Module 2 Structural Health Monitoring

Lecture 3

'static and vibration-based Health monitoring'

SHM has a primary objective
with a basic concept that

current health of a structure can be predicted (or assessed)
through static and Dynamic measurements

In simple terms, when a structure vibrates under influence of any load, (ambient load or service load), then dynamic measurements can be used to characterize the structural condition @ any instant of time

It is necessary to locate the damage, if it is present in the system

location of damage can be done by comparing the structural characteristics @ pre & post - damage conditions

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By this logic,

It is mandatory to impose SHM in all structures of strategic importance

It is not necessary (it is wrong concept) that SHM should be carried out only when damage is perceived.

As structural health can be estimated only by comparing the vibration characteristics, before and after damage, even when the structure is healthy we should impose SHM to obtain vibration characteristics before (any perceived) damage

If all structures are designed to cater to the varying dynamic characteristics, then why SHM is necessary when no damage occurs?

Dynamic characteristics vary significantly with the following

- changes in loading pattern (re-distribution)
- ageing of the material (material degradation)
  - change in Mass and change in stiffness
- changes in support conditions with period of time

- Very important to periodically update the vibration characteristics of all structures (strategic importance)

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only based on pre-damaged condition & comparing it with the damaged state, health of the structure can be assessed.

(1) <u>Static-based SHM</u>

SHM, foremost step is damage identification.

Damage identification can be also done through Dead load redistribution

Basic hypothesis is that

dead load of the structural system will get redistributed <u>automatically</u> when damage occurs in the structural system.

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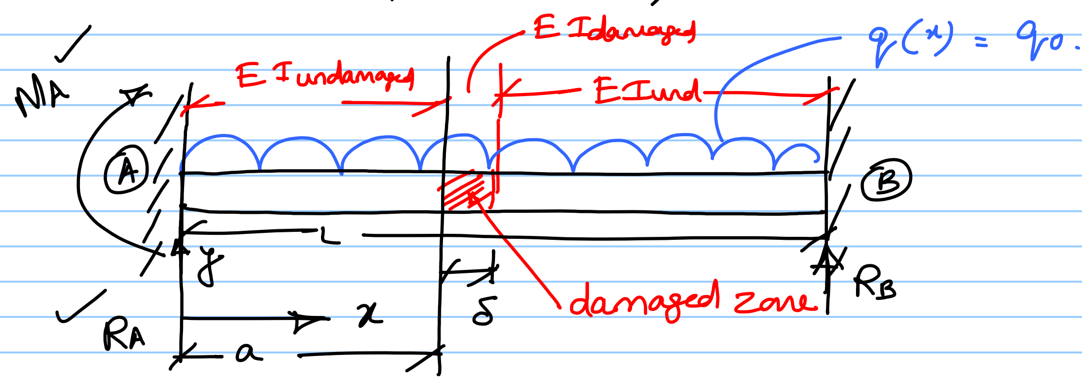
Measurements required ?

Stress and strain, developed due to dead load are used as input to identify the damage

– Static Load Test

Algorithm :

Let us consider a fixed beam, as shown in the Figure below



- Let us perceive that there is a damage @ distance 'a' from the left hand support
- Let the damage region be of span δ

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Damage Zone length = δ

Damage location = a

Moment @ any section x-x, in general, is given by the following expression:

$$M_x = + R_A(x) - \frac{q}{2} x^2 + M_A \quad \text{———} \quad (1)$$

Let EI_{und} = Modulus of Rigidity of the undamaged portion

EI_{dam} = " " of the damaged portion

We already know that,

$$EI \frac{d^2y}{dx^2} = M_x \quad \text{———} \quad (2)$$

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Integrating Eq (2) once, we get:

$$EI\frac{dy}{dx} = \frac{d(M_x)}{dx} + C_1 \quad \text{——} \quad (3)$$

further integrating,

$$EI\,y = \frac{d^2(M_x)}{dx^2} + C_1x + C_2 \quad \text{——} \quad (4)$$

Substituting for M_x from the Eq (1),

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} - \frac{q x^3}{6} + M_A x + C_1$$

✓ $$EI\,y = \frac{R_A x^3}{6} - \frac{q x^4}{24} + \frac{M_A x^2}{2} + C_1x + C_2$$

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a) Region $0 \le x \le a$

$$EI_{und}\,(y_1) = \frac{R_A x^3}{6} - \frac{q x^4}{24} + \frac{M_A x^2}{2} + C_1 x + C_2 \qquad (0 \le x \le a)$$

b) for region $a \le x \le a+\delta$,

$$EI_{damaged}\,(y_2) = \frac{R_A x^3}{6} - \frac{q x^4}{24} + \frac{M_A x^2}{2} + C_3 x + C_4 \qquad (a \le x \le \delta + a)$$

c) for region $[(a+\delta) \le x \le L]$,

$$EI_{undamaged} = \frac{R_A x^3}{6} - \frac{q x^4}{24} + \frac{M_A x^2}{2} + C_5 x + C_6. \qquad (a+\delta \le x \le L)$$

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Boundary conditions

$$y_1(0) = 0$$
$$y_2(0) = 0$$
$$y_3(L) = 0. \qquad (1)$$

$$y_1(a) = y_2(a)$$

$$y_2(a+\delta) = y_3(a+\delta)$$

$$\Rightarrow$$

$$\epsilon(x) = \frac{M_x\, y}{E I_{unda}}$$

$$\epsilon(x) = \frac{M_x\, y}{E I_{damaged}}$$

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Damage Severity Index (α)

$$\alpha = \frac{EI_{und} - EI_{dam}}{EI_{und}} \; ; \qquad 0 < \alpha < 1 .$$

Now, the problem is reduced to minimize the function

$$f(\alpha, \delta, a) = \sum_{j=1}^{k} \left| \frac{\Delta \epsilon_j^{k} - \Delta \epsilon_j^{M}}{\Delta \epsilon_j^{M}} \right|$$ subjected to the condition

$$0 \le a + \delta \le L$$

$$0 \le \alpha \le 1$$

$$\delta \le L$$

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Beam is divided into small lengths (l) – discretization

$$l = \left(\frac{L}{n}\right)$$

$$\Rightarrow \quad \delta = l = (L/n)$$

$$\Rightarrow \quad a = a_i = \delta(i-1) \quad \text{for } i = 1, 2 \cdots n$$

Hence minimize

$$f(\alpha, a_i) = \sum_{j=1}^{k} \left| \frac{\Delta\epsilon_j^{t} - \Delta\epsilon_j^{n}}{\Delta\epsilon_j^{n}} \right|$$

subjected to

$a_i = 1, 2 \cdots n$

for $0 \le \alpha \le 1$

(2) vibration-based damage detection

Hypothesis : Structural damage can be characterized by local modifications of stiffness

- modification in stiffness, in turn affects the modal parameters.

Procedure:

Member is subjected to an external excitation

It can be forced vibration (for model in Expt)

(or)

ambient vibration under natural loading cases.

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- Modal parameters are estimated from the vibration data
- These parameters are used as input for damage identification (damage detection)

Let us consider

- changes in modal parameter as ΔV
- stiffness reduction factor (SRF) as $\{\alpha\}$
- weightage of each term in the stiffness matrix as $\bar{W}$
- Analytical data be "A"
- Experimental data be "E"

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Damage identification can be done as follows:

$$J = \left\| \bar{W} \left\{ \Delta V^{Analy}(\{\alpha\}) - \Delta V^{Exp} \right\} \right\|^2$$

The problem here is to minimize the above function, J. subject to the condition that

$$1 \leq \underset{\uparrow}{\alpha} \leq 0 \quad \text{is valid}$$

stiffness reduction factor

Expanding:
$$J = \left\{ \Delta V^{Analy}(\{\alpha\}) - \Delta V^{Exp} \right\}^T (\bar{W})^2 \left\{ \Delta V^{Analy}(\{\alpha\}) - \Delta V^{Exp} \right\}$$

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Damage detection and quantification can be assessed from three objective function, as given below:

$$J = \sum_{i=1}^{nm} \bar{W}_{\lambda_i}^2 \left[\left(\frac{\lambda_i(\{\alpha\}) - \lambda_i}{\lambda_i} \right)^{Analy} - \left(\frac{\lambda_i^{Damaged} - \lambda_i^{undam}}{\lambda_i^{undam}} \right)^{Exp} \right]^2$$

where # of measured modes is nm

λ_i = i^{th} eigenvalue

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(b) <u>Mode shape</u> changes

change is mode shape characteristics can be done using the foll relationship:

$$J = \sum_{i=1}^{nm} \bar{W}_{\phi i}^{2} \sum_{j=1}^{np} \left(\left[\phi_{ij}(\{\alpha\}) - \phi_{ij} \right]^{Analy} - \left[\phi_{ij}^{Damaged} - \phi_{ij}^{undam} \right]^{Exp} \right)^2$$

where # measured points $= np$

ϕ_{ij} - j^{th} component of i^{th} mass is the <u>normalized</u> mode shape

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(iii) Frequency changes, combined with mode shape

Following function is valid:

$$J = \sum_{i=1}^{nm} \bar{w}_{\lambda i}^{2} \left(\left[\frac{\lambda_i(\{\alpha\}) - \lambda_i^{\circ}}{\lambda_i^{\circ}} \right]^{Analy} - \left[\frac{\lambda_i^{Damaged} - \lambda_i^{undy}}{\lambda_i^{\,under}} \right]^{Exp} \right)^2$$

$$+ \sum_{i=1}^{nm} \bar{w}_{\phi i}^{2} \sum_{j=1}^{np} \left(\left[\phi_{ij}(\{\alpha\}) - \phi_{ij} \right]^{Analy} - \left[\phi_{ij}^{Dam} - \phi_{ij}^{undr} \right]^{Exp} \right)^2$$

Summary

- static method of SHM
  - governing Eqn to minimize
- vibration-based SHM
  - frequency change
  - change in mode shapes
  - combination of the above

damaged & undamaged conditions