Module 5 : Pulse propagation through third order nonlinear optical medium

Lecture 37 : Solitons in optical fibers

Objectives

In this lecture you will learn the following

- The pulse propagation in nonlinear dispersive medium
  (a) Qualitative picture of the interplay of self-phase modulation and group velocity dispersion.
  (b) Theoretical description-Nonlinear Schrödinger equation.
- Solutions of nonlinear Schrödinger equation- Fundamental and higher order solitons.

Solitons in optical fibers

Thus far we have examined, separately, the effects of group velocity dispersion (GVD) and self phase modulation (SPM) phenomena on the propagation of light pulse in Lectures 35 and 36. The former one-GVD is responsible for the pulse broadening. It can also result in pulse compression if the pulse is chirped favorably. The latter one -SPM chirps the pulse by enriching its spectral content without affecting its intensity profile. It is then natural to ask what happens to the propagating pulse when both the effects are present in the medium, simultaneously. We will explore this question first qualitatively and then with mathematical rigor. Let us consider the following cases:

1. **Role of positive GVD and SPM:**

   SPM leads to the spectrally enriched positively chirped pulse by lowering of the leading edge frequencies i.e. red shifted, and raised for the trailing edge frequencies i.e. blue shifted. Consequently, propagation through a medium with positive GVD, the high frequency components in the trailing edge of the pulse will increasingly lag behind the low frequency components in the leading edge upon propagation as if these two end are being pull apart. This results in the enhanced temporal broadening of the pulse (see figure 37.1).

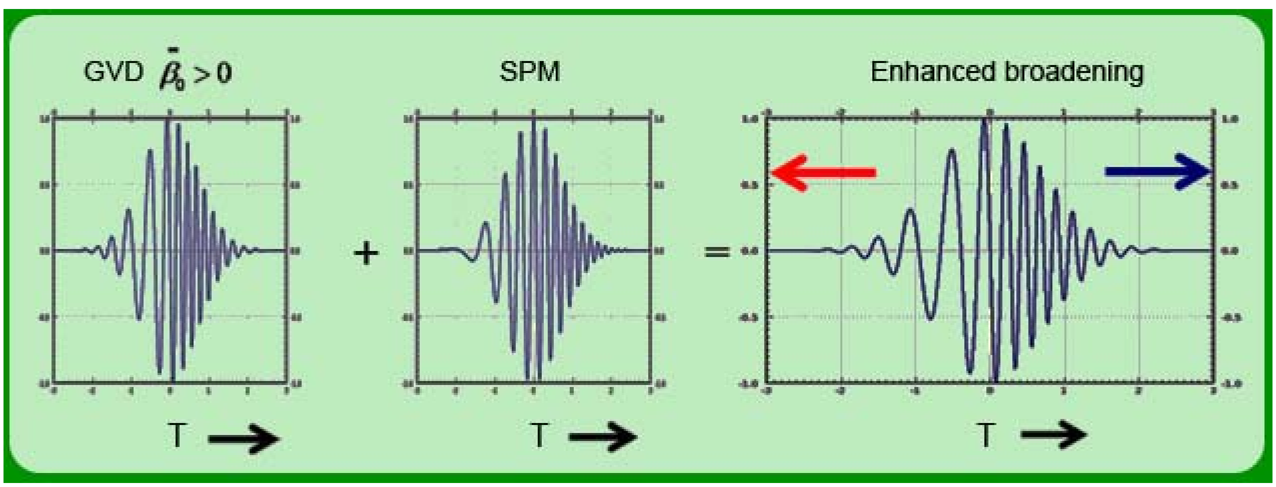

Figure 37.1

2. **Role of -ve GVD and SPM**
   In the light of the of the discussion of lecture#35, we can see that the positive chirp of spectrally enriched pulse produced by SPM can be removed with subsequent pulse compression. The idea of using nonlinearity to compensate for the broadening effect of GVD was first put forward by Hassegawa and Tappert in a theoretical paper [1]. In this case, the high frequency components from the back end try to catch with the low frequency component from the front end due to negative GVD. Consequently, the pulse is compressed as shown in figure 37.2.

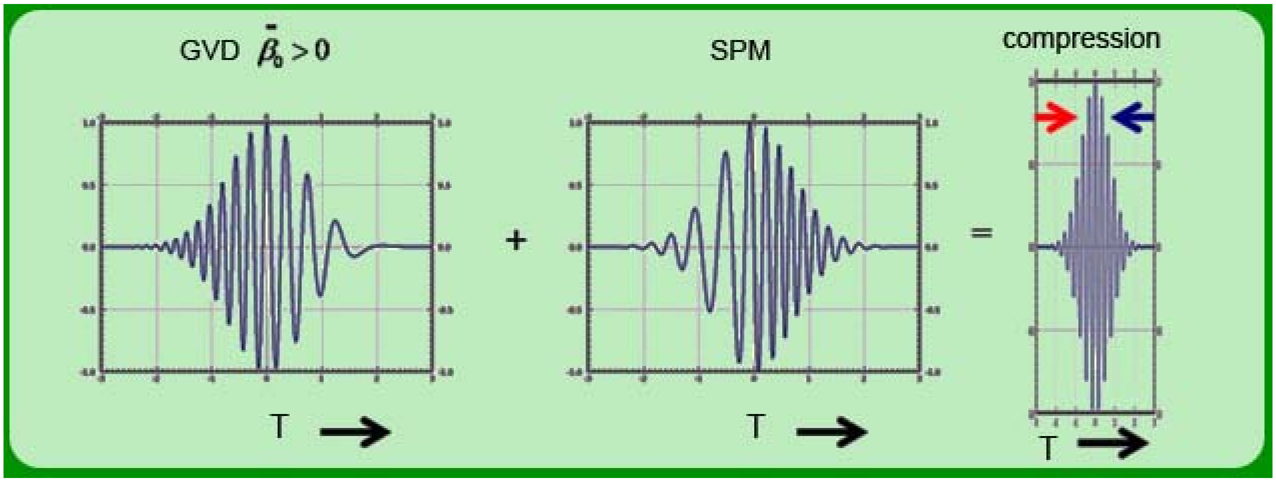

Figure 37.2

If the two effects, balance each other, the pulse will retain a constant shape which moves like a particle and result in a pulse which retains its shape on propagation which is dynamically and structurally stable like a particle and is called a fundamental soliton. Such a particle like state of the pulse will have strong implications for the transport of signal or information. We will discuss such applications in the next lecture. It is also possible that it rebuilds its shape periodically in space. This is the case of higher order solitons.

The evolution of the pulse under the influence of the GVD and SPM is described by the partial differential equation which was derived in lecture #36 (equation (36.32) for lossless medium

$$i\frac{\partial U}{\partial Z} = \frac{k_0 n_2}{A_{eff}}|U|^2 U - \frac{1}{2}\ddot{\beta}_0 \frac{\partial^2 U}{\partial T^2} \tag{37.1}$$

We apply the following transformation

$$u = \left\{ \frac{k_0 n_2 \tau^2}{|\ddot{\beta}_0| A_{eff}} \right\}^{1/2} U; \qquad z = \frac{|\ddot{\beta}_0|}{\tau^2} Z \text{ and } t = \frac{T}{\tau} \tag{37.2}$$

Where τ is a measure of pulse width.

$$\frac{\partial^2}{\partial T^2} = \frac{1}{\tau^2}\frac{\partial^2}{\partial t^2}; \; \frac{\partial}{\partial Z} = \frac{\ddot{\beta}_0}{\tau^2}\frac{\partial}{\partial z} \tag{37.3}$$

Thus

$$i\frac{\partial u}{\partial z} = |u|^2 u - \frac{1}{2}\frac{\ddot{\beta}_0}{|\ddot{\beta}_0|}\frac{\partial^2 u}{\partial t^2} \tag{37.4}$$

As $\ddot{\beta}_0$ can be either +ve or –ve, we have

$$i\frac{\partial u}{\partial z} = |u|^2 u \mp \frac{1}{2}\frac{\partial^2 u}{\partial t^2} \tag{37.5}$$

or

$$i\frac{\partial u}{\partial z}\pm\frac{1}{2}\frac{\partial^2 u}{\partial t^2}-|u|^2 u=0 \tag{37.6}$$

This is the nonlinear Schrodinger Equation (NLSE). In this equation with –ve sign corresponds to $\ddot{\beta}_0<0$ i.e. negative group velocity dispersion. NLSE has a solution of the form

$$u(z,t)=f(t)e^{i\phi(z)} \tag{37.7}$$

where $f(t)$ is the functional form of the pulse envelope and $\phi(z)$ is the phase term. The phase term has been included as SPM generated by the nonlinear term grows with distance.
A simple solution is given by

$$u(z,t)=u_0\,e^{iaz}\,\mathrm{sech}(t) \tag{37.8}$$

substituting this in equation 37.6, we get

$$i\,i\,a u_0\,e^{iaz}\,\mathrm{sec}h\,t=-\frac{1}{2}\frac{\ddot{\beta}_0}{|\ddot{\beta}_0|}u_0 e^{iaz}\;\mathrm{sech}\,t\left\{\tanh^2 t-\mathrm{sech}^2 t\right\}$$
$$+(|u_0|^2\,\mathrm{sech}^2 t)u_0\,e^{iaz}\,\mathrm{sech}\,t$$

Thus

$$2a=\frac{\ddot{\beta}_0}{|\ddot{\beta}_0|}\left\{\tanh^2 t+\mathrm{sech}^2 t\left\{-1-\frac{2|u_0|^2\,|\ddot{\beta}_0|}{\ddot{\beta}_0}\right\}\right\} \tag{37.9}$$

Equation(37.8) is a solution of NLSE (37.6) provided

$$a=\frac{\ddot{\beta}_0}{2|\ddot{\beta}_0|}\text{ and }-1-\frac{2|u_0|^2|\ddot{\beta}_0|}{\ddot{\beta}_0}=1 \tag{37.10}$$

Or

$$a=\frac{\ddot{\beta}_0}{2|\ddot{\beta}_0|}\text{ and }|u|^2=\frac{\ddot{\beta}_0}{|\ddot{\beta}_0|} \tag{37.11}$$

As $|u|^2$ represents power, it has to be +ve. Hence $\ddot{\beta}_0$ has to be –ve.

Thus $u(z,t)=u_0\,e^{iaz}\,\mathrm{secht}(t)$ is a solution of nonlinear Schrödinger equation for –ve GVD.

$$i\frac{\partial u}{\partial z}=\frac{1}{2}\frac{\partial^2 u}{\partial t^2}+|u|^2 u \tag{37.12}$$

and

$$u(z,t)=u_0\,e^{-i\frac{z}{2}}\,\mathrm{sech}(t) \tag{37.13}$$

In term of $U(Z,T)$ we have

$$U(Z,T) = U_0 e^{i\left[\frac{\ddot{\beta}_0}{2\tau^2}\right]Z} \operatorname{sech}(T/\tau) \tag{37.14}$$

This solution represents the pulse envelop propagating in the fiber that does not disperse and retains its shape and hence is a soliton as described earlier. Figure 37.1 shows soliton propagation in one soliton period.

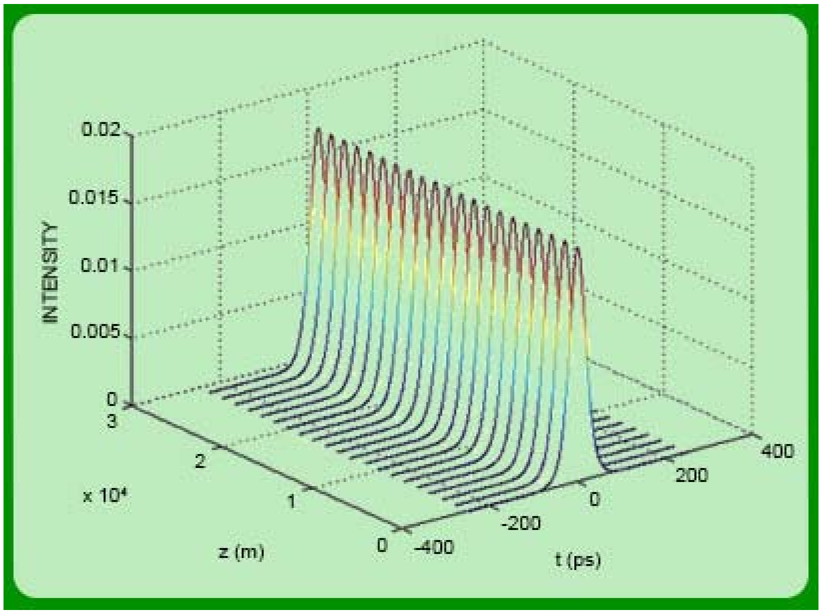

Figure 37.3

Since

$$u(z,t) = \left\{\frac{k_0 n_2 \tau^2}{|\ddot{\beta}_0| A_{eff}}\right\}^{1/2} U(Z,T); \tag{37.15}$$

$$z = \frac{|\ddot{\beta}_0|}{\tau^2} Z \tag{37.16}$$

and

$$t = \frac{T}{Z} \tag{37.17}$$

where

$$|u|^2 = -\frac{\ddot{\beta}_0 A_{eff}}{n_0 n_2 \tau^2} \tag{37.18}$$

and $\frac{\ddot{\beta}_0}{n_2}\ is -ve$, under these conditions

1. Soliton width is independent of "Z"

2. The amplitude of the solution is not arbitrary but uniquely specified by $\ddot{\beta}_0, n_2$ and τ

$$U(Z,T) = U_0 e^{i\left[\frac{\ddot{\beta}_0}{2\tau^2}\right]Z} \operatorname{sech}(T/\tau) \tag{37.19}$$

For soliton the phase term is

$$\exp\left[i\frac{\ddot{\beta}_0}{2\tau^2}Z\right] = \exp\left[-i\frac{n_2 k_0}{A_{eff}}|U_0|^2 Z\right] \tag{37.20}$$

In comparison with the phase term for SPM

$$\exp\left[-i\frac{n_2 k_0}{A_{eff}}|U(Z,T)|^2 Z\right] = \exp\left[-i\frac{n_2 k_0}{A_{eff}}|U_0|^2 \underbrace{\left[f(Z,T)\right]^2}_{\text{Functional shape of the pulse}} Z\right] \tag{37.21}$$

Unlike the SPM phase, the_soliton phase is not a function of time dependent pulse envelope. It rather depends on the squared magnitude of its constant amplitude

Now since

$$P = |U_0|^2 = -\frac{\ddot{\beta}_0 A_{eff}}{n_2 k_0 \tau^2} \tag{37.22}$$

and

$$\ddot{\beta}_0 = \frac{-\lambda_0^2 D}{2\pi c} \tag{37.23}$$

$$\Rightarrow P = \frac{\lambda_0^2 D A_{eff}}{2\pi c k_0 n_2 \tau^2} = \frac{\lambda_0^3 D A_{eff}}{4\pi^2 c n_2 \tau^2} \tag{37.24}$$

Using $(\Delta t)_{\text{FWHM}} = 1.762\,\tau$ gives

$$P = \frac{0.776\lambda_0^3 D A_{eff}}{4\pi^2 c n_2 (\Delta t)^2_{\text{FWHM}}} \tag{37.25}$$

Now soliton phase, is given by $\phi(t) = \frac{\ddot{\beta}_0}{2\tau^2}Z$. We define “Soliton Period” Z_0 such that

$$\phi(z) = \frac{\pi}{4} = \frac{\ddot{\beta}_0}{2\tau^2}Z_0 \tag{37.26}$$

Or

$$Z_0 = \frac{\pi\tau^2}{2\ddot{\beta}_0} \tag{37.27}$$

Or in terms of the dispersion parameter D and FWHM width of the pulse

$$Z_0 = \frac{0.322\pi^2 c(\Delta t)^2_{FWHM}}{\lambda_0^2 D} \tag{37.28}$$

Energy of the soliton can be written as

$$E = P\int_{\infty}^{\infty} \mathrm{Sech}^2(T/\tau)dT = 2\tau P$$

Using $\int_{-\infty}^{\infty} \mathrm{Sech}^2(T/\tau)dT = 2\int_0^{\infty} \mathrm{Sech}^2(T/\tau)dT = 2\frac{\tanh(T/\tau)}{1/\tau}\Big|_o^{\infty}$

$$E = \frac{2P\Delta t}{1.762} = 1.135P\Delta t = \frac{0.881\lambda_0^3 D A_{eff}}{\pi^2 c n_2 (\Delta t)_{FWHM}} \tag{37.29}$$

Note that the Soliton energy is related to its pulse width. Thus, the soliton power, energy and period are

$$P = \frac{0.776\lambda_0^3 A\,|D|}{\pi^2 c n_2 (\Delta t)^2};\ E = \frac{0.881\lambda_0^3\,|D|\,A}{\pi^2 c n_2 \Delta t};\ Z_0 = \frac{0.322\pi^2 c\Delta t^2}{\lambda_0^2\,|D|} \tag{37.30}$$

To get a feel of various parameters, let us consider some typical numerical examples. For a typical fiber having $A = 50\mu m^2$ and operation at $\lambda_0 = 1.55\mu m$

$(\text{minimum loss, } \alpha_{\text{dB}} = 0.2dB/km)$

These expressions become

$$P = \frac{1.53\,|D|}{(\Delta t)^2};\ E = \frac{0.97\,|D|}{\Delta t};\ z_0 = \frac{0.3(\Delta t)^2}{|D|} \tag{37.31}$$

where Δt is FWHM in ps and $|D|$ is in ps/nm/km, P is in watts, E is in pJ, Z_0 is in km.

Taking typical fiber loss value of 0.2db/km $\Rightarrow L_{eff} = \frac{1}{\alpha} = 22km$

$For\ \Delta t = 70\,ps \qquad and\ |D| = 15\,ps/nm/km$

$P = 4.7mW;\ E = 0.2\,pJ;\quad Z_0 = 98km$

As $Z_0 > L_{eff}$ the soliton propagation will not manifest. The optical pulse will behave as an ordinary pulse and broaden on propagation.

Consider yet another numerical example

$For\ \Delta t = 7\,ps \qquad and\ |D| = 15\,ps/nm/km$

$P = 0.47W;\ E = .2pJ;\quad Z_0 = 0.98km$

Now as $Z_0 < L_{eff}$ the soliton effects will manifest.

An important issue to scrutinize is how critical is the balance between the GVD and SPM for the soliton propagation. To put it differently let us ask the question:

What happens when a noisy pulse is launched into the fiber?

It has been shown that when a noisy pulse is propagates over many soliton periods in the fiber; the output pulse is "clean" and has most of the energy of the input pulse as represented in the figure 37.2. Excess energy in the absence of loss results in low level background.

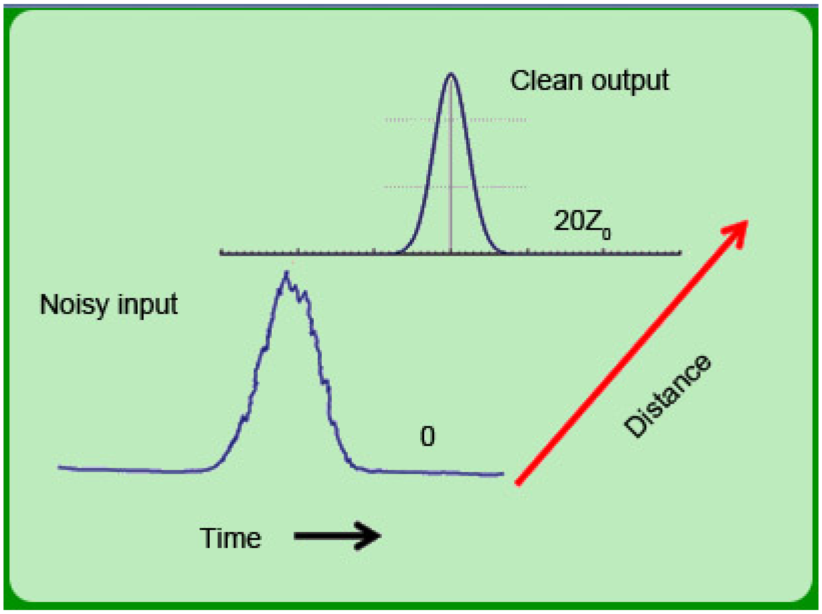

Figure 37.4

Hence formation of solitons is a forgiving process i.e. balances between the nonlinearity and dispersion need not to be exact.
We still need to address the question related to the limits of this flexibility.

Let $R = \int u(t)\,dt$ be the input pulse area and R_0 be area of the soliton corresponding to input pulse duration then a pulse having area in the range $\frac{R_0}{2} < R < \frac{3R_0}{2}$ will evolve into a soliton.

If $R > R_0$, soliton width will be shorter than input pulse width.
If $R < R_0$, Soliton width will be greater than input pulse width.
Also, Non-soliton parts of the pulse are stripped off and propagate as a dispersive wave.

What happens to the pulse if $R > \frac{3R_0}{2}$?

We will discuss this scenario in the following.

& $P_N = N^2 P_1 = (1.3)^2 \times 2.1\,mW = 3.55\,mW$

SOLITON PROPAGATION

First experimental observation of optical soliton was reported by Mollenauer et al [2].They used a synchronously pumped modelocked color centre Laser (F^{2+}:NaC1) operating at 1.55μm (anomalous dispersion regime) in an experimental setup shown in the figure 37.5(taken from ref. 2) to study the nonlinear pulse propagation in single mode optical fibers as a function input pulse peak powers. Values of typical experimental parameters are as given below. Input and output pulses were characterized, alternately, in the same autocorrelator.

$$(\Delta t)_{FWHM} = 7\,ps;\quad \lambda = 1.5\,\mu m \;\; and \;\; P_{avg} = 1W$$
$$L = 700m;\ D = -16\,ps/nm/km$$

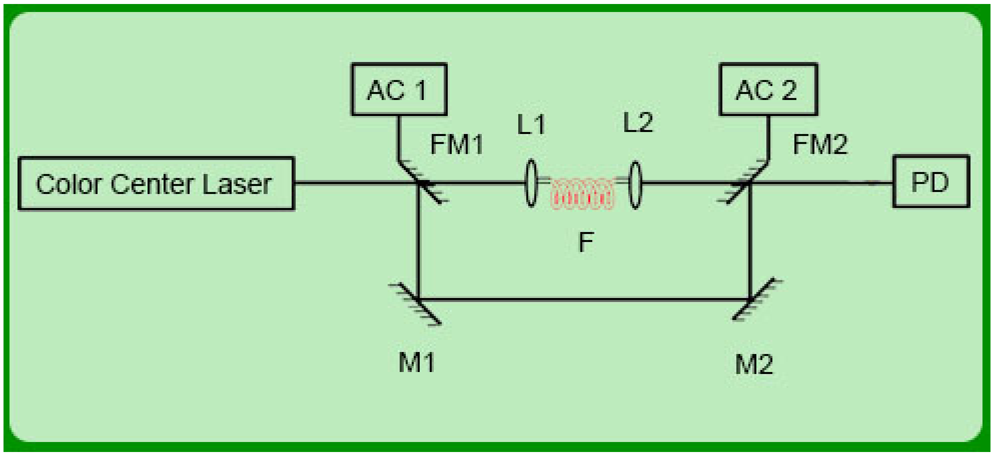

Figure 37.5:$M_{1,2}$-Mirrors, $FM_{1,2}$-Flip Mirrors, $AC_{1,2}$ – actuators to flip mirrors FM_1 and FM_2, $L_{1,2}$-Lenses to couple laser to Fiber F, PD- pulse diagnostic system comprising of pulse autocorrelator and spectromenter.

Observed fiber autocorrelation traces of output pulse shapes at certain critical peak power levels are shown in figure 37.6. Corresponding to an input pulse (figure 37.6a), the output pulse shape (figure 37.6b) observed for the input pulse power P=0.3W agrees with the dispersion broadened pulse. Continuous narrowing in the pulse shape was observed as the peak power of the pulse is raised.

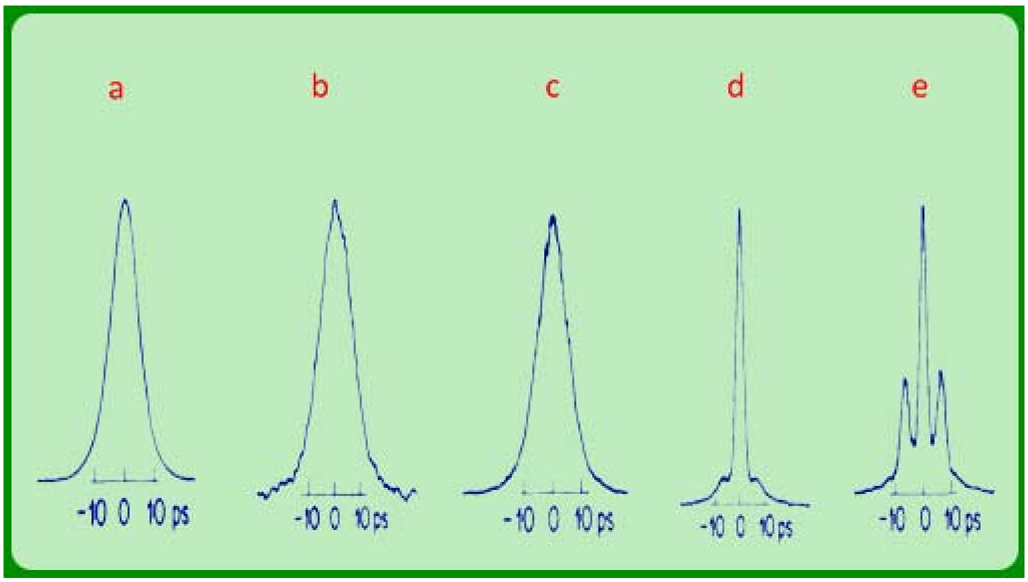

Figure 37.6

At P = 1.2W, pulses essentially retained their original shape (see figure 37.6c). At still higher powers these were observed to further narrow to a minimum value of ~2ps at P=5W (figure 37.6d) but riding on a broad pedestal. With power increasing further the broad base began to rise and split into well resolved structure as seen in figure 37.6e corresponding to input pulse power P=11.4W. The autocorrelation trace shown in figure 37.6e corresponds to a pulse with two fold splitting which represents an N=2 soliton and will be discussed further in following.

These pulse envelop functions, of course, should follow from the solutions of the nonlinear Schrodinger equation (37.12). Satsuma et-al studied the analytical solutions of this equation with the initial condition

$$U(0,T) = A\,\mathrm{sech}(T) \tag{37.32}$$

and found the self maintaining solutions in the sense that their pulse shapes are periodic with propagation whenever A takes an integer value N. For N=1, the input pulse remains invariant forever. The fiber length of 760 meters used in Ref.2 corresponds to the half soliton period. figures 37.7 and 37.8 show the propagation dynamics of N=2 and N=3 solitons. A comparison of the pulse shapes at $Z_0/2$ in these with observed ones readily identifies the pulses in figure 37.6c and 37.6d with N=2 and N=3 solitons. Pulse shape in figure 37.7a is the fundamental(N=1) soliton as its shape remains invariant. It can be noticed that the higher order soliton of order N will manifest at power

$$P_N = N^2 P_1$$

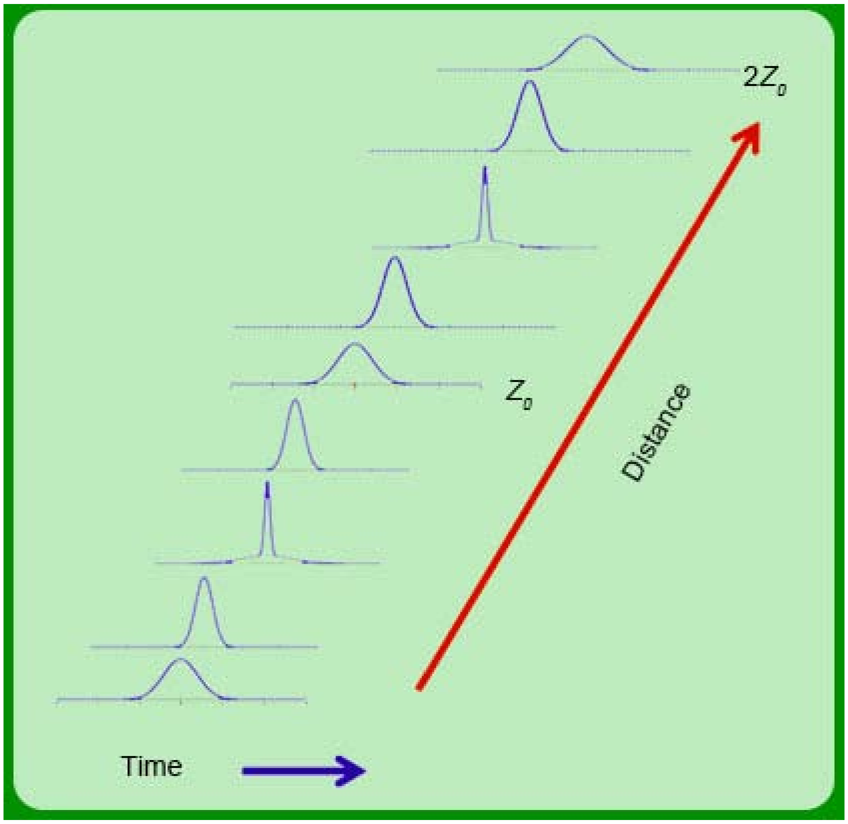

Figure 37.7a Propagation of N=2 soliton

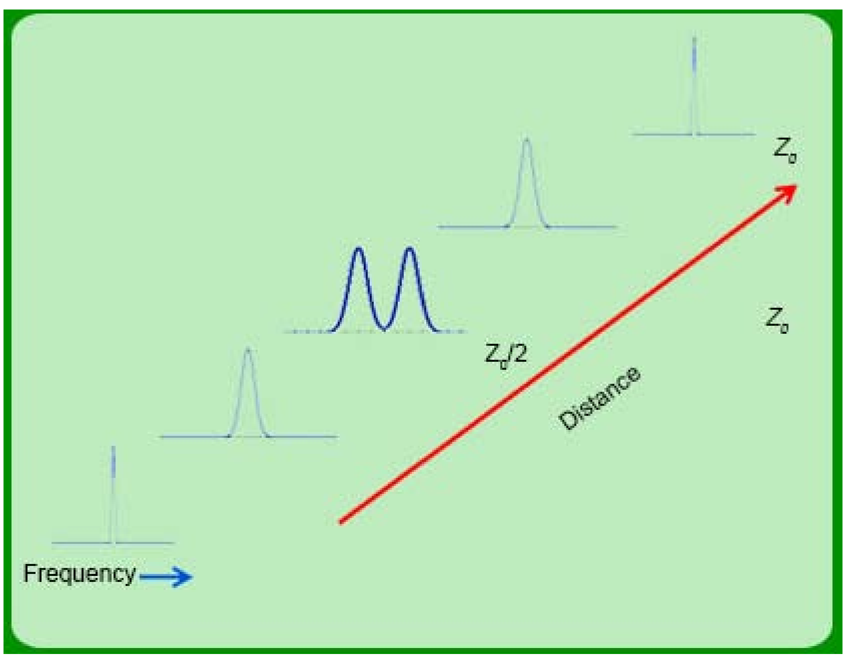

Figure 37.7b Spectral evolution of N = 2 soliton on propagation

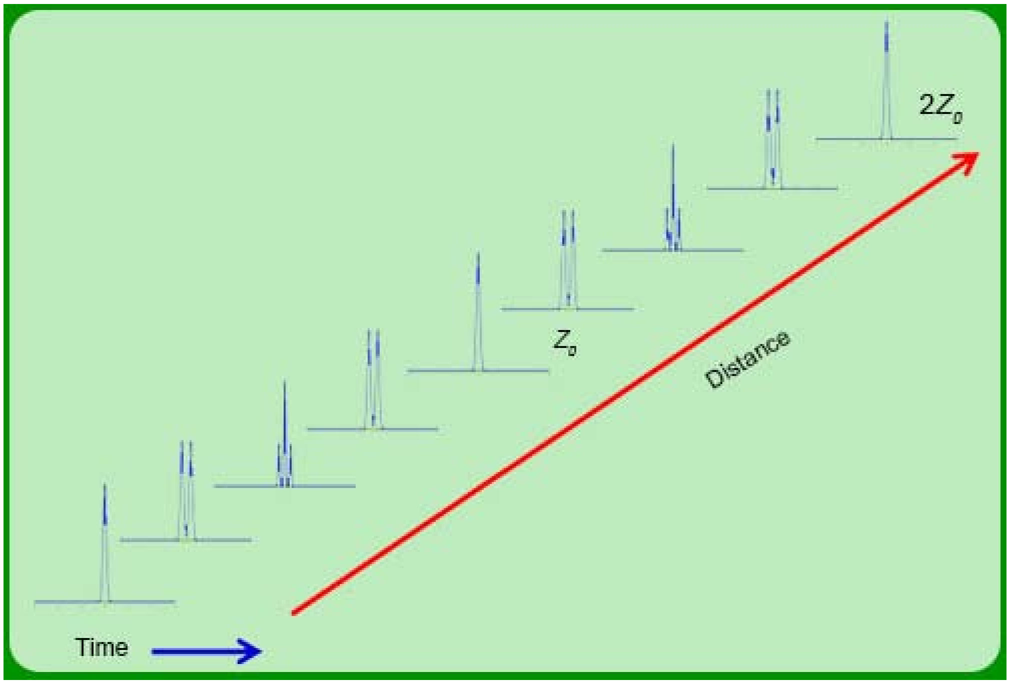

Figure 37.8 Propagation dynamics of N=3 soliton

Also note that the compressed pulse at $Z_0/2$ for N=2 soliton sits on a broad pedestal which contains about as much energy as the spike. It is because during initial narrowing stage evolution of higher order soliton is dominated by SPM which induces linear chirp only over the central part of the pulse. Thus central part is compressed by the GVD whereas nonlinear chirp in the wings is unaffected and appears as a broad pedestal.

It can be noticed from figure 37.7b that the spectrum of N=2 soliton initially broadens and then contracts and periodically returns to original form i.e. in soliton evolution new frequencies are periodically created and destroyed.

References:

1. A. Hasegawa and F. Tappert, Appl. Phys. Lett., **23**,142(1973)
2. F. Mollenauer, R. H. stolen and J. P. Gordon, Phys. Rev. Lett., **45**, 1095(1980)

Recap

In this lecture you have learnt the following

- The pulse propagation in nonlinear dispersive medium
  (a) Qualitative picture of the interplay of self-phase modulation and group velocity dispersion.
  (b) Theoretical description-Nonlinear Schrödinger equation.
- Solutions of nonlinear Schrödinger equation- Fundamental and higher order solitons, conditions of formation and propagation dynamics.