Ideas and methods of condensed matter theory.

This is a graduate level course on the modern understanding of phases of quantum matter at low temperature, and phase transitions between these phases. This modern understanding has as its bedrock the Wilsonian renormalization group idea that physics at a given length-scale is best understood in terms of an *effective model* appropriate to that length scale. The course is self-contained except for material taught in the usual undergraduate Quantum Mechanics and Statistical Physics courses (although all relevant information is reviewed as and when needed). Passing familiarity with the contents of a typical Solid State Physics course will be an advantage but is not a prerequisite.

The course consists of just over thirty lectures of ninety minutes duration each, and is based on material taught in a one semester elective course for a combined class of fourth year Engineering Physics and second year MSc students at IIT Bombay, although some additional material has been included from a graduate level course taught by the author to Ph.D students in the TIFR graduate program. It is thus suitable both as a second year elective in an MSc program or a final year elective in a four year B.S. program, as well as part of the required course-work for entrants to a Ph.D programme.

The course is designed as six modules (including this introductory module that overviews the course). The second module of five lectures dwells on a review of statistical mechanics, a conceptual preview of important new ideas, and an introduction to the systems of interest to us in this course, as well as an introduction to linear response theory.

The third module of eight lectures develops the path integral representation for the statistical mechanics of quantum spin systems, explains where such model Hamiltonians arise in the study of interesting Mott insulating phases of matter, derives the low energy effective theory (quantum rotor model) that is used for the description of quantum antiferromagnetism, and discusses the main experimental probes used in the study of such systems.

Using the derivation of the Heisenberg Hamiltonian for a Mott insulator as motivation, the next module of six lectures develops the formalism of second quantization and coherent state path integrals for describing the statistical mechanics of many-particle quantum systems, and provides a brief introduction to doped Mott insulators, and strongly interacting Bose fluids in optical lattice-trap potentials.

After this, the next seven lecture module uses the effective theory of quantum antiferromagnets (quantum rotor model) as an example that illustrates the analysis of stability of phases, the Mermin-Wagner theorem on the absence of continuous symmetry breaking in two or fewer dimensions at $T > 0$, and the Wilsonian renormalization group approach to phases,phase transitions, and critical properties.

In the next four lecture module, the planar rotor model is introduced as a low-energy theory of the bosons at commensurate densities in lattice potentials, and superfluid hydrodynamics is developed as the long-wavelength theory of the superfluid phase of the planar rotor model. Then, the role of vortices in destroying superfluidity in $d = 2$ at $T > 0$ is explained by analyzing their energetics and contribution to the entropy. Finally the Kosterlitz Thouless renormalization group treatment of the vortex gas is used to derive consequences for the superfluid stiffness in the vicinity of the superfluid insulator transition in two dimensional systems. This provides a self-contained four lecture treatment of Kostelitz-Thouless theory and superfluid insulator transitions in bosonic systems.

Below, I provide the detailed lecture-by-lecture breakup of the course contents.

- Module I. Overview of course
  - 1. Overview of course and index of topics (this lecture).
- Module II. Conceptual overview and linear response theory.
  - 2. Review of undergraduate Statistical Physics (ensemble theory etc), and preview of new ideas (emergent phenomena).
  - 3. Overview of important ideas needed for a deeper study of phase transitions and critical phenomena in low temperature matter (spontaneously broken symmetry, long range order, Goldstone modes, broken ergodicity, diverging timescales etc).
  - 4. Analysis of thermodynamic fluctuations. Linear response theory: Derivation of linear response kernel from time dependent perturbation theory.
  - 5. Linear response theory: Properties of linear response kernel (analyticity in upper half plane, Kramers-Kronig type dispersion relations).

- 6. Linear response theory: Fluctuation-dissipation theorem. Broad-brush introduction to systems of interest in condensed matter physics.

- Module III. Quantum antiferromagnets: Path integral description and introduction.

  - 7. Introduction to path integral representation of the partition function: Coherent state basis for quantum spins.
  - 8. Coherent state path integral for quantum spin systems: Derivation and connection to statistical mechanics of higher dimensional classical spin systems.
  - 9. Coherent state path integral for quantum spin systems: Analysis and characterization of Berry phases for paths.
  - 10. Introduction to the physics of Mott insulators and derivation of the Heisenberg Hamiltonian for the low energy physics of Mott insulators. Example of parent compounds of high-T_c cuprate superconductors.
  - 11. Derivation of long-wavelength effective theory for quantum antiferromagnets: Expanding about short-range antiferromagnetic ordered configurations.
  - 12. Derivation of long-wavelength effective theory for quantum antiferromagnets: Role of Berry phases and dynamical terms.
  - 13. Berry phase effects in $d = 1$ and $d = 2$: Haldane Gap versus gapless power-law antiferromagnetism in $d = 1$, and deconfined quantum criticality in $d = 2$.
  - 14. Experimental probes of quantum antiferromagnets: Susceptibility, heat capacity, NMR Knight shift, NMR $1/T_1$, and inelastic neutron scattering.

- Module IV. Many-body formalism and introduction to strongly correlated bosonic and fermionic systems.

  - 15. Second quantized formalism for representing many-particle bosonic or fermionic systems.

- 16. Second quantized formalism: Representing external potentials and interactions. Hubbard and tJ models for doped Mott insulators. Qualitative description of the physics of doped Mott insulators.
- 17. Coherent states for bosonic and fermionic systems, and path integral representation.
- 18. Coherent state path integral for bosonic and fermionic systems and Green functions.
- 19. Boson hubbard model for ultra-cold bosonic atoms in optical lattice potentials. Mott-lobes, and superfluid insulator transitions.
- 20. Effective field theory for superfluid insulator transitions in the bosonic Hubbard model.

• Module V. Phase and phase transitions of the quantum rotor model.

- 21. Phases of the quantum rotor model: ordered phase, and spin wave excitations in the ordered phase.
- 22. Stability analysis of the ordered phase: Absence of symmetry breaking in dimension $d \leq 2$ at $T > 0$, or dimension $d = 1$ at zero temperature. Quantum paramagnetic state and its excitations.
- 23. Proof of Mermin-Wagner theorem for $d = 2$, $T > 0$ quantum rotor model.
- 24. Introduction to the renormalization group idea: Poor-man's scaling for the quantum rotor model. Technical aspects of the implementation of this idea.
- 25. Poor-man's scaling for the quantum rotor model: Derivation of RG flow equations and elementary consequences: Haldane gap for integer spin chains in $d = 1$.
- 26. Field renormalization, critical properties in $2 + \epsilon$ expansion. Critical line for planar rotors in $d = 1$ at $T = 0$, and for the classical two dimensional xy model.
- 27. Introduction to quantum critical phenomenology at $T > 0$. Relevance to experiment. Poor man's scaling predictions for the effect of temperature fluctuations in the $O(N)$ rotor model at criticality.

- Module VI. Superfluidity and the Kosterlitz Thouless transition.
  - 28. Planar rotor model as low energy effective model at commensurate density: Mott Insulator vs superfluid phases of rotor model. Continuum hydrodynamic description of superfluid phase. Sound-wave and plasmon excitations.
  - 29. Vortex excitations in superfluid phase of rotor model.
  - 30. More on vortices: Statistical mechanics of the vortex gas, and renormalization of superfluid stiffness.
  - 31. Kosterlitz-Thouless renormalization group treatment of vortex gas. Consequences for superfluid films.

Suggested reference texts

- ‘Basic Notions of Condensed Matter Physics’, by P. W. Anderson, Advanced Book Classics, reprinted by Westview Press (1997).
- ‘Interacting Electrons and Quantum Magnetism’, by A. Auerbach, Graduate Texts in Contemporary Physics, Springer, second reprint (1998).
- Negele and Orland’s ‘Quantum Many-particle Systems’, by J. Negele and H. Orland, Advanced Book Classics, reprinted by Westview Press (1998).
- ‘Quantum Phase Transitions’, by S. Sachdev, Cambridge University Press, second edition (2011).