

Module 3 : Sampling and Reconstruction

Lecture 29 : Inverse Discrete Time Fourier Transform

Objectives:

Scope of this Lecture:

In the previous lectures we built up concepts of sampling , discrete time signal processing and Discrete Fourier Transform. The next logical step is to study the Inverse Discrete Fourier Transform. In this lecture we indulge in the various IDFT related concepts.

- The equation for **Inverse Discrete Time Fourier Transform** for a discrete periodic signal .
- Inverse DTFT for the *Cross-Correlation* between 'x[n]' and 'h[n]'.
- Parseval's Relation For discrete time periodic signals .

Inverse DTFT :

DTFT of a discrete periodic signal x[n] by period N is given by :

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$X(\omega)$ is periodic in ω with period 2π .

The Fourier coefficients of this periodic function are given by

$$x[n] = \frac{1}{2\pi} \int_{(2\pi)} X(\omega) e^{j\omega n} d\omega$$

The above equation is referred as Inverse DTFT equation .

$$\text{Inverse DTFT equation : } x[n] = \frac{1}{2\pi} \int_{(2\pi)} X(\omega) e^{j\omega n} d\omega$$

InverseDTFT is also periodic in ω with period 2π

$$\text{The fundamental frequency} = \frac{2\pi}{2\pi} = 1$$

Now ,

x[n] is the nth Fourier series co-efficient of $X(\omega)$.

$$\therefore x[-n] = \frac{1}{2\pi} \int_{(2\pi)} X(\omega) e^{-j\omega n} d\omega$$

$$\Rightarrow x[n] = \frac{1}{2\pi} \int_{(2\pi)} X(\omega) e^{j\omega n} d\omega$$

Now,

$$\begin{aligned} X(\omega) &= \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \\ &= \sum_{n=-\infty}^{\infty} x[n] e^{\overline{j\omega n}} \end{aligned}$$

$\therefore X(\omega)$ is the "projection" of inner product of x[n] with $e^{j\omega n}$.

$$\therefore x[n] = \frac{1}{2\pi} \int_{2\pi} X(\omega) . e^{j\omega n} d\omega$$

can be interpreted as $X(\omega)$ multiplied by vectors of $e^{j\omega n}$ and summed over.

Now inverse DTFT for the cross co-relation between sequences $x[n]$ and $h[n]$ can be written as :

$$\sum_{n=-\infty}^{\infty} \{x[n+m] \overline{h[n]}\} \xrightarrow{\text{inverse DTFT}} \frac{1}{2\pi} \int_{(2\pi)} X(\omega) \overline{H(\omega)} e^{j\omega m} d\omega$$

Put $m=0$, then

$$\sum_{n=-\infty}^{\infty} \{x[n] \overline{h[n]}\} = \frac{1}{2\pi} \int_{(2\pi)} X(\omega) \overline{H(\omega)} d\omega$$

i.e dot product of sequences $x[n]$ and $h[n]$ = dot product of DTFT's of $x[n]$ and $h[n]$.

in particular put $x[n] = h[n]$, then

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{(2\pi)} |X(\omega)|^2$$

The above is Parseval's relation for discrete time periodic signals .

here $\sum_{n=-\infty}^{\infty} |x[n]|^2$ is the total energy in $x[\cdot]$ and $\frac{1}{2\pi} \int_{(2\pi)} |X(\omega)|^2$ is the energy spectral density of $x[\cdot]$

Conclusion:

In this lecture you have learnt:

- DTFT is periodic in ω with period 2π and also , the I-DTFT is periodic in n with period 2π .
- THE DTFT $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$. $\therefore X(\omega)$ is the "projection" of inner product of $x[n]$ with $e^{j\omega n}$.
- Parseval's Relation for Discrete periodic signals : $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{(2\pi)} |X(\omega)|^2$

Congratulations, you have finished Lecture 29.