Module 23 : Non-linear fiber optics II

Lecture : Non-linear fiber optics II

Objectives

In this lecture you will learn the following

- Pulse propogation in Non-Linear optical fiber
- Dispersive Regime
- Non-Linear Regime
- Solitons
- Practical Aspects of Solitonic communication

- **PULSE PROPAGATION IN NON-LINEAR OPTICAL FIBER**

  The pulse evolution on an optical fiber in general is governed by the Non-linear Schrodinger equation (NSE) .

$$\frac{\partial A}{\partial z}+\beta_1\frac{\partial A}{\partial t}-j\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial t^2}+\frac{\alpha}{2}A+j\gamma|A|^2A=0$$

  The different terms in the equation describe different effects like the group velocity, group velocity dispersion (GVD), fiber loss, and fiber non-linearity.

  Since we are interested here in the pulse evolution, the delay in pulse arrival is of less consequence. We therefore define a moving time frame which moves with the group velocity along the fiber. In this frame, the time is given as

$$T=t-\frac{z}{v_g}=t-\beta_1 z$$

  Where v_g is the group velocity of the pulse.

  The NSE in the moving time frame naturally would not have the second term since the pulse appears stationary in the moving frame. The NSE therefore becomes

$$\frac{\partial A}{\partial z} - j\frac{1}{2}\beta_2\frac{\partial^2 A}{\partial T^2} + \frac{\alpha}{2}A + j\gamma |A|^2 A = 0$$

Let us now assume that the input pulse to the fiber is **Gaussian in shape** with width τ_0 and power P.

Let us define two important characteristic lengths related to the dispersion and non-linearity as

$$\textbf{Dispersion Length} = L_D = \frac{\tau_0^2}{|\beta_2|} = \frac{\tau_0^2 2\pi c}{|D|\lambda^2}$$

$$\textbf{Non-linearity Length} = L_{NL} = \frac{1}{\gamma P}$$

The two lengths essentially tell us the distance a pulse has to travel on the fiber to show the respective effect.

We can also define the effective length the pulse travels on the fiber (in presence of loss)

$$L_{eff} \approx 1/\alpha$$

The is the length which the pulse travels before is power is unacceptably small.

In the following discussion, let us assume the fiber to be loss-less, i.e. $\alpha = 0$ **.**

The NSE then becomes

$$\frac{\partial B}{\partial z} - j\frac{\operatorname{sgn}(\beta_2)}{2L_D}\frac{\partial^2 B}{\partial T^2} + j\frac{1}{L_{NL}}|B|^2 B = 0$$

Where B is the normalized pulse. $A(z,T) = \sqrt{P}\,B(z,T)$.

We can now define different regimes of pulse propagation depending upon the length of the fiber, L.

1. $L \ll L_D, L \ll L_{NL}$: In this case neither the dispersion nor the non-linearity plays a role in pulse propagation and the fiber is merely a medium of energy transport.

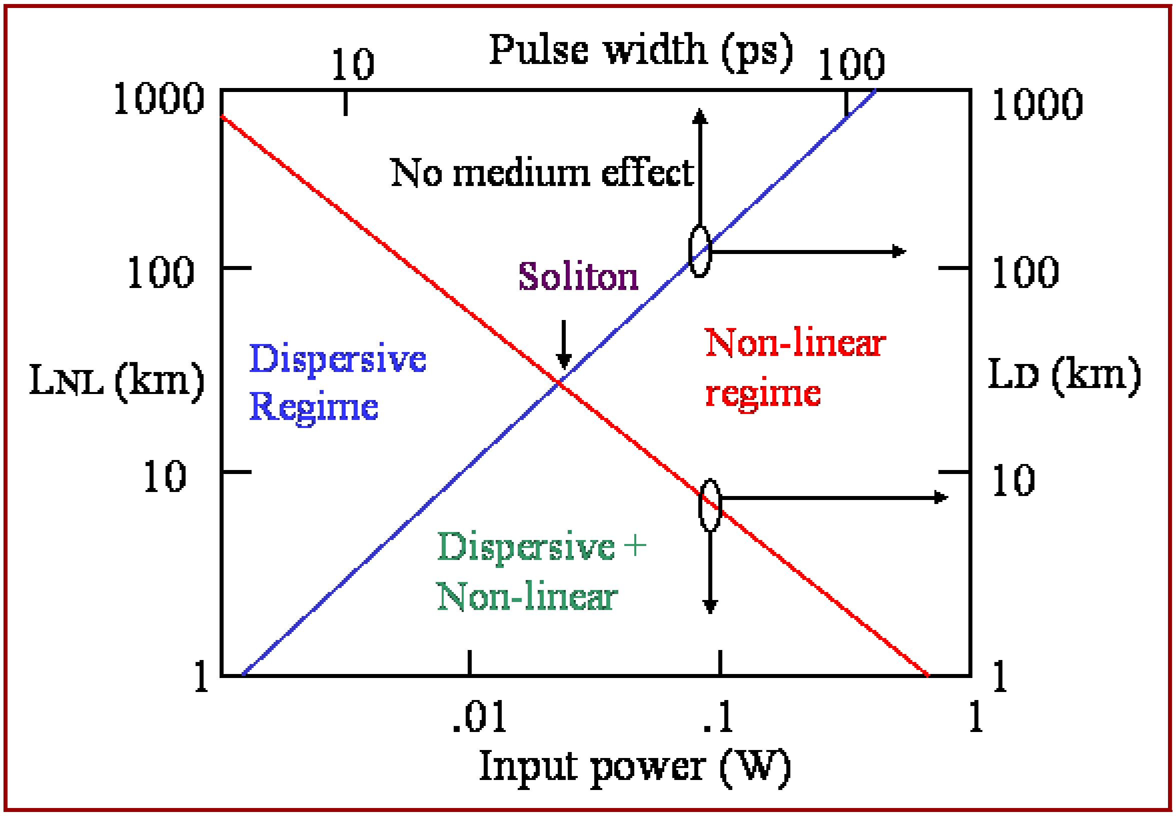
2. $L \geq L_D, L \ll L_{NL}$: This is **Dispersive regime** . The broadens as it propagates on the fiber. Since dispersion length is proportional to the square of the pulse width, for narrower pulses, the dispersion length is smaller and therefore the dispersive effect are significant.

3. $L \ll L_D, L \geq L_{NL}$: This **Non-linear regime** . In this case we get a phenomenon called the self-phase modulation. The pulse spectrum expands but the pulse in time domain remains unchanged. Since the non-linearity length is inversely proportional to the pulse power, for high power but for not so narrow pulses this condition prevails.

4. $L \geq L_D, L \geq L_{NL}$: In this situation both, the dispersion and non-linearity play a role in

pulse propagation, and there is possibility of canceling the two effects giving what is called the **Soliton** .

The four regimes are shown in Fig.



The Fig. shows the variation of dispersion and non-linearity lengths as a function of pulse width and pulse power for $\gamma = 1.3\ \mathrm{W}^{-1}\mathrm{Km}^{-1}$, $D = 22$ ps/km/nm at 1550nm .

- **Dispersive Regime**

In this regime the NSE has just the dispersion term.

$$\frac{\partial B}{\partial z} = j\,\frac{\mathrm{sgn}(\beta_2)}{2L_D}\frac{\partial^2 B}{\partial T^2}$$

Let the input Gaussian pulse be given as

$$B(0,T) = e^{-T^2/2\tau_0^2}$$

The NSE can be solved by taking Fourier transform, to give solution

$$B=\frac{1}{\sqrt{1+j\frac{\operatorname{sgn}(\beta_2)z}{L_D}}}\exp\left(\frac{T^2}{2\tau_0^2(1+j\frac{\operatorname{sgn}(\beta_2)z}{L_D})}\right)$$

We can write the expression in amplitude and phase form as

$$B(z,T)=|B(z,T)|e^{j\varphi(z,T)}$$

Where,

$$|B(z,T)|=\frac{1}{\sqrt{1+(z/L_D)^2}}e^{-\frac{T^2}{2\tau_0^2(1+(z/L_D)^2)}}$$

$$\varphi(z,T)=\frac{\operatorname{sgn}(\beta_2)(z/L_D)}{1+(z/L_D)^2}\frac{T^2}{\tau_0^2}-\tan^{-1}(z/L_D)$$

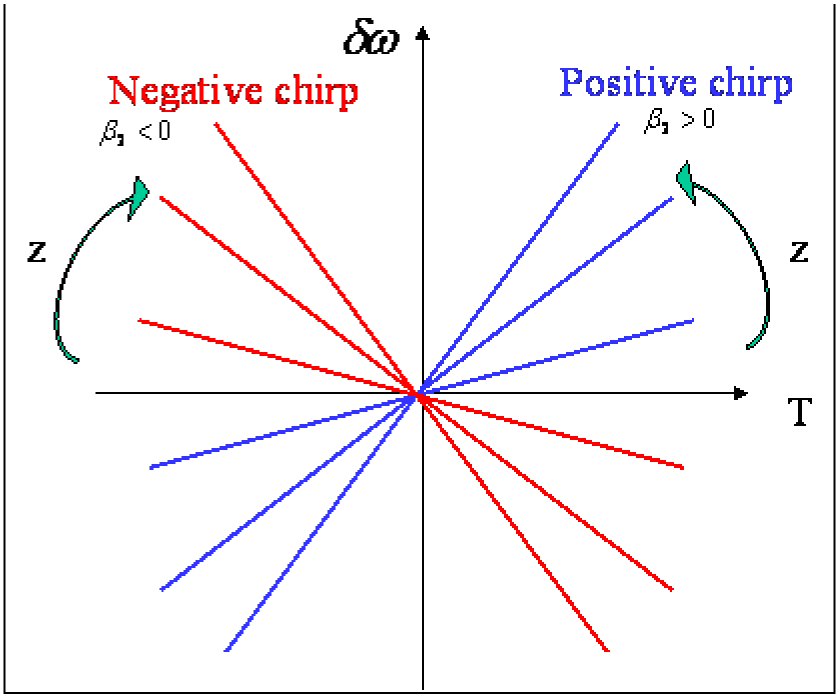
Two important things can be observed here.

- **The shape of the Gaussian pulse remains Gaussian as the pulse travels along the fiber in dispersive regime. However, the pulse width increases with distance. Over a distance z, the pulse width is $\tau_0\sqrt{1+(z/L_D)^2}$. It should also be noted that the pulse always broadens irrespective of the sign of β_2. We can then see that the dispersion length is that distance over which the pulse width increases to $\sqrt{2}$ times of its original value.**

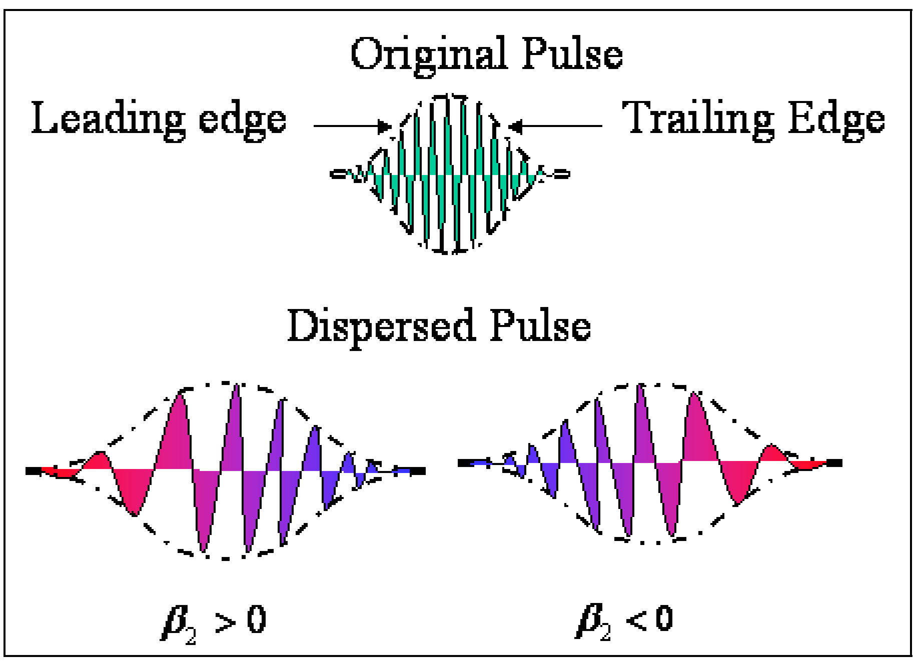
- **Internally the pulse gets phase modulated since the phase of the pulse is a function of T. Since the rate of change of phase is the frequency, we get change in pulse carrier frequency as**

$$\delta\omega=\frac{\partial\varphi}{\partial T}=\frac{\operatorname{sgn}(\beta_2)(z/L_D)}{1+(z/L_D)^2}\frac{T}{\tau_0^2}$$

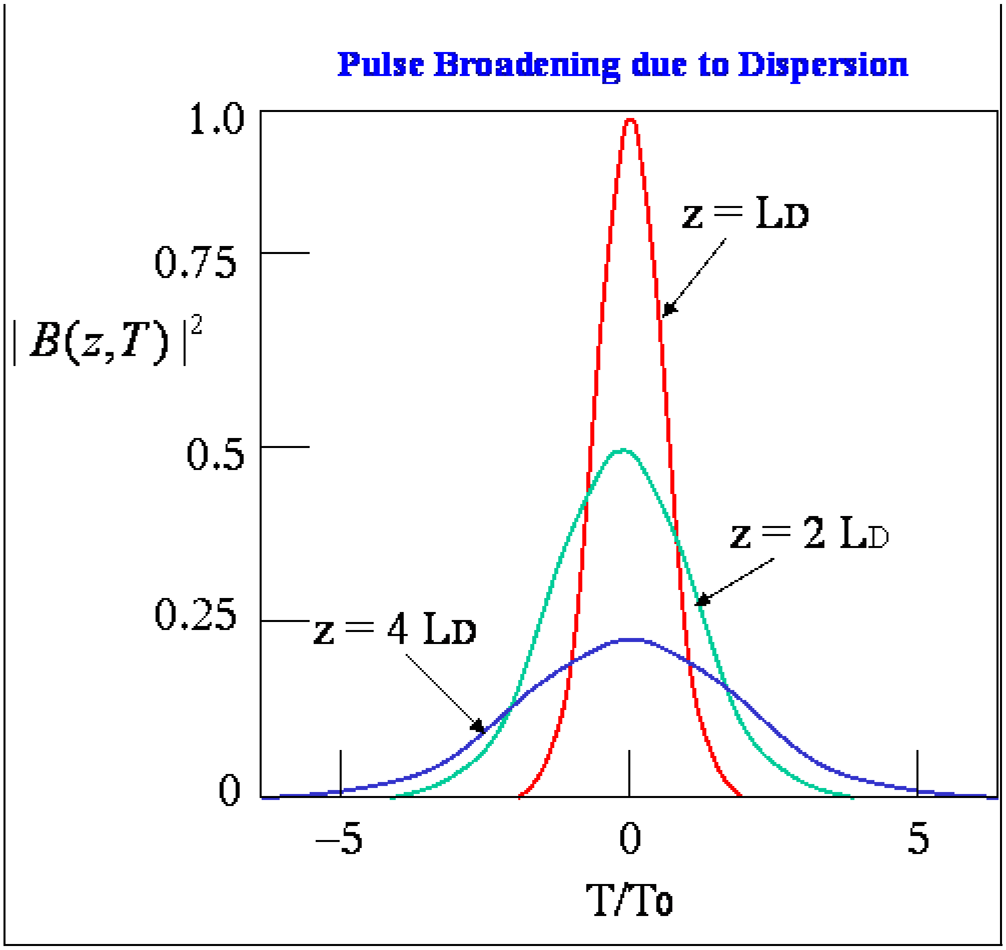
The frequency varies linearly as a function of time, T. It is then said that the pulse is linearly chirped. For positive β_2, the frequency is higher on the trailing edge of the pulse and is lower at the leading edge of the pulse as shown in Fig. The reverse happens when β_2 is negative. **If β_2 is negative the dispersion is called anomalous.**



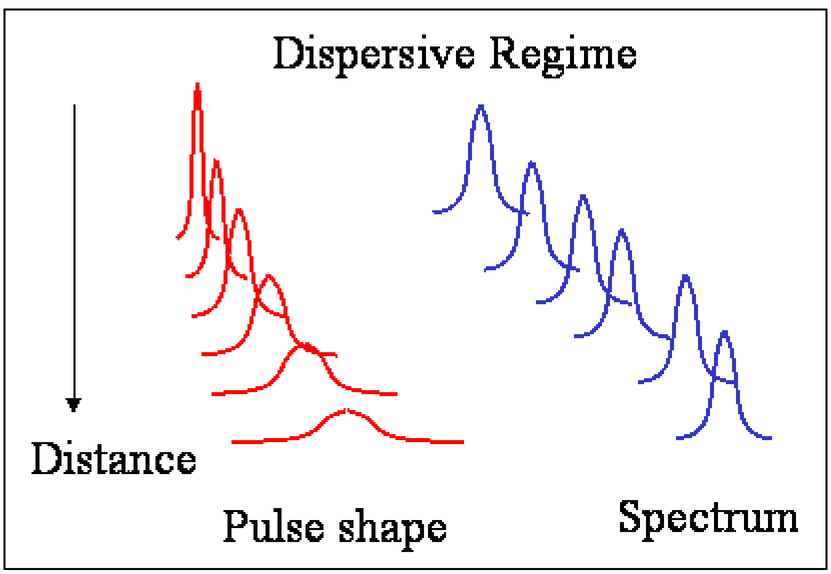
δω
Negative chirp
$\beta_2 < 0$
Positive chirp
$\beta_2 > 0$
z
z
T



Original Pulse
Leading edge
Trailing Edge
Dispersed Pulse
$\beta_2 > 0$
$\beta_2 < 0$



In Dispersive regime the pulse broadens but its spectrum remains same. Only different frequencies get separated in time due to dispersion, but there are now new frequencies generated as shown in Fig.



- **Non-Linear Regime**

In the non-linear regime, the non-linearity length is smaller than the fiber length and the dispersion length is larger than the fiber length.

The NSE in this case is given as

$$\frac{\partial B}{\partial z} = -j\frac{1}{L_{NL}}|B|^2 B$$

The solution to this equation is

$$B(z,T) = B(0,T)\, e^{j\varphi_{NL}(z,T)}$$

Where the non-linear phase

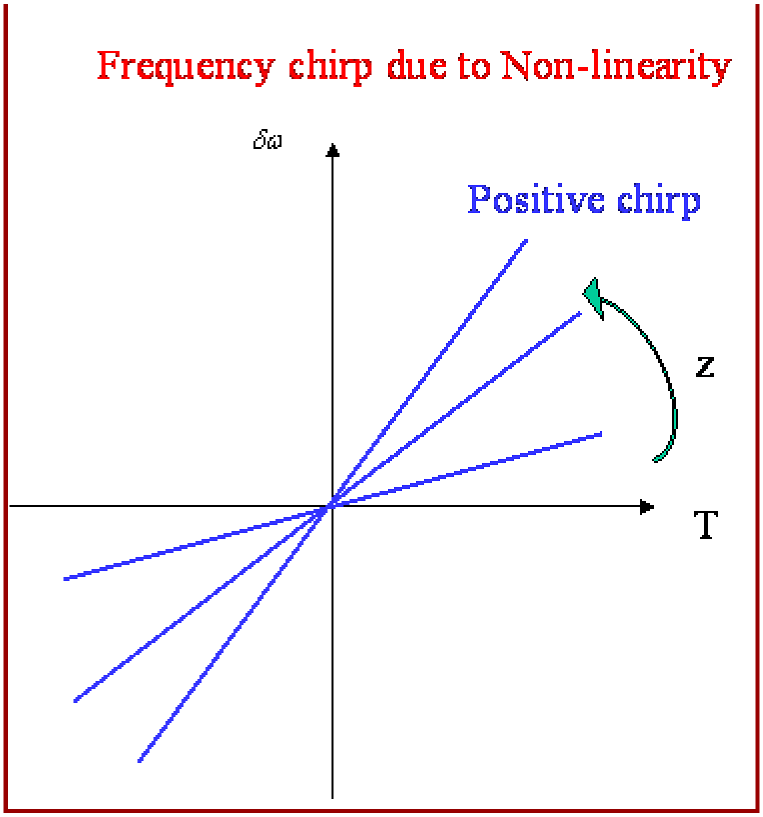
$$\varphi_{NL}(z,T) = -|B(0,T)|^2 z / L_{NL}$$

The phase of the optical carrier is dependent on the location on the pulse. The phase profile is same as the intensity profile of the pulse. The pulse is therefore phase modulated due to its own intensity profile. This phenomenon is called the **Self-Phase Modulation (SPM).**

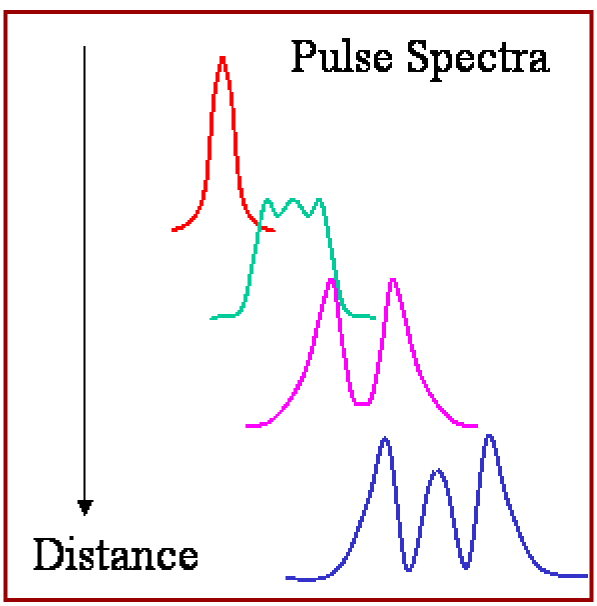
For this phase modulation, there is corresponding frequency modulation. The frequency deviation as a function of time T is

$$\delta\omega(z,T) = \frac{\partial\varphi_{NL}}{\partial T} = -\frac{\partial|B(0,T)|^2}{\partial T}\frac{z}{L_{NL}}$$

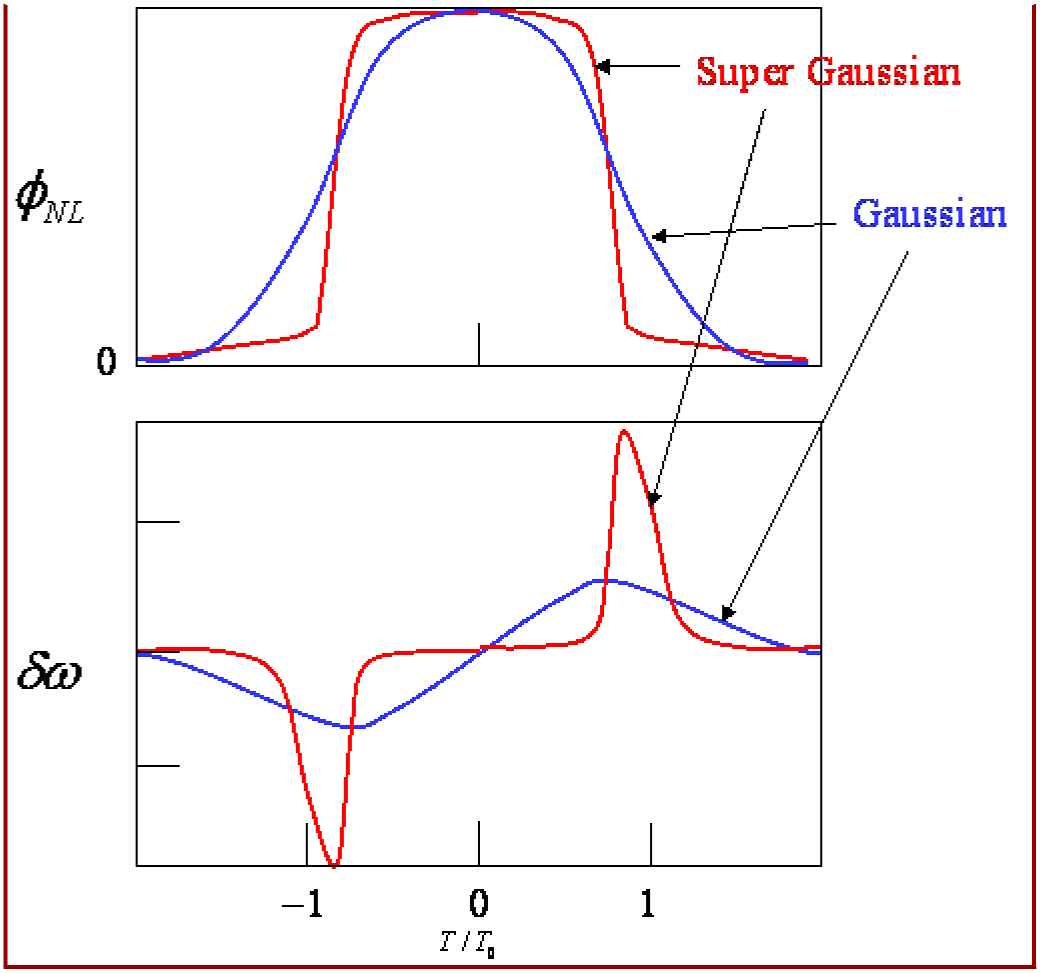
The frequency deviation is proportional to the rate of change of the pulse envelop. For positive T (trailing edge of the pulse), the slope of the envelop is negative, and therefore the frequency deviation is positive. For negative T (leading edge of the pulse) the frequency deviation is negative.



The spectrum of the pulse broadens with distance as the new frequencies are created.



For a Gaussian and super-Gaussian pulses the frequency chirp is given in the Fig.



Where the super-Gaussian pulse is defined as

$$B(0,T) = \exp\left[-\left(\frac{T}{\tau_0}\right)^{2m}\right]$$

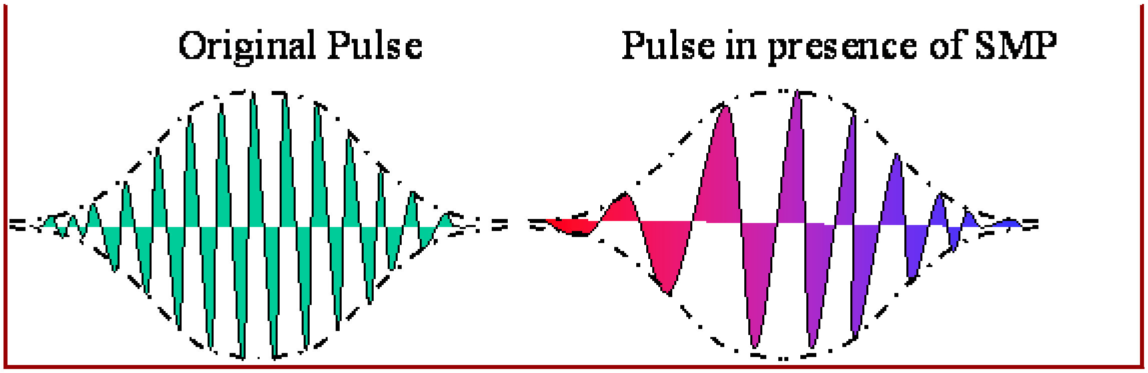
Super-Gaussian pulses with large m can be used to represent rectangular pulses.

As the figure shows, the frequency deviation is zero at the center of the pulse, it increases as we shift off center due to increase in the slope of the envelop reaches the maximum, and then decreases as we shift further due reduction of the power and hence the non-linearity.

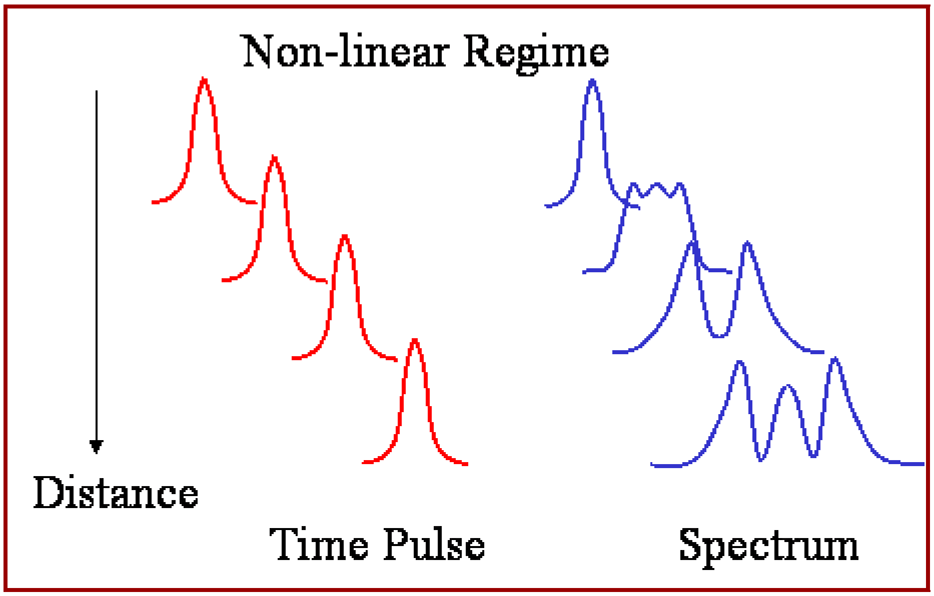
Following important things can be noted about the SPM.

- **Due to SMP the frequency chirp is always positive.**

- **The pulse shape in time domain remains unaltered.** That is, as seen in the previous module, the spectrum broadens due to non-linearity, but the pulse shape in time domain remains unaffected. The pulse then looks as shown in Fig.



• **With distance, the frequency chirp slope increases and the spectrum gets broadened.**



The important thing to note is that if dispersion is anomalous and non-linearity is present, the frequency chirps created by two effects are opposite in nature. It is the possible that the two chirps cancel giving chirp-free pulse. In this case neither the pulse nor its spectrum broadens. The undistorted pulse is called the ‘ **SOLITON'**

- **Solitons**

Waves like Tsunami and their destructive effects are well known to mankind.

Waves with similar features but with less dramatic effect can be created inside an optical fiber. These waves can be exploited for high speed long distance optical communication. In 1991 the optical solitons inside an optical were experimentally demonstrated.

The deployment of soliton based optical communication systems still are not feasible but the technology is progressing to make the solitonic communication a reality.

In the presence of non-linearity and dispersion, there is possibility of undistorted pulse propagation for infinite distance.

Let define normalized distance and normalized time as

$$\xi = z / L_D$$
$$\tau = T / \tau_0$$

The Schrodinger equation with dispersion and non-linearity, can be written as

$$\frac{\partial B}{\partial \xi} - j\frac{\operatorname{sgn}(\beta_2)}{2}\frac{\partial^2 B}{\partial \tau^2} + jN^2 |B|^2 B = 0$$

Where the parameter N is defined as

$$N^2 = L_D / L_{NL} = \frac{\gamma P \tau_0^2}{|\beta_2|}$$

The solution to the NSE in this case is the Soliton. For $N = 1$, we get the fundamental soliton and for higher values of N we get the higher order solitons. Of course, the higher order solitons need higher optical power.

The solution of the NSE for the fundamental soliton is

$$B(\xi, \tau) = \operatorname{sec}h(\tau)\, e^{-j\xi/2}$$

It indicated that if a secant hyperbolic pulse is launched inside an optical fiber, it can travel undistorted for infinite distance (of course in absence of loss).

The fundamental soliton has a very special wave shape, the **secant hyperbolic function .**

Let us see how the two frequency chirps, one due to non-linearity and other due to the dispersion get cancelled for secant hyperbolic pulse shape.

The NSE due for only non-linear term is

$$\frac{\partial B}{\partial \xi} = -j N^2 |B|^2 B$$

The non-linear phase is

$$\varphi_{NL} = -|B(0,\tau)|^2 \xi = -\operatorname{sec}h^2(\tau)\,\xi$$

The NSE due only anomalous dispersion can be written as

$$\frac{\partial B}{\partial \xi} = j\frac{\operatorname{sgn}(\beta_2)}{2}\frac{\partial^2 B}{\partial \tau^2} = -j\left(\frac{1}{2B}\frac{\partial^2 B}{\partial \tau^2}\right)B$$

The phase due to dispersion is

$$\varphi_D = -\left(\frac{1}{2B}\frac{\partial^2 B}{\partial\tau^2}\right)\xi = -\left(\frac{1}{2\sec h(\tau)}\frac{\partial^2 \sec h(\tau)}{\partial\tau^2}\right)\xi = -\left(\frac{1}{2} - \sec h^2(\tau)\right)\xi$$

The sum of the non-linear and the dispersion phase is

$\varphi_{NL} + \varphi_D = -\xi/2$ Independent of time.

The resultant frequency chirp therefore is zero and the pulse spectrum and consequently its shape remains unchanged.

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Practical Aspects of Solitonic communication

There are certain practical issues in realizing solitonic communication.

- **Initial Pulse shape**

In practice however, the optical pulses do not have this special shape. This is not a problem, since in presence of non-linearity, the pulse has self re-shaping capability. However, during reshaping of the pulse, the power gets distributed, and it is possible that after re-shaping the peak power may not be sufficient to give N=1. In that case the soliton will not be formed and the pulse will disperse for ever.

If initially the pulse power is adjusted to get N > 1, then after re-shaping the pulse will still have enough power and the fundamental soliton will be formed.

The solitons therefore can be easily observed inside a single mode optical fiber.

- **Fiber Loss**

One of the major issue related to the solitonic communication is the fiber loss. In presence of loss, the pulse power reduces making the non-linearity weaker. The soliton then loses its shape quickly and the pulse starts dispersing.

There are various possibilities for the loss compensation.

- The optical power can be increased at the beginning of the fiber link such that at the receiving end, the power is still enough to make N > 1. This however makes the initial power very and higher order soliton may get formed which has much distorted pulse shape.

- Provide lumped periodic amplification along the fiber length. This is more feasible since the power does not increase to make N > 2, and the same time the power does not reduce to make N < 1. The soliton may change the shape during propagation since the non-linearity and the dispersion does not perfectly cancel each other at every point on the fiber, but overall the soliton may survive over a long length.

- Provide distributed amplification so the pulse power is maintained more or less constant through out the propagation. Raman amplification is quite suitable for this purpose.

- **Soliton-Soliton Interaction**

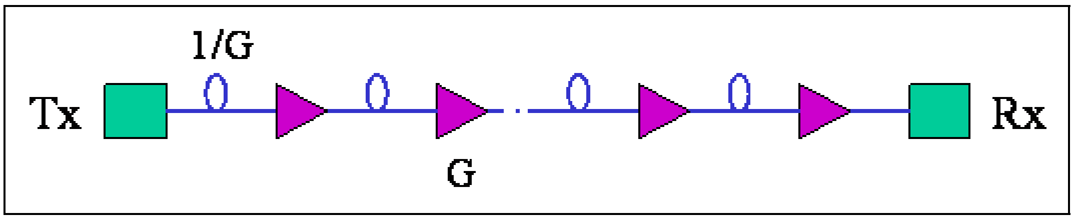
The solitons behave like charged particles. They show attraction and repulsion between them. If the solitons are close in time, they may get attracted to each other and colloid with each other. However, during the collision they do not lose their identity and they separate again as they travel. The solitons therefore show the pulsating behaviour over distance.

The interaction between the solitons can be avoided if the separation between the solitons is 7-8 times the soliton width.

For soolitonic communication, the data bits should be RZ and the duty cycle must be 10-15%. The data rate is then only 10-15% of the bandwidth used by the data.

- **Effect of Noise**

In a long communication link with amplifiers placed periodically (as shown in Fig.) to just compensate the fiber loss, the ASE accumulates.



The noise figure of the total system can be written as

$$F(G) = \frac{1}{G}\left[\frac{(G-1)}{\ln G}\right]^2$$

The ASE superimposed on a soliton leads to a random change in the amplitude and phase of the soliton. The phase change alters the frequency of the soliton in random manner. The frequency change gets converted into the change in arrival time of the soliton due to dispersion. Therefore the ASE leads t a timing jitter of the soliton. This effect is called the **Gordan-Haus effect.**

The variance of the timing jitter is

$$\sigma^2(\mathbf{ps}^2) = 3.6\times 10^{-6} n_{sp} F(G) \frac{\alpha(\mathbf{/km}) D(\mathbf{ps/km/nm})}{A_{eff}(\mathbf{\mu m}^2)\tau(\mathbf{ps})} Z^3(\mathbf{km}^3)$$

Where $n_{sp} \geq 1$ is the population inversion factor of the amplifier, α is the fiber loss, D is the dispersion, Z is the link length, A_{eff} is the effective area of the fiber, and τ is the soliton width. The units for each parameters are mentioned in the brackets.

For a link 10000Km long, the maximum data rate for a dispersion shifted fiber and amplifier spacing of 50km, is about 10Gb/s.

he 10Gb/s seems to be a upper limit for the data transmission over long haul optical links.

Recap

In this lecture you have learnt the following

- Pulse propogation in Non-Linear optical fiber
- Dispersive Regime
- Non-Linear Regime
- Solitons
- Practical Aspects of Solitonic communication

Congratulations, you have finished Module 23. To view the next lecture select it from the left hand side menu of the page