Module 4 : Third order nonlinear optical processes

Lecture 32 : Stimulated Brillouin Scattering

Objectives

In this lecture you will learn the following

- Explain the mechanism of yet another stimulated inelastic scattering process, stimulated Brillouin scattering.
- Discuss the phase matching condition and its implications for the scattering phenomena.
- Describe the process dynamics using coupled wave equations.

Stimulated Brillouin Scattering

In the preceding lectures, we have introduced the stimulated Raman scattering, a subset of the induced inelastic scattering. These processes were described as four-wave -mixing process where material excitation such as molecular vibrations or the optical phonons in solids replaces one of the participating electromagnetic waves. There is another similar process where instead of the optical phonons, acoustic phonons participate in the nonlinear wave coupling. This type of the inelastic scattering is called Brillouin scattering, named after its discoverer Brillouin who reported the scattering of light from thermally excited acoustic waves in 1922.

The phenomenon of stimulated Brillouin scattering(SBS) was first published by Chiao, Towens and Stoicheff in 1964. Classically, SBS can be described as a parametric interaction between the pump wave $\left(\omega_p, k_p\right)$, generated acoustic wave $\left(\omega_s, k_s\right)$, and the scattered light wave $\left(\omega_a, k_a\right)$. The pump wave generates an acoustic(compression) wave by electrostriction, the resulting density variations produces the traveling periodic modulation of the refractive index of the medium which acts like a grating moving with the acoustic velocity v_a. The pump wave is scattered by this grating into a Doppler shifted Stokes wave.

In the quantum mechanical picture SBS is described as the process of annihilation of pump quanta $\hbar\omega_p$ with simultaneous creation of a scattered photon $\hbar\omega_s$ and a acoustical phonon $\hbar\omega_a$ unlike localized molecular vibrations acoustic phonons are the collective excitations of the entire system with definite momentum and energy. Conservation of both energy and momentum between both photons and phonons is described by

$$\omega_s = \omega_p \mp \omega_a \qquad (32.1)$$

$$\vec{k_s} = \overrightarrow{k_p} \mp \overrightarrow{k_a} \qquad (32.2)$$

where the negative sign refers to the Stokes and the positive sign to the anti-Stokes. Here k_s, k_p and k_a represent the propagation vectors of the scattered and pump waves and acoustic phonons, respectively. ω_s, ω_p and ω_a represent their corresponding frequencies. Due to the thermal motion of the constituent atoms and molecules in the solid acoustic phonons are already populated at normal temperature, Stokes and anti-Stokes waves, therefore, have comparable intensities.

The phae velocities of the acoustic and pump are given by

$$v_a = \frac{\omega_a}{k_a} \qquad (32.3)$$

and

$$v_p = \frac{c}{n_p} = \frac{\omega_p}{k_p} \qquad (32.4)$$

where n_p is the refractive index of the medium at the pump wavefrequency.

In solids

$$\frac{v_a}{v_p} \sim 10^{-5} \text{ typically.}$$

also

$$|k_a| \ll |k_p|$$

$$\Rightarrow \omega_a \ll \omega_p, n_p \cong n_p, v_a \approx v_p \text{ and } |k_p| \approx |k_b|$$

Momentum conservation is diagrammatically depicted in figure 32.1.

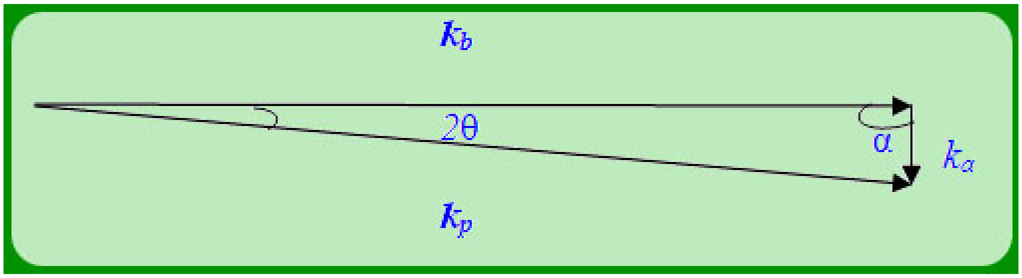

Figure 32.1 Momentum conservation of pump, Stokes and acoustic phonons in SBS

It can be seen from this that

$$2\theta + 2\alpha = \pi \tag{32.5}$$

or

$$\theta = \pi / 2 \tag{32.6}$$

k_b, k_p and k_a almost constitute an isosceles triangle. From this diagram we can write the following relation between k_p and k_a

$$\frac{k_a}{\sin 2\theta} = \frac{k_p}{\sin(90 - \theta)} = \frac{k_p}{\cos\theta} \tag{32.7}$$

Or

$$k_a = \left(\frac{\sin 2\theta}{\cos\theta}\right) k_p \quad = (2\sin\theta) k_p \tag{32.8}$$

Thus

$$\begin{aligned} \omega_b &= \omega_p \mp \omega_a \\ &= \omega_p \left(1 \mp 2\frac{v_a}{v_p} \sin\theta\right) \end{aligned} \tag{32.9}$$

It is thus seen that unlike the Raman scattering, frequency shift here is angle dependent. In the forward direction $\theta = 0$, there is no scattering as $\omega_a = 0$ and $k_a = 0$. The largest frequency shift occurs for the backward direction $\theta = \pi / 2$.
$\Rightarrow$ the pump and stimulated waves run in opposite directions.

Evidently, the three fields namely, the pump, Brillion scattered wave and the acoustic waves are involved in this nonlinear process akin to the case # 3 described in the lecture 29 on general theory of four-wave -mixing. The problem at hand is an example of backward parametric wave coupling. The nonlinear polarizations arise from the density variations in the dielectric constant of the medium by electrostriction. For calculating these, let us consider the total optical field acting an the medium due to pump and scattered waves as

$$E(z,t)=\left[E_p(z,t)e^{i(k_p z-\omega_p t)}+E_b(z,t)e^{i(k_b z-\omega_b t)}+c\cdot c.\right] \tag{32.10}$$

where the subscripts *p* and *b* refer to the pump and the Brillion scattered waves, respectively.
Acoustic field can be described in terms of material density distribution

$$\zeta(z,t)=\rho_0+\left[\rho(z,t)\mathrm{e}^{i(uz-\Omega t)}+c\cdot c_{-}\right] \tag{32.11}$$

where ρ_0 is mean density of care medium, $\Omega=\omega_p-\omega_b$ and $q=2k_p$

Acoustic wave equation gives

$$\frac{\partial^2\zeta}{\partial t^2}-\Gamma'\nabla^2\frac{\partial\zeta}{\partial t}-v^2\nabla^2\zeta=\nabla\cdot f \tag{32.12}$$

where *v* is acoustic wave velocity, Γ' is the damping parameter of the acoustic wave and f is the force/ volume

$$f=\nabla p \tag{32.13}$$

$$p=-\frac{1}{2}\epsilon_0\gamma\left\langle E^2\right\rangle \tag{32.14}$$

Here p is the electrostrictive pressure and $\gamma=\rho\frac{\partial\varepsilon}{\partial\rho}$ is the electrostrictive coefficient.

$$\nabla\cdot f=\epsilon_0\gamma q^2\left[E_pE_b^*\mathrm{e}^{i(qz-\Omega t)}+c\cdot c_{-}\right] \tag{32.15}$$

Substituting (32.11) and (32.15) in (32.12)

$$-2i\Omega\frac{\partial\rho}{\partial t}+\left(\omega_b^2-\Omega^2-i\Omega\Gamma_b\right)\rho-2iqv^2\frac{\partial\rho}{\partial z}=\epsilon_0\gamma q^2E_pE_b^* \tag{32.16}$$

$\Gamma_b=q^2\Gamma'$ is the inverse of the phonon life time.

Since acoustic phonon frequency ~5GHz and Γ_b = 100 MHz, The attenuation coefficient Γ_b/v >> gain for the Brillouin scattering. Consequently, the phonons are absorbed over distances much shorter than for significant variation in the source term on r.h.s.

Hence, one an neglect the $\frac{\partial\rho}{\partial z}$ term on r.h.s.

In the steady state

$$\rho(z,t)=\epsilon_0\gamma q^2\frac{E_pE_b^*}{\left(\omega_b^2-\Omega^2-i\Omega\Gamma_b\right)} \tag{32.17}$$

The nonlinear polarizations that drives the pump and Brillouin waves can be written as

$$\begin{aligned}P_{NL}(\omega_p)&=\epsilon_0\gamma\frac{\rho}{\rho_0}E_b\\&=\frac{\epsilon_0\gamma}{\rho_0}\frac{E_p\left|E_b\right|^2}{\omega_b^2-\Omega^2-i\Omega\Gamma_b}\end{aligned} \tag{32.18}$$

and

$$\begin{aligned}P_{NL}(\omega_b)&=\epsilon_0\gamma\frac{\rho}{\rho_0}E_p\\&=\frac{\epsilon_0\gamma}{\rho_0}\frac{E_b\left|E_p\right|^2}{\omega_b^2-\Omega^2-i\Omega\Gamma_b}\end{aligned} \tag{32.19}$$

The equations governing the spatial evolution of the optical fields can be written in the steady state case as

$$\frac{\partial E_p}{\partial z} = \mathrm{i}\,\frac{\epsilon_0 \omega \gamma^2 q^2}{2nc\rho_0}\,\frac{|E_b|^2 E_p}{\omega_b^2 - \Omega^2 - \mathrm{i}\Omega\Gamma_b} \tag{32.20}$$

and

$$\frac{\partial E_b}{\partial z} = -\mathrm{i}\,\frac{\epsilon_0 \omega \gamma^2 q^2}{2nc\rho_0}\,\frac{|E_p|^2 E_b}{\omega_b^2 - \Omega^2 - \mathrm{i}\Omega\Gamma_b} \tag{32.21}$$

Here $\omega = \omega_p \approx \omega_b$,

Optical field evolution in terms of their intensities upon propagation can be written as

$$\frac{\mathrm{d}I_p}{\mathrm{d}z} = -g_b^{'} I_p I_b \tag{32.22}$$

and

$$\frac{\mathrm{d}I_b}{\mathrm{d}z} = -g_b^{'} I_p I_b \tag{32.23}$$

(sign on the R. H. S. of equation (32.23) has been reversed for the generated wave because it runs in the backward direction).

where we have used the abbreviation

$$g_b^{'} = \frac{\gamma^2 \omega^2}{nvc^3 \rho_0 \Gamma_b} \cdot \frac{(\Gamma_b / 2)^2}{(\omega_b - \Omega)^2 + \left(\frac{\Gamma_b}{2}\right)^2} \tag{32.24}$$

Which describes the SBS gain coefficient which peaks at ω_b.

Combining equations (32.22) and (32.23), we can write

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(I_p - I_b\right) = 0 \tag{32.25}$$

Or

$$I_p(z) - I_b(z) = I_p(0) - I_b(0) = \text{Constant=C} \tag{32.26}$$

Its spectrum is described by the equation similar to equation() for the Raman gain. For the case of undepleted pump i.e. the weak conversion regime, integration of the equation() gives for the medium of length *L*.

$$I_b(z) = I_b(L)\exp\left(g_b^{'} I_p (L - Z)\right) \tag{32.27}$$

Where *L* is the length of the medium.

$\Rightarrow$ exponential growth of the backscattered wave.

If no input at ω_b is present I_b grows from the spontaneously emitted Brillouin scattered wave.

For the strong coupling regime-- case of pump depletion—equations can still be integrated analytically. The dynamics of the pump and stimulated Brillouin scattered waves is described by the following equation.

$$I_p(z) = \frac{C}{1 - \frac{I_b(0)}{I_p(0)}\exp\left(-Cg_b^{'} z\right)} \tag{32.28}$$

and

$$I_b(z) = \frac{C}{\frac{I_p(0)}{I_b(0)}\exp\left(-Cg_b^{'}(L-z)\right)-1} \qquad (32.29)$$

$\Rightarrow$ In stimulated Brillouin scattering the scattered wave grows with distance of propagation at the cost of pump depletion. The spatial evolution of the Brillouin scattered wave propagating in the backward direction is shown in the figure 32.2(a) and (b) for the case of undepleted pump and depleted pump. Zel'dovich and coworkers demonstrated phase conjugation property of SBS in 1972 (Lecture 25)

SBS process requires serious considerations in the design of fiber optic communication system. We will pay attention to these in our next lecture.

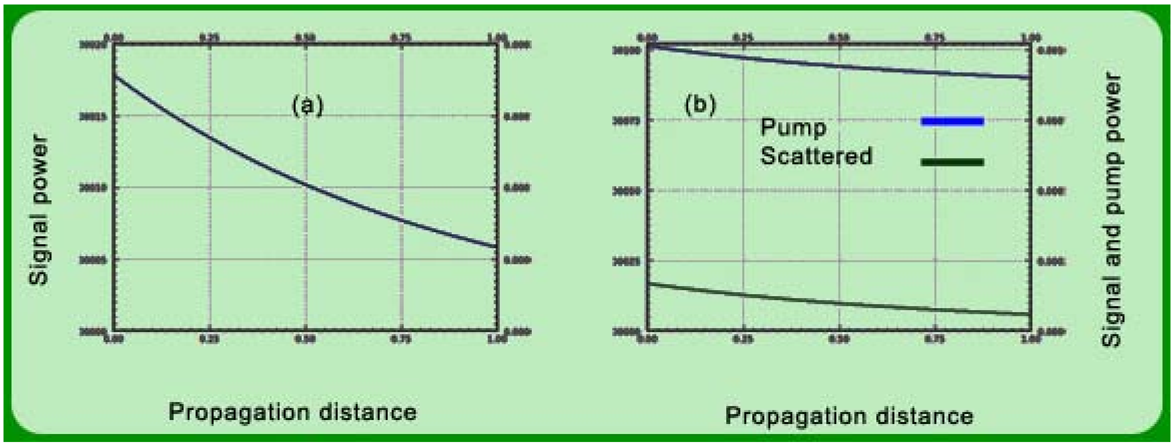

Figure 32.2 growth of incident signal P_{signal} = 58.2 µW at z=L and P_{pump} = 1mW incident at z = 0 in 1km long single mode fiber having mode dia of 7.5 µm (a) undepleted pump $g_b'P_p$ = 1.19/km (b) depleted pump C=0.83e-4 W and C g_b' = 0.94/km.

Recap

In this lecture you have learnt the following

- Stimulated scattering of light by acoustic phonons, the stimulated Brillouin scattering.
- Mechanism behind this process in simple physical picture.
- We have examined the energy and momentum conservation and their implications for the process.
- It has been shown that unlike stimulated Raman scattering process, the frequency shift is angle dependent and there is no stimulated Brillouin scattering in the forward direction. Largest frequency shift occurs in the backward direction.
- Dynamics of the process has been analyzed analytically using coupled wave equations in weak and strong scattering regimes.