

Material Science

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Chapter 4. Mechanical Properties of Metals

Most of the materials used in engineering are metallic in nature. The prime reason simply is the versatile nature of their properties those spread over a very broad range compared with other kinds of materials. Many engineering materials are subjected to forces both during processing/fabrication and in service. When a force is applied on a solid material, it may result in translation, rotation, or deformation of that material. Aspects of material translation and rotation are dealt by *engineering dynamics*. We restrict ourselves here to the subject of material deformation under forces. Deformation constitutes both change in shape, *distortion*, and change in size/volume, *dilatation*. Solid material are defined such that change in their volume under applied forces in very small, thus deformation is used as synonymous to distortion. The ability of material to with stand the applied force without any deformation is expressed in two ways, i.e. strength and hardness. Strength is defined in many ways as per the design requirements, while the hardness may be defined as resistance to indentation of scratch.

Material deformation can be permanent *or* temporary. Permanent deformation is irreversible i.e. stays even after removal of the applied forces, while the temporary deformation disappears after removal of the applied forces i.e. the deformation is recoverable. Both kinds of deformation can be function of time, or independent of time. Temporary deformation is called *elastic deformation*, while the permanent deformation is called *plastic deformation*. Time dependent recoverable deformation under load is called *anelastic deformation*, while the characteristic recovery of temporary deformation after removal of load as a function of time is called *elastic aftereffect*. Time dependent i.e. progressive permanent deformation under constant load/stress is called *creep*. For *visco-elastic* materials, both recoverable and permanent deformations occur together which are time dependent. When a material is subjected to applied forces, first the material experiences elastic deformation followed by plastic deformation. Extent of elastic- and plastic- deformations will primarily depend on the kind of material, rate of load application, ambient temperature, among other factors. Change over from elastic state to plastic state is characterized by the yield strength (σ_0) of the material.

Forces applied act on a surface of the material, and thus the force intensity, force per unit area, is used in analysis. Analogous to this, deformation is characterized by percentage

change in length per unit length in three distinct directions. Force intensity is also called *engineering stress* (or simply *stress*, s), is given by force divided by area on which the force is acting. *Engineering strain* (or simply *strain*, e) is given by change in length divided by original length. Engineering strain actually indicates an average change in length in a particular direction. According to definition, s and e are given as

$$s = \frac{P}{A_0}, e = \frac{L - L_0}{L_0}$$

where P is the load applied over area A , and as a consequence of it material attains the final length L from its original length of L_0 .

Because material dimensions changes under application of the load continuously, engineering stress and strain values are not the true indication of material deformation characteristics. Thus the need for measures of stress and strain based on instantaneous dimensions arises. Ludwik first proposed the concept of, and defined the *true strain* or *natural strain* (ϵ) as follows:

$$\epsilon = \sum \frac{L_1 - L_0}{L_0} + \frac{L_2 - L_1}{L_1} + \frac{L_3 - L_2}{L_2} + \dots$$

$$\epsilon = \int_{L_0}^L \frac{dL}{L} = \ln \frac{L}{L_0}$$

As material volume is expected to be constant i.e. $A_0 L_0 = AL$, and thus

$$\epsilon = \ln \frac{L}{L_0} = \ln \frac{A_0}{A} = \ln(e + 1)$$

There are certain advantages of using true strain over conventional strain or engineering strain. These include (i) equivalent absolute numerical value for true strains in cases of tensile and compressive for same intuitive deformation and (ii) total true strain is equal to the sum of the incremental strains. As shown in *figure-4.1*, if $L_1 = 2 L_0$ and $L_2 = 1/2 L_1 = L_0$, absolute numerical value of engineering strain during tensile deformation (1.0) is different from that during compressive deformation (0.5). However, in both cases true strain values are equal ($\ln [2]$).

True stress (σ) is given as load divided by cross-sectional area over which it acts at an instant.

$$\sigma = \frac{P}{A} = \frac{P}{A_0} \frac{A_0}{A} = s(e + 1)$$

It is to be noted that engineering stress is equal to true stress up to the elastic limit of the material. The same applies to the strains. After the elastic limit i.e. once material starts deforming plastically, engineering values and true values of stresses and strains differ. The above equation relating engineering and true stress-strains are valid only up to the limit of uniform deformation i.e. up to the onset of necking in tension test. This is because the relations are developed by assuming both constancy of volume and homogeneous distribution of strain along the length of the tension specimen. Basics of both elastic and plastic deformations along with their characterization will be detailed in this chapter.

4.1 Elastic deformation and Plastic deformation

4.1.1 Elastic deformation

Elastic deformation is reversible i.e. recoverable. Up to a certain limit of the applied stress, strain experienced by the material will be the kind of recoverable i.e. elastic in nature. This elastic strain is proportional to the stress applied. The proportional relation between the stress and the elastic strain is given by *Hooke's law*, which can be written as follows:

$$\sigma \propto \varepsilon$$

$$\sigma = E\varepsilon$$

where the constant E is the modulus of elasticity or Young's modulus,

Though Hooke's law is applicable to most of the engineering materials up to their elastic limit, defined by the critical value of stress beyond which plastic deformation occurs, some materials won't obey the law. E.g.: Rubber, it has nonlinear stress-strain relationship and still satisfies the definition of an elastic material. For materials without linear elastic portion, either *tangent modulus* or *secant modulus* is used in design calculations. The tangent modulus is taken as the slope of stress-strain curve at some specified level, while secant module represents the slope of secant drawn from the origin to some given point of the σ - ε curve, as shown in *figure-4.1*.

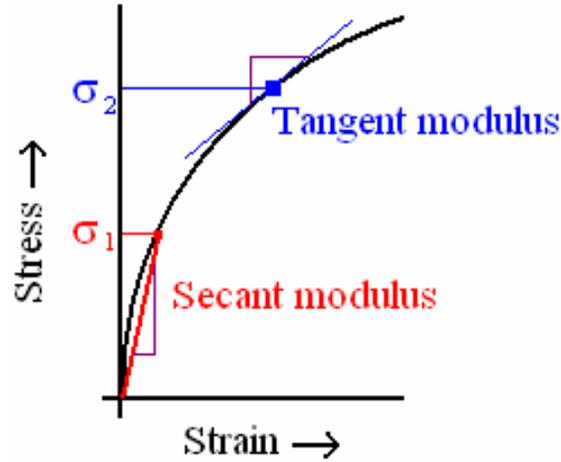


Figure-4.1: Tangent and Secant moduli for non-linear stress-strain relation.

If one dimension of the material changed, other dimensions of the material need to be changed to keep the volume constant. This lateral/transverse strain is related to the applied longitudinal strain by empirical means, and the ratio of transverse strain to longitudinal strain is known as *Poisson's ratio* (ν). Transverse strain can be expected to be opposite in nature to longitudinal strain, and both longitudinal and transverse strains are linear strains. For most metals the values of ν are close to 0.33, for polymers it is between 0.4 – 0.5, and for ionic solids it is around 0.2.

Stresses applied on a material can be of two kinds – normal stresses, and shear stresses. Normal stresses cause linear strains, while the shear stresses cause shear strains. If the material is subjected to torsion, it results in torsional strain. Different stresses and corresponding strains are shown in *figure-4.2*.

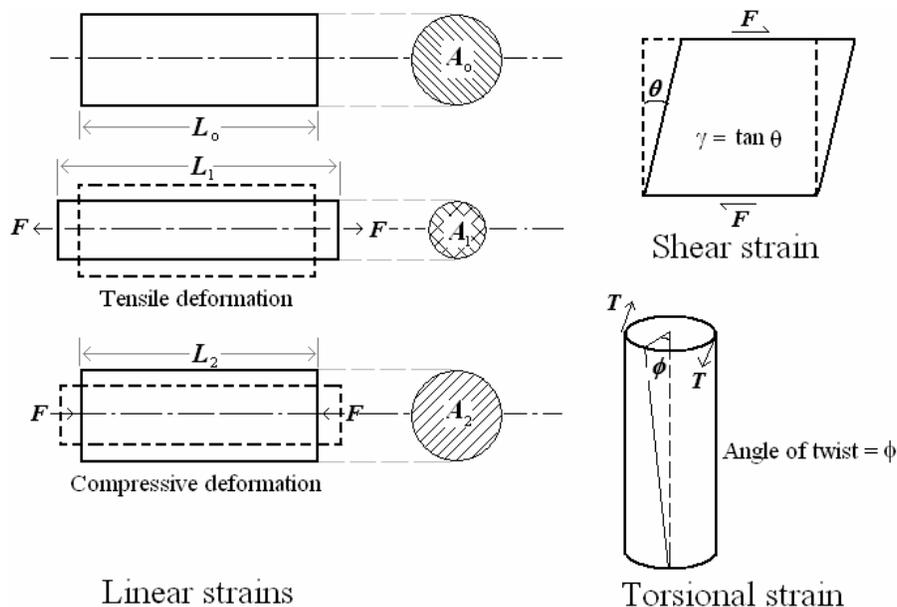


Figure 4.2: Schematic description of different kinds of deformations/strains.

Analogous to the relation between normal stress and linear strain defined earlier, shear stress (τ) and shear strain (γ) in elastic range are related as follows:

$$\tau = G\gamma$$

where G is known as *Shear modulus* of the material. It is also known as *modulus of elasticity in shear*. It is related with Young's modulus, E , through Poisson's ratio, ν , as

$$G = \frac{E}{2(1+\nu)}$$

Similarly, the *Bulk modulus* or *volumetric modulus of elasticity* K , of a material is defined as the ratio of hydrostatic or *mean stress* (σ_m) to the volumetric strain (Δ). The relation between E and K is given by

$$K = \frac{\sigma_m}{\Delta} = \frac{E}{3(1-2\nu)}$$

Let σ_x , σ_y and σ_z are linear stresses and ε_x , ε_y and ε_z are corresponding strains in X-, Y- and Z- directions, then

$$\sigma_m = \frac{\sigma_x + \sigma_y + \sigma_z}{3}$$

Volumetric strain or *cubical dilatation* is defined as the change in volume per unit volume.

$$\Delta = (1 + \varepsilon_x)(1 + \varepsilon_y)(1 + \varepsilon_z) - 1 \approx \varepsilon_x + \varepsilon_y + \varepsilon_z, \Delta = 3\varepsilon_m$$

where ε_m is *mean strain* or *hydrostatic (spherical) strain* defined as

$$\varepsilon_m = \frac{\varepsilon_x + \varepsilon_y + \varepsilon_z}{3}$$

An engineering material is usually subjected to stresses in multiple directions than in just one direction. If a cubic element of a material is subjected to normal stresses σ_x , σ_y , and σ_z , strains in corresponding directions are given by

$$\varepsilon_x = \frac{1}{E} [\sigma_x - \nu(\sigma_y + \sigma_z)], \varepsilon_y = \frac{1}{E} [\sigma_y - \nu(\sigma_x + \sigma_z)], \text{ and } \varepsilon_z = \frac{1}{E} [\sigma_z - \nu(\sigma_x + \sigma_y)]$$

$$\text{and } \sigma_x + \sigma_y + \sigma_z = \frac{E}{1-2\nu}(\varepsilon_x + \varepsilon_y + \varepsilon_z)$$

The strain equation can be modified as

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] = \frac{1+\nu}{E}\sigma_x - \frac{\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$$

After substituting the sum-of-stress into the above equation, stress can be related to strains as follows:

$$\sigma_x = \frac{E}{1+\nu}\varepsilon_x + \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_x + \varepsilon_y + \varepsilon_z) = \frac{E}{1+\nu}\varepsilon_x + \lambda(\varepsilon_x + \varepsilon_y + \varepsilon_z) = 2G\varepsilon_x + \lambda\Delta$$

where λ is called Lamé's constant.

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

Using the above equations, it is possible to find strains from stresses and vice versa in elastic range.

The basis for elastic deformation is formed by reversible displacements of atoms from their equilibrium positions. On an atomic scale, elastic deformation can be viewed as small changes in the inter-atomic distances by stretching of inter-atomic bonds i.e. it involve small changes in inter-atomic distances. Elastic moduli measure the *stiffness* of the material. They are related to the *second* derivative of the inter-atomic potential, or the first derivative of the inter-atomic force vs. inter-atomic distance (dF/dr) (*figure-4.3*). By examining these curves we can tell which material has a higher modulus.

Elastic modulus can also be said as a measure of the resistance to separation of adjacent atoms, and is proportional to the slope of the inter-atomic force Vs inter-atomic distance curve (*figure-4.3*). Hence values of modulus of elasticity are higher for ceramic materials which consist of strong covalent and ionic bonds. Elastic modulus values are lower for metal when compared with ceramics, and are even lower for polymers where only weak covalent bonds present.

Moreover, since the inter-atomic forces will strongly depend on the inter-atomic distance as shown in *figure-4.3*, the elastic constants will vary with direction in the crystal lattice i.e. they are anisotropic in nature for a single crystal. However, as a material consists of number of randomly oriented crystals, elastic constants of a material can be considered as isotropic.

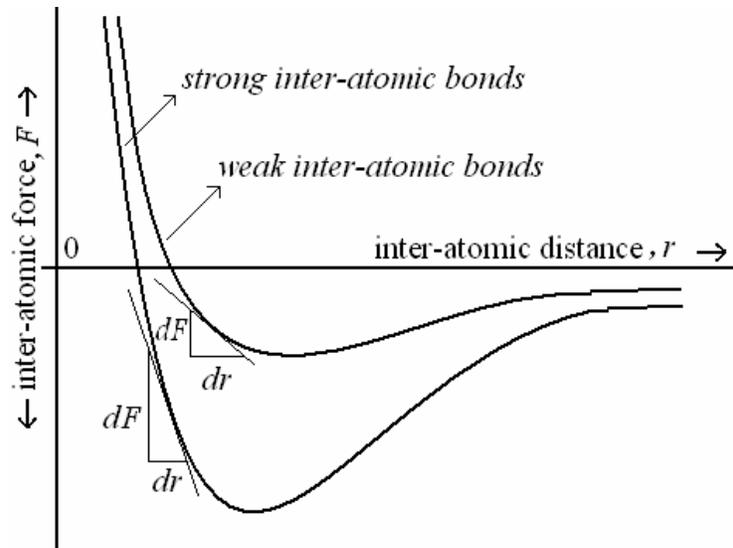


Figure-4.3: Graph showing variation of inter-atomic forces against inter-atomic distance for both weak and strong inter-atomic bonds.

The elastic moduli are usually measured by direct static measurements in tension or torsion tests. For more precise measurements, dynamic techniques are employed. These tests involve measurement of frequency of vibration or elapsed time for an ultrasonic pulse to travel down and back in a specimen. Because strain cycles occur at very high rates, no time for heat transfer and thus elastic constants are obtained under adiabatic conditions. Elastic modulus obtained under isothermal and adiabatic conditions are related as follows:

$$E_{adiabatic} = \frac{E_{iso}}{1 - \frac{E_{iso} T \alpha^2}{9c}}$$

where α is volume coefficient of thermal expansion, and c is the specific heat. It can be observed that with increasing temperature, the modulus of elasticity diminishes. This is because, intensity of thermal vibrations of atoms increases with temperature which weakens the inter-atomic bonds.

4.1.2 Plastic deformation

When the stress applied on a material exceeds its elastic limit, it imparts permanent non-recoverable deformation called plastic deformation in the material. Microscopically it can be said of plastic deformation involves breaking of original atomic bonds, movement of atoms and the restoration of bonds i.e. plastic deformation is based on irreversible displacements of atoms through substantial distances from their equilibrium positions. The mechanism of this deformation is different for crystalline and amorphous materials. For crystalline materials, deformation is accomplished by means of a process called slip that involves motion of dislocations. In amorphous materials, plastic deformation takes

place by viscous flow mechanism in which atoms/ions slide past one another under applied stress without any directionality.

Plastic deformation is, as elastic deformation, also characterized by defining the relation between stresses and the corresponding strains. However, the relation isn't simpler as in case of elastic deformation, and in fact it is much more complex. It is because plastic deformation is accomplished by substantial movement of atomic planes, dislocations which may encounter various obstacles. This movement becomes more complex as number slip systems may get activated during the deformation.

The analysis of plastic deformation, and the large plastic strains involved is important in many manufacturing processes, especially forming processes. It is very difficult to describe the behavior of metals under complex conditions. Therefore, certain simplifying assumptions are usually necessary to obtain an amenable mathematical solution. Important assumptions, thus, involved in theory of plasticity are neglecting the following aspects:

- (i) Anelastic strain, which is time dependent recoverable strain.
- (ii) Hysteresis behavior resulting from loading and un-loading of material.
- (iii) Bauschinger effect – dependence of yield stress on loading path and direction.

The relations describing the state of stress and strain are called *constitutive equations* because they depend on the material behavior. These relations are applicable to any material whether it is elastic, plastic or elastic-plastic. Hooke's law which states that strain is proportional to applied stress is applicable in elastic range where deformation is considered to be uniform. However, plastic deformation is indeed uniform *but* only up to some extent of strain value, where after plastic deformation is concentrated the phenomenon called *necking*. The change over from uniform plastic deformation to non-uniform plastic deformation is characterized by ultimate tensile strength (σ_u). As a result of complex mechanism involved in plastic deformation and its non-uniform distribution before material fractures, many functional relations have been proposed to quantify the stress-strain relations in plastic range.

A true stress-strain relation plotted as a curve is known as *flow curve* because it gives the stress (σ) required to cause the material to flow plastically to any given extent of strain (ϵ) under a set of conditions. Other important parameters affecting the stress-strain curve are: rate at which the load is applied / strain rate ($\dot{\epsilon}$), and the temperature of the material (T , in K) i.e.

$$\sigma = fn(\epsilon, \dot{\epsilon}, T, \text{microstructure})$$

Following are the most common equations that describe the material flow behavior:

$$\sigma = K \epsilon^n$$

where K – is strength coefficient, and n – is strain hardening exponent. The strain-hardening exponent may have values from $n=0$ (perfectly plastic solid) to $n=1$ (elastic solid). For most metals n has values between 0.10 and 0.50. The *Power law equation* described above is also known as *Holloman-Ludwig equation*. Another kind of power equation which describes the material behavior when strain rate effect is prominent:

$$\sigma = K \dot{\epsilon}^m$$

where m – is the index of strain-rate sensitivity. If $m=0$, the stress is independent of strain rate. $m=0.2$ for common metals. If $m=0.4-0.9$, the material may exhibit super-plastic behavior – ability to deform by several hundred percent of strain without necking. If $m=1$, the material behaves like a viscous liquid and exhibits Newtonian flow.

Deviations from the above power equations are frequently observed, especially at low strains ($\epsilon < 10^{-3}$) or high strains ($\epsilon \gg 1,0$). One common type of deviation is for a log-log plot of Power equation to result in two straight lines with different slopes. For data which do not follow the Power equation, following equation could be used

$$\sigma = K(\epsilon_0 + \epsilon)^n$$

where ϵ_0 - is the strain material had under gone before the present characterization.

Another common variation from Power law equation, also known as *Ludwig equation*, is:

$$\sigma = \sigma_0 + K \epsilon^n$$

where σ_0 - is the yield strength of the material.

Many other expressions for flow curve are available in literature. The true strain used in the above equations should actually be the plastic strain value, given by the following equation, where elastic strains can be safely neglected because plastic strains are very much when compared with elastic strain values.

$$\epsilon_p = \epsilon - \epsilon_e = \epsilon - \frac{\sigma_0}{E} \approx \epsilon$$

Both temporary elastic deformation and permanent plastic deformation are compared in *tabel-4.1*.

Table-4.1: *Elastic deformation Vs Plastic deformation.*

Elastic deformation	Plastic deformation
Reversible	Not reversible
Depends on initial and final states of	Depends on loading path

stress and strain	
Stress is proportional to strain	No simple relation between stress and strain
No strain hardening effects	Strain hardening effects

4.2 Interpretation of tensile stress-strain curves

It is well known that material deforms under applied loads, and this deformation can be characterized by stress-strain relations. The stress-strain relation for a material is usually obtained experimentally. Many kinds of experiments those differ in way of loading the material are standardized. These include tension test, compression test (upsetting), plane strain compression test, torsion test, etc.

The engineering tension test is commonly used to provide basic design information on the strength characteristics of a material. Standardized test procedure is explained by ASTM standard E0008-04. In this test a specimen is subjected to a continually increasing uni-axial tensile force while measuring elongation simultaneously. A typical plot of load-elongation is given in the *figure-4.4*. The curve also assumes the shape of an engineering stress – engineering strain curve after dividing the load with initial area and the elongation with initial length. The two curves are frequently used interchangeably as they differ only by constant factors. The shape and relative size of the engineering stress-strain curve depends on material composition, heat treatment, prior history of plastic deformation, strain rate, temperature and state of stress imposed on specimen during the test.

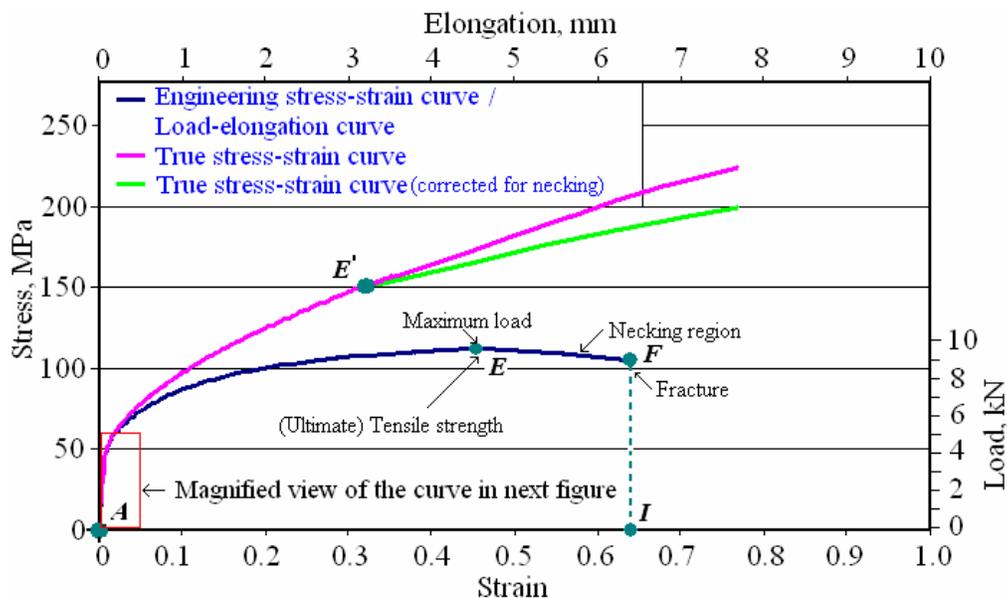


Figure-4.4: Typical load – elongation / engineering stress – engineering strain / true stress – true strain curve.

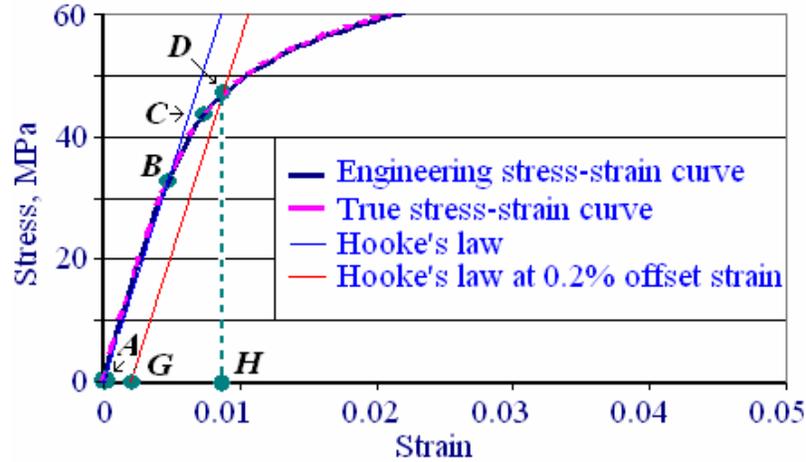


Figure-4.5: Magnified view of initial part of stress-strain curve.

As shown in *figure-4.4*, at initial stages load is proportional to elongation to a certain level (elastic limit), and then increases with elongation to a maximum (uniform plastic deformation), followed by decrease in load due to necking (non-uniform plastic deformation) before fracture of the specimen occurs. Magnified view of the initial stage of the curve is shown in *figure-4.5*. Along the segment *AB* of the curve, engineering stress is proportional to engineering strain as defined by Hooke's law, thus *point-B* is known as proportional limit. Slope of the line *AB* gives the elastic modulus of the material. With further increase in stress up to *point-C*, material can still be elastic in nature. Hence *point-C* is known as elastic limit. It is to be noted that there is no point where exactly material starts deform plastically i.e. there is no sharp point to indicate start of the yield. Otherwise *point-C* can also be called yield point. Thus it is common to assume that stress value at 0.2% offset strain as yield strength (σ_0), denoted by *point-D*. This offset yield strength is also called proof stress. Proof stress is used in design as it avoids the difficulties in measuring proportional or elastic limit. For some materials where there is essentially no initial linear portion, offset strain of 0.5% is frequently used. In *figure-4.5* distance between *points-B, C and D* is exaggerated for clarity. For many materials it is difficult to make any difference between *points-B and C*, and *point-D* also coincides with *point-B/C*. After the yield point, *point-D*, stress reaches a maximum at *point-E* (tensile strength (σ_f) as *figure-4.4*) till where plastic deformation is uniform along the length of the specimen. Stress decreases hereafter because of onset of necking that result in non-uniform plastic deformation before specimen fractures at *point-F*, fracture limit. Yield strength and tensile strength are the parameters that describe the material's strength, while percent elongation and reduction in cross-sectional area are used to indicate the material's ductility – extent of material deformation under applied load before fracture.

Percent elongation (*e*) and reduction is area (*r*) are related as follows:

$$\text{Volume constancy} \quad A_0 L_0 = AL$$

Percent elongation $e = \frac{L - L_0}{L_0}$

Reduction in area $r = \frac{A_0 - A}{A_0}, \frac{A_0}{A} = \frac{1}{1 - r}$

Relation $e = \frac{L - L_0}{L_0} = \frac{L}{L_0} - 1 = \frac{A_0}{A} - 1 = \frac{1}{1 - r} - 1 = \frac{r}{1 - r}$

Other important parameters from the engineering stress-strain curve are – *resilience* and *toughness*. Resilience is defined as ability of a material to absorb energy when deformed elastically and to return it when unloaded. Approximately this is equal to the area under elastic part of the stress-strain curve, and equal to *area-ADH* in *figure-4.5*. This is measured in terms of modulus of resilience (U_r)– strain energy per unit volume required to stress the material from zero stress to yield stress ($\sigma_0 = s_0$).

$$U_r = \frac{1}{2} s_0 e_0 = \frac{1}{2} s_0 \frac{s_0}{E} = \frac{s_0^2}{2E}$$

where e_0 – it elastic strain limit. Toughness (U_t) of the materials is defined as its ability to absorb energy in the plastic range. In other terms, it can be said to equal to work per unit volume which can be done on the material without causing it to rupture. It is considered that toughness of a material gives an idea about both strength and ductility of that material. Experimentally toughness is measured by either *Charpy* or *Izod impact tests*. Numerically the toughness value is equal to the area under the stress-strain curve, *area-AEFI*, approximately of rectangular shape.

$$U_t \approx s_u e_f \approx \frac{s_0 + s_u}{2} e_f$$

For brittle materials (those have high elastic modulus and low ductility), stress-strain curve is considered to assume the shape of parabola, thus

$$U_t \approx \frac{2}{3} s_u e_f$$

where s_u – ultimate tensile strength and e_f – strain at fracture.

As explained in earlier section, engineering stress-strain curve is not true representative of the material behavior. But, the flow curve (true stress-true strain curve) represents the basic plastic-flow characteristics of the material. More upon, special feature of the flow curve is that any point on the curve can be considered as yield point i.e. if load is removed and then reapplied, material will behave elastically throughout the entire range of reloading. It can be said from the relations between engineering stress-strain and true

stress-strains the true stress-true strain curve is always to the left of the engineering curve until the maximum load is reached. *Point-E'* on represents the corresponding location on true stress-strain curve to ultimate tensile stress *point-E* on engineering stress-strain curve. After the *point-E'*, flow curve is usually linear up to fracture, and in some cases its slope decreases continually up to fracture.

In correspondence to different variables defined based on engineering stress-strain curve, following parameters are defined based on flow curve:

True stress at maximum load $\sigma_u = \frac{P_{\max}}{A_u}, s_u = \frac{P_{\max}}{A_o}, \epsilon_u = \ln \frac{A_0}{A_u} \Rightarrow \sigma_u = s_u \frac{A_0}{A_u} = s_u e^{\epsilon_u}$

True fracture strain $\epsilon_f = \ln \frac{A_0}{A_f} = \ln \frac{1}{1-r}$

It is not possible to calculate ϵ_f from estimated values of e_f because the relation between them is not valid beyond the onset of necking.

True uniform strain $\epsilon_u = \ln \frac{A_0}{A_u}$

The uniform strain is useful in estimating the formability of metals from the results of a tension test. The condition ' $\epsilon_u = n$ ' represents the onset of necking.

True local necking strain $\epsilon_n = \ln \frac{A_u}{A_f}$

It is important to note that the rate of strain hardening ($d\sigma/d\epsilon$) is not identical to the strain-hardening exponent (n). They differ as follows:

$$n = \frac{d(\log \sigma)}{d(\log \epsilon)} = \frac{d(\ln \sigma)}{d(\ln \epsilon)} = \frac{\epsilon}{\sigma} \frac{d\sigma}{d\epsilon} \quad \text{or} \quad \frac{d\sigma}{d\epsilon} = n \frac{\sigma}{\epsilon}$$

4.3 Yielding under multi-axial stress, Yield criteria, Macroscopic aspects of plastic deformation and Property variability & Design considerations

4.3.1 Yielding under multi-axial stress

Once the necking starts to form i.e. material starts to deform plastically but in non-uniform mode, uni-axial state of stress turns into multi-axial (tri-axial) stress state. The necked region is in effect a mild notch. The chief effect of the notch is not in introducing

a stress concentration but in producing a tri-axial state of stress i.e. introduction of transverse stresses. As a result of tri-axial state of stress, yield stress becomes greater than the uni-axial yield stress, σ_0 , because it is more difficult to spread the yielded zone in the presence of tri-axial stresses. Thus, the average true stress at the neck is higher than the stress which would be required to cause flow in simple tension prevailed.

Bridgman put forward mathematical analysis to calculate the true stress from measured stress in axial direction under tri-axial stress condition. His analysis is based on the following assumptions: counter of the neck is approximated by the arc of a circle; cross-section of the necked region remains circular; von Mises yield criterion is applicable; strains are constant over the cross-section of the neck. Bridgman's correction is applicable from the onset of necking for flow curve i.e. from *point-E'* shown in *figure-4.4*. Corrected yield stress under tri-axial state of stress is given as follows:

$$\sigma = \frac{(\sigma_x)_{avg}}{(1 + 2R/a) [\ln(1 + a/2R)]}$$

where $(\sigma_x)_{avg}$ measured stress in the axial direction, a – smallest radius in the neck region, R – radius of the curvature of neck (*figure-4.6*).

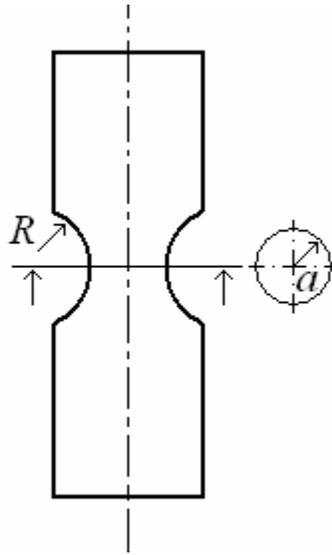


Figure-4.6: Geometry of necked region in cylindrical specimen under tensile load.

4.3.2 Yield criteria

It is known that material yields under condition of applied stress(es). In uni-axial loading, like in a tension test, yield occurs i.e. macroscopic plastic flow starts at yield stress, σ_0 . However the situation is much more complicated in presence of multi-axial stresses. It could be expected that yielding condition under multi-axial stresses to be a function of particular combination of principal stresses.

Presently available yield criteria are essentially empirical relations. Thus, it needs to satisfy some experimental observations. These include: hydrostatic component of stress state should not influence the stress where the yield occurs; it must be an invariant function i.e. independent of the choice of axes. These lead to the statement that yield criterion must be some function of invariants of stress deviator.

There are two generally accepted criteria are in use for predicting the onset of yielding in ductile materials: von Mises or Distortion-energy criterion and Maximum-shear-stress or Tresca criterion.

von Mises or Distortion-energy criterion: it states that yielding occur when the second invariant of the stress deviator J_2 exceeded some critical value.

$$J_2 = k^2$$

where $J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$; σ_1, σ_2 and σ_3 are principal stresses.

It implies that yield condition is not dependent on any particular stress, but instead it depends on all three principal stresses. And as the criterion is based on differences of normal stresses, it is independent of hydrostatic stress component. In energy terms, it can be said that yielding occurs when the *distortion energy* reaches a critical value. Distortion energy is part of total strain energy per unit volume that is involved in change of shape as opposed to a change in volume.

To evaluate the constant k , let's consider the yield in uni-axial tension test i.e. $\sigma_1 = \sigma_0, \sigma_2 = \sigma_3 = 0$. Thus,

$$\begin{aligned} \frac{1}{6}(\sigma_0^2 + \sigma_0^2) &= k^2 \Rightarrow \sigma_0 = \sqrt{3}k \\ \Rightarrow \sigma_0 &= \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1}{2}} \end{aligned}$$

To identify the constant k , by considering the state of stress in pure shear (torsion test): $\sigma_1 = \tau, \sigma_2 = -\tau, \sigma_3 = 0$.

$$\frac{1}{6}(\sigma_1^2 + \sigma_1^2 + 4\sigma_1^2) = k^2 \Rightarrow \sigma_1 = k$$

i.e. k represents the yield stress under pure shear, whereas σ_0 represents the yield stress under uni-axial tension. These two yield stresses can be related as follows:

$$k = \frac{1}{\sqrt{3}}\sigma_0 = 0.577\sigma_0$$

von Mises yield criterion can also be interpreted as yielding occurs if octahedral shear stress reaches a critical value. This is shear stress on octahedral plane which makes equal angles with all three principal axes. It also represents the mean square of the shear stress averaged over all orientations in the solid.

Cosine of angle between normal to a face of octahedron and a nearest principal axis is $1/\sqrt{3}$, i.e. the angle is $54^\circ 44'$. Normal octahedral stress (σ_{oct}) is equivalent to hydrostatic component of the stress system. Thus it can result in yielding, but shear octahedral stress (τ_{oct}) do.

$$\sigma_{oct} = \sigma_m = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

$$\tau_{oct} = \frac{1}{3} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

$$\Rightarrow \tau_{oct} = \frac{\sqrt{2}}{3} \sigma_0 = 0.471 \sigma_0$$

Corresponding octahedral strains are given as follows:

$$\varepsilon_{oct} = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3}, \gamma = \frac{2}{3} [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]^{1/2}$$

Maximum-shear-stress or Tresca criterion: this criterion states that yielding occurs once the maximum shear stress of the stress system reaches the value of shear stress in uni-axial tension test.

If σ_1, σ_2 and σ_3 are principal stresses arranged in descending order, maximum shear stress is given as

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

Though this criterion is simpler than the von Mises yield criterion, here it is necessary to know before-hand the maximum and minimum principal stresses from the stress system.

As with the earlier criterion, under uni-axial tension test conditions ($\sigma_1 = \sigma_0, \sigma_2 = \sigma_3 = 0$),

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = \tau_0 = \frac{\sigma_0}{2} \Rightarrow \sigma_1 - \sigma_3 = \sigma_0$$

Under pure shear stress conditions ($\sigma_1 = -\sigma_3 = k, \sigma_2 = 0$),

$$k = \frac{\sigma_1 - \sigma_3}{2} = \frac{1}{2} \sigma_0$$

Thus the other mathematical form for the criterion can be

$$\sigma_1 - \sigma_3 = \sigma_1' - \sigma_3' = k$$

Note that the yield locus for the Tresca criterion falls inside of the von Mises yield ellipse. The two yielding criteria predict the same yield stress for conditions of uni-axial stress and balanced bi-axial stress ($\sigma_1 = \sigma_3$). The greatest divergence between the two criteria occurs for pure shear ($\sigma_1 = -\sigma_3$) where yield stress from von Mises criterion ($\frac{1}{\sqrt{3}} \sigma_0$) is 15.5% greater than the yield stress from Tresca criterion ($\frac{1}{2} \sigma_0$).

4.3.3 Macroscopic aspects of plastic deformation

In single crystal, plastic deformation is accomplished by the process called slip, and sometimes by twinning. The extent of slip depends on many factors including external load and the corresponding value of shear stress produced by it, the geometry of crystal structure, and the orientation of active slip planes with the direction of shearing stresses generated. Schmid first recognized that single crystals at different orientations but of same material require different stresses to produce slip. The dependence of various factors has been summarized using a parameter – critical resolved shear stress, τ_R , given as

$$\tau = \frac{P \cos \lambda}{A / \cos \phi} = \frac{P}{A} \cos \phi \cos \lambda = \sigma \cos \phi \cos \lambda$$

$$\Rightarrow m = \cos \phi \cos \lambda$$

where P – external load applied, A – cross-sectional area over which the load applied, λ – angle between slip direction and tensile axis, ϕ – angle between normal to the slip plane and the tensile axis and m – Schmid factor.

Shear stress is maximum for the condition where $\lambda = \phi = 45^\circ$. If either of the angles are equal to 90° , resolved shear stress will be zero, and thus no slip occurs. If the conditions are such that either of the angles is close to 90° , crystal will tend to fracture rather than slip. Single crystal metals and alloys are used mainly for research purpose and only in a few cases of engineering applications.

Almost all engineering alloys are polycrystalline. Gross plastic deformation of a polycrystalline specimen corresponds to the comparable distortion of the individual grains by means of slip. This involves generation, movement and (re-)arrangement of dislocations. During deformation, mechanical integrity and coherency are maintained along the grain boundaries; that is, the grain boundaries are constrained, to some degree,

in the shape it may assume by its neighboring grains. Let's assume that the grains are equi-axed before deformation, or have approximately the same dimension in all directions. If the material is imparted with deformation in a particular direction, the grains become elongated along the direction in which the specimen was extended, as shown in *figure-4.7*.

Another noticeable aspect of plastic deformation is arrangement of dislocations forming cell like configurations with clear areas in the centers of the cells. With increasing deformation, the cell structure becomes denser. In early stages of deformation, slip is essentially confined to primary glide planes and thus dislocations form coplanar arrays. As deformation proceeds, cross slip comes into picture, leading to tangled network of dislocations forming cell walls, *figure-4.8*. Cell size decreases with increase in deformation but reaches a fixed size depending on the material, the strain, the strain rate and the temperature of deformation. In materials with low stacking fault energy where cross-slip is difficult, development of cell structure is less pronounced.

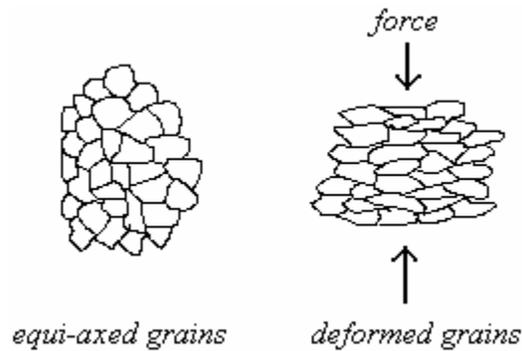


Figure-4.7: Grain elongation under applied external forces.

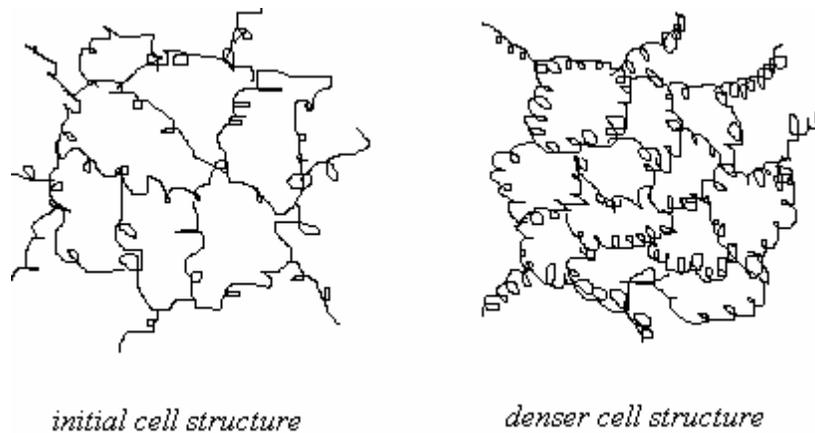


Figure-4.8: Schematic view of cell structure.

Schematic presentation of macroscopic observation of a tensile specimen is shown in *figure-4.9*. As explained earlier, initially specimen undergoes elastic deformation, followed by uniform plastic deformation. After attaining maximum load capacity, non-

uniform plastic deformation sets-in. this leads to formation of macroscopic defects, like cracks, which causes eventual fracture failure of the sample.

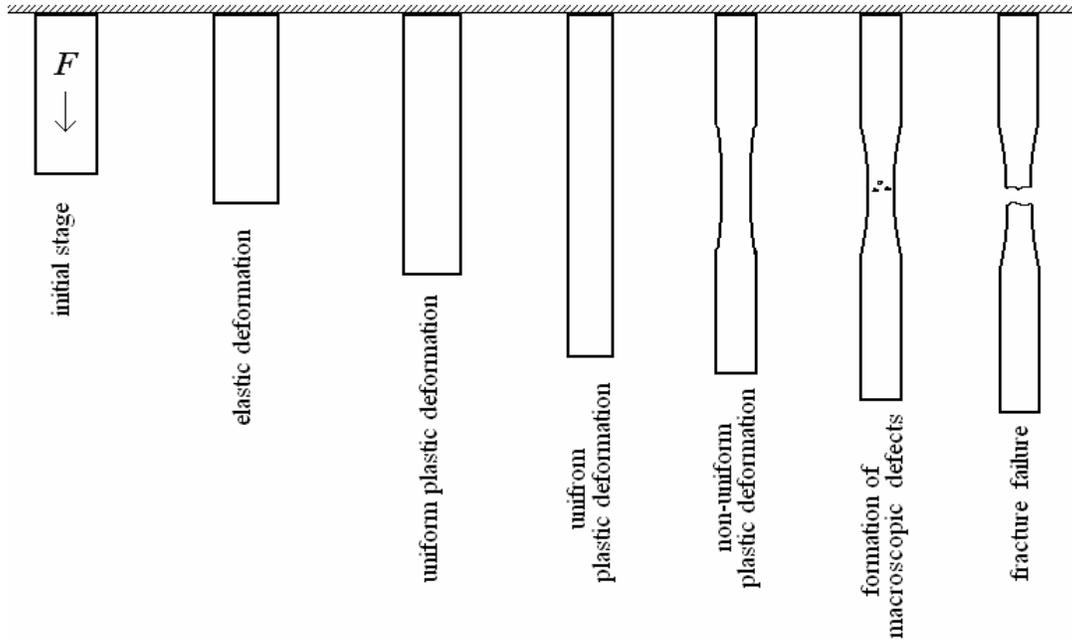


Figure-4.9: Schematic macroscopic presentation of plastic deformation in tensile specimen.

4.3.4 Property variability & Design consideration

Scatter in measured properties of engineering materials is inevitable because of number of factors such as test method, specimen fabrication procedure, operator bias, apparatus calibration etc. In spite of property variation, some typical value is desirable. Most commonly used typical value is by taking an average of the data. The average (\bar{x}) of the a parameter, x_i , is given as

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

where n is the number of samples, and x_i is the discrete measurement.

In some instances, it is desirable to have an idea about the degree of variability, scatter, of the measured data. Most common measure of this degree of variability is the standard deviation, s , which is given by

$$s = \left[\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1} \right]^{1/2}$$

A large value for s means a high degree of scatter. Scatter is usually represented in graphical form using error bars. If a parameter is averaged to \bar{x} , and the corresponding standard deviation is s , the upper error bar limit is given by $(\bar{x} + s)$, while the lower error bar is equal to $(\bar{x} - s)$.

To account for variability of properties, designers use, instead of an average value of, say, the tensile strength, the probability that the yield strength is above the minimum value tolerable. This leads to the use of a safety factor $N > 1$. Thus, a working stress or safe stress would be $\sigma_w = \sigma_y / N$.

Alternatively, a design factor (N') is defined to account for allowances thus protect against unanticipated failure. If calculated yield stress is σ_c , then the design stress σ_d can be given as $\sigma_d = N' \sigma_c$, where N' is greater than unity. Thus the material to be used for a particular application is chosen so as to have strength at least as high as the value of σ_d .

Utilization of design stress is usually preferred since it is based on the anticipated maximum applied stress instead of the yield strength of the material. In design it is important to use an appropriate value for N . In general, values for factor of safety range from 1.2 to 4.0. Higher the value of N , lesser will be the efficiency of design or material usage i.e. either too much material or a material having a higher than necessary strength will be used.

Selection of N will depend on a number of factors, including economics, previous experience, the accuracy with which mechanical forces and material properties may be determined and most importantly on the consequences of failure in terms of loss of life or property damage. If failure would result in loss of life, the factor of safety should be increased. In military equipment where light weight may be a prime consideration, the factor of safety may be lower than in commercial equipment. For static loading, as in a building, the factor of safety would be lower than in a machine, which is subjected to vibration and fluctuating stresses.

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