Module 4 : Laplace and Z Transform
Lecture 31 : Z Transform and Region of Convergence

Objectives:

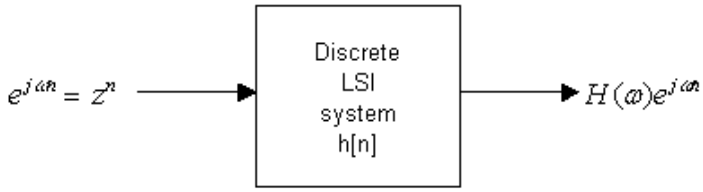
Scope of this lecture:

We have already seen the implementation of Fourier Transform and Laplace Transform for the study of Continuous Time (C.T.) signals and systems. Now our interest lies in frequency domain analysis and design of **Discrete Time** (D.T.) signals and systems. The Z-Transform provides a valuable technique for frequency domain analysis of D.T. signals and design of **DT-LTI** systems. Further Z-Transform offers an extremely convenient and compact way to describe digital signals and processors. Numerical problems are presented for a better understanding of the relevant concepts involved.

- We shall look at the definition of Z-transform .
- The need to consider Region of Convergence (ROC) with suitable illustrations
- The nature of ROC's in both Laplace Transform and Z-transform Domains..

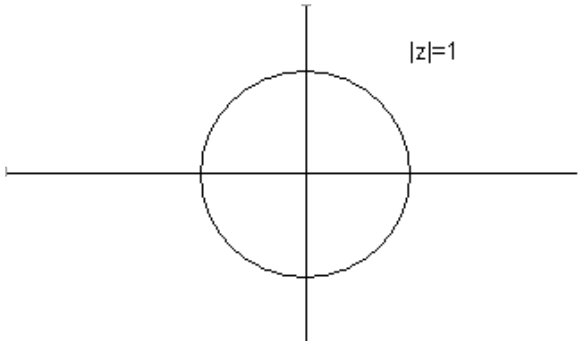
Z-transform

The response of a linear time-invariant system with impulse response h[n] to a complex exponential input of the form z^n can be represented in the following way :



where $H(\omega) = \sum h[n]e^{-j\omega n}$

In the complex z-plane , we take a circle with unit radius centered at the origin.



$\Rightarrow z = e^{j\omega}$
is a unit circle
in z-plane

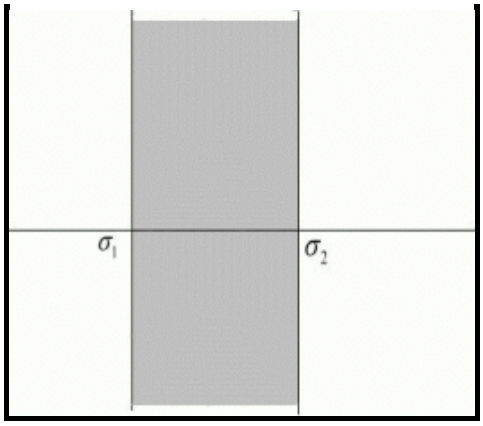
H(w) is periodic with period 2Π with respect to ' w ' .

When we replace **z** by $e^{j\omega}$,we get periodicity of 2Π in the form of a circle.

Nature of Region of Convergence

Laplace Transform:

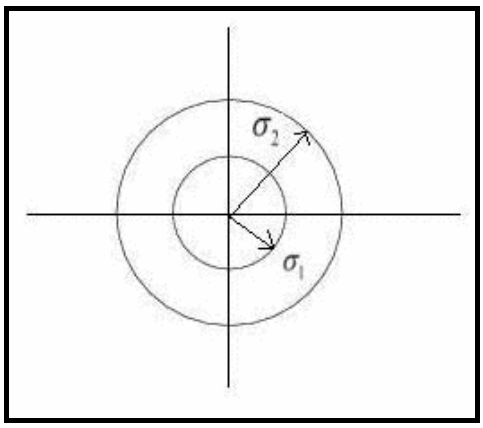
The ROC of the Laplace transform **X(s)** of a two-sided signal lies between two vertical lines in the s-plane.



$\sigma_1$and σ_2 depend only on **real part of s**. For a right-sided signal $\sigma_2 \to \infty$ and the corresponding ROC is referred to as right-half plane. Similarly for a left-sided signal $\sigma_1 \to -\infty$. This ROC is referred to as left-half plane. When **x(t)** is two-sided i.e; of infinite extent for both t > 0 and t < 0 ; both σ_1 and σ_2 are finite and the ROC thus turns out to be a vertical strip in the s-plane.

Z-transform:

The ROC of **X(z)** of a two sided signal consists of a ring in the z-plane centered about the origin.



σ_1 and σ_2 depend only on **magnitude of z**. As in the case of Laplace transform $\sigma_2 \to \infty$ for a right-sided sequence and $\sigma_1 \to 0$ for a left-sided sequence. If **x[n]** is two-sided ;the ROC will consist of a ring with both σ_1 and σ_2 finite and non-zero.

Conclusion:

Flash File

Congratulations, you have finished Lecture 31.