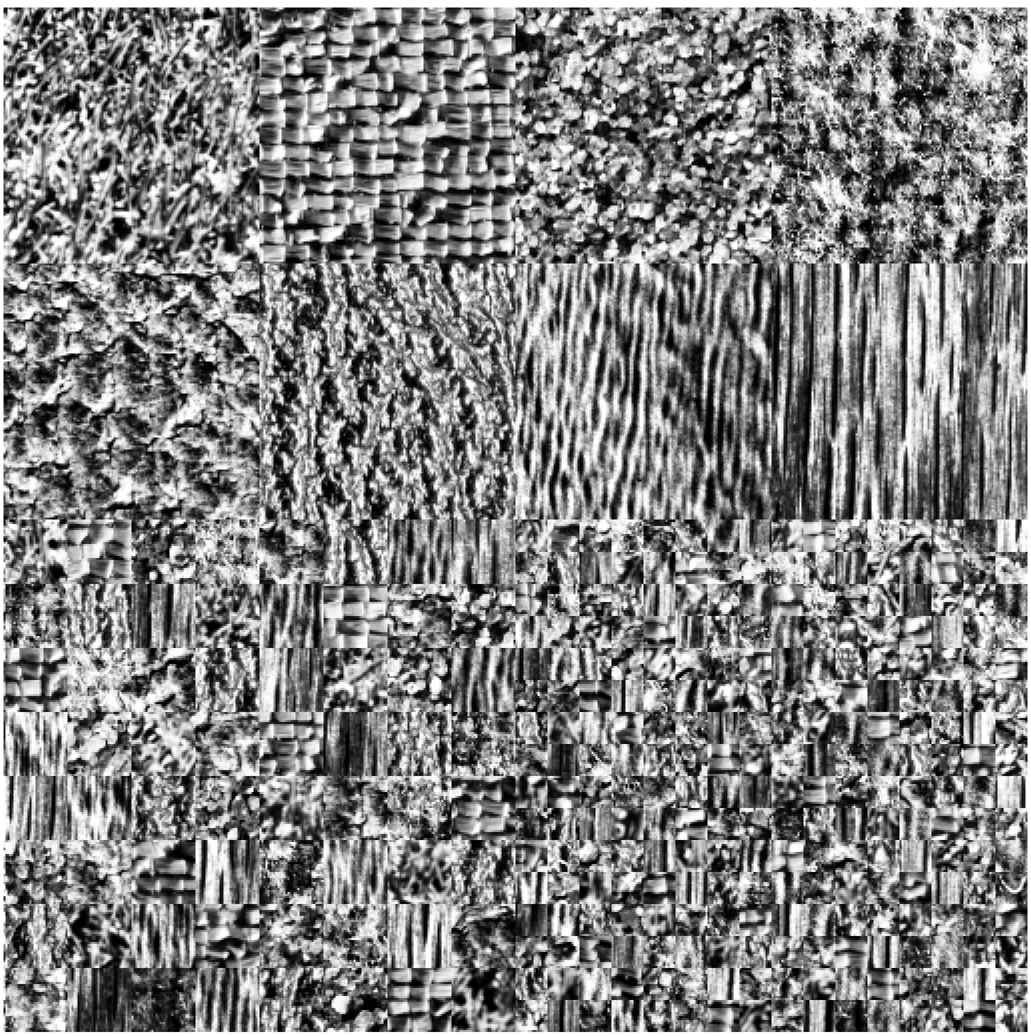

***Fig 2.2 Texture image examples. Contains less redundancy***

Rect.raw

***Fig 2.3 Example Image containing one rectangular foreground object against background***

Fig.2.3 is a simple image consisting of one rectangular foreground object of uniform intensity against a dark background of uniform intensity. To represent this image without incurring any loss of data, we need to include only the following information:

- Background intensity.
- Foreground intensity.
- Size and position of the foreground object.

If the intensity levels are represented by one byte, size (height and width) by two bytes and position (x-y coordinates of a reference corner point) by two bytes, we require just 6 bytes of data, as compared to the original 256 x 256 pixel array, consisting of 65536 bytes. In this case, very high compression is achievable. Now, look at Fig.2.4. It is a synthetically generated random-dot image. Can you perceive any redundancy? There is hardly any! It is not possible to predict any pixel from its immediate neighbors. To represent this image without incurring any loss, we need to include each pixel value individually and hence, there is no redundancy.
Natural images however are not as simple as that of Fig.2.3 and not as complex as that of Fig.2.4. Definitely, the extent of redundancy varies from image to image.

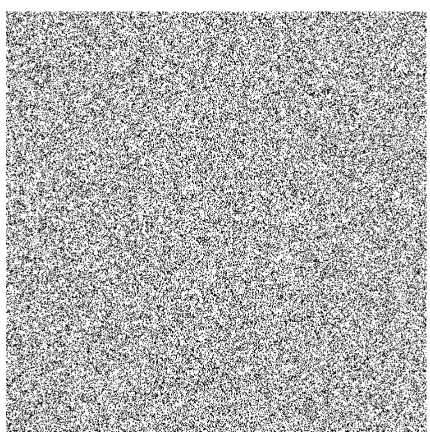

***Fig.2.4 Random Dot Image***

### 2.1.1 Classification of Redundancies

Redundancies in images may be categorized as follows –

**(a) Statistical Redundancy**

Statistical redundancy occurs due to the fact that pixels within an image tend to have very similar intensities as those of its neighborhood, except at the object boundaries or illumination changes. For still images, statistical redundancies are essentially spatial in nature. For natural two-dimensional images, redundancies are present along both the x- and y-dimensions. Video signals exhibit yet another form of statistical redundancy and that is temporal. For video, intensities of same pixel positions across successive frames tend to be very similar, unless there is large amount of motion present. In this lesson however, we focus our attention to only one form of statistical redundancy and that is spatial redundancies of images. Presence of statistical redundancies in natural images allows efficient representation in the transformed output (to be discussed in Section-2.4.1 of this lesson).

**(b) Psychovisual Redundancy**

Psychovisual redundancy arises due to the problem of perception. Our eyes are more responsive to slow and gradual changes of illumination than perceiving finer details and rapid changes of intensities. Hence, to what extent we should preserve the details for our perception and to what extent we can compromise on the quality of reconstructed image that we perceive is essentially carried out by exploiting the psychovisual redundancy. As we shall see later, psychovisual redundancy has been well studied and its exploitation has been included within the multimedia standards.

## 2.2 Lossless and lossy image compression

In general, image compression schemes can be broadly classified into two categories, viz,

(a) Lossless Image Compression

(b) Lossy Image Compression

### *2.2.1* Lossless Image Compression

As the name implies, lossless image compression schemes exploit redundancies without incurring any loss of data. Thus, the data stream prior to encoding and after decoding is exactly the same and no distortion in the reconstruction quality is observed. Lossless image compression is therefore exactly reversible.

Lossless compression is achieved through the exploitation of statistical redundancy. For example, if we transform the image into a string of symbols prior to encoding and then assign shorter code words to more frequently occurring symbols and longer code words to less frequently occurring symbols, then we can achieve compression and at the same time, the encoding process can be exactly reversed during decoding, since there is an one-to-one mapping between the symbols and their codes.

We shall learn more about lossless compression in subsequent lessons, when we study schemes like run-length encoding, entropy coding, Ziv-Lempel coding etc. It may be noted that lossless image compression schemes can achieve only limited extent of bandwidth reduction for data transmission, but preserves the quality of the image, without suffering any distortion.

### *2.2.2* Lossy Image Compression

Contrary to lossless image compression, lossy image compression schemes incur loss of data and hence suffer a loss of quality in reconstruction. Like lossless image compression, the image is first transformed into a string of symbols, which are quantized to a discrete set of allowable levels. In this scheme, it is possible to achieve significant data compression, but quantization being a many-to-one mapping is irreversible and exact reconstruction is never possible. Yet, if the loss in reconstruction quality is acceptable to our visual perception, we may accept this scheme in the interest of achieving very significant degree of compression.

It is to be noted that lossy compression schemes essentially exploit the psychovisual redundancy. While designing the quantizers, it must be known where we can tolerate loss of quality and where we can not. This requires considerable studies related to psychovisual observations and quantization tables prepared for standards like JPEG, MPEGs, H.26x etc, results of such studies have been taken into consideration.

We shall understand lossy image compression schemes better in subsequent lessons when we study the quantizers in depth and see how the quantization tables are used in multimedia standards.

## 2.3 Measuring the quality of reconstructed images

The reconstructed images obtained through lossy compression/de-compression schemes are never exactly the same as the original. The simplest measure often employed to measure the quality of such reconstructed images is *Mean Square Error (MSE).* For an M x N original image array *f(i,j),* where *i* and *j* are the row and column indices, *(i=0,1,……,M-1; j=0,1,……,N-1)*, if $f'(i,j)$ is the reconstructed image array of the same size, the *MSE* is given by

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left[f(i,j) - f'(i,j)\right]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1)$$

The mean squared error essentially measures the noise in the reconstruction. The reconstruction quality is also measured with *Signal-to-Noise Ratio (SNR)* and the *Peak Signal-to-Noise Ratio (PSNR)*. In the former, the noise power is measured with respect to the actual signal power, whereas in the latter, the noise power is measured with respect to the peak signal power, considering the intensities in the range of 0-255. These measures are given by

$$SNR = 20\log_{10} \frac{\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[f(i,j)]^2}{\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[f(i,j) - f'(i,j)]^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.2)$$

$$PSNR = 20\log_{10} \frac{255^2 MN}{\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}[f(i,j) - f'(i,j)]^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.3)$$

Both these measures are expressed in the units of decibels (dB). It may however be noted that these measures often do not correspond well to the perceptual quality, but in absence of any perceptual measure, which is essentially subjective in nature, one can use these measures to judge the quality of reconstruction.

## 2.4 Elements of Image Compression System

Due to the high degree of statistical (spatial) and psychovisual redundancies present in a natural image, it can be compressed without significant degradation of the visual quality. A typical image compression system consists of the following elements –

(a) Transformer, (b) quantizer and

(c) coder. An image compression system is often referred to in the literature as image encoder.

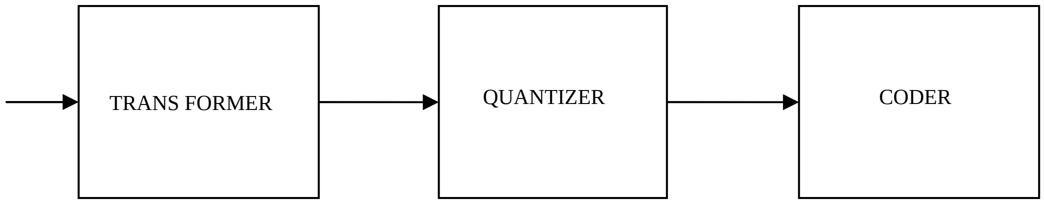


***Fig 2.5 Elements of image encoding system***

The roles of these blocks are explained below.

### 2.4.1 Transformer

This block transforms the original input data into a form that is more amenable to compression. The transformation can be local, involving pixels in the neighbourhood or global, involving the full image or a block of pixels. The example of local transformation is linear predictive coding followed by Differential Pulse Code Modulation (DPCM), which we are going to discuss in the next lesson. Global transformation techniques use Discrete Fourier Transforms (DFT), Discrete Cosine Transforms (DCT), Karhunen-Love Transforms (KLT), Discrete Wavelet Transforms (DWT) etc, which we are going to discuss in the subsequent lessons. The transformer block transforms the original spatial domain signal into another spatial domain signal of reduced dynamic range, as is done in DPCM or into the transform domain, where only a few coefficients contain bulk of the energy and efficient compression is possible. Please note that this block in itself does not perform any compression and is lossless.]

### 2.4.2 Quantizer

The quantizer follows the transformer block in image compression systems and generates a limited number of symbols that can be used in the representation of the transformed signal. It is a many-to-one mapping which is irreversible. Quantizers are of two basic types-

(i) **Scalar quantization –** refers to element-by-element quantization of data

(ii) **Vector quantization -** refers to quantization of a block at a time. Quantization exploits psychovisual redundancy and achieves significant bit reduction. It is the only block in image compression system, which is lossy. We shall discuss quantizers with further details in subsequent lessons.

### 2.4.3 Coder

Coders assign a code word, a binary bit-stream, to each symbol at the output of the quantizer. The coder may employ

(i) Fixed-length coding (FLC), which have codeword length fixed, irrespective of the probabilities of occurrence of quantized symbols or,

(ii) Variable length coding (VLC) , also known as entropy coding, assigns code words in such a way as to minimize the average length of the binary representation of the symbols. This is achieved by assigning shorter code words to the more probable symbols.We shall study VLC with greater details in the subsequent lessons.

## 2.5 Elements of Image De-compression system

The output of an image compression system is a bit-stream in an encoded form and hence can not be displayed. In an image communication system, where the image is transmitted from one place to another, the encoded bit-stream is sent through a communication channel, which is ideally lossless but is noisy and hence lossy for all practical channels. At the receiver end, the encoded bit-stream received through the communication channel has to be decoded before it can be displayed. The image de-compression system, also known as image decoding system should do exact reversal of the processes adopted during encoding.

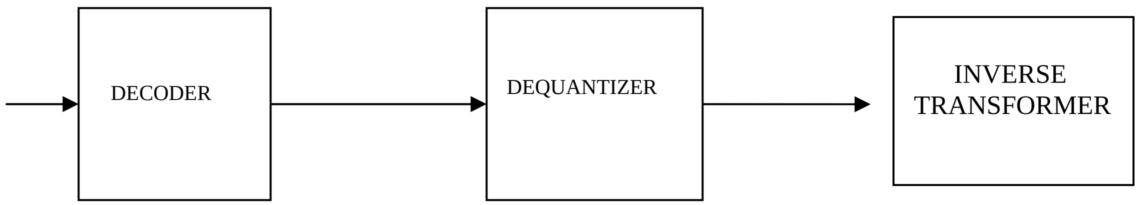


***Fig 2.6 Elements of Image decoding system***

An image de-compression system consists of the following elements –

(a) **Image decoder –** Performs exact reversal of the coder in image compression system. This block extracts the quantized coefficients.

(b) **De-quantizer-** Performs inverse of the quantization operation in image compression. Since quantizer itself is lossy, de-quantization can never exactly recover the transformed coefficients.

(c) **Inverse Transformer –** Performs exact reversal of the transformation operation carried out in the corresponding image compression system. The output of this block can be used for display.

## Questions

**NOTE:** The students are advised to thoroughly read this lesson first and then answer the following questions. Only after attempting all the questions, they should click to the solution button and verify their answers.

## PART-A

A.1. What are the types of redundancies present in an image?

A.2. Distinguish between lossless and lossy image compression schemes.

A.3. Write down the mathematical definitions of –

(i) Mean Square Error, (ii) Signal to Noise Ratio and

(iii) Peak Signal to Noise Ratio. Do these measures always correspond to our subjective assessment about image quality?

A.4. Name the elements of an image compression system.

A.5. Name the elements of an image de-compression system.

A.6. Outline the roles of each block in an image encoding system.

## PART-B: Multiple Choice

In the following questions, click the best out of the four choices.

B.1 A still image has the following form(s) of statistical redundancy.

(A) Spatial redundancy.

(B) Temporal redundancy.

(C) Spatial as well as temporal redundancy.

(D) Neither spatial nor temporal redundancy.

B.2 A still image with uniform intensity exhibits

(A) Best spatial redundancy.

(B) Worst spatial redundancy.

(C) Best temporal redundancy.

(D) Worst temporal redundancy.

B.3 Entropy coding is a

(A) Lossless coding using psychovisual redundancy.

(B) Lossy coding using psychovisual redundancy.

(C) Lossless coding using statistical redundancy.

(D) Lossy coding using statistical redundancy.

B.4 In image compression systems, quantizer is a

(A) Lossless element using psychovisual redundancy.

(B) Lossy element using psychovisual redundancy.

(C) Lossless element using statistical redundancy.

(D) Lossy element using statistical redundancy.

B.5 For a lossless encoder-channel-decoder combination, the PSNR of the reconstructed image measured in dB will be

(A) Zero

(B) 20-dB

(C) 40-dB

(D) Infinity.

B.6 A quantizer at the encoder performs

(A) One-to-one mapping.

(B) One-to-many mapping.

(C) Many-to-one mapping.

(D) Many-to-many mapping.

B.7 An inverse quantizer at the decoder performs

(A) One-to-one mapping.

(B) One-to-many mapping.

(C) Many-to-one mapping.

(D) Many-to-many mapping.

B.8 Variable length coding (VLC) achieves compression by

(A) Using psychovisually motivated quantization tables.

(B) Assigning shorter code words to more probable symbols.

(C) Assigning longer code words to more probable symbols.

(D) Using transformations such as DFT, DCT, DWT etc.

B.9 Mean square error of reconstructed image in lossy compression schemes depends upon

(A) All the blocks in image compression/ de-compression system.

(B) Quantizer, but is image independent.

(C) Quantizer, VLC and the image.

(D) Quantizer and the image.

## PART-C: Problems

C-1. A given 4 x 4 image array and its corresponding reconstructed image array obtained through lossy compression scheme are given below:

| | | | |
|---|---|---|---|
| 148 | 129 | 133 | 89 |
| 153 | 138 | 103 | 84 |
| 155 | 141 | 92 | 78 |
| 162 | 139 | 86 | 81 |

Original

| | | | |
|---|---|---|---|
| 146 | 130 | 133 | 85 |
| 155 | 139 | 105 | 84 |
| 154 | 142 | 98 | 80 |
| 162 | 139 | 84 | 78 |

Reconstructed

Calculate the (a) *MSE,* (b) *SNR* and (c) *PSNR* of the reconstructed image array.

## SOLUTIONS

A.1 Redundancies in images can be classified into two basic types:

a) Statistical redundancy.

b) Psychovisual redundancy.

A.2

| Sr.No. | Lossless Compression | Lossy Compression |
|---|---|---|

| | | |
|---|---|---|
| 1. | There is no loss of data | There is always loss of data. |
| 2. | Exactly reversible. | Not reversible. |
| 3. | Exploits statistical redundancy. | Exploits psychovisual redundancy. |

A.3 Refer to equations (2.1) to (2.3) in Section-2.3.
No, these measures need not correspond well with the subjective assessment of quality.

A.4 The elements of an image compression system are –

a) Transformer.

b) Quantizer.

c) Coder.

A.5 The elements of an image de-compression system are –

a) Decoder.

b) De-quantizer.

c) Inverse transformer.

A.6 Refer to Section-2.4.1, Section-2.4.2 and Section-2.4.2 .

B.1 (A) B.2 (A) B.3 (C) B.4 (B) B.5 (D)

B.6 (C) B.7 (A) B.8 (B) B.9 (D).

C.1 For a 4 x 4 image, M=N=4.

$$MSE = \frac{1}{MN}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left[f(i,j) - f'(i,j)\right]^2 = \frac{1}{4^2}\left[\begin{pmatrix} (148-146)^2 + (129-130)^2 + (133-133)^2 + (89-85)^2 \\ +(153-155)^2 + (138-139)^2 + (103-105)^2 + (84-84)^2 \\ +(155-154)^2 + (141-142)^2 + (92-98)^2 + (78-80)^2 \\ +(162-162)^2 + (139-139)^2 + (86-84)^2 + (81-78)^2 \end{pmatrix}\right]$$

$$= \frac{1}{16}(4+1+16+4+1+4+1+1+36+4+4+9)$$

$$= 5.31$$

$$SNR = 20\log_{10} \frac{\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left[f(i,j)\right]^2}{\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left[f(i,j) - f^{'}(i,j)\right]^2}$$

$$= 20\log_{10} \frac{242249}{85}$$

$$= 69.09$$

$$PSNR = 20\log_{10} \frac{255^2 MN}{\sum_{i=0}^{M-1}\sum_{j=0}^{N-1}\left[f(i,j) - f^{'}(i,j)\right]^2} \ldots$$

$$= 20\log_{10} \frac{1040400}{16}$$

$$= 96.26$$