

# Module 10 : Receiver Noise and Bit Error Ratio

## Lecture : Receiver Noise and Bit Error Ratio

### Objectives

In this lecture you will learn the following

- Receiver Noise and Bit Error Ratio
- Shot Noise
- Thermal Noise
- Signal-to-noise Ratio (SNR)
- Thermal Noise Dominated Receiver
- Shot Noise Dominated Receiver
- Bit-Error Ratio (BER)
- Minimum Average Power

### Receiver Noise and Bit Error Ratio

- The optical receiver converts incident optical power into electrical current through the photo-diode.
- The relation between the optical power and the photo-current is linear, i.e.

$$I_p = \mathfrak{R} P_{in}$$

where  $\mathfrak{R}$  is the responsivity of the photo-detector,  $I_p$  is the photo-current and  $P_{in}$  is the incident optical power.

- The interaction of photon with the matter in a photo-detector however is a statistical process. That means the linear relationship is valid in an average sense. If we go to micro level there are fluctuations in the photo-current around the mean. These fluctuations are called the noise in the receiver.
- There are fundamental mechanisms of noise:
  - 1) Shot noise: This is due to statistical fluctuations in the optical signal itself and the statistical interaction process.
  - 2) Thermal noise: This is due to the thermal motion of the electrons inside the electronic circuits and the amplifiers following the photo-detector.
- The presence of noise degrades the signal quality. For the analog system the signal quality is measured by the parameter, signal-to-noise ratio (SNR), and for a digital system the signal quality is measured by the parameter, bit error ratio (BER).
- The analysis of noise therefore very important in a communication link. Here first we investigate the properties of the two noises mentioned above and then calculate the BER for the digital signals.

### Shot Noise

- The shot noise is because of the random fluctuation in the photo-carrier generation. The shot noise was first investigated by Schottky in 1918.
- In the presence of the noise the photo current can be written as

$$I(t) = I_p + i_s(t)$$

where  $I(t)$  is the instantaneous current of the photo detector, and  $i_s(t)$  is current fluctuation due to the shot noise.

- The noise current  $i_s(t)$  is stationary random process with Poisson statistics.
- The spectral density of the shot noise is practically constant over a very wide band and hence the **noise is almost white**.

- The variance of the shot noise is

$$\sigma_s^2 = \langle i_s^2(t) \rangle = 2 q I_p B$$

where,  $q$  is the electronic charge, and  $B$  is the effective receiver bandwidth. The variance of the noise essentially gives the rms noise power.

- The receiver bandwidth depends upon at what location in the receiver the noise is measured. If the noise is measured just after the photo-detector, the bandwidth is equal to the intrinsic bandwidth of the detector. However, if the noise is measured at the end of the receiver, then the transfer function of the receiver  $H(f)$  comes into picture and the noise variance is given as

$$\sigma_s^2 = \langle i_s^2(t) \rangle = 2 q I_p \int_0^\infty |H(f)|^2 df$$

- The integral essentially gives the effective bandwidth of the system.
- All photo-diodes generate a small amount of current even in the absence of optical signal. This is due to the stray light falling on the detector and also due to thermal generation of the carriers. This current is called the **dark current** ,  $I_d$ .
- The total current in the presence of signal then is the sum of the signal photo-current and the dark current. In a binary digital signal the dark current is generally negligible for bit 1. For bit 0 however, since the optical signal is very low (ideally zero), the dark current is to be taken into account.
- Since variance of the shot noise is proportional to the average signal current, the shot noise essentially is a **multiplicative noise** . The shot noise therefore is a multiplicative white noise.

### Thermal Noise

- The thermal noise is because of the random thermal motion of the electrons inside a conductor. The thermal is added in the load resistor connected to the photo-diode.
- The thermal noise also has constant spectral density over a very wide frequency range and

consequently, thermal noise is also white.

- The thermal noise is modeled by Gaussian process. Since this noise is added externally to the photo-current, this noise is additive in nature.

- The variance of the thermal noise is given as

$$\sigma_T^2 = \langle i_T^2(t) \rangle = \frac{4 K T B}{R_L}$$

where  $K$  is the Boltzmann constant,  $T$  is the temperature of the receiver, and  $R_L$  is the load resistor connected to the photo-diode. The variance gives the thermal noise power.

- The thermal noise does not depend on the signal. Hence it is same for both 1 and 0 bit of the binary signal.

- The amplifier following the load resistor also adds the noise. The character of this noise is similar to the thermal noise.

- The contribution of the amplifier noise can be accounted for by a parameter called the Noise Figure  $F_n$  of the amplifier. The variance of the thermal noise in presence of amplifier is

$$\sigma_T^2 = \langle i_T^2(t) \rangle = \frac{4 K T B}{R_L} F_n$$

- It may be noted that the thermal noise can be reduced by using high load resistance.

This indeed gives loading problem for the following amplifier. However, that is an

- independent issue. Generally high input trans-conductance amplifiers are used after the photo-detectors.

### Signal-to-noise Ratio (SNR)

- The SNR is a ratio of the signal power to the total noise power.

- To get total noise power, we assume that the shot noise is approximately Gaussian with of course mean equal to the average photo-current. Then since the shot and thermal processes are independent Gaussian random processes, the variance of the total noise is equal to the sum of the variances of the two noises.
- The average mean square value of the photo-current fluctuation is therefore

$$\sigma^2 = \langle \Delta I^2 \rangle = \sigma_s^2 + \sigma_T^2 = 2 q I_p B + \frac{4 K T B}{R_L} F_n$$

- The signal to noise ratio is

$$SNR = \frac{\text{Average signal power}}{\text{noise power}} = \frac{I_p^2}{\sigma^2}$$

$$SNR = \frac{\mathfrak{R}^2 P_{in}^2}{2 q I_p B + \frac{4 K T B}{R_L} F_n}$$

- Depending upon the optical signal the thermal noise may dominate over the shot noise and vice versa. Since shot noise is proportional to the signal, the shot noise is dominant at high signal levels, typically higher than -20 dBm.

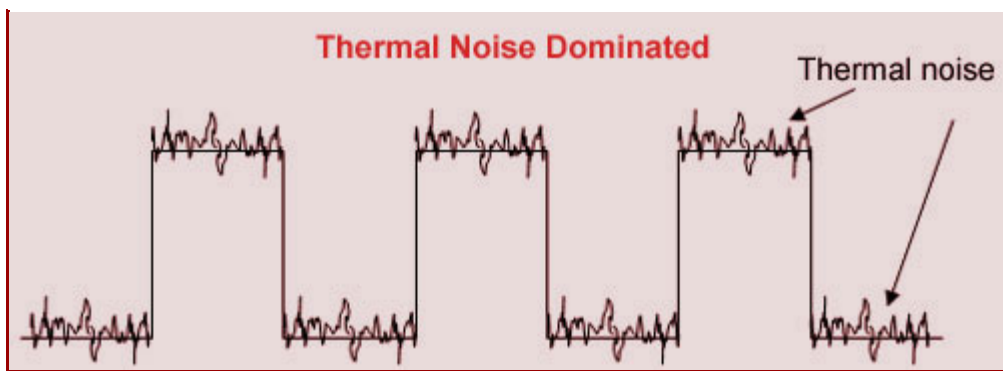
### Thermal Noise Dominated Receiver

- The signal levels at most of the receivers is between -60 dBm to -30 dBm, most of the receivers are thermal noise dominated, i.e.  $\sigma_T \gg \sigma_s$ . The SNR in this case is given as

$$SNR = \frac{\mathfrak{R}^2 P_{in}^2 R_L}{4 K T B F_n}$$

- The expressions suggests that the SNR can be improved by
  - 1) By increasing the signal power ( SNR is proportional to the square of the signal power)
  - 2) By increasing the load resistance





- The thermal noise is quantified by a parameter, called the **noise equivalent power (NEP)** which is defined as

$$NEP = \frac{P_{in}}{\sqrt{B}} = \left( \frac{4 K T F_n}{R_L R^2} \right)^{1/2}$$

- The inverse of NEP is called the **detectivity** of the receiver.
- Typical value of NEP for good receiver lies in the range  $1-10 \text{ pW} / \sqrt{\text{Hz}}$ .
- From the knowledge of the NEP, one can obtain the signal power needed to achieve a desired SNR.
- Since bandwidth is proportional to the data rate in a digital system, the power requirement for a given SNR is proportional to the square of the bandwidth, i.e.

$$P_{in} \propto \sqrt{B}$$

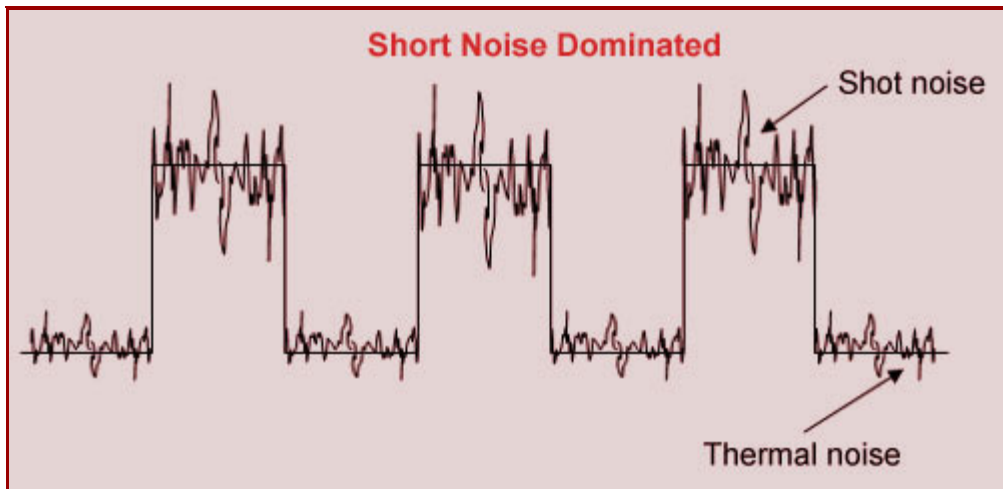
### Shot Noise Dominated Receiver

- If the signal amplitude is large ( this can happen for the receivers which are close to the transmitter in an optical network) the shot noise dominates over the thermal noise, i.e.

$$\sigma_S \gg \sigma_T.$$

- The SNR in this case

$$SNR = \frac{\Re P_{in}}{2 q B}$$



- For shot noise limited regime the load resistance has no role to play in SNR improvement. Also the SNR improves linearly with the signal power.
- The signal power requirement in the case increases linearly with bandwidth, i.e.

$$P_{in} \propto B$$

- This very interesting, that at low signal powers, the receiver performance improves better with signal power than at high signal powers.

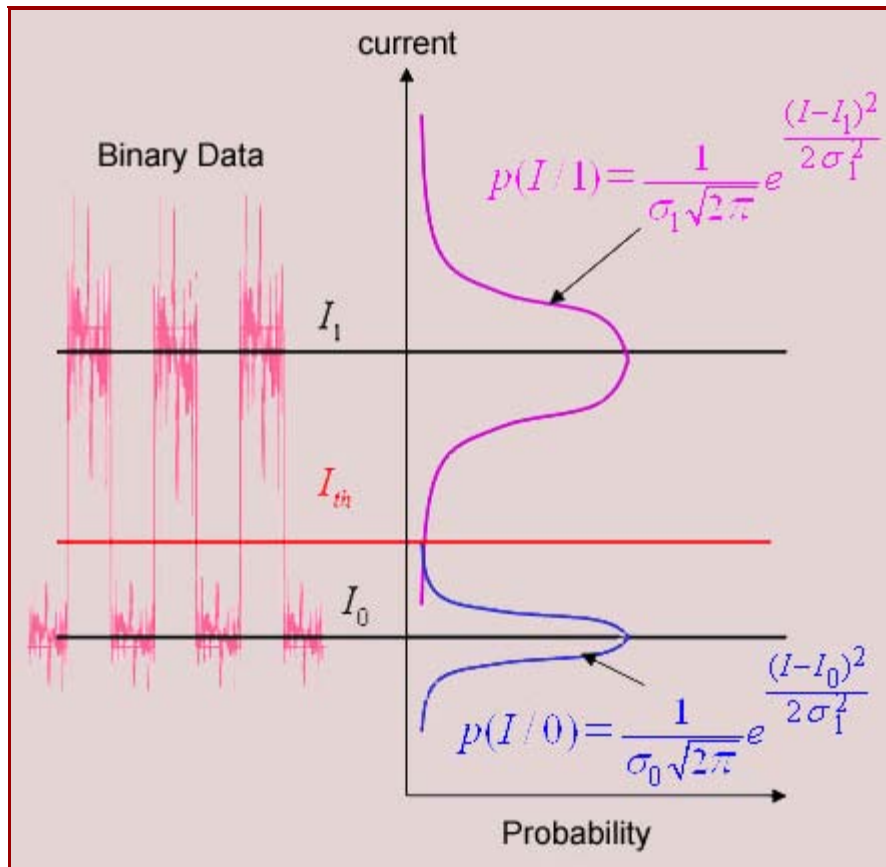
### Bit-Error Ratio (BER)

- The performance of digital receiver is measured by a parameter called the Bit-Error Ratio (BER). The BER tells the fraction of the bits which are wrongly detected, i.e.

$$BER = \frac{\text{No. of bits in error}}{\text{Total bits transmitted}} = \frac{\text{Wrong bits per sec}}{\text{Data rate in bits per sec}}$$

- For a satisfactory performance the BER has to be less than  $10^{-9}$ .
- The Figure shows the signal current when bit-0 is transmitted and when bit-1 is transmitted. The figure also shows the probability density function of the current in the two

binary states.



- For BER calculation it is assumed that the noise is almost Gaussian with standard deviations  $\sigma_0$  and  $\sigma_1$ , and means  $I_0$  and  $I_1$  respectively for the 0 and 1 binary levels. For an optical receiver, in general the two standard deviations are different. For thermal noise dominated regime the two become same.

- The decision threshold is  $I_{th}$ . That is

$$\text{If } I \geq I_{th} \quad \text{bit} = \mathbf{1}$$

$$\text{If } I < I_{th} \quad \text{bit} = \mathbf{0}$$

- So a **bit error occurs**

when bit-0 is transmitted and  $I \geq I_{th}$ .

When bit-1 is transmitted and  $I < I_{th}$ .



- The BER for an unbiased data ( a data which has statistically equal number of 0 and 1 bits), the BER is given as

$$BER = \frac{1}{2} (P(0/1) + P(1/0))$$

where  $P(0/1)$  is the probability of error in bit-1, i.e. probability of current remaining below the threshold when actually bit-1 has been received.

- $P(1/0)$  is the probability of error in bit-0, i.e. probability of current becoming greater than or equal to the threshold when actually bit-0 has been received.

#### Bit-Error Ratio (BER) - contd

- The two probabilities are given as

$$P(0/1) = \frac{1}{\sigma_1 \sqrt{2\pi}} \int_{-\infty}^{I_{th}} \left[ -\frac{(I - I_1)^2}{2\sigma_1^2} \right] dI = \frac{1}{2} \operatorname{erfc} \left( \frac{I_1 - I_{th}}{\sigma_1 \sqrt{2}} \right)$$

$$P(1/0) = \frac{1}{\sigma_0 \sqrt{2\pi}} \int_{I_{th}}^{\infty} \left[ -\frac{(I - I_0)^2}{2\sigma_0^2} \right] dI = \frac{1}{2} \operatorname{erfc} \left( \frac{I_{th} - I_0}{\sigma_0 \sqrt{2}} \right)$$

where the complementary error-function ( $\operatorname{erfc}(x)$ ) is defined as

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-y^2) dy$$

- There are standard tables available for the complementary error functions.
- The BER for the binary data is then given as

$$BER = \frac{1}{4} \left[ \frac{1}{2} \operatorname{erfc} \left( \frac{I_1 - I_{th}}{\sigma_1 \sqrt{2}} \right) + \frac{1}{2} \operatorname{erfc} \left( \frac{I_{th} - I_0}{\sigma_0 \sqrt{2}} \right) \right]$$

- The BER is function of the decision threshold current. The threshold is to be optimized to get the minimum BER.
- **Minimum BER occurs when the threshold is**

$$I_{th opt} = \frac{\sigma_0 I_1 + \sigma_1 I_0}{\sigma_0 + \sigma_1}$$

- For thermal dominated regime,  $\sigma_0$  and  $\sigma_1$  are equal and the threshold is mean of  $I_0$  and  $I_1$ . That is the threshold is half way between the 0 and 1 levels.

- For shot noise dominated regime,  $\sigma_0 \ll \sigma_1$ , and the threshold comes closer to  $I_0$ .

- For the optimum threshold, the BER becomes

$$BER = \frac{1}{2} \operatorname{erfc} \left( \frac{Q}{\sqrt{2}} \right)$$

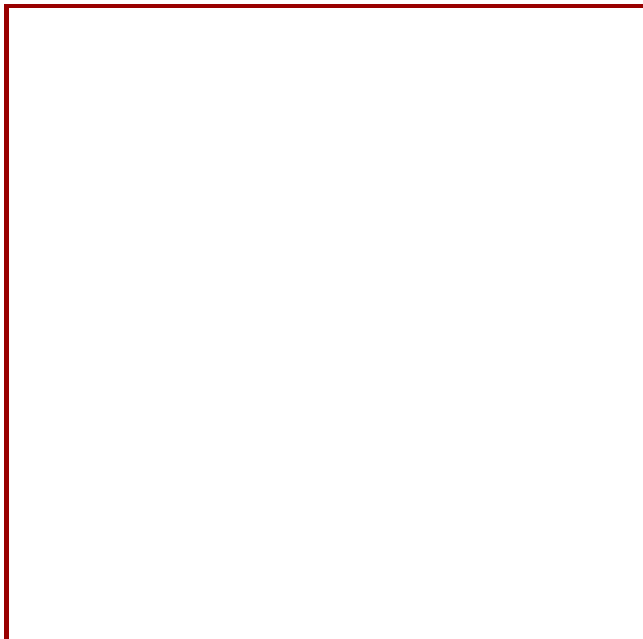
where, Q is defined as

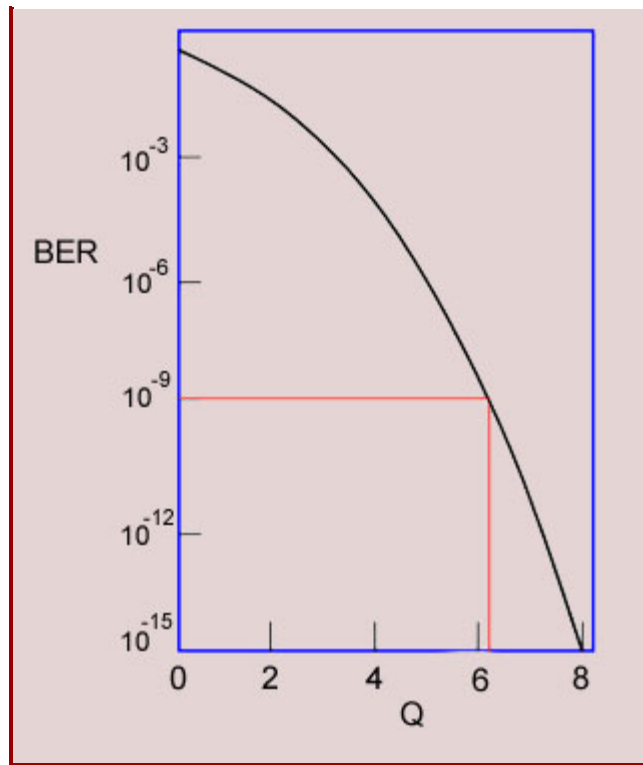
$$Q = \frac{I_1 - I_0}{\sigma_1 + \sigma_0}$$

- The quantity essentially gives the fractional noise margin of the received binary signal.

- For  $Q > 3$ , the BER expression can be approximated to get

$$BER \approx \frac{1}{Q\sqrt{2\pi}} \exp \left( -\frac{Q^2}{2} \right)$$





- The BER is a very steep function of Q. For change in Q by 1, the BER drops almost by 3 orders of magnitude.  
**For a BER of better than  $10^{-9}$ , Q should be  $> 7$ .**

#### Minimum Average Power

- Assuming the dark current to be zero,  $I_0 = 0$ . We then get

$$Q = \frac{I_1}{\sigma_1 + \sigma_0} = \frac{2\mathfrak{R}P_{av}}{\sigma_1 + \sigma_0}$$

where it is assumed that the data is unbiased and the average power is half of that corresponding to bit-1.

- For BER of  $10^{-9}$ , the minimum average power is

$$P_{\min} = \frac{7(\sigma_1 + \sigma_0)}{2\mathfrak{R}}$$

- For **thermal noise dominated regime**,  $\sigma_0 = \sigma_1$ ,  $\Rightarrow Q = I_1 / 2\sigma_1$ . The average signal current is  $I_p = I_1 / 2$ , giving  $Q = I_p / \sigma_1$ .

- The minimum SNR then is

$$SNR = \frac{I_p^2}{\sigma_1^2} = Q^2 = (7)^2 = 49 = 17 \text{ dB}$$

- For **Shot noise limited regime**,  $\sigma_0 \ll \sigma_1$ ,  $\Rightarrow Q = I_1 / \sigma_1 = 2I_p / \sigma_1$ .

- The minimum SNR then is

$$SNR = \frac{I_p^2}{\sigma_1^2} = Q^2 / 4 = (7)^2 / 4 \approx 12 = 11 \text{ dB}$$

## Recap

In this lecture you have learnt the following

- Receiver Noise and Bit Error Ratio
- Shot Noise
- Thermal Noise
- Signal-to-noise Ratio (SNR)
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- Bit-Error Ratio (BER)
- Minimum Average Power

Congratulations, you have finished Module 10. To view the next lecture select it from the left hand side menu of the page