Module 3 : Sampling & Reconstruction
Lecture 21: Sampling

Objectives:

Scope of this lecture:

Modern Communication would not have been possible without the development of sampling theory. The sampling theory provides means and ways for processing the Continuous Time (C.T.) data in digital domain. Thus sampling theorem provides the bridge between CT and DT signals. By sampling we mean taking the instantaneous values of CT signal at a regular interval of time. The topics covered in this lecture are listed below:

- The concept of sampling of a signal .
- The notion of **apriori** information & its use to represent a signal economically .
- The most common approach towards economical signal representation.

What is Sampling?

Sampling is a methodology of representing a signal with less than the signal itself.

We can do better than just describing a signal by specifying the value of the dependent variable for each possible value of the independent variable. The concept is explained with the following examples where **'x(t)'** is the **dependent variable** and '**t**' is the **independent variable.**

Let $x(t) = A_o \sin(\omega t + \phi_o)$

Here '**x(t)**' is defined by a sinusoidal relation with a phase constant , amplitude and angular frequency. Now the knowledge of these three parameters suffices to describe '**x(t)**' completely. Thus we are able to compute '**x(t)**' without depending on the independent variable '**t**'.

Consider another example given below:

$$x(t) = a_o + a_1 t + \dots\dots\dots\dots\dots + a_N t^N$$

Here **x(t)** is a polynomial in 't' of degree 'N' and can be computed completely if we know the coefficients $a_0, a_1, a_2 \dots a_N$.

Thus we observe that the **apriori** information we had that allowed us to represent these signals. In the first case we knew that '**x(t)**' is a pure sinusoid and in the second case we knew that it was a polynomial of degree '**N**'.

Thus, as a method of using Apriori information available to represent a signal economically is one way of defining **sampling**.

A Common Approach for Signal Representation:

The approach most often used to economically represent a signal is to look at the values of the **dependent variable** as a set of properly chosen values of the **independent variable** such that these 'tuples' and the 'apriori' information can be used to reconstruct the signal completely.

Lets say we know that some signal 'x(t)' is a pure sinusoid described by the three quantities amplitude (A_o) , angular frequency (ω_0),and phase constant (ϕ_0). For 't_1, t_2 & t_3' values of 't' we get the following three independent equations. :

$$A_o \sin(\omega_0 t_1 + \phi_0) = x(t_1)$$
$$A_o \sin(\omega_0 t_2 + \phi_0) = x(t_2)$$
$$A_o \sin(\omega_0 t_3 + \phi_0) = x(t_3)$$
$$(t_1, t_2 \text{ and } t_3 \text{ should be such that these equations are independent})$$

From the observed values of the signal **$x(t_1)$**, **$x(t_2)$** and **$x(t_3)$** at **t_1**, **t_2** and **t_3**, the parameters of the signal A_o, ω_0 and ϕ_0 can be determined

Consider another example:

Let **x(t)** be a polynomial of order 'N' which is represented mathematically as shown below. It is further represented in the form of a matrix where the LHS is the 'apriori' information.

$$\text{Let} \quad x(t) = \sum_{k=0}^{N} a_k t^k$$

It is necessary and sufficient to measure $x(t_i)$; $i = 0,1,\ldots\ldots,N$ with $t_i \neq t_j$ ($\forall\, i \neq j$)

We have (N+1) equations in (N+1) unknowns :- a_i where $i = 0,1,\ldots\ldots,N$

$$\sum a_k t_i^k = x(t_i) \qquad \text{where } i = 0,1,\ldots\ldots,N$$

Thus we observe that, this system can be solved as the determinant of the square matrix on the LHS so long as $t_i \neq t_j \;\forall\, i \neq j$.

$$\begin{bmatrix} 1 & t_0 & t_0^2 & \cdots & t_0^N \\ 1 & t_1 & t_1^2 & \cdots & t_1^N \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & t_N & t_N^2 & \cdots & t_N^N \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} x(t_0) \\ x(t_1) \\ \vdots \\ x(t_N) \end{bmatrix}$$

Thus given the 'apriori' information, the entire information about the signal is contained in its value at N + 1 distinct points.

You have seen two examples, where 'apriori' information, and "samples" of a signal at certain values of the independent variable help us reconstruct the signal completely.

But If you have no Apriori information you can do no better than to represent the signal as it is.

Even knowing about the continuity of a signal is 'apriori' information. Further we can talk of the relative measure of the 'apriori' information. This can be done by observing the size of the set in which that signal occurs. The larger the set, the lesser *the* 'apriori' information we have. For example, knowing that the signal is sinusoidal is much larger an 'apriori' information than knowing that it is continuous as the set of sine functions is much smaller than the set of continuous functions.

The main challenge in sampling and reconstruction is to make the best use of 'apriori' information in order to represent a signal by its samples most economically.

In the next lecture, we focus on a special class of signals those that are Band-limited (this is the 'apriori' information we shall have) and see how such signals can be reconstructed from their samples.

Conclusion:

From this lecture you have learnt :

- Sampling is a method of using 'apriori' information about a signal to represent it economically.
- The most common approach in sampling and reconstruction is to describe the signal by specifying its value at selected points on the time axis ('t') such that this and the 'apriori' information can be used to reconstruct the signal completely.
- The main challenge in sampling & reconstruction is to make the best use of the apriori information available to represent a signal most economically.

Congratulations, you have finished Lecture 21.