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Module 2

Lect 7: Damage identification using lumped mass.

Change in Eigenvalues – can enable damage detection

Let us consider dynamic eqⁿ of forces acting @ the j^{th} storeys.

By considering all the storeys above j^{th} storey, we get:

$$k_{i(j)}\, d_{i(j)}(t) + C_{i(j)}\, \dot{d}_{i(j)}(t) = f_{i(j)}(t) \quad \text{— (1)}$$

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where $k_{i(j)}$ and $C_{i(j)}$ are stiffness and damping parameters of the j^{th} storey

$f_{i(j)}(t)$ is the inertia force acting on the storey mass of j^{th} storey and all the storey above it.

$$f_{i(j)}(t) = -\sum_{k=j}^{N} m_k \ddot{x}_{i(k)}(t) \quad \text{—— (2)}$$

$$= -\omega_i^2 e^{\omega_i t} \sum_{k=j}^{N} m_k \phi_{i(k)} \quad \text{—— (3)}$$

where N - # of storey

m_k - mass of the k^{th} storey

$d_{i(j)}(t)$ — is relative displacement b/w j^{th} & $(j-1)^{th}$ storey

For i^{th} mode of vibration, this can be expressed as

$$d_{i(j)}(t) = x_{i(j)}(t) - x_{i(j-1)}(t)$$

$$= \left\{ \phi_{i(j)} - \phi_{i(j-1)} \right\} e^{\omega_i t} \quad \text{——— (4)}$$

$$\dot{d}_{i(j)}(t) = \dot{x}_{i(j)}(t) - \dot{x}_{i(j-1)}(t)$$

$$= \left\{ \phi_{i(j)} - \phi_{i(j-1)} \right\} \omega_i \, e^{\omega_i t} \quad \text{——— (5)}$$

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where $\phi_{i(j)}$ & ω_i – are eigenvector & eigenvalue of i^{th} mode

Once eigenvalue & eigenvector are measured for i^{th} storey,

Eq (1) can be formulated for j^{th} storey @ each time interval.

Using short data length, which is approximately equal to ω_1 of the building.

Eq (1) can be solved.

One can repeat this procedure for other storey to determine [K] & [M] parameter of each storey.

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Salient points

(1) This method assumes a known storey mass

(2) only applicable for lumped mass, shear building model.

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(3) Method to identify damage using Element Modal stiffness

In a linear, undamped structure, i^{th} modal stiffness is given by:

$$K_i = \bar{\Phi}_i^T [K] \bar{\Phi}_i \quad \text{——} \quad (1)$$

where $\bar{\Phi}_i$ is the i^{th} mode shape vector

$[K]$ is the complete stiffness matrix of the entire structure

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Combination of j^{th} member to i^{th} modal stiffness is given by:

$$k_{ij} = \bar{\phi}_i^T [k_j] \bar{\phi}_i$$

Where k_j - j^{th} member contribution to $[k]$

Fraction of modal energy of i^{th} mode, contributed by j^{th} member is called <u>Modal sensitivity</u>.

Modal sensitivity is given by:

$$F_{ij} = \frac{k_{ij}}{k_i}$$

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The above Eqn is valid for undamaged structure.

This is modified for a damaged structure. Modal sensitivity for a damaged structure is given by:

$$F_{ij}^{*} = \frac{K_{ij}^{*}}{K_i^{*}}$$

$$K_{ij}^{*} = \bar{\Phi}_i^{*T} [K_j^{*}] \cdot \bar{\Phi}_i^{*}$$

$$K_i^{*} = \hat{\Phi}_i^{*T} [K^{*}] \bar{\Phi}_i^{*}$$

$$K_j = E_j [K_{j0}]$$

$$K_j^{*} = E_J^{*} [K_{j0}]$$

Where E_j, E_j^* represent material stiffness properties, related to undamaged & damaged state of the structure

$[k_{jo}]$ is assumed to be unchanged even after damage occurs.

Basic assumption of this method is that

Modal sensitivity for i^{th} mode & j^{th} member remains unchanged before & after damage.

Mathematically

$$\frac{F_{ij}}{F_{ij}^*} = \frac{k_{ij}^* \, k_i}{k_i^* \, k_{ij}} = 1$$

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Damage Index, β_j for the j^{th} member is defined as:

$$\beta_j = \frac{E_j}{E_j^*}$$

Substituting from the earlier eqn

$$\beta_j = \frac{\gamma_{ij}^* \, k_i}{\gamma_{ij} \, k_i^*}$$

$$= \frac{\bar{\Phi}_i^{*T} \, [k_{j0}] \, \bar{\Phi}_i^* \, k_i}{\bar{\Phi}_i^T \, [k_{j0}] \, \bar{\Phi}_i \, k_i^*}.$$

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Damage Index, can also be approximated as below.

$$\beta_j \cong \left[\frac{\bar{\Phi}_i^{*T} [k_{j0}] \bar{\Phi}_i^{*} + \sum_{k=1}^{NE} \bar{\Phi}_i^{*T} [k_{j0}] \bar{\Phi}_i^{*}}{\bar{\Phi}_i^{T} [k_{j0}] \bar{\Phi}_i + \sum_{k=1}^{NE} \bar{\Phi}_i^{T} [k_{j0}] \bar{\Phi}_i} \right] \left(\frac{k_i}{k_i^*} \right)$$

Normalized damage indicator is given by:

$$Z_j = \frac{\beta_j - \bar{\beta}}{\sigma_\beta}$$ where $\bar{\beta}$ mean value of β

σ_β std deviation of β.

Severity of damage can be estimated as:

$$E_j^* = E_j \left(1 + \frac{dE_j}{E_j}\right)$$

$$= E_j (1 + \alpha_j)$$

where $\alpha_j = \dfrac{\gamma_{ij} \, k_i^*}{\gamma_{ij}^* \, k_i} - 1$

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(4) Damage identification using Modal Strain Energy

This is a Two stage process:

1st stage: to locate the damage using change in modal strain energy of the element

2nd stage: Extent of damage is determined by iterative scheme

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Modal strain energy of j^{th} element of both undamaged & damaged structure is given by:

$$MSE_{ij} = \Phi_i^T K_j \Phi_i \quad \text{——} \quad (1)$$

$$MSE_{ij}^d = \Phi_{di}^T K_{dj} \Phi_{di} \quad \text{——} \quad (2)$$

where d subscript identifies damaged state

where K_j element stiffness matrix of j^{th} element

Φ_i is the i^{th} mode shape

Change in Modal strain Energy ratio is indicated as MSECR

$$MSECR_j^i = \frac{|MSE_{ij}^d - MSE_{ij}|}{MSE_{ij}} \qquad (3)$$

Eq (3) is a meaningful indicator of damaged elements

Damaged elements will have a significant change in the stiffness.

- stiffness will be degraded

Change in stiffness for the damaged element can be expressed as a fractional change of element stiffness matrix.

$$K^d = K + \sum_{j=1}^{L} \Delta k_j \qquad (4)$$

$$= K + \sum_{j=1}^{L} \alpha_j k_j \quad \text{——— (5)}$$

$$\left(\text{valid for } -1 < \alpha_j \leq 0\right)$$

Now, change is Modal strain Energy is expressed as below:

$$MSEC_{ij} = 2\Delta \Phi_i^T k_j \Phi_i + \alpha_j \Phi_i^T k_j \Phi_i \quad \text{——— (6)}$$

In the above eqn, α_j is unknown.

To start with, this is assumed to be zero & iteration is set in.

Thus,

$$MSEC_{ij} = 2\Delta \bar{\Phi}_i k_j \Phi_i \quad \text{——— (7)}$$

Modal strain Energy change can be determined for both damaged & undamaged states. By using the appropriate (k_j & $\underline{\Phi_i}$) - for damaged case, one can use k_{dj}, Φ_{id}.

In the undamped case, following Eqn holds good:

$$\left[(K + \Delta K) - (\omega_i + \Delta\omega_i) M \right] \left[\Phi_i + \Delta\Phi_i \right] = 0. \quad \text{———} \quad (8)$$

Now, $\Delta\Phi_i$ is expressed as a linear combination of mode shape of undamped system & is given by:

$$\Delta\Phi_i = \sum_{k=1}^{n} d_{ik} \Phi_k \quad \text{———} \quad (9)$$

sub (9) in Eq (8) & neglecting higher order terms, we get

$$d_{ir} = - \frac{\Phi_r^T \Delta K \Phi_i}{(\omega_r - \omega_i)} \quad \text{for} \quad r \neq i \quad \text{—} \quad (10)$$

MSEC is then given by :

$$MSEC_{ij} = 2\Phi^T k_j \left(\sum_{r=1}^{n} - \frac{\Phi_r^T \Delta k \Phi_i}{(\omega_r - \omega_i)} \Phi_r \right) \quad \text{for } r \neq i \qquad —(1)$$

The earlier Eqn (4-5) of MSEC can be simplified as below:

$$MSEC_{ij} = \sum_{j=1}^{L} 2\alpha_p \Phi_i^T k_j \left(\sum_{r=1}^{n} - \frac{\Phi_r^T k_p \Phi_i}{\omega_r - \omega_i} \Phi_r \right) \text{ for } r \neq i \qquad —(12)$$

Once the damage is located, then damage sensitivity can be determined as below:

$$\begin{Bmatrix} MSEC_{i1} \\ MSEC_{i2} \\ \vdots \\ MSEC_{ij} \end{Bmatrix} = \begin{bmatrix} \beta_{11} & \beta_{12} & \cdots & \beta_{1p} \\ \beta_{21} & \ddots & & \\ \vdots & & \ddots & \\ \beta_{j1} & & & \beta_{jp} \end{bmatrix} \begin{Bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_p \end{Bmatrix} \qquad (13)$$

where p is # of suspected damaged sites (locations)

j is # of elements considered to compute MSEC

$$\beta_{st} = -2\sum_{r=1}^{n} \Phi_i^T k_s \frac{\Phi_r^T K_i \Phi_i}{(\omega_r - \omega_i)} \Phi_r \quad \text{for } r \neq i$$

where β_{st} is the element sensitivity w.r.t. of MSEC

MSEC are experimentally measured and then substituted in Eq (13). to obtain the fractional change in stiffness of the damaged system (elements)

Once, initial value of α_p (as referred in Eq 12) is obtained, values of MSEC can be updated for each iteration, until convergence is reached

Summary

- Damage identification using Lumped mass
- " " element modal stiffness
- " " modal strain Energy method

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