



# Module 5



# Basics of energy conversation cycles



# Heat Engines and Efficiencies



- The objective is to build devices which receive heat and produce work (like an aircraft engine or a car engine) or receive work and produce heat (like an air conditioner) in a sustained manner.
- All operations need to be cyclic. The cycle comprises of a set of processes during which one of the properties is kept constant ( $V, p, T$  etc.)



# Heat Engines (contd...)

- A minimum of 3 such processes are required to construct a cycle.
- All processes need not have work interactions (eg: isochoric)
- All processes need not involve heat interactions either (eg: adiabatic process).



# Heat Engines (Contd...)



- A cycle will consist of processes: involving some positive work interactions and some negative.
- If sum of +ve interactions is  $>$  -ve interactions the cycle will produce work
- If it is the other way, it will need work to operate.
- On the same lines some processes may have +ve and some -ve heat interactions.



# Heat Engines (Contd...)

- Commonsense tells us that to return to the same point after going round we need at one path of opposite direction.
- I law does not forbid all heat interactions being +ve nor all work interactions being -ve.
- But, we know that you can't construct a cycle with all +ve or
- All -ve Q's nor with all +ve or all -ve W's
- Any cycle you can construct will have some processes with
- Q +ve some with -ve.



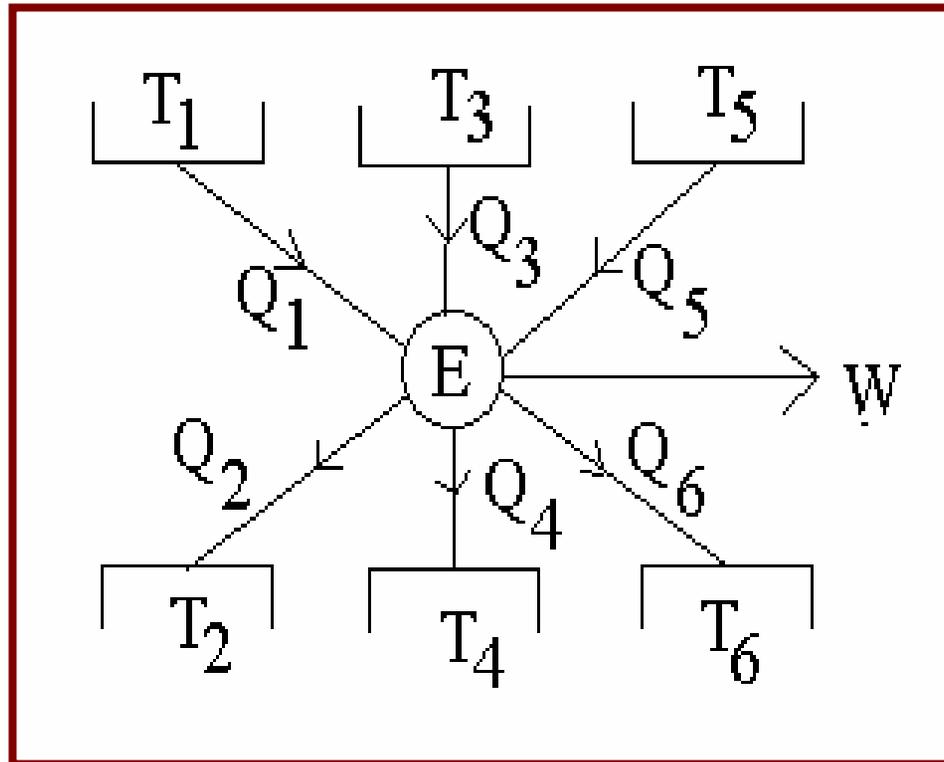
# Heat Engines (Contd...)



- Let  $Q_1, Q_3, Q_5 \dots$  be +ve heat interactions (Heat supplied)
- $Q_2, Q_4, Q_6 \dots$  be -ve heat interactions (heat rejected)
- From the first law we have
- $Q_1 + Q_3 + Q_5 \dots - Q_2 - Q_4 - Q_6 \dots = \text{Net work delivered } (W_{\text{net}})$
- $\Sigma Q_{+ve} - \Sigma Q_{-ve} = W_{\text{net}}$
- The efficiency of the cycle is defined as  $\eta = W_{\text{net}} / \Sigma Q_{+ve}$
- Philosophy → What we have achieved ÷ what we have spent to achieve it



# Heat Engines (Contd...)





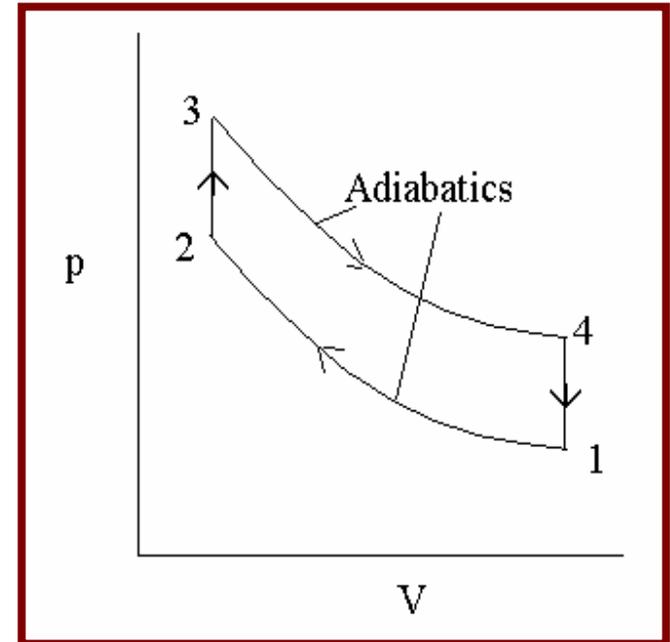
# Otto Cycle



Consider the **OTTO Cycle** (on which your car engine works)

It consists of two isochores and two adiabatics

- There is no heat interaction during 1-2 and 3-4
- Heat is added during constant volume heating (2-3)  $Q_{2-3} = c_v (T_3 - T_2)$
- Heat is rejected during constant volume cooling (4-1)  $Q_{4-1} = c_v (T_1 - T_4)$
- Which will be negative because  $T_4 > T_1$





# Otto Cycle (Contd...)



- Work done =  $c_v (T_3 - T_2) + c_v (T_1 - T_4)$
- The efficiency =  $[c_v(T_3 - T_2) + c_v(T_1 - T_4)] / [c_v(T_3 - T_2)]$   
 $= [(T_3 - T_2) + (T_1 - T_4)] / [(T_3 - T_2)]$   
 $= 1 - [(T_4 - T_1) / (T_3 - T_2)]$



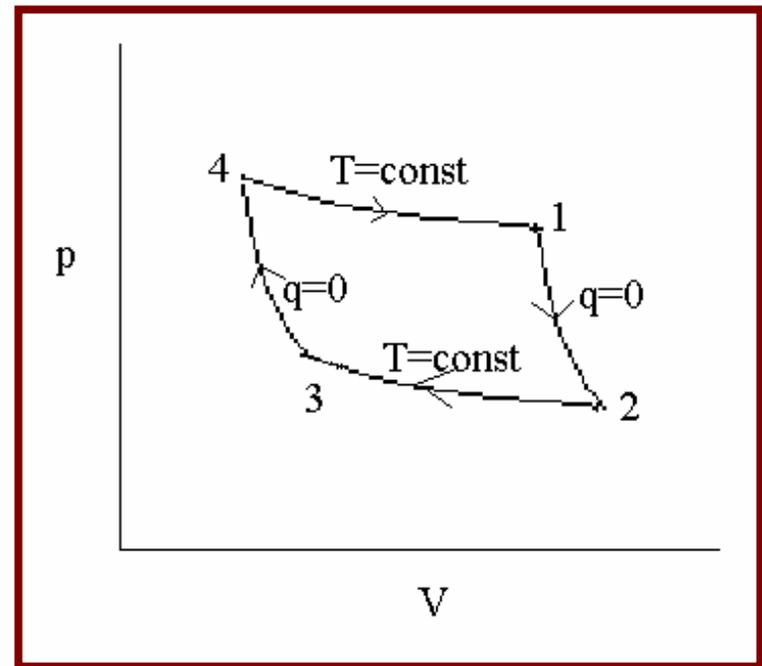
# Carnot Cycle



Consider a **Carnot cycle** - against which all other cycles are compared

It consists of two isotherms and two adiabatics

- Process 4-1 is heat addition because  $v_4 < v_1$
- Process 2-3 is heat rejection because  $v_3 < v_2$





# Carnot Cycle (contd..)



<u>Process</u>	<u>Work</u>	<u>Heat</u>
1-2	$(p_1v_1 - p_2v_2)/(g-1)$	0
2-3	$p_2v_2 \ln (v_3/v_2)$	$p_2v_2 \ln (v_3/v_2)$
3-4	$(p_3v_3 - p_4v_4)/(g-1)$	0
4-1	$p_4v_4 \ln (v_1/v_4)$	$p_4v_4 \ln (v_1/v_4)$
Sum	$(p_1v_1 - p_2v_2 + p_3v_3 - p_4v_4)/(g-1)$	
	$+ RT_2 \ln (v_3/v_2)$	$RT_2 \ln (v_3/v_2)$
	$+ RT_1 \ln (v_1/v_4)$	$+ RT_1 \ln (v_1/v_4)$

But,  $p_1v_1 = p_4v_4$  and  $p_2v_2 = p_3v_3$

Therefore the first term will be 0

*!!We reconfirm that I law works!!*



# Carnot Cycle (contd..)



We will show that  $(v_2/v_3) = (v_1/v_4)$

1 and 2 lie on an adiabatic

so do 3 and 4

$$p_1 v_1^\gamma = p_2 v_2^\gamma$$

$$p_4 v_4^\gamma = p_3 v_3^\gamma$$

Divide one by the other  
/  $p_3 v_3^\gamma$  (A)

$$(p_1 v_1^\gamma / p_4 v_4^\gamma) = (p_2 v_2^\gamma / p_3 v_3^\gamma)$$

$$(p_1/p_4) (v_1^\gamma / v_4^\gamma) = (p_2/p_3) (v_2^\gamma / v_3^\gamma)$$

But  $(p_1/p_4) = (v_4/v_1)$  because 1 and 4 are on the same isotherm

Similarly  $(p_2/p_3) = (v_3/v_2)$  because 2 and 3 are on the same isotherm



# Carnot Cycle (contd..)



Therefore A becomes

$$(v_1 / v_4)^{\gamma-1} = (v_2/v_3)^{\gamma-1}$$

which means

$$(v_2/v_3) = (v_1/v_4)$$

Work done in Carnot cycle =  $RT_1 \ln (v_1/v_4) + RT_2 \ln (v_3/v_2)$

$$= RT_1 \ln (v_1/v_4) - RT_2 \ln (v_2/v_3)$$

$$= R \ln (v_1/v_4) (T_1 - T_2)$$

Heat supplied =  $R \ln (v_1/v_4) T_1$

The efficiency =  $(T_1 - T_2)/T_1$

In all the cycles it also follows that Work done = Heat supplied - heat rejected



# Carnot Cycle (contd..)

Carnot engine has one  $Q +ve$  process and one  $Q -ve$  process. This engine has a single heat source at  $T_1$  and a single sink at  $T_2$ .

If  $Q +ve > Q -ve$ ;  $W$  will be  $+ve$  It is a heat engine

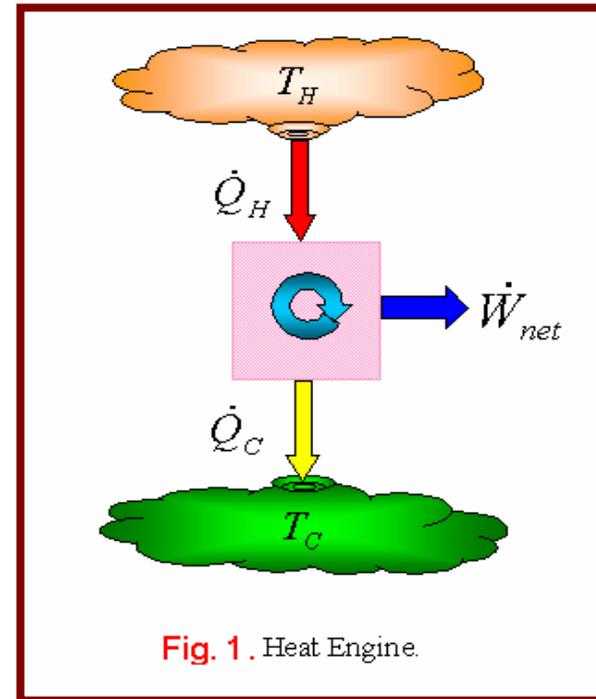
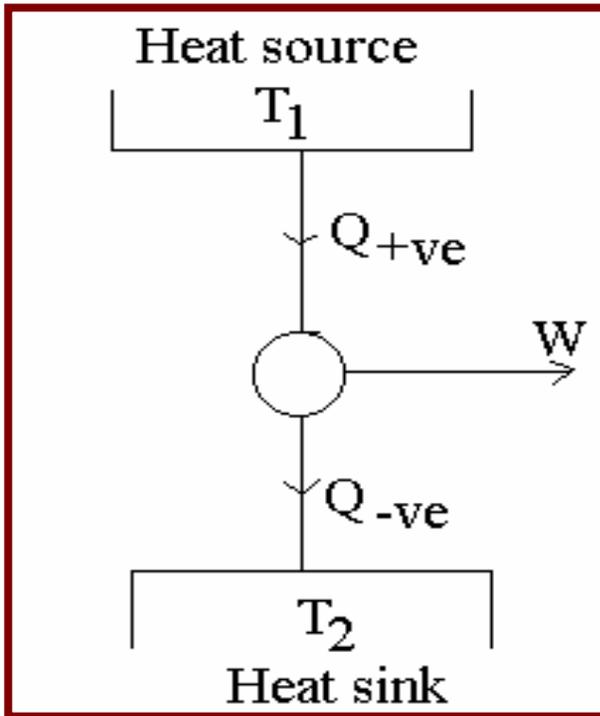


Fig. 1. Heat Engine.



# Carnot Cycle (contd..)



It will turn out that Carnot efficiency of  $(T_1 - T_2)/T_1$  is the best we can get for any cycle operating between two fixed temperatures.



# Carnot Cycle (contd..)

$Q_{+ve} < Q_{-ve}$   $W$  will be -ve It is not a heat engine

Efficiency is defined only for a work producing heat engine  
not a work consuming cycle

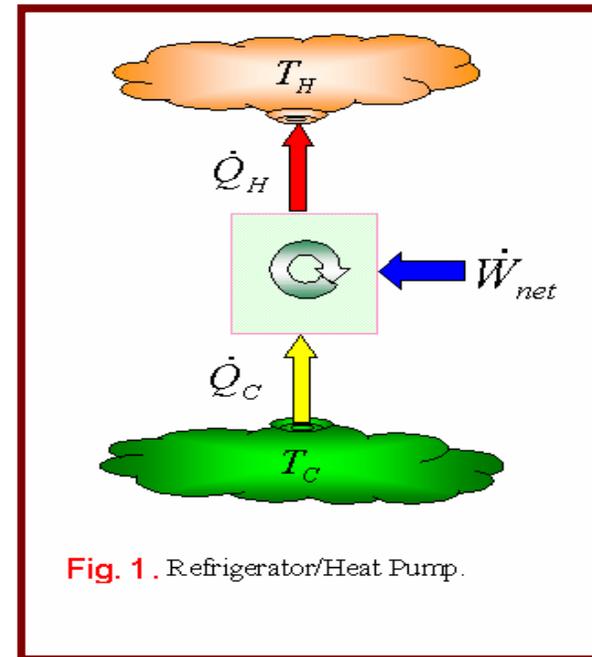
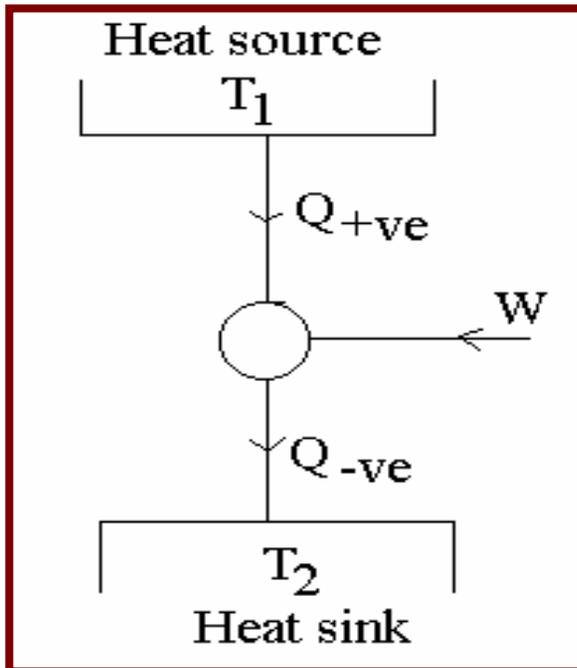


Fig. 1. Refrigerator/Heat Pump.



# Carnot Cycle (contd..)



**Note:** We can't draw such a diagram for an Otto cycle because there is no single temperature at which heat interactions occur