

Module 2 : Nonlinear Frequency Mixing

Lecture 12 : Three Wave Mixing-2

Objectives

In this lecture we will look at

- Solutions for 3 wave mixing coupled wave equations.
- Comparison between sum and difference frequency generation.
- Optical parametric oscillator.

The sum frequency generation(SFG) and difference frequency generation (DFG) are both described by the same set of coupled differential equations. To see these two processes in some detail, we begin with Eqs.(11.19)-(11.22) for the amplitudes of the three waves and their relative phase angle q . We eliminate the amplitudes of the signal and idler waves by using the two Manley-Rowe constants be denoted by m_1 and m_2 .

$$m_1 = v_p^2 + (\omega_p / \omega_s) v_s^2 \quad (12.1)$$

and

$$m_2 = v_p^2 + (\omega_p / \omega_i) v_i^2 \quad (12.2)$$

Since the three amplitudes are connected by the conservation of power flow, these two constants are not independent and are related by

$$\omega_s m_1 + \omega_i m_2 = \omega_p \quad (12.3)$$

The independent Manley Rowe constant determines the ratio of the fractional powers in the input waves. In the sum frequency generation process the ratio of the two Manley Rowe constants is simply the ration of the two initial photon fluxes. One of them is larger than one and the other smaller than one. If the two initial photon fluxes are equal both the Manley-Rowe constants are equal to one. In this case, there is negligible power in both the fundamental and the idler waves when maximum conversion to the pump wave occurs. Conversely, this is the DFG case when we start from a strong pump and very small power in the signal and idler waves.

Apart from the two Manely Rowe contants, corresponding to Eq. in SHG we have

$$\Gamma = v_p v_s v_i \cos \theta + (\Delta S / 2) v_p^2 \quad (12.4)$$

To verify that this is a constant ,we take the derivative of the right hand side with respect to z , the normalized length and use eqs (11.19) to (11.22) and show that the derivative vanishes. The solution for the pump frequency amplitude is then given by

$$v_p^2(\zeta) = v_{p2}^2 + (v_{p0}^2 - v_{p2}^2) \sin^2((v_{pc}^2 - v_{p2}^2)^{1/2} (\omega_s \omega_i / \omega_p)^{1/2} (\zeta + \zeta_0), \gamma) \quad (12.5)$$

where

$$v_{pc}^2 \geq v_{p0}^2 \geq v_{p2}^2 \quad (12.6)$$

are the three roots of the cubic equation

$$v_p^2 (m_1 - v_p^2) (m_2 - v_p^2) - (\omega_p^2 / \omega_s \omega_i) (\Gamma - \Delta S v_p^2 / 2)^2 = 0 \quad (12.7)$$

and the modulus γ is given by

$$\gamma^2 = (v_{p0}^2 - v_{p2}^2) / (v_{pc}^2 - v_{p2}^2) \quad (12.8)$$

Knowing v_p , we can find the signal and idler amplitudes from equations (12.1) and (12.2).

These are shown for a typical example in Figure 12.1.

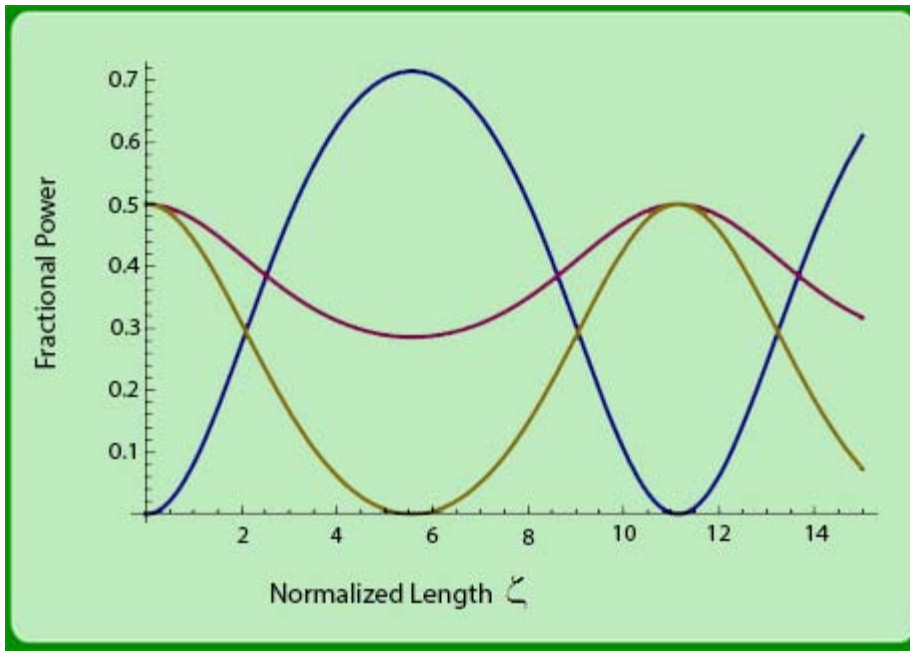


Figure 12.1 Normalized power in 3 waves as functions of the scaled length ζ . Initially the power is equally divided between two waves the signal wave (magenta with $\omega_s=0.3\omega_p$) and the idler wave (mustard; $\omega_i=0.7\omega_p$) with zero power in the pump wave (blue: ω_p). First in the sum frequency generation process the pump power grows at the cost of the two input waves till the idler is fully depleted. Then the idler and the signal waves grow from the pump wave in a difference frequency process.

We observe:

- The solutions are periodic with the period depending on the ratio of powers in the idler and signal when the pump wave has its minimum amplitude ν_{pz} .
- If we start with only the signal and idler waves, $\nu_{pz} = 0$ since the initial conditions ensure that. It is easy to verify that in this case 0 is indeed one of the roots of eq (12.7).
- One can get $\nu_{ps} = \nu_{pc} = 1$ only if the $\nu_{pz} = 0$ and $m_1 = m_2 = 1$, which corresponds to equal photon fluxes in the signal and idler waves. In this case, $\gamma = 1$ and the sn function becomes hyperbolic tangent function.
This has the important physical consequence that the *maximum conversion to SFG always occurs in a finite crystal length except when we start with equal photon fluxes in the s and i- waves.*
- This is obviously not possible in a DFG experiment!

To further explore this asymmetry between SFG and DFG let us consider the phase-matched propagation of the three waves. The corresponding equations (11.7) to (11.9) now become:

$$\frac{\partial A_s^*}{\partial z} = \frac{\omega_s^2 K}{k_s \cos^2 \alpha_s} A_p^* A_i \quad (12.9)$$

$$\frac{\partial A_i^*}{\partial z} = \frac{\omega_i^2 K}{k_i \cos^2 \alpha_i} A_p^* A_s \quad (12.10)$$

$$\frac{\partial A_p}{\partial z} = -\frac{\omega_p^2 K}{k_p \cos^2 \alpha_p} A_s A_i \quad (12.11)$$

where K is the (real) coupling constant given by equation (11. 10):

$$K = 2\epsilon_0 \tilde{\chi}^{(2)}(-\omega_p, \omega_s, \omega_i) : \hat{a}_p \hat{a}_s \hat{a}_i$$

From this it is obvious that the coupling coefficient can be positive or negative but away from all resonances it is always real. On the other hand if any of the fields has zero amplitude its phase is arbitrary. In such a situation the phase of the wave with zero amplitude has to be such that it can grow.

We now consider the situation in which one of the two incident waves is much stronger than the other. In SFG, the two input waves are ω_s and ω_i . For $|A_i| \gg |A_s|$, one could assume the idler beam amplitude as constant. Then the equations for the pump and signal beam can be decoupled by differentiating both sides of eq (12.9) and substituting from (12.11) to get

$$\frac{\partial^2 A_s}{\partial z^2} = -\frac{\omega_s^2 \omega_p^2 K^2 |A_i|^2}{k_s k_p \cos^2 \alpha_p \cos^2 \alpha_s} A_s = -\beta^2 A_s \quad (12.12)$$

where β is a real constant given by

$$\beta = \frac{\omega_s \omega_p 2 \varepsilon_0 \tilde{\chi}^{(2)}(-\omega_p, \omega_s, \omega_i) : \hat{a}_p \hat{a}_s \hat{a}_i |A_i|}{\sqrt{k_s k_p} \cos \alpha_p \cos \alpha_s}$$

and similarly for the pump wave we obtain

$$\frac{\partial^2 A_p}{\partial z^2} = -\beta^2 A_p \quad (12.13)$$

The solutions for both these equations are sinusoidal with a period determined by β . With the initial conditions that the pump wave has no amplitude initially, the solution of eq (12.11) is given by:

$$A_s(z) = A_s(0) \cos \beta z$$

where $A_s(0)$ is the initial amplitude of the wave at frequency ω_s . The amplitude of the generated wave at ω_p is obtained from the Manley-Rowe Relation

A physical situation well described by this limit is the up-conversion of a very weak infrared signal to a visible wave. The maximum conversion here occurs in a finite distance which can be controlled by the strong input idler wave typically in the visible region. Thus in this process we obtain a photon in the visible (ω_p) where very sensitive detectors are available for a photon in the infrared where detectors are not so sensitive.

The solutions for DFG, with a strong pump are equally instructive. If the depletion of the pump wave is neglected, the coupled wave equations for the idler and signal wave can be decoupled easily to yield

$$\frac{\partial^2 A_s}{\partial z^2} = \frac{\omega_s^2 \omega_i^2 K^2 |A_p|^2}{k_s k_i \cos^2 \alpha_i \cos^2 \alpha_s} A_s \quad (12.15)$$

and

$$\frac{\partial^2 A_i}{\partial z^2} = \frac{\omega_s^2 \omega_i^2 K^2 |A_p|^2}{k_s k_i \cos^2 \alpha_i \cos^2 \alpha_s} A_i \quad (12.16)$$

The solutions for these show exponential growth with the gain factor

$$G = \frac{\omega_s \omega_i K |A_p|}{\sqrt{k_s k_i} \cos \alpha_i \cos \alpha_s} \quad (12.17)$$

With the initial conditions $A_i(0) = A_0$ and $A_s(0) = 0$, the solutions are:

$$A_i(z) = A_i(0) \cosh(Gz) \quad (12.18)$$

and

$$A_s(z) = \frac{\omega_s \cos \alpha_s}{\omega_i \cos \alpha_i} \sqrt{(k_i / k_s)} A_i(0) \sinh(Gz) \quad (12.19)$$

In contrast to the SFG case, here both the beams increase, and they continue to increase till the pump wave is exhausted. Ofcourse, when both the signal and the idler waves are initially absent, this theory based on classical description of fields predicts no transfer of energy. But an infinitesimal signal will grow

exponentially with a gain coefficient G . This infinitesimal signal can come from vacuum fluctuations in quantum theory description of optical fields or from thermal noise. As in a laser, if such a nonlinear optical medium pumped by a strong laser wave is put in a Fabri-Perot resonator, if the round trip gain exceeds loss, it will become an oscillator at the resonator frequency. Let us compare the two systems.

- In a laser the pump wave can be coherent –as in a laser pumped laser- or incoherent as in a discharge pumped laser or a flash lamp pumped laser. Even in laser pumped laser the coherence of the laser is generally unimportant. In an OPO, the coherence of the pump wave is crucial.
- In a laser, the resonator is resonant at the lasing or oscillator frequency. In the OPO, it can be singly resonant if the feedback is provided only on the signal frequency, or doubly resonant if both the idler and the signal are provided feedback. In the later case, the oscillator length has to be such that it supports longitudinal modes of both the waves –signal and idler.
- In a laser, only a part of the input energy is converted into the desired wave. In an OPO, the whole pump can be converted into the two output waves. The remaining energy has to be removed as thermal energy. In an OPO, the heat removal issues are minimal, since absorption of energy is negligible.
- A laser may be a narrow band laser, with little scope for tunability. An OPO is intrinsically tunable.
- In an OPO, a pump wave at frequency ω_p , can split into waves at any two frequencies ω_s and ω_i such that $\omega_s + \omega_i = \omega_p$. The frequency selection is provided by the phase-matching condition. That pair of signal and idler frequencies is produced for which $k_s + k_i = k_p$.
- Tuning of the phase-matched pair of frequencies is obtained by rotating the nonlinear optical crystal or in some cases, especially, $LiNbO_3$, by varying the temperature of the crystal. In contrast, in a laser the frequency selection is mainly obtained by adding frequency selection elements in the resonator.
- Phase matching also ensures that emission in direction other than the desired one is negligible, unlike a laser where spontaneous emission is more or less isotropic.
- Although we have described only a Fabry-Perot resonator ring lasers are often used in OPO's.

Typical experimental arrangement of components for singly resonant OPO is shown in Figure 12.2 below

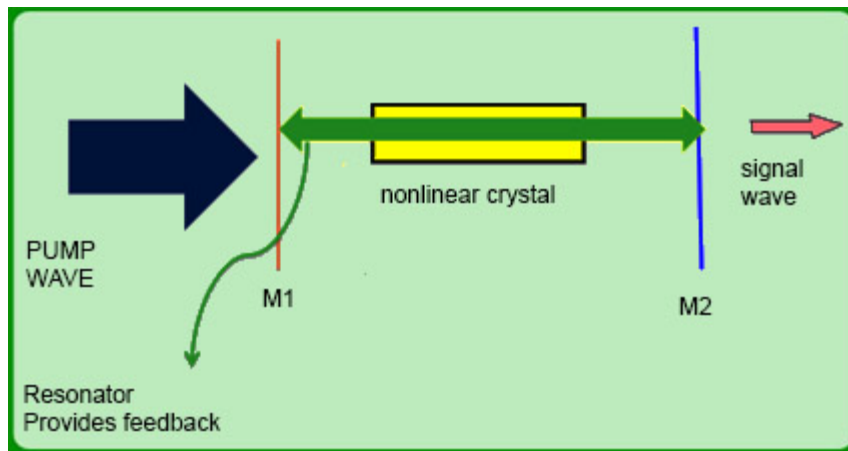


Figure 12.2: Schematic for the singly resonant optical parametric oscillator. The mirror M1 is transmitting at the pump wavelength and highly reflecting at the signal wavelength. The mirror M2 is partially reflecting at the signal wavelength, the reflectivity being optimized to obtain maximum out put at signal wavelength.

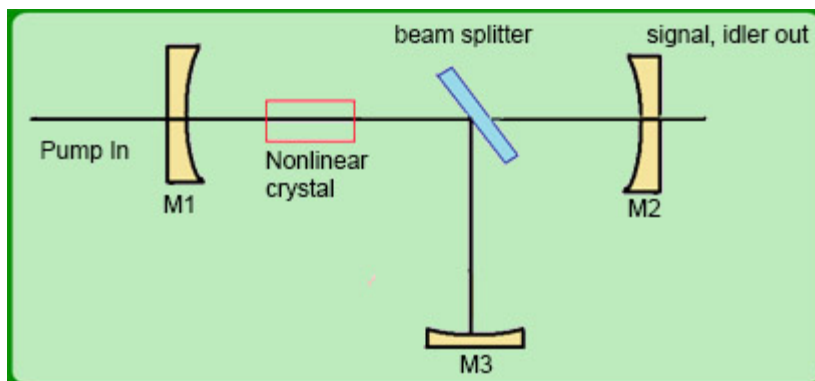


Figure 12.3 Schematic diagram for the doubly resonant OPO.

In the doubly resonant OPO shown schematically in Figure 12.2 mirrors M1 and M2 form the resonator for the signal, while M1 and M3 form the resonator for the idler. The mirror M1 has thus to be transmitting for the pump wave and highly reflecting for the signal wave as well as the idler wave. Mirror M2 is partially reflecting for the signal wave while mirror M3 is reflecting for the idler wave. The beam splitter splits the signal wave from the idler wave. If type II birefringent phase-matching is used the signal and idler have perpendicular polarization allowing the use of a polarizing prism for beam splitter. When signal and idler waves have the same polarization (type I birefringent phase-matching or quasi-phase-matching) a dichroic mirror has to be used for beam splitting. The translation of mirrors M2 and M3 are also used to ensure longitudinal mode matching. The curvature of the three mirrors are chosen to ensure maximum overlap of the 3 waves in the nonlinear crystal.

OPO's have greatly influenced the development of coherent light sources specially because they allow wide spectral coverage using a single efficient solid-state laser source. For example starting with a diode laser pumped Nd:YAG a coherent source covering the entire visible and mid infrared range can be made. We will discuss one such source later.

RECAP:

- SFG and DFG, although described by the same set of 3 coupled equations behave quite differently.
- SFG commonly used for frequency up-conversion of weak infrared signals, gives at most one visible photon for each infrared photon.
- In DFG a nonlinear crystal pumped by a powerful laser is an amplifier for the signal and idler waves.
- Optical parametric oscillator is obtained when feedback is provided for the signal and/or idler waves.