Lecture – 5

TITLE: Electromagnetic Radiation – Matter Interactions

Page-1

Objectives

⇨ In the previous lecture, we have learnt that the radiation or light is following the Wave-Particle dual nature

⇨ It can be treated as electromagnetic wave or particle nature like photon

⇨ Similarly, the electrons can also be treated as particle and also as wave corresponding to its momentum for describing the stable structure of atom.

⇨ When both of them interact with each other, we have to understand the mechanism to follow for describing the experimental observations such as Compton effect, absorption and emission of light by atoms.

⇨ In this lecture, the different kinds of treatment to understand the light-matter interactions are described. We will start with the classical phenomenon and then proceed to understand the quantum mechanical descriptions.

Classical treatment:

⇨ In classical theory as shown in Figure – 5.1, we model the atom as a heavy nucleus with electron attached to it with spring → the binding force between them.

⇨ The resonant frequency for this system is ω_0

⇨ If we treat the light as wave then the electric field can be represented as

$$\vec{E} = \vec{E}_0 Sin\omega t$$

Equation – 5.1

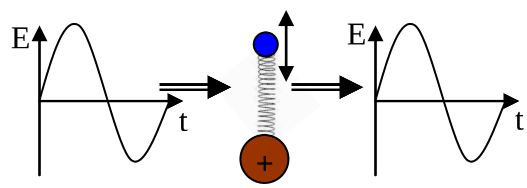


Figure – 5.1

⇨ The oscillating electric field will force the electron to oscillate. The displacement of electron with respect to nucleus will produce an oscillating dipole.

⇨ It is known that an oscillating dipole emit electromagnetic radiation with the same frequency of vibration. This emission of light is known as scattering of light by matter.

⇨ However, if the incident radiation frequency matches with the resonant frequency of the system (ω_0) then the resonance occurs. Energy transfer takes place.

⇨ The oscillating dipole $p(t) = -e\,x(t)$

Equation – 5.2

Where e is the charge of the electron and *x(t)* is the time dependent displacement of electron with respect to nucleus.

When this dipole oscillates, it emits radiation. As time passes by, due to the radiation loss the emission dies or decays. This phenomenon can be modeled as a damping oscillator whose solution will be *x(t)*.

⇨ The differential equation of motion of the damping oscillator with damping constant γ

$$\ddot{x}(t) + \gamma\dot{x}(t) + \omega_0^2 x(t) = 0$$

Equation – 5.3

Where $\omega_0^2 = k/m$, here k is the force constant of the spring.

With the initial values i.e. at $t = 0$
the displacement $x(0) = x_0$ and the velocity $\dot{x}(0) = 0$, the solution is

$$x(t) = x_0 e^{-(\gamma/2\omega)t}[Cos\omega t + (\gamma/2\omega)Sin\omega t]$$

Equation – 5.4

The frequency $\omega = (\omega_0^2 - \frac{\gamma^2}{4})^{1/2}$ of the damped oscillation is slightly lower than the frequency ω_0 of the undamped case.

Taking $\gamma \ll \omega_0$, we get $x(t) = x_0 e^{-(\gamma/2\omega)t} Cos\omega_0 t$.

Equation – 5.5

Line profile of the emitted radiation

The damped oscillation $x(t)$ can be described as a superposition of monochromatic oscillations with slightly different frequencies ω and amplitude $A(\omega)$

$$x(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} A(\omega) e^{i\omega t} d\omega$$

Equation – 5.6

The amplitude $A(\omega)$ can be calculated by $A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(t) e^{-i\omega t} dt$

Equation – 5.7

The intensity

$$I(\omega - \omega_0) = A(\omega)A^*(\omega) = I_0 \frac{1}{(\omega - \omega_0)^2 + (\gamma/2)^2}$$

Equation – 5.8

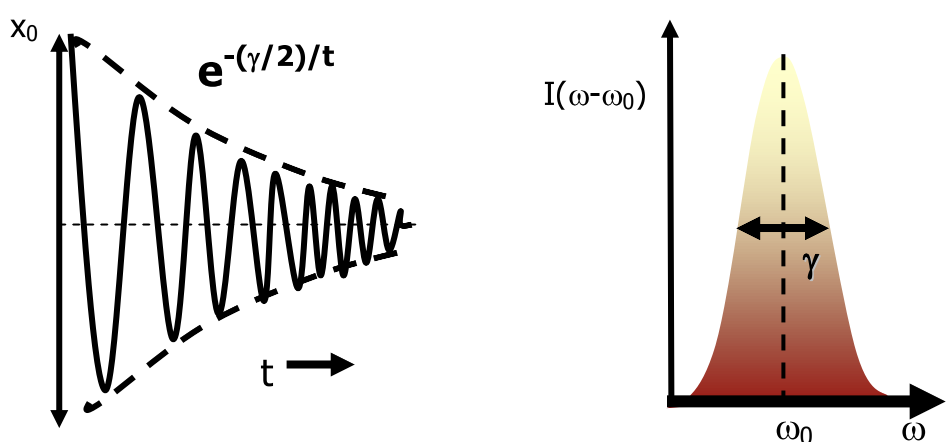


Figure – 5.2

Momentum of a Photon

Momentum vector magnitude $= |p| = \dfrac{E}{c}$

E is the energy.

$|p| = \dfrac{E}{c} = \dfrac{h\nu}{c} = h\lambda$ parallel to the direction of propagation.

From relativity, we know that

$$E = mc^2$$

$$p = mV$$

$$m = \frac{m_0}{\sqrt{1 - V^2/c^2}}$$

$$p = mV = \frac{E}{c^2}V$$

Equation – 5.9

Minkowski's four dimensional space & time (E & three comp. of p)

If the system is confined to a single particle so, $E^2 - p^2c^2$ is invariant under changes of ref. frame.

$$E^2 - p^2c^2$$

$$= E^2 - \frac{E^2}{c^4}V^2c^2 = E^2\left(1 - \frac{V^2}{c^2}\right)$$

$$= m^2c^4\left(1 - V^2/c^2\right)$$

$$= \frac{m_0^2 c^4}{\left(1 - V^2/c^2\right)}\left(1 - V^2/c^2\right) = m_0^2 c^4$$

Equation – 5.10

Photon is a quantity of energy. If we consider photon as a particle, then the energy $E = mc^2$, with this infinite energy E.

So the only way of reconciling a speed $V = c$ with a finite energy is to assume rest mass $m_0 = 0$.

So, $E^2 - p^2c^2 = 0$; $p = E/c$

Rest energy $= m_0 c^2 = 0$ for photon

So, $K.E. = E - m_0 c^2 = E$ so for photon all energy is K.E.

Elastic Collisions of Photons – Compton Effect

The X-ray of wavelength λ_0 was incident on a target. With a crystal diffractometer, the wavelengths of the scattered X-ray were measured by changing the angle θ as shown in Figure – 5.3.

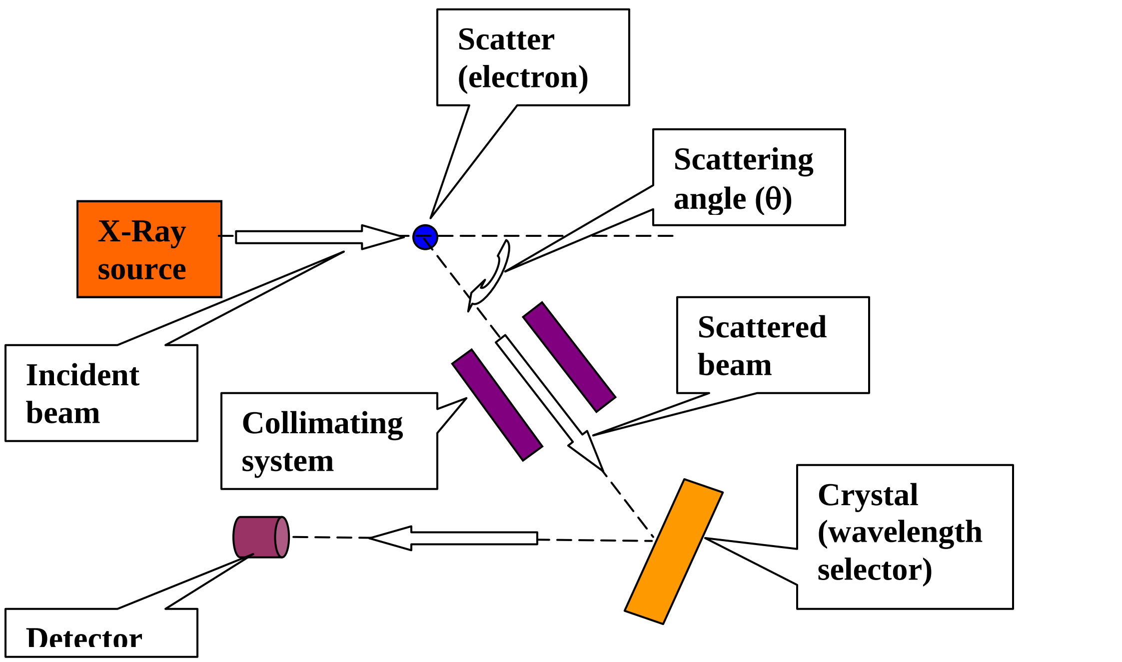
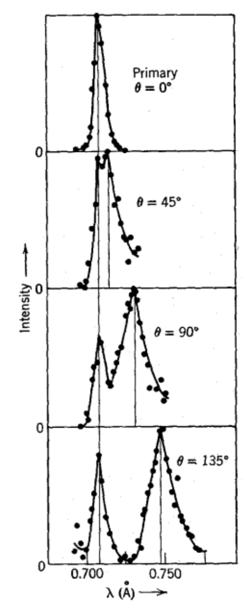


Figure – 5.3

Scattered radiation is composed by two lines:

(i) a component at the incident wavelength λ_0, called the Thomson component. Scattered radiation has the same frequency as the incident radiation.

(ii) a component of different wavelength $\lambda = \lambda_0 + \partial\lambda$.



<u>Conclusion from experiment</u>

(i) $\delta\lambda = \lambda - \lambda_0$ is always positive.

(ii) $\delta\lambda$ is an increasing function of θ.

(iii) $\delta\lambda$ is independent of and the composition of the material used for scattering.

Explanation

Elastic scattering between photons and free electrons (weakly bound to atoms can be considered as free electrons) i.e. binding energy $<< h\nu_0$.

Elastic Scattering $\rightarrow$ K.E. is conserved

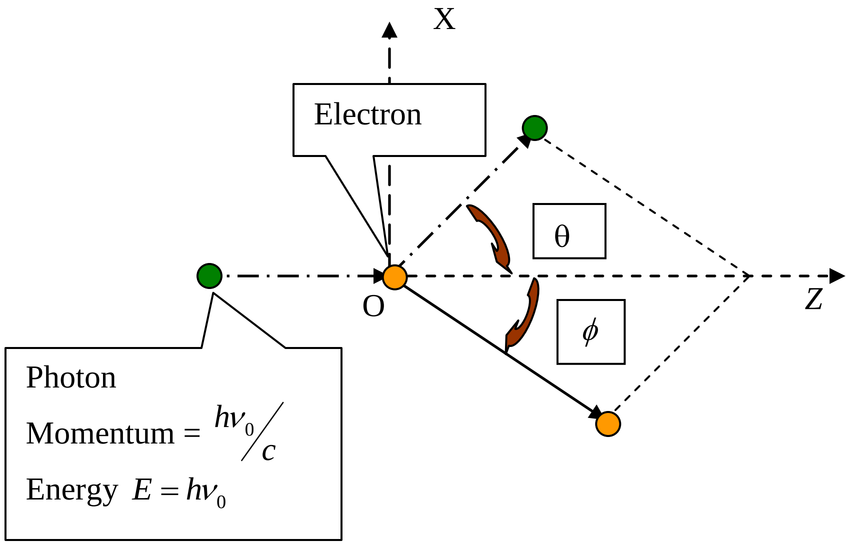


Figure – 5.4

We have taken into account the fact that p & E are related by relativistic invariance $E^2 - p^2c^2 = m_0^2c^4$.

		Before Collision	After Collision
Photon		Energy $h\nu_0 = h\, c/\lambda_0$	$h\nu = h\, c/\lambda$
		Momentum $h\, \nu_0/c = h/\lambda_0$ along Oz	$h\nu/c = h/\lambda$
Electron		Energy $= m_0c^2$	$\omega = mc^2$ $= \sqrt{\left(p^2c^2 + m_0^2c^4\right)}$
		Momentum = Zero	p making an angle ϕ with OZ

<u>Conservation of energy and momentum</u>

Energy $h\nu_0 + m_0 c^2 = h\nu + \sqrt{\left(p^2c^2 + m_0^2 c^4\right)}$

Equation – 5.11

Momentum

along OZ $$\frac{h\nu_0}{c} = \frac{h\nu}{c}\cos\theta + p\cos\phi$$

along OX $$0 = \frac{h\nu}{c}\sin\theta + p\sin\phi$$

$$p\cos\phi = \frac{h\nu_0}{c} - \frac{h\nu}{c}\cos\theta$$
$$= \frac{h}{c}\left(\nu_0 - \nu\cos\theta\right)$$
$$p\sin\phi = -\frac{h}{c}\nu\sin\theta$$

Equation – 5.12

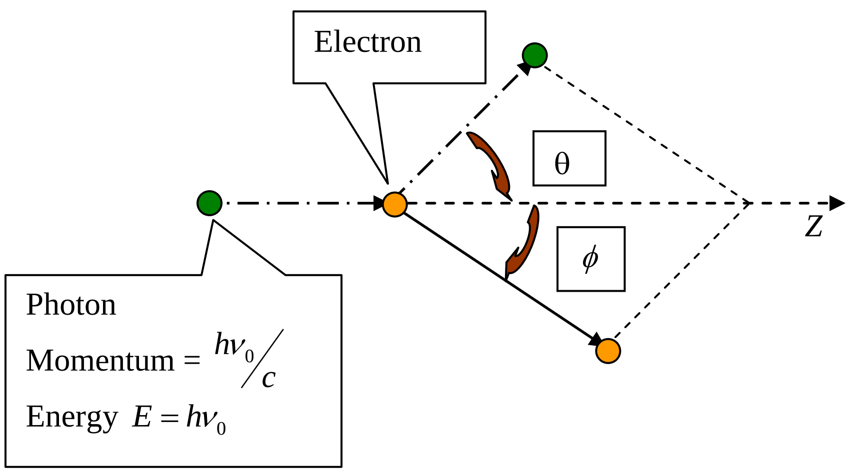


Figure – 5.5

$$p^2 = \frac{h^2}{c^2}(\nu_0 - \nu\,\cos\theta)^2 + \frac{h^2}{c^2}\nu^2\cos^2\theta$$

$$= \frac{h^2}{c^2}\left(\nu_0^2 + \nu^2\cos^2\theta - 2\nu\nu_0\cos\theta + \nu^2\sin^2\theta\right)$$

$$= \frac{h^2}{c^2}\left(\nu_0^2 + \nu^2 - 2\nu\nu_0\cos\theta\right)$$

$$\Rightarrow p^2c^2 = h^2\left(\nu_0^2 + \nu^2 - 2\nu\nu_0\cos\theta\right)$$

Equation – 5.13

From 1st equation $p^2c^2 = \left[h(\nu_0 - \nu) + m_0c^2\right]^2 - m_0^2c^4$

Equation – 5.14

Equating Equation – 5.13 & Equation – 5.14,

$$h^2\left(\nu_0^2 + \nu^2 - 2\nu\nu_0\cos\theta\right) = \left[h(\nu_0 - \nu) + m_0c^2\right]^2 - m_0^2c^4$$

$$= h^2(\nu_0 - \nu)^2 + \cancel{m_0^2c^4} + 2h(\nu_0 - \nu)m_0c^2 - \cancel{m_0^2c^4}$$

$$\Rightarrow \cancel{h^2\nu_0^2} + \cancel{h^2\nu^2} - 2h^2\nu\nu_0\cos\theta = \cancel{h^2\nu_0^2} + \cancel{h^2\nu^2} - 2h^2\nu\nu_0 + 2h(\nu_0 - \nu)m_0c^2$$

$$\Rightarrow 2h(\nu_0 - \nu)m_0c^2 = 2h^2(1 - \cos\theta)\nu\nu_0$$

$$\Rightarrow \nu_0 - \nu = \frac{\cancel{2}h\cancel{^2}}{\cancel{2h}\,m_0c^2}(1 - \cos\theta)\nu\nu_0$$

$$\Rightarrow \frac{\nu_0}{\nu\nu_0} - \frac{\nu}{\nu\nu_0} = \frac{h}{m_0c^2}(1 - \cos\theta) \qquad \text{where } m_0 = 9.1\times10^{-31}Kg$$

$$\Rightarrow \frac{1}{\nu} - \frac{1}{\nu_0} = \frac{h}{m_0c^2}(1 - \cos\theta)$$

$$\Rightarrow \lambda - \lambda_0 = \frac{h}{m_0c}(1 - \cos\theta)$$

$$\Rightarrow \delta\lambda = \frac{h}{m_0c}(1 - \cos\theta) \qquad \begin{cases} \theta = 0 \;; & \partial\lambda = 0 \\ \theta = \pi \;; & \partial\lambda = \dfrac{2h}{m_0c} \\ \theta = 90^o; & \partial\lambda = \dfrac{h}{m_0c} = \Lambda = Compton\,wavelength \end{cases}$$

Page-9

So, we get

$$\lambda - \lambda_0 = \partial\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

Equation – 5.15

(i) $\partial\lambda$ is positive.

(ii) independent of λ_0.

(iii) depends on θ only.

Hard X-ray $\lambda = 1\, pm$

$h\nu = 1 MeV$

Energy of Compton Wavelength,

$$\frac{hc}{\Lambda} = \frac{hc}{\frac{h}{m_0 c}} = m_0 c^2 \quad \Rightarrow \text{rest mass of the electron}$$

$h\nu_0 << m_0 c^2$ $\lambda_0 >> \Lambda$ $\partial\lambda << \lambda_0$ photons gives up very little energy.

$h\nu_0 > m_0 c^2$ $\lambda_0 < \Lambda$ $\partial\lambda > \lambda_0$ photons gives up most of its energy.

Equation – 5.16

Absorption of Photons (Inelastic Collision)

Let us define $h\nu_{12}$ is the energy difference between the two states of an atom with energy E_2 and E_1 ($E_2 > E_1$ ground state) $E_2 - E_1 = h\nu_{12}$.

If there is a collision between an atom and a photon as shown in Figure – 5.6 and after the collision they form one particle, then the initial momentum of the incident must be preserved by the final single particle (momentum conservation). Or we can say that after collision the atom must possess some kinetic energy that it did not have previously.

This kinetic energy can have been taken from the energy of the incident photon.

This is possible only when $h\nu > h\nu_{12}$. We have to determine the relationship between ν and ν_{12}.

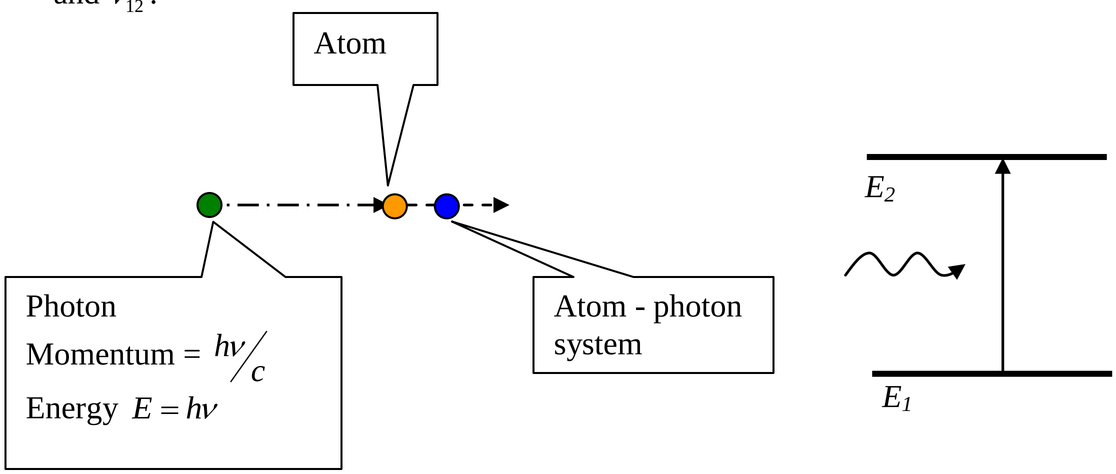


Figure – 5.6

Considering the properties of the relativistic particles:

		Before Collision	After Collision
Photon	Energy	$h\nu$	
	Momentum	$h\nu/c$	
Atom	Energy	$E_1 = m_1 e^2$ $m_1 \rightarrow$ rest mass	$W = \sqrt{\left(P^2c^2 + m_2^2 e^4\right)}$
	Momentum	Zero	P where, $m_2 = \dfrac{E_2}{c^2} = \dfrac{E_1 + h\nu_{12}}{c^2}$

From the conservation of energy

$$hv + E_1 = \sqrt{\left(P^2c^2 + m_2^2e^4\right)}$$

Equation – 5.17

$$Momentum\ \ p = {hv}/{c}\ \text{same as photon}$$

Equation – 5.18

$$E_1 + hv_{12} = m_2c^2$$

From Equation – 5.17,

$$\left(E_1 + hv\right)^2 = p^2c^2 + m_2^2c^4$$
$$\Rightarrow E_1^2 + 2E_1hv + h^2v^2 = p^2c^2 + \left(E_1 + hv_{12}\right)^2$$
$$= \frac{h^2v^2}{c^2}c^2 + E_1^2 + 2E_1hv_{12} + h^2v_{12}^2$$
$$\Rightarrow v = \frac{2hv_{12}E_1}{2E_1h}\left[1 + \frac{h^2v_{12}^2}{2E_1hv_{12}}\right] = v_{12}\left[1 + \frac{hv_{12}}{2E_1}\right]$$
$$= v_{12}\left[1 + \frac{hv_{12}}{2m_1c^2}\right]$$

Equation – 5.19

So the incident photon frequency must be greater than the theoretical frequency of the spectral lines $\left(\dfrac{E_2 - E_1}{h} = \nu_{12}\right)$.

$$\left.\begin{array}{ll} h\nu_{12} & 1 \; to \; 10\, eV \\ m_1 c^2 & 10 \; to \; 100\, GeV \end{array}\right\} \qquad 10^{-10}$$

<u>Comment:</u>

(i) In the optical transitions $h\nu_{12} << m_1 c^2$ typically, $\dfrac{h\nu_{12}}{m_1 c^2} \sim 10^{-10}$ which means the shift will be very small compared to other broadening $\left(10^{-6}\right)$ cannot be observed. Resonant condition is applicable i.e. $E_2 - E_1 = h\nu$.

(ii) When $h\nu_{12}$ is not very small compared to $m_1 c^2$, the shift is appreciable.

Emission of Photons:

		Before Collision	After Collision
Photon	Energy	~	$h\nu$
	Momentum	~	$h\nu/c$ along Oz
Atom	Energy	E_2	$W=\sqrt{\left(P^2c^2+m_1^2c^4\right)}$
	Momentum	0 $m_2 = E_2/c^2$	p along Oz $m_1 = E_1/c^2 = \frac{E_2+h\nu_{12}}{c^2}$

$$p+\frac{h\nu}{c}=0 \qquad \Rightarrow p^2c^2=h^2\nu^2$$

$$E_2 = W + h\nu$$

$$\Rightarrow E_2 - h\nu = W$$

$$\Rightarrow E_2^2 + h^2\nu^2 - 2h\nu E_2 = p^2c^2 + m_1^2c^4$$

$$= h^2\nu^2 + m_1^2c^4$$

$$\Rightarrow E_2^2 - 2h\nu E_2 = m_1^2c^4 = \left(E_2 - h\nu_{12}\right)^2$$

$$\Rightarrow E_2^2 - 2h\nu E_2 = E_2^2 + h^2\nu_{12}^2 - 2E_2h\nu_{12}$$

$$\Rightarrow 2h\nu E_2 = 2E_2h\nu_{12} - h^2\nu_{12}^2$$

$$\Rightarrow \nu = \frac{2E_2h\nu_{12}}{2hE_2} - \frac{h^2\nu_{12}^2}{2E_2h} = \nu_{12} - \frac{h\nu_{12}^2}{2E_2}$$

$$\Rightarrow \nu = \nu_{12}\left[1 - \frac{h\nu_{12}}{2m_2c^2}\right]$$

Equation – 5.20

Application to $\gamma-\text{rays}$

$\gamma-\text{rays} \rightarrow \text{Very High Energy}$

$$10^{19} \text{ to } 10^{22} Hz$$

Nuclear transitions also very high frequency

$h\nu_{12} < m_1 c^2$

Emission of $\gamma-\text{ray}$ photon is interpreted as the radiative transition between two energy states of the nucleus (analogous to optical transition).

$\textit{Frequency Displacement} > \textit{Line Width}$

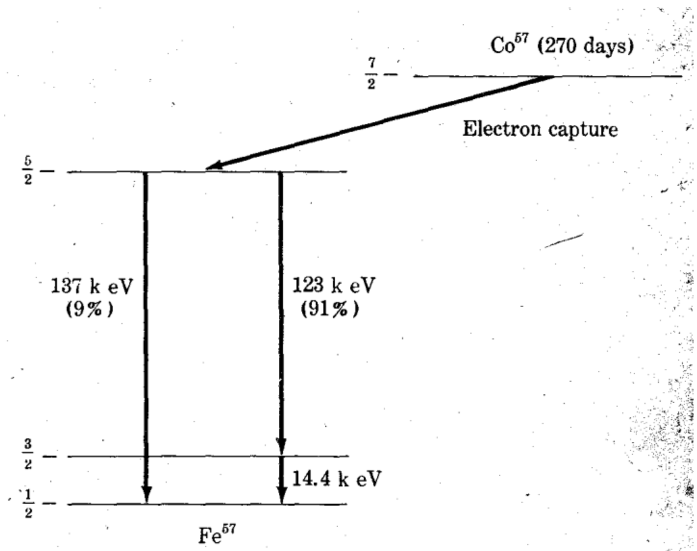


Figure – 5.7

Resonance absorption and emission is not possible as in the case of optical transition. By increasing temperature it is possible. So to get the information about the nucleus from this is difficult.

What Mössbaur discovered that by cooling the source and the absorber the intensity absorption increased.

It means that, below a certain temperature emitted or absorbing nucleus is embedded in a crystalline lattice and it is the white crystal which recoils.

$\textit{Mass of Crystal} >> \textit{Mass of Nucleus} \sim 6\times10^{23}$

The velocity of recoil is negligible.

No recoil energy loss $\rightarrow$ extremely sharp lines (natural width) could be obtained.

$Debye-Waller\ Factor$

$$f = \exp\left[-\frac{E_0}{K\Theta}\left(\frac{3}{2}+\frac{\pi^2 T^2}{\Theta}\right)\right]$$

where, $T = Abs.\ Temp.$

$K = Boltzmann's\ Const.$

$\Theta = Degree\ Temp.\ of\ \ the\ Solid$

Equation – 5.21

There are three main hyperfine interactions that can be observed by Mössbauer spectroscopy. They are (i) Isomer Shift, (ii) Quadrupole Splitting and (iii) Nuclear Zeeman Splitting.

For example :

The isomer shift of $Fe^{57}\left(3/2 \rightarrow 1/2 = 14.4\,KeV\right)$ in Ferricinium bromide is $+2\times10^{-8}\,eV$.

$$h(\nu-\nu_0) = h\partial\nu = 2\times10^{-8}\,eV$$

$$h\nu_0 = 14.4\times10^{3}\,eV$$

$$\frac{\nu}{\nu_0} = \frac{c}{c-v}$$

$$\Rightarrow \frac{\delta\nu}{\nu_0} = \frac{v}{c-v} \approx \frac{v}{c}$$

$$\Rightarrow v = \frac{\partial\nu}{\nu_0}\times c = \frac{2\times10^{-8}}{14.4\times10^{3}}\times3\times10^{10}$$

$$= \frac{6\times10^{2}}{14.4\times10^{3}} = 0.04\,cm/s$$

Mössbaur Spectrum of Ferricinium bromide

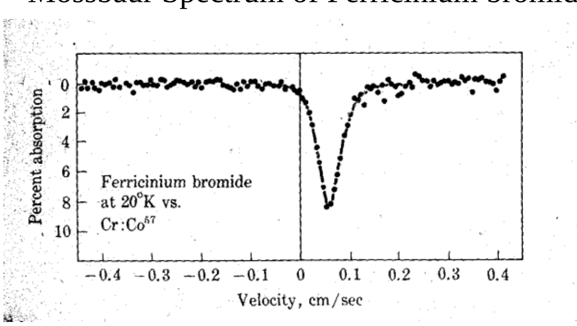


Figure – 5.8

Recap

In this lecture, we came to know that the classical physics has limitation to describe the several experimental observations such as Compton effect and absorption and emission of light by atoms.

We understood the different kinds of treatment for describing the light-matter interactions: both the classical phenomenon and then the quantum mechanical descriptions.
We also come to know the recoil energy required when both light and matter interact.
Based on this, we came to know the application of this recoil energy in Mossbaur spectroscopy.