Objectives

In this lecture you will learn the following

- Linear Shift-Invariant systems, and their importance
- The discrete time unit impulse
- Signals as a linear combination of shifted unit impulses
- The unit impulse response
- Obtaining an arbitrary response from the unit impulse response for LSI systems

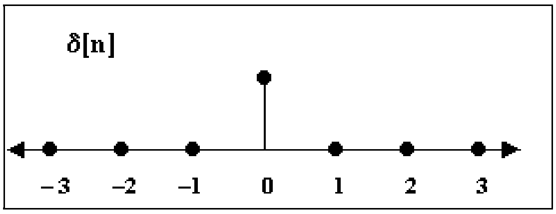
Linear Shift-Invariant systems:

Linear Shift-Invariant systems, called LSI systems for short, form a very important class of practical systems, and hence are of interest to us. They are also referred to as Linear Time-Invariant systems, in case the independent variable for the input and output signals is time. Remember that linearity means that is $y_1(t)$ and $y_2(t)$ are responses of the system to signals $x_1(t)$ and $x_2(t)$ respectively, then the response to $ax_1(t) + bx_2(t)$ is $ay_1(t) + by_2(t)$.

Shift invariance implies that the response of the system to $x_1(t - t_0)$ is given by $y_1(t - t_0)$ for all values of t and t_0. Linear systems are of interest to us for primarily two reasons: first, several real-life systems can be well approximated by linear systems. Second, linear systems come with several properties which make their analysis simple. Similarly, shift-invariant systems allow us to use simpler math to analyse the system. As we proceed with our analysis, we will point out cases where some results (which are rather intuitive) are valid for only LSI systems.

The unit impulse (discrete time):

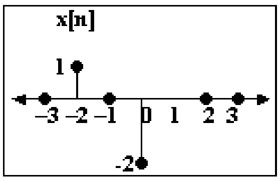
How do we go on with studying the responses of systems to various signals? It would be great if we can study the response of the system to one (or a few) signal(s) and predict the responses to all signals. It turns out that LSI systems can in fact be treated in such manner. The signal whose response we study is the unit impulse signal. If we know the response of the system to the unit impulse (called, for obvious reasons, the unit impulse response), then the system is completely characterized - we can find the response of the system to all possible inputs. This follows rather intuitively in discrete signals, so let us begin our analysis with discrete signals. In discrete signals, the unit impulse is a signal which has zero values everywhere except at one point, where its values is 1. Typically, this point is taken to be the origin (n=0).

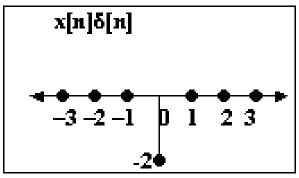


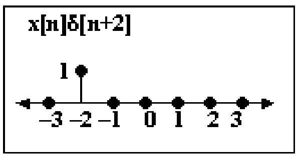
The unit impulse is denoted by the Greek letter delta δ . For example, the above impulses are denoted by $\delta[n]$ and $\delta[n-4]$ respectively.

Note: We are towards invoking shift invariance of the system here - we have shifted the signal $\delta[n]$ by 4 units.

We can thus use $\delta[n]$ to pick up a certain point from a discrete signal: suppose our signal x[n] is multiplied by $\delta[n-k]$ then the value of $x_1[n] = x[n]\delta[n-k]$ is zero at all point except $n=k$. At this point, the value of $x_1[k]$ equals the value $x[k]$.



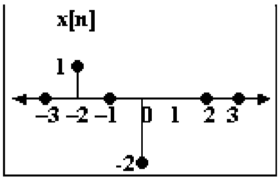


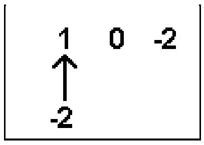


Now, we can express any discrete signal as a sum of several such terms:

$$\sum_{k=-\infty}^{+\infty} x[n].\delta[k]$$

This may seem redundant now, but later we shall find this notation useful when we take a look at convolutions etc. Here, we also want to introduce a convention for denoting discrete signals. For example, the signal x[n] and its representation are shown below :





The number below the arrow shows the starting point of the time sequence, and the numbers above are the values of the dependent variable at successive instants from then onwards. We may not use this too much on the web site, but this turns out to be a convenient notation on paper.

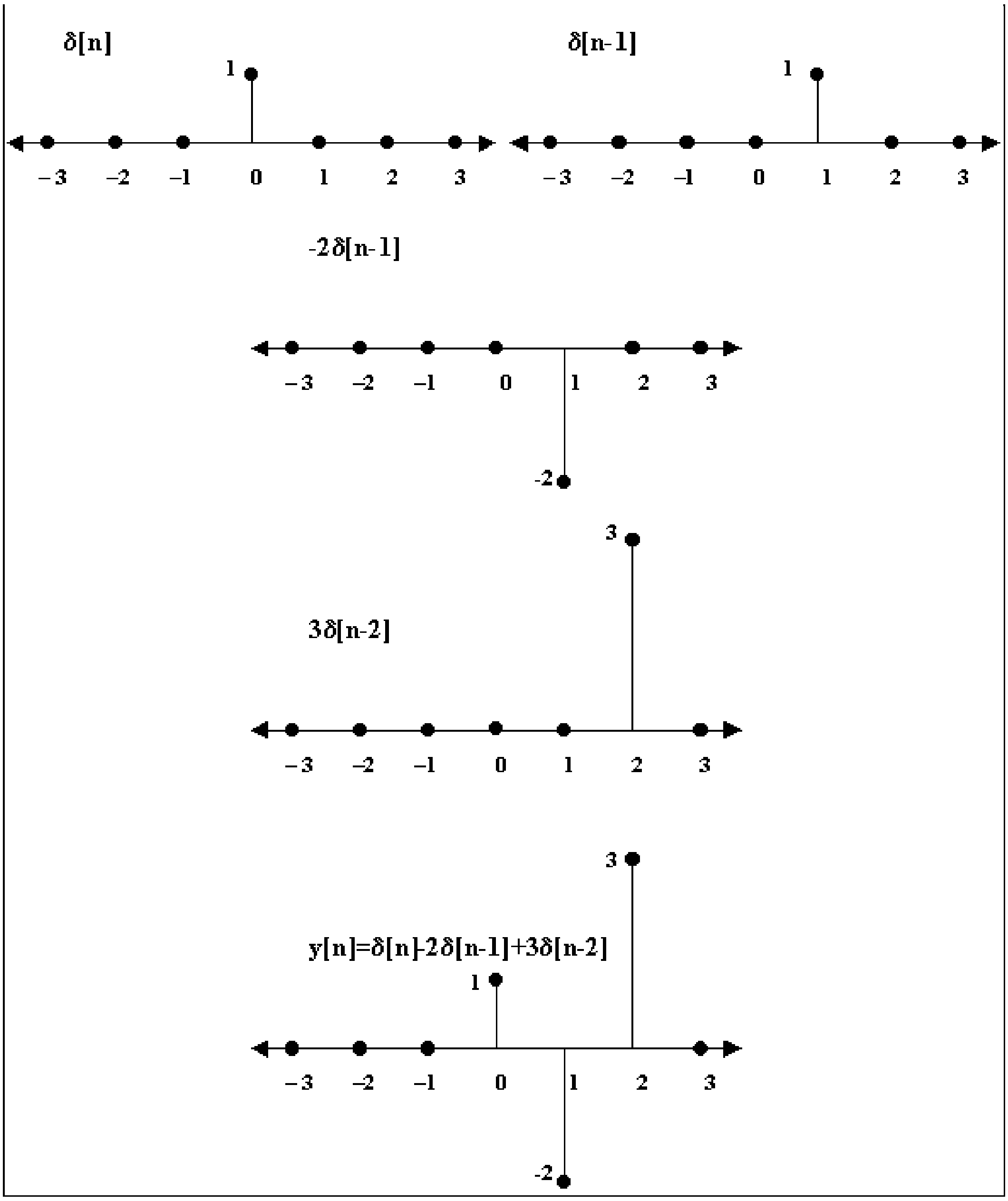
The unit impulse response:

The response of a system to the unit impulse is of importance, for as we shall show below, it characterizes the LSI system completely. Let us consider the following system and calculate the unit step response to it: **y[n] = x[n] - 2x[n-1] + 3x[n-2]**. Now, we apply a unit step **x[n]=**d**[n]** to the system and calculate the response :

$$y[n] = x[n] - 2x[n-1] + 3x[n-2] \mid x[n] = \delta[n]$$

n	x[n]	x[n-1]	x[n-2]	y[n]
..., -1	0	0	0	**0**
0	1	0	0	**1**
1	0	1	0	**-2**
2	0	0	1	**3**
3, ...	0	0	0	**0**

The graphical calculation and the response are as follows :

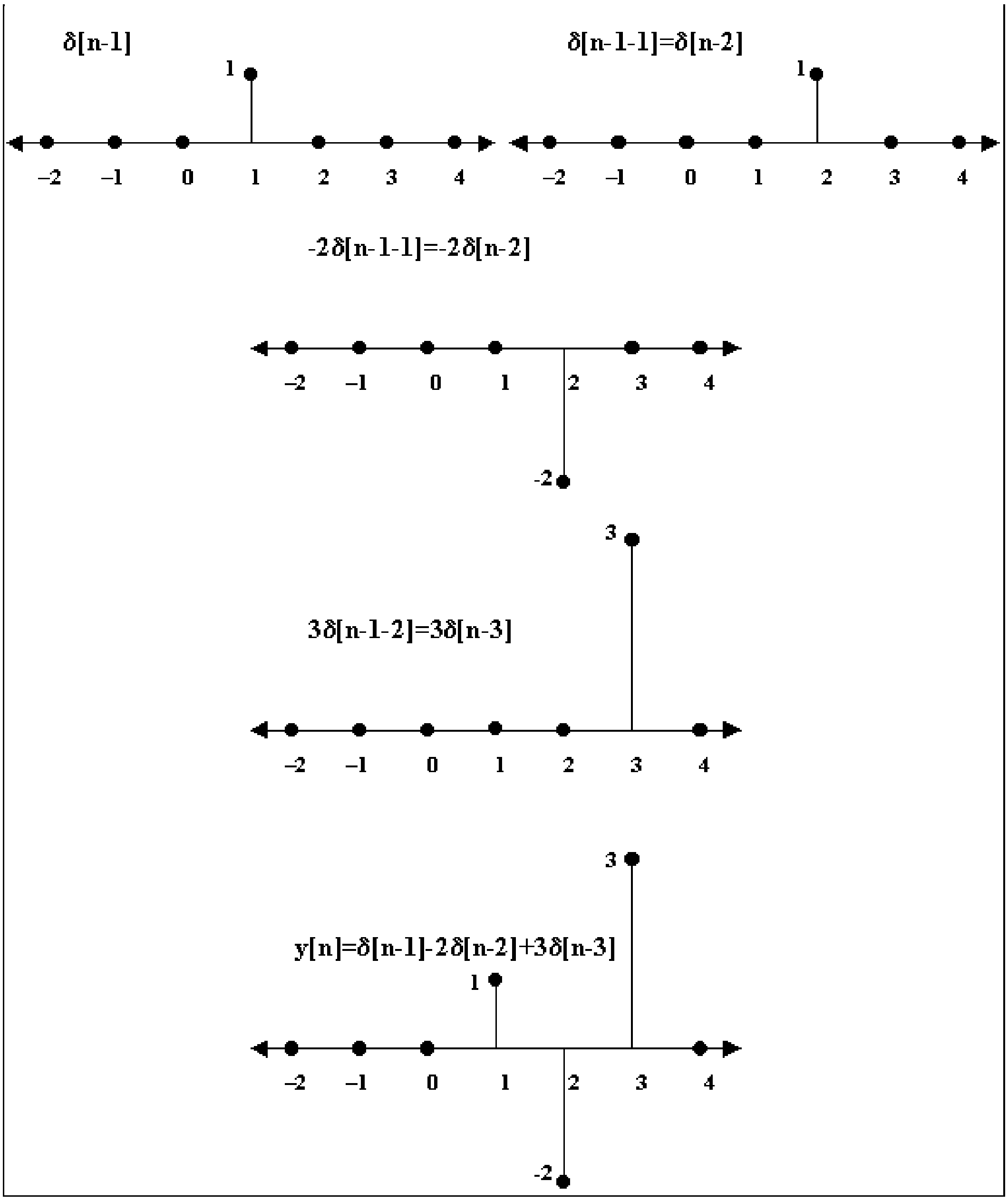

δ[n]
δ[n-1]
-2δ[n-1]
3δ[n-2]
y[n]=δ[n]-2δ[n-1]+3δ[n-2]

Arbitrary input signals:

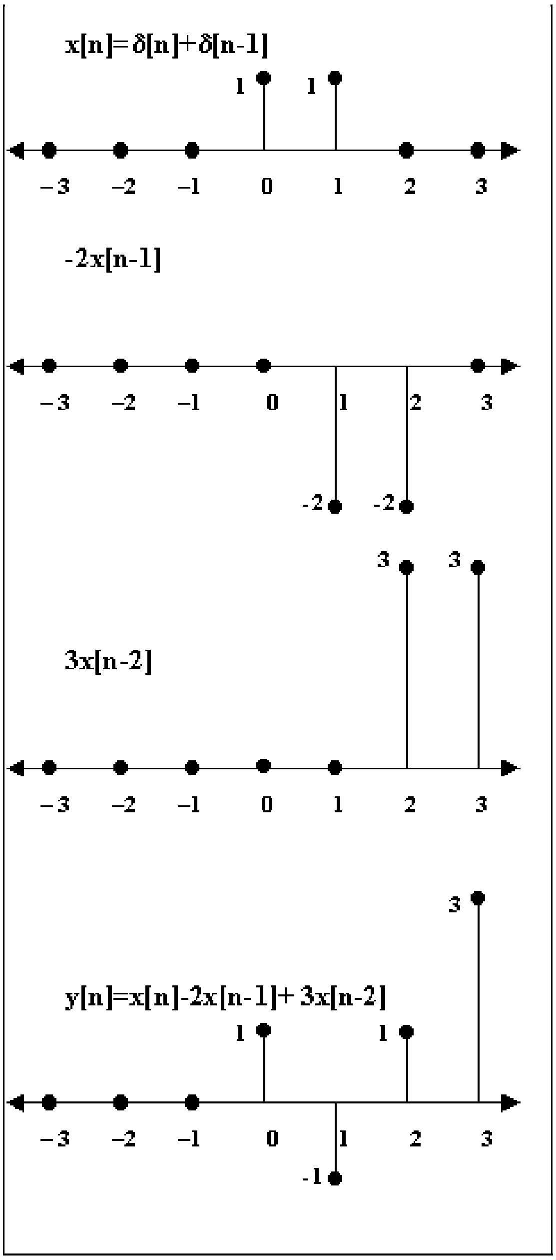
Now let us consider some other input, say **x[0]=1, x[1]=1** and **x=0** for n other than 0 and 1. What will be the response of the above LSI system to this input? We calculate the response in a table as below

y[n] = x[n] - 2x[n-1] + 3x[n-2]		**x[n]=** $\delta[n]$ **+** $\delta[n-1]$	
n	**y_1[n] from** $\delta[n]$	**y_2[n] from -** $\delta[n-1]$	**y[n] = y_1[n] + y_2[n]**
..., -1	0	0	**0**
0	1	0	**1**
1	-2	1	**-1**
2	3	-2	**1**
3	0	3	**3**
4, ...	0	0	**0**

Ah! What we have actually done, is applied the additive (linear), homogenous (linear) and shift invariance properties of the system to get the output. First, we decomposed the input signal as a sum of known signals: first being the unit step $\delta[n]$. The second signal is derived from the unit step by shifting it by **1**. Thus, our input signal is as shown in the figure below. Then, we invoke the LSI properties of the system to get the responses to the individual signals: the first calculation is show above, while the calculation of response for $\delta[n-1]$ is shown below.



Finally, we add the two responses to get the response **y[n]** of the system to the input **x[n]**. The image below shows the final response with an alternative method of calculating it:



This brings us up to the concept of **convolutions**, covered in detail in a later section.

Conclusion:

In this lecture you have learnt:

- Discrete time LSI systems and their importance
- The discrete time unit impulse as a building block
- Expressing signals as a linear combination of shifted unit impulses
- What is the unit impulse response?
- Expressing arbitrary responses as a linear combination of shifted unit impulse responses

Congratulations, you have finished Lecture 7.