Module
2
Mechanics of Machining

Lesson 9

Analytical and Experimental determination of cutting forces

Instructional Objectives

At the end of this lesson, the student would be able to

(i) Develop and use equations for estimation of major cutting force components in turning under
- Orthogonal cutting
- Oblique cutting

(ii) Evaluate analytically the major cutting forces in
- Drilling
- Plain milling

(iii) Identify the needs and purposes of measurement of cutting forces

(iv) State the possible methods of measurement of cutting forces.

(i) Development of equations for estimation of cutting forces

The two basic methods of determination of cutting forces and their characteristics are :

(a) Analytical method : enables estimation of cutting forces characteristics : -
- easy, quick and inexpensive
- very approximate and average
- effect of several factors like cutting velocity, cutting fluid action etc. are not revealed
- unable to depict the dynamic characteristics of the forces.

(b) Experimental methods : direct measurement characteristics : -
- quite accurate and provides true picture
- can reveal effect of variation of any parameter on the forces
- depicts both static and dynamic parts of the forces
- needs measuring facilities, expertise and hence expensive.

The equations for analytical estimation of the salient cutting force components are conveniently developed using Merchant's Circle Diagram (MCD) when it is orthogonal cutting by any single point cutting tool like, in turning, shaping, planning, boring etc.

Development of mathematical expressions for cutting forces under orthogonal turning.

- **Tangential or main component, P_Z**

This can be very conveniently done by using Merchant's Circle Diagram, MCD, as shown in Fig. 9.1

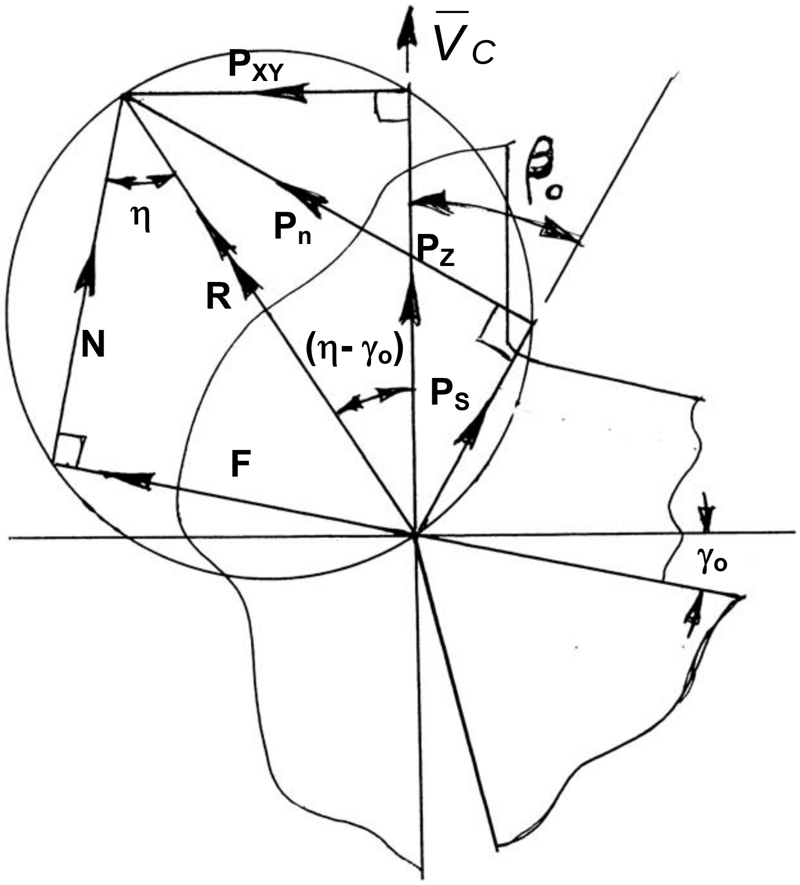

Fig. 9.1 *Forces involved in machining and contained in Merchant's Circle.*

From the diagram in Fig. 9.1,

$$P_Z = R\cos(\eta - \gamma_o) \tag{9.1}$$

$$P_s = R\cos(\beta_o + \eta - \gamma_o) \tag{9.2}$$

Dividing Eqn. 9.1 by Eqn. 9.2,

$$P_Z = \frac{P_s \cos(\eta - \gamma_o)}{\cos(\beta_o + \eta - \gamma_o)} \tag{9.3}$$

It was already shown that,

$$P_s = \frac{ts_o\tau_s}{\sin\beta_o} \tag{9.4}$$

where, τ_s = dynamic yield shear strength of the work material.

Thus, $$P_Z = \frac{ts_o\tau_s\cos(\eta - \gamma_o)}{\sin\beta_o\cos(\beta_o + \eta - \gamma_o)} \tag{9.5}$$

For brittle work materials, like grey cast iron, usually, $2\beta_o + \eta - \gamma_o = 90^o$ and $\boldsymbol{\tau_s}$ remains almost unchanged.
Then for turning brittle material,

$$P_Z = \frac{ts_o\tau_s\cos(90^o - 2\beta_o)}{\sin\beta_o\cos(90^o - \beta_o)}$$

or, $P_Z = 2ts_o\tau_s\cot\beta_o$ (9.6)

where, $\cot\beta_o = \zeta - \tan\gamma_o$ (9.7)

$$\zeta = \frac{a_2}{a_1} = \frac{a_2}{s_o\sin\phi}$$

It is difficult to measure chip thickness and evaluate the values of ζ while machining brittle materials and the value of $\boldsymbol{\tau_s}$ is roughly estimated from

$\boldsymbol{\tau_s}$ = 0.175 BHN (9.8)

where, BHN = Brinnel Hardness number.
But most of the engineering materials are ductile in nature and even some semi-brittle materials behave ductile under the cutting condition.
The angle relationship reasonably accurately applicable for ductile metals is

$\beta_o + \eta - \gamma_o = 45^o$ (9.9)

and the value of $\boldsymbol{\tau_s}$ is obtained from,

$\boldsymbol{\tau_s}$ = 0.186 BHN (approximate) (9.10)

or = $0.74\sigma_u\varepsilon^{0.6\Delta}$ (more suitable and accurate) (9.11)

where, σ_u = ultimate tensile strength of the work material
ε = cutting strain
$\cong \zeta - \tan\boldsymbol{\gamma_o}$
and Δ = % elongation

Substituting Eqn. 9.9 in Eqn. 9.5,

$P_Z = ts_o\tau_s(\cot\beta_o + 1)$ (9.12)

Again $\cot\beta_o \cong \zeta - \tan\gamma_o$

So, $\mathbf{P_Z = ts_o\tau_s(\zeta - \tan\gamma_o + 1)}$ (9.13)

- **Axial force, P_X and transverse force, P_Y**

From MCD in Fig. 9.1,

$P_{XY} = P_Z\tan(\eta - \gamma_o)$ (9.14)

Combining Eqn. 9.5 and Eqn. 9.14,

$$P_{XY} = \frac{ts_o\tau_s\sin(\eta - \gamma_o)}{\sin\beta_o\cos(\beta_o + \eta - \gamma_o)} \quad (9.15)$$

Again, using the angle relationship $\beta_o + \eta - \gamma_o = 45^o$, for ductile material

$P_{XY} = ts_o\tau_s(\cot\beta_o – 1)$ (9.16)

or $\mathbf{P_{XY} = ts_o\tau_s(\zeta - \tan\gamma_o – 1)}$ (9.17)

where, $\tau_s = 0.74\sigma_u\varepsilon^{0.6\Delta}$ or 0.186 BHN

It is already known,

$P_X = P_{XY}\sin\phi$
and $P_Y = P_{XY}\cos\phi$

Therefore, $\mathbf{P_X = ts_o\tau_s(\zeta - \tan\gamma_o – 1)\sin\phi}$ (9.18)

and $\mathbf{P_Y = ts_o\tau_s(\zeta - \tan\gamma_o – 1)\cos\phi}$ (9.19)

- **Friction force, F, normal force, N and apparent coefficient of friction μ_a**

Again from the MCD in Fig. 9.1

$$F = P_Z\sin\gamma_o + P_{XY}\cos\gamma_o \tag{9.20}$$

and $$N = P_Z\cos\gamma_o - P_{XY}\sin\gamma_o \tag{9.21}$$

and, $$\mu_a = \frac{F}{N} = \frac{P_Z \sin\gamma_o + P_{XY}\cos\gamma_o}{P_Z\cos\gamma_o - P_{XY}\sin\gamma_o} \tag{9.22}$$

or, $$\boldsymbol{\mu_a = \frac{P_Z \tan\gamma_o + P_{XY}}{P_Z - P_{XY}\tan\gamma_o}} \tag{9.23}$$

Therefore, if P_Z and P_{XY} are known or determined either analytically or experimentally the values of F, N and μ_a can be determined using equations only.

- **Shear force P_s and P_n**

Again from the MCD in Fig. 9.1

$$P_s = P_Z\cos\beta_o - P_{XY}\sin\beta_o \tag{9.24}$$

and $$P_n = P_Z\sin\beta_o + P_{XY}\cos\beta_o \tag{9.25}$$

From P_s, the dynamic yield shear strength of the work material, τ_s can be determined by using the relation,

$$P_s = A_s\tau_s$$

where, A_s = shear area $= \dfrac{ts_o}{\sin\beta}$

Therefore, $$\tau_s = \frac{P_s\sin\beta_o}{ts_o}$$

$$= \frac{(P_Z\cos\beta_o - P_{XY}\sin\beta_o)\sin\beta_o}{ts_o} \tag{9.26}$$

- **Cutting forces in turning under oblique cutting**

In orthogonal cutting, the chip flows along the orthogonal plane, π_o and all the forces concerned, i.e., P_Z, P_{XY}, F, N, P_S and P_n are situated in π_o and contained in the MCD. But in oblique cutting the chip flow is deviated from the orthogonal plane and a force develops along the cutting edge and hence MCD (drawn in π_o) is not applicable. However, since it is a single point tool, only one force will really develop which will have one component along the cutting edge in oblique cutting.

Fig. 9.2 shows how the only cutting force, R can be resolved into

Either, P_X, P_Y and P_Z; which are useful for the purpose of measurement and Design of the M – F – T system

or, P_l, P_m and P_n; which are useful for the purpose of design and stress analysis of the tool and determination of chip-tool interaction in oblique cutting when the chip does not flow along π_o.

For convenience of analysis, the set of force components are shown again in Fig. 9.3 where the cutting force R is resolved into two components R_C and R_r as

$$\overline{R} = \overline{R}_C + \overline{R}_r \qquad (9.27)$$

where, R_C is taken in cutting plane, π_C and R_r in reference plane, π_R.

From Fig. 9.3, the forces in π_C are related as,

$$P_n = P_Z\cos\lambda - P_h\sin\lambda \qquad (9.28)$$

$$P_l = P_Z\sin\lambda + P_h\cos\lambda \qquad (9.29)$$

Where, P_n is acting normal to the cutting edge and P_l is acting along the cutting edge. P_h is an imaginary component along Y_o axis.

Similarly the forces on π_R in Fig. 9.3 are related as,

$$P_m = P_X\sin\phi + P_Y\cos\phi \qquad (9.30)$$

and $$P_h = -P_X\cos\phi + P_Y\sin\phi \qquad (9.31)$$

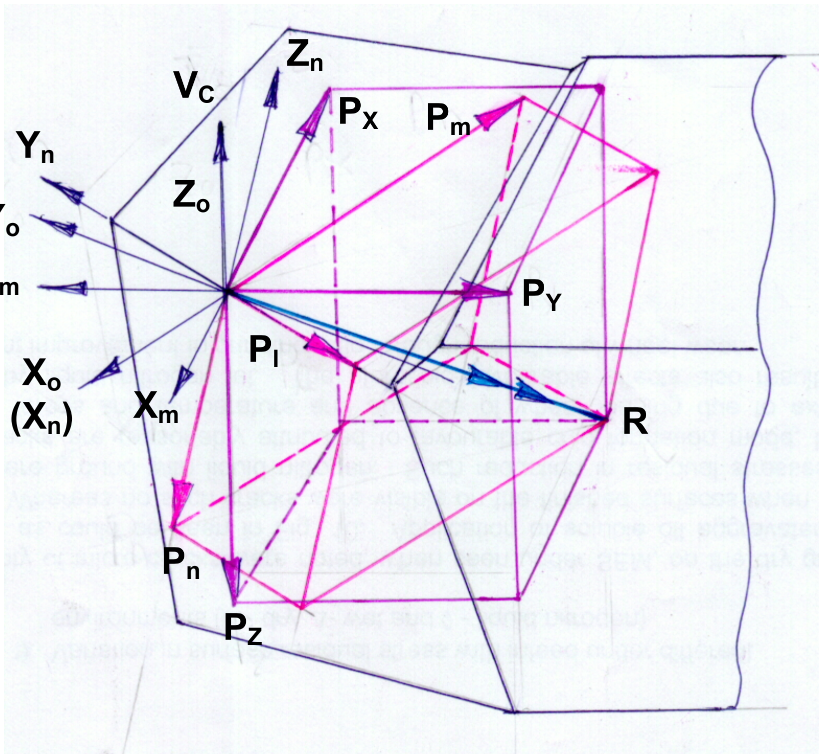

Fig. 9.2 Resolving the cutting force in oblique cutting (turning)

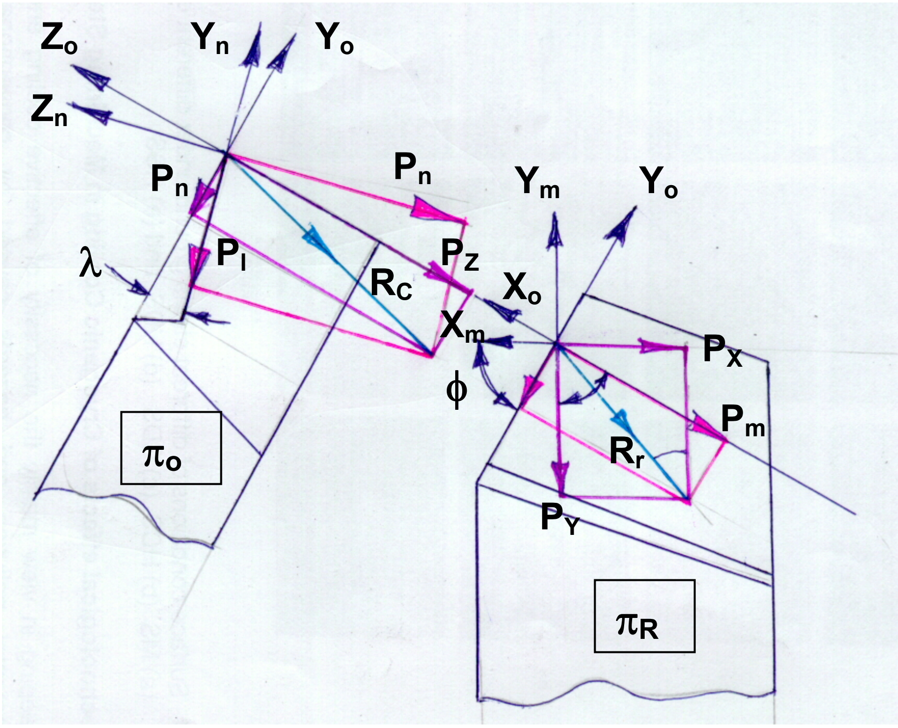

Fig. 9.3 *Resolved components of the cutting force in oblique cutting.*

From equations 9.28 to 9.31, the following three expressions are attained.

$$P_l = -P_X \cos\phi \cos\lambda + P_y \sin\phi \cos\lambda + P_Z \sin\lambda \qquad (9.32)$$

$$P_m = P_X \sin\phi + P_y \cos\phi \qquad (9.33)$$

and $$P_n = P_X \cos\phi \sin\lambda - P_Y \sin\phi \sin\lambda + P_Z \cos\lambda \qquad (9.34)$$

The equations 9.32, 9.33 and 9.34 may be combined and arranged in matrix form as

$$\begin{bmatrix} P_l \\ P_m \\ P_n \end{bmatrix} = \begin{bmatrix} -\cos\phi\cos\lambda & \sin\phi\cos\lambda & \sin\lambda \\ \sin\phi & \cos\phi & 0 \\ \cos\phi\sin\lambda & -\sin\phi\sin\lambda & \cos\lambda \end{bmatrix} \begin{bmatrix} P_X \\ P_Y \\ P_Z \end{bmatrix} \qquad (9.35)$$

The equation 9.35 is very important and useful for evaluating the force components P_l, P_m and P_n from the measured or known force components P_X, P_Y and P_Z in case of oblique cutting.

By inversion of the Eqn. 9.35, another similar matrix form can be developed which will enable evaluation of P_X, P_Y and P_Z, if required, from P_l, P_m and P_n if known other way.

Under oblique cutting, the coefficient of friction, μ_a is to be determined from

$$\mu_a = \frac{F''}{N'} = \frac{\dfrac{F'}{\cos\rho_c}}{N'}$$; ρ_c = chip flow deviation angle $\cong \lambda$

where, F′ and N′ are to be determined from the values of P_n and P_m as,

$$F' = P_n \sin\gamma_n + P_m \cos\gamma_n \quad (9.36)$$

and $$N' = P_n \cos\gamma_n - P_m \sin\gamma_n \quad (9.37)$$

Therefore, under oblique cutting,

$$\mu_a = \frac{P_n \tan\gamma_n + P_m}{\cos\lambda(P_n - P_m \tan\gamma_n)} \quad (9.38)$$

(iii) Analytical Estimation of cutting forces in drilling and milling.

(a) Cutting forces in drilling.

In drilling ductile metals by twist drills, the thrust force, P_X and torque, T can be evaluated using the following equations (Shaw and Oxford) :

$$P_X = K_{x1}.H_B.s_o^{0.8}d^{0.8} + K_{x2}.H_B.d^2 \quad \text{kg} \quad (9.39)$$

and $$T = K_t.H_B.s_o^{0.8}d^{1.8} \quad \text{kg – mm} \quad (9.40)$$

Where, K_{x1}, K_{x2} and K_t are constants depending upon the work material. H_B is Brinnel Hardness and d is drill diameter (mm).

As for example, for steels of $H_B \leq 250$ and $d_c/d = 0.18$ [d_c = chisel edge diameter, mm]

Eqn. 9.39 and 9.40 become

$$P_X = 0.195 H_B s_o^{0.8} d^{0.8} + 0.0022 H_B d^2 \quad (9.41)$$

and $$T = 0.087 H_B s_o^{0.8} d^{1.8} \quad (9.42)$$

The drilling torque and thrust can also be roughly evaluated using following simpler equations:

$$T = C_1 d^x s_o^y \quad \text{(kg – mm)} \quad (9.43)$$

and $$P_X = C_2 d^{x'} s_o^{y'} \quad \text{(kg)} \quad (9.44)$$

Table 9.1 typically shows the approximate values of the constants C_1 and C_2 and the exponents x, y, x′ and y′ for some common engineering materials (Fe-based):

Table 9.1 Constant and exponents in drilling.

Work material	C_1	C_2	x	y	x′	y′
Plain carbon and low alloy steels	35 ~ 55	85 ~ 160	2.0	0.6 ~ 0.8	1.0	0.7
Cast iron BHN 150 ~ 190	20 ~ 23	50	1.9	0.8	1.0	0.8

(b) Cutting forces in Plain milling

In plain or slab milling, the average tangential force, P_{Tavg}, torque, T and cutting power, P_C can be roughly determined irrespective of number of teeth engaged and helix angle, by using the following expressions :

$$P_{Tavg} = \frac{C_p}{\pi} . \frac{B.s_o^x.d^y.Z_C}{D_C^z} \quad \text{kg} \quad (9.45)$$

$$T = P_{Tavg} x \frac{D_C}{2} \quad \text{kg – mm} \quad (9.46)$$

$$\text{and } P_C = P_{Tavg} x V_C \quad \text{kg-m/min} \qquad (9.47)$$

$$= \frac{9.81.P_{Tavg} x V_C}{60x1000} \quad \text{kW}$$

There are several other equations available (developed by researchers) for evaluating milling forces approximately under given cutting conditions.

(iv) Needs and Purposes of Measurement of Cutting Forces

In machining industries and R & D sections the cutting forces are desired and required to be measured (by experiments)

- for determining the cutting forces accurately, precisely and reliably (unlike analytical method)
- for determining the magnitude of the cutting forces directly when equations are not available or adequate
- to experimentally verify mathematical models
- to explore and evaluate role or effects of variation of any parameters, involved in machining, on cutting forces, friction and cutting power consumption which cannot be done analytically
- to study the machinability characteristics of any work – tool pair
- to determine and study the shear or fracture strength of the work material under the various machining conditions
- to directly assess the relative performance of any new work material, tool geometry, cutting fluid application and special technique in respect of cutting forces and power consumption
- to predict the cutting tool condition (wear, chipping, fracturing, plastic deformation etc.) from the on-line measured cutting forces.

(v) General methods of measurement of cutting forces

(a) Indirectly

- from cutting power consumption
- by calorimetric method

Characteristics

- o inaccurate
- o average only
- o limited application possibility

(b) Directly

Using tool force dynamometer(s)

Characteristics

- o accurate
- o precise / detail
- o versatile
- o more reliable

Exercise – 9

[Problems and solutions]

Q.1 If, in orthogonal turning a tool of $\gamma_o = 0^o$ and $\phi = 90^o$, the force components, P_X and P_Z are measured to be 400 N and 800 N respectively then what will be the value of the apparent coefficient (μ_a) of friction at the chip tool interface at that condition?

[solve using equations only]

Solution :

It is known that, $\mu_a = F/N$

where, $F = P_Z\sin\gamma_o + P_{xy}\cos\gamma_o$ and $N = P_Z\cos\gamma_o - P_{xy}\sin\gamma_o$

Now, $P_{xy} = P_X/\sin\phi = 400/\sin 90^0 = 400\text{N}$.

$\sin\gamma_o = \sin 0^0 = 0$

$\cos\gamma_o = \cos 0^0 = 1$.

$\mu_a = P_{xy}/P_Z = 400/800 = 0.5$ Ans.

Q.2 Determine without using MCD, the values of P_S (shear force) and P_N using the following given values associated with a turning operation : $P_Z = 1000$ N, $P_X = 400$ N $P_Y = 200$ N, $\gamma_o = 15^o$ and $\zeta = 2.0$

Solution :

The known relations are:

$P_S = P_Z\cos\beta_o - P_{XY}\sin\beta_o$

$Pn = P_Z\sin\beta_o + P_{XY}\cos\beta_o$

Let b_0(shear angle) from

$\tan\beta_o = \cos\gamma_o/(\zeta - \sin\gamma_o)$

$= \cos 15^0/(2.0 - \sin 15^0) = 0.554$

$\therefore$ $\beta_o = 29^0$; $\cos\beta_o = 0.875$

and $\sin\beta_o = 0.485$

$$P_{XY} = \sqrt{(P_X)^2 + (P_Y)^2} = \sqrt{(400)^2 + (200)^2} = 445 \text{ N}$$

So, $P_S = 1000\text{x}0.875 - 445\text{x}0.485 = 659$ N

and $P_n = 1000\text{x}0.485 + 445\text{x}0.875 = 874$ N

Q. 3 During turning a steel rod of diameter 150 mm by a carbide tool of geometry;

0^o, $—12^o$, 8^o, 6^o, 15^o, 60^o, 0 (mm)

at speed 560 rpm, feed 0.32 mm/rev. and depth of cut 4.0 mm the followings were observed :

P_Z= 1000 N, P_Y= 200 N, a_2=0.8 mm

Determine, without using MCD, the expected values of F, N, μ, P_S, P_n, τ_s, cutting power and specific energy requirement for the above mentioned machining operation.

Solution :

- $P_{XY} = P_X/\sin\phi = 200/\cos60^o = 400$ N
- $F = P_Z\sin\gamma_o + P_{XY}\cos\gamma_o$;

Here $\gamma_o = - 12^o$ \ $\sin\gamma_o = - 0.208$ and $\cos\gamma_o = 0.978$

$F = 1000(-0.208) + 400(0.978) = 600$ N ans.

and $N = P_Z\cos\gamma_o — P_{XY}\sin\gamma_o$

$= 1000(0.978) — 400(-0.208)$

$= 1060$ N answer

So, $\mu_a = F/N = 600/1060 = 0.566$ answer

- $P_S = P_Z\cos\beta_o — P_{XY}\cos\beta_o$

where $\beta_o = \tan^{-1}(\cos\gamma_o/(\zeta—\sin\gamma_o))$

Here, $\zeta = a_2/(s_o\sin\phi) = 0.8/(0.32\text{x}\sin60^o) = 2.88$

$\beta_o = \tan^{-1}\{(0.978/(2.88+0.208)\} = 17.6^o$

So, $P_S = 1000\text{x}\cos(17.6^o) – 400\text{x}\sin(17.6^o) = 832$ N **answer**

and $P_N = 1000\sin(17.6^o) + 400\cos(17.6^o) = 683$ N **answer**

- $P_S = (ts_o\ \tau_s)/\sin\gamma_o$

$\therefore \tau_s = P_S\sin\gamma_o /(ts_o) = 832\sin(17.6^o)/(4\text{x}0.32)$

$= 200$ N/mm^2 answer

- Cutting power, $P_C = P_Z.V_C$

where $V_C = \pi DN/1000 = \pi\text{x}150\text{x}560/1000 = 263$ m/min

$\therefore P_C = 1000\text{x}263$ N-m/min = 4.33 KW **answer**

- Specific energy consumption, E_C

E_C= power/MRR = $(P_Z.V_C)/(V_C.s_o.t)$ N-m/m-mm^2

= 1000x263 (Joules/min)/{263x0.32x4x1000(mm^3/min)}

= 0.78 Joules/mm^3 **answer**

Q.4 During turning a steel rod of diameter 100 mm by a ceramic tool of geometry:

0^o, $—10^o$, 8^o, 7^o, 15^o, 75^o, 0.5 (mm)

at speed 625 rpm, feed 0.36 mm/rev. and depth of cut 5.0 mm the average chip thickness was found to be 1.0 mm. Roughly how much power will be consumed in the above mentioned machining work if;

(i) the work material is semi-ductile

(ii) Brinnel hardness number of the work material is 240 (kg/mm^2)

Solution :

Cutting power, $P_C = P_Z . V_C$ N.m/min.

V_C= Cutting Velocity = πDN/1000 m/min.

= (π x100x625)/1000 =196 m/min.

$P_Z = ts_o\tau_s\cos(\eta - \gamma_o)/\{\sin\beta_o.\cos(\beta_o + \eta - \gamma_o)\}$

For semi-ductile materials, the angle relationships that may be taken

$2\beta_o + \eta - \gamma_o = \pi/2$ [Earnst & Merchant]

Then, $P_Z = 2ts_o\cot\beta_o$

Get shear angle, β_o from,

$\tan\beta_o = (\cos\gamma_o) / (\zeta - \sin\gamma_o)$

where, $\zeta = a_2/a_1 = a_2/s_o\sin\phi = 1.0/(0.36.\sin75^0) = 2.87$

$\beta_o = \tan^{-1}\{\cos(-10^0)/(2.87-\sin(-10^0)\} = 17.9^0$

$\therefore$Shear strength, τ_s = 0.186 BHN

= 0.186x240x9.81 N/mm^2

= 424N/mm^2

So, P_Z= 2 x5x0.36x424xcot(17.9^0) = 4697 N. **<u>Ans</u>.**