

# Module 8 : Laser- II

## Lecture : Laser- I

### Part-I

#### Objectives

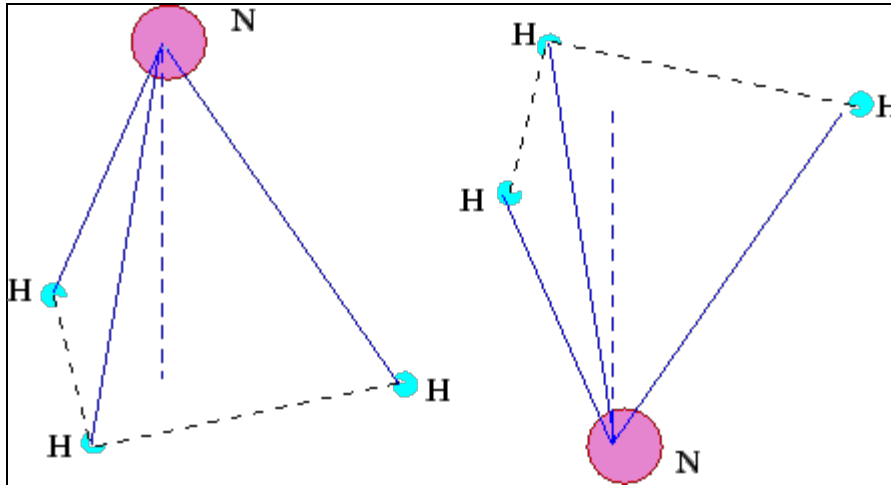
In this lecture you will learn the following

- Maser
- Rate Equations for a Two Level System
- Three Level Laser
- Rate Equations for a Three Level System
- Four Level Laser
- Rate Equations for a Four Level System
- Fabry - Perot Cavity and Laser Oscillation
- Gain Saturation
- Properties of Laser Beam
- Spatial Coherence

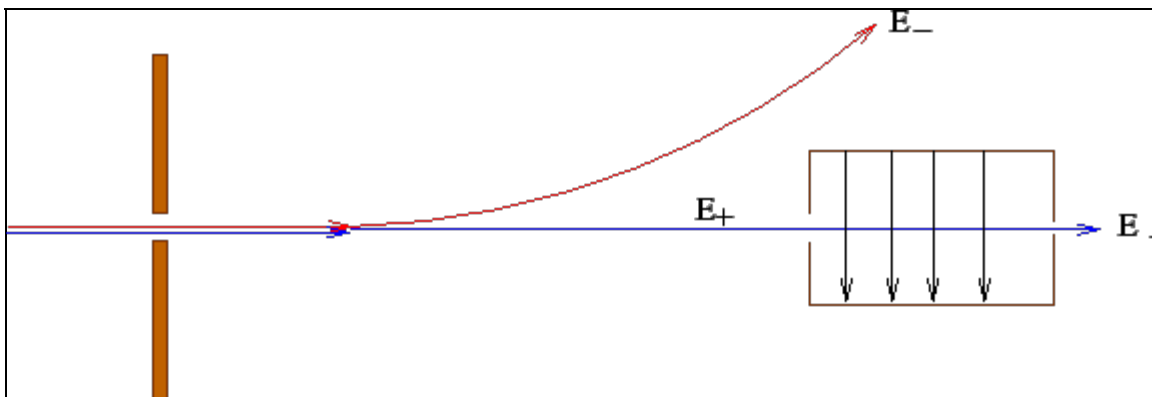
#### 2.7 MASER:

The frequency of microwave photons is  $10^{13}$  Hz, corresponding to an energy of the order of 0.01 eV. The energy is of the same order as that of the thermal energy of air molecules. In such a case, the population of the excited states is comparable to that of the ground state at room temperatures. The process of stimulated emission can then be used to amplify microwave signal. MASER is an acronym for **Microwave Amplification by Stimulated Emission of Radiation** .

Ammonia Maser is such a device for generating electromagnetic waves. Ammonia molecule has two resonant states with a small energy difference  $\sim 0.1$  eV. Geometrically, the two states may be pictured as follows. The three hydrogen atoms are at the vertices of an equilateral triangle which forms the base of a pyramid with the nitrogen atom at the apex of the pyramid. The nitrogen atom may be in two possible positions, either **above** the hydrogen plane or **below** it. (Physically, the two states are distinguished by the direction of their dipole moment in the presence of an electric field.) The molecules make transition from one state to another by absorption or emission of radiation.



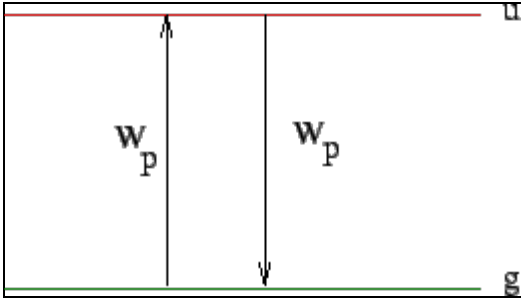
The principle of ammonia maser is to separate the two types of molecules which have different energies  $E_+$  and  $E_-$ . This is done by subjecting the beam to an inhomogeneous electric field in a transverse direction.



The higher energetic beam is passed through a cavity to which it delivers energy. This is done by having a time varying electric field  $E = E_0 \cos \omega t$  in the cavity. If the frequency of the electric field is tuned such that  $\omega = 2\pi(E_+ - E_-)/h$ , resonance condition is satisfied and the molecules make a radiative transition from states with higher energy to that with lower energy.

**Rate Equations for a Two Level System:**

Consider a two level system with the upper level  $u$  and the ground level  $g$ . In order that laser transition may occur, we need a population inversion. As at normal temperatures, the population of lower level is more than that at the upper level, atoms must be pumped into the upper level by providing them energy equal to the energy difference between the two levels.



If  $W_p$  is the pumping induced transition rate for  $g \rightarrow u$  or  $u \rightarrow g$  and  $\tau$  is the natural lifetime of atoms in the upper level, the rate equation for the two levels may be written as follows :

$$\frac{dN_u}{dt} = -\frac{dN_g}{dt} = W_p(N_g - N_u) - \frac{N_u}{\tau}$$

where  $N_g + N_u = N = \text{constant}$ . In the steady state, the time derivatives vanish, and we have

$$N_u = N \frac{W_p \tau}{1 + 2W_p \tau}$$

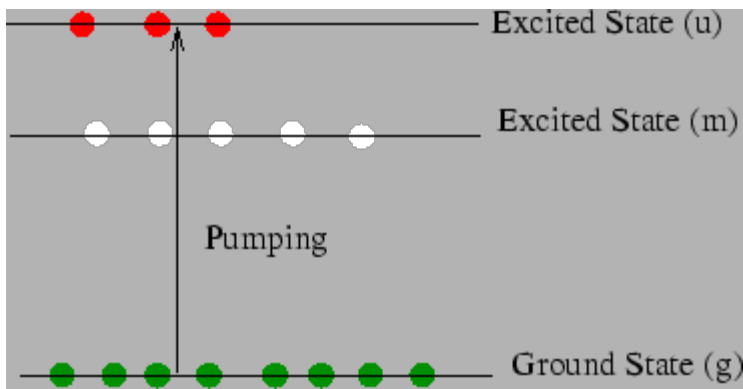
In order that a population inversion may take place, we must have  $N_u > N/2$ . However, one sees that as the intensity is increased the population in the upper level at best approaches this number as its maximum. Thus **population inversion is not possible in a two level system**.

## 2.9

### Three Level Laser:

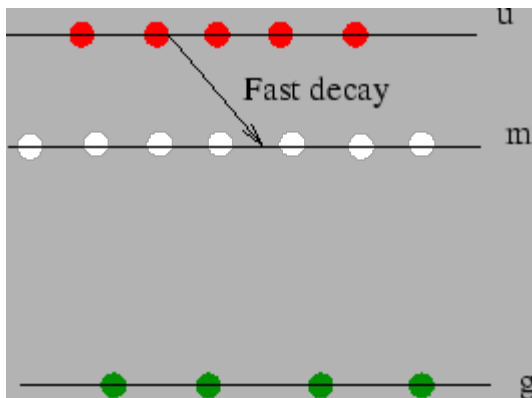
For optical frequencies, population inversion cannot be achieved in a two level system. In

- 1956 Bloembergen proposed a mechanism in which atoms are **pumped** into an excited state  $u$  by an external source of energy ( such as by an electric pulse or by optical illumination).

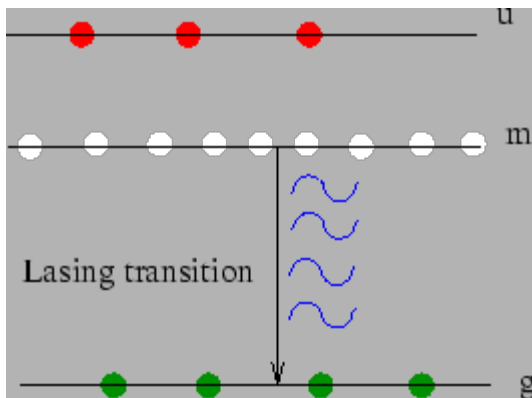


- The system, in addition to the state  $u$ , has an excited state  $m$  which is a **metastable** state, i.e. a state in which the atom has a long life time. Atoms from the upperlevel  $u$  decays spontaneously to this metastable state  $m$ . Life time in the level  $m$  is such that the rate of spontaneous decay from level  $m$  to the ground level  $g$  is slower than the rate at

which atoms decay from  $u$  to  $m$ . This results in a population inversion between the metastable level and the ground state.



- The emitted photons are confined to a laser cavity to stimulate further emission from the excited atoms. Ruby laser works on the principle of a three level system. The pumping power required for such a system is very high because more than half of the ground state atoms have to be pumped into the upper level to achieve population inversion.



A few additional points to be noted about the three level system are :

- As  $N_m \simeq N_g = N/2$  and  $N_m - N_g \ll N$ , once the population inversion is achieved,

the power required to maintain it is small.

$u \rightarrow m$  transition is generally radiationless, the energy being given away to the lattice.

•

As more than half of the atoms are to be raised to the level  $m$ , the probability of

• spontaneous emission is also much higher.

• Laser transition occurs from the metastable state to the lowest possible state which are well separated. This leads to low efficiency.

### 2.9.1 Rate Equations for a Three Level System :

In a three level system, the laser transition is from the metastable state  $m$  to the ground state  $g$ . In the following analysis, we will ignore the effects of degeneracy.

As in the case of two level system, we denote by  $W_p$ , the transition rate induced by

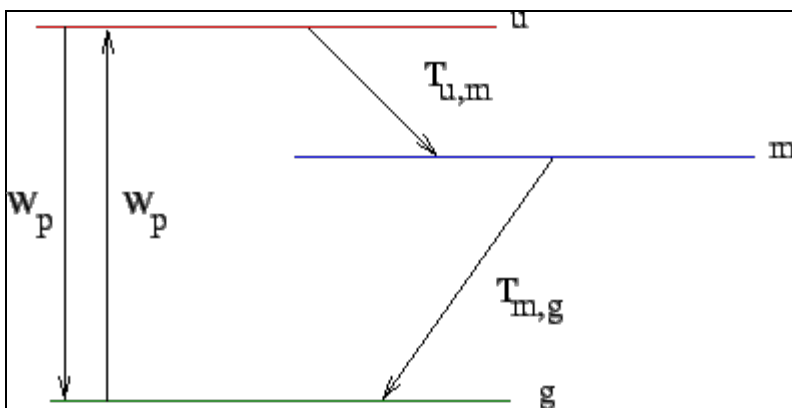
pumping from the ground state ( $g$ ) to the top level  $m$  or vice versa.  $W_p$  is clearly

proportional to the pumping rate. As the metastable state is long lived, we assume that

$\tau_{u,m}$  is much smaller than either  $\tau_{u,g}$  or  $\tau_{m,g}$ , where  $\tau_{i,j}$  denotes the lifetime of transition

$i \rightarrow j$ . As a result, the population of the upper level is nearly zero, and correspondingly,

$$N = N_u + N_m + N_g \simeq N_m + N_g$$



The rate equation for the level  $u$  may be written as

$$\frac{dN_u}{dt} = W_p(N_g - N_u) - \frac{N_u}{\tau_{u,m}}$$

the equation for the metastable level  $m$  is

$$\frac{dN_u}{dt} = \frac{N_u}{\tau_{u,m}} - \frac{N_m}{\tau_{m,g}}$$

The rate equation for the ground state is obtained from the above two equations by number conservation

$$\frac{dN_g}{dt} = -\frac{d}{dt}(N_u + N_m) = -W_p(N_g - N_u) + \frac{N_m}{\tau_{m,g}}$$

In the steady state, each of the time derivatives is zero, which gives

$$N_u = \frac{W_p \tau_{u,m}}{W_p \tau_{u,m} + 1} N_g$$

$$N_m = \frac{W_p \tau_{m,g}}{W_p \tau_{u,m} + 1} N_g$$

The equation for population inversion becomes

$$\frac{\Delta N}{N} = \frac{N_m - N_u}{N} = \frac{W_p \tau_{m,g}(1 - \beta) - 1}{W_p \tau_{m,g}(1 + 2\beta) + 1} (A)$$

where

$$\beta = \frac{N_u}{N_m} = \frac{\tau_{u,m}}{\tau_{m,g}} (B)$$

For population inversion to occur, the numerator of eqn. (A) must be positive,

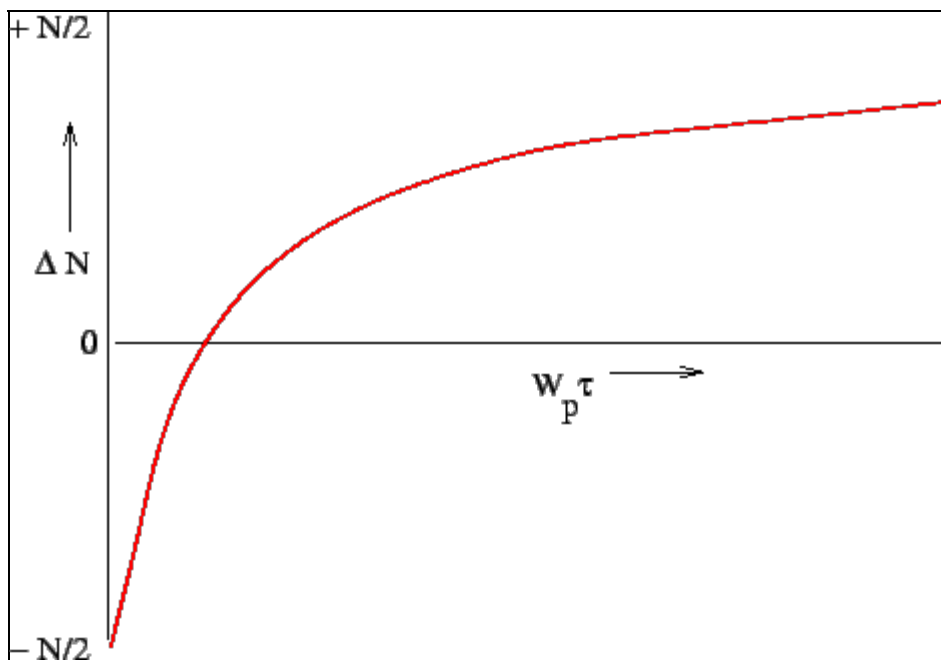
$$W_p \tau_{m,g}(1 - \beta) > 1, \text{ i.e.,}$$

$$W_p \tau_{m,g} > \frac{1}{1 - \beta} \quad (C)$$

From (B),  $\beta < 1$  as the lifetime for  $m \rightarrow g$  transition is longer than that for  $u \rightarrow m$  transition. Thus  $W_p \tau_{m,g} > 1$  is a sufficient condition for population inversion. A minimum pumping power determined by (C) is required to ensure population inversion. For  $\beta = 0$ ,

$$\frac{\Delta N}{N} = \frac{W_p \tau_{m,g} - 1}{W_p \tau_{m,g} + 1}$$

As  $N_g$  is ground level, its value is initially very large and at least  $N/2$  atoms must be transferred to the upper level achieve population inversion. Hence a three level system is not very energy efficient.

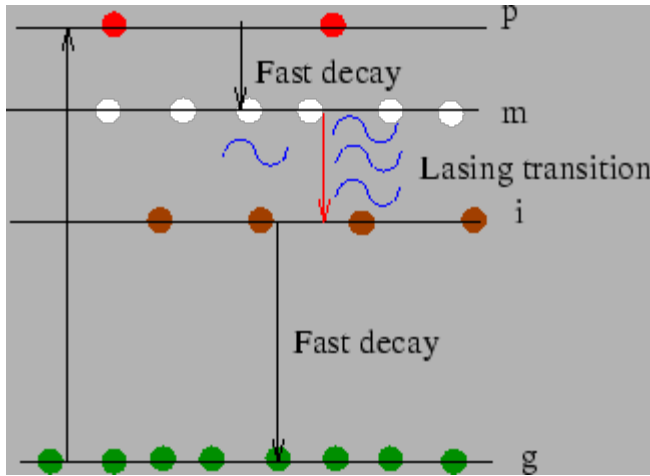


## 2.10 Four Level Laser :

One of the most popular and low cost lasers is helium-neon laser, which works on the basis of a four level system with two levels intermediate between the ground state  $g$  and the pumping level  $p$ .

The ground state atoms are electrically pumped to a short lived state  $p$ . Atoms from this

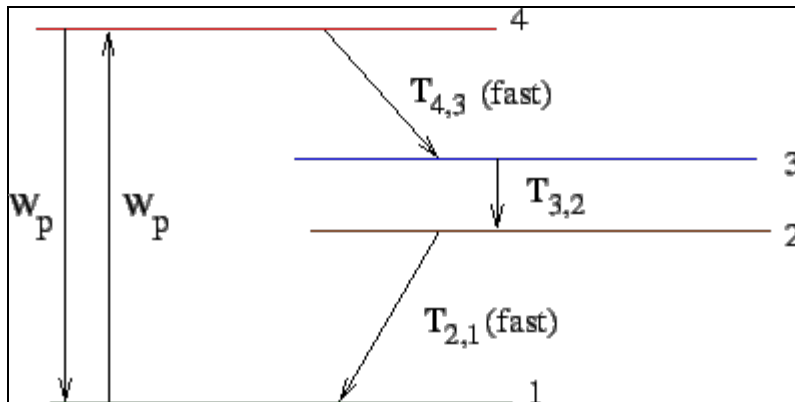
state undergo fast decay to a metastable state  $m$ . Between  $m$  and  $g$ , yet another short lived excited state  $i$  exists. A population inversion takes place between this intermediate state  $i$  and the metastable state  $m$ , between which the lasing transition occurs.



### 2.10.1 Rate Equations for a Four Level System :

We denote the ground state as level 1 and the pumping level as level 4. Lasing transition occurs between levels 2 and 3 (  $2 \rightarrow 3$  ). Let  $W_p = W_{1 \rightarrow 4} = W_{4 \rightarrow 1}$  be pumping

induced transition rate between the ground level and the uppermost level. The lifetime of the lasing transition  $\tau_{3,2}$  is much greater than all other lifetimes in the problem.



Let  $\tau$  be the lifetime of the level 4 for decay into any of the lower states. As the decay constants are additive, we have

$$\frac{1}{\tau_4} = \frac{1}{\tau_{4,1}} + \frac{1}{\tau_{4,2}} + \frac{1}{\tau_{4,3}}$$

A similar convention will be used to denote the lifetime of a state against decay to any of



the lower levels. Thus

$$\frac{1}{\tau_3} = \frac{1}{\tau_{3,2}} + \frac{1}{\tau_{3,1}}$$

The rate equations may now be written as follows :

$$\frac{dN_4}{dt} = W_p(N_1 - N_4) - \frac{N_4}{\tau_4} \quad (1)$$

$$\frac{dN_3}{dt} = \frac{N_4}{\tau_{4,3}} - \frac{N_3}{\tau_3} \quad (2)$$

$$\frac{dN_2}{dt} = \frac{N_4}{\tau_{4,2}} + \frac{N_3}{\tau_{3,2}} - \frac{N_2}{\tau_{2,1}} \quad (3)$$

These are three independent equations in four variables as the total number  $N = N_1 + N_2 + N_3 + N_4$ .

We will now solve these equations in steady state by putting the time derivatives to be zero. From Eqn. (1), we get,

$$N_4 = \frac{W_p \tau_4}{1 + W_p \tau_4} N_1 \quad (4)$$

From Eqn. (2), we get

$$N_3 = \frac{\tau_3}{\tau_{4,3}} N_4 \quad (5)$$

Eqn. (3) gives,

$$N_2 = \frac{\tau_{2,1}}{\tau_{4,2}} N_4 + \frac{\tau_{2,1}}{\tau_{3,2}} N_3$$

$$= \left( \frac{\tau_{2,1}}{\tau_{4,2}} \frac{\tau_{4,3}}{\tau_3} + \frac{\tau_{2,1}}{\tau_{3,2}} \right) N_3 \equiv \beta N_3 \quad (6)$$

where

$$\left( \frac{\tau_{2,1}}{\tau_{4,2}} \frac{\tau_{4,3}}{\tau_3} + \frac{\tau_{2,1}}{\tau_{3,2}} \right) \quad (7)$$

If  $\beta < 1$ , there will be more population in the upper laser level than in the lower one, leading to a population inversion. Using the above, and after straight forward simplification, we get

$$N = \frac{1 + 2W_p\tau_4 + (1 + \beta)W_p \frac{\tau_4\tau_3}{\tau_{4,3}}}{1 + W_p\tau_4} N_1$$

and the population difference  $N_3 - N_2 = \Delta N$  given by

$$\Delta N = (1 - \beta) \frac{\tau_3}{\tau_{4,3}} \frac{W_p\tau_4}{1 + W_p\tau_4} N_1$$

Combining the above two equations

$$\boxed{\frac{\Delta N}{N} = \frac{(1 - \beta) W_p \frac{\tau_3\tau_4}{\tau_{4,3}}}{1 + \left[ (1 + \beta) + 2 \frac{\tau_{4,3}}{\tau_3} \right] W_p \frac{\tau_4\tau_3}{\tau_{4,3}}}}$$

Let us define **Quantum Efficiency**  $\eta$  as the fraction of atoms excited from the ground state (1) which ultimately results in stimulated emission.  $\eta$  is thus a product of the fraction which arrives at the upper laser level (3) and the fraction of atoms in the upper laser level which make **radiative transition** to the lower laser level (2).

Fraction of atoms in level 4 which arrive at level 3 is given by the ratio  $\tau_4/\tau_{4,3}$  and the fraction of atoms in level 3 which radiatively make a transition to level 2 is  $\tau_3/\tau_{3,2}^{rad}$ . We have used a superscript **rad** to indicate that only the radiative component of the transition from 3 to 2 is considered. Thus

$$\eta = \frac{\tau_4}{\tau_{4,3}} \frac{\tau_3}{\tau_{3,2}^{rad}}$$

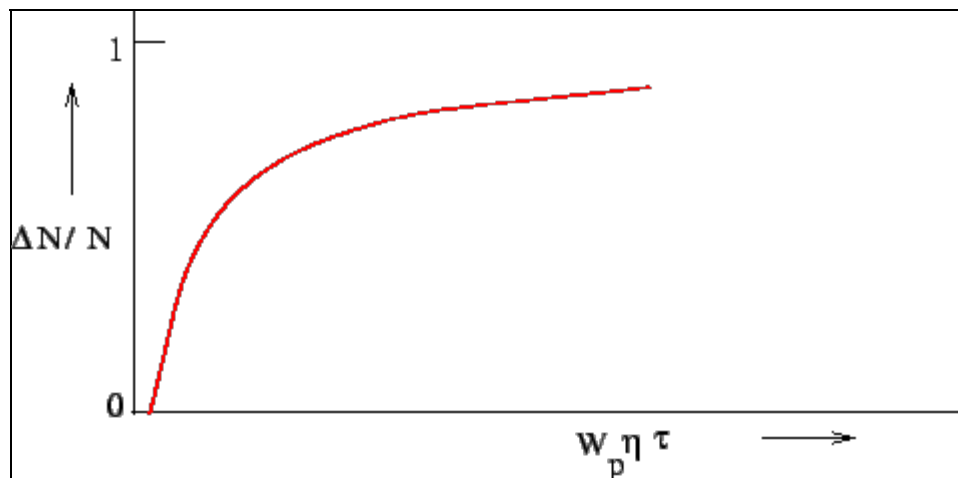
Substituting this in the previous equation, we get, for  $\Delta N/N$

$$\frac{\Delta N}{N} = \frac{(1 - \beta) W_p \eta \tau_{3,2}^{rad}}{1 + \left[ (1 + \beta) + 2 \frac{\tau_{4,3}}{\tau_3} \right] W_p \eta \tau_{3,2}^{rad}}$$

To ensure that most of the atoms excited by pumping participate in laser transition, the life time in the level 3 must be the longest. Using  $\tau_{4,3} \ll \tau_3$ , we may ignore this term in the denominator of the above. Further,  $\tau_{3,2} \gg \tau_{2,1}$ . Using these, one can see that  $\beta$  is small and approaches zero.

Thus  $\Delta N/N$  remains positive and population inversion occurs even for very small pumping power. For  $\beta \rightarrow 0$ , the expression for  $\Delta N/N$  is given by

$$\frac{\Delta N}{N} = \frac{W_p \eta \tau_{3,2}^{rad}}{1 + W_p \eta \tau_{3,2}^{rad}}$$



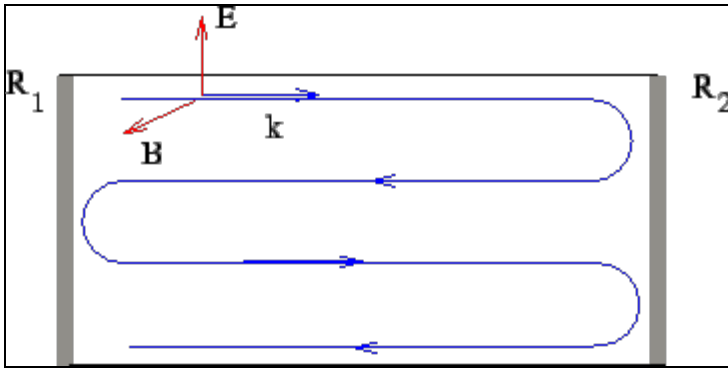
## 2.11 Fabry - Perot Cavity and Laser Oscillation:

Optical resonators play an important role in amplifying laser beam. Fabry-Perot resonators ( **etalons** ) are optical cavities which are used for the purpose.

A simple Fabry-Perot etalon consists of two partially reflecting parallel mirrors with reflectances  $R_1$  and  $R_2$  separated by a length  $L$ . For simplicity, consider an

electromagnetic wave of amplitude  $E_i$  incident normally on one of the faces of the etalon.

The ray suffers multiple reflection, bouncing to and fro between the mirrors. At each reflection, the field amplitude gets reduced by a factor  $r_1 = \sqrt{R_1}$  or  $r_2 = \sqrt{R_2}$  as the case may be.



In addition, if the wave vector of the field is  $k$ , the wave suffers a phase change  $2kL$  (this is in addition to the phase change due to reflection) If  $t_1$  and  $t_2$  are the transmission coefficients of the mirror, the transmitted amplitude is given by

$$E_t = t_1 t_2 E_i + t_1 t_2 r_1 r_2 E_i e^{2ikL} + t_1 t_2 (r_1 r_2)^2 E_i e^{4ikL} + \dots$$

$$= t_1 t_2 E_i \frac{1}{1 - r_1 r_2 e^{2ikL}}$$

The transmitted intensity is given by

$$I_t = |E_t|^2 = |t_1 t_2|^2 E_i^2 \frac{1}{1 - r_1 r_2 e^{2ikL}} \frac{1}{1 - r_1 r_2 e^{-2ikL}}$$

$$= I_0 T_1 T_2 \frac{1}{(1 - r_1 r_2)^2 + 4 r_1 r_2 \sin^2 kL}$$

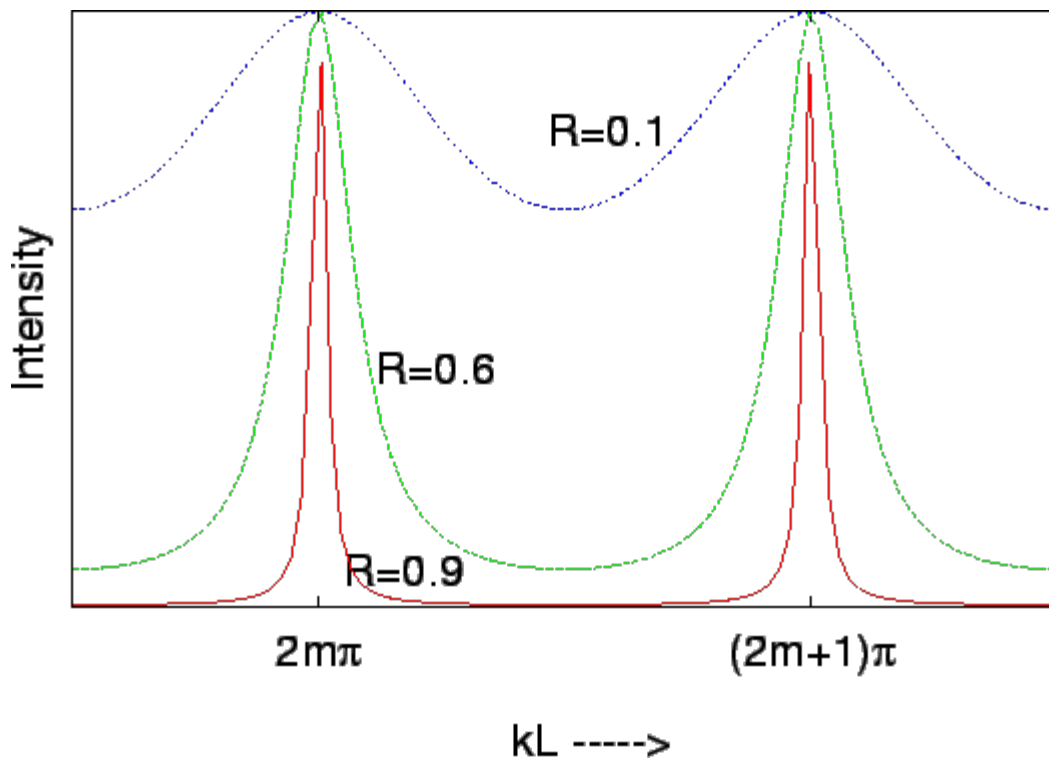
where  $T_1$  and  $T_2$  are the transmittance. If the reflectances of both the mirrors are the same, and, using  $T + R = 1$ , we have

$$I = I_0 \frac{1}{1 + \frac{4R}{(1-R)^2} \sin^2 kL}$$

$$= I_0 \frac{1}{1 + F \sin^2 kL}$$

$F = 4R/(1 - R)^2$  is called **finesse**.  $1/(1 + F \sin^2 kL)$  is known as **Airy function**. For

non-normal incidence, a similar expression is valid with  $kL$  being replaced by  $kL \cos \theta$  where  $\theta$  is the angle of incidence. The intensity pattern for three values of reflectance is shown.



Note that at the resonance, i.e., when  $kL = m\pi$ , the transmission intensity is maximum. The FWHM of the intensity pattern can be obtained by looking at any of the peaks since the shape of intensity function repeats itself. Consider the peak near  $kL = 0$ . For large values of  $R$ , the peak is very sharp and we may replace  $\sin kL$  by  $kL$  itself. Thus FWHM is given by

$$\frac{1}{1 + R(kL)^2} = \frac{1}{2}$$

which gives  $kL = \pm 1/\sqrt{F}$ , so that FWHM is  $2/\sqrt{F}$ .

A Fabry-Perot cavity can, therefore, be used to select wavelength by suitably adjusting the mirror separation, the wavelengths at which resonances occur are given by

$$\lambda_{\max} = \frac{2L}{m}$$

where  $m$  are integers. As  $m$  increases, the wavelengths decrease. Using the speed of light in the medium to be  $c/\mu$ , where  $\mu$  is the refractive index of the medium, the resonance frequencies are

$$\nu_{\max} = \frac{cm}{2\mu L}$$

which are equi-spaced. Each possible standing wave satisfying the above is called a cavity mode  $m$  being the **mode number**.

If now, an active medium with a gain coefficient  $g$  is put inside the Fabry-Perot cavity, in one round trip the optical power  $P$  is

$$P = P_0 R_1 R_2 e^{2g(\nu_s)L}$$

Optical amplification occurs if  $P > P_0$ , i.e., if

$$g(\nu_s) \geq \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right)$$

The righthand side of the above is termed **threshold gain**  $g_{th}(\nu_s)$ .

## 2.12 Gain Saturation:

Clearly, gain cannot continue indefinitely. For each photon added to the field, one atom is removed from the upper laser level and one is added to the lower level. Thus population inversion reduces, which in turn, reduces gain. There is a competition between pumping and stimulated emission rate. Pumping builds up population of the upper level till threshold is reached and stimulated emission starts.

Let us consider a simple rate equation for the population of the upper level. Let  $S$  be the number density of photons per unit time. In a gain medium  $S = S_0 e^{\int g dx}$ . The population

of the upper level is given by

$$\frac{dN_u}{dt} = R_u - \frac{N_u}{\tau} - \sigma N_u S(A)$$

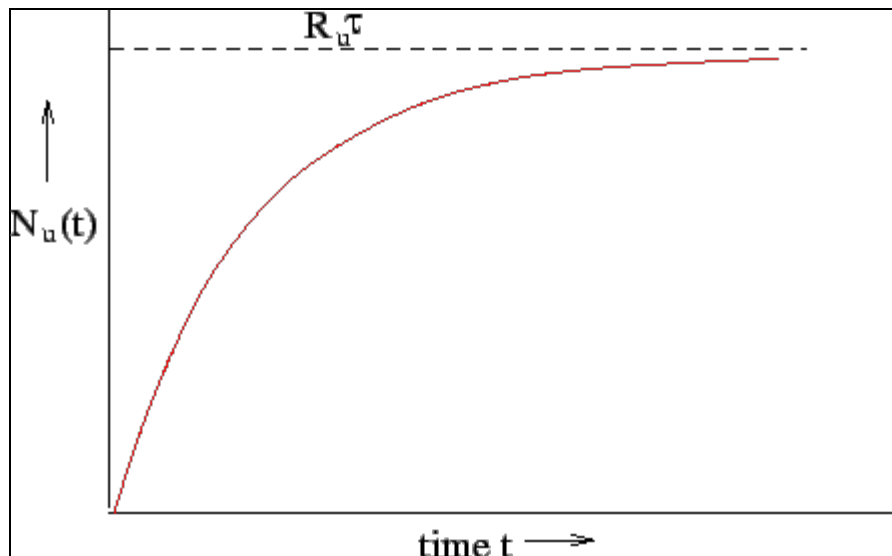
where,  $R_u$  is the pumping rate to the upper level,  $\tau$  is the natural lifetime of the excited atoms and  $\sigma$  is the gain cross section.

In the absence of stimulated emission  $\sigma = 0$ , so that the population in the steady state is given by

$$N_u^0 = R_u \tau$$

The population reaches the steady state in an exponential fashion

$$N_u(t) = R_u \tau (1 - e^{-t/\tau})$$



In the presence of light, the steady state solution to equation (A) is obtained by equating the right hand side of (A) to zero. The steady state population of the upper level is given

by

$$N_u = \frac{N_u^0}{1 + \sigma \tau S}$$

We may rewrite the above as

$$N_u = \frac{N_u^0}{1 + S/S_{\text{sat}}} \equiv \frac{N_u^0}{1 + I/I_{\text{sat}}}$$

where  $I_{\text{sat}}$  is the saturation intensity, which is equal to the intensity for which the gain is reduced by a factor of 2.

## 2.13

### Properties of Laser Beam:

Laser beams are characterized by the following special properties :

- **Coherence:** Laser beam is highly coherent, i.e, different parts of the beam maintain a phase relationship for a long time. this results in interference effect. When a laser beam reflects off a surface, the reflected light can be seen to have bright regions separated by dark regions.

#### Temporal Coherence:

One can define a **coherence time**  $\Delta \tau$  after which the phase correlation between two waves which were initially in phase ( or between two points in the same wave which had a known phase difference) drops significantly.

The reason for loss of coherence is that an optical source does not emit a continuous wave for all time to come. Thermal sources, for instance, have a typical life time of

$\Delta t = 10^{-7}$  seconds so that a wave which seems continuous, actually consists of a sequence of waves which are typically  $c\Delta t \simeq 0.3\text{m}$  long which no phase correlation between parts of one wave train and another.

Temporal coherence is essentially a measure of monochromaticity of the beam. To see this consider a wave train which is emitted for a finite duration. Let the wave disturbance be represented by

$$F(t) = f_0 e^{-2\pi i \nu_0 t} \text{ for } |t| \leq \frac{\Delta}{2}$$

$$= 0, \text{ otherwise}$$

- The wave is not strictly monochromatic, as would have been the case if the wave train was for indefinite duration. The spread in frequency in the wave is found by taking a Fourier transform of the wave train,

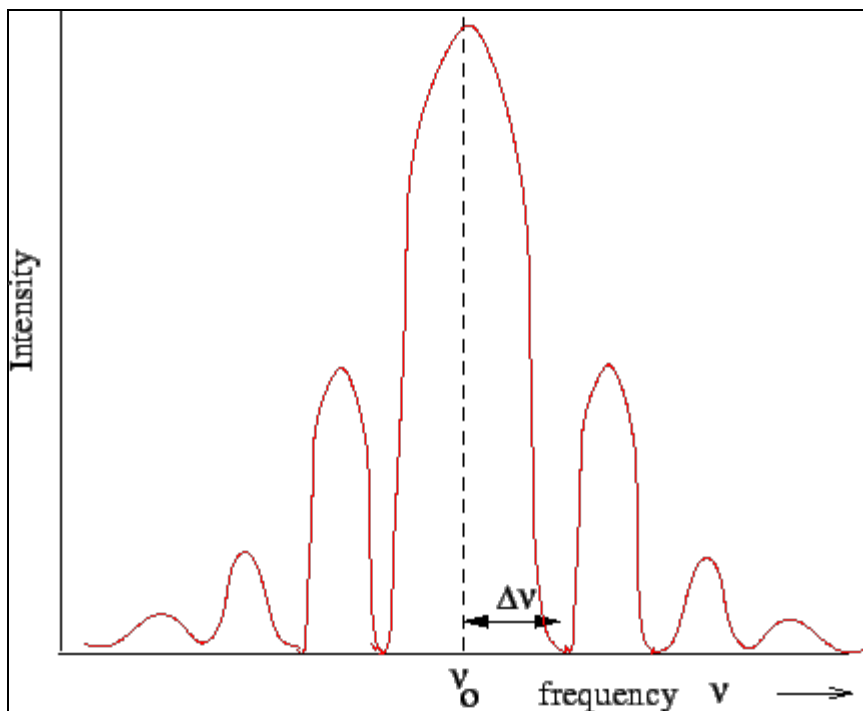


$$F(\nu) = f_0 \int_{-\Delta/2}^{\Delta/2} e^{-2\pi i(\nu - \nu_0)t} dt$$

$$= f_0 \Delta t \left[ \frac{\sin \pi(\nu - \nu_0) \Delta t}{\pi(\nu - \nu_0) \Delta t} \right]$$

- The intensity pattern which is given by the square of  $F(\nu)$  is shown. Though there is no unique way of defining the frequency spread, we note that the first zero of the intensity pattern occurs at  $\nu - \nu_0 = \pm 1/\Delta t$ . Thus one can define the spread of frequency to be given by  $\Delta\nu = 1/\Delta t$ . This is simply a statement of the classical uncertainty principle connecting the band width to coherence time  $\Delta t$ ,

$$\Delta t \Delta \nu \sim 1$$



- The temporal coherence length can be defined as  $l = c\Delta t$ , so that
- $l = \frac{c}{\Delta\nu} = \frac{\lambda^2}{\Delta\lambda}$

- where  $\lambda$  is the mean wavelength and  $\Delta\lambda$  is the spread of wavelengths about this mean, which is also a measure of the degree of monochromaticity of the wave.

### Exercise :

A He-Ne laser operating at 630 nm has an emission width of  $\delta\lambda = 10^{-5}$  nm. Calculate the temporal coherence length. (Ans. 400 m)

It may be noted that a laser beam is highly monochromatic with the spread of wavelength being very small.

### Exercise:

An Argon laser operating in single mode has a linewidth of 7.5 MHz. Calculate its coherence length. (Ans. 40 m)

### Spatial Coherence:

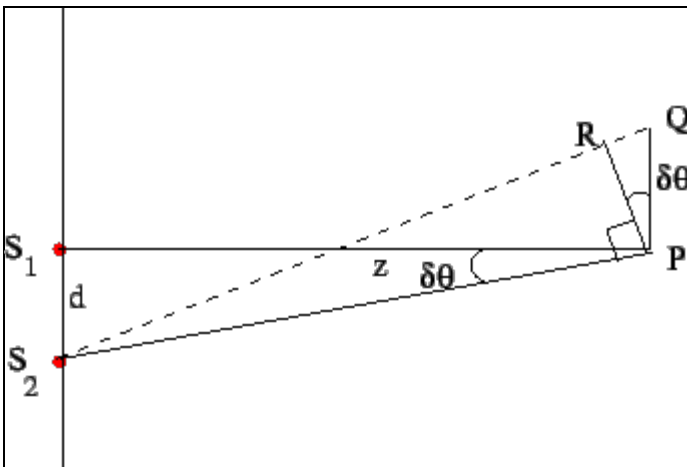
Spatial coherence describes the distance over which phase correlation exists between different points in the same wave in a direction perpendicular to the direction of observation. Spatial correlation arises because a source is never really a point source. Consider two point

sources  $S_1$  and  $S_2$  at a distance  $d$  from each other along the y-axis, as shown.

Suppose the waves arriving at the point P which is at a distance  $z$  along the direction perpendicular to the line joining the sources are coherent. The phase difference between waves arriving at the point Q which is at a lateral distance  $x$  is  $S_2Q - S_1Q$ . Using

straightforward geometry, we can see that the path difference is given by

$$\delta l = x \frac{d}{z}$$



- We may arbitrarily define the spatial coherence length  $x$  to be the lateral distance at which the waves are out of phase by  $\pi$ . Thus

- $x \frac{d}{z} = \frac{\lambda}{2}$

- The spatial coherence length is given by

-

$$x = \frac{\lambda z}{2d}$$

- Using the above argument, if we have a source of circular shape of diameter  $D$ , the lateral coherence length is
- $l_c = \frac{\lambda z}{2D}$
- As an illustration, we note that a source of 100  $\mu\text{m}$  diameter emitting at 100 nm has a spatial coherence length of  $2.5 \times 10^{-4}\text{m}$ .
- **Directionality:** Laser beam is highly collimated and can travel long distances without significant spread in the beam cross section. As the collimated beam propagates, the beam spreads out. The full angle beam divergence  $\Delta\phi$  is defined as the amount by which the beam diameter ( $D$ ) increases over a distance  $z$  after leaving the source

$$\Delta\phi = \frac{D}{z}$$

Taking the beam profile to be gaussian, the intensity varies as  $I = I_0 e^{-r^2}$ , where  $r$  is the distance from the centre of the beam where the intensity ( $I_0$ ) is maximum. We define beam radius as the radial distance over which the intensity decreases by a factor  $e^2 \simeq 7.389$  from its maximum value at the centre. Thus nearly 94% of the energy is concentrated within the beam radius.

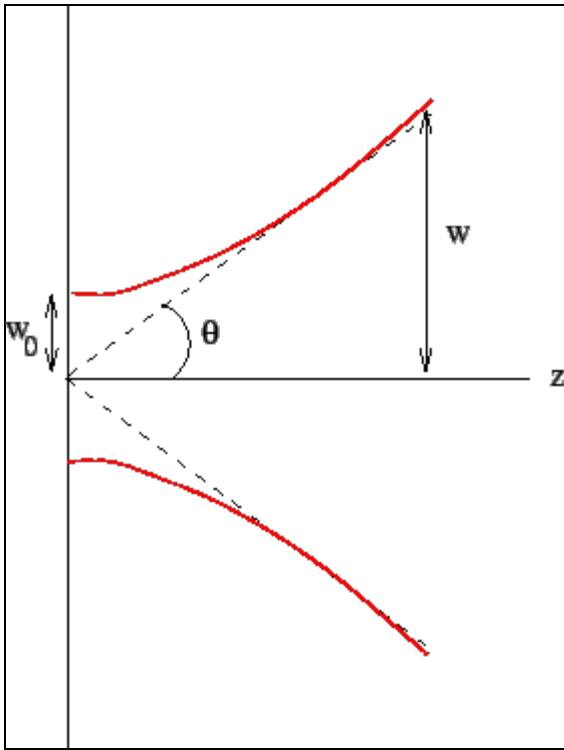
The angular divergence of the beam is given by

$$\theta = \frac{\lambda}{\pi w_0}$$

where  $w_0$  is the radius of the beam at the point where the beam leaves the laser. The radius of the beam varies with the distance as

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2}$$

where  $z_0 = \pi w_0^2 / \lambda$ .



**Example:**

A 10 mW He-Ne laser operating at 633 nm has a spot size of 10mm. Assuming a Gaussian beam find the beam radius and intensity at a distance of 100m from the source.

**Solution:**

The beam radius is half the spot size. The beam divergence (half angle) is given by

$$\theta = \frac{\lambda}{\pi w_0} = \frac{633 \times 10^{-9}}{\pi \times 5 \times 10^{-3}} = 4.03 \times 10^{-5} \text{ rad} = 0.0023^\circ$$

The radius increases with distance  $z$  as

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_0}\right)^2} = w_0 \sqrt{1 + \left(\frac{z\theta}{w_0}\right)^2} = 6.4 \text{ mm}$$

Area of the spot is  $\pi w^2 = 20.1 (\text{mm})^2$ . The intensity of the spot =  $10^{-2} / 20 \times 10^{-6} = 500 \text{ W/m}^2$ .

## Recap

In this lecture you have learnt the following

- Maser
  - Rate Equations for a Two Level System
  - Three Level Laser
  - Rate Equations for a Three Level System
  - Four Level Laser
  - Rate Equations for a Four Level System
  - Fabry - Perot Cavity and Laser Oscillation
  - Gain Saturation
  - Properties of Laser Beam
  - Spatial Coherence
- 

## Part-II

### Objectives

In this lecture you will learn the following

- Types of Lasers and Applications
- Helium-Neon Laser
- Carbon dioxide Laser
- Argon-ion Laser
- Semiconductor Lasers
- Homojunction Laser (Laser Diode)

- Heterojunction Laser
- Properties of GaAs Laser
- Vertical Surface Emitting Laser
- Noise in a Laser Source

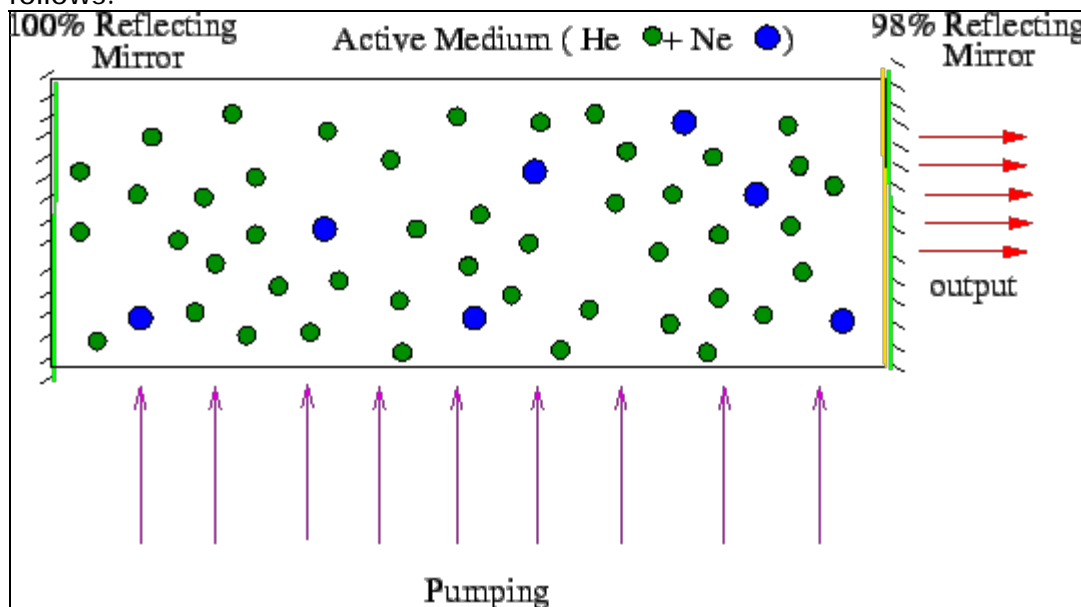
## 2.14 Types of Lasers and Applications:

Lasers have found wide applications in areas as diverse as optical communications, medical surgery, welding technology, entertainment electronics etc. What makes such veritable use of lasers possible is the highly collimated nature of the laser beams and the consequent possibility of delivering a very high energy density in a limited region of space. Depending on the material used for the **active medium**, lasers are broadly classified as (i) conventional or gas lasers (ii) solid state lasers (iii) liquid lasers and (iv) semiconductor lasers. Among the gas lasers, some of the most commonly used ones are Helium - Neon laser, Carbon dioxide laser and Argon- ion laser.

### 2.14.1 Helium-Neon Laser:

Helium-neon laser consists of an active medium of a gas mixture with about 80% He and 20% Ne, kept in a glass chamber at at low pressure. The ends of the chamber are silvered with one end having a perfectly reflecting mirror while the other end has a mirror which reflects 98%.

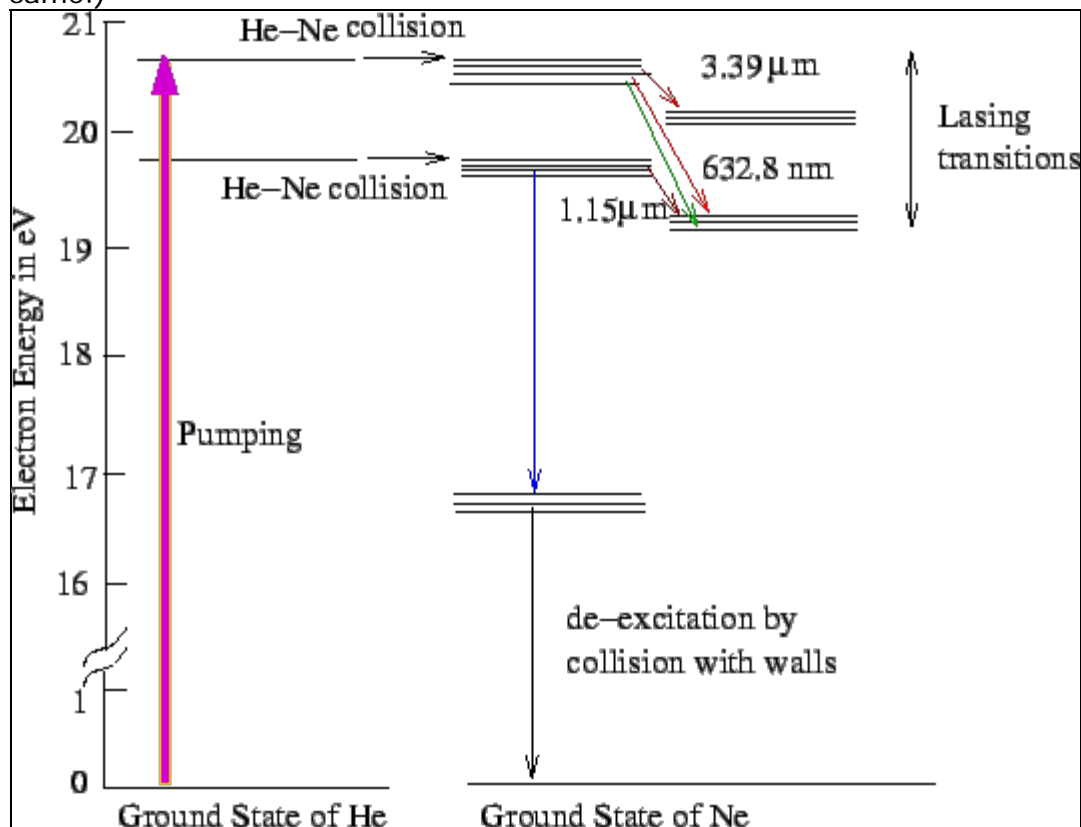
Pumping helium atoms to their excited states is provided by electrical discharge at about 1 keV. The mirrors reflect light back and forth extending the path travelled by by light which increases the probability of stimulated emission. The emergent laser light is primarily in the red region of spectrum at  $\lambda = 632.8\text{nm}$ . The principle of lasing is as follows.



Among many excited states of helium, one of the states is a metastable (long lived) state with an energy which is 20.6 eV above the ground level. (An electron in this level is not permitted to return to the ground state by emission of a photon as it would violate conservation of angular momentum.) The ground state of neon has an electronic configuration of  $2p^6$ . Neon atoms have various excited levels of which there is a set of

levels corresponding to a configuration  $2p^55s$  which are coincidentally removed from the

ground state of neon by 20.6 eV with a small spread of 0.04 eV. Helium atoms which are pumped into the excited state may collide with the neon atoms in their ground states and transfer their energy to the neon atoms, taking the latter to their excited levels. The small energy spread of 0.04 eV can be accounted for by the kinetic energy of colliding atoms. The following figure shows the transitions that takes place. (The figure shows additional energy levels of helium and neons which are also involved, the principle, however, remains the same.)



Neon has lower lying energy levels at about 18.7 eV above its ground state corresponding to the atomic configuration  $2p^5 3p$ . At any instant there are more atoms in the  $2p^5 5s$  than in any of the lower levels, resulting in a population inversion. Lasing transition takes place between the  $2p^5 5s$  level and the  $2p^5 3p$  level which emits in the red at a wavelength of 632.8 nm. Lasing also occurs in infrared and far infrared with emissions at  $3.39 \mu\text{m}$  and  $1.15 \mu\text{m}$  as shown in the figure. Less prominent emissions in the green part of the spectrum (543 nm) also takes place. Helium-neon laser, which is a low cost device is not particularly an energy efficient device, its energy output being a few milliwatts whereas the pumping power is between 10 to 100 watts. However, its primary utility lies in the coherence and directionality of the beam as well as the energy that can be delivered over a small area because the power, though small, is concentrated over a small beam diameter giving a power density between  $0.1$  to  $1 \text{ kW/m}^2$ . Coherence of the beam is useful in interferometric and holographic applications. Collimation and the ability to traverse long distances is used in measuring and sensing devices, as barcode scanners etc. As the emission is in the red - green region of the visible region, He-Ne lasers are used as tools in advertising in light shows and in entertainment electronics.

#### 2.14.2 Carbon dioxide Laser:

Carbon dioxide laser was invented by C. K. N. Patel of Bell Laboratories in 1964. It is basically a three level system, though a fourth level is also involved in the transition. It

contains a mixture of gases of  $\text{CO}_2$ ,  $\text{N}_2$  and He, approximately in the ratio 1:1:8. Note

that inspite of the name, the primary constituent is helium. In case of  $\text{CO}_2$  laser, in addition to the quantized electronic levels of the atoms, the vibrational and the rotational states of the molecules are also involved in the transition. Electrical discharge is used to excite the nitrogen molecules to higher excited states, which are long lived and cannot decay by emission of photons. The excited nitrogen molecules collide with carbon dioxide molecules which happen to have a second excited level (the pumping level) precisely at the excitation energy of the nitrogen molecules. A population inversion occurs and lasing transition takes place in far infrared ( $\sim 10.6 \mu\text{m}$ ) and also at  $9.4 \mu\text{m}$ . Helium has an

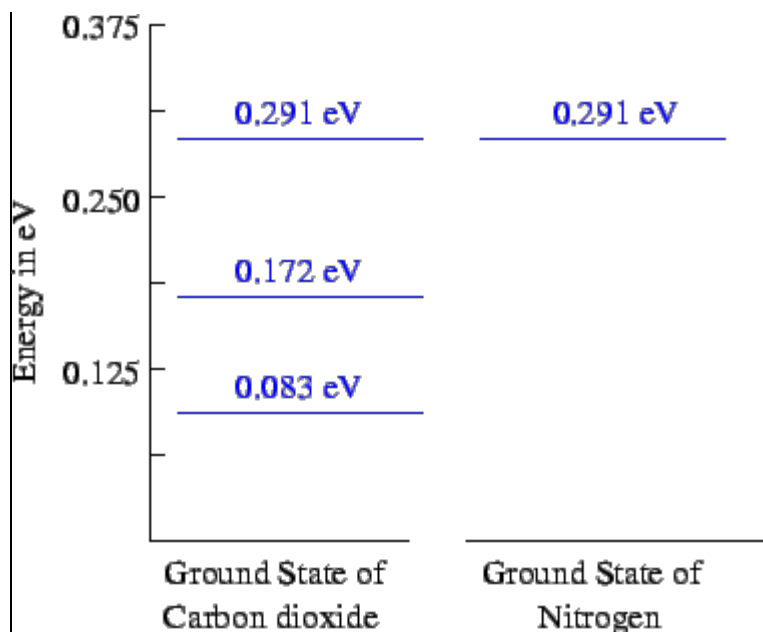
important role to play in the laser operation. Carbon dioxide molecules to return to the ground state by collision with helium atoms. In addition, helium improves the thermal conductivity of the gas mixture without which the gas would become hotter and would have an increased population in the excited levels negating the effect of population inversion.

#### Exercise:

The adjacent figure shows the energy levels of nitrogen and carbon dioxide molecules.

Using this, explain laser action in  $\text{CO}_2$  lasers.





Nitrogen-carbon dioxide laser system is the most efficient and powerful among all the

lasers, the output efficiency being about 12%. Because of such high power,  $\text{CO}_2$  lasers are used in industrial applications like cutting and welding. It has been used extensively in military applications for rangefinding through a technology called LIDAR (Light Detection and Ranging), which is very similar to RADAR which uses radio waves in that the distance of an approaching aircraft may be determined by measuring time delay in arrival of a laser

pulse at its source after reflection.  $\text{CO}_2$  lasers have been used in medical applications in thoracic and retinal surgery.

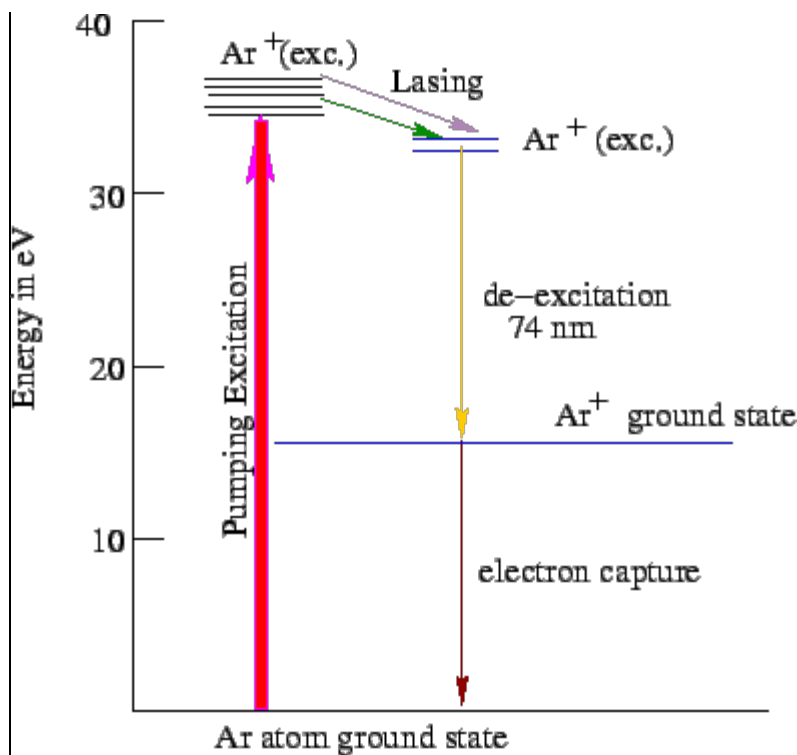
### 2.14.3 Argon-ion Laser :

Argon-ion laser is an example of a **continuous wave (CW)** laser, which is a type of laser in which a coherent beam is generated continuously as an output. Because of this it has applications in communication technology. It can be operated in CW mode at about 25 different wavelengths from 350 nm in the ultraviolet to 520 nm in the blue green, but the strongest lines are 488 nm and 514 nm. Because of such prominent lines in the visible region, the laser is used in light shows.

The energy levels of neutral argon as well as that of a singly ionized argon ( $\text{Ar}^+$ ) is shown alongside. The neutral atom is pumped into its excited state. As we are dealing with noble gases, the excitation energy is very high. Ionization of neutral Ar atoms requires a voltage pulse of about 10 kV. The process of ionization creates argon ions in their ground as well as excited states. The ground state of  $\text{Ar}^+$ , which has a configuration of  $3s^2 3p^5$ , is about 15.75 eV above the ground level of neutral Ar.

Stimulated emission occurs between the  $3s^2 3p^4 4p^1$  excited state of the ion and the

$3s^2 4p^4 4s^1$  excited of the ion. The ion in the latter excited state drops to the ground state of the ion by a spontaneous emission at 74 nm. From this state electron capture returns the ion to the ground state of the neutral atom.



High energy required for ionization makes the laser rather energy inefficient. However, high energy outputs up to about 20 W can be obtained in CW mode. Because of this, argon ion lasers have applications in scientific studies where high power is required. In addition, the laser is used extensively in ophthalmic and general surgery. **2.14.4 Solid State Lasers:** Typical examples of solid state lasers are Ruby lasers, Nd-glass laser etc. Ruby laser

consists of rods of ruby, which are  $\text{Al}_2\text{O}_3$  with about 0.05% Cr with a highly polished mirror at one end and a semi-transparent mirror at the other. A xenon flash bulb is used to excite chromium atoms to their excited states. Lasing transition at 694.3 nm takes place between states of chromium. Pulsed beam with bursts lasting  $1.2 \times 10^{-14}$  seconds can be

generated with such a laser. Power output of solid state lasers are high and they have wide variety of applications like cutting, welding, printing and xeroxing, medical and surgical applications etc.

## 2.15 Semiconductor Lasers:

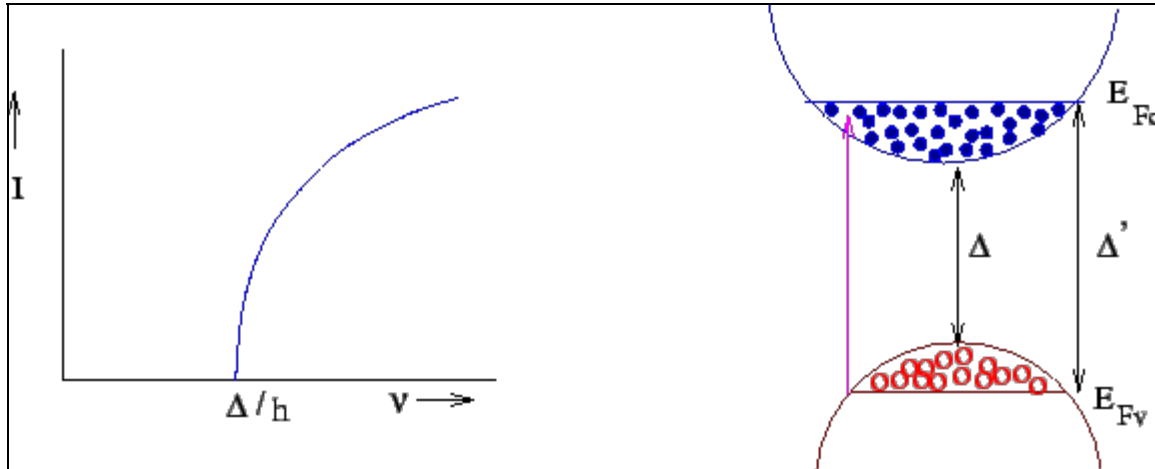
Semiconductor lasers make use of junction between different semiconductors as the active medium. Laser action is achieved by heavily doping the junction which ensures availability of a large concentration of electron hole pairs for recombination. Ends of the device are polished so that spontaneously emitted light travels back and forth enabling stimulated emission. Emission wavelengths span a wide range from near red into far infrared. Power output of semiconductor lasers can be from a few milliwatts to several watts under cw conditions while pulsed power of several hundreds of watts may be made available. Semiconductor lasers have wide range of applications. These include their use in communication systems, environmental sensing, audio compact discs, laser printing etc. The principle of semiconductor laser is basically that of p-n junction. The junctions are either of differently doped but of the same material or between material having different band gaps. The former are known as **homojunction laser** while the latter are called **heterojunction lasers**. Homojunction lasers are not often used. However, they serve to illustrate the principle.

The essential difference between semiconductor lasers and lasers based on atomic energy levels is that the carriers in a semiconductor are distributed throughout the material. As a

result, while in case of a collection of atoms, the system interacts only with a probe beam whose frequency is equal to the energy difference between atomic levels, a semiconductor would interact with a probe (photon) as long as the energy of the photon is greater than the band gap. If  $\nu$  is the incident frequency and  $\Delta$  is the band gap, the absorption coefficient is proportional to the density of states at the energy  $\hbar\nu - \Delta$ , i.e., to

$\sqrt{\hbar\nu - \Delta}$ . The intensity of the probe beam diminishes, following Beer's law (

$$I = I_0 e^{-\alpha z}).$$



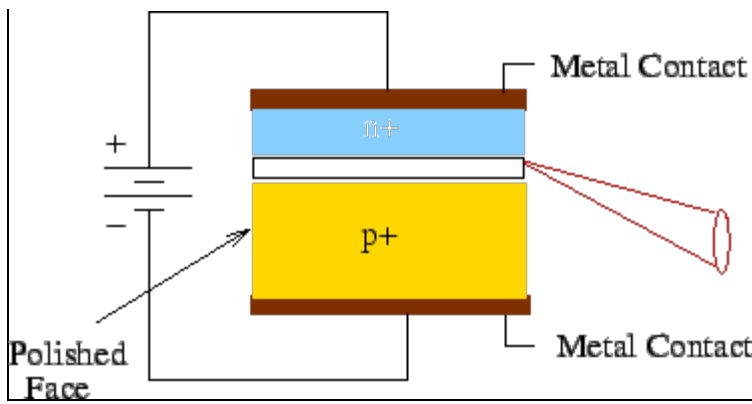
Suppose we use an optical pump to promote electrons from the valence band to the conduction band so that there is an appreciable population of electrons in the conduction band and holes in the valence band, up to the levels marked  $E_{Fc}$  and  $E_{Fv}$  in the

diagram. If we send a probe signal of frequency  $\nu$  such that  $\Delta/h \leq \nu \leq \Delta'/h$  through

the material, the probe would **see** a population inversion in the sense that there are more electrons in the upper level than in the lower level resulting in a gain.

### 2.15.1 Homojunction Laser (Laser Diode):

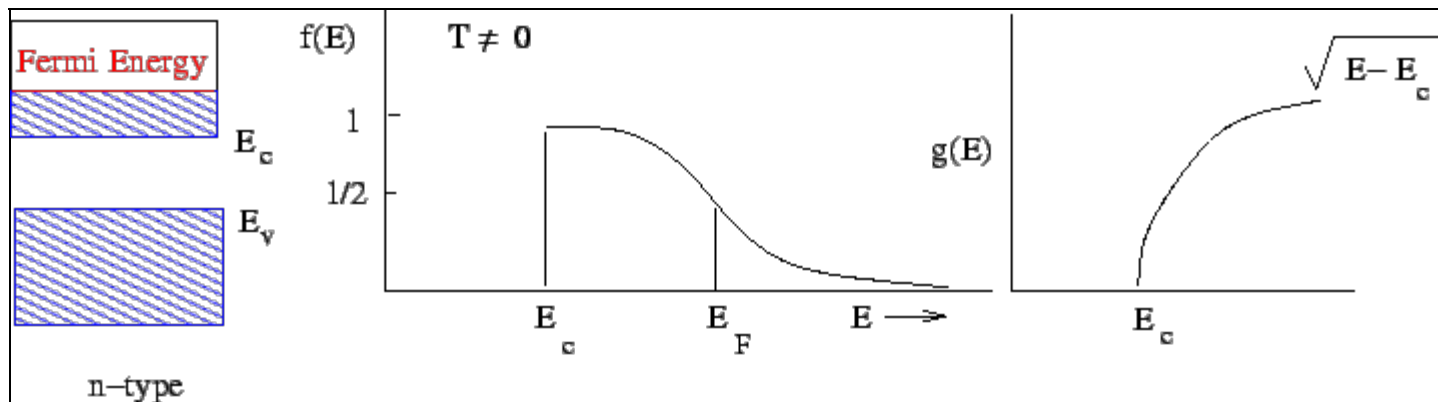
Homojunction lasers are made of p-n junctions of either the same material or of material having similar bandwidth. Population inversion can be achieved by heavily doping a p-type material with electrons or n-type material with holes.



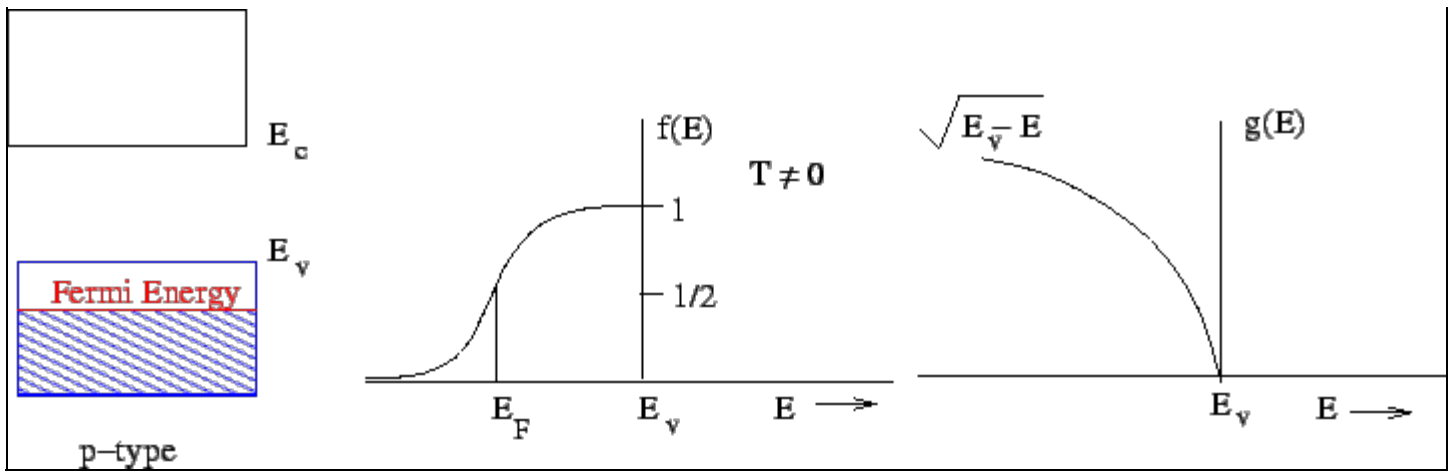
Consider a p-type semiconductor. When the doping is light, there are acceptor states near the top of the valence band. Electrons are thermally excited from valence band to the acceptor states, leaving holes in the valence band. As the doping concentration is small, the probability of two holes trying to occupy the same state is small and the effect of Pauli principle may be ignored. Likewise, in an n-type semiconductor, when the doping level is low, number of states in the conduction band being much larger than the number of donor electrons, the states are sparsely populated.

As the dopant concentration increases, the donors and acceptor levels form bands which overlap respectively with the conduction band and the valence band. Such a semiconductor is said to be **degenerate**.

The Fermi level of a degenerate n-type semiconductor is pushed into the conduction band and lies on the top of the donor impurity band. Likewise, the



Likewise, the Fermi level of a p-type degenerate semiconductor is pushed into the valence band and lies at the bottom of the acceptor band. The distribution function (at  $T > 0$ ) and the density of states are shown along with the schematic band diagram.



Consider a p-type degenerate semiconductor.  $f(E)$  is the probability that a state of energy  $E$  is occupied by an electron. It follows that  $1 - f(E)$  is the probability that a state of energy  $E$  is occupied by a hole. Recalling that

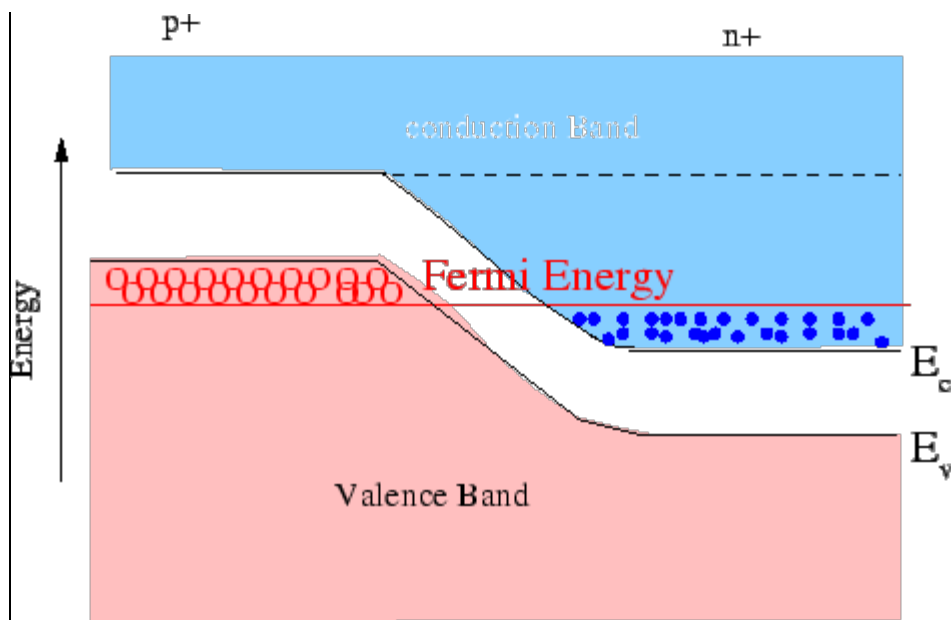
$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

if  $E \gg E_F$ , the denominator is large.  $f(E)$  is, therefore, small. Thus, if the Fermi level is inside the valence band, for  $E > E_F$ , the probability of holes can be large. Likewise, for an n-type degenerate semiconductor, the probability of occupation of electrons in the conduction band can be large.

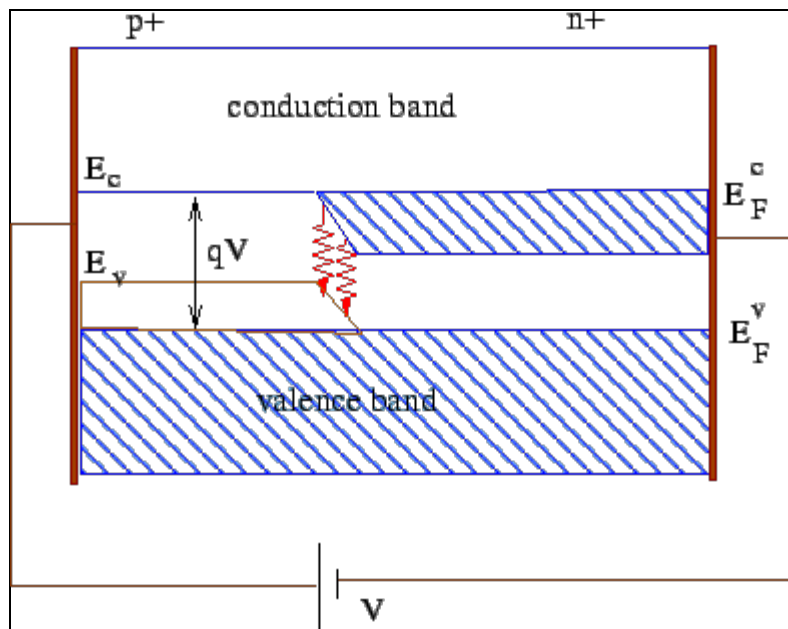
**i p-n Junction of Degenerate Semiconductor :** Consider a p-n junction of degenerate semiconductors with a heavily doped n-type on one side and a heavily doped p-type on the other. In the absence of an applied voltage, the Fermi level is the same throughout, i.e.,  $E_{Fp} = E_{Fn} = E_F$ . The Fermi level lies inside the valence band of the p-side and inside

the conduction band of the n-side. A contact potential  $V_B \simeq \Delta/q$  is built across the

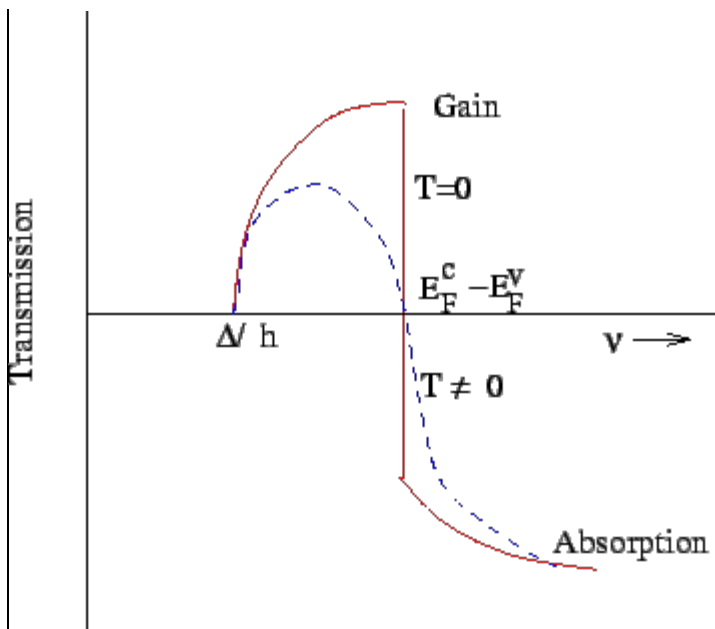
junction which prevents electrons on the n-side to diffuse into the conduction band on the p-side. Similarly, the holes on the p-side of the junction face a barrier to diffuse to the n-side.



When a forward bias  $V$  is applied (i.e., p-side positive and n-side negative)  $E_{Fp}$  is lowered and  $E_{Fn}$  is raised, the separation  $E_{Fn} - E_{Fp} = qV$ . The barrier potential is reduced and electrons from the degenerate n-side diffuse to the p-region through the depletion layer. Similarly, the holes from the p-side move towards n-side.



As a result the junction is no longer depleted. Near the junction an incoming signal **sees** a population inversion with more electrons at a higher level. If the signal frequency is larger than the frequency corresponding to the band gap, there will be gain for frequencies smaller than  $E_F^c - E_F^v$  and absorption for larger frequencies.

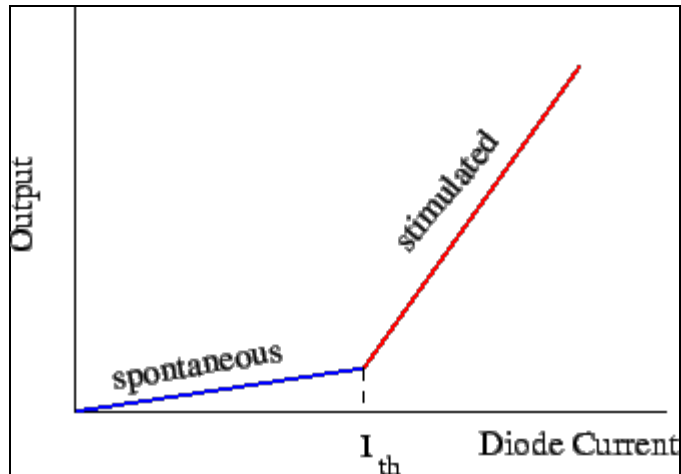


Gain is proportional to the carrier population above the threshold value. Since diode current is proportional to the carrier density, one can define a **threshold current**  $I_{th}$

above which lasing occurs.

The problem of homojunction laser is that the threshold current density is rather large ( $\sim 400 \text{ A/mm}^2$ ) which would require a good heat sink for continuous removal of generated heat. Homojunction lasers are, therefore, used in "pulsed mode" or at temperature below 100 K.

Another disadvantage of homojunction laser is that the junction region is wide which results in a divergent beam.



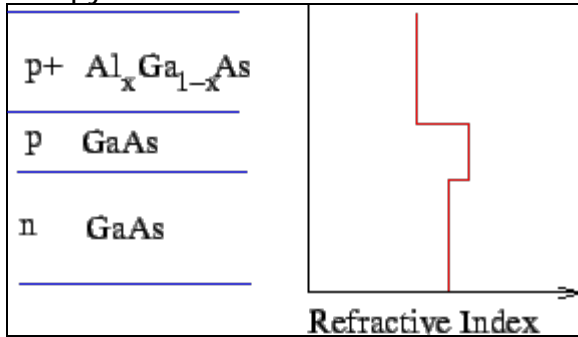
### 2.15.2 Heterojunction Laser:

We have seen that homojunction lasers suffer from two disadvantages.

- Wide beam width
- Large threshold current requiring low operating temperatures.

Reduction of threshold current may be achieved by confining carriers to a small region around the junction.

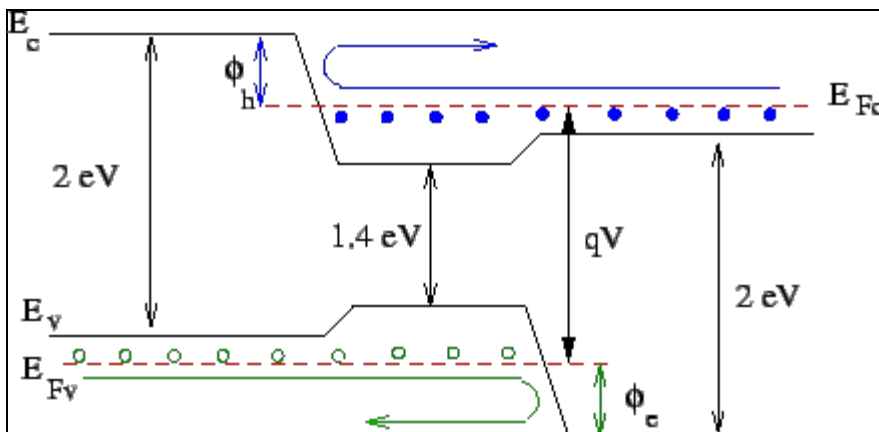
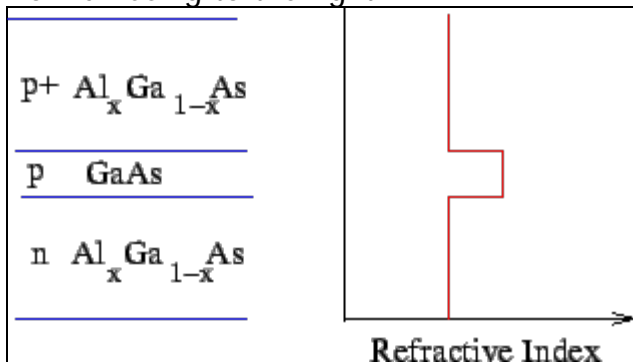
A heterojunction consists of two single crystal semiconductors with different bandgap energies. Because of different refractive index on either side of the junction, the carriers are confined in the junction layer and recombine. The choice of material for junction is decided by lattice matching. GaAs with a lattice constant of 5.65 Å has nearly the same lattice constant as AlAs which has a lattice constant of 5.66 Å. Thus in  $\text{Al}_x\text{Ga}_{1-x}\text{As}$ , the Al atoms occupy Ga sites.



In the single heterojunction shown above, the difference between the refractive indices of n-type and p-type GaAs is small. A double heterojunction is generally preferred as it leads to better confinement.

In the schematic double heterojunction shown here the intermediate GaAs layer has higher bandwidth material  $\text{Al}_x\text{Ga}_{1-x}\text{As}$  on either side. This results in the carriers getting

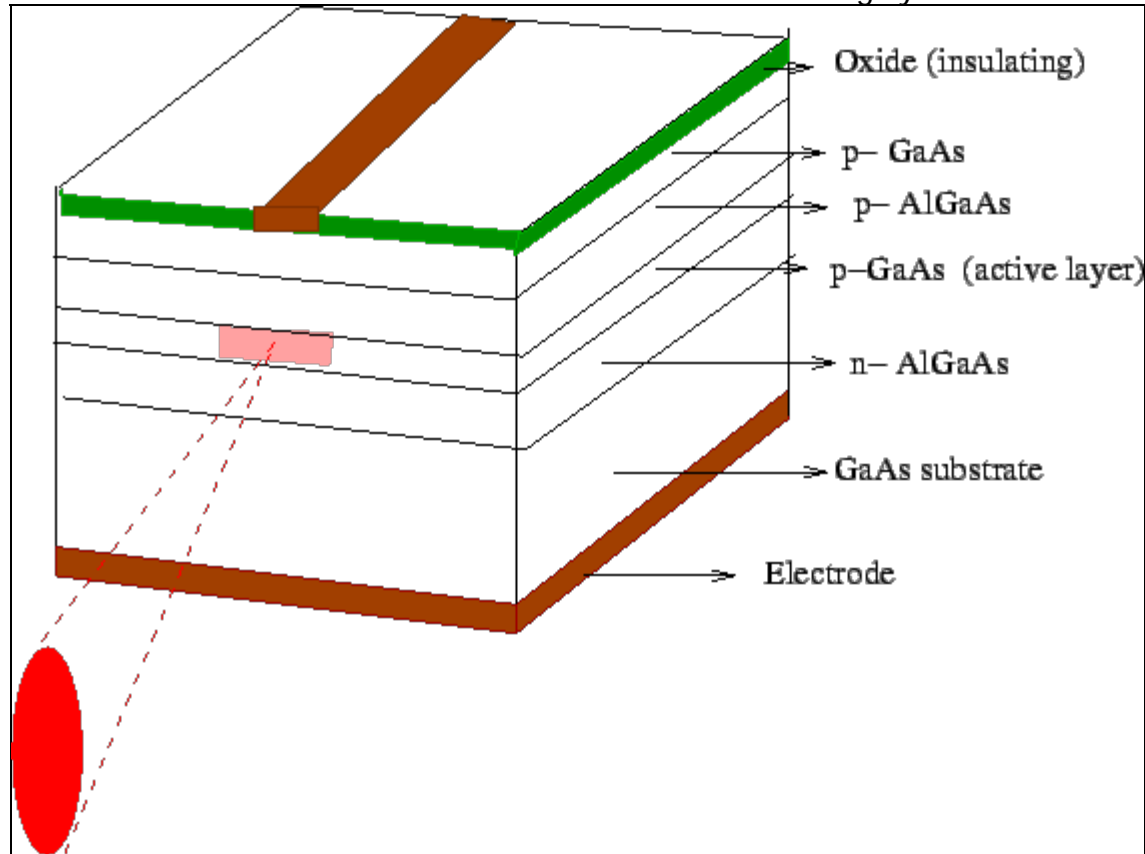
confined to the central layer as the electrons which diffuse to the junction from the n-side find a barrier which stops them from diffusing to the left. Similarly, the holes are prevented from diffusing to the right.



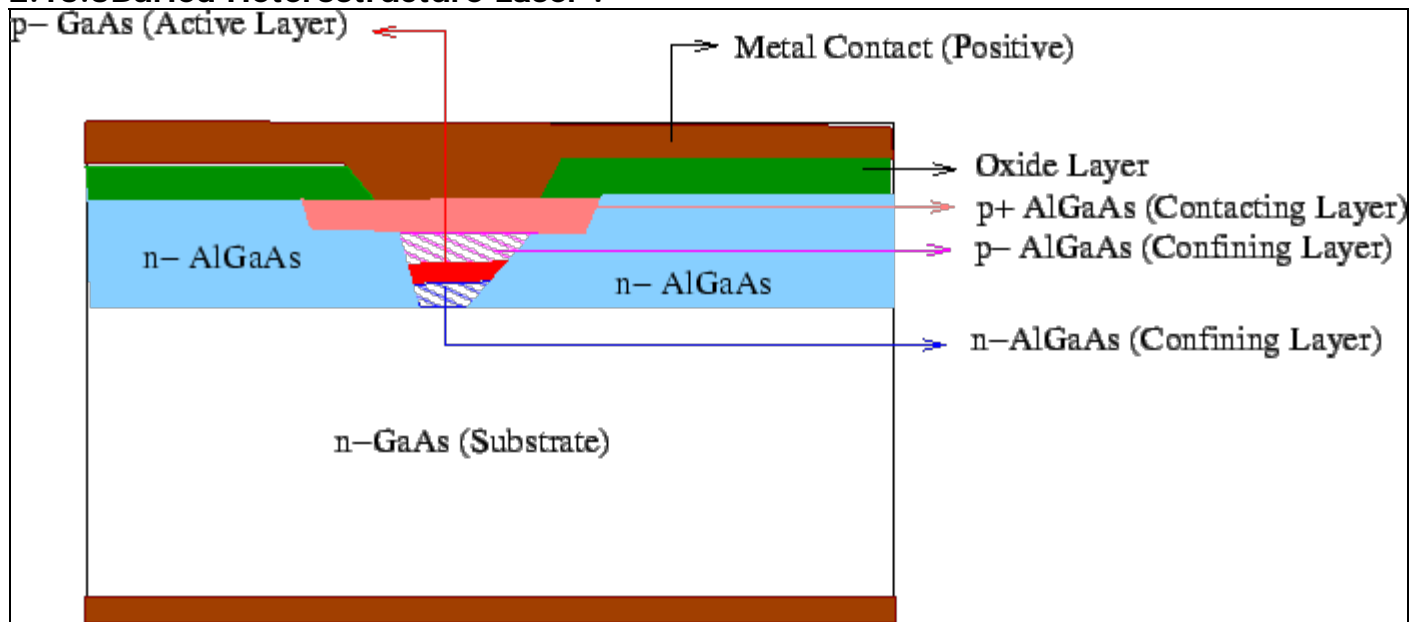
The figure below shows a schematic figure of a double heterojunction laser with **stripe**



**geometry.** In this geometry the top electrode does not cover the top surface of the structure. Instead, the top is covered with an insulating layer of silicon dioxide and the contact of the electrode is made over a narrow channel roughly 10 microns wide.



### 2.15.3 Buried Heterostructure Laser :



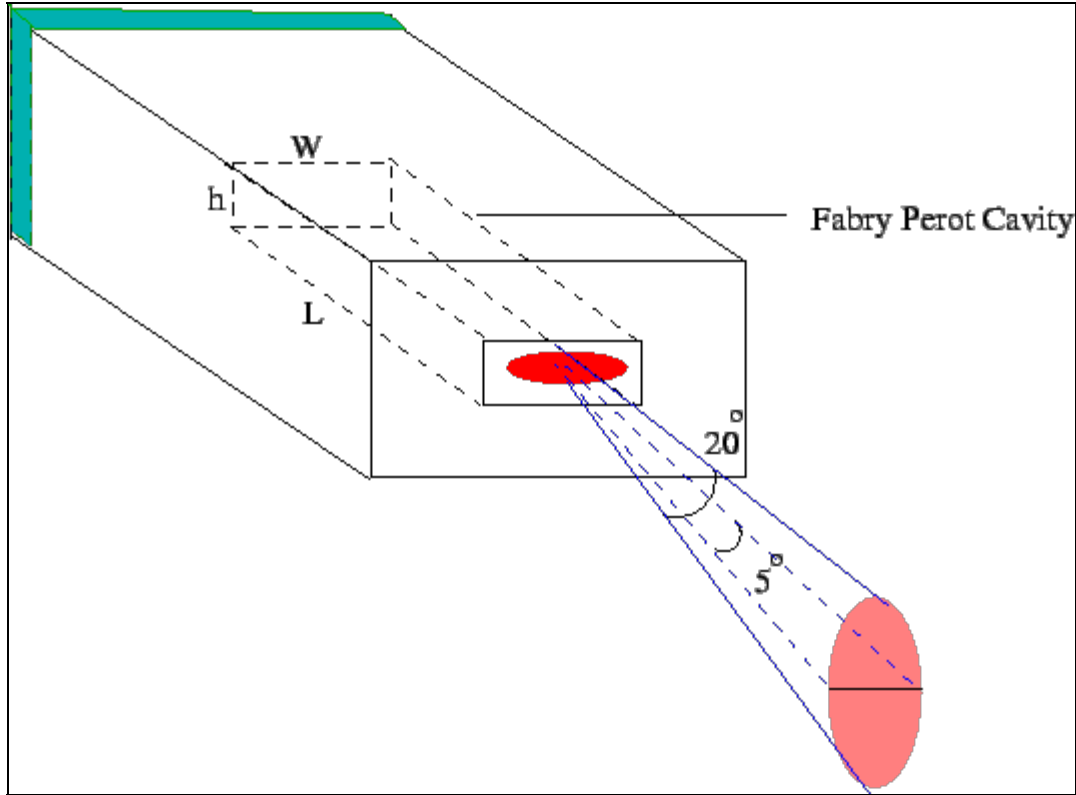
Laser confinement can be further improved by **index guided** mechanism in which the optical power is confined by a lateral variation of the refractive index. The principle is the same as for obtaining vertical confinement in DH laser in that a higher band gap material (AlGaAs) is etched both laterally and vertically. As the active layer is surrounded by a material of lower refractive index, it provides a waveguide. The threshold current is

reduced to less than 10 mA because of further reduction in stripe width.

## 2.16 Properties of GaAs Laser:

### Modes in Laser Cavity:

The stripe geometry essentially implies that the active region is a rectangular cavity of length  $L$ , width  $w$  and height  $h$ . This results in only certain modes being excited.



The resonance condition is given by the dimension of the cavity along any of the three directions being equal to multiple of half wavelength. For a longitudinal mode, the condition is given by

$$m \frac{\lambda_m}{2} = L$$

where  $\lambda_m$  is the wavelength in the medium and  $m$  is an integer. In terms of the free space wavelength  $\lambda$  and refractive index  $\mu$ , the condition is

$$m \frac{\lambda}{2\mu} = L$$

Similar conditions apply in the other two directions.

$$n \frac{\lambda}{2\mu} = w$$

$$l \frac{\lambda}{2\mu} = h$$

Each mode is, therefore, described by a set of three integers  $(m, n, l)$ . When  $w$  and  $h$  are sufficiently small, only the lowest transverse mode (TEM<sub>00</sub>) is excited.

**Example:**

A heterostructure diode has a cavity length of 500 microns. The peak wavelength of radiation is at 870 nm and the refractive index is 4. Find the index of the longitudinal mode and the separation  $\Delta\lambda$ .

**Solution:**

Using  $m = 2\mu L/\lambda$ , we get  $m \simeq 4598$ .  $\Delta\lambda$  is given by the wavelength separation between two consecutive modes

$$\Delta\lambda = \frac{2\mu L}{m} - \frac{2\mu L}{m+1} \approx \frac{2\mu L}{m^2} = \frac{\lambda^2}{2\mu L}$$

Substituting values,  $\Delta\lambda = 0.19\text{nm}$ .

The number of modes excited depends on the FWHM above the gain threshold. An index guided structure has a shorter cavity length which increases the spacing between adjacent mode frequencies so that fewer modes can be supported within the envelope of the optical gain.

**Example:**

The gain envelope of a DH structure has FWHM of 5 nm wavelength. Use the data of the previous example to compute the number of longitudinal modes that may be excited in the cavity.

**Solution:**

From the previous example, we have,  $\Delta\lambda = 0.19\text{nm}$ . As FWHM is 5 nm, the number of modes is  $5/0.19 \approx 26$ . The following exercise shows that the number of modes is reduced if the cavity length is decreased.

**Exercise:**

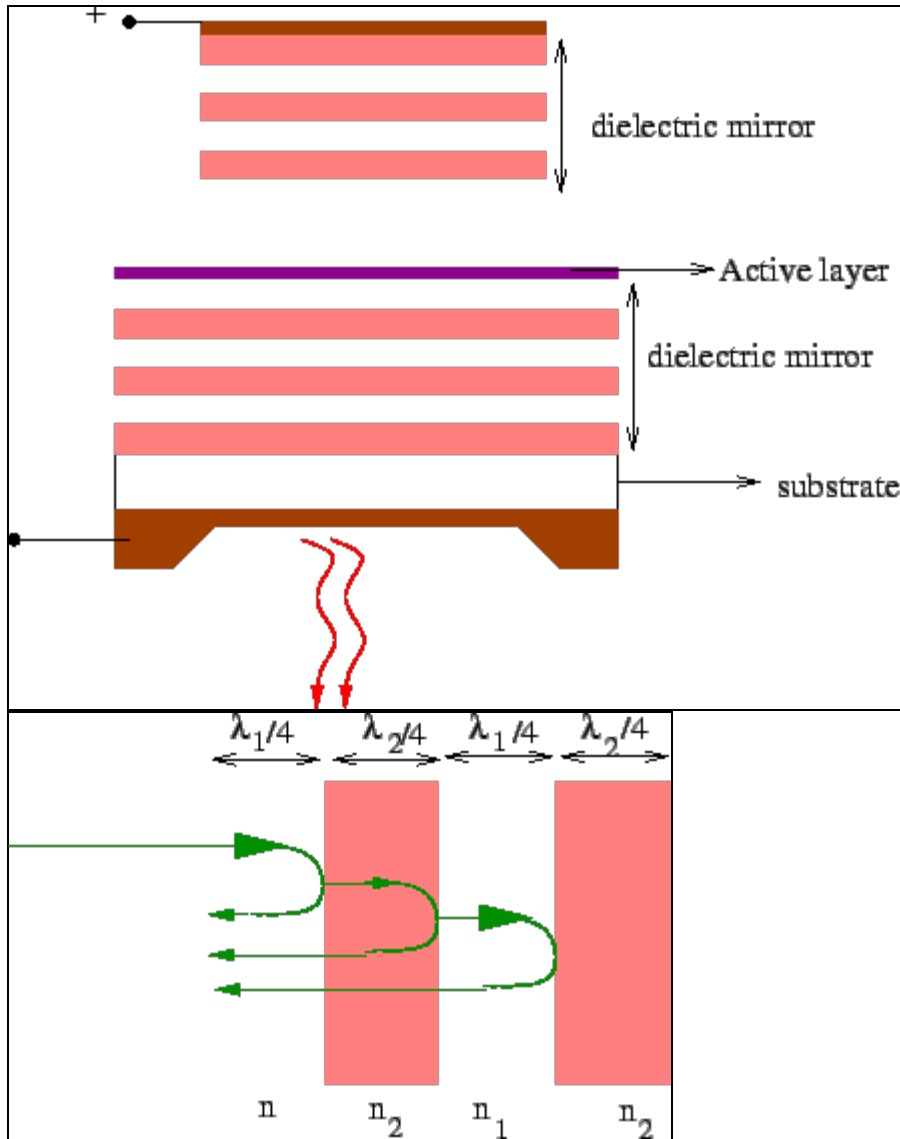
Determine the number of longitudinal modes excited if the cavity length in the above example is reduced to 200 microns. (Ans. 11)

When the diode current increases, the optical power also increases. Above the threshold region, the spectrum becomes sharper with the increase in optical power and fewer modes are excited. GaAs laser does not have as good directionality and monochromaticity

properties as for a gas laser. However, they have certain advantages which make them attractive :

### 2.17 Vertical Surface Emitting Laser :

Vertical Surface Emitting Laser (VCSEL, pronounced *Vixel*) is a semiconductor device which emits beam of a small circular cross section perpendicular to the substrate.



The device consists of a vertical sandwich of a p-type multilayer, an active region and an n-type multilayer. The two multilayers provide mirrors for resonant cavity and are called **distributed Bragg reflectors**. Wavelength tuning is possible by a proper choice of semiconductors and dopants while fabricating the multilayers. The features of VCSEL are as follows:

- System has high gain and low turn on voltage. By high speed turning on and off it can be modulated to a speed greater than 10 Gbps.
- As the output beam has circular cross section, it is easy to couple to a fiber.
- The beam has low divergence.

- Emission takes place from surface rather than from the edge.
- A thinner active region leads to a lower threshold current.

## 2.18 Noise in a Laser Source :

The process of optical communication has three essential components, viz., the source, the medium of propagation (fiber) and the detection mechanism. Each one of these contribute to corruption of the signal because of noise associated with it.

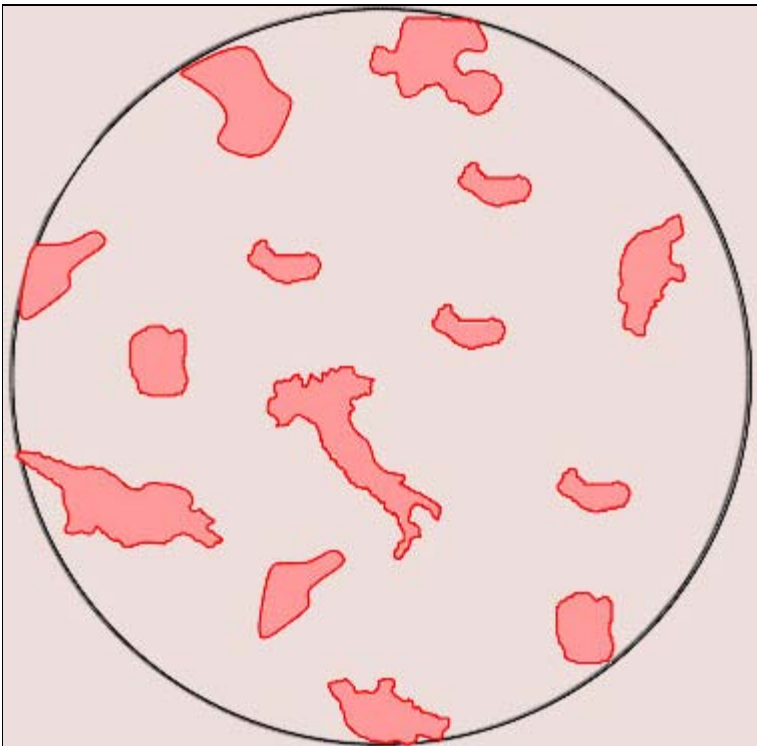
The operating characteristics of laser diodes cause three types of noise, which, in turn, will affect the receiver output. The primary noise sources for the transmitter are :

- Modal or speckle noise.
- Mode-partition noise.
- reflection noise.

### Speckle Noise :

When light from a coherent laser source is launched into a multimode fiber, several modes are simultaneously excited in the fiber. As long as these modes maintain their relative phase coherence, the radiation pattern in any plane is due to interference between such modes. The interference pattern takes the shape of **speckle** , as shown in the figure.

The number of speckles is approximately equal to the number of modes propagating. Laser speckles appear whenever an optically rough surface is illuminated by a highly coherent light provided the roughness of the surface is at least of the order of wavelength of light used.



The optical disturbance at any point along the path consists of wavelets arising from a different element of the surface. Because of different sizes of surface granularities, such wavelets have different phases. Speckles look like noise, but in reality, they are not. According to Gabor, winner of Nobel prize for invention of holography, ``it is not really noise, it is information that we do not want." In this case, the unwanted information is the granularity of the surface. Speckle is not without its use. A new branch of optical measurement technique, known as **speckle metrology** uses speckle pattern in non-destructive testing of a surface.

**Speckle noise**, therefore, is not speckle itself but a variation in the speckle pattern due to temporal variation of the coherent optical signal. A second cause of such noise is presence of elements which make propagation mode-selective. Such time varying speckle pattern falling on a photodetector produces a time varying noise in the received signal, which degrades its performance.

The way to eliminate modal noise is to use single mode fibers for coupling to laser sources and to use LEDs for multimode fibers (which do not produce speckles).

### **Mode-Partition Noise :**

Mode partition noise is due to intensity fluctuations in the longitudinal modes of a laser diode. We have seen that the cavity modes in a laser (not to be confused with fiber modes) are defined by a unique wavelength given by

$$\underline{\lambda}$$

where  $2nL = m\lambda$  ( $m = 1, 2, \dots$ ) is the refractive index of the medium and  $n$  is the length of the laser cavity. Output from a laser diode could be from more than one longitudinal mode. The total power output coupled to the fiber is constant in time but the distribution of power among various cavity modes changes with time. This fluctuation in power at various times is called the mode partition noise. This is the primary source of noise in a single mode fiber.

### **Reflection Noise :**

Light coupled to a fiber may be reflected back from various surfaces, like fiber junctions, and re-enter the laser cavity. Such reflected light can couple with the cavity modes and change their phase. Reflection noise can be eliminated by using index-matching fluids at the joints.

### **Spontaneous Emission Noise :**

In addition to above, a laser source has noise due to spontaneous emission of radiation between the lasing levels. Unlike stimulated emission, such emissions are not coherent. Spontaneous emissions cause a spectral width to the laser output and also reduces SNR at the output of laser amplifiers.

## References :

1. S.O. Kasap, Optoelectronics and Photonics, Prentice Hall, N.J. (2001)
2. A. Yariv, Introduction to Optical Electronics, Holt, Rinehart and Winston, Inc. (1991)
3. K. A. Jones, Introduction to Optical Electronics, Harper & Row (1987)
4. C. R. Pollock, Fundamentals of Optoelectronics, Irwin (1995)
5. J. Wilson and J. F. B. Hawkers, Optoelectronics - An Introduction, Prentice Hall (1993)
6. J. T. Verdeyen, Laser Electronics, Prentice Hall (1989)
7. W. T. Silvast, Laser Fundamentals, Cambridge (2004)

## Recap

In this lecture you have learnt the following

- Types of Lasers and Applications
- Helium-Neon Laser
- Carbon dioxide Laser
- Argon-ion Laser
- Semiconductor Lasers
- Homojunction Laser (Laser Diode)
- Heterojunction Laser
- Properties of GaAs Laser
- Vertical Surface Emitting Laser
- Noise in a Laser Source