

Module 4 : Third order nonlinear optical processes

Lecture 21 : Self focusing Refraction and absorption

Objectives

In this lecture

we will explore an interesting phenomenon which is consequence of nonlinear refraction. We will study the following:

1. The transverse effects on the intense laser beam propagation in the nonlinear propagation.
2. A simple physical picture to explain self focusing and self trapping phenomena.
3. Theoretical formulation of these phenomenon.

Self focusing

An intense laser beam with in homogeneous transverse intensity profile, propagating through a plane parallel slab of a nonlinear medium can exhibit focusing by itself. This phenomenon of self focusing was first observed by Kelly in 1965(1).

Towards the simple physical picture of this phenomenon, let us consider the passage of a cylindrical laser beam with Gaussian intensity profile through a nonlinear medium with intensity dependent refractive index

$$n(I) = n_0 + n_2 I \quad (21.1)$$

where n_0 is the linear refractive index, I is the intensity and n_2 is the coefficient of nonlinear index of refraction which we consider here to be positive. The refractive index profile in the medium will bear an imprint of the Gaussian beam intensity profile i.e. the refractive index in the medium decreases radially outward, the maximum of it being on the axis.

This retards the inner part of the beam more than the peripheral part like a positive lens and causes the beam to focus. As the beam shrinks, the increased intensity brings in even larger refractive index differential across the transverse profile of the beam, and sets up a positive feedback to accelerate the beam focusing. However, with the reduction in the beam size, increased divergence due to the diffraction counteracts the self focusing effect. If the two effects balance each other, the laser beam will travel with its beam profile unaltered just like in a waveguide. This limiting case is called self-trapping. Above the critical power required for self-trapping the self-focusing effect dominates over diffraction leading to the collapse of the beam and can lead to the optical damage of the medium

We will consider the transverse effects on the propagation of an intense monochromatic laser beam characterized by its linearly polarized electric field

$$\tilde{E} = E(r_T, z, t) e^{i(kz - \omega t)} \quad (21.2)$$

propagating along the z-axis through a third order nonlinear isotropic medium with its refractive index given by.

$$n^2 = n_0^2 + 4n_0 n_2 |E|^2 \quad (21.3)$$

r_T in equation(21.2) refers to the transverse coordinates. Assuming that the field depends weakly on the transverse coordinates, its propagation will be described by

$$\nabla^2 \tilde{E} - \frac{n^2}{c^2} \partial_t^2 \tilde{E} = 0 \quad (21.4)$$

OR

$$\left[\left(\partial_x^2 + 2ik - k^2 \right) E + \nabla_T^2 E - \frac{1}{c^2} \left(n_0^2 + 4n_0 n_2 |E|^2 \right) \left(\partial_t^2 - 2i\omega \partial_t - \omega^2 \right) E \right] e^{i(kz - \omega t)} = 0 \quad (21.5)$$

Where $\nabla_r^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\partial_z = \frac{\partial}{\partial z}$ and $\partial_t = \frac{\partial}{\partial t}$.

Expanding it further using $k = \frac{\omega n_0}{c}$, we get

$$\left[\left(\partial_z^2 + 2ik - k^2 \right) E + \nabla_r^2 E - \frac{1}{c^2} \left(n_0^2 + 4n_0 n_2 |E|^2 \right) \left(\partial_t^2 - 2i\omega \partial_t - \omega^2 \right) E \right] e^{i(kz - \omega t)} = 0 \quad (21.6)$$

using $k = \frac{\omega n_0}{c}$, we get

$$\left[\nabla_r^2 E + 2ik \left(\partial_z + \frac{n_0}{c} \partial_t \right) E + 4n_0 n_2 |E|^2 E \right] = 0 \quad (21.7)$$

Transforming the coordinates to

$$z' = z$$

and

$$t' = t - \frac{n_0}{c} z$$

it is easy to show that

$$\left(\partial_z + \frac{n_0}{c} \partial_t \right) E(z, r_T, t) = \partial_{z'} E(z', r_T, t') \quad (21.8)$$

The transformed wave equation can then be written as

$$2ik \partial_{z'} E(z', r_T, t') = \left(\nabla_r^2 + 4n_0 n_2 |E|^2 \right) E(z', r_T, t') \quad (21.9)$$

One can easily identify this equation with the nonlinear Schrödinger equation where the role of time t is played by z . If the field E depends on both the transverse coordinates x and y , no closed form solutions are possible.

However, if the field depends on one transverse variable, say x , only, the solution can be readily written as

$$E(x, z') = E_0 \text{sech} \left(\frac{x}{x_0} \right) e^{ikz'} \quad (21.10)$$

where $k = 2n_0 n_2 |E_0|^2$ and $x_0 = \left(\frac{n_0}{2n_2 k^2 |E_0|^2} \right)^{\frac{1}{2}}$ represents the width of the field's transverse profile.

Solution in equation (21.10) describes a field whose transverse profile remains invariant upon propagation. Such a field is called a spatial soliton.

A useful approach to solve the problem of wave propagation is based upon the eikonal equation. It is the nonlinear partial differential equation obtained by approximating the wave equation using the WKB theory and describes the evolution of the ray trajectory or the wavefront. To get the eikonal equation, we use the ansatz

$$E(\vec{r}) = A_0(\vec{r}) e^{-iS(\vec{r})} \quad (21.11)$$

where S is the eikonal or action. Both A_0 and S are real quantities.

Separating it in to the real and imaginary parts gives us two equations

$$\partial_x^2 A_0 + \nabla_T (A_0^2 \nabla_T S) = 0 \quad (21.12)$$

and

$$2\partial_x S + (\nabla_T S)^2 = \frac{\nabla_T^2 A_0}{k^2 A_0} + 2 \frac{\bar{n}_2 |E|^2}{n_0} \quad (21.13)$$

Equation (21.12) corresponds to an equation of continuity which describes the conservation of energy. S represents the wave front and we can trace the ray by following its gradient.

Equation (21.13) describes the influences of the diffraction and self focusing (the first and second terms on the r.h.s., respectively) on the distortion of the warfront.

If at some point $z=z_0$, these two effects balance each other and the wave front is planar i.e. $\nabla_T S = 0$ then

$$\partial_x S = 0 \text{ and } \partial_x A_0 = 0$$

The wave propagates in the medium with plane wave front and constant transverse profile or in other words as spatial soliton. A small variation in the laser power by way of absorption, scattering or fluctuation from its critical value P_{cr} can result in the imbalance of the two effects and can destroy the soliton causing the beam to either self focus or diverge. Analytical solutions for the self focusing can be obtained for the Kerr nonlinearity⁽²⁾.

We will rather skip the rigorous mathematical apparatus and derive the approximate expressions for the critical power, a P_{cr} and self focusing distance based on the simple paraxial optics approximation.

Let us consider a Gaussian laser beam of radius w_0 incident on the medium where refractive index is described by equation (21.1). Ignoring the effect of diffraction, intensity dependent refractive index causes the beam to focus at a distance z_{sf} from the input face as shown in the figure 21.1

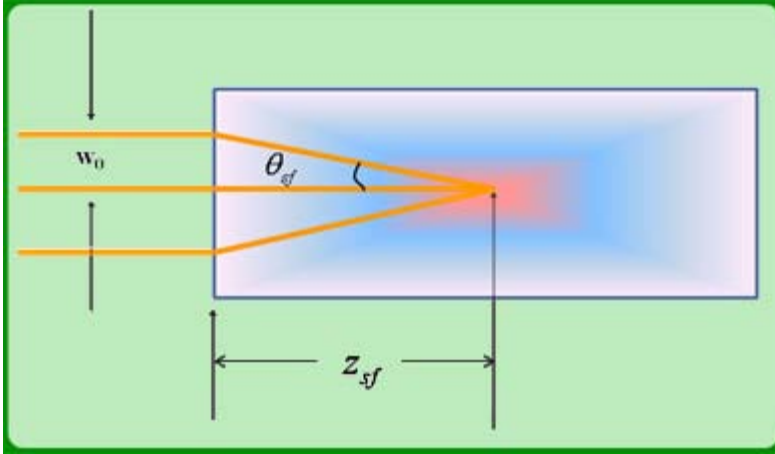


Figure 21.1

We approximate the ray trajectories in the medium to be straight lines. The ray traveling along the beam axis sees the refractive index given by equation (21.1) whereas the peripheral rays where the intensity of the Gaussian beam ~ 0 , the refractive index is n_0

According to Fermat's principle the optical path for all the rays from a wave front at the input face to the focus point is same.

$$(n_0 + n_2 I) z_{sf} = \frac{n_0 z_{sf}}{\cos \theta_{sf}} \quad (21.14)$$

where θ_{sf} is the half angle that the focal point subtends at the beam aperture at the input face.

For small angle θ_{sf}

$$\cos \theta_f = 1 - \frac{\theta_f^2}{2} \quad (21.15)$$

from equations (21.14) and (21.15)

$$\theta_f = \sqrt{\frac{2n_2 I}{n_0}} \quad (21.16)$$

and

$$z_f = \frac{w_0}{\theta_f} = w_0 \left(\sqrt{\frac{n_0}{2n_2 I}} \right) \quad (21.17)$$

However, as the beam size shrinks in the medium, the diffraction effect starts manifesting more and more strongly and the self focusing angle is reduced to

$$\theta = \left(\theta_f^2 - \theta_{diff}^2 \right)^{1/2} \quad (21.18)$$

where the diffraction angle

$$\theta_{diff} = 0.61 \frac{\lambda}{n_0 d} \quad (21.19)$$

In equation(21.8), d is beam diameter. Self trapping occurs when the two effects just balance each other i.e.

$$\theta_{diff} = \theta_f \quad (21.20)$$

This happens at the intensity

$$I_{cr} = \frac{(0.61)^2 \lambda^2}{2n_0 n_2 d^2} = \frac{4P_{cr}}{\pi d^2} \quad (21.21)$$

where P_{cr} is the critical beam power for self trapping

Hence

$$P_{cr} = \frac{0.61^2 \pi \lambda^2}{8n_0 n_2} \approx \frac{\lambda^2}{8n_0 n_2} \quad (21.22)$$

Self focusing will take place when. In this case,

$$\begin{aligned} z_f &= \frac{w_0}{\theta} \\ &= \frac{2n_0 w_0^2}{\lambda} \frac{1}{\sqrt{\frac{P}{P_{cr}} - 1}} \end{aligned} \quad (21.23)$$

Notice that unlike many other nonlinear optical phenomena, self focusing depends on the power of the beam and not at its intensity. If the beam power $P \gg P_{cr}$ a multimode beam breaks up in to multiple filaments. Formation of trapped filament can also take place. For a medium with negative value of n_2 , self defocusing occurs. In this case, the wave front distortion is opposite to that for self focusing case. Many other interesting effects associated with pulse propagation such as self steepening of wavefront etc. occur, which are outside the scope of this lecture.

References:

1. P.L. Kelly Phys. Rev. Lett. **15**,1005(1965).
2. Y. R. Shen, "Principles of Nonlinear Optics"(John Willey & Sons, New York 1984).

Recap

In this lecture

1. We have explained the phenomena of self focusing and self trapping of intense laser beam propagating through optical Kerr medium in terms of a simple physical picture.
2. The evolution of the transverse spatial profile has been formulated using wave equation. It is then reduced to nonlinear Schrodinger equation and the corresponding spatial soliton solution in 1-d transverse case is identified.
3. To study the influence of the two competing effects of diffraction and self focusing on the distortion of the wave front an eikonal equation has been derived and discussed.
4. Critical power for self trapping and the focusing distance have been derived.