Module 5 : Pulse propagation through third order nonlinear optical medium

Lecture 35 : Pulse propagation in a linear dispersive medium

Objectives

In this lecture we will investigate the following

1. Propagation of a transform limited pulse in linear but dispersive medium.
2. Propagation of a chirped pulse in linear but dispersive medium and prospects of pulse compressor.
3. Grating pulse compressor.

Pulse propagation in a linear dispersive medium

To study the effect of dispersion on the broadening of pulse envelope upon propagation, we will consider the example of a Gaussian pulse envelop. Gaussian pulse at $z = 0$ is given by

$$E(z=0,t) = e^{-i\omega_0 t} e^{-\left(\frac{t}{\tau}\right)^2} \tag{35.1}$$

$$= e^{-i\omega_0 t} \int_{-\infty}^{\infty} F(\Omega) e^{-i\Omega t} d\Omega \tag{35.2}$$

Where

$$F(\Omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-(t\tau)^2} e^{i\Omega t} d\Omega \tag{35.3}$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{\left(\frac{t^2}{\tau^2}+i\Omega t\right)} d\Omega \tag{35.4}$$

Using

$$\int_{-\infty}^{\infty} e^{-(ax^2+bx+c)} dx = \sqrt{\frac{\pi}{a}} e^{[(b^2-4ac)4a]} \tag{35.5}$$

We can write

$$F(\Omega) = \frac{\tau}{2\sqrt{\pi}} e^{-\left(\Omega\tau/_2\right)^2} \tag{35.6}$$

After propagating distance z in a medium, the electric field evolves to

$$E(z,t) = e^{-i\omega_0 t} \int_{-\infty}^{\infty} F(\Omega) e^{-i\Omega t} e^{i\beta(\omega)z} d\Omega \tag{35.7}$$

Note that the propagation constant β is frequency dependent.

$$\beta(\omega_0+\Omega) = \beta_0 + \dot{\beta}_0\Omega + \frac{1}{2}\ddot{\beta}_0\Omega^2 + \cdots \tag{35.8}$$

For $z = L$, we have

$$E(L,t) = e^{-i(\omega_0 t-\beta_0 L)} \int_{-\infty}^{\infty} F(\Omega) \exp\left[-i\Omega\left(t-\dot{\beta}_0 L-\frac{1}{2}\ddot{\beta}_0\Omega L\right)\right] d\Omega \tag{35.9}$$

$$= e^{-i(\omega_0 t-\beta_0 L)} u(L,t) \tag{35.10}$$

Where the envelop function is

$$u(L,t) = \int_{-\infty}^{\infty} F(\Omega) \exp\left[-i\Omega\left(t-\dot{\beta}_0 L-\frac{1}{2}\ddot{\beta}_0\Omega L\right)\right] d\Omega \tag{35.11}$$

$$= \frac{\tau}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left[-\Omega^2\left(\frac{\tau^2}{4} + i\frac{\ddot{\beta}_0 L}{2}\right) + i\Omega\left(\dot{\beta}_0 L - t\right)\right] d\Omega$$

$$= \frac{\tau}{2\pi} \sqrt{\frac{\pi}{\left(\frac{\tau^2}{4} + \frac{i\ddot{\beta}_0 L}{2}\right)}} \exp\left[\frac{-\left(\dot{\beta}_0 L - t\right)^2}{4\left(\frac{\tau^2}{4} + i\ddot{\beta}_0 \frac{L}{2}\right)}\right]$$

$$= \sqrt{\frac{1}{1 + 2i\ddot{\beta}_0 \frac{L}{\tau^2}}} \exp\left[\frac{-(t - \dot{\beta}_0 L)^2}{\tau^2 + 2i\ddot{\beta}_0 L}\right] \tag{35.12}$$

Let $\left(t - \dot{\beta}_0 L\right) = T$

$$u(L,t) = \sqrt{\frac{1}{1 + 2i\ddot{\beta}_0 \frac{L}{\tau^2}}} \exp\left(\frac{-T^2}{\tau^2\left(1 + \frac{2i\ddot{\beta}_0 L}{\tau^2}\right)}\right) \tag{35.13}$$

$$= \sqrt{\frac{1}{1 + 2i\ddot{\beta}_0 \frac{L}{\tau^2}}} \exp\left[\frac{-T^2}{\tau^2 + \left(\frac{2i\ddot{\beta}_0 L}{\tau}\right)^2}\right] \exp\left(\frac{2i\ddot{\beta}_0 L T^2}{\tau^4 + \left(2\ddot{\beta}_0 L\right)^2}\right) \tag{35.14}$$

The last exponential factor in the equation above is the phase term

$$\tau'^2 = \tau^2 + \left[2\frac{\ddot{\beta}_0}{\tau} L\right]^2 \tag{35.15}$$

We can write

$$u(L,t) = \sqrt{\frac{1}{1 + 2i\ddot{\beta}_0 \frac{L}{\tau^2}}} \exp\left[-\frac{T^2}{\tau'^2}\right] \exp\left(\frac{2i\ddot{\beta}_0 L T^2}{\tau^4 + \left(2\ddot{\beta}_0 L\right)^2}\right) \tag{35.16}$$

Equation (35.16) shows that the Gaussian pulse remains a Gaussian as it propagates. However its width after propagating a distance *L* broadens to

$$\tau' = \tau\sqrt{1 + \left(2\ddot{\beta}_0 \frac{L}{\tau^2}\right)^2} \tag{35.17}$$

In optical communication systems, the signal pulses propagate over huge distances.

Consequently, the broadening can easily reach an extent of spoiling the signal definition itself as we will show in an example to follow.

In practice, we do not measure the electric field of light, rather we measure the intensity, *I* which is related to the electric field, E as

$$I \propto |E|^2$$

Given $E \propto e^{-t^2\tau^2}$ for the Gaussian pulse

$$I \propto e^{-2t^2\tau^2}$$

It is more appropriate, in practice, to measure the pulse duration in terms of FWHM of intensity profile rather than (1/e) width. The FWHM of input pulse t_i is related to its (1/e) width as

$$\tau_i(FWHM) = \tau\sqrt{2\ln 2} \tag{35.18}$$

FWHM width τ_0 of a Guassian pulse after propagating through distance *L* in the dispersive media will be

$$\tau_0 = \sqrt{2\ln 2}\tau \sqrt{1 + \left(\frac{2\ddot{\beta}_0 L}{\tau^2}\right)^2}$$

$$= \tau_i \sqrt{\left(1 + \left(\frac{4\ln 2\ddot{\beta}_0 L}{\tau_i^2}\right)^2\right)}$$

$$= \tau_i \left(\sqrt{1 + (2.77\ddot{\beta}_0 L \tau^2{}_i)^2}\right) \tag{35.19}$$

In terms of the dispersion parameter D

$$\tau_0 = \tau_i \sqrt{\left(1 + \left(\frac{2.77\lambda_0^2 DL}{2\pi c \tau_i^2}\right)\right)} \tag{35.20}$$

Quite often when dealing with pulse propagation in optical fibers, one expresses λ_0 in terms of μm; *DL* in ps/nm and τ_i in ps. In terms of these units, the input and output pulse widths are related as

$$\tau_0 = \tau_i \sqrt{\left(1 + \left(\frac{1.47\lambda_0^2 DL}{\tau_i^2}\right)\right)} \tag{35.21}$$

Example:

Let us consider τ_i = 10 ps, *DL* = 450 ps/nm and *L* = 30 km

The width of the output pulse will be

τ_0 = 150 ps = 15 $\times$ τ_i !!

which is a drastically broadened pulse.

The natural question then to be asked is:

For a given fiber length, what is optimum shortest input pulse width?

- If an extremely short pulse is chosen then it will result in large pulse broadening due to its huge spectral bandwidth.
- If a long input pulse width is chosen then at best $\tau_0 = \tau_i$ which will grossly limit the information carrying capacity of the fiber.

It can be shown that

$$(\tau_0)_{min} = \sqrt{2}\tau_{optimum} \tag{35.22}$$

or

$$\tau_{optimum} = \lambda_0 \sqrt{(1.47 DL)} \tag{35.23}$$

Thus for the example given above $(\tau_0)_{min}$ = 54 ps for $\tau_{optimum}$ = 39 ps.

Generally term "Dispersion" length, L_D is used to define fiber length corresponding pulse broadening by a factor of $\sqrt{2}$.

$$L_D = \tau'^2 / 2\ddot{\beta}_0 \tag{35.24}$$

$$= \frac{\tau'}{I \cdot 47 D\lambda_0^2} \tag{35.25}$$

An inspection of the equation will reveal the following key aspects of the pulse propagation through a dispersing media

- Broadening does not depend upon the sign of $\ddot{\beta}_0$.
- since there is no broadening for $\ddot{\beta}_0 = 0$, it is desirable that we operate near zero dispersion wavelength λ_D.

One can rewrite the electric field for the optical pulse after traversing a distance z through dispersive medium

$$E(z,t) = |u(z,t)|\exp\{\mathrm{i}\phi(z,t)\} \tag{35.26}$$

where the phase $\varphi\ (z,\ t)$ is given as

$$\phi(z,t) = \omega_0 t + \frac{2\ddot{\beta}_0 z T^2}{\tau^4 + \left(2\ddot{\beta}_0 z\right)^2} \tag{35.27}$$

The instantaneous frequency is given as

$$\begin{aligned} \omega(z,t) &= \frac{\partial\phi(z,t)}{\partial t} \\ &= \omega_0 + \frac{4\ddot{\beta}_0 z\left(t - \dot{\beta}_0 z\right)}{\tau^4 + \left(2\ddot{\beta}_0 z\right)^2} \\ &= \omega_0 + \frac{4\ddot{\beta}_0 z T}{\tau^4 + \left(2\ddot{\beta}_0 z\right)^2} \end{aligned} \tag{35.28}$$

The first term in the above equation, is the original frequency and the second term is the frequency that varies linearly in time. It is called the linear frequency chirp or sweep. This frequency sweep arises as the different frequency waves constituting the pulse undergo relative de-phasing due to their different speeds of propagation through the dispersive medium. It should be noted that the chirp depends upon the sign of the group velocity dispersion parameter $\ddot{\beta}_0$. Remember that

- In the normal dispersive regime $\ddot{\beta}_0 > 0$.
- In the anomalous dispersive regime $\ddot{\beta}_0 > 0$.

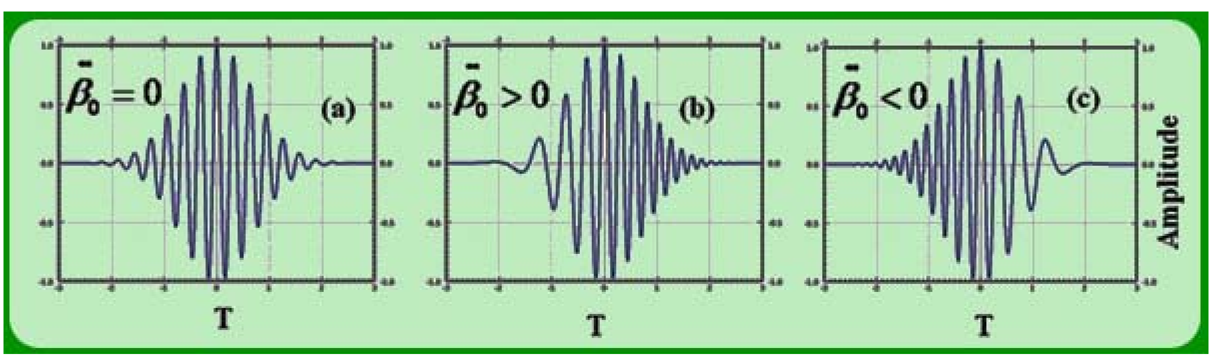

Figure 35.1

The chirp or the frequency sweep between the leading and trailing edges in these two regimes is shown in the figure (35.1) below.

so far we have considered the propagation of a transform limited Gaussian pulse.

A transform limited pulse is one without any chirp at $z=0$. Transform limited pulses are characterized by

$$(\Delta\omega)\tau \sim 1 \tag{35.29}$$

Where $\Delta\omega$ is the spectral width at 1/e-points. of intensity profile.

Practical laser sources usually do not produce the chirp free pulses. Envelop function of a chirped Gaussian pulse entering the dispersive medium is given by

$$u(0,t) = \exp\left[\frac{(1+\mathrm{i}\xi)}{\tau^2}t^2\right] \tag{35.30}$$

where ξ is the chirp parameter.

Unlike that in the case of the transform limited pulse in this case, the pulse broadening is governed by the sign of chirp parameter ξ and the group velocity dispersion parameter $\ddot{\beta}_0$.

Exercise I

Show that the output pulse width for a chirped Gaussian pulse after propagating a distance *L* in the dispersive medium is given by

$$\tau_0 = \tau\left[\left[1+\frac{2\xi\cdot\ddot{\beta}_0 L}{\tau^2}\right]^2+\left[\frac{2\ddot{\beta}_0 L}{\tau^2}\right]^2\right]^{1/2}$$

An important consequence of this is that it is possible to compress or ensure no broadening of pulse

From the equation given in the exercise above, the condition for pulse compression i. e. $\tau_0 < \tau$ can be written immediately as

$$\left(\frac{1+2\xi\ddot{\beta}_0 L}{\tau^2}\right)^2+\left[\frac{2\ddot{\beta}_0 L}{\tau^2}\right]^2 < 1 \qquad (35.31)$$

$$\ddot{\beta}_0 L < -\frac{\xi\tau^2}{(1+\xi^2)} \qquad (35.32)$$

This condition dictates that for pulse compression, we need to operate in anomalous dispersion regime

Grating pulse compressor:

Amplification of ultrashort laser pulses is a complex process. To avoid detrimental nonlinear optical effects and component damage at the intensity level that can easily reach during the course of amplification, chirped pulse amplification (CPA) is used. In this process, the ultrashort pulses from the oscillator are first stretched to several hundred picosecond prior to amplification and then recompressed to their near initial pulse width after the required level of amplification. In the following we will describe the optical systems for the realization of pulse compression and stretching.

Let us consider a positively chirped pulse incident on a pair of parallel gratings as shown in figure (35.2).

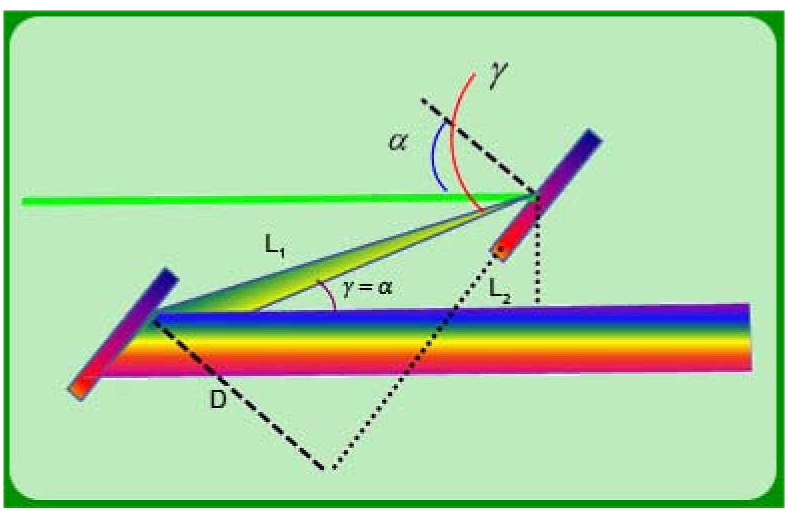

Figure 35.2

Upon diffraction at the first grating G_1, the blue front of the pulse spectrum is diffracted at smaller angle than its red counterpart which belongs to the leading edge *t* < 0. It can be seen in this figure that the blue part covers a shorter path so that the trailing edge catches up with the leading edge and the pulse is thus compressed. Thus the parallel grating pair produces a negative dispersion.

Different wave length components are diffracted by the grating according to the grating equation

$$\sin\gamma = \frac{2\pi c}{\omega d} - \sin\alpha \tag{35.33}$$

where α is the angle of incidence, γ is the diffraction angle and d is the grating period. After propagating distance

$$L(\omega) = L_I(\omega) + L_2(\omega) \tag{35.34}$$

, a frequency dependent group delay τ_g and a phase shift ϕ is introduced such taht.

$$\cos\left(\omega t + \phi\right) = \cos\left(\omega t - \beta(\omega) z\right) \tag{35.35}$$

Correspondingly, the time delay

$$\begin{aligned} \tau_g(\omega) &= \frac{L(\omega)}{v_g} = L\dot{\beta}_0 = \frac{L}{c} \\ &= -\frac{\mathrm{d}\phi}{\mathrm{d}\omega} \end{aligned} \tag{35.36}$$

One can readily see from the geometry of the arrangement.

$$\begin{aligned} L(\omega) &= L_1 + L_2 = L_1\left(1 + \frac{L_2}{L_1}\right) \\ &= D\,\frac{\left(1 + \cos(\gamma - \alpha)\right)}{\cos r} \\ &= \frac{D\left(1 + \cos\gamma\cos\alpha + \sin\gamma\sin\alpha\right)}{\cos\gamma} \\ &= D\left(\sec\gamma + \cos\alpha + \tan\gamma\sin\alpha\right) \end{aligned} \tag{35.37}$$

Therefore,

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}\omega^2} = -\frac{1}{c}\frac{\mathrm{d}L(\omega)}{\mathrm{d}\gamma}\frac{\mathrm{d}\gamma}{\mathrm{d}\omega} \tag{35.38}$$

$$\begin{aligned} &= -\frac{D}{c}\sec^2\gamma(\sin\gamma + \sin\alpha)\frac{\mathrm{d}\gamma}{\mathrm{d}\omega} \\ &= -\frac{D}{c}\sec^2\gamma\left(\frac{2\pi c}{\omega d} - \sin\alpha + \sin\alpha\right)\frac{\mathrm{d}\gamma}{\mathrm{d}\omega} \\ &= -\frac{2\pi D}{\omega d}\sec^2\gamma\frac{\mathrm{d}\gamma}{\mathrm{d}\omega} \\ &= \frac{4\pi^2 cD}{\omega^3 d^2}\cdot\left[1 - \left(\frac{2\pi c}{\omega d} - \sin\alpha\right)^2\right]^{-3/2} > 0 \end{aligned} \tag{35.39}$$

Here, we have made use of equations (35.33) to write

$$\sec^2\gamma = \left[1 - \left(\frac{2\pi c}{\omega d} - \sin\alpha\right)^2\right]^{-1}$$

$$\frac{\mathrm{d}\gamma}{\mathrm{d}\omega} = -\frac{2\pi c}{\omega^2 d}\left[1 - \left(\frac{2\pi c}{\omega d} - \sin\alpha\right)^2\right]^{-1/2}$$

Equation (35.6)

$\Rightarrow \ddot{\beta}_0 < 0$ and hence a pulse which is positively chirped due to propagation in a normally dispersive medium can be compressed again by the negative dispersion in a grating pair compressor described above.

It can be seen easily seen that the anti-parallel grating pair will produce positive dispersion and hence, it will result in the pulse streaming which again is useful device for ultrashort pulse amplification. Instead of a grating pan prism pair can also be used for this application. However, due to lower dispersion in prism, compression ratio is smaller.

Recap:

In this lecture we have learned about the following

- Propagation of a transform limited pulse in linear but dispersive medium.
- Propagation of a chirped pulse in linear but dispersive medium and prospects of pulse compressor.
- Grating pulse compressor.