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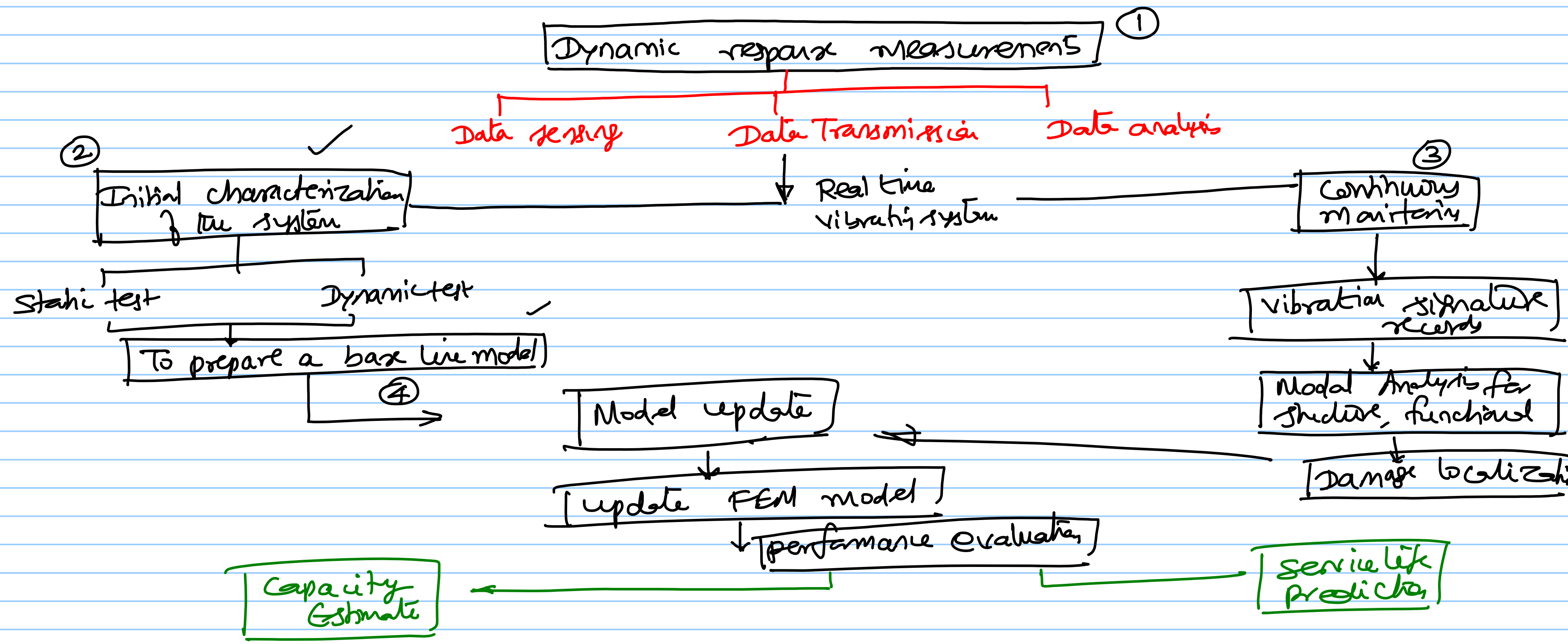
Module 2

Lecture 6 : SHM methods – I

Flow chart – vibration-based monitoring



I. Method using frequency & mode shapes

- change in structural characteristics can be readily identified by noticing the change in natural frequencies

- change in $[K]$, $[M]$ changes eigenvalue, which can be modelled as:

$$\{Z\} = [F]\{\alpha\} + [G]\{\beta\} \qquad (1)$$

where $\{Z\}$ = vector of measured frequency changes

$\{\alpha\}$, $\{\beta\}$ vectors of the changes in the system $\left\langle \begin{array}{l} \text{stiffness} \\ \text{mass} \end{array} \right.$ respectively

$[F]$, $[G]$ sensitivity matrices

To calculate the changes in $[K]$ & $[M]$ i.e. to compute $\{\alpha\}$, $\{\beta\}$, one need to calculate the sensitivity matrices.

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Either be calculated theoretically from the Eigenvalues of the st system

or

they can be computed numerically using 'perturbation method' with Finite Element model

- studies conducted earlier show that change in Mass matrix, before and after damage is neglible

- Therefore, sensitivity Equation, can be reformulated as below:

$$Z_i = \sum_{j=1}^{NE} F_{ij}\, \alpha_j \qquad (2)$$

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where Z_i - fractional change of i^{th} eigenvalue (frequency)

α_j = fractional reduction in i^{th} stiffness parameter

F_{ij} is expressed as fraction of Modal Strain Energy for i^{th} mode, stored in j^{th} element of the structure

F_{ij} can be expressed as below:

$$F_{ij} = \frac{\{\phi_i\}^T [k_j] \{\phi_i\}}{\{\phi_i\}^T [K] \{\phi_i\}} \quad \text{———} \quad (3)$$

where $[K]$ & $[k_j]$ are global & elemental stiffness matrices, respectively.

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Once, stiffness matrix of the complete system and mode shape are known, then F_{ij}, as seen from Eq(3) can be generated numerically.

To obtain the relative damage, sensitivity Eqn can be reformulated as:

$$\frac{Z_m}{Z_n} = \frac{\sum_{j=1}^{NE} F_{mj}\,\alpha_j}{\sum_{j=1}^{NE} F_{nj}\,\alpha_j} \quad \text{——} \quad (4)$$

Suppose, only ① element is damaged, then the above Eqn reduces to

$$\frac{Z_m}{Z_n} = \frac{F_{mq}}{F_{nq}} \quad \text{——} \quad (5)$$

which is unique for the q^{th} location. Damage can be now located

Error index is given by:

$$e_{ij} = \frac{Z_m}{\sum_{k=1}^{NM} Z_k} - \frac{F_{mq}}{\sum_{k=1}^{NM} F_{kq}} \quad \text{— (6)}$$

$e_{ij} = 0$, which indicates damage @ the j^{th} location.

Damage sensitivity is given by:

$$\frac{\delta \omega_i^2}{\omega_i^2} = \eta \, S_{ik} \left(\frac{a_k}{H}\right)_i^2 \quad \text{— (7)}$$

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where $\dfrac{\partial \omega_i^2}{\omega_i^2}$ is fractional change in Eigenvalue in the i^{th} mode

$\left(\dfrac{a_k}{H}\right)_i$ is dimensionless crack size, which is normalized to the depth of the member (H).

η - shape factor, accounting for geometry of the mass

S_{ik} is sensitivity of k^{th} location in the i^{th} modal strain energy

If there is a fractional change in Eigenvalue of the system, which is measured experimentally, then we can easily determine the crack size from Eq (7).

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Eq (7) has a limitation:

If only one damage is present, Eq (7) can be used to locate the damage

In case of multiple damage locations, it is not applicable

Based upon the changes in [K] & [M] one can observe changes in Eigenvalues, before & after damage

- crack/damage location
- crack size

etc

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II To estimate mass & stiffness of a shear model building from the model test

Let undamped shear building, expressed by the Eqn of motion as below:

$$M \ddot{x} + K x = F \quad \text{———} \quad (8)$$

Characteristic Eqn to determine Eigenvalues mode shape is given by:

$$(K - \omega_i^2 M) \phi_i = 0.$$

where ω_i - Eigen value

ϕ_i - corresponding mode shape

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Stiffness matrix of the shear building is given by:

$$[K] = \begin{bmatrix} k_1+k_2 & -k_2 & - & - & - & - \\ -k_2 & k_2+k_3 & -k_3 & - & - & - \\ \vdots & & & & & \\ \vdots & & & -k_{n-1} & k_{n-1}+k_n & -k_n \\ & - & - & - & -k_n & k_n \end{bmatrix}$$

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Mass matrix is given by

$$[M] = \begin{bmatrix} m_1 & - & - & - & \cdot \\ 0 & m_2 & - & - & - \\ 0 & \ddots & & & \\ & & \ddots & & \\ & & & & m_n \end{bmatrix}$$

Expanding the above equations for l & r modes and reorganizing in terms of stiffness and mass parameters, following Eqn's generated:

$$\begin{bmatrix} \phi_{l_1} & -\omega_l \phi_{l_1} & \phi_{l_1} - \phi_{l_2} & \cdots & & \\ \phi_{r_1} & -\omega_r \phi_{r_1} & \phi_{r_1} - \phi_{r_2} & \cdots & & \\ \vdots & & & \ddots & & \\ \vdots & & & & \phi_{l_n} - \phi_{l_{n-1}} & \omega_l \phi_{l_n} \\ \vdots & & & & \phi_{r_n} - \phi_{r_{n-1}} & \omega_r \phi_{r_n} \end{bmatrix} \begin{Bmatrix} k_1 \\ m_1 \\ k_2 \\ m_2 \\ \vdots \\ k_n \\ m_n \end{Bmatrix} = \begin{Bmatrix} 0 \end{Bmatrix}.$$

(or) Simply

$$[B]\{b\} = \{0\}$$

If $m_n = 1$, then the above equation will reduce to the form as below,

$$[B']\{b'\} = \begin{Bmatrix} 0 \\ \vdots \\ \omega_\ell \phi_{\ell n} \\ \omega_r \phi_{rn} \end{Bmatrix}$$

where $[B']$ is of the order $(2n-2)\times(2n-2)$ in which the last 2 rows & columns of $[B]$ are eliminated

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$\{b'\}$ is $(2n-2)\times(1)$ vector in which last (2) members of $\{b\}$ are eliminated

Now, solving for the unknown mass & stiffness parameters (k_i, m_i) we get:

$$\{b'\} = \left([B']^T [B']\right)^{-1} [B']^T \begin{Bmatrix} 0 \\ \vdots \\ \omega_\ell \, \phi_{\ell n} \\ \omega_r \, \phi_{rn} \end{Bmatrix}$$

$$k_n = \frac{\omega_\ell \, \phi_{\ell n}}{\phi_{\ell n} - \phi_{\ell n-1}}$$

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Mass and stiffness parameters, obtained from the above set of equations are relative values of m_n ($\because$ m_n is considered to be Unity)

Advantages of this method

(1) Only mode shape & frequency of (2) modes are required

(2) This can be applicable only to shear model buildings

(3) This is valid only for undamped systems

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Summary

- SHM methods
- flow chart - for vibration - based monitoring
- i) using frequency/ mode shapes, how can damage be located?
  ii) How to obtain [M] & [K] for shear-model building with any 2 frequencies/ mode shapes.

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