Module 1 : Introduction and Background Material

Lecture 5 : Introduction to Lasers

Objectives

In this lecture we will look at

- Fabry Perot resonator with gain.
- Stimulated emission.
- Population inversion.

Consider a Fabry-Perot resonator composed of two parallel mirrors separated by distance d with a medium filling the space between the mirrors. Let the amplitude reflectivity of the first mirror be r_1 and its transmission t_1. Since a mirror can give phase changes we will assume both r_1 and t_1 to be complex quantities.

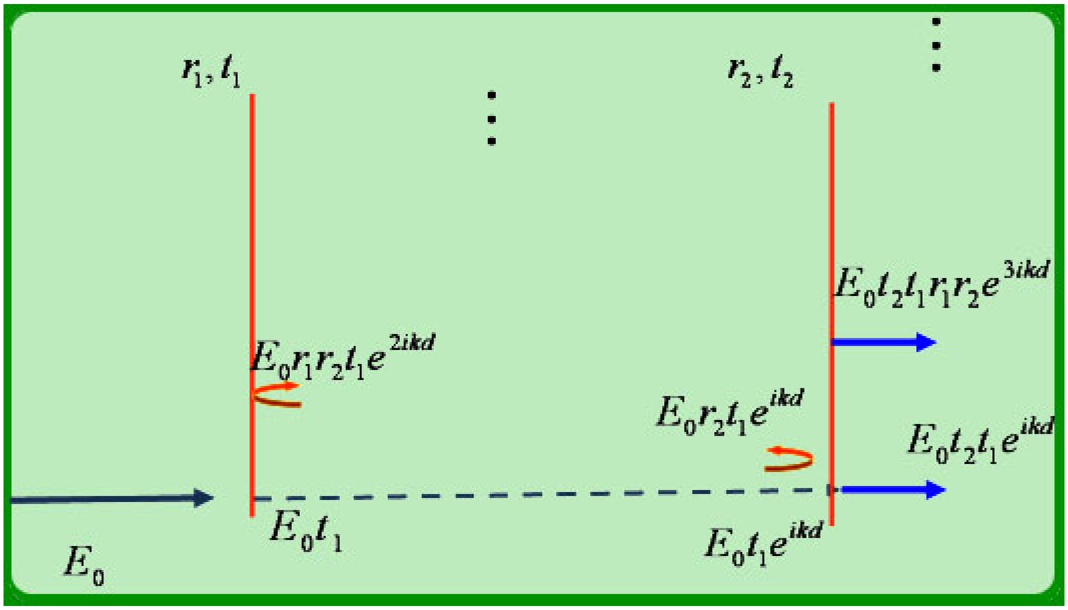

Figure 5.1: Multiple beam interference in a Fabry-Perot interferometer

- If an electromagnetic wave with an electric field amplitude E_0 is incidence normally on the first mirror it will transmit an amplitude $E_0 t_1$.
- Travelling to the second mirror with reflectivity r_2 and its transmission t_2 this amplitude become $E_0 t_1 e^{ikd}$ where k is propagation vector if the medium filling the space between the two mirrors is neither absorbing nor amplifying the propagation constant k will be real. If the medium is absorbing, the constant *k* has a positive imaginary part. If it is amplifying the imaginary part of *k* is negative.
- At the second mirror the amplitude of the transmitted wave is $E_0 t_2 t_1 e^{ikd}$ and that of the refracted wave is given by $E_0 r_2 t_1 e^{ikd}$ which become $E_0 r_2 t_1 e^{2ikd}$ on reaching the first mirror.
- The reflected amplitude will now be $E_0 r_1 r_2 t_1 e^{2ikd}$ The corresponding transmitted beam at the second mirror has an amplitude $E_0 t_2 t_1 r_1 r_2 e^{3ikd}$.
- In this way we can write the total transmitted field by adding the amplitudes after each round trip. We get

$$E_T = E_0 t_1 t_2 e^{ikd}(1 + r_1 r_2 e^{2ikd} + (r_1 r_2)^2 e^{4ikd} + \cdots) \qquad (5.1)$$

where we recall that $2kd = \delta$ is the round trip phase difference when *k* is real that is for an empty Fabry –Perot resonator. We will now assume that *k* is, in general, a complex number. The series for E_t is an infinite geometric series which is easily summed to give the following result

$$
\begin{aligned}
E_T &= E_0 t_1 t_2 e^{ikd}(1 + r_1 r_2 e^{2ikd} + (r_1 r_2)^2 e^{4ikd} + \cdots) \\
&= \frac{E_0 t_1 t_2 e^{ikd}}{1 - r_1 r_2 e^{2ikd}}
\end{aligned} \tag{5.2}
$$

Here we observe that if the denominator vanishes we can get finite transmitted amplitude E_t even with an infinitesimal value of E_0. When that happens, we say that oscillations can set in *i.e.* the arrangement can produce a finite output, starting from an infinitesimal input which may be provided by noise. That an amplifier with a positive feedback becomes an oscillator is well known in electronics. Obviously this can happen only if energy is supplied to the system in some way. This is the lasing condition. We can observe several interesting consequences of this simple result.

First, we look at the oscillation condition that is when the denominator in eq (5.2) vanishes

$$1 - r_1 r_2 e^{2ikd} = 0 \tag{5.3}$$

Since r_1 and r_2 are complex we let

$$r_1 r_2 = |r_1 r_2| e^{i\phi_r} \tag{5.4}$$

and

$$k = k_R + ik_I \tag{5.5}$$

where the subscripts R and I depict the real and imaginary parts. This implies that the oscillation condition become

$$1 - |r_1 r_2| e^{2ik_R d} e^{i\phi_r} e^{-k_I d} = 0 \tag{5.6}$$

This is possible only if we have

$$1 = |r_1 r_2| e^{-k_I d} \tag{5.7}$$

and

$$2k_R d + \phi_r = 2m\pi \tag{5.8}$$

where *m* is an integer. Eq (5.7) implies that $k_I < 0$ because $|r_1 r_2| < 1$. This first condition tells us that we need amplification or gain to have an optical oscillator. The second given by eq (5.8) tells that the oscillator can only radiate mode of the Fabry Perot resonator i.e. those frequencies for which the round trip phase difference is an integral multiple of 2π This condition means that the amplitudes after successive reflections need to add in phase to form a mode of the resonator. The field then builds up inside the resonator i.e. between the two mirrors.

How do we obtain gain in a medium ?

We had earlier seen that in the generalized Lorentz model the induced polarization is given by

$$\chi(\omega) = \sum_{i,j} f_{i\to j} \frac{Ne^2}{\varepsilon_0 m(\omega_{i\to j}^2 - \omega^2 - i\omega\gamma_{i\to j})} \tag{5.9}$$

where the summation is over all transitions with *i* and *j* denoting the initial and final states, respectively. $f_{i\to j}$ is called the oscillator strength of the transition corresponding to frequency $\omega_{i\to j}$. This form of susceptibility can be derived using time dependent perturbation theory in quantum mechanical description of the matter. The damping term then represents various scattering mechanisms would also be different for different transitions and depend on several factors like impurities, temperature defects etc.

We now look for condition under which $\chi(\omega)$ and hence k can have a negative imaginary negative part. For this we follow Einstein's original discussion of stimulated emission by considering the interaction of light with atoms using the Bohr model for the hydrogen like atoms. {we will later derive this using perturbation theory on mixed quantum states in Lecture 15}.

In the Bohr model of the atom:

An atom could make a transition from a lower stationary state to a higher state by absorbing a photon

Or it could emit a radiation of frequency ω with $\hbar\omega = E_2 - E_1$ the energy difference between the two states and make a transition from a higher state to a lower state.

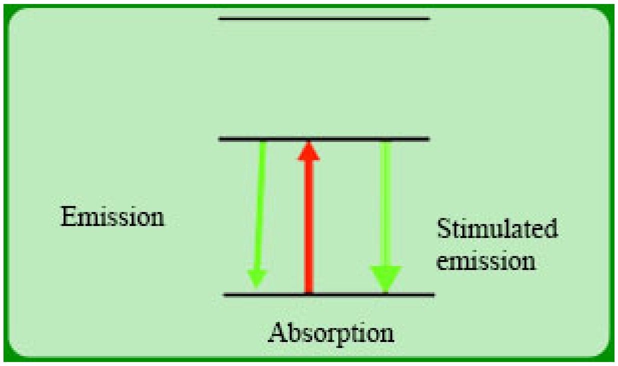

Figure 5.2

It seems OK to assume that an interaction of light with charges in atom can cause an upward transition it is difficult to follow why an atom in a "stationary" should by itself make a transition to a lower state by emitting radiation. Indeed this "spontaneous" emission by atom can be satisfactorily understood only when we treat radiation also quantum mechanically.

Seeking to make a consistent picture of light matter interaction, Einstein argued in 1917, that there must also be a process called stimulated emission in which an atom is stimulated to make a downward transition by incident photons. He showed that this was necessary to reconcile Planck's radiation the Boltzmann thermal distribution and Bohr's model. It is worthwhile to look at this derivation.

Einstein's arguments for the need for stimulated emission:

In 1900 Planck had shown that spectrum of a Black body (radiation from a body in thermal equilibrium at Temp T) is given by

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{\exp(h\nu / kT) - 1} \qquad (5.10)$$

where $\rho(\nu)d\nu$ is the energy per unit volume between frequencies ν and $\nu + d\nu$.

1905 Einstein argued that radiation consists of quanta each with energy $h\nu$ since the threshold for the photoelectric Effect identifying depends on frequency not intensity.

1913: Bohr presented this model for the Hydrogen atom spectrum

$$h\nu = E_i - E_F$$

In 1917, 8 years before Schrodinger and Heisenberg pioneered quantum mechanics, Einstein Considered atoms with two energy levels

$$E_1, \text{ and } E_2 = E_1 + h\nu$$

with

$N_1 = \text{number of atoms in } E_1$

$N_2 = \text{number of atoms in } E_2$

For atom photon interaction he considered 3 processes:

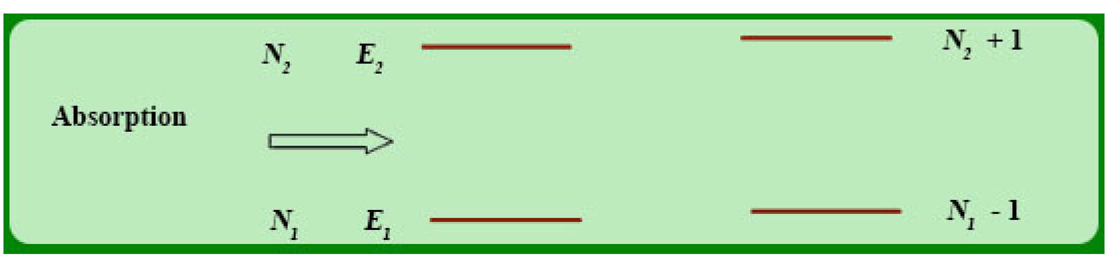

Figure 5.3

with related change in population given by

$$\left.\frac{dN_1}{dt}\right|_{absorption} = -B_{12}N_1\rho(\nu) \tag{5.11}$$

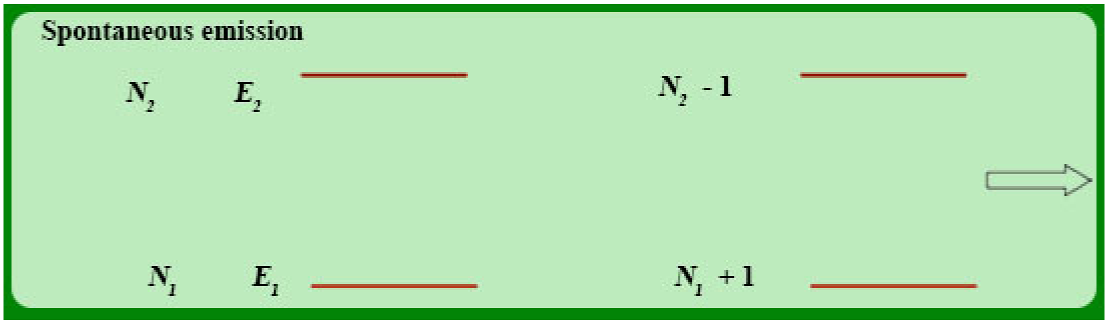

Figure 5.4

with related change in population given by

$$\left.\frac{dN_1}{dt}\right|_{sp-emission} = -AN_2 \tag{5.12}$$

and, **Stimulated Emission**

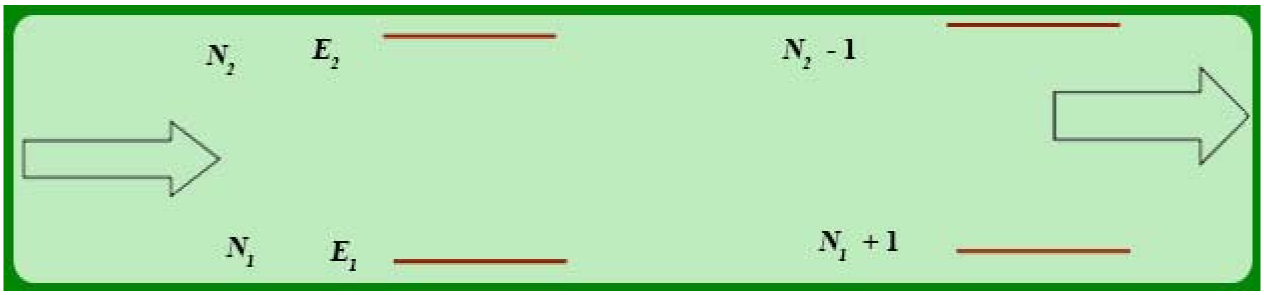

Figure 5.5

with related change in population given by

$$\left.\frac{dN_1}{dt}\right|_{St-emission} = +B_{21}N_1\rho(\nu) \tag{5.13}$$

In equilibrium, we need

$$\left.\frac{dN_1}{dt}\right|_{net} = 0$$

$$\Rightarrow AN_2 + B_{21}N_2\rho(\nu) = B_{12}N_1\rho(\nu) \tag{5.14}$$

N_2 / N_1 should be determined by Boltzmann is Boltzmann's law

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT} = e^{-h\nu/kT} \tag{5.15}$$

From these two equations (5.14) and (5.15) we get

$$\Rightarrow \rho(\nu) = \frac{AN_2}{B_{12}N_1 - B_{21}N_2} = \frac{A}{B_{12}e^{h\nu/kT} - B_{21}} \tag{5.16}$$

If this has to agree with the Planck's radiation law (5.10) we must have

$$B_{12} = B_{21} = B = A\frac{c^3}{8\pi h\nu^3} \tag{5.17}$$

We note:

- If stimulated emission did not exist we should have got $B_{21} = 0$.
- Instead we got $B_{12} = B_{21}$. Thus, we see that Stimulated Emission is exactly the reverse process of absorption.
- In Bohr's model spontaneous emission was included although it appears contrary to the idea that

all energy levels are stationary states.

- Stimulated emission appears naturally in semi-classical treatment of radiation interaction with atoms i.e. when atom is treated quantum mechanically and radiation is treated classically.
- Spontaneous emission cannot be explained unless electromagnetic fields are also quantized.
- Later, we will show that population inversion leads to ***coherent*** amplification of an electromagnetic wave.

Now since, upward and downward transitions between two levels have the same probability, it follows that the net absorption is proportional to $N_1 - N_2$. For stimulated emission to win we need larger number of atoms in the upper level E_2 than in E_1. Consider a collection of atom with energy level as shown in figure. Most of the atom will be in the ground state at room temperature as shown the figure 5.6(a). When we raise the temperature of the gas more thermal energy become available and the number of atom in the exited state increases as indicated in the figure 5.6(b). Ignoring complications like degeneracy of energy level in atoms, the ratio of no of atoms in the excited state to that in the ground state is given by the Boltzmann factor $e^{-(E_2-E_1)/k_bT}$ and can therefore never exceed 1. If by some trick we can have more atoms in the higher level than the lower level stimulated emission will dominate and thereby we can get gain at the resonance frequency $\left(E_2 - E_1\right)/h$. This, as we saw, cannot happen in a system in thermal equilibrium. When this does happen we say that the population is inverted, it then provides a way of amplifying light coherently.

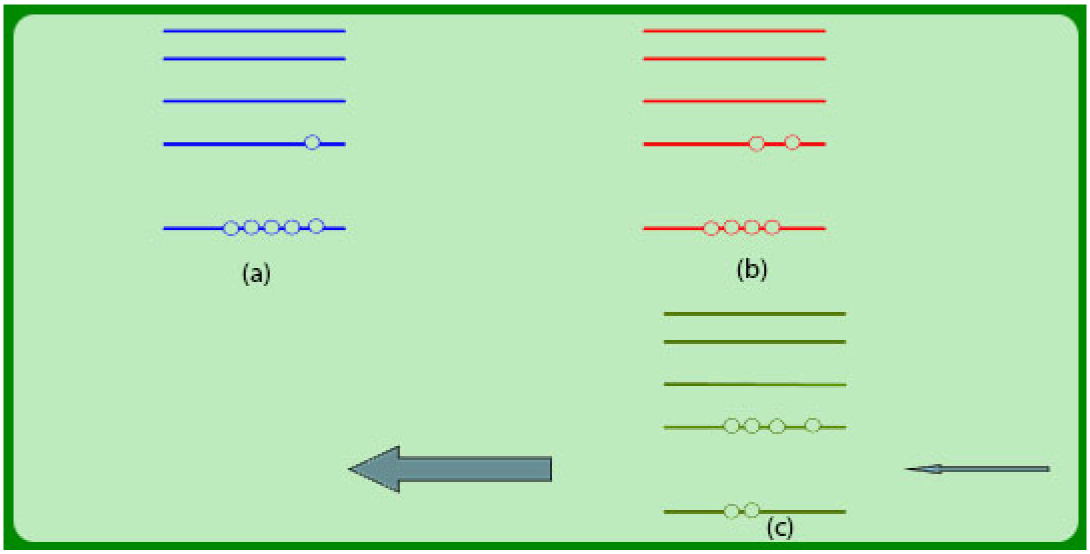

Figure 5.6

Thus we arrive at the principle of laser action if we enclose a medium with inverted population (i.e. a medium in which the number of atoms in the higher state is more than the number of atoms in the lower state) we can obtain amplification of coherent light.

LASER MEDIA

Various types of mechanisms have been exploited to obtain population inversion. Briefly, these are:

1. Solids, crystals as well as glass doped with suitable ions- generally transition metal ions like Cr, Ti or rare earth ions like Nd. They cover visible and near infrared regions.
2. Gases or gas mixtures can have population inversion in electric discharges. Atomic system such as Ar^{++}, $He-Ne$ and laser operate in and visible region usually in cw mode while Cu –vapour laser operates in the visible region in pulsed mode. Molecular gases like CO_2 and CO provide lasers which can operate in continuous wave (cw) as well as pulsed mode in the mid infrared region. CO_2 lasers are capable of delivering very high cw powers and are often used in industrial applications.
3. Organic dyes provide broad band lasing and can therefore be turned over a wide spectral range. Their use is now reducing because of enormous development of tunable laser based on nonlinear optical effects.
4. Semiconductor diode efficiency, compactness and amenability to modern manufacturing techniques to reduces costs enormously. They are probably the most rapidly developing laser system.

In the next lecture we look at some of the properties of laser beams.

RECAP

In this lecture we have learned about the

- A Fabry Perot interferometer becomes an oscillator if a medium with gain is placed between the two mirrors forming the interferometer.
- Only those frequencies can lase which correspond to round trip phase difference of an integral multiple of 2π.
- Population inversion is required to obtain gain.