Module 3 : Sampling & Reconstruction
Lecture 26 : Ideal low pass filter

Objectives:

Scope of this Lecture:

We saw that the ideal low pass filter can be used to reconstruct the original Continuous time signal from its samples. However, due to non-availability of an ideal low pass filter and problems associated with it, we look at an alternative method of reconstruction using a Zero-Order-Hold filter.

- How to tackle the problem of low pass filter not being ideal .
- To know about Zero Order sampling .
- To study Response of the Hold Filter .
- To study the types of distortions in the Hold Filter.
- Reconstruction of Signal in zero order Hold Filter.

How can we tackle the problem of Low pass Filter not being ideal ?

$|H(f)|$ =**Magnitude Response of the filter**

$\sphericalangle H(f)$=**Phase Response**

Normally we want ,

$\sphericalangle H(f)$ to be zero .(This is what we want IDEALLY)

Next best thing that we can do is we can have some linear phase variation, i.e. constant time delay for all frequencies.

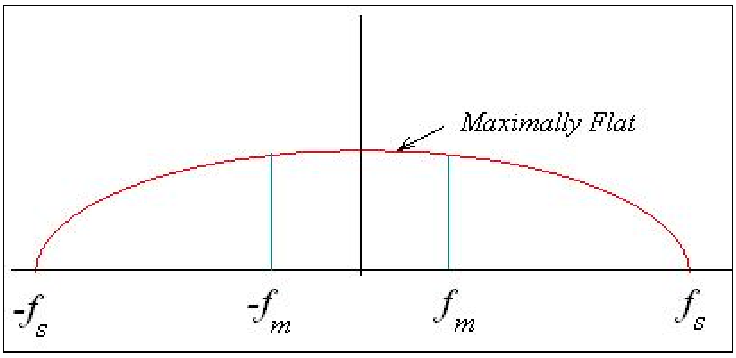
$$\sphericalangle H(f) = K_o f$$

Unfortunaltely analog filters can NEVER give linear phase response.

We can design analog filters as near to an ideal filter in terms of magnitude response, but can not really make ideal filter.

How can we solve this problem?

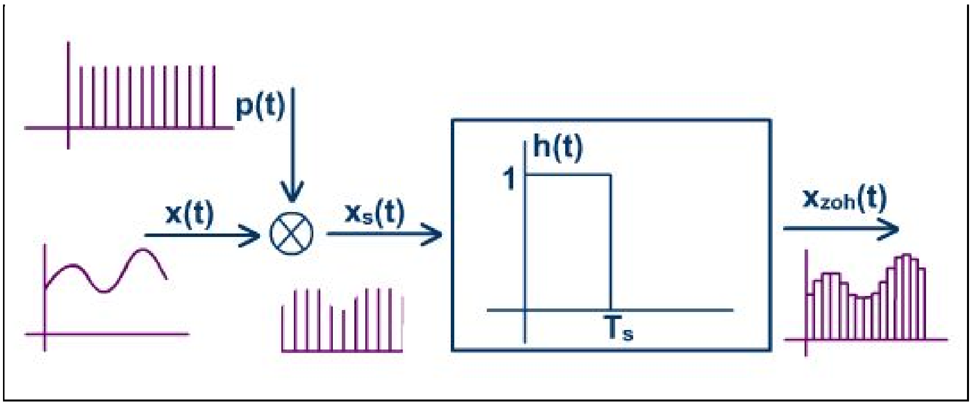
What we have to do is to get a Maximally Flat Sampling, i.e. fs>>2fm



This is what we can do using Hold Filters, which is referred to as **Zero - Order - Hold Sampling.** It is a staircase approximation of the analog signal.

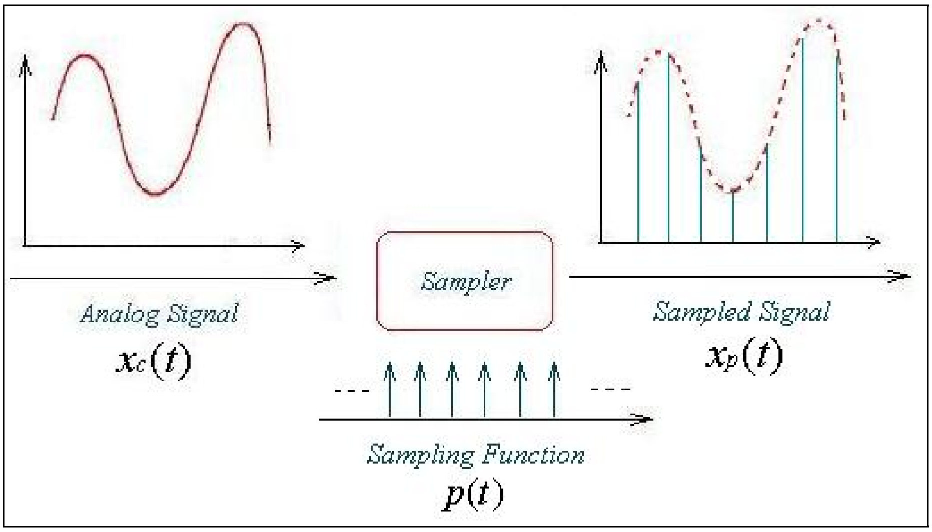
How it works?

In practice, analog signals are sampled using zero - order - hold (ZOH) devices that hold a sample value constant until the next sample is acquired. This is also called as **flat - top sampling**. This operation is equivalent to ideal sampling followed by a system whose impulse response is a pulse of unit height and duration T_s(to stretch the incoming pulses). This is illustrated in Figure below :



Reconstruction of signal in Zero Order Hold Filter

The analog Signal (continuous - time signal) is multiplied with a periodic impulse train, referred to as Sampling Function. A sampled signal is then obtained as shown in figure below.

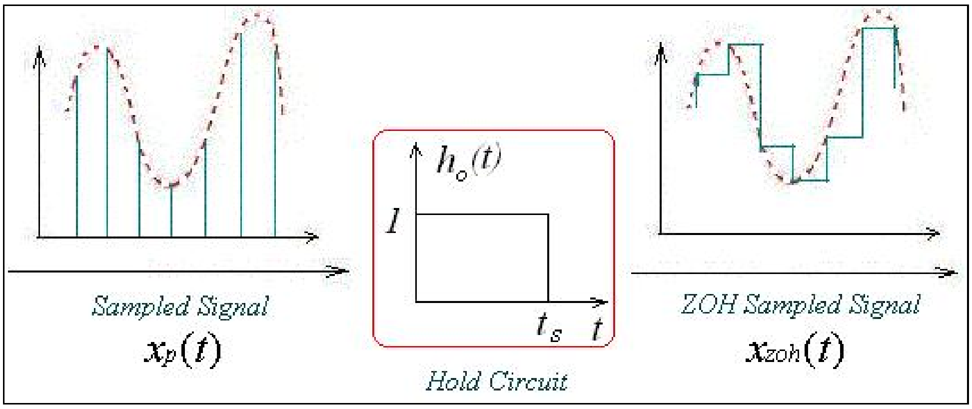


The ideally sampled signal $x_p(t)$ is the product of the impulse train p(t) and the analog signal $x_c(t)$ and is written as

$$x_p(t) = x_c(t)p(t)$$

$$\boxed{p(t) = \sum_{-\infty}^{\infty} \delta(t - nt_s)}$$

$$x_p(t) = \sum_{-\infty}^{\infty} x(nt_s)\delta(t - nt_s) = \sum_{n=-\infty}^{\infty} x[n]\delta(t - nt_s)$$



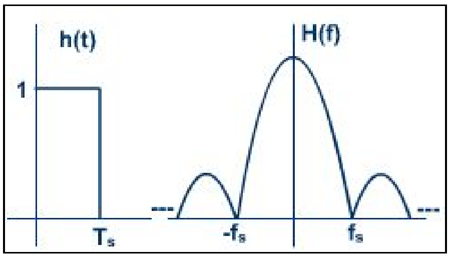
The ZOH Sampled Signal $x_{ZOH}(t)$ can be regarded as the convolution of $h_o(t)$ and a sampled signal $x_p(t)$

$$
\begin{aligned}
x_{ZOH}(t) &= h_o(t) * x_p(t) \\
&= h_o(t) * \left[\sum_{n=-\infty}^{\infty} x(nt_s)\delta(t - nt_s) \right] \\
&= \sum_{n=-\infty}^{\infty} x(nt_s)\, h_o(t - nt_s)
\end{aligned}
$$

Distortion in Zero-order-hold sampling :

The transfer function H(f) of the zero - order - hold circuit is the Sinc function

$$
\begin{aligned}
H(f) &= T_s \operatorname{sinc}(fT_s)\ e^{-j\pi f T_s} \\
&= \frac{1}{f_s} \operatorname{sin c}\left(\frac{f}{f_s}\right)\ e^{-j\pi \frac{f}{f_s}}
\end{aligned}
$$



Since the spectrum of the ideally sampled signal is $f_s \sum X(f - kf_s)$

The spectrum of the zero- order - hold sampled signal $x_{ZOH}(t)$ **is given by the product**

$$
\begin{aligned}
X_{ZOH}(f) &= H(f) \cdot f_s \sum_{k=-\infty}^{\infty} X(f - kf_s) \\
&= \operatorname{Sinc}\left(\frac{f}{f_s}\right) e^{-j\pi f / f_s} \sum_{k=-\infty}^{\infty} X(f - kf_s)
\end{aligned}
$$

This spectrum is illustrated in Figure shown below :

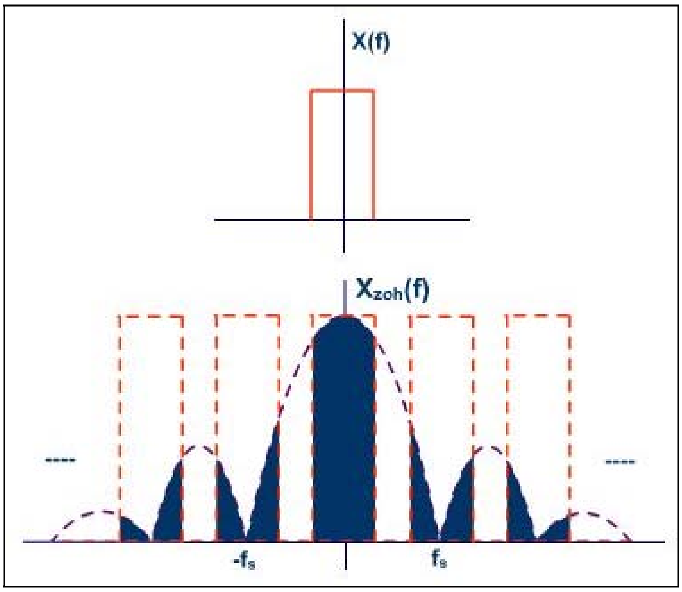


Figure : Spectrum of a zero - order - hold sampled signal

The term sinc(f / f_s) attenuates the spectral images X(f - k f_s) and causes their distortion.

There are two types of distortion :-

a) Aliased Component Distortion : Aliased Component distortion can be corrected, if required by cascading another better lowpass filter.

b) Baseband Spectrum Distortion (Sinc Distortion) : Baseband Spectrum Distortion is corrected by an Equalizer. An Equalizer is an LSI system with Fourier Transformable impulse response which acts like an inverse 1 / H (f) to another LSI system, at least in a certain range of frequencies. Equalizer is also used to correct channel imperfections in a communication system.

The higher the sampling rate f_s, the less is the distortion in the spectral image X(f) centered at origin.

An ideal lowpass filter with unity gain over $-0.5\ f_s \leq\ f \leq 0.5\ f_s$ recovers the distorted signal.

$$\tilde{X}(f) = X(f)\mathrm{Sinc}(\frac{f}{f_s})e^{-j\pi f/f_s} \quad -0.5 f_s \leq f \leq 0.5 f_s$$

To recover X(f) with no amplitude distortion, we must use a compensating filter that negates the effects of the Sinc distortion by profiling a concave shaped magnitude spectrum corresponding to the reciprocal of the Sinc function over the principal period $|\ f\ | \leq 0.5\ f_s$

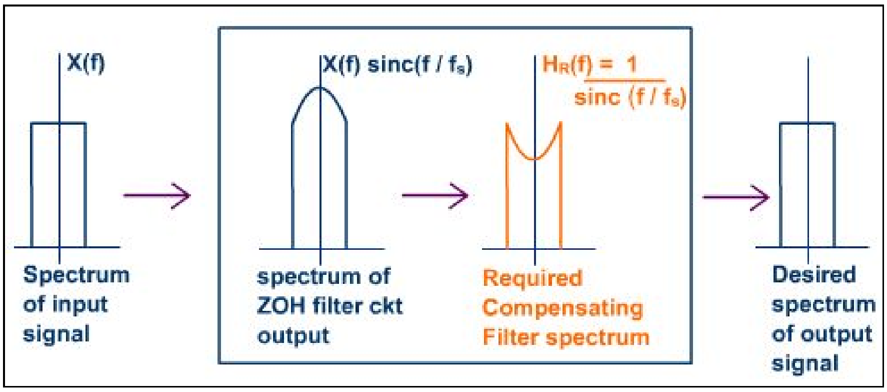


Figure : Spectrum of a filter that compensates for Sinc distortion

The magnitude spectrum of the compensating filter is given by

$$|H_r(f)| = \frac{1}{\mathrm{Sinc}(\frac{f}{f_s})} \quad |f| \leq 0.5 f_s$$

Reconstruction of signal in Zero Order Hold Filter

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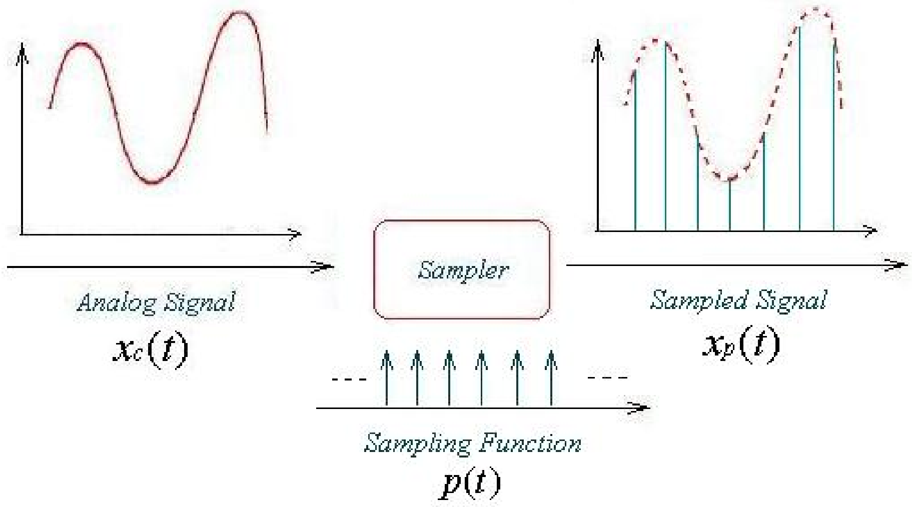


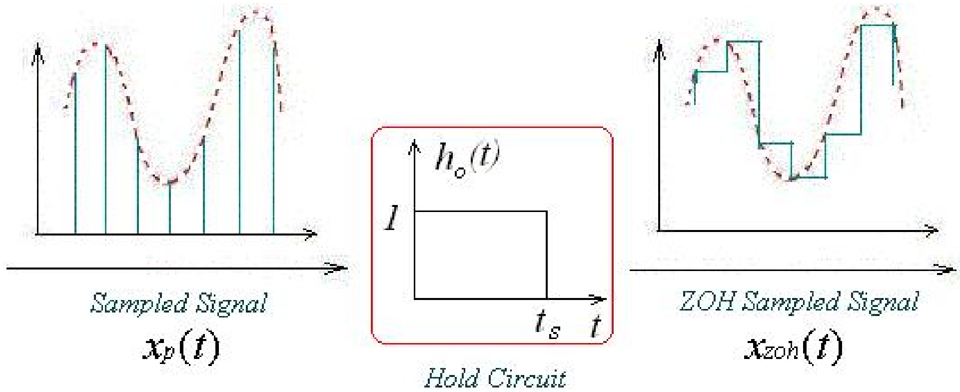
$$p(t)=\sum_{-\infty}^{\infty}\delta(t-nt_s)$$

The ideally sampled signal xp(t) is the product of the impulse train p(t) and the analog signal xc(t) and may be written as

$$x_p(t)=x_c(t)p(t)$$

$$x_p(t)=\sum_{-\infty}^{\infty}x(nt_s)\delta(t-nt_s)=\sum_{n=-\infty}^{\infty}x[n]\delta(t-nt_s)$$

$$x_p(t)=\sum_{n=-\infty}^{\infty}x[n]\delta(t-nt_s)$$



The ZOH Sampled Signal xZOH (t) can be regarded as the convolution of ho(t) and a sampled signal xp(t)

$$\begin{aligned} x_{ZOH}(t) &= h_o(t) * x_p(t) \\ &= h_o(t) * \left[\sum_{n=-\infty}^{\infty} x(nt_s)\delta(t-nt_s)\right] \\ &= \sum_{n=-\infty}^{\infty} x(nt_s)\, h_o(t-nt_s) \end{aligned}$$

Conclusion:

In this lecture you have learnt:

- analog filters can NEVER give linear phase response .
- Hence, we can design analog filters as near to an ideal filter in terms of magnitude response, but can not really make ideal filter .
- Hold Filters can be used to get approximation by a **Maximally Flat Sampling** i.e $\mathbf{f_s >> 2f_m}$.

There are 2 types of distortion: Baseband Spectrum Distortion (Sinc Distortion)

& Aliased Component Distortion.

- The ZOH Sampled Signal $\mathbf{x_{ZOH}(t)}$ can be regarded as the convolution of $\mathbf{h_o(t)}$ and a sampled signal $\mathbf{x_p(t)}$.

Congratulations, you have finished Lecture 26.