



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Failure Analysis and Prevention

Industrial Engineering Tools for FA: Reliability II

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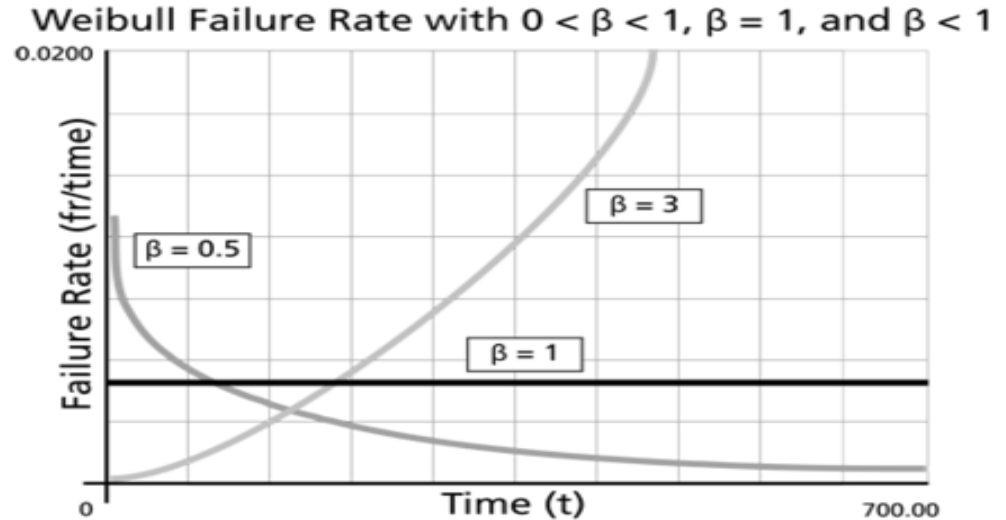
Statistical Aspects

Failure distributions Applicable to Reliability:

- ☐ Exponential distribution.
- ☐ Normal distribution.
- ☐ Weibull distribution.

Reliability Curves:

- ☐ The failure rate as a function of time.

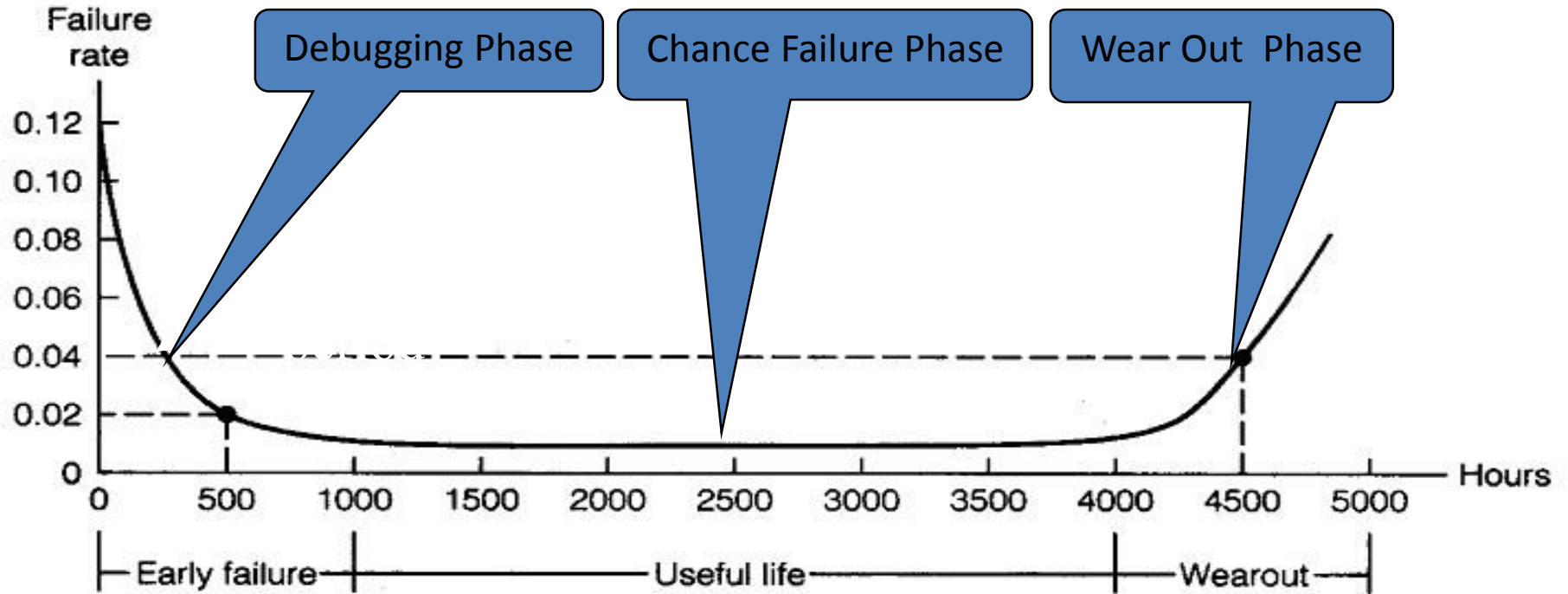


Life History Curve

- ❑ The curve, sometimes referred to as the “bathtub” curve, is a comparison of failure rate with time.
- ❑ It has three distinct phases:
 - ❑ The debugging phase.
 - ❑ The chance failure phase.
 - ❑ The wear-out phase.



Life History Curve



Life History Curve

1. The debugging phase:

It is characterized by marginal and short-life parts that cause a rapid decrease in the failure rate.

It may be part of the testing activity prior to shipment for some products.

The Weibull distribution $\beta < 1$ is used to describe the occurrence of failures.



Life History Curve

2. The chance failure phase:

Failures occur in a random manner due to the constant failure rate. The Exponential and the Weibull distributions $\beta = 1$ are best suited to describe this phase.

3. The wear-out phase:

Is depicted by a sharp raise in failure rates. The Normal distribution and the Weibull distribution $\beta > 1$ are used to describe this phase.



Normal Failure Analysis

- The Normal distribution.

$$R(t) = 1.0 - \int_0^t f(t)dt$$

$$R(t) = 1.0 - P(t)$$

$R(t)$: Reliability at time t

$P(t)$: Probability of failure



Exponential Failure Analysis

Exponential distribution:

$$Rt = e^{-t/\theta}$$

Where:

t : Time or cycles.

θ : Mean life.



Exponential Model - Example

The response time X at a certain on-line computer terminal has an exponential distribution with expected response time equal to 5 sec.

- (a) What is the probability that the response time is at most 10 seconds?
- (b) What is the probability that the response time is between 5 and 10 seconds?

Exponential Model - Example

The $E(X) = 5 = \theta$.

The probability that the response time is at most 10 sec is:

$$\begin{aligned}P(X \leq 10) &= F(10, 0.2) \\&= 1 - e^{-10/5} \\&= 1 - 0.135 \\&= 0.865\end{aligned}$$

$$\text{or } P(X > 10) = 0.135$$

The probability that the response time is between 5 and 10 sec is:

$$\begin{aligned}P(5 \leq X \leq 10) &= (1 - e^{-2}) - (1 - e^{-1}) \\&= 0.233\end{aligned}$$



Weibull Failure Analysis

- ❑ Used for the debugging phase ($\beta < 1$) and the chance failure phase ($\beta = 1$).
- ❑ $\beta = 1$, the Weibull equals the exponential
- ❑ $\beta = 3.4$, the Weibull approximates the Normal

$$Rt = e^{-(t/\theta)^\beta}$$

Where β is the Weibull slope.



Weibull Distribution: Example

The ultimate tensile strength of steel is X (MPa) at 200 degrees C. If X has a Weibull distribution with parameters $\beta = 20$, and $\theta = 100$. Find:

- (a) $P(X \leq 105)$
- (b) $P(98 \leq X \leq 102)$
- (c) the value of x such that $P(X \leq x) = 0.10$

Weibull Distribution - Example Solution

(a) $P(X \leq 105) = F(105; 20, 100)$

$$= 1 - e^{-(105/100)^{20}} = 1 - 0.070 = 0.930$$

(b) $P(98 \leq X \leq 102)$

$$= F(102; 20, 100) - F(98; 20, 100)$$

$$= e^{-(0.98)^{20}} - e^{-(1.02)^{20}}$$

$$= 0.513 - 0.226 = 0.287$$