Module
4

MULTI-RESOLUTION ANALYSIS

Lesson
11

Multi-resolution Analysis: Theory of Subband Coding

Instructional Objectives

At the end of this lesson, the students should be able to:

1. Show a two-band filter bank for subband coding and decoding of one-dimensional signals.
2. Define analysis and synthesis filters.
3. Explain the need for downsampling after the analysis and upsampling before the synthesis.
4. Determine the z-transforms of downsampled and upsampled signals.
5. Determine the z-transform of the reconstructed signal.
6. Derive the conditions for error-free reconstruction of the signal.
7. Show that the analysis and synthesis filter banks for error-free reconstruction fulfill the conditions of bi-orthogonality.
8. Extend the idea of subband coding for a two-dimensional four-band filter bank.
9. Repetitively apply four-band split on a given image using FIR analysis filter bank.
10. Synthesize an image from the subbands using FIR synthesis filter bank.

11.0 Introduction

In lesson-10, we had introduced the idea of multi-resolution image analysis using scaling and wavelet functions. It was shown that the scaling functions provide low frequency analysis of the signals to obtain approximations and the wavelet functions provide high-frequency analysis of the signals to extract the details. In this lesson, we present a more general approach towards subband analysis and synthesis. The basic theories presented herein would provide an insight into the design requirements of the filter banks needed for analysis and synthesis. The conditions for error-free reconstruction of signals will be derived and the concepts will be extended for two-dimensional signals, i.e. images. In the next lesson, we are going to show how Discrete Wavelet Transforms (DWT), to be defined later would perfectly fit into the subband coding and decoding requirements. It should however be noted that the theories presented for subband coding is a general one and DWT is just one of the ways, but not the only one to perform subband coding.

11.1 Two-band Analysis of Signals

Before we discuss about the subband analysis of images, let us first consider the simplest example of a band-limited one-dimensional signal, having a cut-off frequency $\omega_c = \pi$, that is, exactly half of the sampling frequency $\omega_s = 2\pi$. Suppose, we pass this signal through a bank of two digital filters – the first one being a low-pass filter having impulse response $h_0(n)$ and upper cut-off frequency $\omega_0 = \pi/2$ and the second one being a high-pass filter having impulse response $h_1(n)$ and lower cut-off frequency $\omega_1 = \pi/2$.
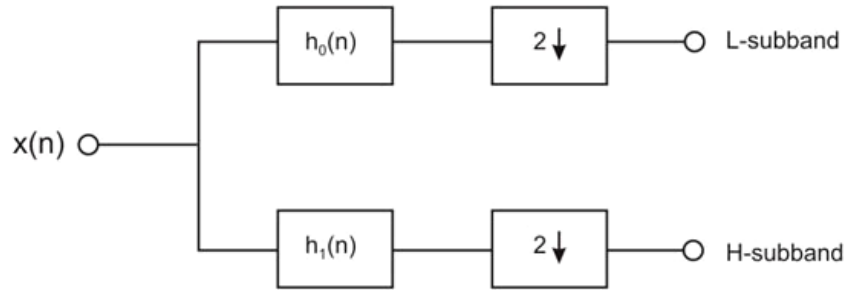

Fig11.1: Two-Band analysis filter

Fig.11.1 shows the block diagram of the filter bank, which essentially analyzes the signal into two subbands, whose spectral responses are as shown in Fig.11.2.
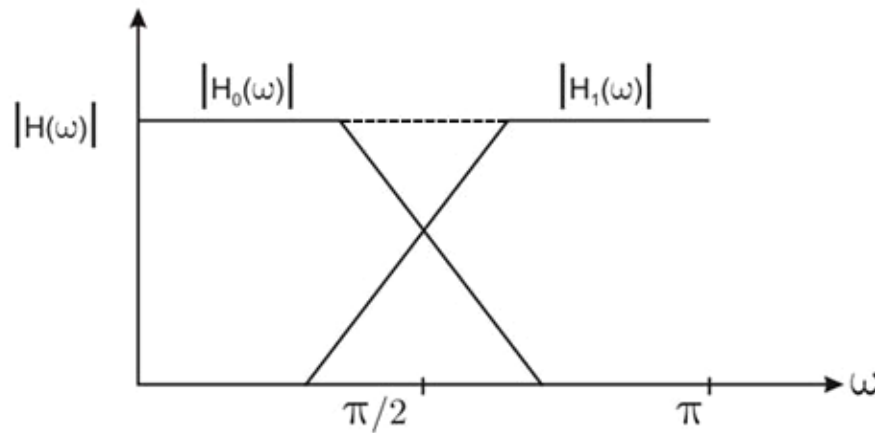

Fig11.2: Spectral response of two-bands of analysis filter.

Since the filter banks perform subband analysis, the filters are known as analysis filters.

If we perform subband analysis on *N* samples of an input sequence $x(n)$ for $n = 0,1,\cdots\cdots,N-1$, both the low-frequency and high-frequency analysis filter outputs will have *N* samples. However, it should be noted that since the

bandwidth of the signal at each of the analysis filter outputs is only one-half of the original signal, the analysis filter outputs can be sampled at half the original Nyquist rate. In other words, half of the samples at the analysis filter outputs are redundant and hence every one out of two consecutive samples at the filter output can be dropped. Therefore, the analysis filter outputs are downsampled by a factor of two, as shown in the block diagram.
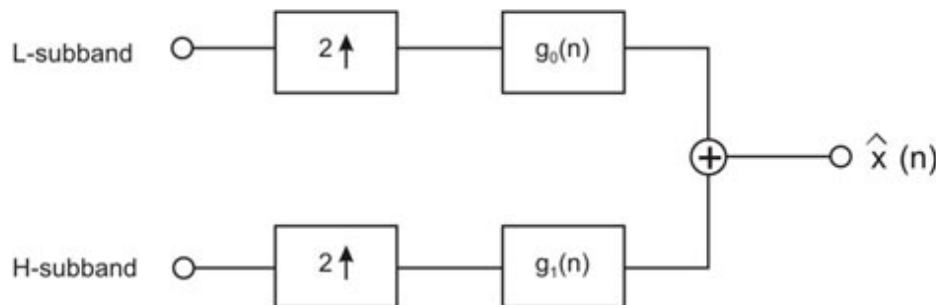
11.2 Two-band Synthesis of Signals

Fig11.3: *Two-band synthesis filter.*

Synthesis is just the reverse of analysis, as shown in fig.11.3. The low-pass and high-pass filtered subbands are first upsampled by a factor of two, so that each of the subband filters, having impulse responses $g_0(n)$ and $g_1(n)$ respectively for low and high subbands, generate *N* samples for an input sequence of length *N* samples. The outputs of these two filters are added to generate the reconstructed sequence $\hat{x}(n)$ for $n = 0,1,\cdots\cdots,N-1$. Since the two filter outputs do the synthesis of the signal, these filters are known as *synthesis filters.* The two-band analysis and synthesis can be shown in the form of a combined block diagram of fig.11.4.

11.3 Conditions for perfect reconstruction in analysis-synthesis filters

We are now going to derive the necessary conditions for perfect reconstruction in analysis-synthesis filters. For a perfect reconstruction, we must have $x(n) = \hat{x}(n)$ for $n = 0,1,\cdots\cdots,N-1$ and to achieve this, the analysis and the synthesis filters must fulfill some conditions. Since, filtering involves convolution in the time (or spatial) domain and multiplication in the transform-domain, it is easier to derive the conditions of perfect reconstruction in transform-domain and we use the *z*-transforms for this purpose. The choice of *z*-transform is motivated by the fact

that it can handle sampling-rate changes, as required by us due to the involvement of down-sampling in analysis and up-sampling in synthesis.

11.3.1 z-transform of down-sampled sequence:

The *z*-transform of the original sequence $x(n)$, $n = 0,1,2,\cdots\cdots$ is given by

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)z^{-n} \quad\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.1)$$

where, *z* is a complex variable. Down-sampling $x(n)$ by a factor of two preserves every alternate samples of the original sequence, i.e. $x(2n)$ for $n = 0,1,2,\cdots\cdots$. Its *z*-transform is therefore given by

$$X_{down}(z) = \sum_{n=-\infty}^{\infty} x(2n)z^{-n}$$

$$= x(0)z^0 + x(2)z^{-1} + x(4)z^{-2} + \cdots\cdots + x(-2)z^1 + x(-4)z^2 + \cdots\cdots$$

$$= \frac{1}{2}\left[x(0)z^0 + x(1)z^{-1/2} + x(2)z^{-1} + \cdots\cdots + x(-1)z^{1/2} + x(-2)z^1 + \cdots\cdots\right] \quad .. (11.2)$$

$$+ \frac{1}{2}\left[x(0)z^0 - x(1)z^{-1/2} + x(2)z^{-1} - \cdots\cdots - x(-1)z^{1/2} + x(-2)z^1 + \cdots\cdots\right]$$

$$= \frac{1}{2}\left[X(z^{1/2}) + X(-z^{1/2})\right]$$

11.3.2 z-transform of up-sampled sequence:

In the process of up-sampling, zeros are added for every odd sample and thus, up-sampling a given sequence $x(n)$ for $n = 0,1,2,\cdots\cdots$ can be expressed in terms of the samples of given sequence as

$$x^{up}(n) = \begin{cases} x\left(\dfrac{n}{2}\right) & \text{for } n = 0,2,4,\cdots\cdots \\ 0 & \text{otherwise} \end{cases}$$

The *z*-transform of the up-sampled sequence may be expressed as

$$X^{up}(z) = x(0)z^0 + x(1)z^{-2} + x(2)z^{-4} + \cdots\cdots + x(-1)z^2 + x(-2)z^4 + \cdots\cdots$$
$$= X(z^2)$$
$$\dots\dots\dots\dots (11.3)$$

Without using any analysis or synthesis filter, if we simply cascade a down-sampler and an up-sampler, then the *z*-transform of the reconstructed sequence is given by combining equations (11.2) and (11.3) as

$$\hat{X}(z) = \frac{1}{2}[X(z) + X(-z)] \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \text{(11.4)}$$

The second term of the above equation represents the z-transform of the aliased version of the signal $\hat{x}(n)$. Its inverse z-transform is given by

$$Z^{-1}[X(-z)] = (-1)^n x(n) \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \text{(11.5)}$$

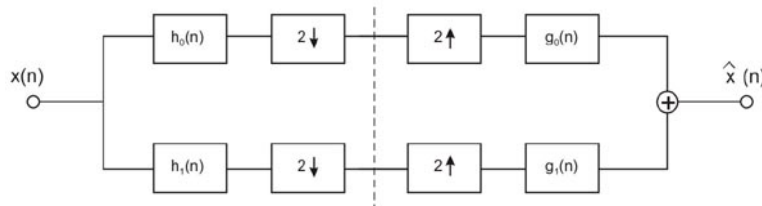### 11.3.3 z-transform of subband coding/decoding:

Fig11.4 Analysis-synthesis of 2-band decomposition and reconstruction

With this background, we can now consider the block-diagram of fig.11.4 and write the z-transform of the reconstructed signal by using equation (11.4) and using the property that convolution in time (or spatial) domain is multiplication in frequency domain as

$$\hat{X}(z) = \frac{1}{2}G_0(z)[H_0(z)X(z) + H_0(-z)X(-z)]$$
$$+ \frac{1}{2}G_1(z)[H_1(z)X(z) + H_1(-z)X(-z)] \quad \dots \dots \dots \dots \dots \dots \dots \dots \text{(11.6)}$$

Re-arranging the terms, we obtain

$$\hat{X}(z) = \frac{1}{2}[G_0(z)H_0(z) + G_1(z)H_1(z)]X(z)$$
$$+ \frac{1}{2}[G_0(z)H_0(-z) + G_1(z)H_1(-z)]X(-z) \quad \dots \dots \dots \dots \dots \dots \text{(11.7)}$$

where the second term involving –z indicates the aliased components.

11.3.4 Conditions for error-free reconstruction of signals:
From equation (11.7), it is easy to derive the necessary conditions for error-free reconstruction of the signal at the analysis-synthesis filter bank output. For error-free reconstruction, we must ensure

$$X(z) = \hat{X}(z) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(11.8)$$

the conditions for which are obtained from equation (11.7) as

$$G_0(z)H_0(z) + G_1(z)H_1(z) = 2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(11.9)$$

and

$$G_0(z)H_0(-z) + G_1(z)H_1(-z) = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(11.10)$$

Equation (11.9) ensures that there is no amplitude distortion in reconstruction and equation (11.10) forces the condition that there is no aliasing. Both these equations can be combined in a matrix form as

$$\begin{bmatrix} G_0(z) & G_1(z) \end{bmatrix} \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix} = \begin{bmatrix} 2 & 0 \end{bmatrix} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots(11.11)$$

We define $\mathbf{H}_m(z) = \begin{bmatrix} H_0(z) & H_0(-z) \\ H_1(z) & H_1(-z) \end{bmatrix}$ as the analysis modulation matrix and

hence, equation (11.11) can be rewritten as

$$\begin{bmatrix} G_0(z) & G_1(z) \end{bmatrix} \mathbf{H}_m(z) = \begin{bmatrix} 2 & 0 \end{bmatrix} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.12)$$

Assuming that the matrix $\mathbf{H}_m(z)$ is non-singular, we can take the transpose of both the sides of equation (11.12) and then pre-multiply by $\left(\mathbf{H}_m^T(z)\right)^{-1}$, we obtain

$$\begin{bmatrix} G_0(z) \\ G_1(z) \end{bmatrix} = \frac{2}{\det(\mathbf{H}_m(z))} \begin{bmatrix} H_1(-z) \\ -H_0(-z) \end{bmatrix} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.13)$$

where $\det(\mathbf{H}_m(z))$ is the determinate of the matrix $\mathbf{H}_m(z)$.
The above equation reveals an interesting fact that $G_0(z)$ is a function of $H_1(-z)$ and $G_1(z)$ is a function of $H_0(-z)$, which means that the analysis and the synthesis filters are cross-modulated. For Finite Impulse Response (FIR) filters,

the determinate of the matrix $\mathbf{H}_m(z)$ is a pure delay and is given by $\det(\mathbf{H}_m(z)) = \alpha z^{-(2k+1)}$. Neglecting the delay, letting $\alpha = 2$ and taking inverse transforms, we obtain

$$g_0(n) = (-1)^n h_1(n)$$
$$g_1(n) = (-1)^{n+1} h_0(n)$$ ……………………………………………………… (11.14)

If $\alpha = -2$, the above expressions are sign-reversed, i.e.

$$g_0(n) = (-1)^{n+1} h_1(n)$$
$$g_1(n) = (-1)^n h_0(n)$$ …………………………………………………….. (11.15)

Hence, for error-free reconstruction, the FIR synthesis filters are cross-modulated copies of analysis filters, with one of the signs reversed.

11.4 Bi-Orthogonality of analysis-synthesis filters

We define the product of the low-pass analysis and synthesis filter transfer functions as
$P(z)$. Hence, from equation (11.13), we get

$$P(z) = G_0(z)H_0(z) = \frac{2}{\det(\mathbf{H}_m(z))} H_0(z)H_1(-z) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.16)$$

Also, noting that $\det(\mathbf{H}_m(z)) = -\det(\mathbf{H}_m(-z))$, we can obtain the product of the high-pass analysis and synthesis filter transfer functions from equation (11.13) as

$$G_1(z)H_1(z) = \frac{-2}{\det(\mathbf{H}_m(z))} H_0(-z)H_1(z) = P(-z) = G_0(-z)H_0(-z) \dots\dots\dots\dots\dots (11.17)$$

Thus, one of the conditions for error-free reconstruction, given by equation (11.9) may be rewritten as

$$G_0(z)H_0(z) + G_0(-z)H_0(-z) = 2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.18)$$

Taking the inverse z-transform of the above, we obtain

$$\sum_k g_0(k)h_0(n-k) + (-1)^n \sum_k g_0(k)h_0(n-k) = 2\delta(n) \dots\dots\dots\dots\dots\dots\dots\dots (11.19)$$

where $\delta(n)$ is unit impulse function having value of unity for $n = 0$ and zero otherwise.

Since, all odd-indexed terms get cancelled, as per equation (11.19) and the even-indexed terms add up in the left-hand side, it is possible to write equation (11.19) in a different form as

$$\sum_k g_0(k)h_0(2n-k) = \langle g_0(k), h_0(2n-k) \rangle = \delta(n) \dots\dots\dots\dots\dots\dots\dots (11.20)$$

Again, using equation (11.17) and changing z to -z, it is also possible to rewrite equation (11.18) as

$$G_1(z)H_1(z) + G_1(-z)H_1(-z) = 2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.21)$$

From the above equation, we can similarly derive
$$\langle g_1(k), h_1(2n-k) \rangle = \delta(n) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.22)$$

Taking equation (11.10) as the starting point, we can also derive two other conditions

$$\langle g_0(k), h_1(2n-k) \rangle = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.23)$$

$$\langle g_1(k), h_0(2n-k) \rangle = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11.24)$$

The four conditions given in equations (11.20), (11.22)-(11.24) can be expressed in a combined form of bi-orthogonality condition as

$$\langle g_i(k), h_j(2n-k) \rangle = \delta(i-j)\delta(n), \quad i,j = \{0,1\} \dots\dots\dots\dots\dots\dots (11.25)$$

Thus, the bi-orthogonality of analysis and synthesis filter responses is essentially the condition for error-free reconstruction.

Bi-orthogonality conditions are fulfilled by following classes of filters: Quadrature Mirror Filters (QMF), Conjugate Quadrature Filters (CQF) and Orthonormal filters. For each class, a prototype filter is constructed, based on the filter

specifications and then the other analysis and synthesis filters are derived from the prototype. For QMF and CQF, the readers are referred to the references provided. The orthonormal class of filters requires some special mention. These filters not only satisfy bi-orthogonality, but also orthonormality, as given by

$$\langle g_i(n), g_j(n+2m)\rangle = \delta(i-j)\delta(m), \qquad i,j = \{0,1\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(11.26)}$$

For orthonormal filters, the impulse response of analysis filters $h_0(n)$, $h_1(n)$ and synthesis filter $g_1(n)$ can be obtained from $g_0(n)$ as

$$\begin{aligned} g_1(n) &= (-1)^n g_0(2K-1-n) \\ h_i(n) &= g_i(2K-1-n) \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. \text{(11.27)}$$

where $2K$ is the number of taps in FIR filter. Examples of orthonormal FIR filters include the Smith and Barnwell filters, Daubechies filters and Vaidyanathan and Hoang filters. Fig.11.5 shows the impulse responses of four 8-tap Daubechies FIR filters, whose coefficients of low-pass analysis filters are shown in Table-11.1. Other filters are derived through equation (11.27).

Table-11.1 Coefficients of Daubechies 8-tap low-pass analysis filter

$h_0(0)$	-0.01059740
$h_0(1)$	0.03288301
$h_0(2)$	0.03084138
$h_0(3)$	-0.18703481
$h_0(4)$	-0.02798376
$h_0(5)$	0.63088076
$h_0(6)$	0.71484657
$h_0(7)$	0.23037781

11.5 Subband decomposition of Images

The idea of subband coding can be extended for two-dimensional signals, if the one-dimensional filters used for analysis and synthesis can be used as two-dimensional separable filters. Fig.11.6 shows the block diagram of a two dimensional four-band analysis filter bank used for subband coding. The two-dimensional signal, i.e. images having discrete set of samples $s(n_1, n_2)$ are first analyzed into low-frequency and high-frequency subbands through FIR analysis filters along the n_1-direction, i.e. along the rows (vertically) and then down-sampled by factors of 2. Each of the resulting subbands are then analyzed into the low and high-frequency subbands along the n_2-direction, i.e. along the columns (horizontally) and we thus obtain four subbands, each of which is further sub-sampled by a factor of two.
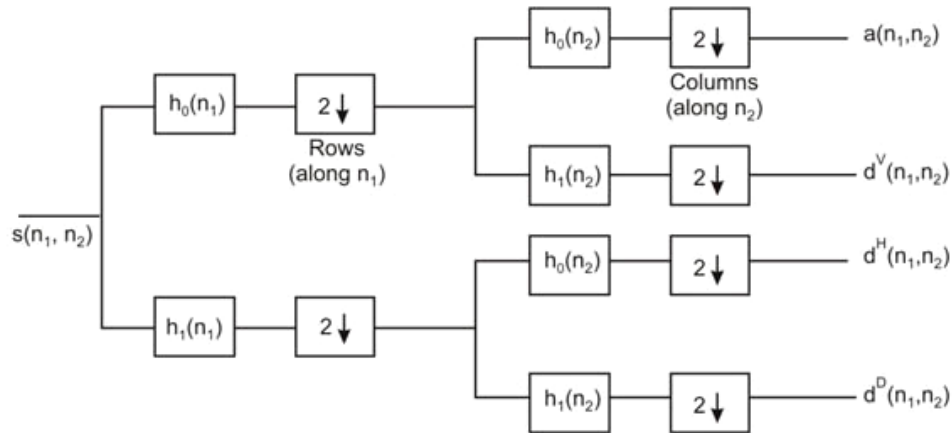

Fig 11.6: *Four-band analysis of images*

Thus, each of the subbands shown in fig.11.6 contain only one-fourth of the original samples in the pixel array. The resulting subband outputs $a(n_1, n_2)$, $d^V(n_1, n_2)$, $d^H(n_1, n_2)$ and $d^D(n_1, n_2)$ contain the approximation, the vertical details, the horizontal details and the diagonal details respectively. In terms of resulting images, these subbands are generally expressed as LL, LH, HL and HH, where the first letter indicates the filter applied in the horizontal direction ("L" for low-pass and "H" for high-pass) and the second letter indicates the filter applied in the vertical direction. Since each of the resulting images contain only one-fourth samples, we can place resulting images in the order shown in fig.11.7 to replace the original image.

Fig.11.7 Dyadic Partitioning of Images

The above principle of four-band decomposition may be applied to one or more of the subbands. Any of the resulting subbands obtained may be further

split into four subbands and so on. Click on any of the subbands in fig.11.7 to get a feeling as to how subband decomposition can be repetitively applied.

Fig11.8: *Example of 4-band decomposition of an image using Daubechies 8-tap FIR filter.*

Fig.11.8 shows an example of the 4-band decomposition of an image using Daubechies 8-tap FIR filter. The block diagram of synthesis filter banks to reconstruct the image is shown in fig.11.9. It is the mirror of the analysis filter banks of fig.11.6 and require an up-sampling by a factor of two before filtering the signal. If the filters satisfy the conditions of bi-orthogonality, the reconstruct is going to be exact. In practice, the image compression systems perform quantization of the subband coefficients and thus, the reconstructed image contains quantization errors.
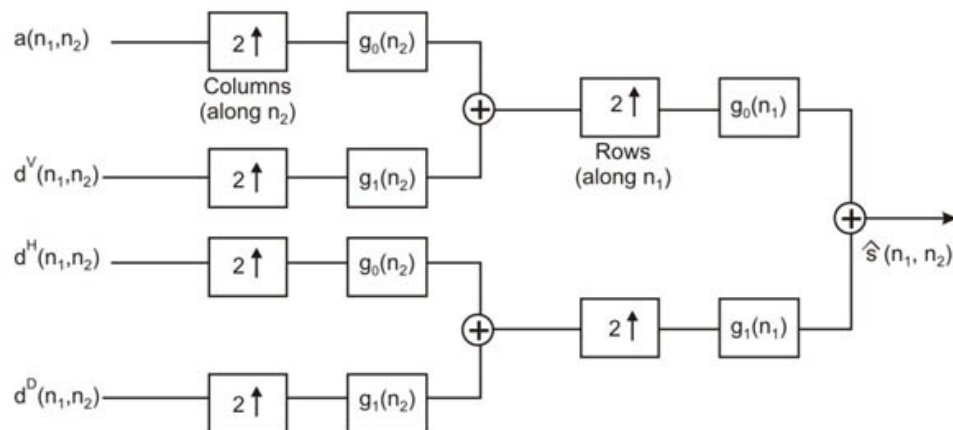

Fig11.9: *Synthesis of four subbands*

11.6 Conclusion

In this lesson, we have developed the theory for subband decomposition, considering one-dimensional signals and two subbands. We also showed the conditions for perfect reconstruction of signals. The concept of filtering was extended for two-dimensional signals and four filter banks and showed the FIR analysis and synthesis filtering to realize these. In the next lesson, we are going to relate the Discrete Wavelet Transforms with subband coding.

Questions

NOTE: The students are advised to thoroughly read this lesson first and then answer the following questions. Only after attempting all the questions, they should click to the solution button and verify their answers.

PART-A

A.1. Show the block diagrams of analysis and synthesis filters to perform two-band subband coding and decoding.

A.2. Why is downsampling done after the analysis filter?

A.3. Why is upsampling done before the synthesis filter?

A.4. Determine the z-transform of a signal downsampled by a factor of two.

A.5. Determine the z-transform of a signal upsampled by a factor of two.

A.6. Derive the z-transform of the reconstructed signal in terms of the z-transforms of the original signal and the filter transfer functions.

A.7. From the above expression, identify the aliased component. How can the aliased component be eliminated?

A.8. From the conditions of perfect reconstruction of signal at the decoder, derive the conditions for bi-orthogonality of analysis and synthesis filters.

A.9. Show the block-diagram of the analysis filter banks used to perform subband decomposition of an image into four subbands. Clearly indicate which subbands extract (a) approximated form of the image, (b) horizontal details, (c) vertical details and (d) diagonal details.

A.10. Show the block-diagram of the synthesis filters corresponding to the above.

PART-B: Multiple Choice

In the following questions, click the best out of the four choices.

B.1 In the following analysis filter, the bandwidth of the signal at the output of the filter-bank #2 in terms of angular sampling frequency is

(A) $[0, \pi/4]$ (B) $[\pi/4, \pi/2]$

(C) $[\pi/2, 3\pi/4]$ (D) $[3\pi/4, \pi]$

B.2 A one-dimensional signal $x(n)$ is downsampled by a factor of 4. The z-transform of the corresponding downsampled signal is given by

(A) $\dfrac{1}{2}[X(z) + X(-z)]$

(B) $\dfrac{1}{2}\left[X\left(z^{\frac{1}{4}} \right) + X\left(-z^{\frac{1}{4}} \right) \right]$

(C) $\dfrac{1}{4}\left[X\left(z^{\frac{1}{4}} \right) + X\left(z^{\frac{1}{2}} \right) + X\left(-z^{\frac{1}{2}} \right) + X\left(-z^{\frac{1}{4}} \right) \right]$

(D) $\dfrac{1}{6}\left[X\left(z^{\frac{1}{4}} \right) + X\left(z^{\frac{1}{2}} \right) + X\left(z^{\frac{3}{4}} \right) + X\left(-z^{\frac{3}{4}} \right) + X\left(-z^{\frac{1}{2}} \right) + X\left(-z^{\frac{1}{4}} \right) \right]$

B.3 A 2-tap FIR synthesis filter $g_0(n)$ is given by $g_0(0) = \dfrac{1}{\sqrt{2}}$ and $g_0(1) = \dfrac{1}{\sqrt{2}}$. It is given that the filter banks $g_0(n)$ and $g_1(n)$ is orthonormal. The FIR filter coefficients $g_1(0)$ and $g_1(1)$ are

(A) $\dfrac{1}{\sqrt{2}}$ and $\dfrac{1}{\sqrt{2}}$.

(B) $-\dfrac{1}{\sqrt{2}}$ and $\dfrac{1}{\sqrt{2}}$

(C) $\dfrac{1}{\sqrt{2}}$ and $-\dfrac{1}{\sqrt{2}}$

(D) $-\dfrac{1}{\sqrt{2}}$ and $-\dfrac{1}{\sqrt{2}}$

B.4 A 5-tap FIR analysis filter coefficients $h_0(n)$ are given as follows:

$h_0(0) = -0.2,\ h_0(1) = 0.3,\ h_0(2) = 0.4,\ h_0(3) = 0.3,\ h_0(4) = -0.2$.

If the value of α for cross-modulation is taken as 2, the filter coefficients $g_1(n)$ are given by

(A) $g_1(0) = 0.2,\ g_1(1) = 0.3,\ g_1(2) = 0.4,\ g_1(3) = 0.3,\ g_1(4) = 0.2$

(B) $g_1(0) = 0.2,\ g_1(1) = 0.3,\ g_1(2) = -0.4,\ g_1(3) = 0.3,\ g_1(4) = 0.2$

(C) $g_1(0) = -0.2,\ g_1(1) = -0.3,\ g_1(2) = 0.4,\ g_1(3) = -0.3,\ g_1(4) = -0.2$

(D) $g_1(0) = -0.2,\ g_1(1) = 0.3,\ g_1(2) = -0.4,\ g_1(3) = 0.3,\ g_1(4) = -0.2$

B.5 Bi-orthogonality condition necessarily implies that the FIR tap lengths of

(A) $h_0(n)$ and $g_1(n)$ must be equal.

(B) $h_0(n)$ and $h_1(n)$ must be equal.

(C) $h_0(n)$ and $g_0(n)$ must be equal.

(D) $h_0(n)$, $h_1(n)$, $g_0(n)$ and $g_1(n)$ must be equal.

B.6 Bi-othogonality and orthonormality conditions together imply that

(A) $h_0(n)$ and $g_1(n)$ must be equal.

(B) $h_0(n)$ and $h_1(n)$ must be equal.

(C) $h_0(n)$ and $g_0(n)$ must be equal.

(D) $h_0(n)$, $h_1(n)$, $g_0(n)$ and $g_1(n)$ must be equal.

B.7 QMF filters are

(A) bi-orthogonal as well as orthonormal.
(B) bi-orthogonal, but not orthonormal.
(C) not bi-orthogonal, but orthonormal.
(D) neither bi-orthogonal, nor orthonormal.

B.8 An image of size 256 x 256 pixels is subband decomposed in several steps to obtain the following subbands
The number of samples in subband-2 is

  (A) 16 x 16.

  (B) 32 x 32.

  (C) 64 x 64.

  (D) 128 x 128.

PART-C:Problems

C-1.
(a) Write a computer program to obtain the four subbands, namely LL, LH, HL and HH from a square image. Use Daubechies' 8-tap, orthonormal, separable, analysis-synthesis filters defined in Section-11.4.

(b) Apply the program on a square image and display your results in the form shown in fig.11.8.

(c) Split the LL subband into four subbands and display your results for two-level decomposition.

(d) Write a computer program to synthesize the image again from the two-level subbands in part (c).

(e) Apply your program on the results of part-(c) and compare reconstructed image with the original image. Is the reconstructed image exactly same as the original one?

SOLUTIONS

A.1
A.2
A.3
A.4
A.5
A.6
A.7
A.8
A.9
A.10

B.1 (C) B.2 (B) B.3 (C) B.4 (B)
B.5 (A) B.6 (D) B.7 (B) B.8 (B)

C.1