

METAL MACHINING



Presented by:

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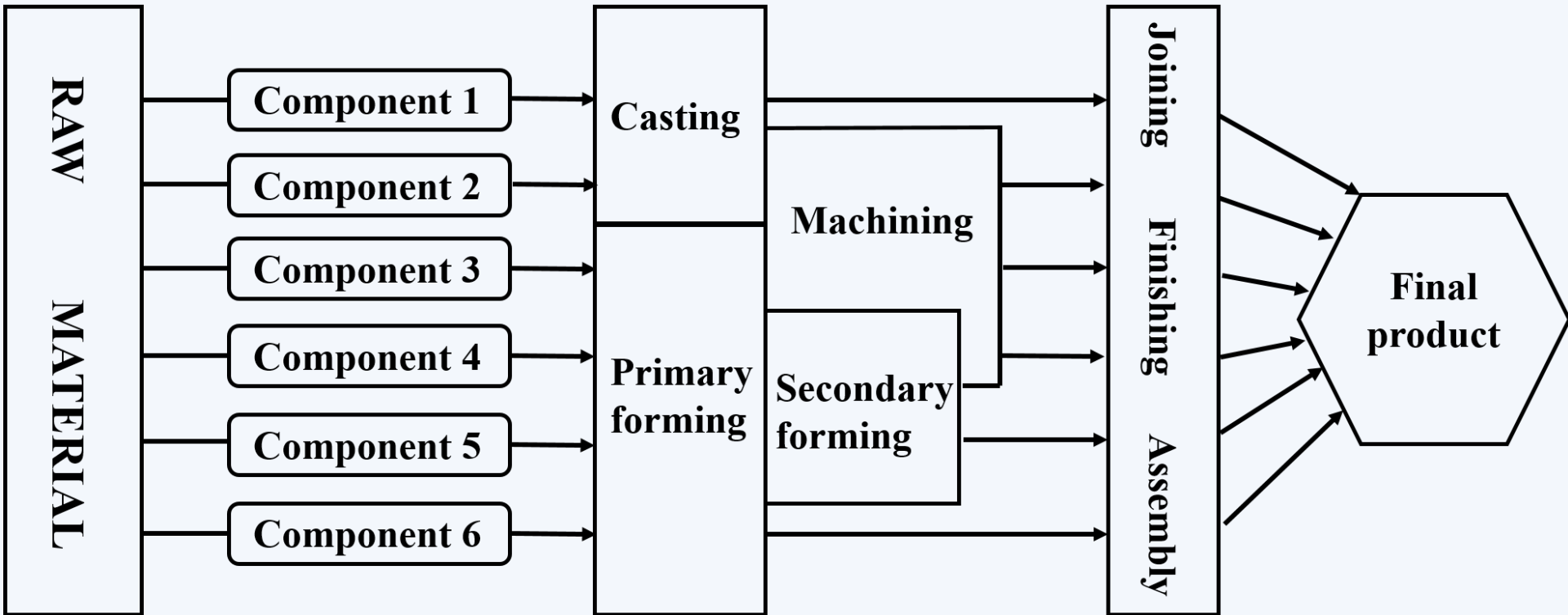
Indian Institute of Technology Kanpur

Kanpur, India





Manufacturing Operation

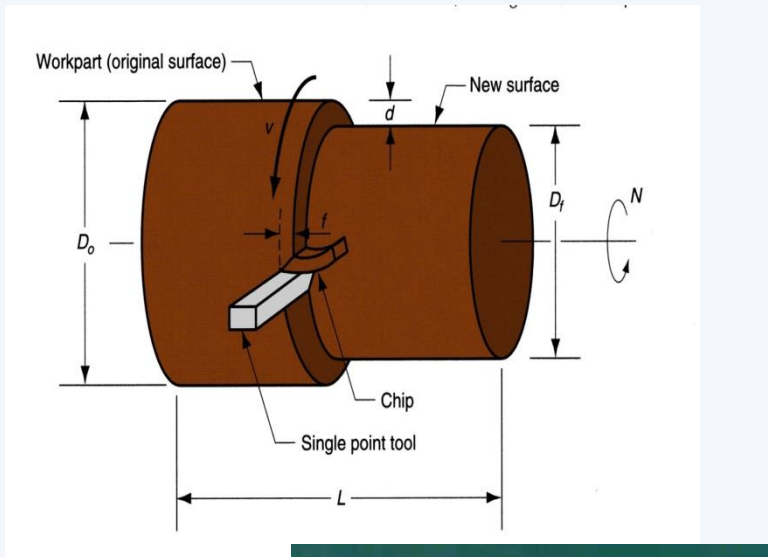
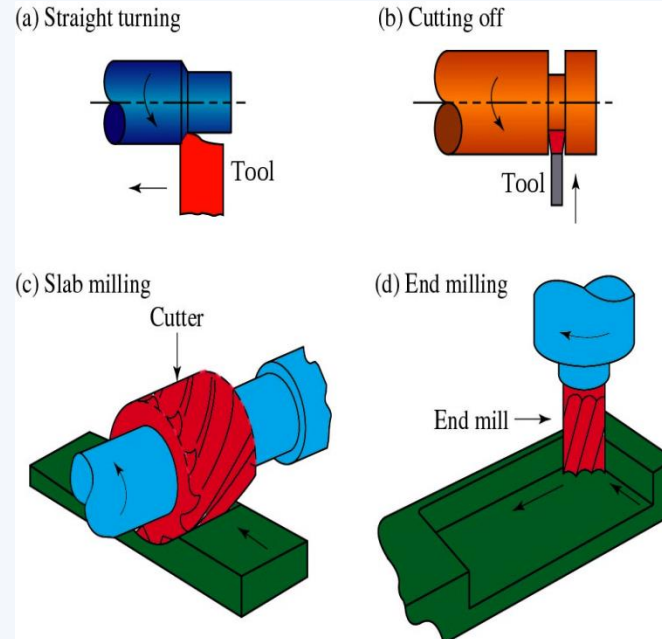


Manufacturing Processes

Machining Operations

In **MACHINING**, the shape, size, finish and accuracy are obtained by removing the excess material from the workpiece surface.

Various surfaces are obtained as an interaction between a workpiece and a cutting tool with the help of a contrivance known as **MACHINE TOOL**.



Advantages and Disadvantages of Machining

- Variety of work materials can be machined.
 - Most frequently used to cut metals
- Variety of part shapes and special geometric features possible, such as:
 - ❑ Screw threads
 - ❑ Accurate round holes
 - ❑ Very straight edges and surfaces
- Good dimensional accuracy and surface finish
- Generally performed after other manufacturing processes, such as casting, forging, and bar drawing

Disadvantages:

Wasteful of material and time consuming



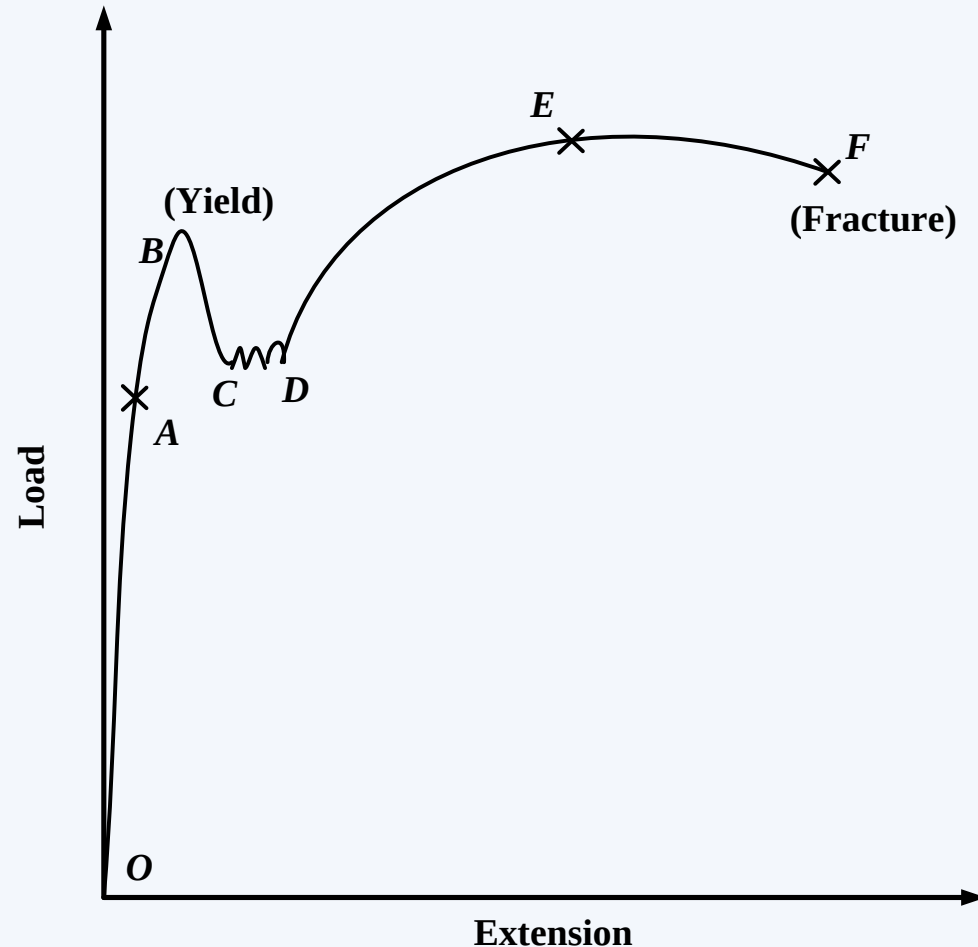
Mechanism of Machining

- Earlier it was believed that when metal is cut, the material merely splits off in front of the tool (similar to chopping of wood)
- Subsequently it was realised that the cutting of metal involved deformation process.
- Further experiments revealed that the deformation was principally one of shear and that the type of chips varied with the work material and cutting conditions.
- Extensive research revealed that the process essentially involves plastic deformation and fracture under high strain rate and high temperature.



Tensile Test – Mild steel bar

- Point A ⇒ Proportionality limit
- Point B ⇒ Elastic limit
- Point C ⇒ Lower yield point
- Point E ⇒ Necking point
- Point F ⇒ Fracture point



Load-extension curve for mild steel

Tensile Test – Mild steel bar

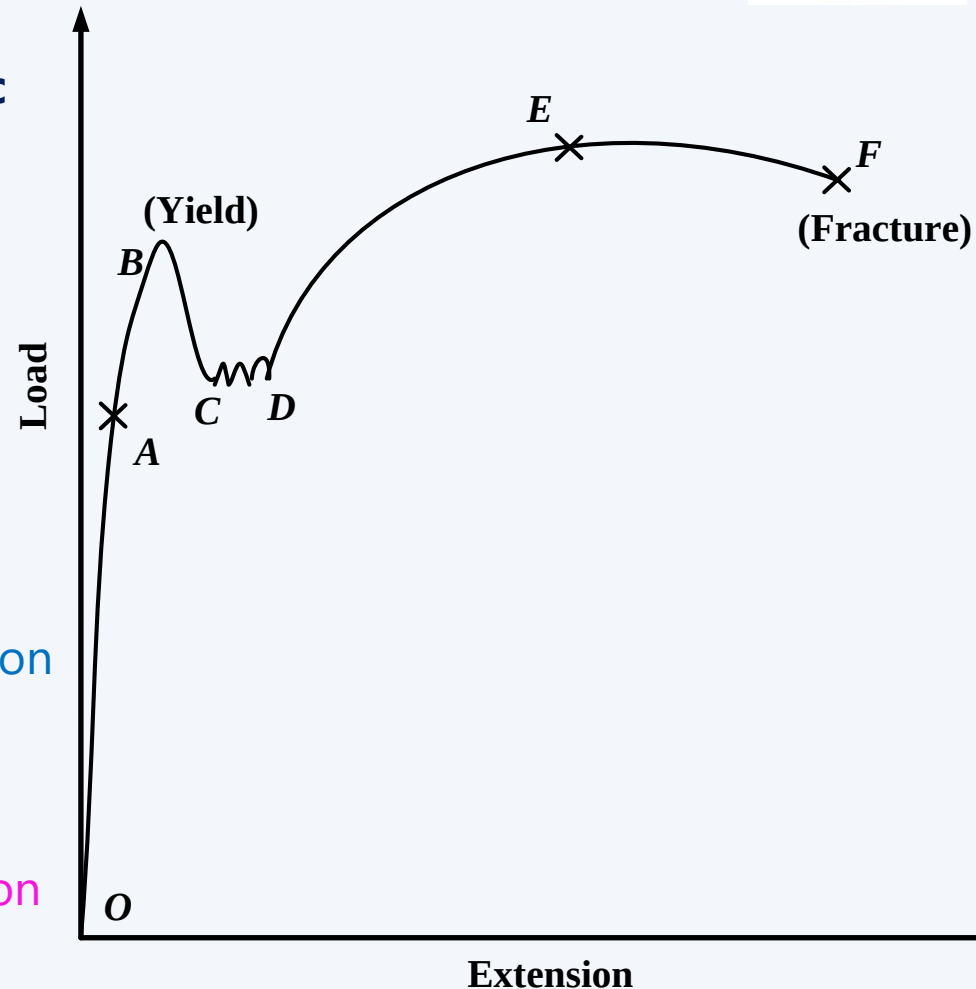
□ Region OA $\Rightarrow$ Material exhibit elastic behaviour

- Increase in length is proportional to the applied load (**linear in nature**)
- When applied load is removed the bar returns to its original length

□ Region AB $\Rightarrow$ Material is still elastic

- Relationship between load and extension no longer remains linear
- Maximum load that can be applied without causing permanent deformation defines the elastic limit, i.e., point 'B'

□ Beyond point 'B' metal no longer behaves elastically and removal of the load leaves the bar with permanent extension (Plastic in nature)



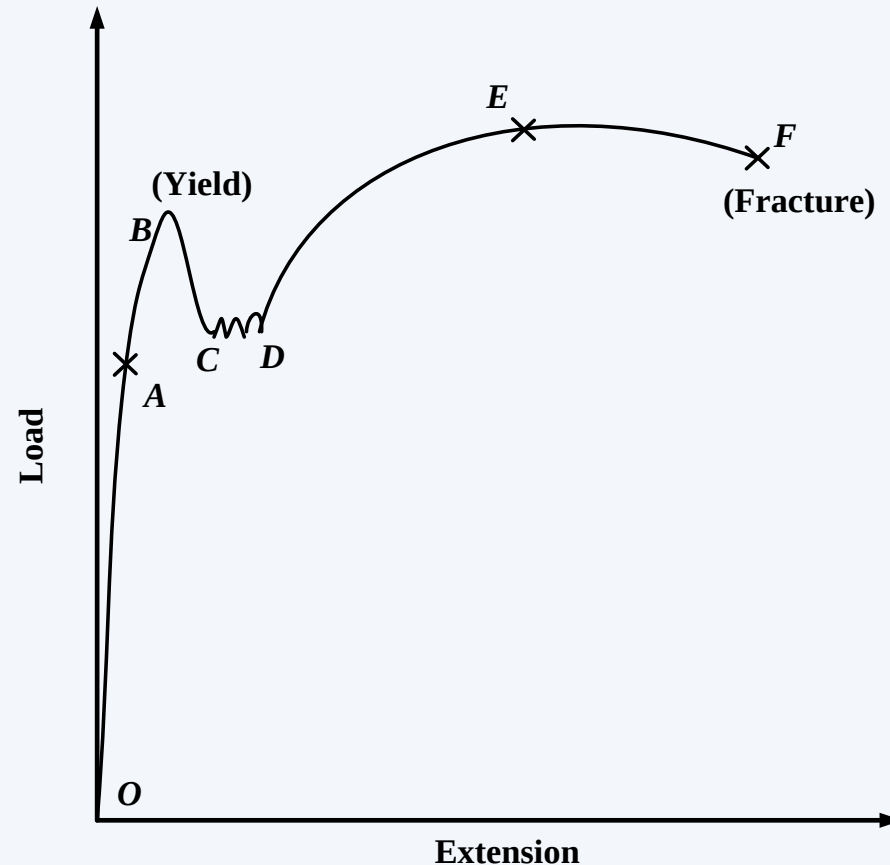
Tensile Test – Mild steel bar

☐ Just above point B $\Rightarrow$ Upper yield point

- Elastic behaviour of the material ceases and yielding starts is called **the yield point**
- Stress at point B is usually called **upper yield stress**

☐ Region CD:

- Yielding in mild steel, is accompanied by a sudden drop in load and increase in length at approximately constant load
 - Stress at point C is usually called **lower yield stress**
- This upper yield stress and lower yield stress is characteristic of mild steel, caused by the presence of small amount of foreign atoms in the material



▪ Carbon

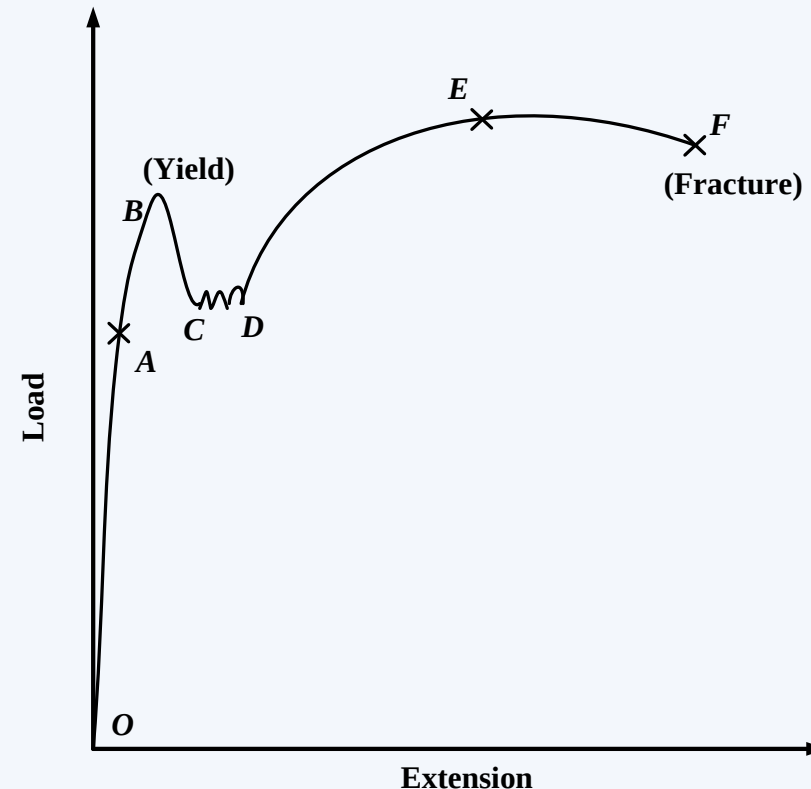
▪ Nitrogen

Tensile Test – Mild steel bar

- Most metals and alloys do not show this type of pronounced initial yielding and change from **ELASTIC** to **ELASTIC-PLASTIC** deformation is gradual

□ Region DE $\Rightarrow$ Strain hardening

- Following the transition from elastic to plastic behaviour, the load required for further deformation increases
- The material being able to withstand the increased load despite the uniform reduction in cross sectional area is called **strain-hardening or work-hardening**

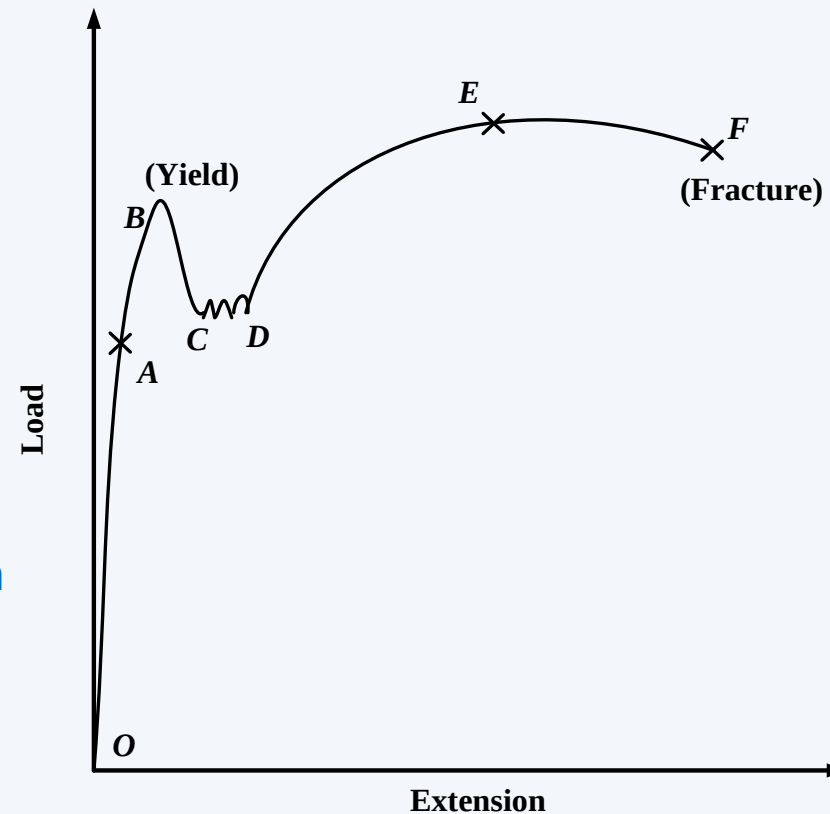


- Two opposing factors operate to determine the load required for a given extension
 - Strain-hardening leads to an increase of load
 - Reduction in cross-sectional area leads to a decrease of load

Tensile Test – Mild steel bar

□ At point E ➡ Necking

- When extension becomes large, the strain hardening is unable to cope up with the decrease in cross-sectional area
- This leads to localized deformation in the specimen and is indicative of an instability condition
- This leads to a decrease in cross-sectional area and increase in the local stress so that further elongation occurs in the thinned portion only and a **neck is developed**



□ At point F ➡ Fracture

- Straining takes place under a complicated and continuously changing triaxial tensile stress system

Stress and Strain

- The mode of deformation of a material under applied load will depend not only on the magnitude of forces and the mechanical properties of the material, but also on the way in which the forces are applied and combined.
- Nominal Stress is defined as the load divided by the original cross sectional area of the bar .
- Engineering Strain is defined as the extension per unit original length.
- The load-extension diagram of a bar with unit cross sectional area and unit gauge length is therefore the nominal Stress-Strain curve for the material.
- Stress, when considered as acting perpendicular to a surface, called Normal or Direct Stress, when parallel to a surface, they are called shear stresses.



Stress and Strain

- At any point in a stressed body the stresses may be resolved into nine components acting on the face of an imaginary cube.
- The axes defining the faces of the cube may be chosen in any direction and for some purpose, it is convenient to choose axes so that the shear stresses are zero. The normal stresses acting are then known as Principal Stresses.
- Mohr's stress-circle construction is a convenient means for relating the stresses on any plane to the principal stress.



If l_0 is the original length of a bar, and
 l is the current gauge length after elongation,
Then, True or Natural Strain, e is defined in terms of the strain increment, i.e.,

$$e = \int_{l_0}^l \frac{dl}{l} = \ln \left(\frac{l}{l_0} \right)$$

The advantage in using the logarithmic definition of strain is that logarithmic strains are additive in sequential process, which is not true of linear strains.

Suppose, a bar is strained during a tensile test from l_0 to l_1 . The true strain,

$$e' = \ln \left(\frac{l_1}{l_0} \right)$$

If the bar is further extended to a length l_2 , the further strain will be

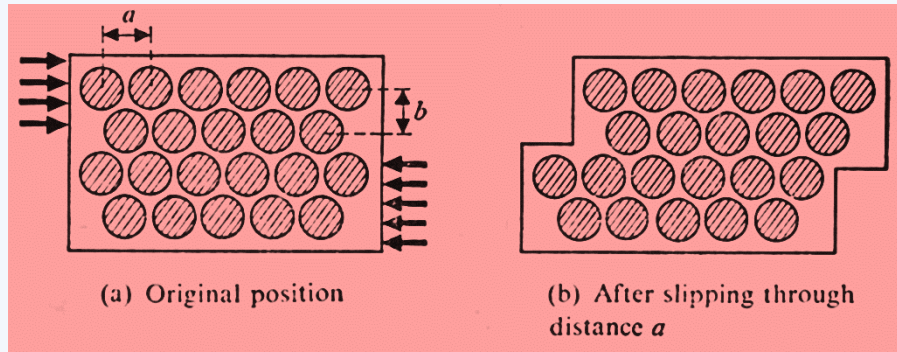
$$e'' = \ln \left(\frac{l_2}{l_1} \right)$$

$$e = e' + e'' = \ln \left(\frac{l_1}{l_0} \right) + \ln \left(\frac{l_2}{l_1} \right) = \ln \left(\frac{l_2}{l_0} \right)$$



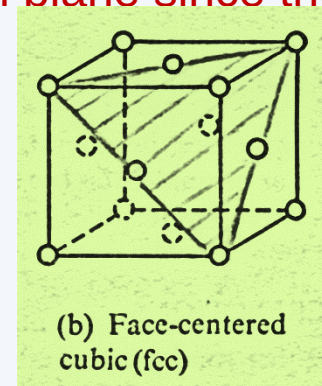
Mechanism of Plastic Deformation

- For plastic deformation to occur, it is necessary to have large scale slipping, where two planes of atoms slip past each other causing one entire section to move relative to another.



- Slip occurs more easily on certain crystallographic planes depending on the crystal structure. These are known as SLIP PLANES.
- Crystallographic planes that are furthest apart are also the ones of the greatest atomic density. Slip tends to occur on such plane since the resistance to slip is then a minimum.

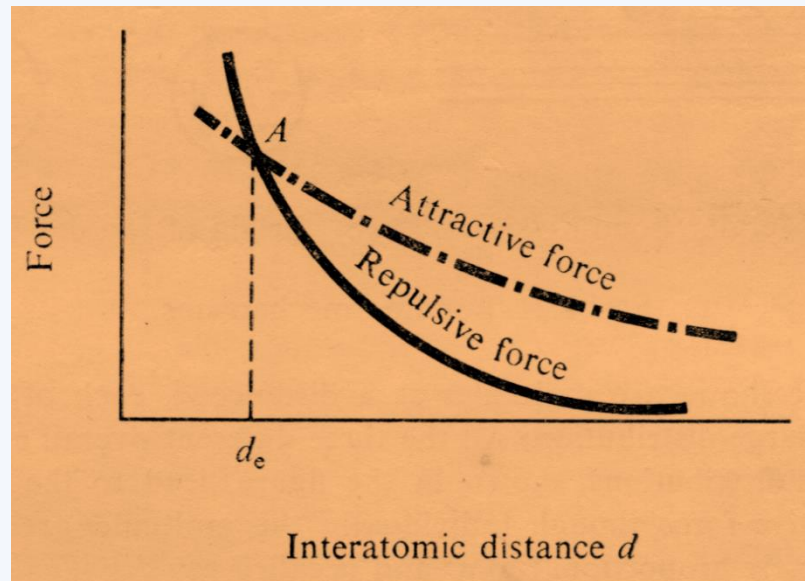
Example:



Mechanism of Plastic Deformation

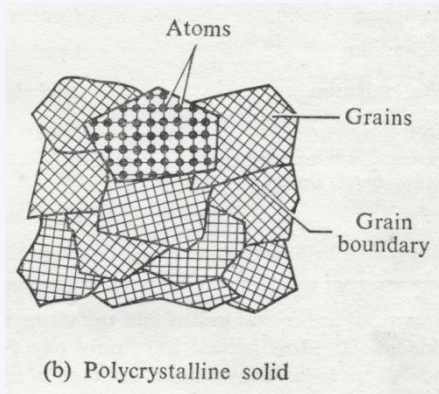
How Slip Occurs?

- When two atoms are sufficiently close to each other, the outer electrons are shared by both the nuclei.
- Result: Attractive force between two atoms and repulsive force when two nuclei come very close to each other.



Mechanism of Plastic Deformation

- These atoms form a polycrystalline solid with atoms in equilibrium position.

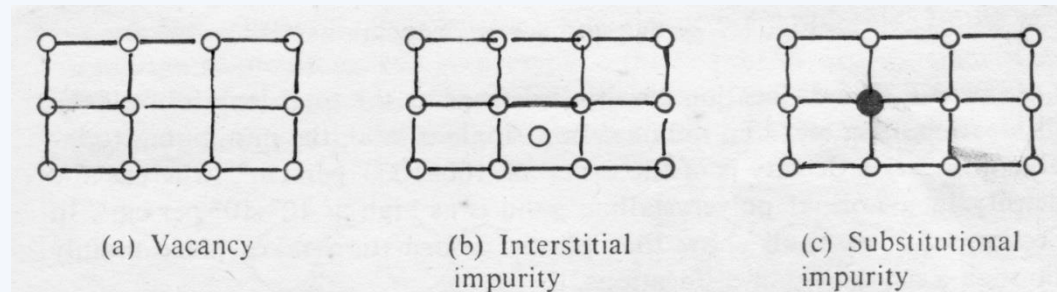


- Crystals are not perfect, i.e, lattices are not without imperfections.

Imperfections:

- Point Defect
- Line Defect
- Surface Defect

Point Defect:



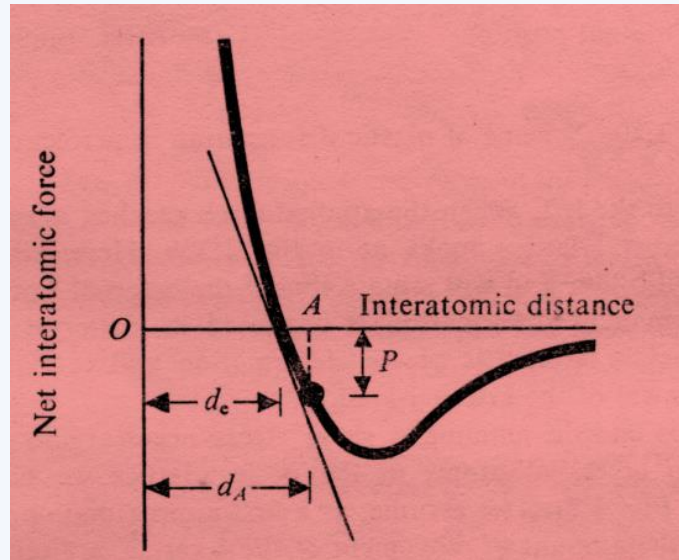
Line Defect (or dislocation): If an imperfection extending along a line has a length much larger than the lattice spacing.

Surface Defect: When an imperfection extends over a surface.

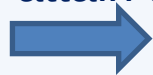


Mechanism of Plastic Deformation

Elastic and Plastic Deformation in Atomic Scale :



- if d_A lies within 5% of d_e , then upon removal of external forces the atoms attain their original position,



Elastic Deformation.

- if d_A becomes more than 5% of d_e , then upon removal of external forces the atoms do not come back to their original position,

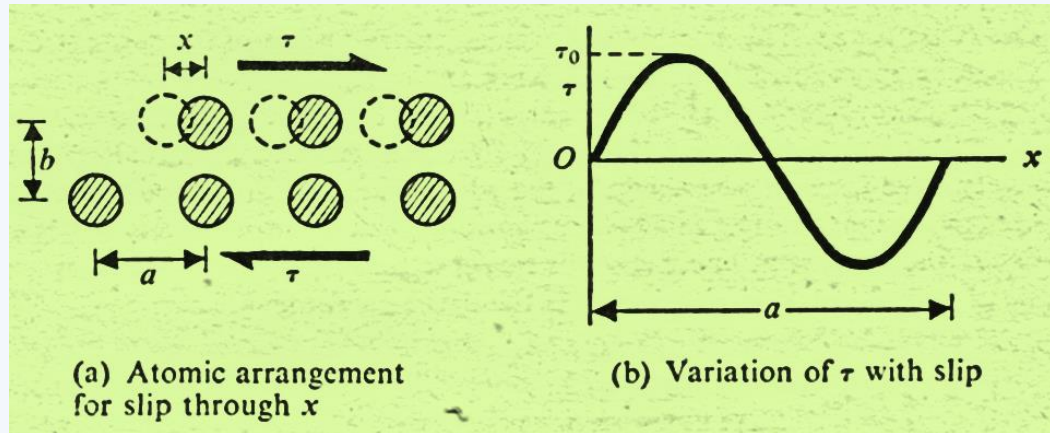


Plastic Deformation.



Mechanism of Plastic Deformation

The amount of Shear Stress necessary to effect the Slip:



Let us assume, $\tau = \tau_0 \sin\left(\frac{2\pi x}{a}\right)$; for small values of $\left(\frac{x}{a}\right)$, it can be expressed as:

$$\tau = \tau_0 \left(\frac{2\pi x}{a}\right) \quad \text{Strain} = \left(\frac{x}{b}\right)$$

Therefore,

$$\tau = \tau_0 \left(\frac{2\pi x}{a}\right) = G \left(\frac{x}{b}\right) \quad \text{or, } \tau_0 = \left(\frac{G}{2\pi}\right) \left(\frac{a}{b}\right)$$

As a rough approximation, let us assume $a = b$, then

$$\tau_0 = \left(\frac{G}{2\pi}\right)$$

In practice the shear stress required is much less.

Basic Machining Parameters

Speed (V) [m/min]

- Relates velocity of the cutting tool to the work piece (Primary motion).

Feed (f) [mm/rev]

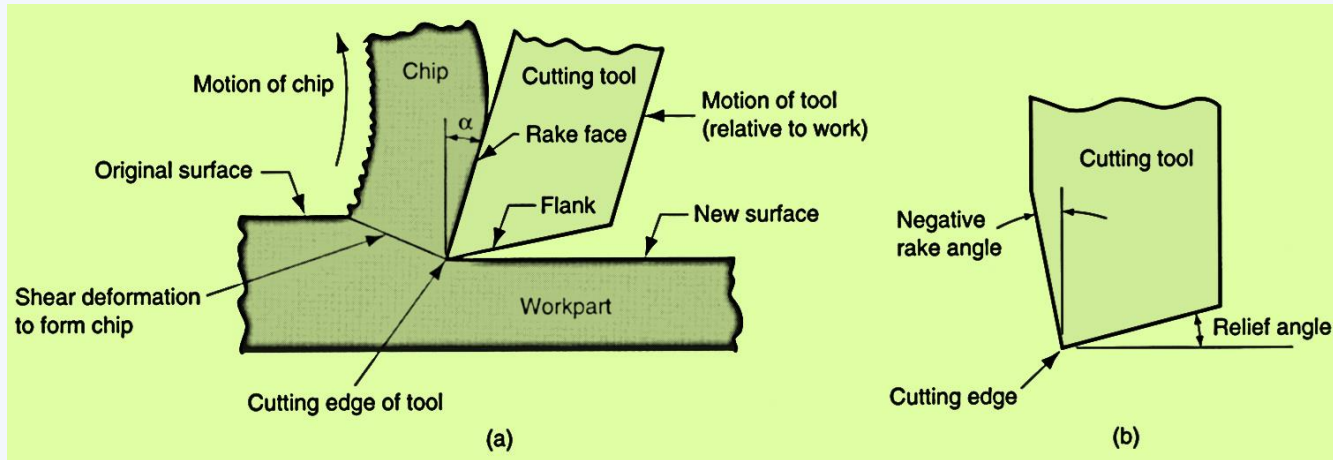
- Movement (advancement) of the tool per revolution of the workpiece

Depth of Cut (d) [mm]

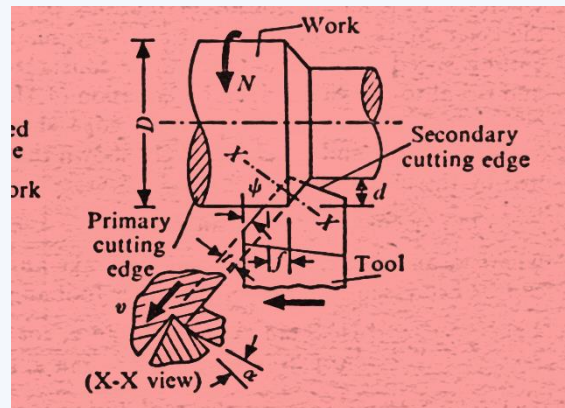
- Distance the tool has plunged into the surface



Chip Formation



Cutting action involves shear deformation of work material to form a chip. As chip is removed, new surface is exposed.

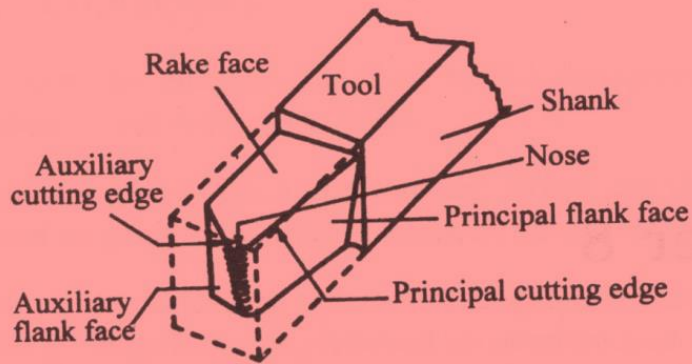


$$t_1 = f \cos \gamma_s$$

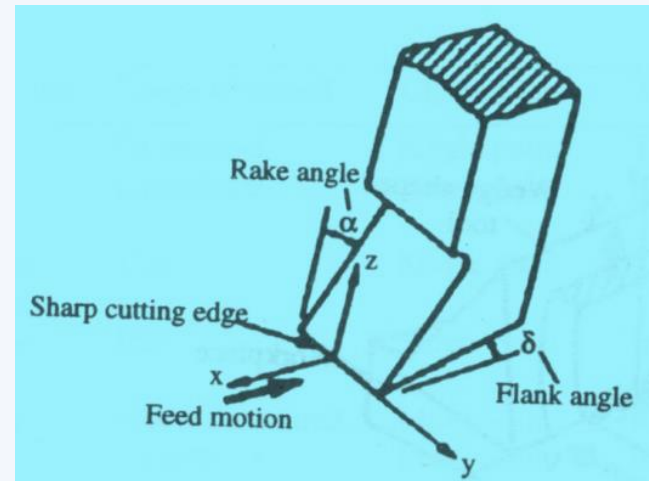
$$W = \frac{d}{\cos \gamma_s}$$



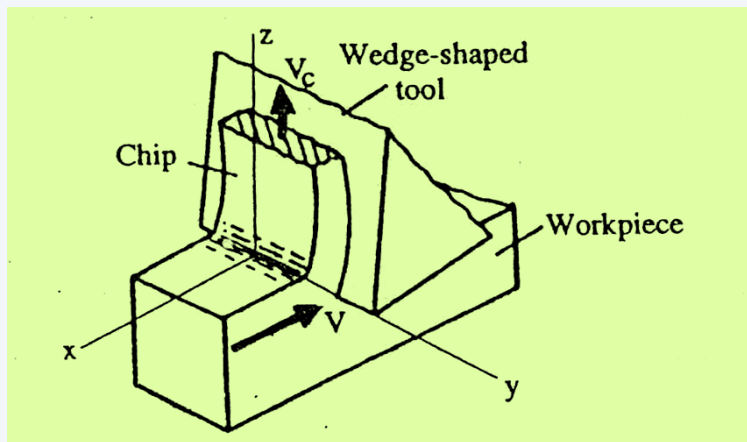
Cutting Tools & Types of Machining



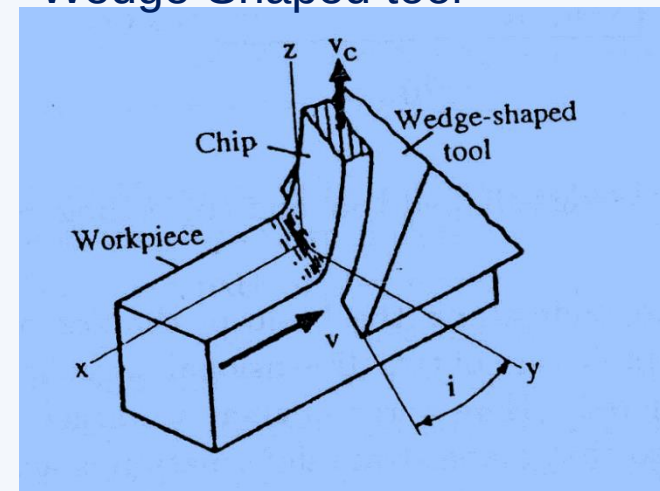
A Typical Lathe Tool



Wedge-Shaped tool



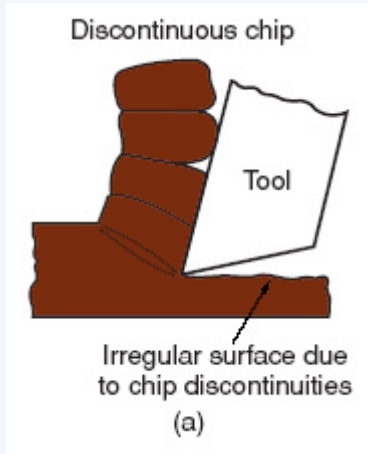
Orthogonal Cutting



Oblique Cutting



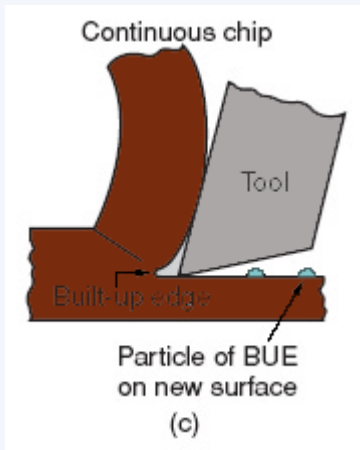
Types of Chips



- Brittle work materials
- Low cutting speeds
- Large feed and depth of cut
- Small rake angle
- High tool-chip friction



- Ductile work materials
- High cutting speeds
- Small feeds and depths
- Large rake angle
- Sharp cutting edge
- Low tool-chip friction



- Ductile materials
- Low-to-medium cutting speeds
- Large feed
- Small rake angle
- Tool-chip friction causes portions of chip to adhere to rake face
- Built up Edge (BUE) forms, then breaks off, cyclically



Built-up Edge

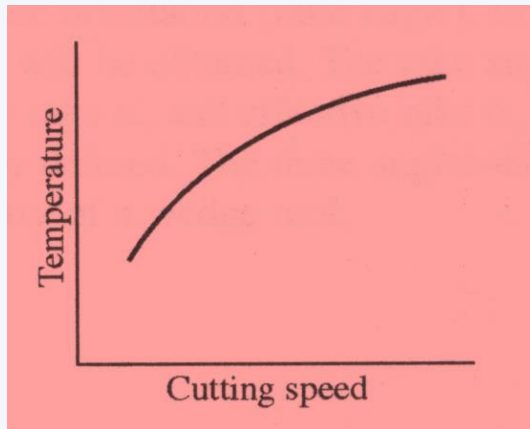
Built-up edge involves:

a) **Welding**, and

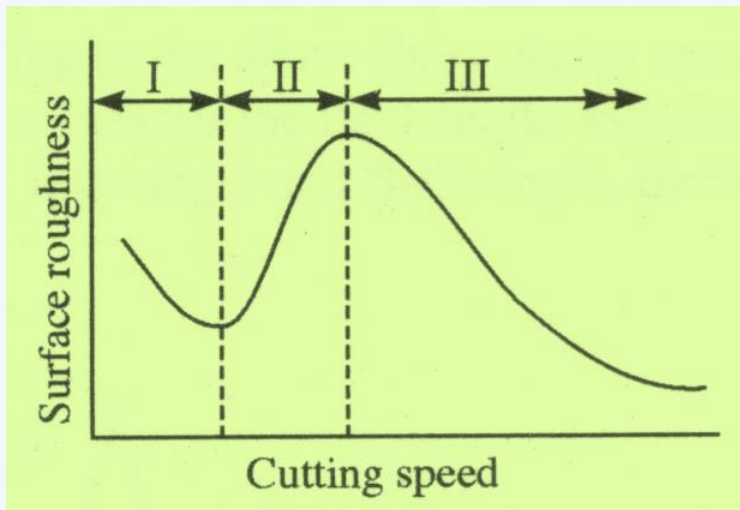
b) **Rupture of chip material**

- Welding is affected by the temperature and the normal pressure near the tool tip
- **Rupture is influenced by strain hardening and thermal softening of the chip material.**





$$\theta \propto \sqrt{V_c}$$



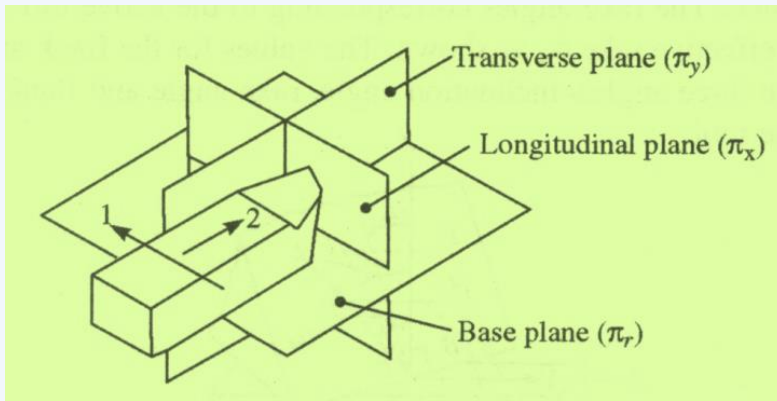
Zone I : Discontinuous chip. Initially poor surface finish. It improves as speed increases and the chip becomes semi-discontinuous.

Zone II : BUE is formed; continue till the recrystallization temperature is reached.

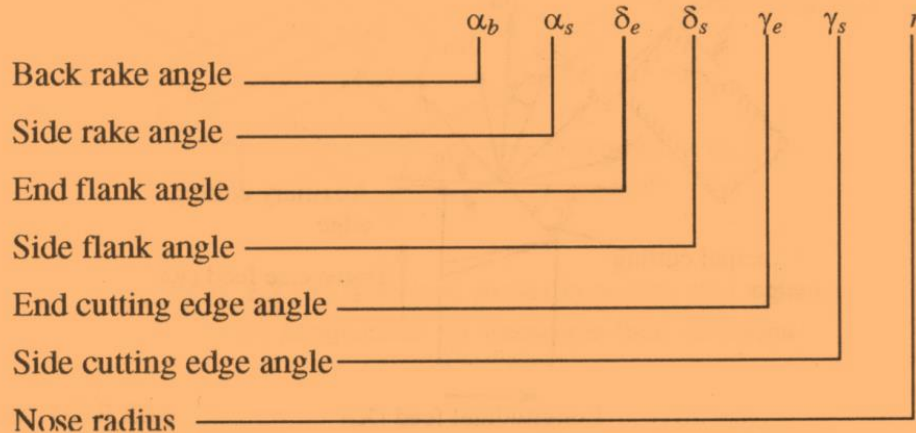
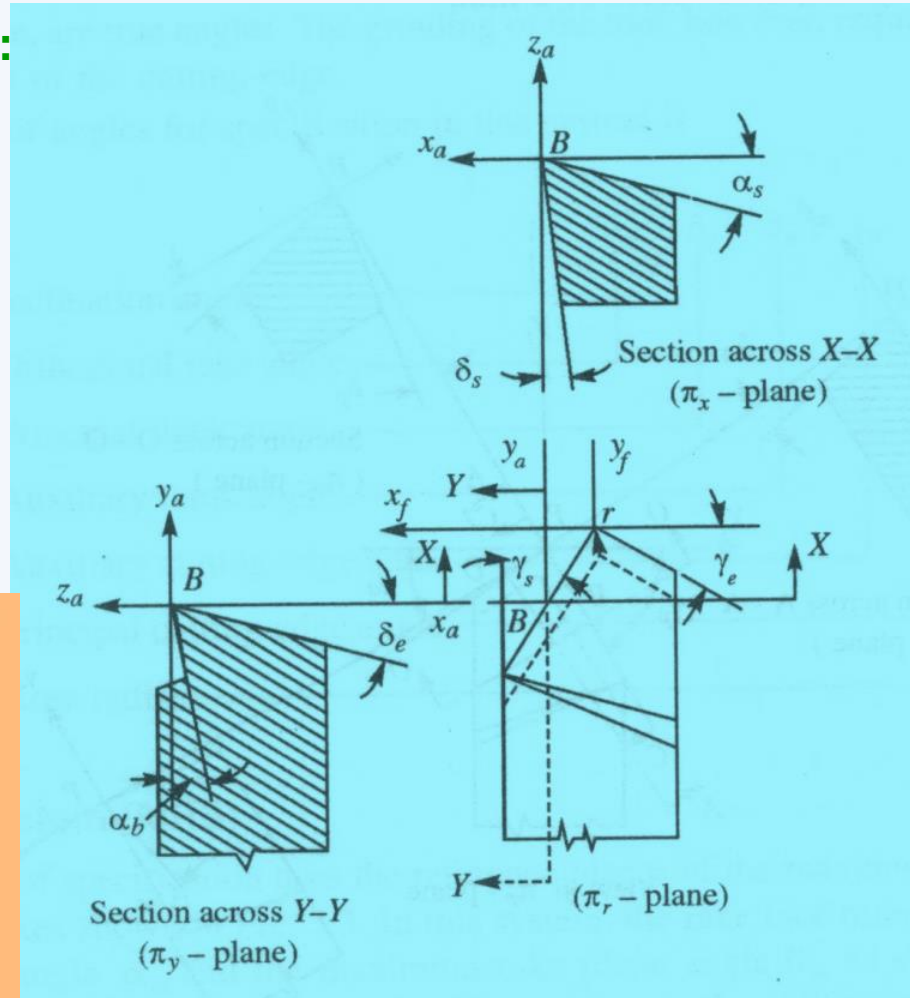
Zone III : Continuous chip without BUE.

Tool Nomenclature

American or Coordinate System (ASA):

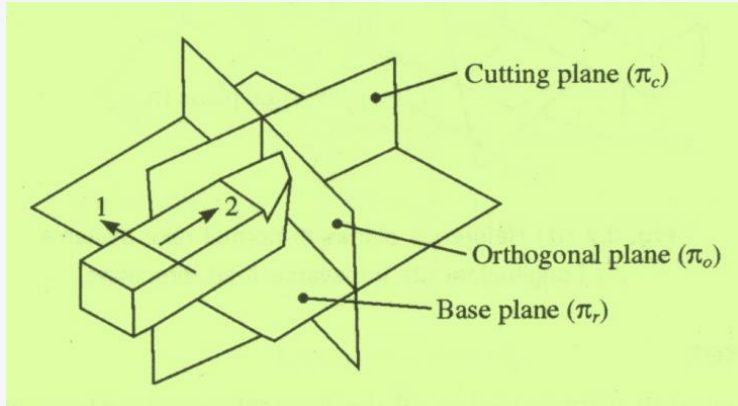


Reference Planes

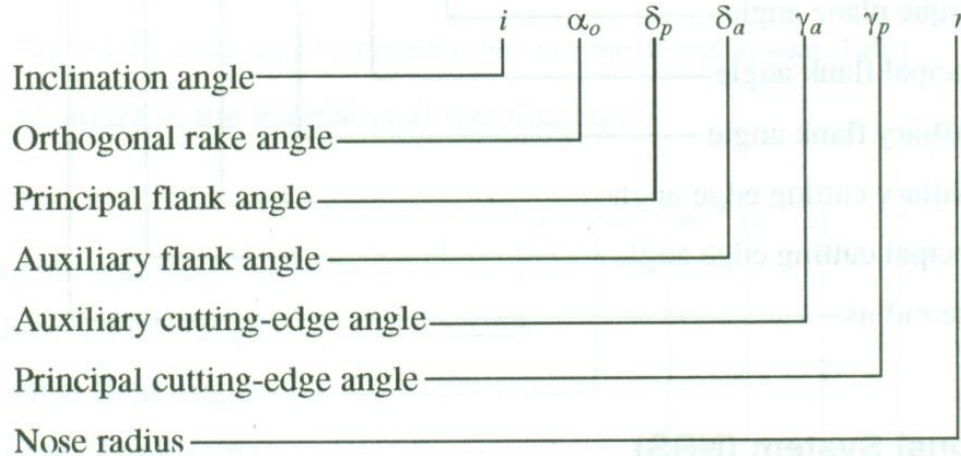
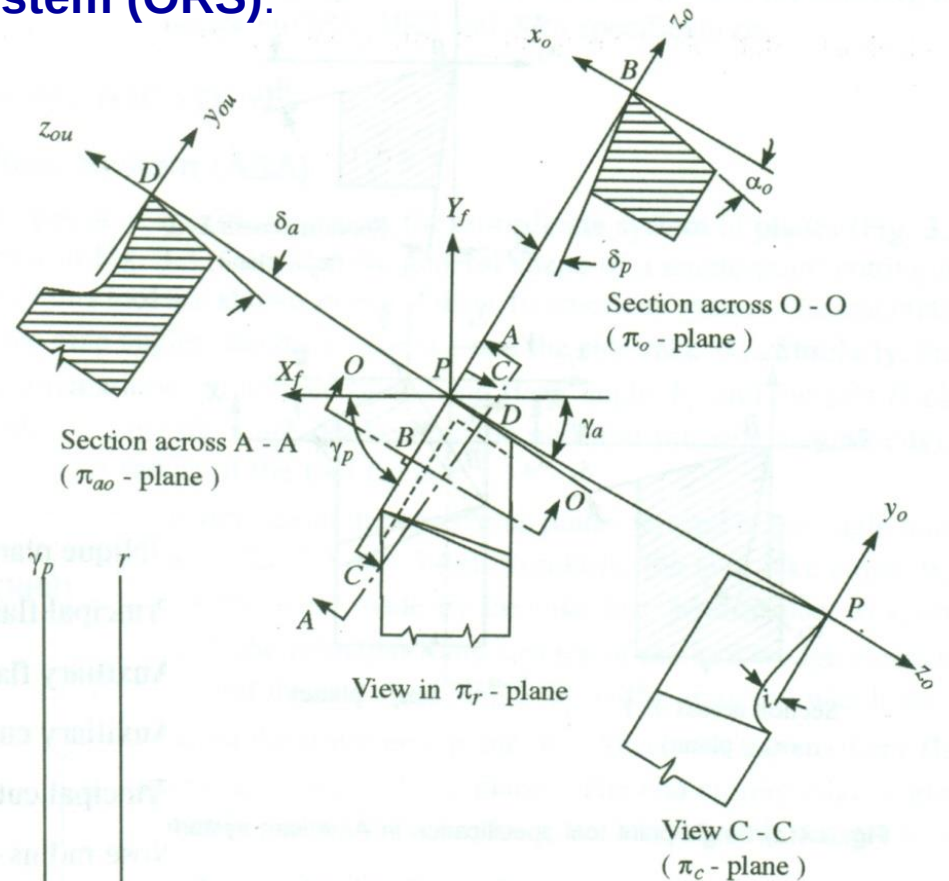


Tool Nomenclature

Continental or Orthogonal Rake System (ORS):

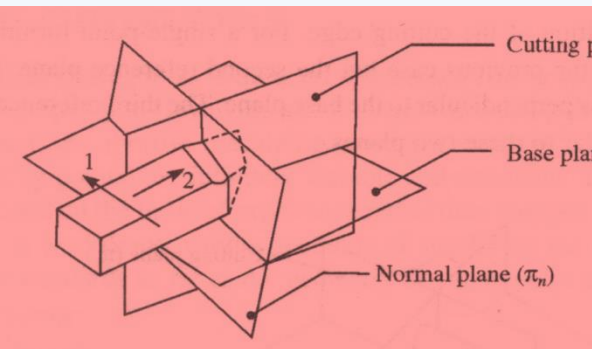


Reference Planes

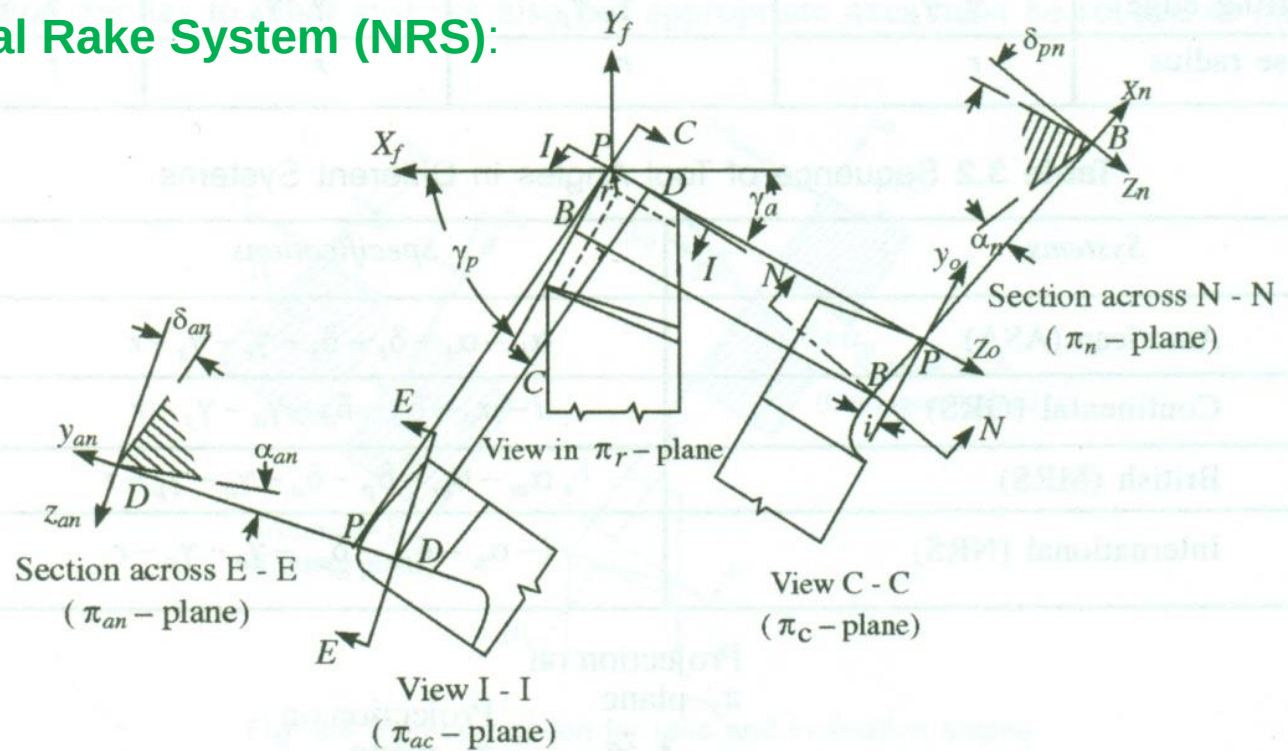


Tool Nomenclature

International or Normal Rake System (NRS):



Reference Planes



Inclination angle	i
Orthogonal rake angle	α_n
Principal flank angle	δ_{pn}
Auxiliary flank angle	δ_{an}
Auxiliary cutting-edge angle	γ_a
Principal cutting-edge angle	γ_p
Nose radius	r

Conversion from ORS to ASA System

$$\tan i = \frac{B_c B'_c}{A_c B'_c} = \frac{T_x T_x'' - T_x T'_x}{AB} = \frac{T_y T_y'' - T_x T'_x}{AB}$$

$$= \frac{A_y T_y'' \tan \alpha_b - T'_x B_x \tan \alpha_s}{AB}$$

$$= \frac{AT}{AB} \tan \alpha_b - \frac{BT}{AB} \tan \alpha_s$$

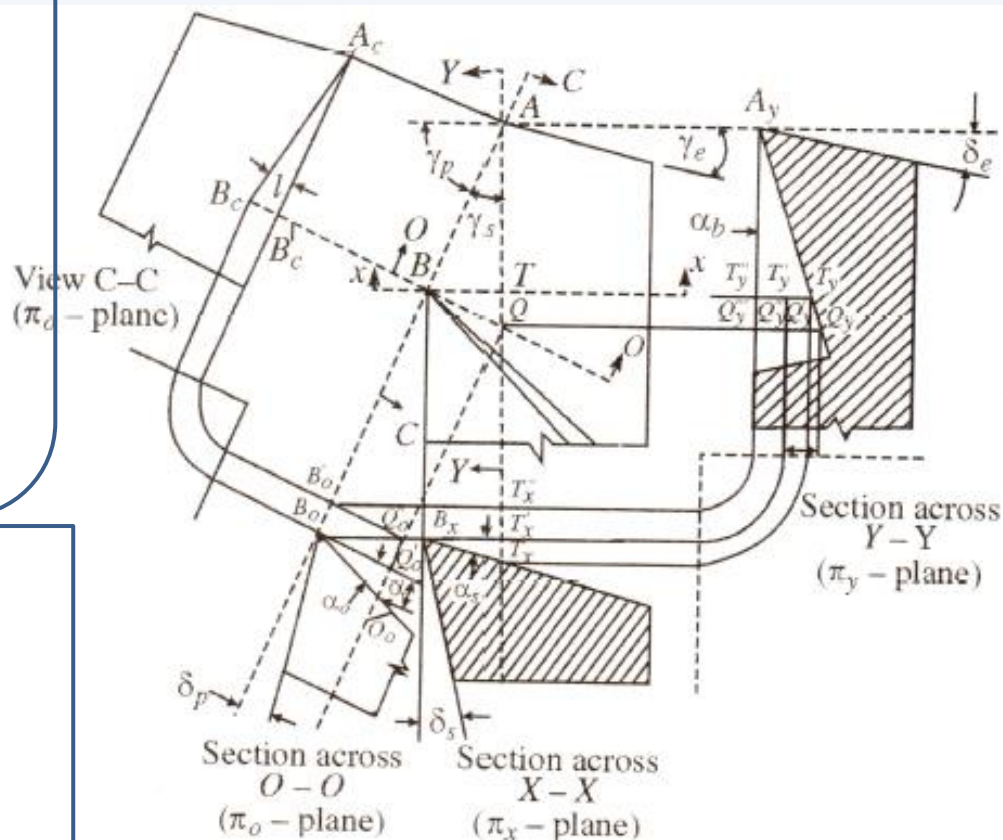
$$\text{So, } \tan i = \sin \gamma_p \tan \alpha_b - \cos \gamma_p \tan \alpha_s$$

$$\tan \alpha_o = \frac{Q_o Q'_o}{B_o Q'_o} = \frac{Q_y Q_y'' - Q_y Q'_y}{B_o Q'_o}$$

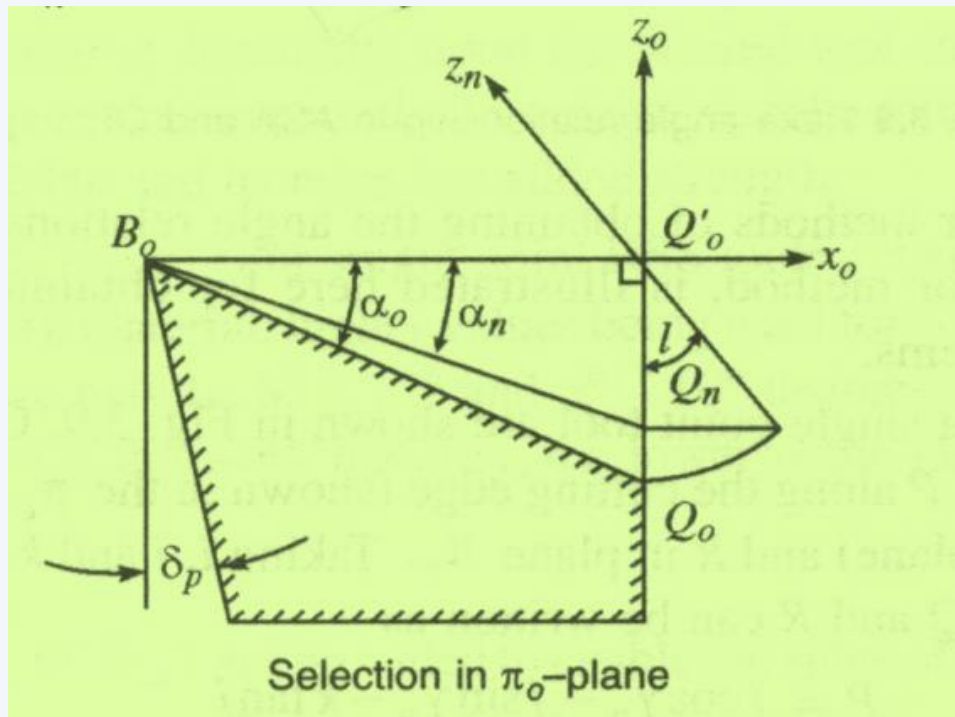
$$= \frac{T_x T'_x - Q_y Q'_y}{BQ} = \frac{B_x T'_x \tan \alpha_s + Q'_x T_y \tan \alpha_b}{BQ}$$

$$= \frac{BT}{BQ} \tan \alpha_s + \frac{QT}{BQ} \tan \alpha_b$$

$$\text{So, } \tan \alpha_o = \sin \gamma_p \tan \alpha_s + \cos \gamma_p \tan \alpha_b$$



Conversion from ORS to NRS System



$$\gamma_p = (90 - \gamma_s)$$

$$\tan \alpha_n = \frac{Q'_o Q_n}{Q'_o B_o} = \frac{Q'_o Q_o \cos i}{Q'_o B_o}$$

$$\Rightarrow \frac{Q_o Q'_o}{Q'_o B_o} \cos i$$

$$\text{or, } \tan \alpha_n = \cos i \tan \alpha_o$$

SELECTION OF TOOL ANGLES

Rake Angles:

- With zero inclination (i) or back rake angle (α_b), the chip will flow parallel to the work surface and may cause removal problem.
- With appropriate inclination angle (i) or back rake angle (α_b), the chip can be made to flow away from the workpiece and strike a suitable chip breaker, curl and break into small fragments.
- Side rake (α_s), orthogonal rake (α_o) and normal rake (α_n) angles influence the cutting forces, power and surface finish. The larger is the rake angle, the lower are the cutting forces and power and better is the surface.





SELECTION OF TOOL ANGLES

- Large rake angles, however, decreases cutting angle between the rake and the principal flank faces, and less metal is available to support the tool, i.e, less strength of the tool and conduct the heat generated due to plastic deformation and friction.
- Rake angles may be positive or negative depending upon the desired tool strength. For rough machining, often carbide tools with negative rakes are used.
- $\alpha_s, \alpha_o, \alpha_n$ usually vary between 5 – 15 degrees for single point HSS tool depending upon the work material. Higher values are used for softer work material like aluminium. In general, harder is the work material, lower is the rake angle.
- For Steel: $\alpha = 8$ to 12 degrees
- For Cast Iron: $\alpha = 5$ to 10 degrees



Tool Angles

Flank Angle

- Minimizes rubbing of flank faces with the machined surface
- Higher values of flank angle will reduce rubbing but also weaken the tool
- Flank angles have no influence on cutting forces and power. So angles large enough to avoid rubbing is generally chosen
- Angle: 5 – 12 degrees for HSS; higher for softer and lower for brittle material



Tool Angles

Cutting Edge Angles:

- Provided to clear the cutting edge from the machined surface
- To Reduce tool chatter
- Affects tool life as well as surface finish



Tool Parameter

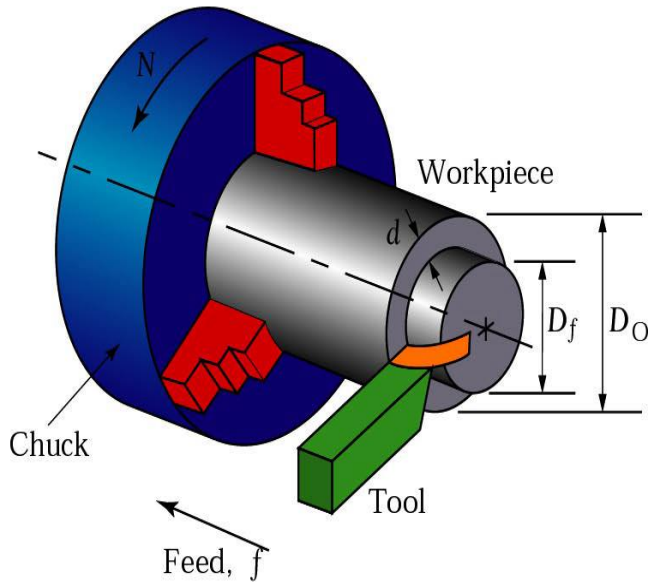
Nose radius

- Improves tool life and surface finish.
 - Large nose radius
 - Increases cutting forces and power
 - Causes chatter (self-excited vibration)
- Recommended value: 1 – 3 mm

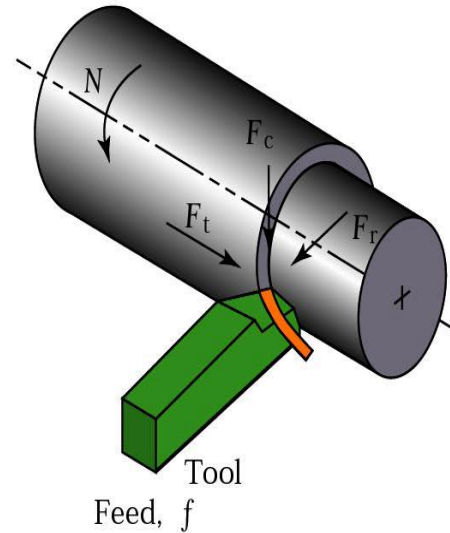


Turning Operation

(a)



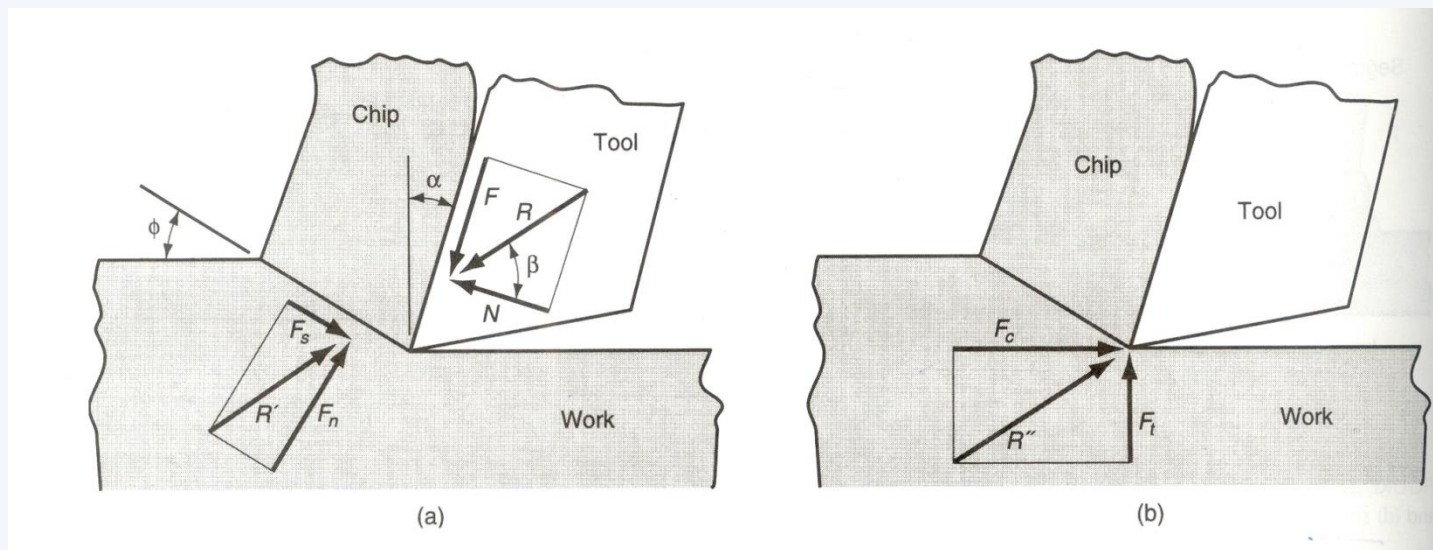
(b)



Schematic illustration of a turning operation showing depth of cut, d , and feed, f . Cutting speed is the surface speed of the workpiece, F_c is the cutting force, F_t is the thrust or feed force (in the direction of feed), F_r is the radial force that tends to push the tool away from the workpiece being machined.



Forces in Machining



F = Frictional force between the tool and chip

N = Normal force

β = Friction angle;

F_s = Shear force

F_n = Normal force to shear

F_c = Cutting force

F_t = Thrust force

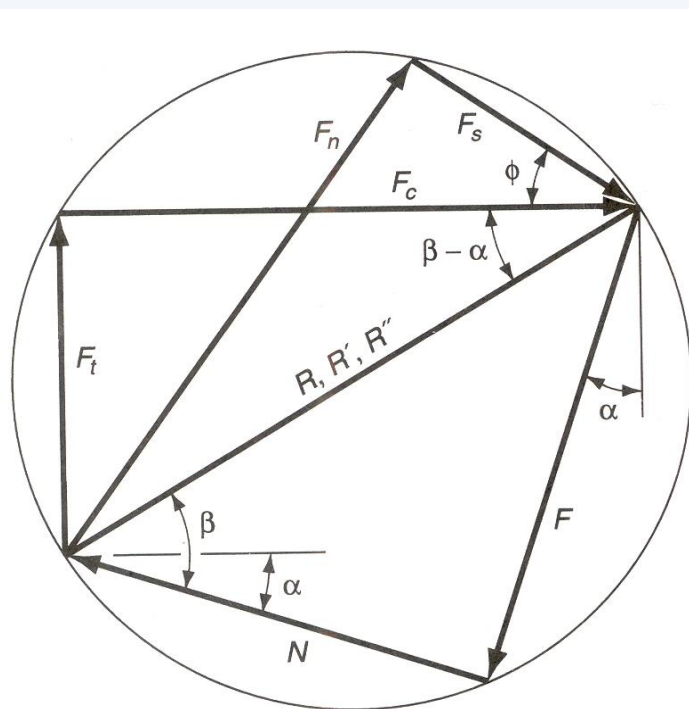
Φ = Shear Plane Angle

Assumptions:

1. The tool tip is sharp, and that the chip makes contact only with rake face of the tool.
2. The cutting edge is perpendicular to the cutting velocity
3. The deformation is two dimensional, i.e, no side spread
4. The deformation takes place in a very thin zone
5. Continuous chip without BUE
6. Workpiece material is rigid and perfectly plastic
7. Coefficient of friction is constant
8. The resultant force on the chip R' applied at the shear plane is equal, opposite and co-linear to the resultant force R applied to the chip at the chip-tool interface.



Merchant's Circle Diagram



$$F_s = R \cos(\phi + \beta - \alpha)$$

$$F_n = R \sin(\phi + \beta - \alpha)$$

$$F_c = R \cos(\beta - \alpha)$$

$$F_T = R \sin(\beta - \alpha)$$

$$F = R \sin \beta$$

$$N = R \cos \beta$$

Expressing through F_c ,

$$F_s = F_c \cos \phi - F_T \sin \phi$$

$$F_n = F_c \sin \phi + F_T \cos \phi$$

$$F = F_c \sin \alpha + F_T \cos \alpha$$

$$N = F_c \cos \alpha - F_T \sin \alpha$$

$$R = \frac{F_c}{\cos(\beta - \alpha)} = \frac{F_s}{\cos(\phi + \beta - \alpha)}$$

$$F_c = \frac{F_s \cos(\beta - \alpha)}{\cos(\phi + \beta - \alpha)}$$

Merchant's First Equation

Shear force, F_S along the shear plane can be written as:

$$F_S = \frac{\omega t_1}{\sin \phi} \tau_s$$

Where, ω is the width of the workpiece under cutting, t_1 is the uncut thickness, and τ_s is the shear strength of the work material

$$F_c = \frac{\omega t_1 \tau_s \cos(\beta - \alpha)}{\sin \phi \cos(\phi + \beta - \alpha)}$$

$$\text{Power, } P = F_c V_c$$

As per nature of taking path of least resistance, during cutting ϕ takes a value such that least amount of energy is consumed, or $P = \text{Min}$.

$$P(\phi) = \frac{\text{Const.}}{\sin \phi \cos(\phi + \beta - \alpha)} = \frac{N_m}{D_n}$$

For least energy,

$$\frac{dD_n(\phi)}{d\phi} = 0$$

$$2\phi + \beta - \alpha = \frac{\pi}{2}$$

Known as Merchant's FIRST EQUATION

Assumptions:

- Tool tip is sharp
- Orthogonal case
- Continuous chip without BUE
- μ along chip-tool contact is constant



Shear Stress and Normal Stress

Shear Stress,

$$\tau = \frac{F_s}{A_s}$$

Where, A_s is the area of shear plane

$$A_s = \frac{\omega t_1}{\sin \phi}$$

$$\tau = \frac{(F_c \cos \phi - F_t \sin \phi) \sin \phi}{\omega t_1}$$

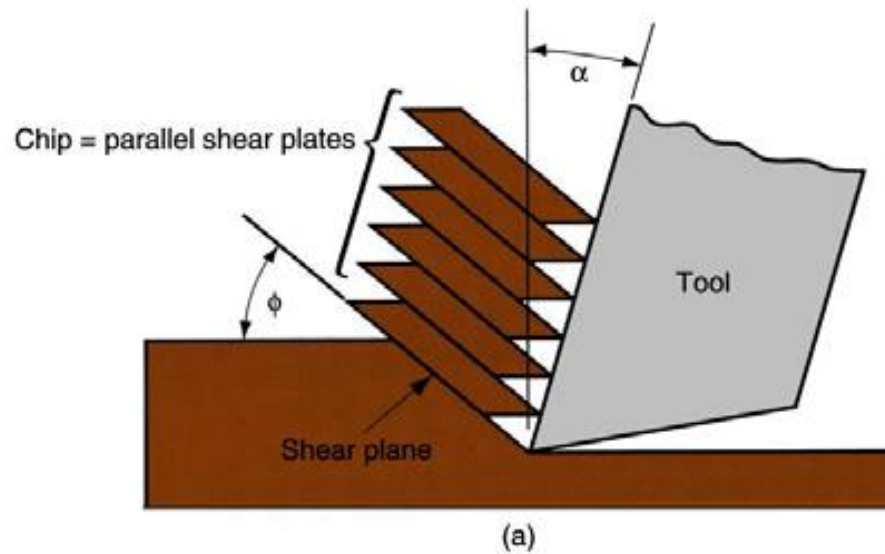
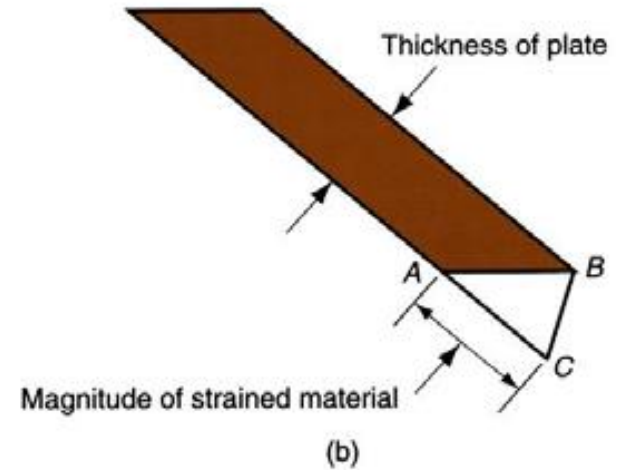
Normal Stress:

$$\sigma = \frac{F_n}{A_s}$$

$$\sigma = \frac{(F_c \sin \phi + F_t \cos \phi) \sin \phi}{\omega t_1}$$

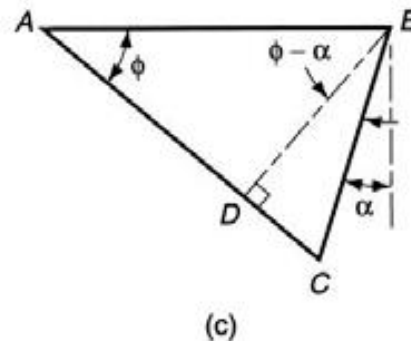
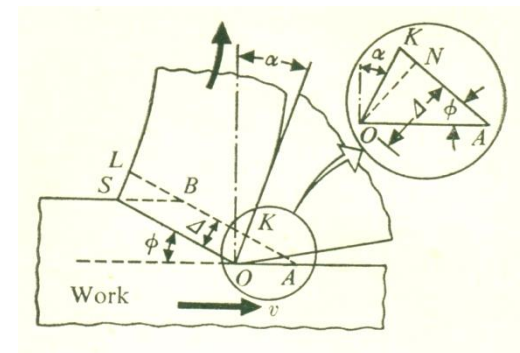


Shear Strain in Chip Formation



$$\gamma = \frac{AC}{BD} = \frac{AD + DC}{BD} = \frac{AD}{BD} + \frac{DC}{BD}$$

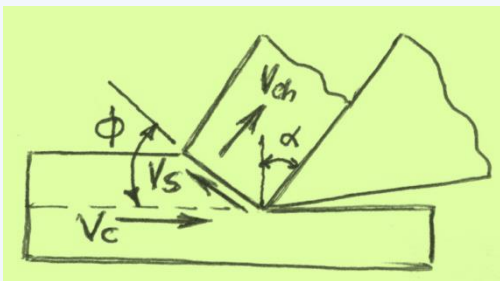
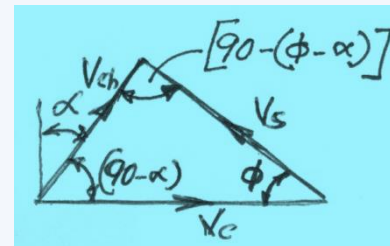
$$\gamma = \tan(\phi - \alpha) + \cot \phi$$



where γ = shear strain, ϕ = shear plane angle, and α = rake angle of cutting tool

Strain Rate

$$\dot{\gamma} = \frac{\Delta S}{\Delta Y} \cdot \frac{1}{\Delta t}$$



$\dot{\gamma}$ Can also be obtained in terms of shear velocity from the velocity diagram

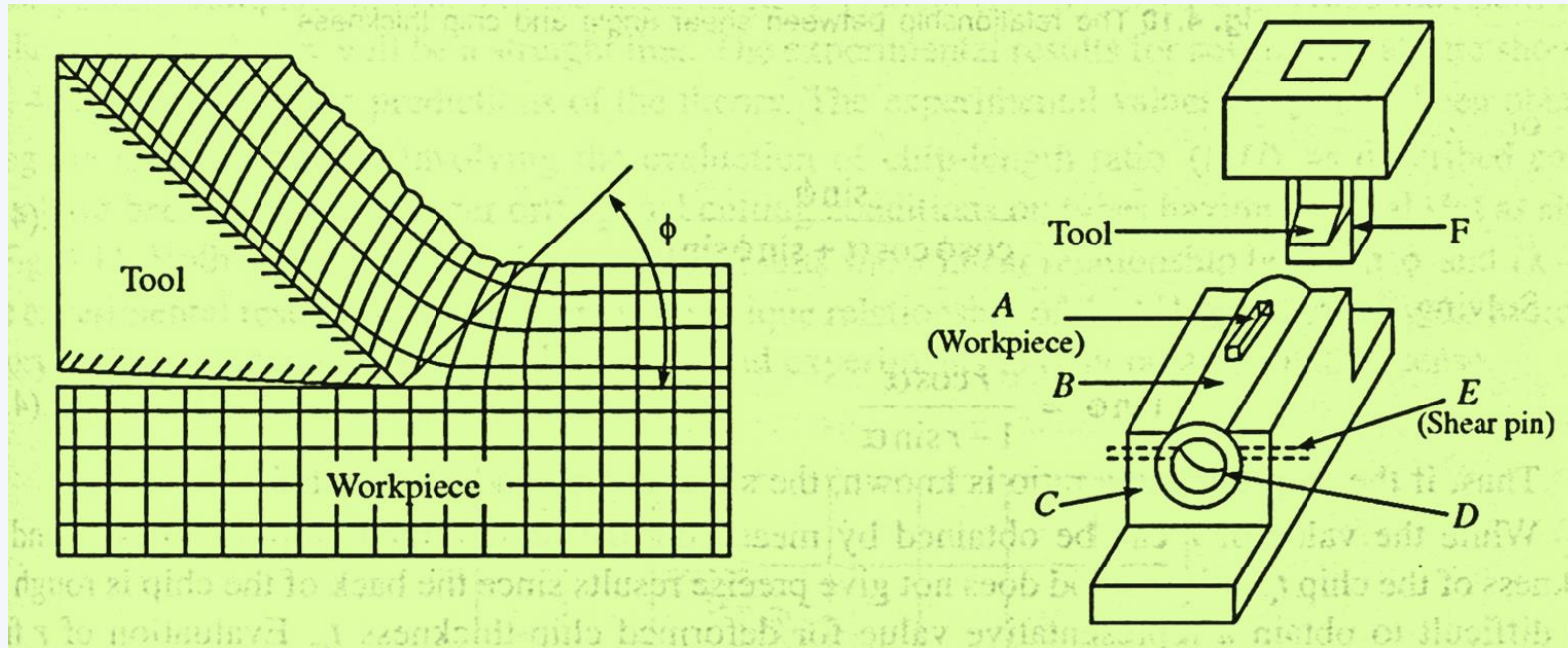
$$\frac{V_s}{\sin(90 - \alpha)} = \frac{V_c}{\sin[90 - (\phi - \alpha)]} = \frac{V_{ch}}{\sin \phi}$$

Therefore, Shear Velocity,

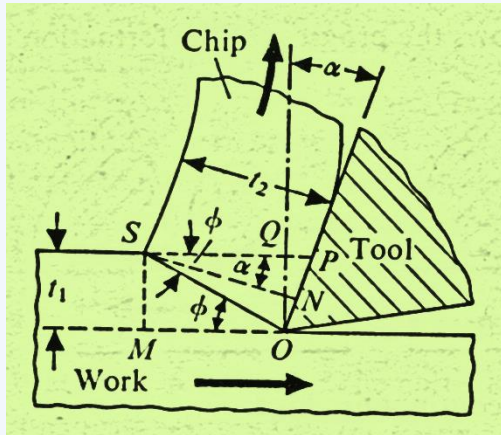
$$\dot{\gamma} = \frac{\Delta S}{\Delta Y}$$

$$V_s = \frac{\cos \alpha}{\cos(\phi - \alpha)} \cdot V_c$$

Measurement of Shear Plane Angle



Shear Plane Angle



$$\text{Chip Thickness Ratio } (\gamma) = \frac{\text{Undeformed Chip Thickness } (t_1)}{\text{Chip Thickness } (t_2)}$$

$$t_1 = OS \sin \phi$$

$$t_2 = OS \cos(\phi - \alpha)$$

$$\text{Therefore, } \gamma = \frac{OS \sin \phi}{OS \cos(\phi - \alpha)} = \frac{\sin \phi}{\cos(\phi - \alpha)}$$

$$\frac{1}{\gamma} = \frac{\cos \phi \cos \alpha + \sin \phi \sin \alpha}{\sin \phi} = \frac{\cos \alpha}{\tan \phi} + \sin \alpha$$

$$\text{or, } \tan \phi = \frac{\gamma \cos \alpha}{1 - \gamma \sin \alpha}$$

Normally, Chip Thickness Ratio = 0.5 – 0.6

Numerical Examples

Problem – 1. A mild steel workpiece is being machined at a cutting speed of 200 m/min with a tool specified as:

$$0^\circ - 8^\circ - 5^\circ - 7^\circ - 15^\circ - 75.5^\circ - 0.05 \text{ inch (ASA)}$$

The depth of cut and the uncut thickness are 0.5 mm and 0.2 mm respectively. If the average value of coefficient of friction between the chip and the tool is 0.5 and the shear stress of the work material is 400 N/mm², then

- Determine the shear plane angle
- Determine the cutting and the thrust components of the machining force

SOLUTION:

$$(a) \quad 2\phi + \lambda - \alpha = 90^\circ \quad \lambda = \tan^{-1} \mu = \tan^{-1}(0.5) = 26.57^\circ$$

$$\therefore \phi = \frac{90 - 26.57 + 8}{2} = 35.715^\circ$$

($\alpha = 8$ since α_s , side rake angle influences the cutting force)



Solution (b): Determine the cutting and the thrust components of the machining force

$$F_s = \frac{wt_1\tau_s}{\sin\phi}; w = \frac{d}{\sin\gamma_p} = \frac{0.5}{\sin(90-\gamma_s)} = \frac{0.5}{\sin(90-75.5)} = 1.997 \approx 2$$

$$F_s = \frac{2 \times 0.2 \times 400}{\sin(35.715)} = 274.09 N$$

$$R = \frac{F_s}{\cos(\phi + \lambda - \alpha)} = \frac{274.09}{\cos(35.715 + 26.57 - 8)} = 469.5 N$$

$$F_c = \frac{2wt_1\tau_s}{\tan\phi}; F_T = \frac{wt_1\tau_s \sin(\lambda - \alpha)}{\sin^2\phi}$$

$$F_c = R \cos(\lambda - \alpha) = 469.5 \cos(26.57 - 8) = 445.06 N$$

$$F_T = R \sin(\lambda - \alpha) = 469.5 \sin(26.57 - 8) = 149.52 N$$

Alternatively:

$$F_c = \frac{F_s \cos(\lambda - \alpha)}{\cos(\phi + \lambda - \alpha)} = \frac{274.09 \cos(26.57 - 8)}{\cos(35.715 + 26.57 - 8)} = \frac{259.82}{0.584} = 445.08 N$$

$$F_T = \frac{F_c \cos\phi - F_s}{\sin\phi} = \frac{445.08 \cos(35.715) - 274.09}{\sin(35.715)} = \frac{87.284}{0.584} = 149.52 N$$



Numerical Examples:

Problem – 2: During orthogonal turning of a mild steel workpiece of 20 mm diameter with 150 rpm with a 0° rake tool, the forces normal to the shear plane and thrust are found to be 85.6 N and 35 N respectively. If the chip thickness is twice the uncut thickness, estimate the power consumption in Watt.

SOLUTION: Given, $D = 20$ mm; $N = 150$ rpm; $F_N = 85.6$ N; $F_T = 35$ N

Therefore, $V_c = \pi DN = \pi \times 20 \times 150 = 9422.77 \text{ mm/min} = 9.427 \text{ m/min} = 0.157 \text{ m/s}$

$$F_c = \frac{F_N - F_T \cos \phi}{\sin \phi} \quad (\text{since, } F_N = F_c \sin \phi + F_T \cos \phi)$$

Given, $r = \frac{t_1}{t_2} = 0.5$; $\tan \phi (\text{for } \alpha = 0) = \frac{r \cos \alpha}{1 - r \sin \alpha} = r = 0.5 \quad \therefore \phi = \tan^{-1}(0.5) = 26.56^\circ$

$$F_c = \frac{F_N - F_T \cos \phi}{\sin \phi} = \frac{85.6 - 35 \cos(26.56)}{\sin(26.56)} = 121.4 \text{ N}$$

$$\therefore \text{Power, } P = F_c V_c = 121.4 \times 9.427 = 1144.4 \text{ Nm/min} = 19.07 \text{ Nm/s} = 19.07 \text{ Watt}$$

Alternative Solution: $F_T = R \sin(\lambda - \alpha)$; or, $35 = R \sin \lambda (\text{for } \alpha = 0)$

$$\text{Also, } F_N = R \sin(\phi + \lambda - \alpha); \text{ or, } 85.6 = R \sin(26.56 + \lambda) \quad \therefore \frac{35}{85.6} = \frac{\sin \lambda}{\sin(26.56 + \lambda)}; \text{ or, } \lambda = 16.08^\circ$$

$$\text{Now, } R = \frac{35}{\sin \lambda} = \frac{35}{\sin(16.08)} = 126.36 \text{ N} \quad F_c = R \cos(\lambda - \alpha) = R \cos \lambda (\text{for } \alpha = 0) = 126.36 \cos(16.08) = 121.42 \text{ N}$$

$$\therefore \text{Power, } P = F_c V = 121.42 \times 0.157 = 19.06 \text{ Nm/s} = 19.06 \text{ Watt}$$



Numerical Examples:

Problem – 3: For an orthogonal cutting, the following data are given:

Cutting Force, $F_c = 980$ N; Thrust Force, $F_T = 440$ N; Rake angle = 10° ;
Chip Thickness = 1.5 mm; Width of Cut = 2 mm; Shear Plane angle = 24°
Determine:

- (a) Strength of the workpiece material;
- (b) Friction angle, without using Merchant's First Relationship.

SOLUTION:

$$\phi = \tan^{-1} \left[\frac{r \cos \alpha}{1 - r \sin \alpha} \right] \quad \tan(24^\circ) - r \tan(24) \sin 10 = r \cos 10$$

$$\text{or, } r = \frac{\tan 24}{\tan 24 \sin 10 + \cos 10} = 0.419 = \frac{t_1}{t_2} \quad \text{Given, } t_2 = 1.5 \text{ mm, } t_1 = 0.419 \times 1.5 = 0.628 \text{ mm}$$

$$A_s = \frac{wt_1}{\sin \phi} = \frac{2 \times 0.628}{\sin 24} = 3.088 \text{ mm}^2 \quad F_s = F_c \cos \phi - F_T \sin \phi = 980 \cos 24 - 440 \sin 24$$

Therefore, the strength of material,

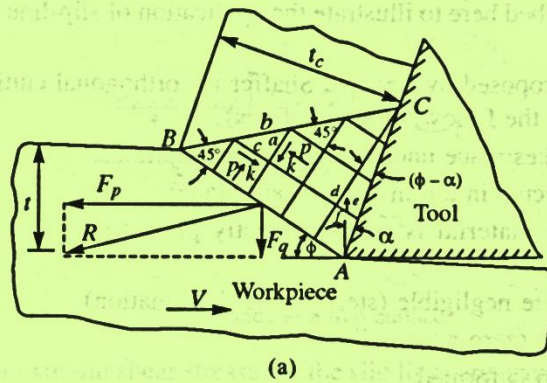
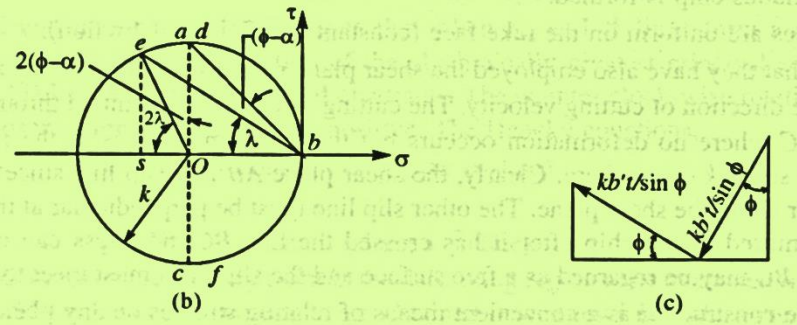
$$K = \frac{F_s}{A_s} = \frac{716.3}{3.088} = 231.96 \frac{\text{N}}{\text{mm}^2} \quad F_s = F_c \cos \phi - F_T \sin \phi = 980 \cos 24 - 440 \sin 24 = 716.3 \text{ N}$$

$$F = F_c \sin \alpha + F_T \cos \alpha = 980 \sin 10 + 440 \cos 10 = 603.5 \text{ N} \quad N = F_c \cos \alpha - F_T \sin \alpha = 980 \cos 10 - 440 \sin 10 = 888.7 \text{ N}$$

$$\mu = \frac{F}{N} = 0.679; \text{ Hence, Friction angle, } \lambda = \tan^{-1}(\mu) = 34.176^\circ$$



Thin Zone Model: Lee & Shaffer Relationship



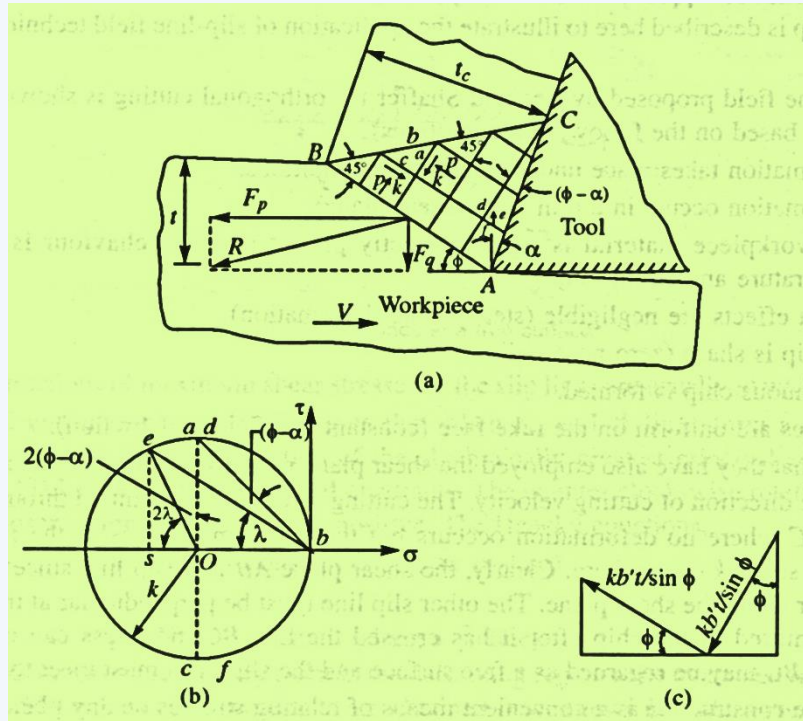
Their solution is based on the following assumptions:

- (i) Deformation takes place under plane strain conditions.
- (ii) Deformation occurs in a thin zone (shear plane).
- (iii) The workpiece material is rigid, perfectly plastic and its behaviour is independent of temperature and strain rate.
- (iv) Inertia effects are negligible (steady-state deformation).
- (v) Tool tip is sharp (zero nose radius).
- (vi) Continuous chip is formed.
- (vii) Stresses are uniform on the rake face (constant coefficient of friction).

The cutting forces are transmitted through the triangular plastic zone ABC where no deformation occurs, because they considered that there must be a stress field within the chip to transmit the cutting forces from the shear plane to the tool face.

- In the ABC, the entire material is in the plastic state (stressed up to yield point)
- Shear plane AB is a slip line since maximum shear stress occurs here.
- Other slip lines must be perpendicular to this line.
- BC is the FREE SURFACE since no force is transmitted to the chip after it has crossed the line BC.
- Slip line must meet this surface at 45 degree.

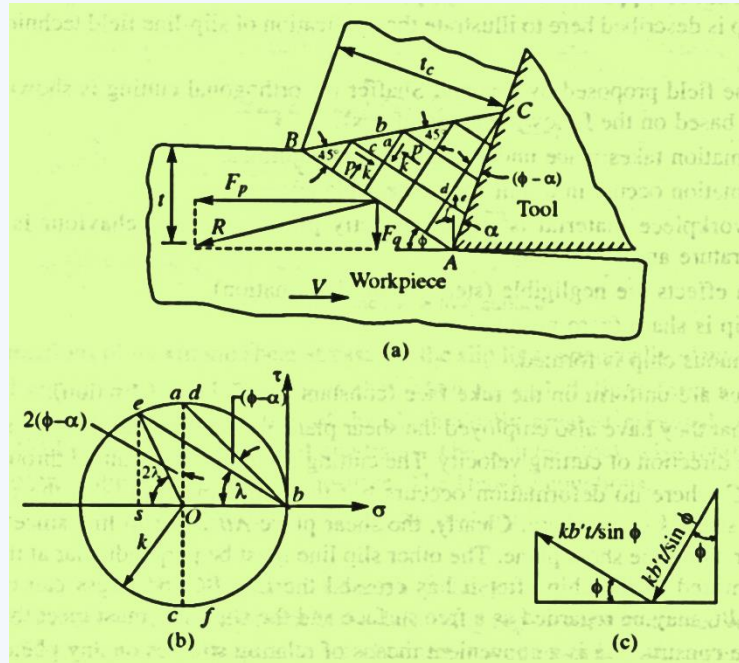
Thin Zone Model: Lee & Shaffer Relationship



- Mohr circle construction is a convenient means of relating stresses on any plane to the Principal Stresses.
- Since plane BC is stress free, the Mohr circle must pass through the origin, b.
- Points a, c, d and f are displaced from b by 90 degree (twice the angle of physical plane)
- Face 'e' is inclined to face 'd' at an angle ' η ', therefore in the stress plane the angle subtended by the arc 'ae' at the centre is ' 2η '

Assuming uniform shear stress τ and normal stress σ on the rake face, the friction angle,

Thin Zone Model: Lee & Shaffer Relationship



$$\lambda = \tan^{-1} \left(\frac{\tau}{\sigma} \right) = \tan^{-1} \left(\frac{es}{sb} \right) = \angle ebs$$

$$\text{Since, } \tan \lambda = \mu = \frac{F}{N}$$

$$\angle eos = 2\lambda$$

$$\text{Therefore, } \angle eos + \angle eod = 2\lambda + 2\eta = 2\lambda + (\phi - \alpha) = \frac{\pi}{2}$$

$$\text{Therefore, } 2\lambda + 2\phi - 2\alpha = \frac{\pi}{2}, \text{ or, } \phi = \frac{\pi}{4} - (\lambda - \alpha) \text{ or, } \phi + \lambda - \alpha = \frac{\pi}{4}$$

Friction in Metal Cutting

The Nature of Sliding Friction:

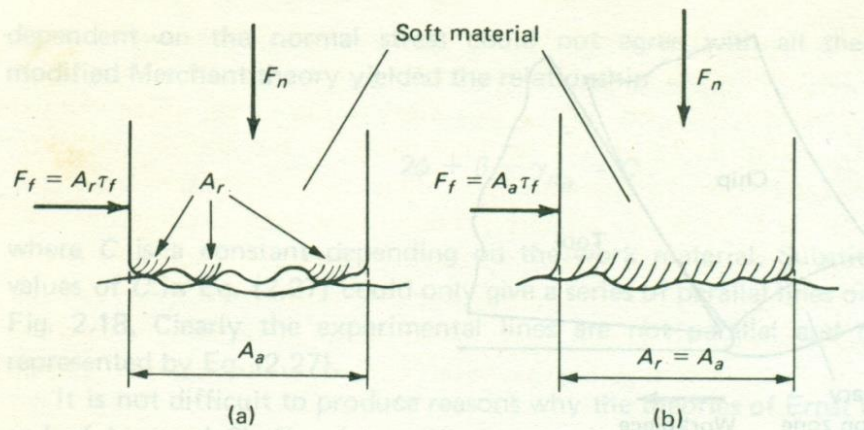


FIG. 2.20 Suggested frictional behavior for a "soft" slider, where F_f = frictional force, F_n = normal force, A_r = real area of contact, A_a = apparent area of contact, and τ_f = shear strength of softer metal. (a) Sliding friction; (b) sticking friction.

$$F = \tau \cdot A_r$$

$$\mu = \frac{F}{N} = \frac{\tau \cdot A_r}{\sigma_y A_a} = \frac{\tau}{\sigma_y}$$

This equation shows that μ is independent of the apparent contact area and since $\frac{\tau}{\sigma_y}$ is constant for a given metal, μ remains constant.

- Since the solid surfaces have asperities, the real area of contact differs from the apparent area (geometrical mating area).
- In case when the load increases, the asperity deformation becomes fully plastic and the real area of contact is then a direct function of the applied load, independent of the apparent area or geometrical area of the surfaces.

$$A_r = \frac{N}{\sigma_y}$$

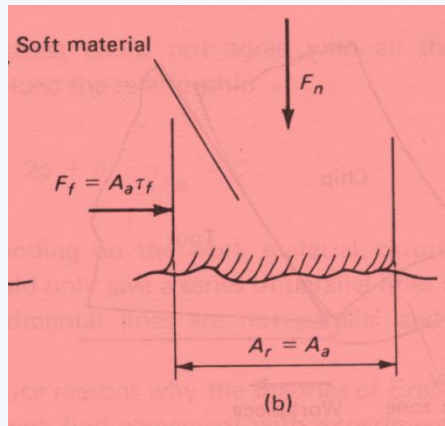
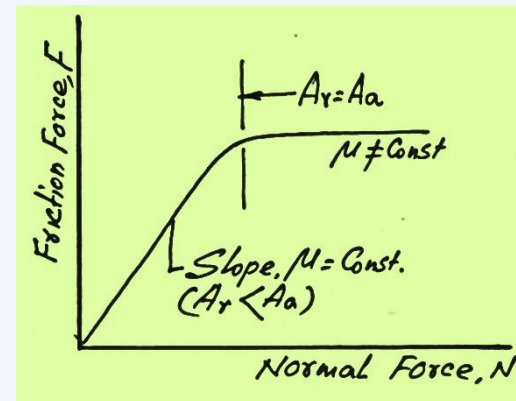
N – Normal Force ; σ_y - Yield stress of the softer material.

During sliding, shearing of the welded asperities occurs, the mechanism described by the Adhesion Theory of Friction.



Friction in Metal Cutting

- In metal cutting, the coefficient of friction can vary considerably.
- The variance of μ results from the very high normal pressure that exists at the chip-tool interface, causing the real area of contact to become equal to the apparent contact area over a portion of the chip-tool interface.



F is now independent of N and the ordinary law of friction no longer apply.

Under these conditions, the shearing action is no longer confined to surface asperities but takes place within the body of the softer metal.



Friction in Metal Cutting

Model of Orthogonal Cutting with a continuous chip and no BUE:
(Zorev's Model)

Normal Stress Distribution on the tool face:

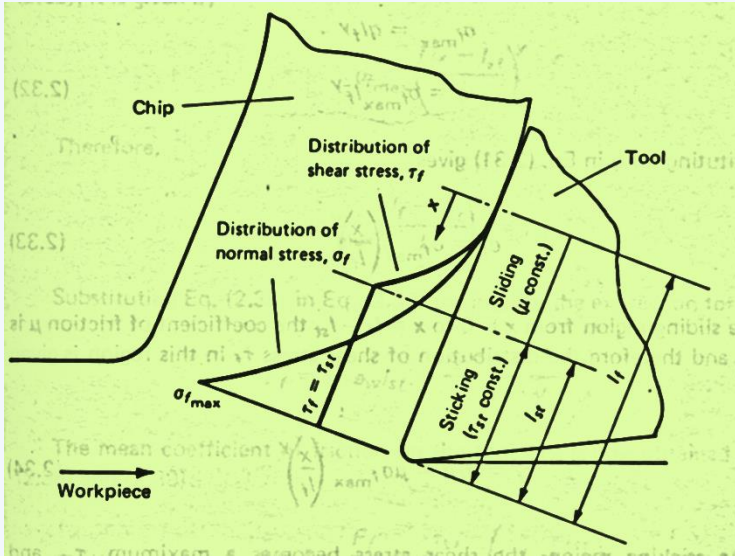
$$\sigma_f = q \cdot X^y$$

X is the distance along the tool face from the point where the chip loses contact with the tool; q, y – Constants.

σ_{fmax} occurs when $X=l_f$, so, $\sigma_{fmax} = q \cdot l_f^y$

$$q = \sigma_{fmax} \cdot l_f^{-y}$$

$$\sigma_f = \sigma_{fmax} \cdot \left(\frac{X}{l_f}\right)^y \dots\dots\dots (1)$$



In the sliding region from $X=0$ to $X=l_f - l_{st}$, the μ is constant and the distribution of shear stress in this region is given by:

$$\tau = \sigma_f \cdot \mu = \mu \cdot \sigma_{fmax} \cdot \left(\frac{X}{l_f}\right)^y$$



Friction in Metal Cutting

From $X = (l_f - l_{st})$ to $X = l_f$, the shear stress becomes maximum, $\tau = \tau_{st}$

Integrating $\sigma_f = \sigma_{fmax} \cdot \left(\frac{X}{l_f}\right)^y$ to get the normal force acting on the tool face gives,

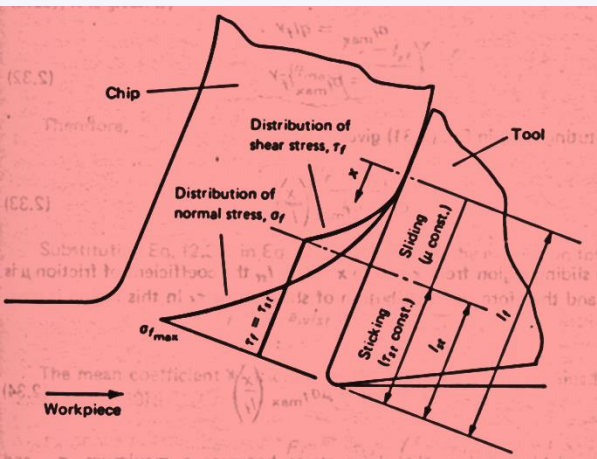
$$N = a_w \int_0^{l_f} \sigma_{fmax} \left(\frac{X}{l_f}\right)^y dx = \frac{\sigma_{fmax} \cdot a_w \cdot l_f}{(1+y)}$$

a_w - width of cut

The Friction Force, F on the tool face can be obtained as:

$$F = a_w \left[\tau_{st} \cdot l_{st} + \int_0^{l_f - l_{st}} \mu \sigma_{fmax} \left(\frac{X}{l_f}\right)^y dx \right]$$

$$= \tau_{st} \cdot a_w \cdot l_{st} + \frac{\mu \sigma_{fmax} a_w (l_f - l_{st})^{1+y}}{l_f^y (1+y)} \dots \dots \dots (2)$$



Friction in Metal Cutting

At the point $X = (l_f - l_{st})$ the normal stress is given by (τ_{st}/μ) . Further, from the equation (1) it is given by:

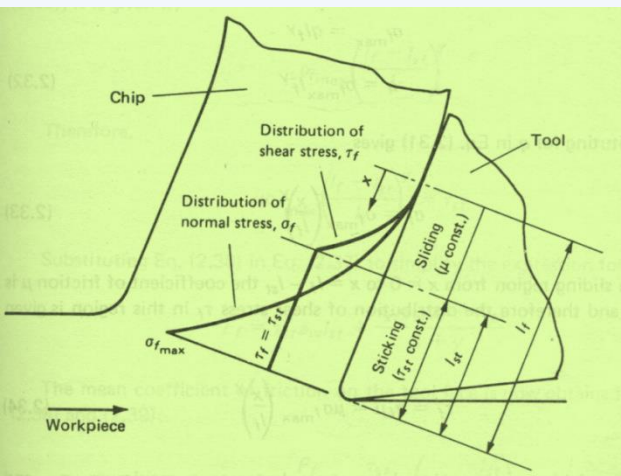
$$\sigma_f = \sigma_{f_{max}} \cdot \left(\frac{X}{l_f}\right)^y \dots [1]$$

Therefore,
$$\mu \sigma_{f_{max}} \left(\frac{l_f - l_{st}}{l_f}\right)^y = \tau_{st} \dots \dots \dots (3) \quad [F = \tau_{st} \cdot a_w \cdot l_{st} + \frac{\mu \sigma_{f_{max}} a_w (l_f - l_{st})^{1+y}}{l_f^y (1+y)} \dots \dots [2]$$

Substituting Eq. (3) into Eq. (2), the expression of F can be simplified as:

$$F_f = \tau_{st} a_w l_{st} + \frac{\tau_{st} a_w (l_f - l_{st})}{1 + y}$$

The mean coefficient of friction on the tool face can now be expressed as :



$$\tan \lambda = \frac{F}{N} = \frac{\tau_{st}}{\sigma_{f_{max}}} \left(1 + y \frac{l_{st}}{l_f}\right) \dots \dots \dots (4)$$

$$[N = a_w \int_0^{l_f} \sigma_{f_{max}} \left(\frac{X}{l_f}\right)^y dx = \frac{\sigma_{f_{max}} \cdot a_w \cdot l_f}{(1+y)}$$

Friction in Metal Cutting

The mean normal stress on the tool face is given by: $\sigma_{fav} = \frac{N}{a_w l_f} = \left(\frac{\sigma_{fmax}}{1 + \gamma} \right)$

Therefore,

$$\sigma_{fmax} = (1 + \gamma) \sigma_{fav}$$

Substituting for σ_{fmax} in Eq. (4) gives:

$$\lambda = \arctan \left\{ \frac{\tau_{st}}{\sigma_{fav}} \frac{\left[1 + \gamma \left(\frac{l_{st}}{l_f} \right) \right]}{1 + \gamma} \right\}$$

In experimental works it is found that the term

$$\tau_{st} \frac{[1 + \gamma (l_{st}/l_f)]}{1 + \gamma}$$

remains sufficiently

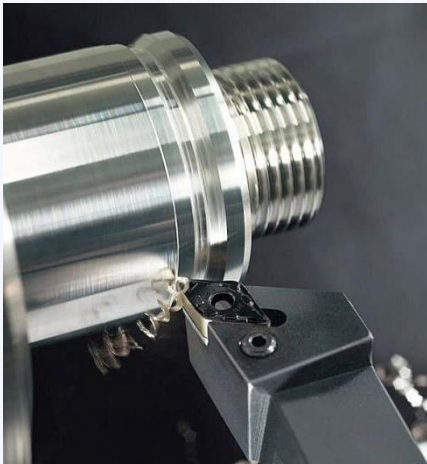
constant for a given material over a wide range of unlubricated cutting condition, and therefore the expression becomes:

$$\lambda = \arctan \frac{K}{\sigma_{fav}}$$

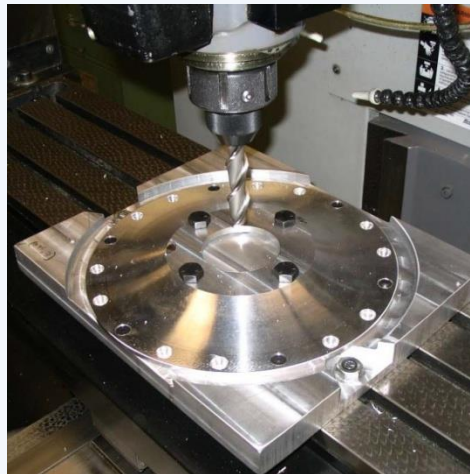
This equation shows that the mean angle of friction is mainly dependent on the mean normal stress on the tool face. This explains the following fact: as working normal rake increases, the component of the resultant tool force normal to the tool face will decrease and therefore, the mean normal stress will decrease and the friction angle will increase.



Practical Machining Operations



TURNING



MILLING



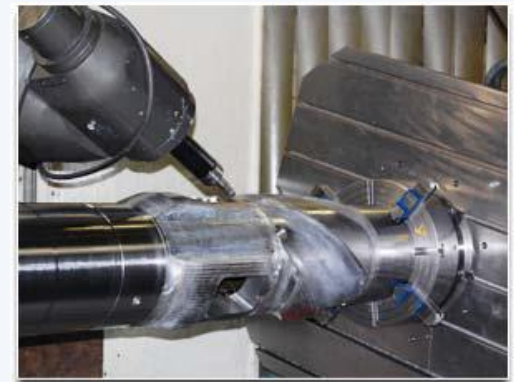
DRILLING



SAWING



BROACHING

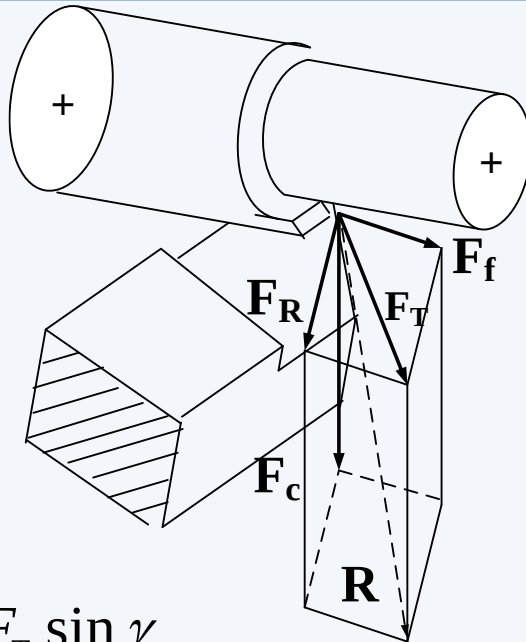


SHAPING/PLANING



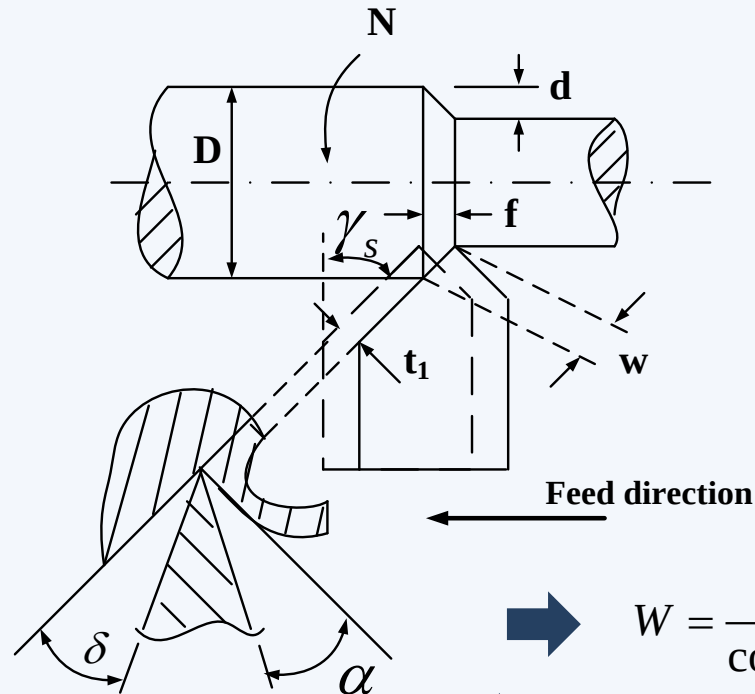
Practical Machining Operations

Turning:



$$\Rightarrow F_R = F_T \sin \gamma_s$$

$$\Rightarrow F_f = F_T \cos \gamma_s$$



$$\Rightarrow W = \frac{d}{\cos \gamma_s}$$

$$\Rightarrow t_1 = f \cos \gamma_s$$

Power, $P = F_c V_c + F_f V_f \approx F_c V_c$

$$P = U_c \times MRR$$

Since, $V_f \ll V_c$

d – depth of cut [mm]
t₁ – Uncut thickness [mm]

Where, U_c – Specific Energy = Energy required to remove a unit Volume of material [Joules/mm³]

Practical Machining Operations

➔ $U_c = U_o (t_1)^{-0.4}$; U_o – Specific energy to remove 1 mm of t_1

➔ $MRR = fd \cdot \frac{\pi DN}{60}$ [mm³/sec]

Material Removal Rate

(area) (velocity)

➔ Number of revolution/pass = $\left(\frac{L}{f} \right)$

➔ Time/pass = $\left(\frac{L}{fN} \right)$

➔ Total Time; T = $\left(\frac{L}{fN} \right) n$

n – number of passes

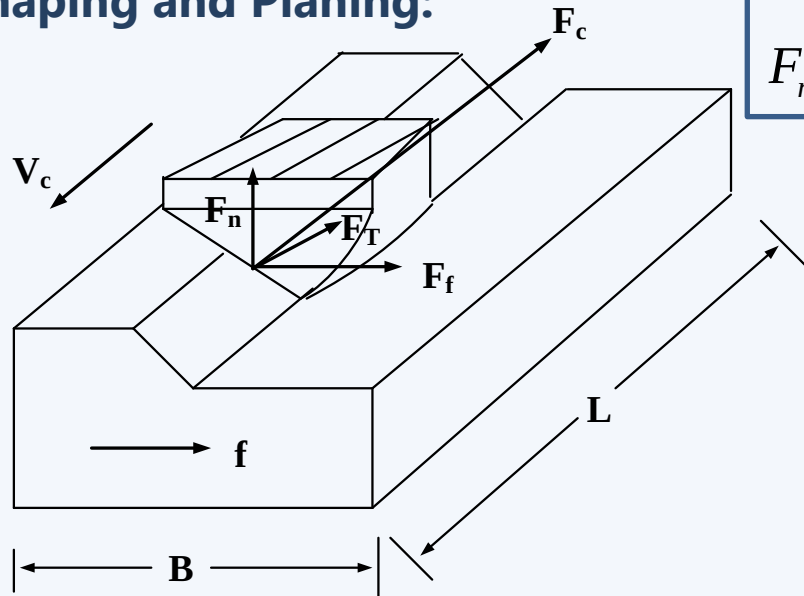
L – cylinder length [mm]

N – spindle speed [rpm]

f – feed [mm/rev]

Practical Machining Operations

Shaping and Planing:

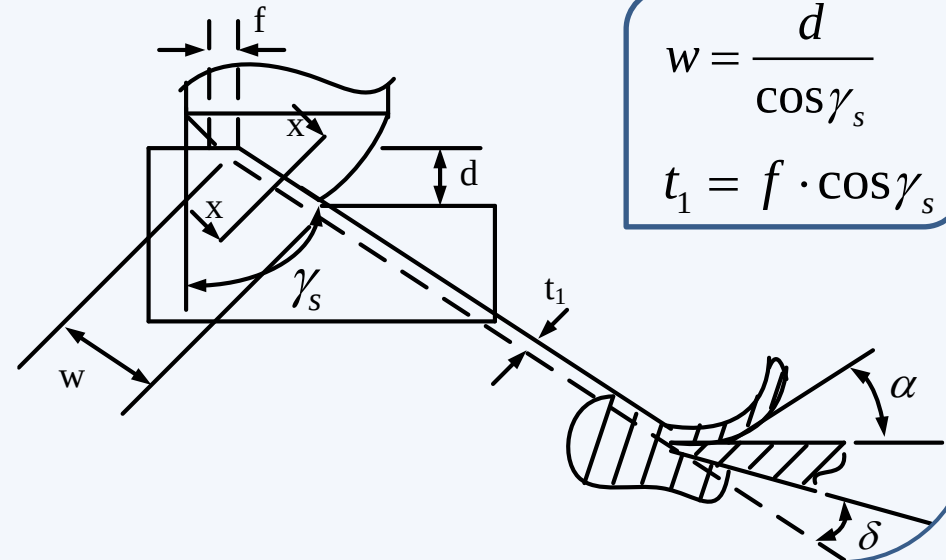


feed = feed/stroke

$$F_f = F_T \cos \gamma_s$$

$$F_n = F_T \sin \gamma_s$$

γ_s - Primary principal cutting edge angle



$$w = \frac{d}{\cos \gamma_s}$$

$$t_1 = f \cdot \cos \gamma_s$$

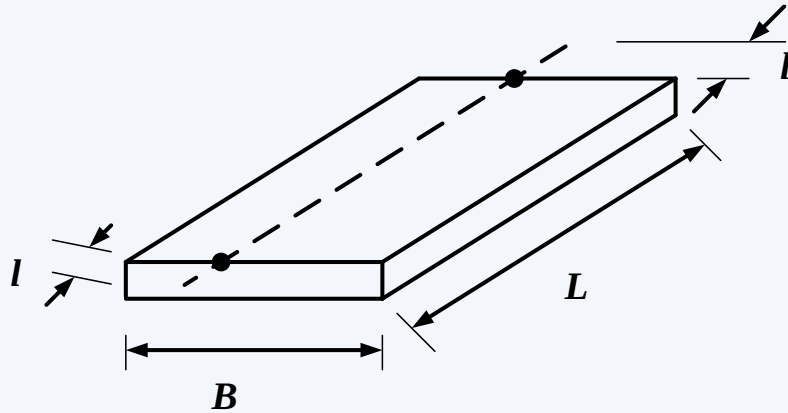
Power,

$$P = U_c \cdot MRR \quad ; \quad MRR = V_c \cdot \overbrace{f \cdot d}^{\text{area}}$$

$$U_c = U_o (t_1)^{-0.4}$$

Average velocity of cutting

Shaping



l = Approach length

V = Average velocity of cutting

f = feed/stroke

r = Quick return ratio

➔ Time for forward motion = $\frac{L + 2l}{V} = t_f$

➔ Time for backward motion = $\frac{t_f}{r} = t_b$

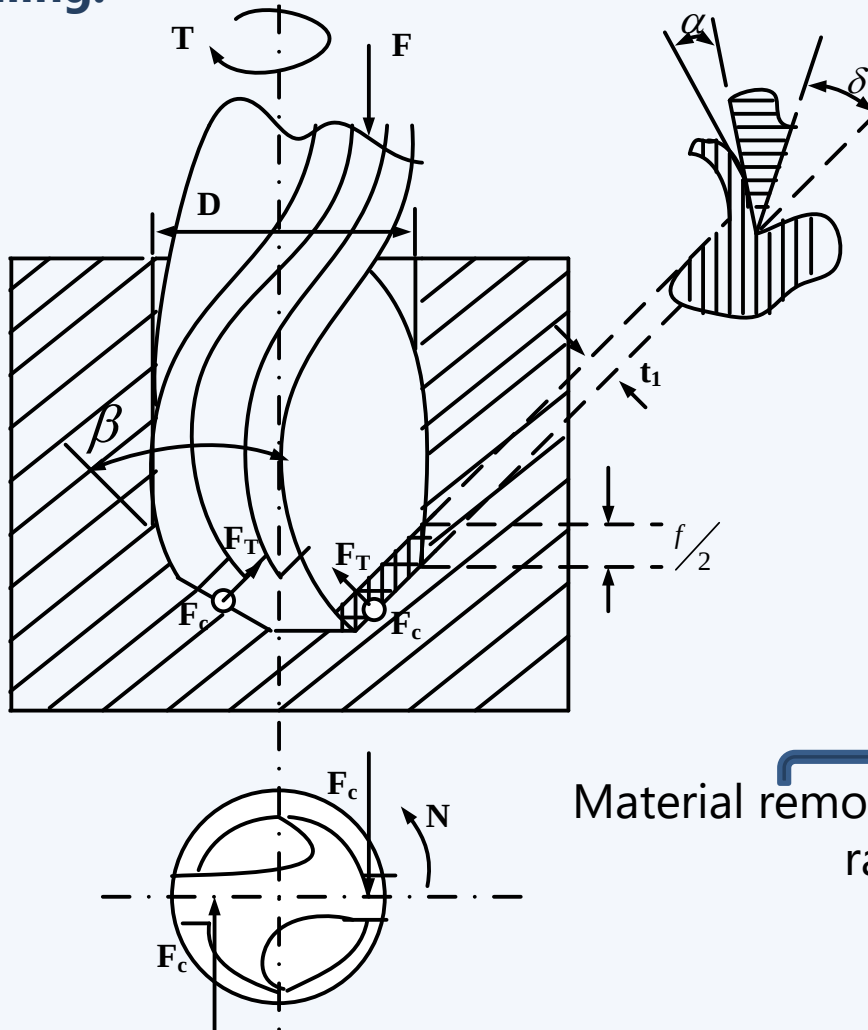
➔ So, total time/stroke = $t_f + t_b = \frac{L + 2l}{V} \left[1 + \frac{1}{r} \right]$

➔ Total number of strokes to cover the surface = $\frac{B}{f}$

So, total time = $\frac{B(L + 2l)}{f \cdot V} \left[1 + \frac{1}{r} \right]$

Practical Machining Operations

Drilling:



$$t_1 = \frac{f}{2} \sin \beta \quad ; \quad w = \frac{D/2}{\sin \beta}$$

Thrust, $F = 2F_T \sin \beta$

Torque, $T = F_c \cdot \frac{D}{2}$

Power, $P = U_c \cdot MRR$

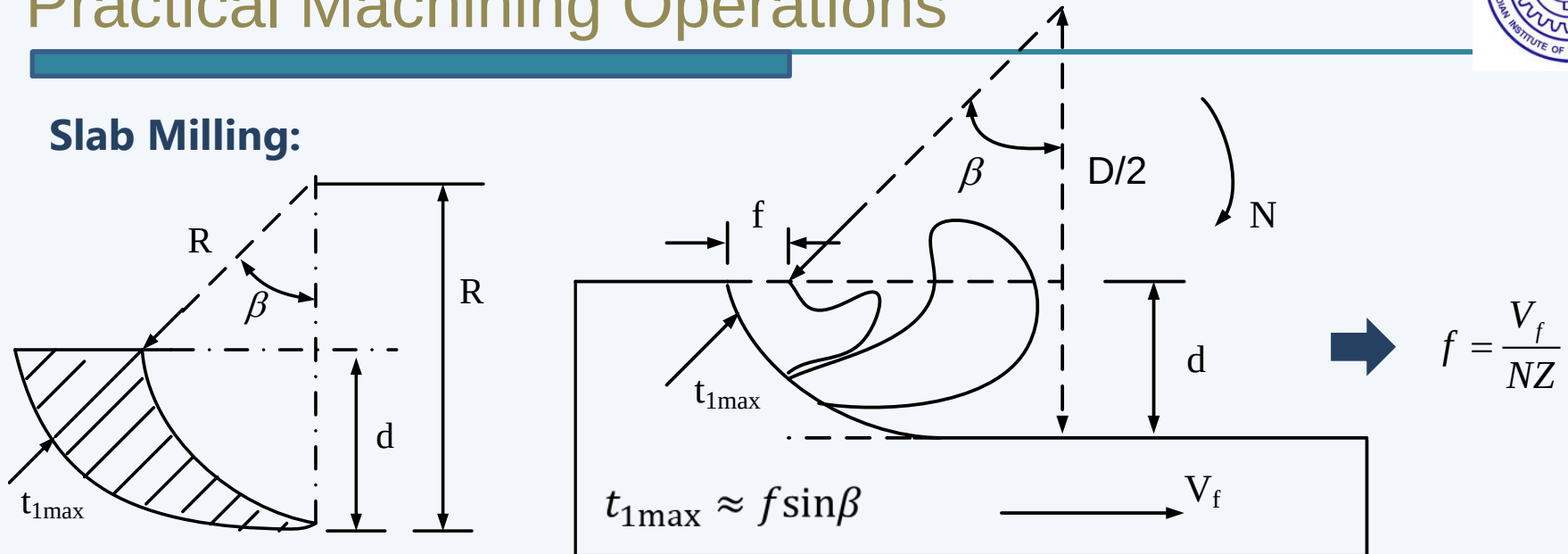
$$U_c = U_o (t_1)^{-0.4}$$

Material removal rate

$$MRR = \underbrace{\frac{N}{60}}_{\text{feed velocity}} \cdot \underbrace{f \cdot \left(\frac{\pi D^2}{4} \right)}_{\text{area}} \quad [\text{mm}^3/\text{min}]$$

Practical Machining Operations

Slab Milling:



$$t_{av} \cong \frac{1}{2} t_{1\max} \quad ; \quad t_{1\max} \approx f \sin \beta$$

$$\cos \beta = \frac{R-d}{R} = \left(1 - \frac{d}{R}\right)$$

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - \left(1 - \frac{d}{R}\right)^2} = \sqrt{\frac{2d}{R} - \frac{d^2}{R^2}} \approx \sqrt{\frac{2d}{R}}$$

$$t_{1\max} = f \sqrt{\frac{2d}{R}}$$

Power, $P = U_c \cdot MRR$

$$U_c = U_o (t_1)^{-0.4}$$

$$MRR = B \cdot d \cdot V_f$$

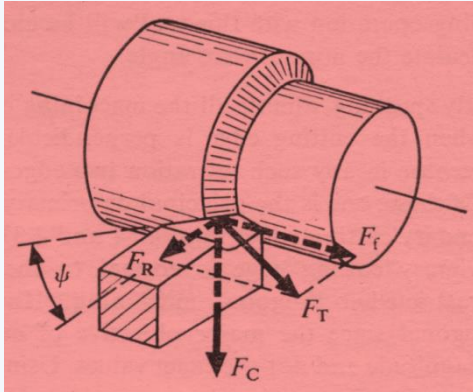
N – RPM

Z – number of teeth

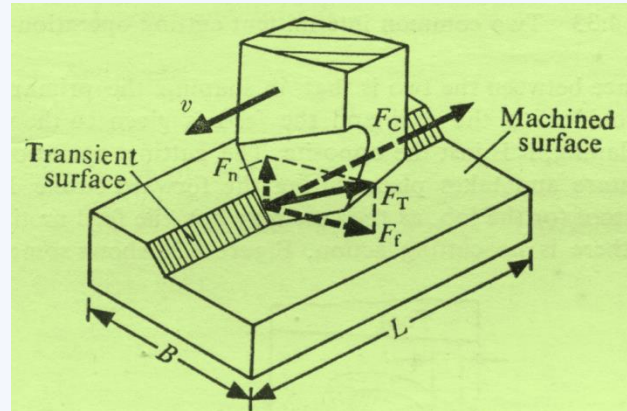
V_f – feed velocity

f – feed/tooth

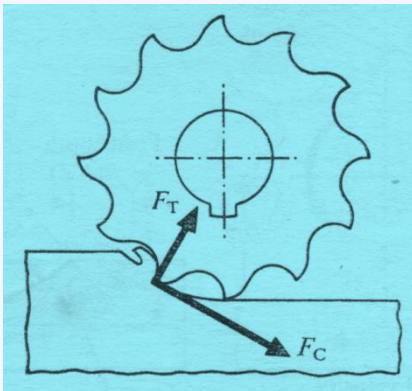
Forces in Machining



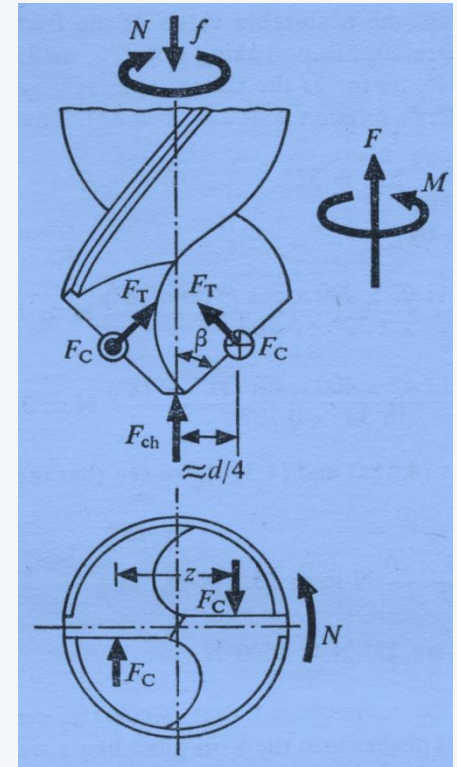
Forces in Turning



Forces in shaping/planing



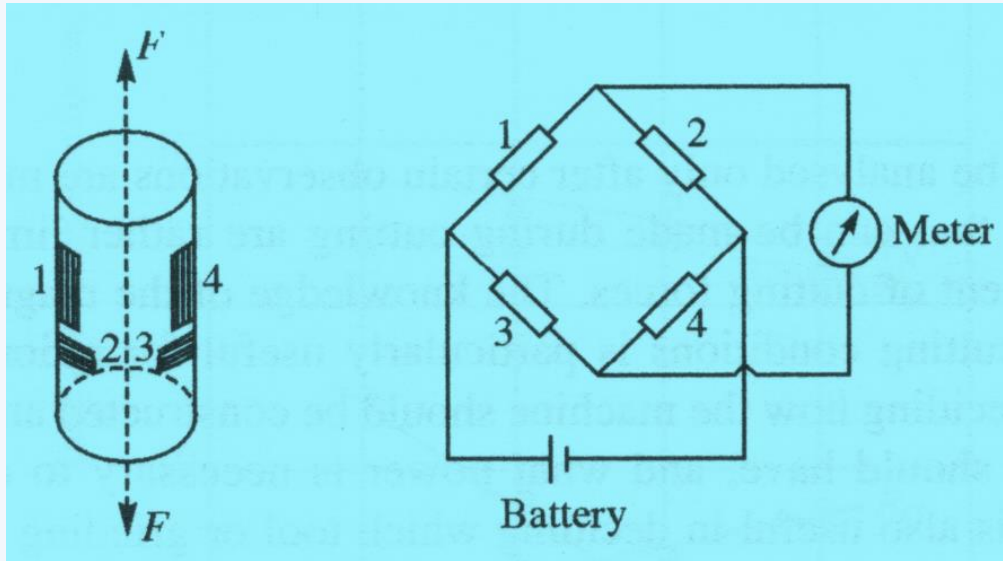
Forces in Milling



Forces in Drilling

Measurement of Cutting Forces

Axially Loaded Member:



Gauges 1 and 4 will measure Axial Strain and the circumferential strain caused by Poisson's effect will be measured by gauges 2 & 3.

The Strain in the gauges will be:

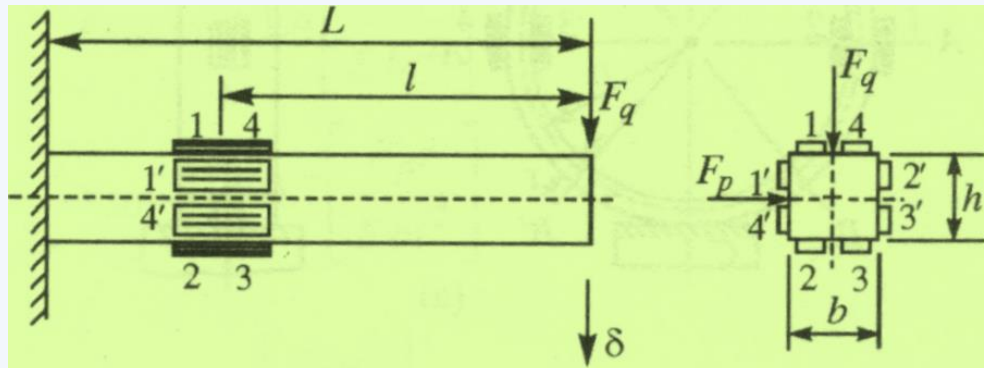
$$e_1 = e_4 = \frac{F}{AE} ; e_2 = e_3 = -\frac{\gamma F}{AE}$$

With this arrangement of the gauges, the bridge output will be insensitive to any loading other than F . Suppose, a bending load causes an additional strain in gauge 1, gauge 4 will then have an equal and opposite strain and the effect will cancel out in the bridge since gauges 1 & 4 are in opposite arms, symmetrically placed.

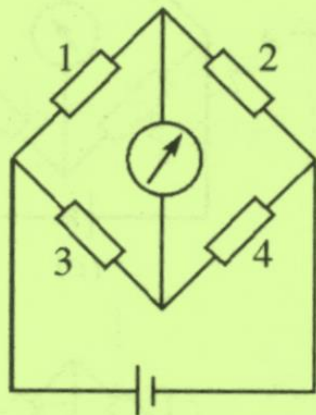
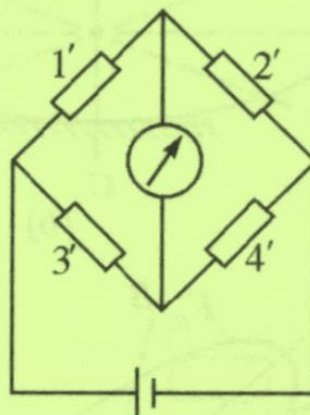
Measurement of Cutting Forces

Cantilever Beam

:



(a)



F_q is measured by gauges 1, 2, 3 & 4

F_p is measured by gauges 1', 2', 3' & 4'

$$e_1 = e_2 = e_3 = e_4 = \frac{6F_q l}{Ebh^2}$$

$$e'_1 = e'_2 = e'_3 = e'_4 = \frac{6F_p l}{Ehb^2}$$

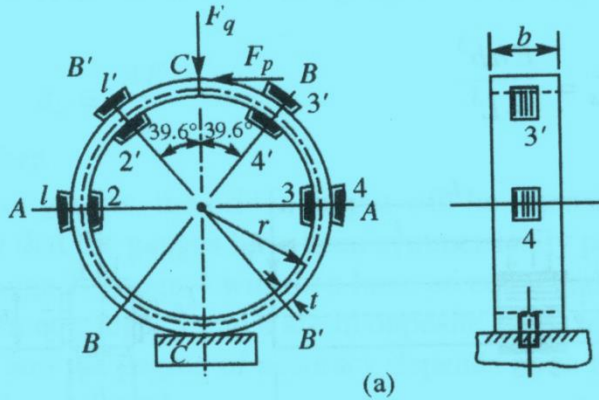
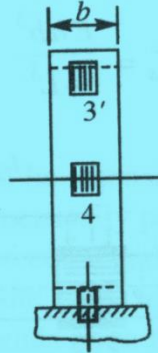
In general:

$$e = \frac{\sigma_i}{E}; \sigma_i = \frac{6M}{bh^2} = \frac{6Fl}{bh^2}$$

$$\text{Therefore, } e = \frac{6Fl}{Ebh^2}$$

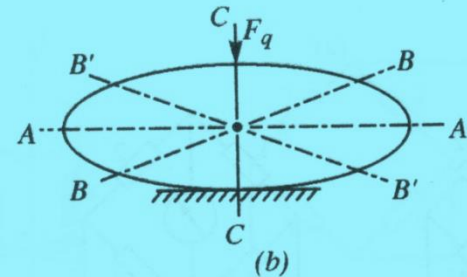
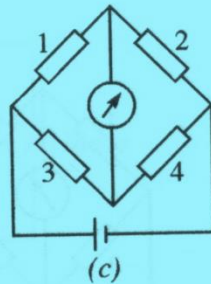
Measurement of Cutting Forces

Rings:

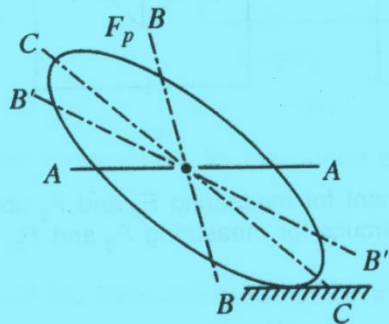
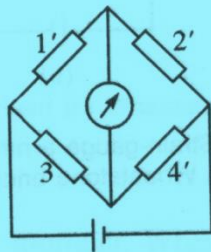


Rings provide a high ratio of sensitivity to stiffness and at the same time have adequate stability against buckling.

When the ring is subjected to normal force, inside of the ring will be subjected to a opposite state of strain from the outside. Hence four active strain gauges can be located at 1,2,3 & 4.



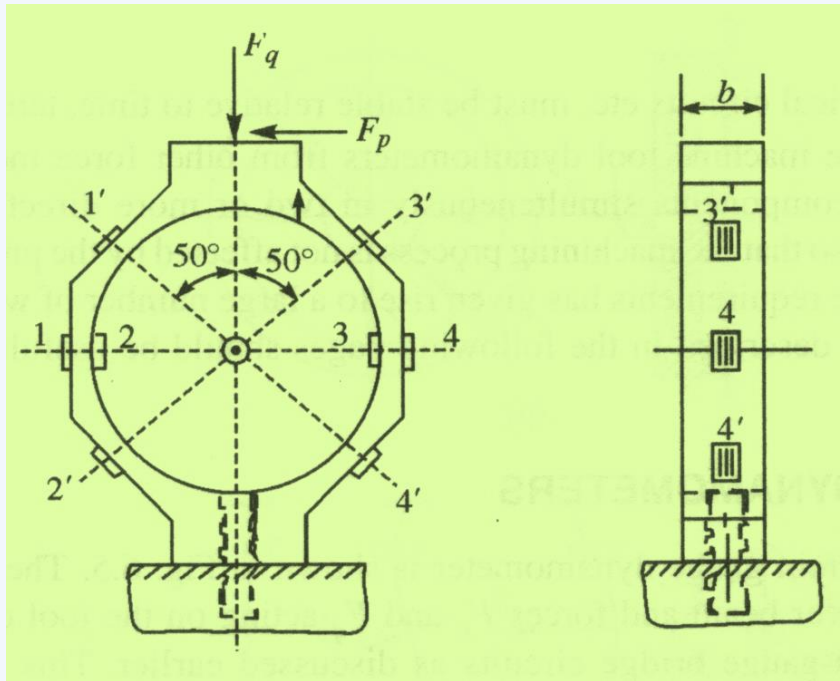
$$e_{90^\circ} = \pm 1.09 \left[\frac{F_q r}{E b t^2} \right] \quad (\text{Thin ring elastic theory})$$



$$e_{39.6^\circ} = \pm 2.31 \left[\frac{F_p r}{E b t^2} \right]$$

Measurement of Cutting Forces

Octagon :



Photoelastic examination of such rings reveal that the strain nodes now occur at 50 and 90 degrees.

F_q is measured by the gauges 1,2,3 & 4, while gauges at 50 degrees pick up F_p .

$$e_{90^\circ} = \pm 1.09 \left[\frac{F_q r}{E b t^2} \right]$$

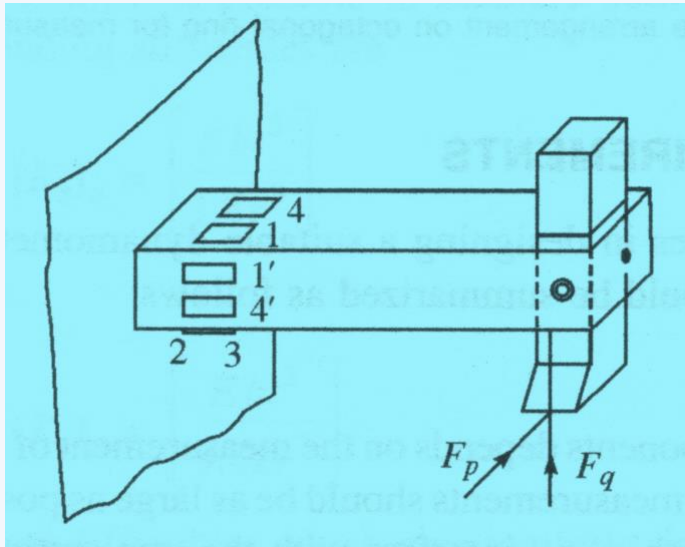
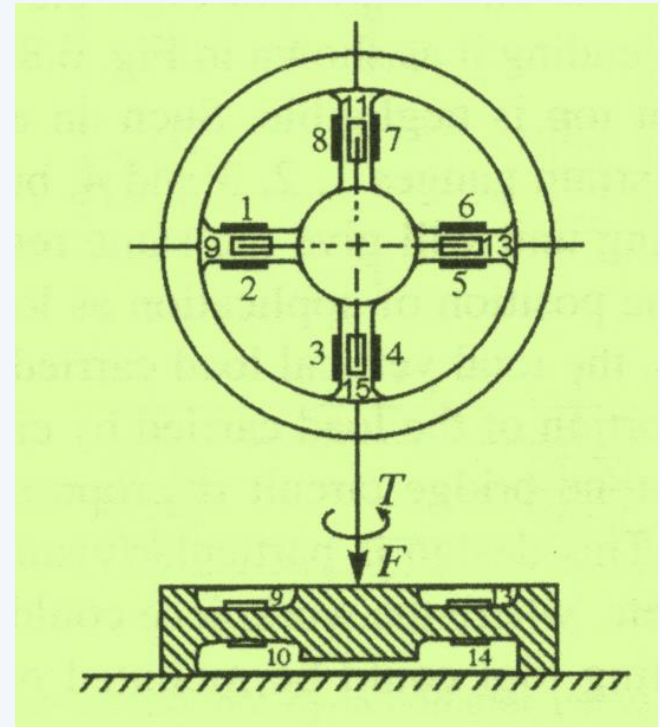
$$e_{50^\circ} = \pm 1.4 \left[\frac{F_p r}{E b t^2} \right]$$

Measurement of Cutting Forces

Dynamometer Requirements:

- a) Sensitivity
- b) Rigidity: Should be sufficiently rigid relative to the rest of the machine tool so that it does not affect the cutting condition by causing excessive vibration and chatter. In order that the interference with cutting condition is small, the natural frequency of the dynamometer should be several times the cutting frequency.
- c) Linearity : The relationship between the magnitude of the cutting forces and the deflection should be perfectly linear without any hysteresis effect.
- d) Interaction : There should be negligible cross sensitivity.
- e) Stability:

Machine Tool Dynamometer

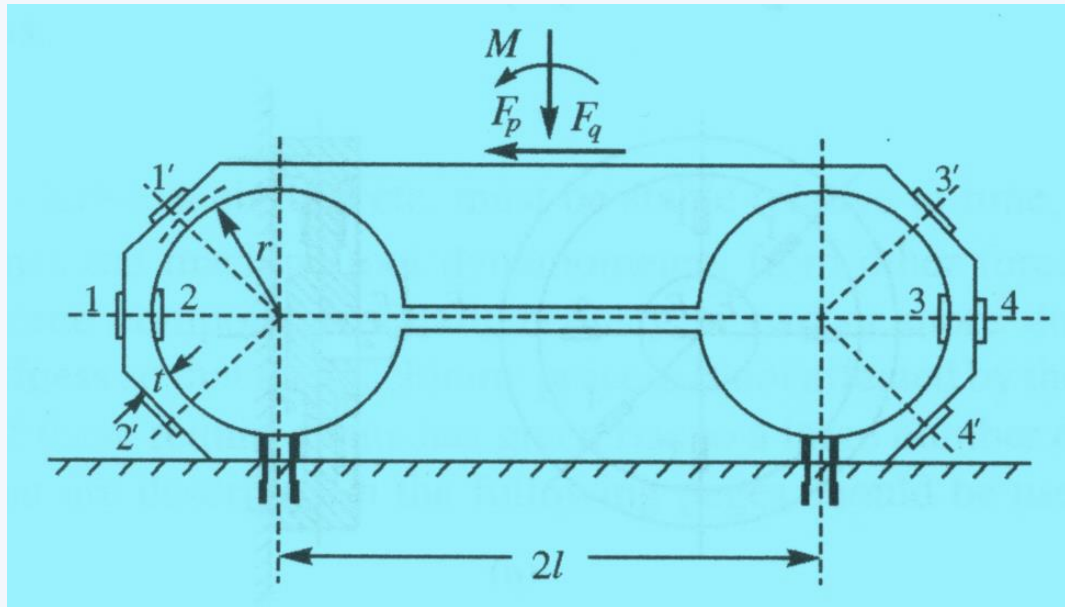


Drilling Dynamometer



Measurement of Cutting Forces

Extended Octagonal Ring Dynamometer :



Circular or Octagonal rings are not useful by itself but can be made sufficiently stable by extending it.

Suitable for Horizontal milling, horizontal surface grinding etc.

Tool Wear & Tool Life

Modes of Tool Failure

- **Fracture failure**

Cutting force becomes excessive and/or dynamic, leading to brittle fracture

- **Temperature failure**

Cutting temperature is too high for the tool material

- **Gradual wear**

Gradual wearing of the cutting tool



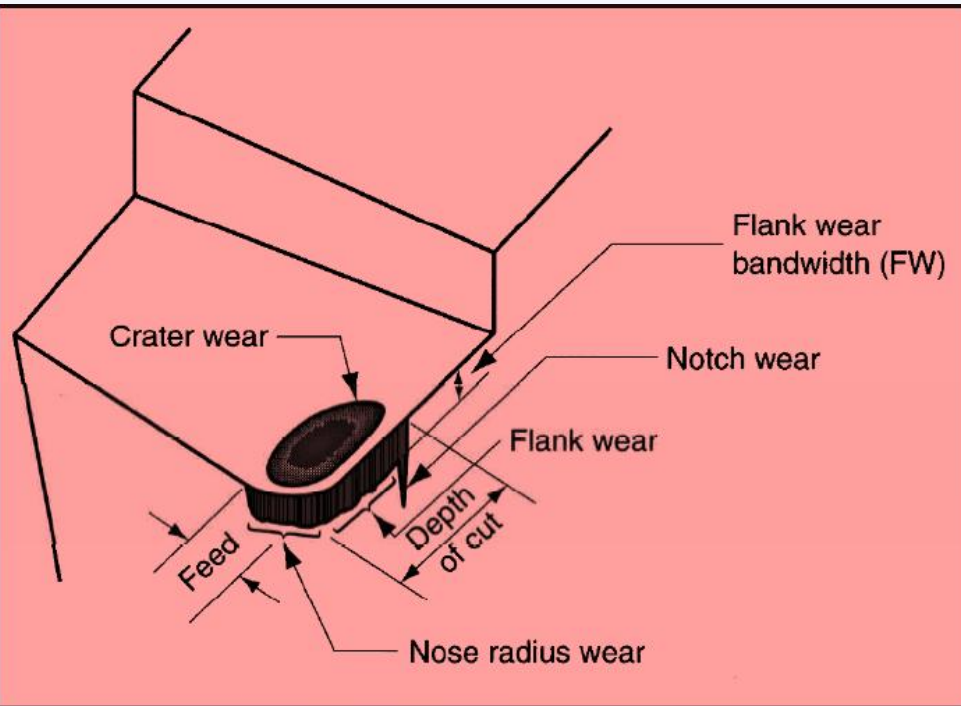
Tool Wear & Tool Life

Mechanisms of Tool Wear:

- **Abrasion:** If one of the surfaces contains very hard particles, then these during the process of sliding may dislodge material from other surfaces by Ploughing action.
- **Adhesion:** When bodies of similar nature are in contact, the asperities tend to get welded. Sliding causes fracture of these welded junctions and material is lost from both the surfaces.
- **Diffusion:** Atoms in a metallic crystal lattice always move from higher concentration to that of low concentration. The process is known as Diffusion.



Tool Wear & Tool Life

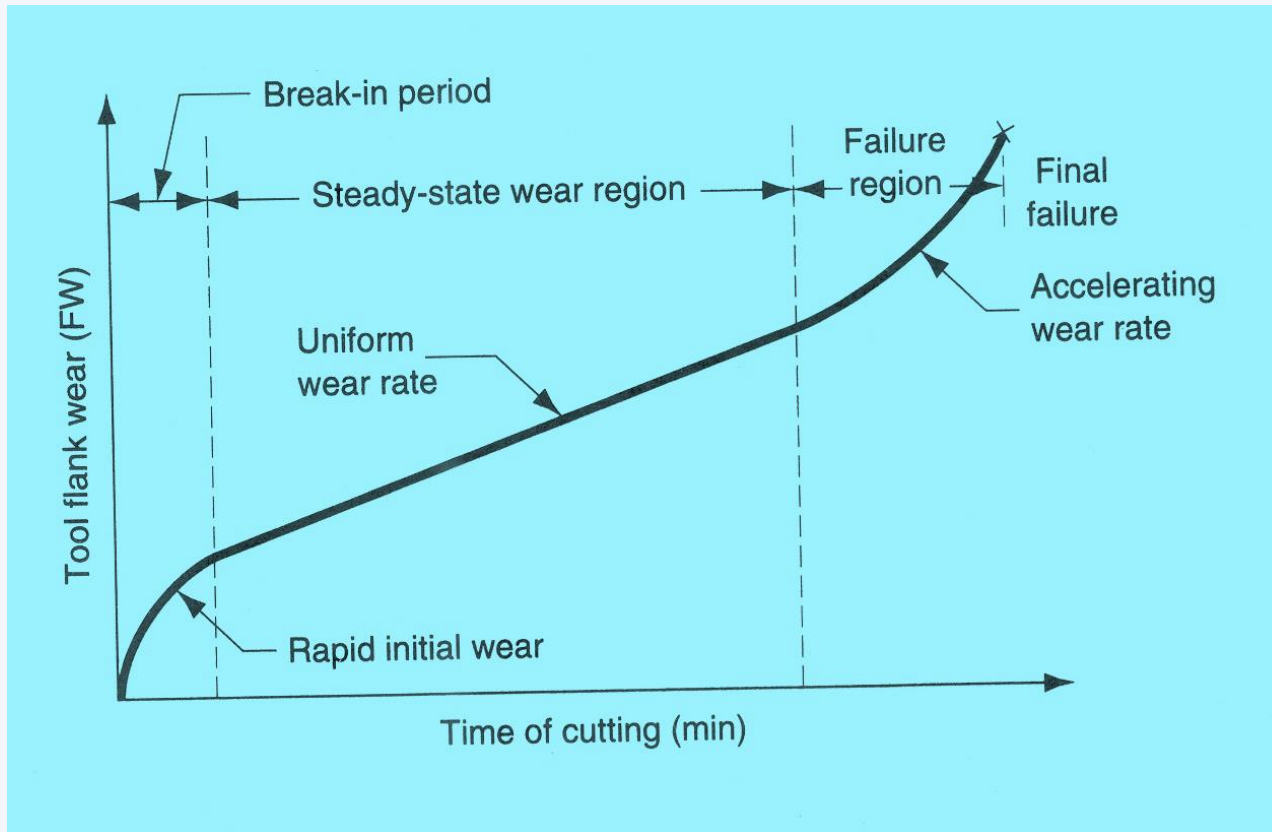


- The progressive wear of a cutting tool first occurs on the flank face in the form of a wear land due to rubbing against the newly machined surface known as FLANK WEAR.
- As a result of chip flowing over the rake face, wear also takes place on the rake surface in the form of a crater known as CRATER WEAR.

- Flank wear is measured by means of a measuring microscope.
- Crater wear can be evaluated by means of a surface analyser.
- Crater wear is formed at higher speeds, when diffusion has been considered to play a major role.



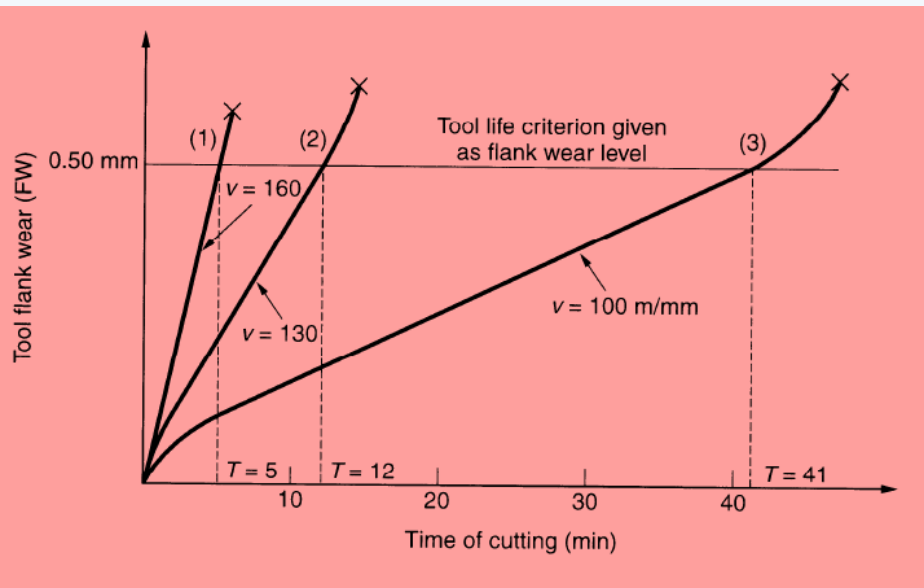
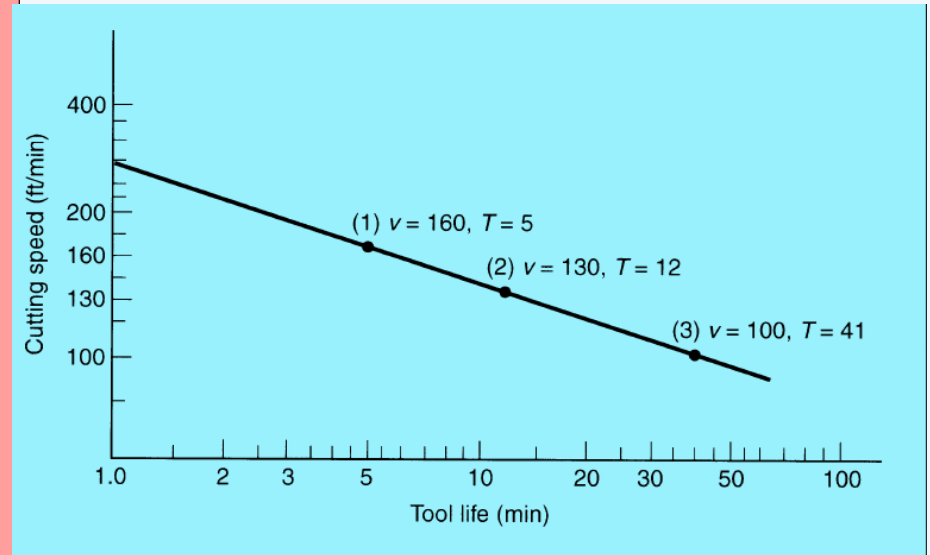
Tool Wear & Tool Life



A tool that no longer performs the desired function (and not unable to cut at all) is said to have reached the end of its useful life or failed. At this point the tool is resharpened and used again.



Tool Wear & Tool Life



Taylor Tool Life Equation:

$$VT^n = C$$

where v = cutting speed; T = tool life; and n and C are parameters that depend on feed, depth of cut, work material, tooling material, and the tool life criterion used

n is the slope of the plot

C is the intercept on the speed axis



Tool Wear & Tool Life

- The variables, Speed, Feed and Depth of cut affect the material removal rate and hence have a direct effect on the production cost.

$$T = \frac{C^{\frac{1}{n}}}{V^{\frac{1}{n}}}$$

T can also be expressed as:

$$T = \frac{C_1}{V^{\frac{1}{n}} f^{\frac{1}{n_1}} d^{\frac{1}{n_2}}}$$

Where, $\frac{1}{n} > \frac{1}{n_1} > \frac{1}{n_2}$, indicates that the cutting speed has greatest effect on tool life followed by feed and then depth of cut.



Tool Wear & Tool Life

Variables Affecting Tool Life:

- The Cutting Conditions
- The Tool Geometry
- The Tool Material
- The Work Material
- The Cutting Fluid



Tool Wear & Tool Life

Effect of Tool Geometry: Rake angle

- Increasing the Rake Angle reduces the cutting force and the cutting temperature resulting in increased tool life.
- However, for large rake angle, tool edge is weakened resulting in increased wear due to chipping of the cutting edge.
- Increased wear is also due to larger temperature since the tool becomes thinner and the area available for heat conduction reduces.
- These conditions give an optimum rake angle which gives the maximum tool life.
- Higher is the strength of workpiece material, lower is the value of optimum rake angle.



Tool Wear & Tool Life

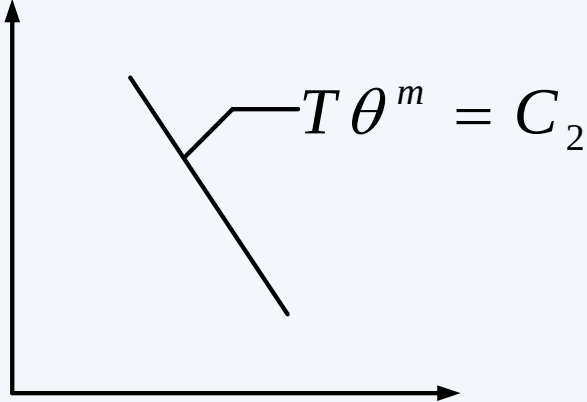
Effect of Tool Geometry: Flank angle

- Increasing the Flank Angle reduces rubbing between tool and the workpiece and hence improves the tool life.
- However, too high a value of flank angle weakens the tool and reduces its life.
- Optimum value of flank angles is also affected by the feed rates. Higher is the feed rate, lower is the optimum value. The flank angle, therefore, should be low if higher feed values are to be used.



Tool geometry Vs Tool life.

Log T (Tool life)

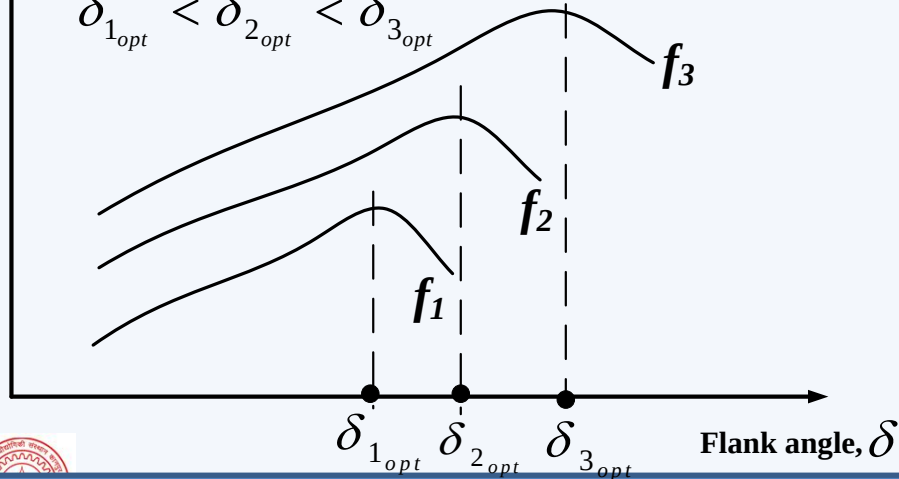


Log θ (Temperature)

Tool life, T

$f_1 > f_2 > f_3$ (feed rate)

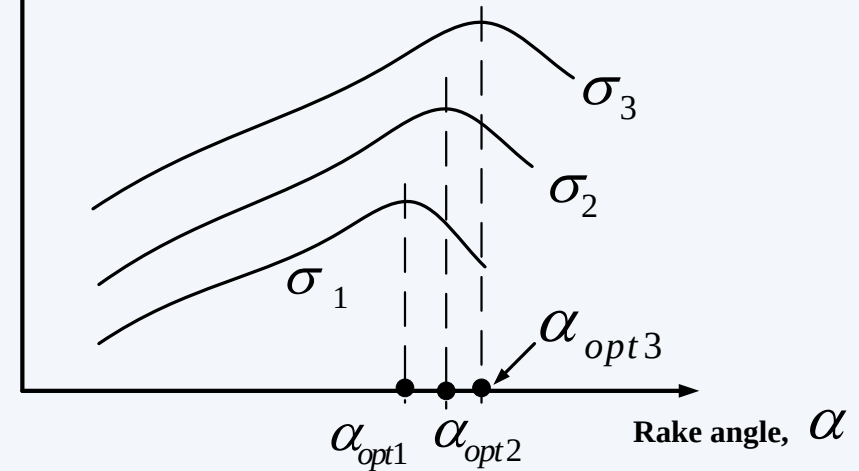
$\delta_{1opt} < \delta_{2opt} < \delta_{3opt}$



Tool life, T

$\sigma_1 > \sigma_2 > \sigma_3$

$\alpha_{opt1} < \alpha_{opt2} < \alpha_{opt3}$



Tool Wear & Tool Life

Tool and Work Material

- Tool material must be at least 35% to 58% harder than the work material.
- High strain rate of deformation and elevated temperature of the work material further complicate the situation.
- With the increase in machining speed, the temperature of both the tool and the work material increases, resulting in a lowered effective hardness of the tool. Unfortunately, the expected fall in the hardness in the work material is neutralized by the higher rate of deformation.
- In general, harder the work material, higher will be the tool wear rate and lower will be the tool life.



Tool Wear & Tool Life

Cutting Fluid

- Cutting fluids are primarily used for decreasing the cutting temperature and the chip-tool interface friction.
- They also serve to keep the workpiece cool to avoid thermal expansion, provide a rust-proof layer on the finished surface and remove chips from the machining area.
- Cutting fluids should have high specific heat and good thermal conductivity, a chemical constituent to form weak junctions, should have a low viscosity and small molecular size, non corrosive and inexpensive.
- At a very high speed, coolant is ineffective.



Numerical Examples

Problem – 1: Tool life tests in turning yield the following data: (1) $v = 100$ m/min, $T = 10$ min; (2) $v = 75$ m/min, $T = 30$ min. (a) Determine the n and C values in the Taylor tool life equation. Based on your equation, compute (b) the tool life for a speed of 90 m/min, and (c) the speed corresponding to a tool life of 20 min.

SOLUTION:

(a) Two equations: (1) $100(10)^n = C$ and (2) $75(30)^n = C$.

$$100(10)^n = 75(30)^n$$

$$\ln 100 + n \ln 10 = \ln 75 + n \ln 30$$

$$4.6052 + 2.3026 n = 4.3175 + 3.4012 n$$

$$4.6052 - 4.3175 = (3.4012 - 2.3026) n$$

$$0.2877 = 1.0986 n \quad \mathbf{n = 0.2619}$$

$$C = 100(10)^{0.2619} = 100(1.8277) \quad \mathbf{C = 182.77}$$

$$(b) 90 T^{0.2619} = 182.77$$

$$T^{0.2619} = 182.77/90 = 2.037$$

$$T = 2.037^{1/0.2619} = 2.037^{3.818} = \mathbf{15.13 \text{ min.}}$$

$$(c) V (20)^{0.2619} = 182.77$$

$$V = 182.77/(20)^{0.2619} = 182.77/2.1914 = \mathbf{83.4 \text{ m/min.}}$$



Numerical Examples

Problem – 2:

Turning tests have resulted in 1-min tool life for a cutting speed $v = 4.0$ m/s and a 20-min tool life at a speed $v = 2.0$ m/s. (a) Find the n and C values in the Taylor tool life equation. (b) Project how long the tool would last at a speed $v = 1.0$ m/s.

SOLUTION:

(a) For data (1) $T = 1.0$ min, then **$C = 4.0$ m/s = 240 m/min.**

For data (2) $v = 2$ m/s = 120 m/min.

$$120(20)^n = 240; \text{ or, } 20^n = 240/120 = 2.0$$

$$n \ln 20 = \ln 2.0; \text{ or, } 2.9957 n = 0.6931 \quad \mathbf{n = 0.2314}$$

(b) At $v = 1.0$ m/s = 60 m/min.

$$60(T)^{0.2314} = 240$$

$$(T)^{0.2314} = 240/60 = 4.0$$

$$T = (4.0)^{1/0.2314} = (4)^{4.3215} = \mathbf{400 \text{ min.}}$$



Numerical Examples

Problem – 3:

In a production turning operation, the workpart is 125 mm in diameter and 300 mm long. A feed rate of 0.225 mm/rev is used in the operation. If cutting speed = 3.0 m/s, the tool must be changed every 5 workparts; but if cutting speed = 2.0 m/s, the tool can be used to produce 25 pieces between tool changes. Determine the Taylor tool life equation for this job.

SOLUTION:

$$(1) T_m = \pi(125 \text{ mm})(0.3 \text{ m}) / (3.0 \text{ m/s})(0.225 \text{ mm}) = 174.53 \text{ s} = 2.909 \text{ min.}$$

$$T = 5(2.909) = 14.54 \text{ min.}$$

$$(2) T_m = \pi(125 \text{ mm})(0.3 \text{ m}) / (2.0 \text{ m/s})(0.225 \text{ mm}) = 261.80 \text{ s} = 4.363 \text{ min.}$$

$$T = 25(4.363) = 109.08 \text{ min.}$$

$$(1) v = 3 \text{ m/s} = 180 \text{ m/min.}$$

$$(2) v = 2 \text{ m/s} = 120 \text{ m/min.}$$



Numerical Examples

$$(1) 180(14.54)^n = C$$

$$(2) 120(109.08)^n = C$$

$$180(14.54)^n = 120(109.08)^n$$

$$\ln 180 + n \ln(14.54) = \ln 120 + n \ln(109.08)$$

$$5.1929 + 2.677 n = 4.7875 + 4.692 n$$

$$5.1929 - 4.7875 = (4.692 - 2.677) n$$

$$0.4054 = 2.0151 n$$

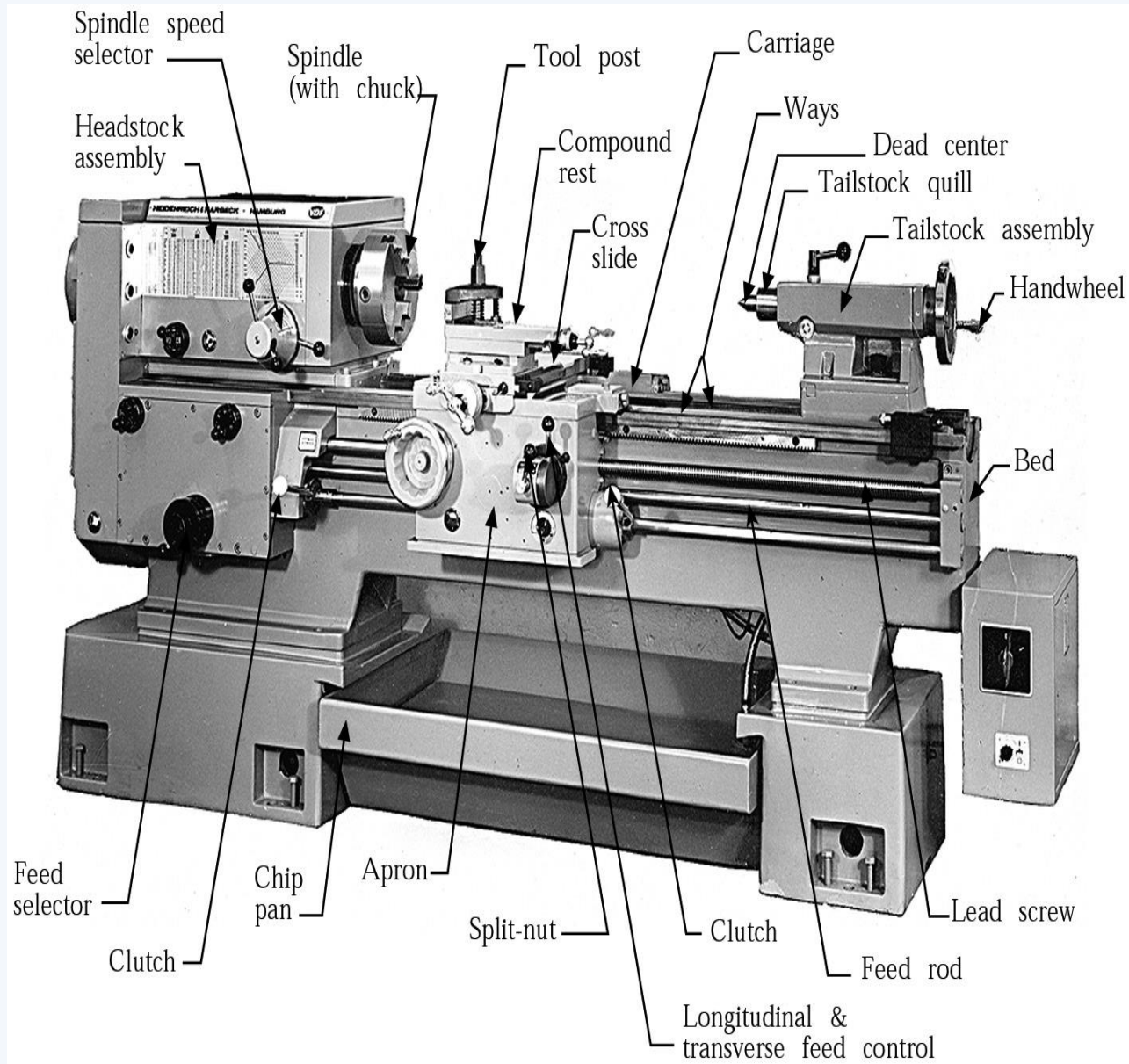
$$\mathbf{n = 0.2012}$$

$$C = 180 (14.54)^{0.2012}$$

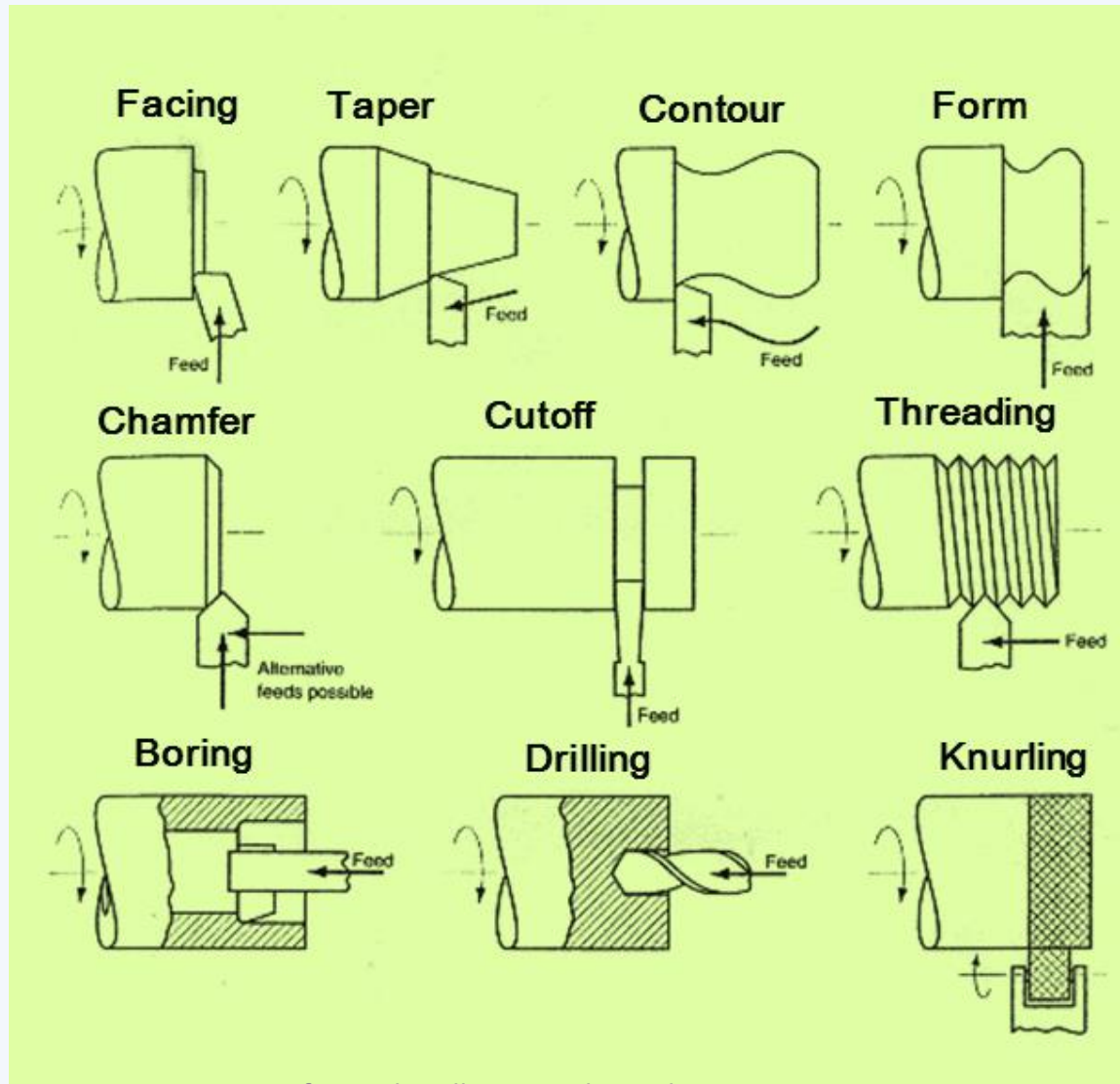
$$\mathbf{C = 308.43}$$



Lathe Machine

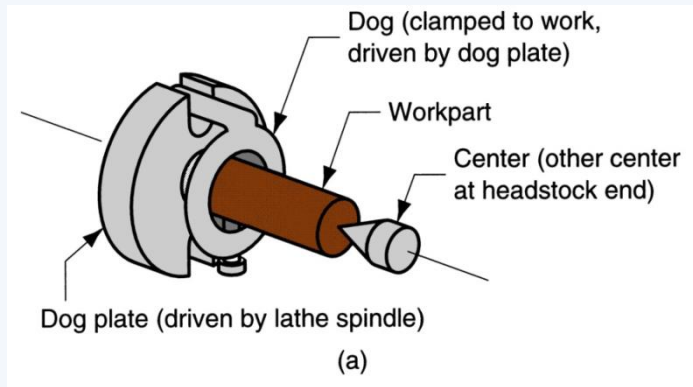


Tool-work interaction

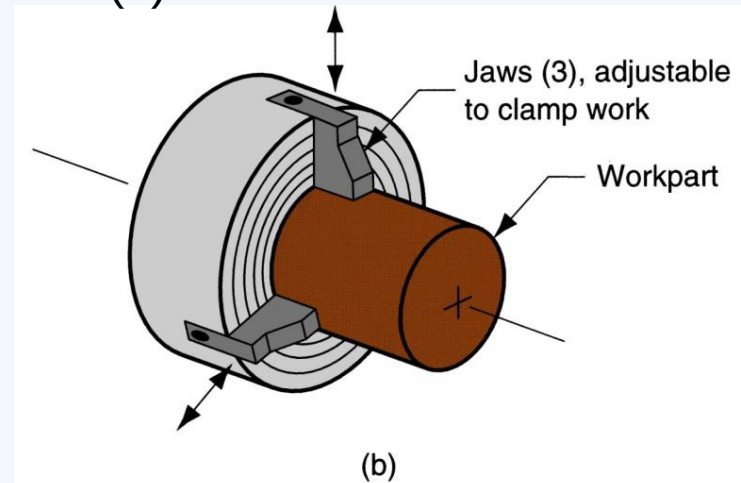


Methods of Holding the Work in a Lathe

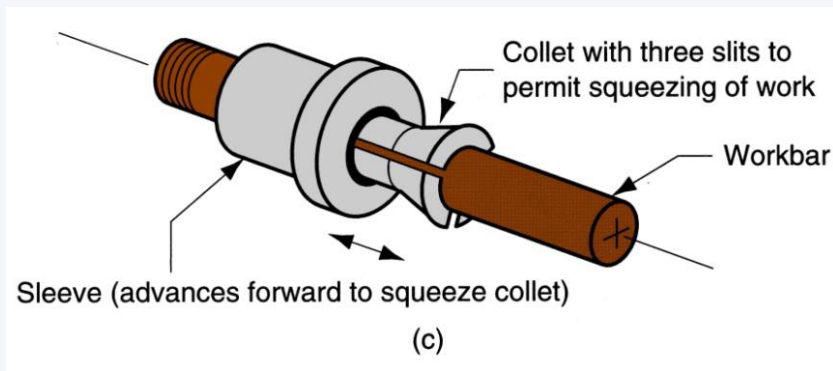
(a) Holding the work between centers



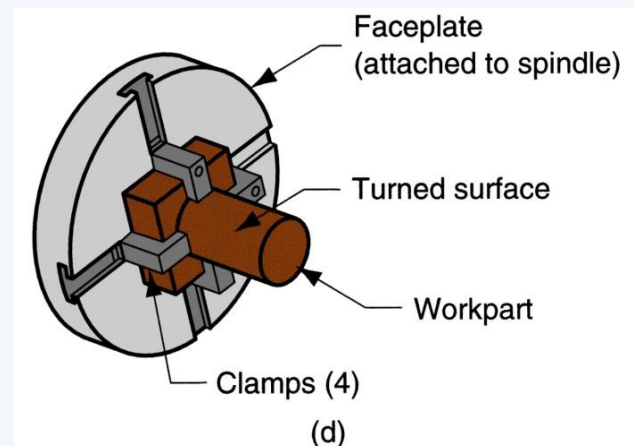
(b) Chuck



(c) Collet



(d) Face plate



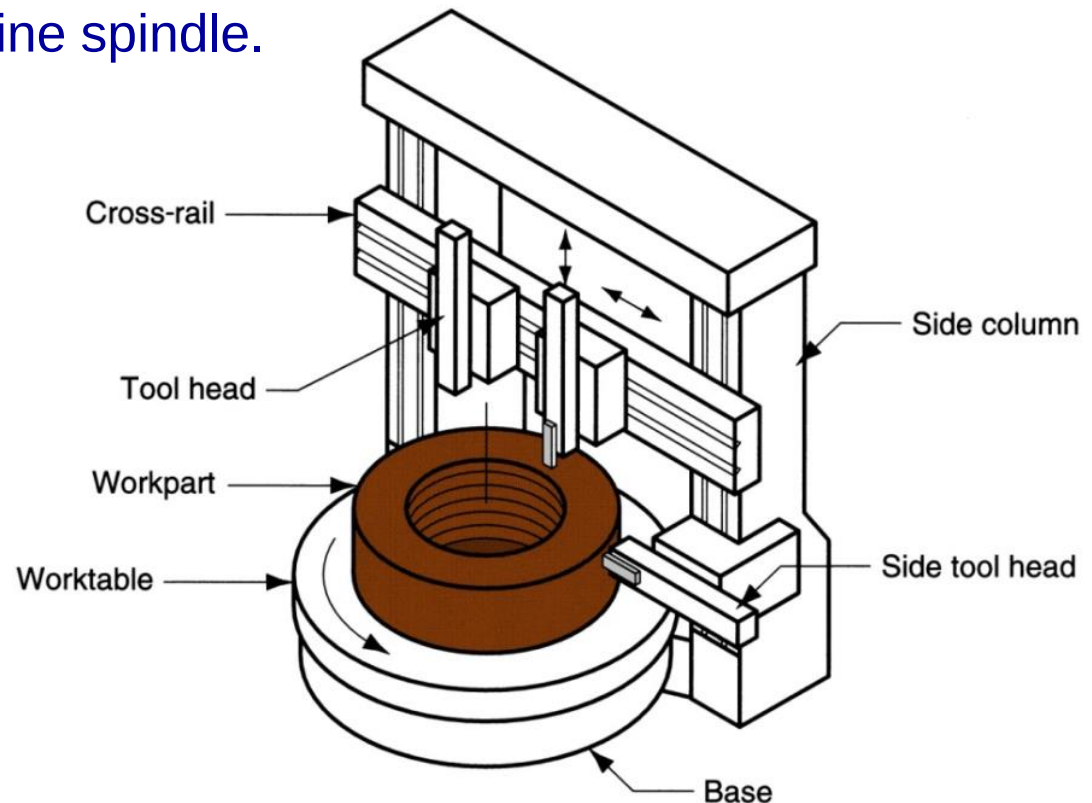
Boring

Difference between boring and turning:

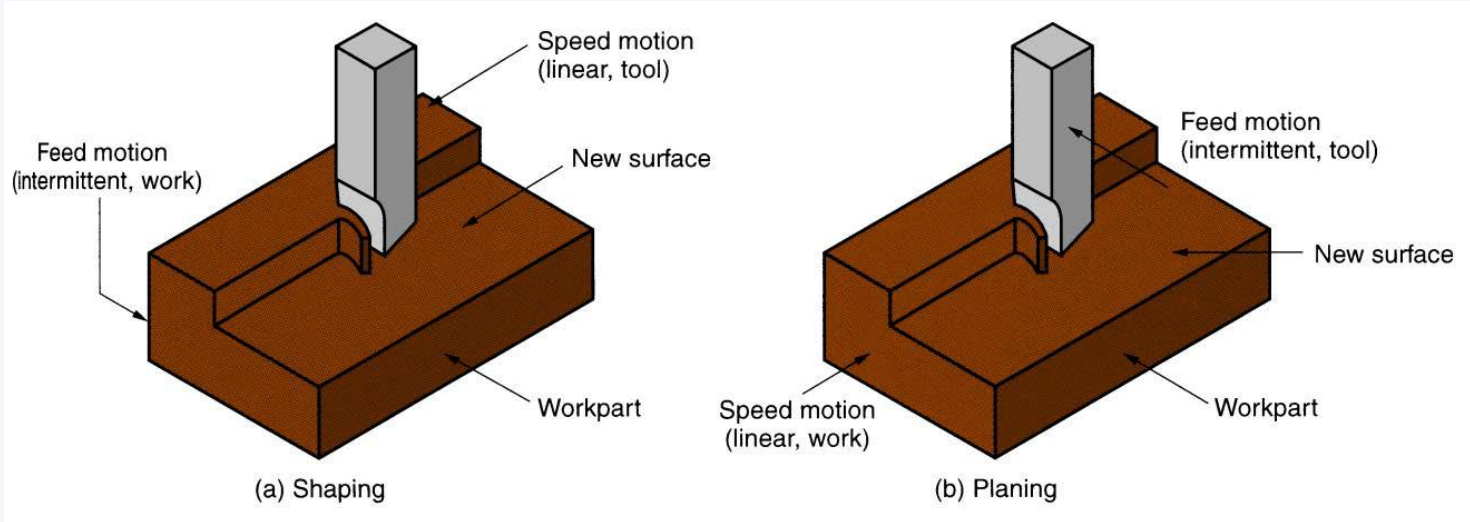
- *Boring is performed on the inside diameter of an existing hole.*
- *Turning is performed on the outside diameter of an existing cylinder*

➤ In effect, boring is an internal turning operation

Boring machines Horizontal or vertical - refers to the orientation of the axis of rotation of machine spindle.



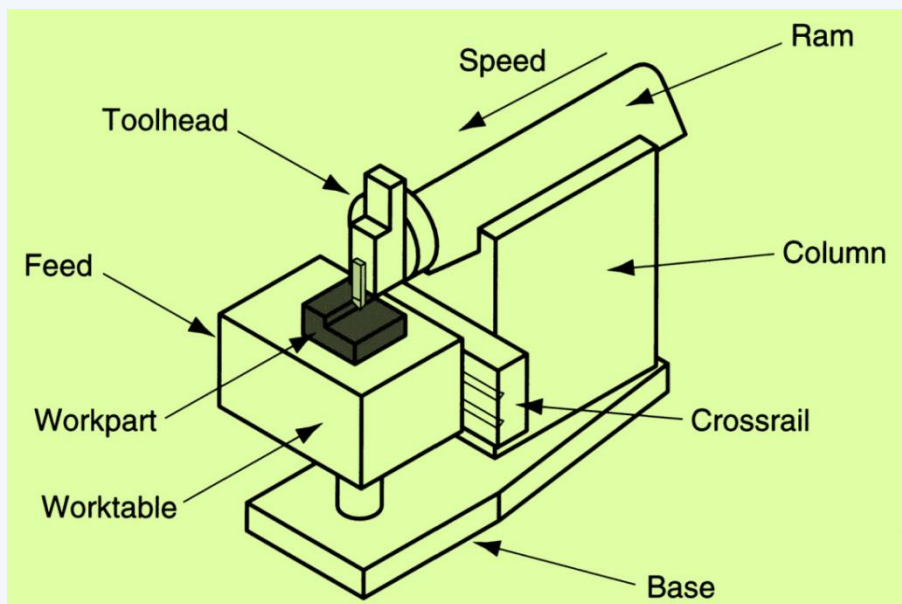
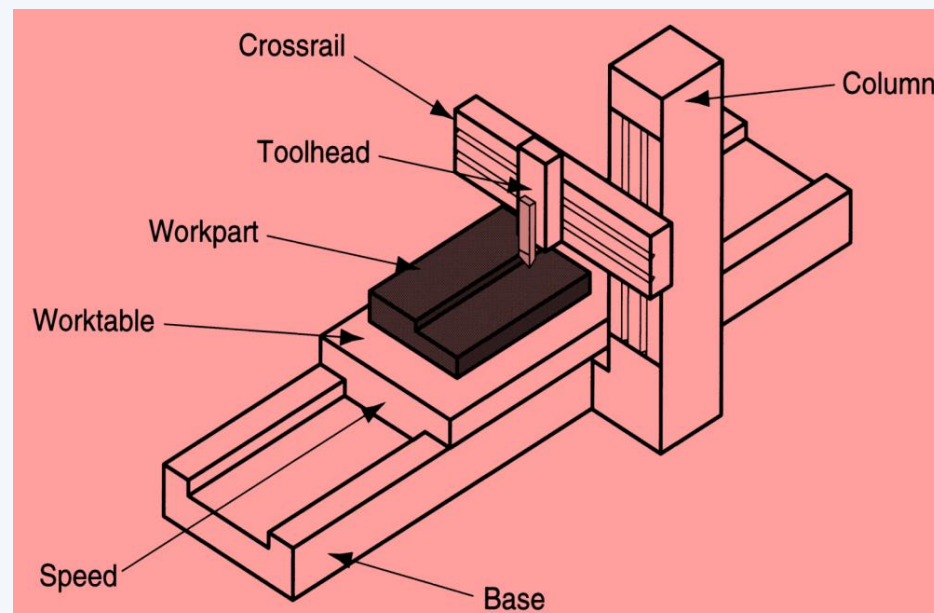
Shaping and Planing



- Both use a single point cutting tool moved linearly relative to the workpart
- A straight, flat surface is created in both operations
- Interrupted cutting ; Subjects tool to impact loading when entering work
- Low cutting speeds due to start-and-stop motion



Shaping and Planing Machines



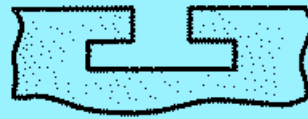
Components of a shaper and planer:



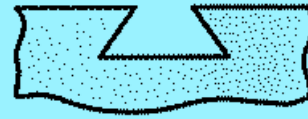
(a)



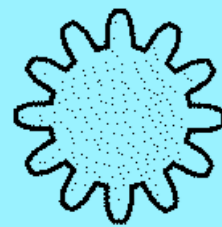
(b)



(c)



(d)

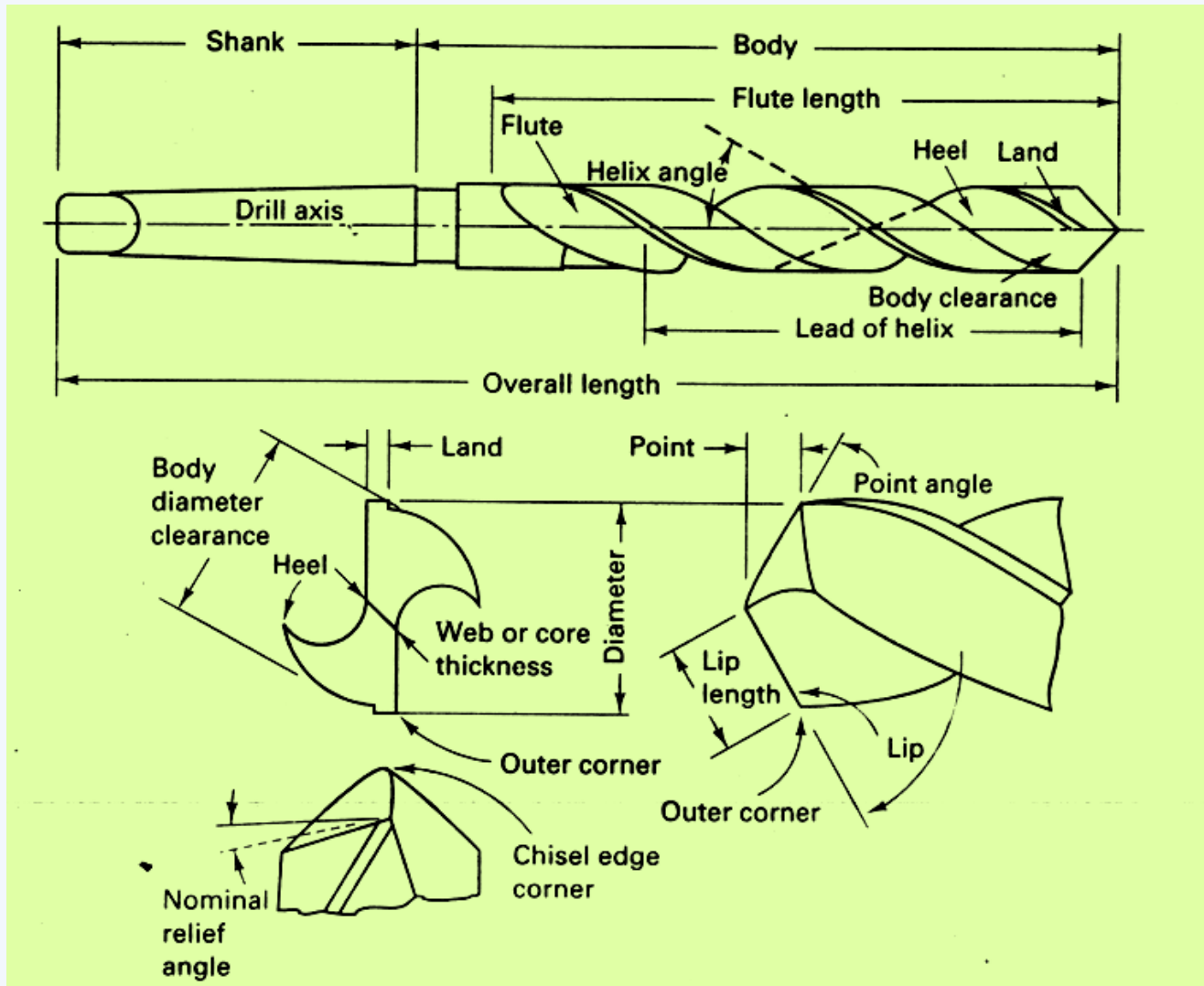


(e)

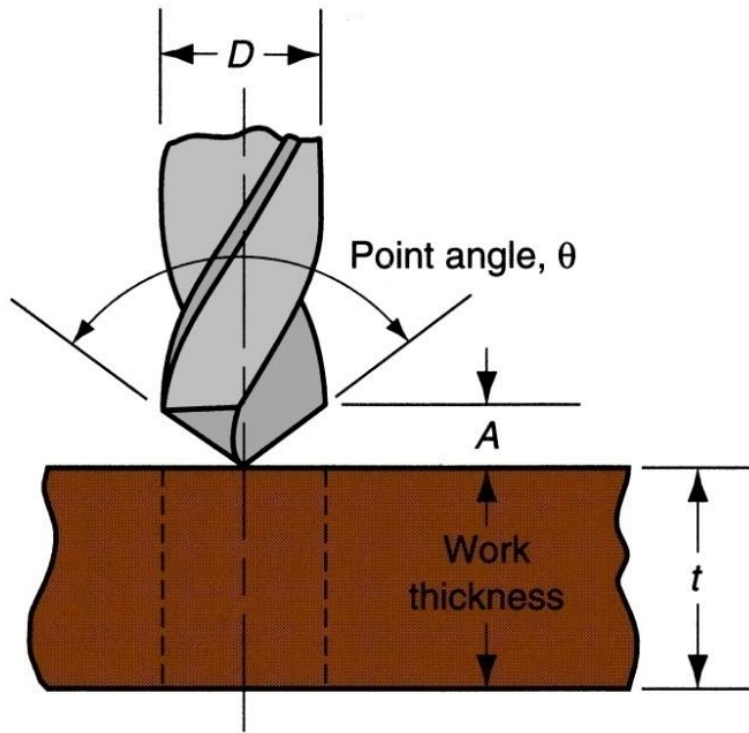


Drilling

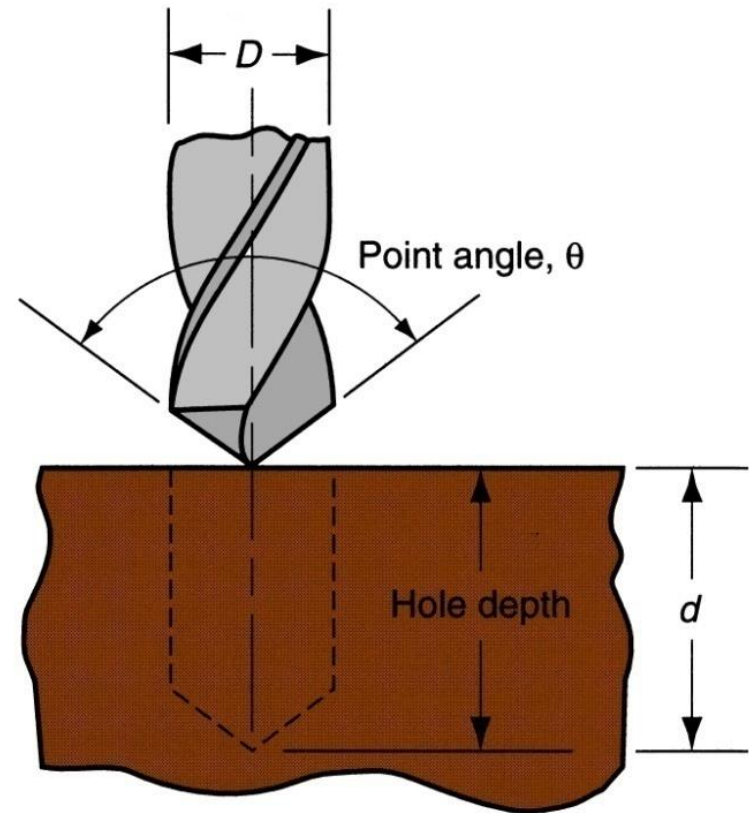
➤ Used for making (drilling) internal holes



Drilling – Through or Blind Holes

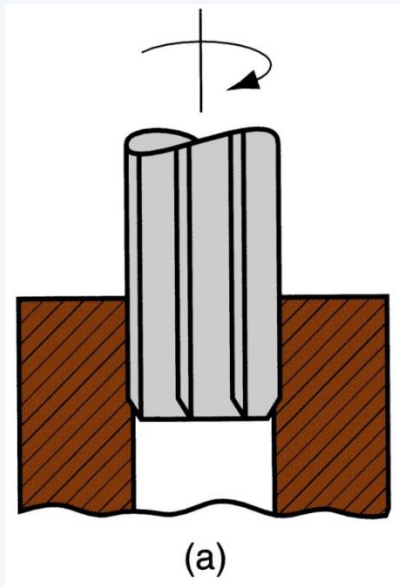
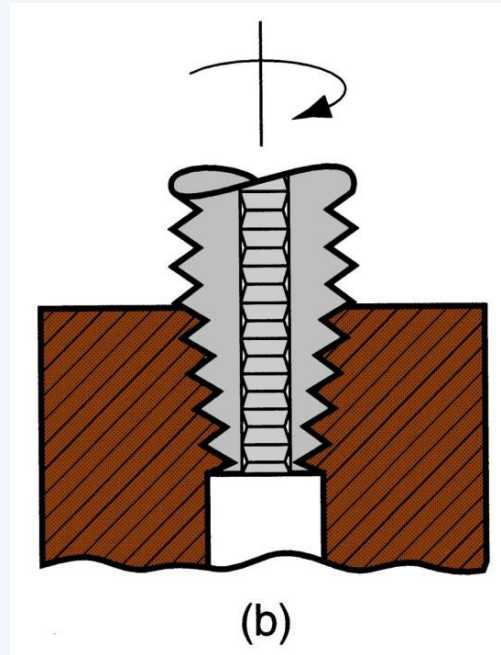
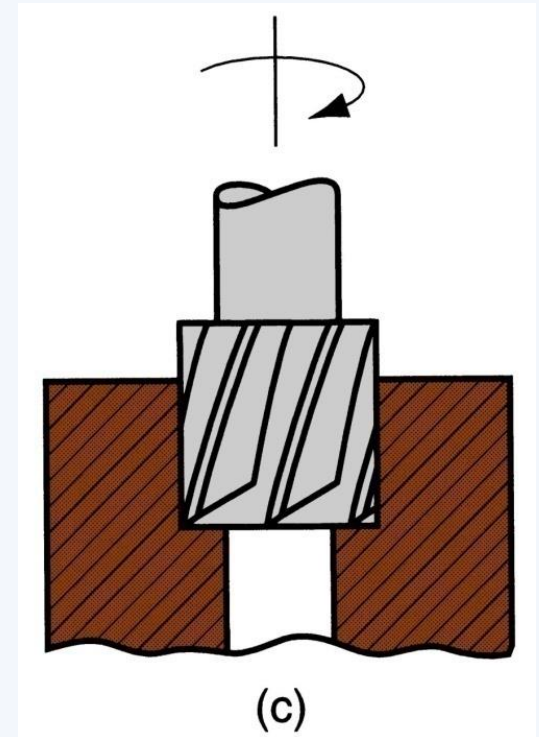


(a)



(b)

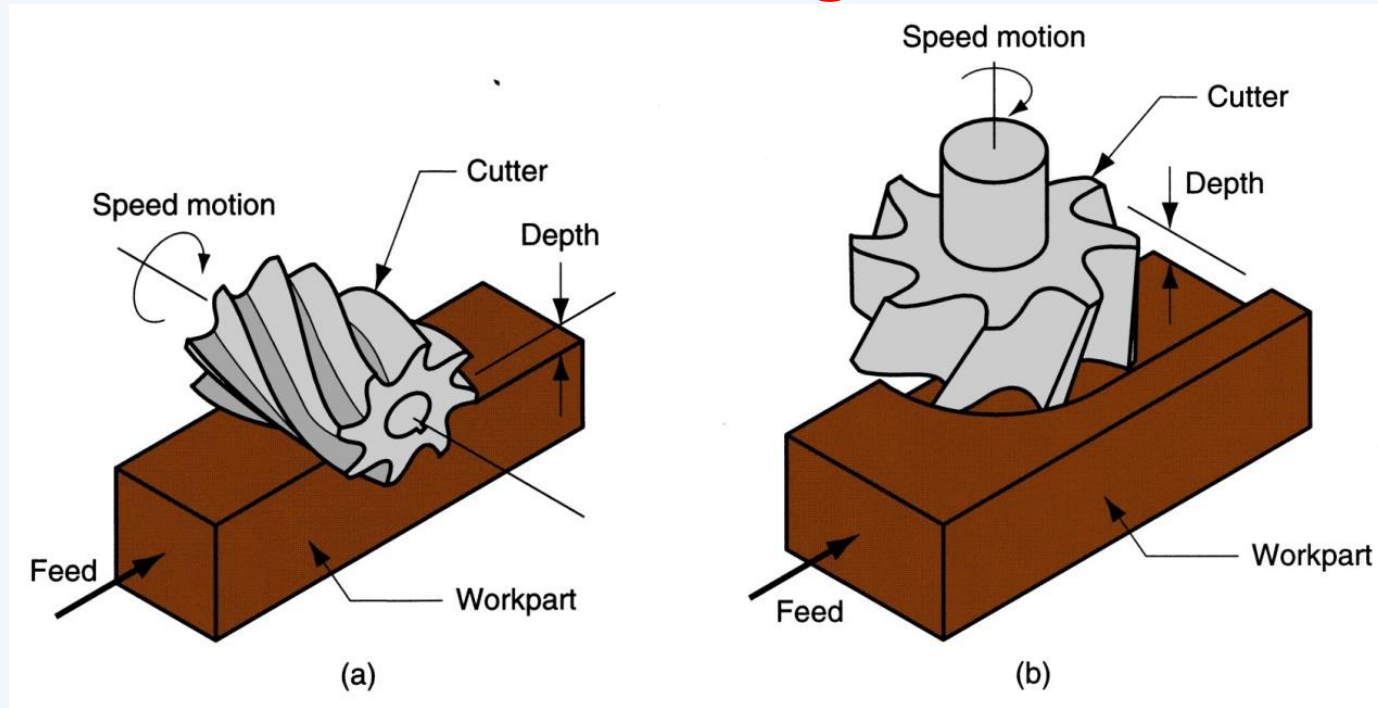
Machining operations performed on a drilling Machine



(a) Reaming (b) Tapping (c) Counterboring



Milling

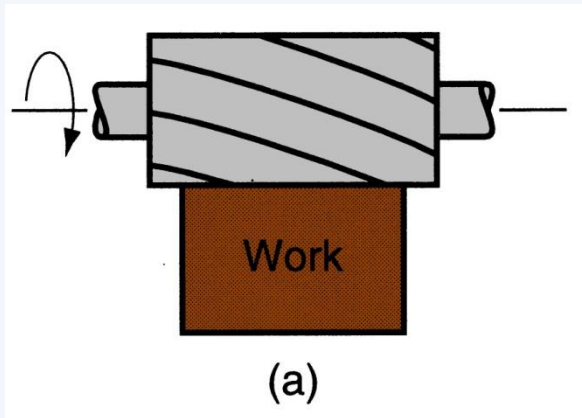
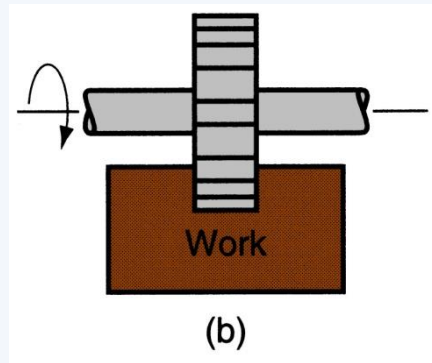
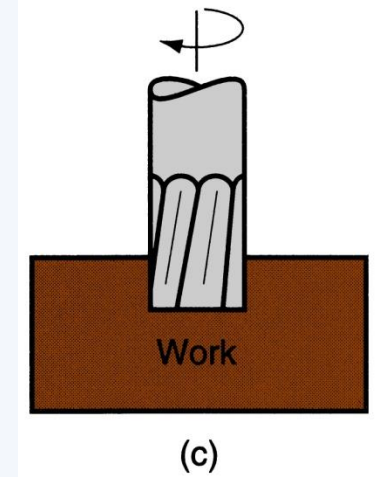


(a) Peripheral milling: Cutter axis is parallel to surface being machined Cutting edges on outside periphery of cutter

(b) Face milling: Cutter axis is perpendicular to surface being milled Cutting edges on both the end and outside periphery of the cutter



Various Milling Operations



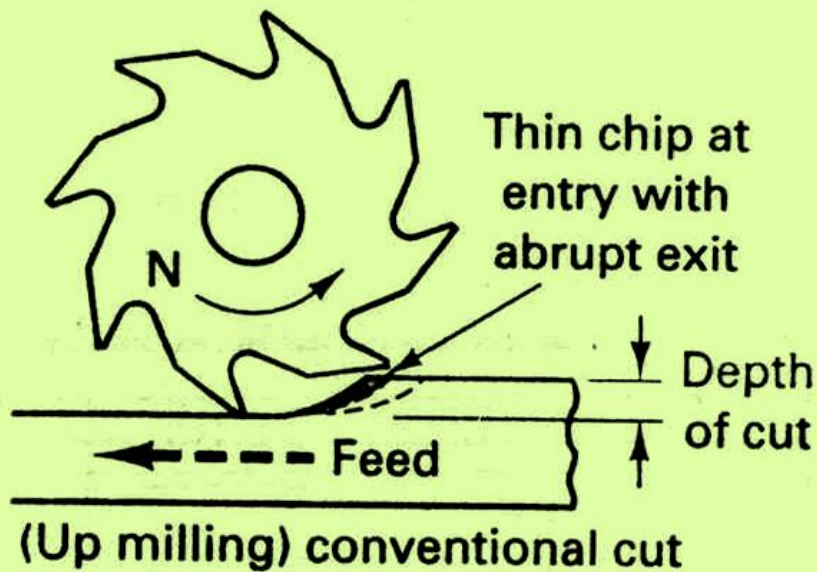
(a) Slab Milling: The basic form of peripheral milling in which the cutter width extends beyond the workpiece on both sides

(b) Slotting: Width of cutter is less than workpiece width, creating a slot in the work

(c) End Milling: Cutter diameter is less than work width, so a slot is cut into part



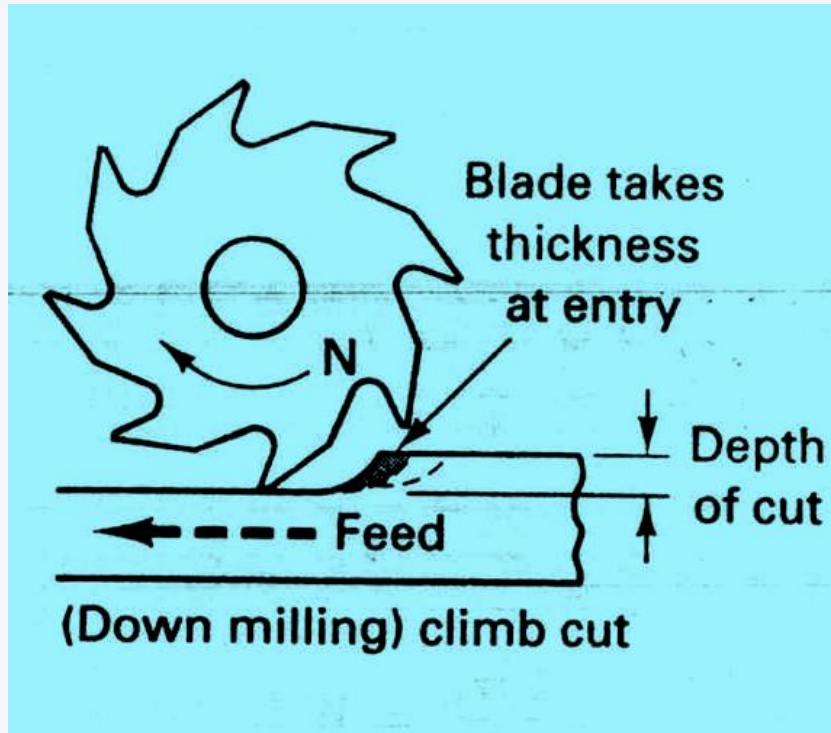
Up Milling



- Common method of milling
- Chip thin in the beginning and increases in thickness
- Work piece has a tendency to be pulled up. Needs clamping
- Tool engagement is not affected by surface characteristics, surface scales
- Chips can be carried into the newly machined surface leading to poor surface finish



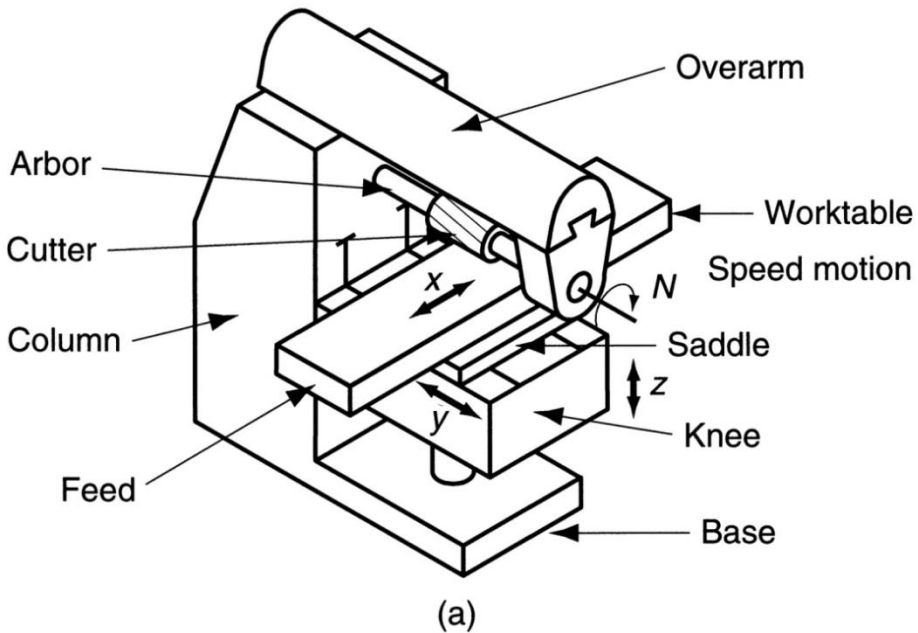
Down Milling



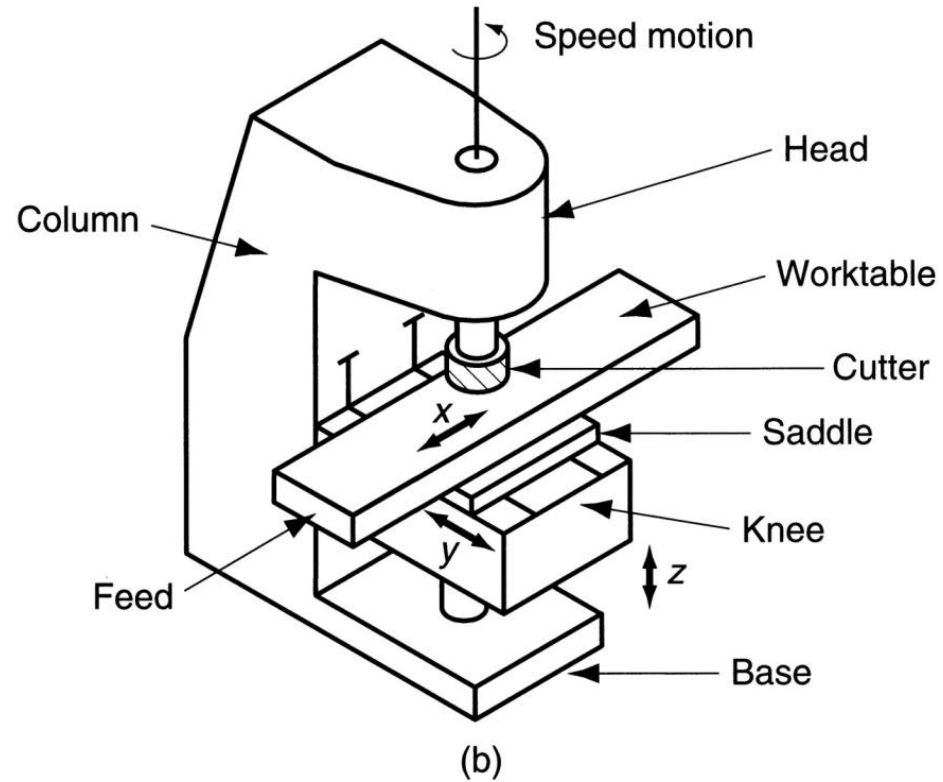
- Cutting starts at the surface where the chip is at its thickest.
- High impact forces require rigid setup
- Cutting force tends to push the work piece down reducing clamping requirements
- Less chatter
- Not suitable for work piece with hard, abrasive scales, as in hot worked metals, forgings and castings.
- Excessive tool wear



Milling Machines



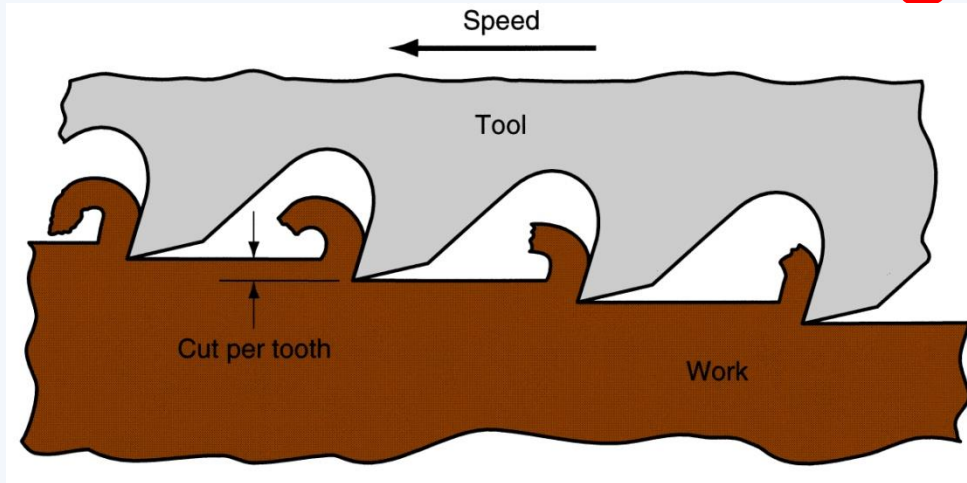
(a) Horizontal Milling Machine



(b) Vertical Milling Machine



Broaching



Moves a multiple tooth cutting tool linearly relative to work in direction of tool axis

Advantages:

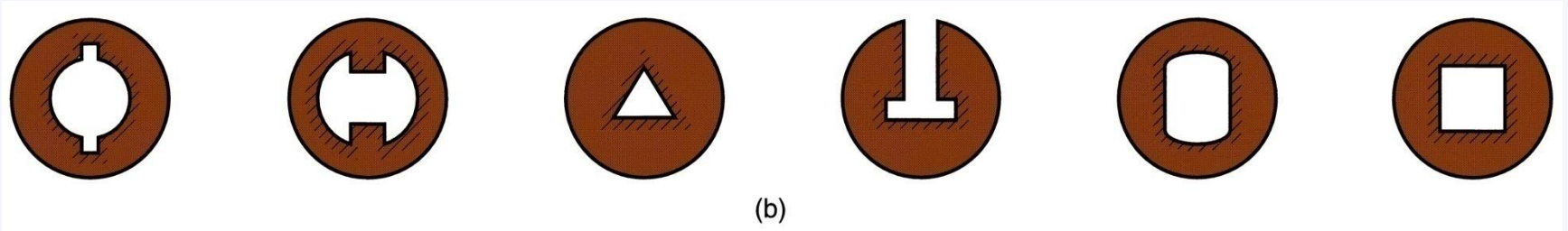
- ❑ Good surface finish
- ❑ Close tolerances
- ❑ Variety of work shapes possible

Cutting tool called a *broach*

➤ Owing to complicated and often custom-shaped geometry, tooling is expensive



Internal Broaching



- Performed on internal surface of a hole
- A starting hole must be present in the part to insert broach at beginning of stroke

Abrasive Machining Processes

- Hard particles like Al_2O_3 , B_4C , SiC etc. abrade the softer materials. Machining processes that use such particles are called Abrasive Machining Processes.
- Material removal by action of hard, abrasive particles usually in the form of a bonded wheel
- Generally used as finishing operations after part geometry has been established conventional machining
- Grinding is most important abrasive processes
- Material removal process in which abrasive particles are contained in a bonded grinding wheel that operates at very high surface speeds
- Other abrasive processes: honing, lapping, super finishing, polishing, and buffing



Abrasive Machining Processes

- The Grinding Wheel consists of abrasive particles and bonding material
- Abrasive particles accomplish cutting
- Bonding material holds particles in place and establishes shape and structure of wheel
- Grinding Wheel Parameters
 - Type of Abrasive material
 - Grain size
 - Wheel grade
 - Wheel structure
 - Bonding material



Abrasive material

Commonly used abrasives in abrasive machining are:

Conventional Abrasives:

- **Aluminum Oxide (A):** Used for grinding Steels, Fe-Alloys, Bronze and other high-strength materials.
- **Silicon Carbide (C):** Used for grinding Cast Iron, Brass, Al, Hard alloys and Carbides.

Superabrasives: Used for very hard materials like Glass, Carbides and Ceramics.

- **Cubic boron nitride (CBN)**
- **Diamond (D)**



Grain Size

- Grain Size is expressed in terms of a SIEVE NUMBER , S_n which corresponds to the number of openings per linear inch.
- The diameter of an abrasive grain, $D_g = \frac{0.6}{S_n}$ inch
- The larger the size of the grains, the more will be material removal, but surface finish will be worse.

<u>Sieve No.</u>	<u>Type of Grain</u>
10-24	Coarse
30-60	Medium
70-180	Fine
220-600	Very Fine



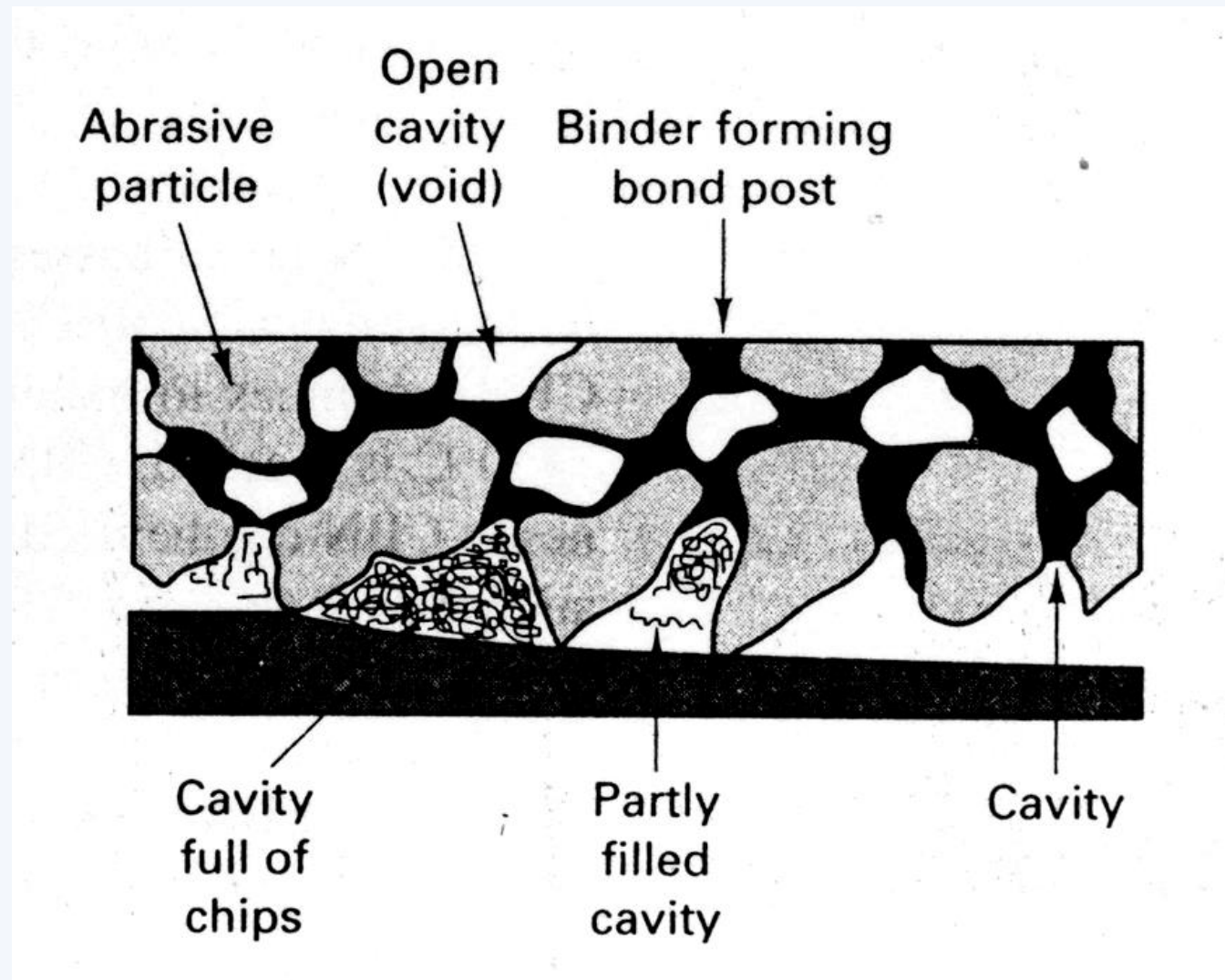
Grade

- Indicates the strength of the binding material.
- When the work material is hard, the grains wear out easily and the sharpness of the cutting edges is quickly lost. This is known as **WHEEL GLAZING**.
- To avoid this problem, a soft wheel should be used.

- ☐ **A-H – Soft Wheel**
- ☐ **J-P – Medium Wheel**
- ☐ **Q-Z – Hard Wheel**



Structure



Structure

- If the voids are too small for the chips, the chips stay in the wheels blocking the voids. This is known as **LOADING** of the wheel.
- **0.....16 – Dense (Closed) to Open structures**

Recommendations:

❑ For Hard work material – Closed Structure

- Open structure means *voids are* relatively large and *grains are* relatively small - recommended when clearance for chips must be provided
- Dense structure means *voids are* relatively small and *grains are* larger - recommended to obtain better surface finish and dimensional control



Bonding Material

- **Vitrified Bond (V)** – Strong and Rigid, commonly used.
- **Resinoid (B)** – Provides shock absorption and elasticity. They are Strong enough.
- **Silicate (S)** – Provides softness (grains dislodge quickly)
- **Shellac (E)** – Used for making thin but strong wheels possessing some elasticity.
- **Rubber Bonds (R)** – For making flexible wheels
- **Metallic Bond (M)** – For diamond wheels only.

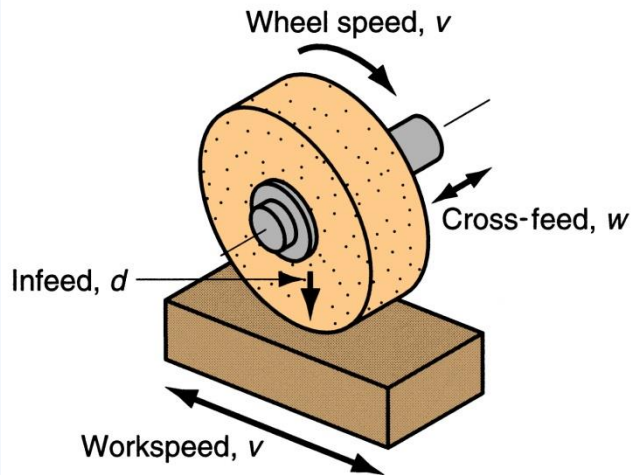


Grinding Wheel Specification

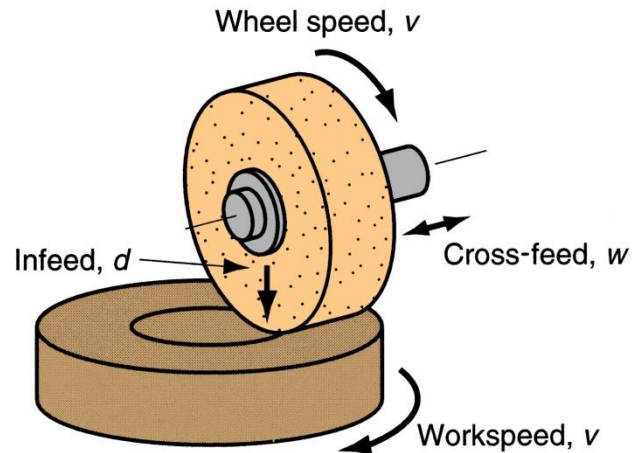
- Standard grinding wheel marking system used to designate abrasive type, grit size, grade, structure, and bond material
 - Example: **A-46-H-6-V**
- Also provides for additional identifications for use by grinding wheel manufacturers



Grinding Processes

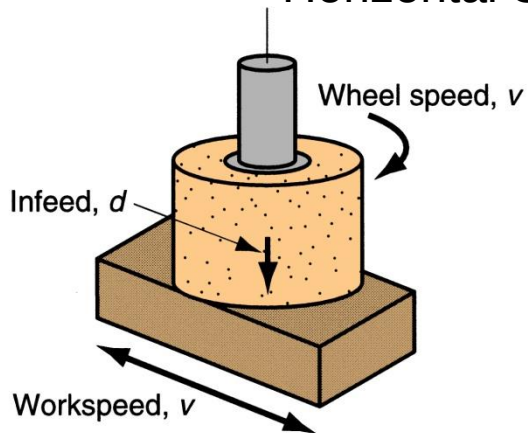


(a)

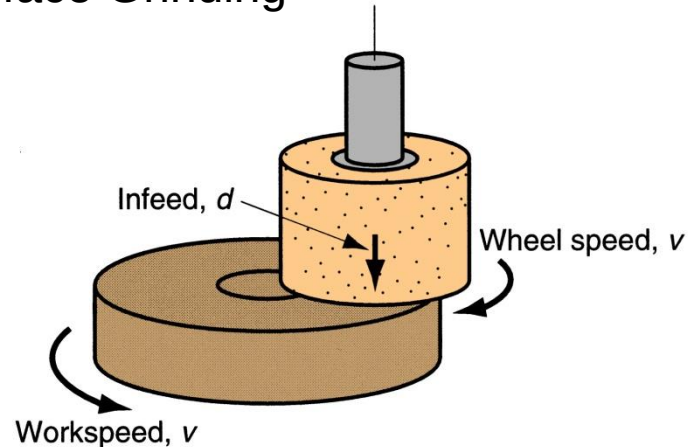


(b)

Horizontal Surface Grinding



(c)

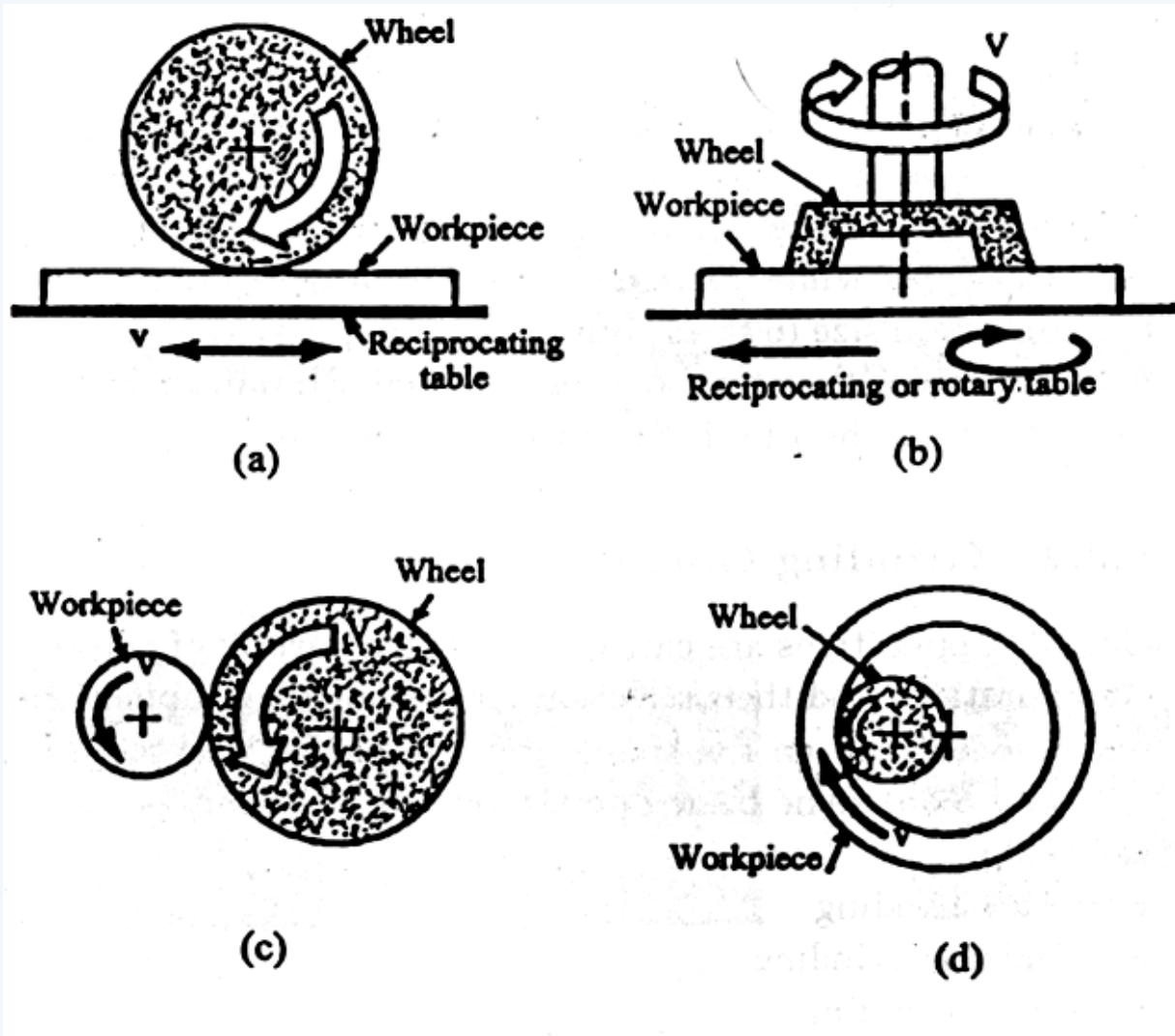


(d)

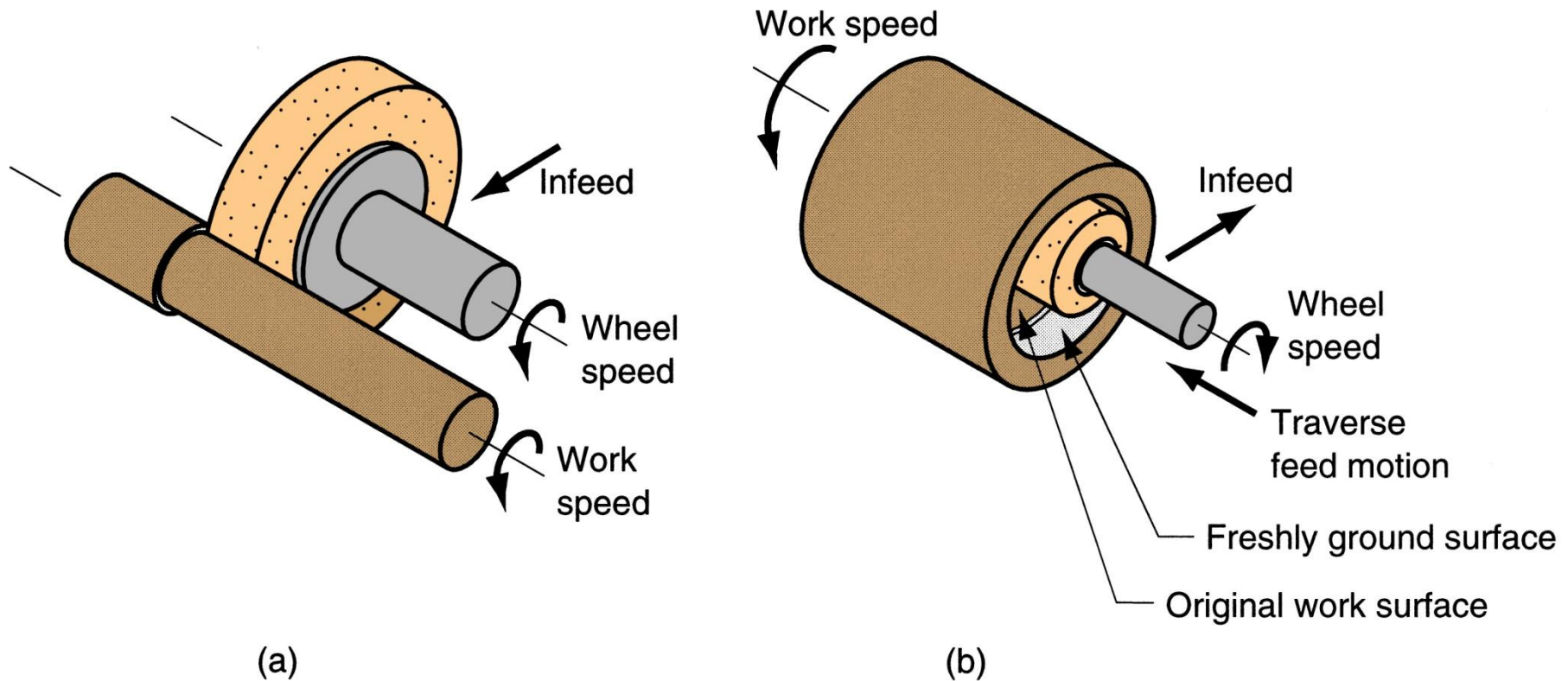
Vertical Surface Grinding



Grinding Processes



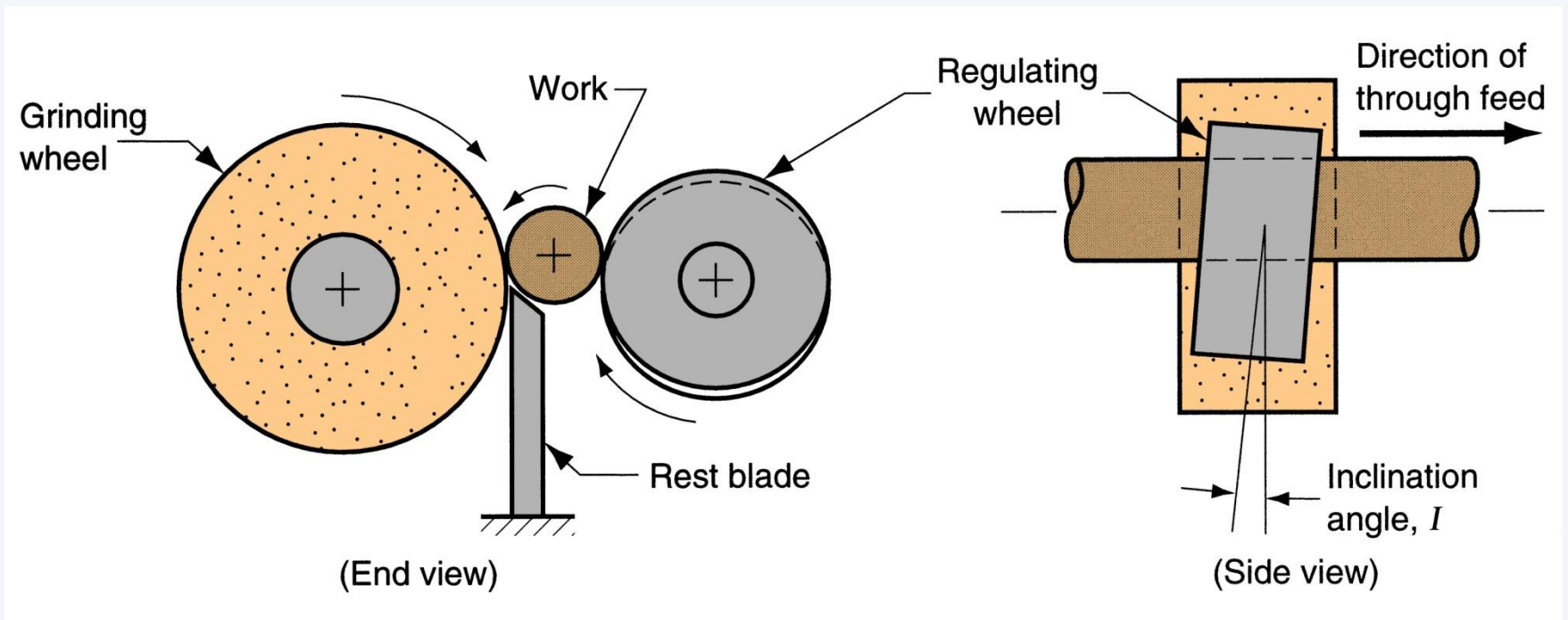
Grinding Processes



Two types of cylindrical grinding:
(a) external, and (b) internal



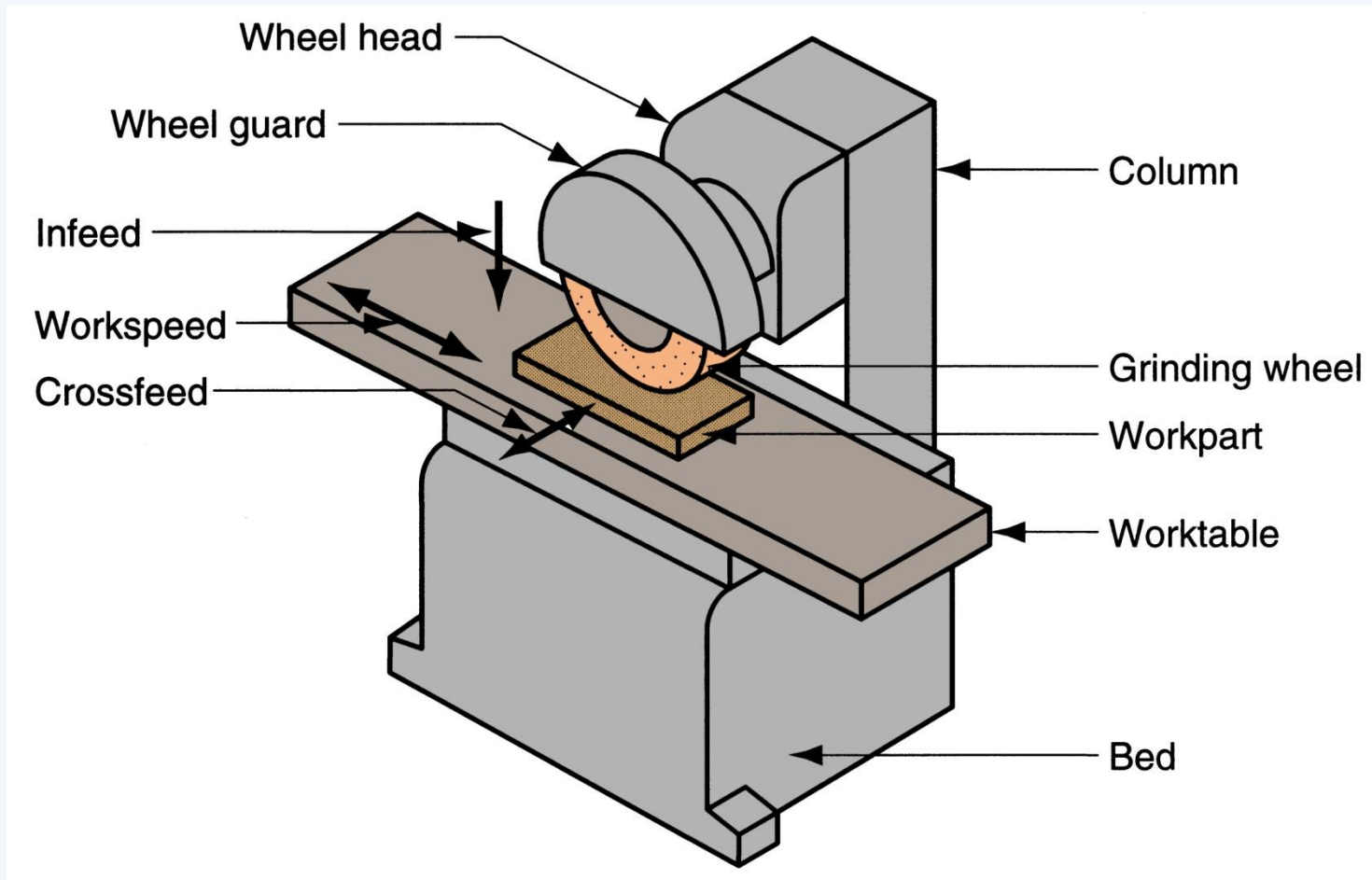
Grinding Processes



External centerless grinding



Grinding Processes

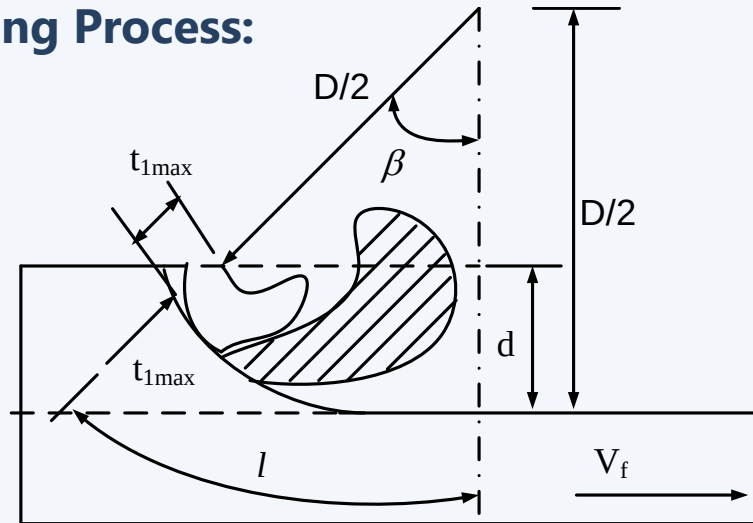
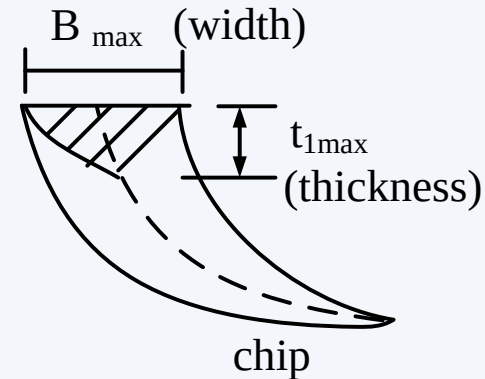


Horizontal Grinding Machine



Mechanics of Grinding Process

Grinding Process:



Grinding Ratio, $\gamma_g = \frac{B_{\max}}{\frac{1}{2} t_{1\max}} \approx 10 - 20$ (in practice)

Volume of a chip = $\frac{1}{2} \cdot B_{\max} \cdot t_{1\max} \cdot l$

$$= \frac{1}{4} t_{1\max}^2 \cdot \gamma_g \cdot l$$

l – average chip length
 d – depth of cut
 D – grinding wheel diameter
 B – width of cut

Mechanics of Grinding Process

- Average chip length, $l = \frac{D}{2} \cdot \beta$

Therefore, 'l' can be expressed in terms of ' β ' as:

$$l = \frac{D}{2} \cdot 2\sqrt{\frac{d}{D}} = \sqrt{Dd}$$

- and, volume of chip $= \frac{1}{4} \gamma_g t_{1\max}^2 \sqrt{Dd}$

- Volume of material removal/sec $= \left[\frac{B \cdot V_f \cdot d}{60} \right]$
 $\approx MRR$

- Number of chips produced/sec $= \left[\frac{\pi \cdot D \cdot N \cdot B \cdot C}{60} \right]$

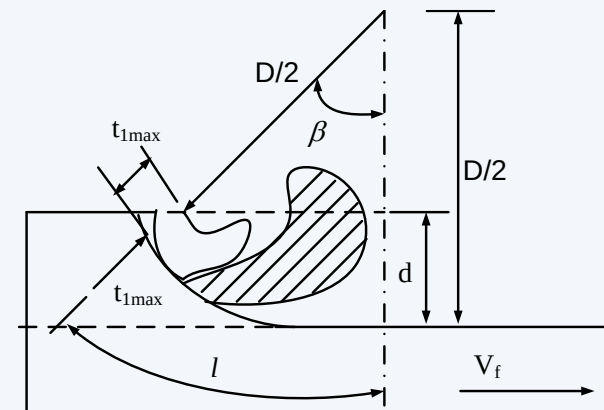
Where, C – concentration of active grains (Number of active grains per unit wheel surface area)

$$\cos \beta = \frac{\left(\frac{D}{2} - d\right)}{D/2} = \left(1 - \frac{2d}{D}\right)$$

Or,

$$\sin \beta = \sqrt{1 - \cos^2 \beta} = \sqrt{1 - 1 + \frac{4d}{D} - \frac{4d^2}{D^2}}$$

$$\therefore \beta \approx 2\sqrt{\frac{d}{D}} \approx \sqrt{\frac{4d}{D}} = 2\sqrt{\frac{d}{D}}$$



Mechanics of Grinding Process

- MRR is in [mm³/sec]

Therefore

$$\frac{1}{4} \gamma_g t_{1\max}^2 \sqrt{Dd} \times \frac{\pi \cdot D \cdot N \cdot B \cdot C}{60} = \frac{B \cdot V_f \cdot d}{60}$$

➔

$$t_{1\max} = \sqrt{\frac{4V_f \sqrt{d}}{\pi DNC \gamma_g \sqrt{d}}}$$

Specific energy, $U_c = U_o (t_{1av})^{-0.4} = U_o \left(\frac{1}{2} t_{1av} \right)^{-0.4}$

Power, $P = \frac{F_c \pi DN}{60 \times 1000} = \frac{B \cdot V_f \cdot d}{60} \times U_c$

➔

$$F_c = \frac{1000 \cdot B \cdot V_f \cdot d \cdot U_c}{\pi DN}$$

Average force per grit, $F'_c = \frac{F_c}{BlC}$

$$\therefore F'_c \propto \frac{V_f d U_o}{DNC d^{0.5} D^{0.5}} \left[\frac{V_f^{0.5} d^{0.25}}{N^{0.5} C^{0.5} D^{0.5} \gamma_g^{0.5} D^{0.25}} \right]^{-0.4}$$

So, $F'_c \propto \frac{U_o V_f^{0.8} d^{0.4} \gamma_g^{0.2}}{N^{0.8} C^{0.8} D^{1.2}}$

$\therefore F'_c$ can be increased by increasing V_f , d , or by decreasing N and D

Specific Energy in Grinding

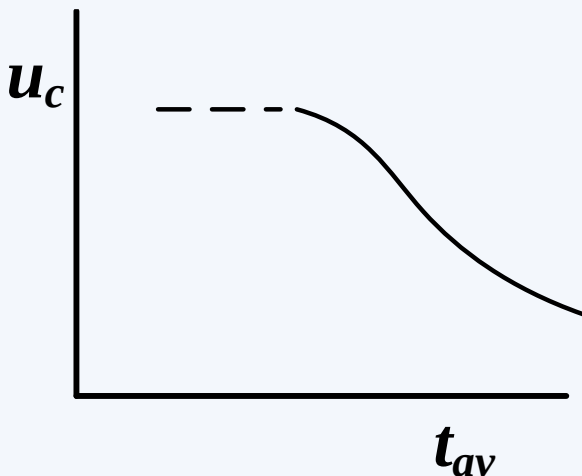
- **Specific energy** can be calculated through the total tangential force on the wheel as:

$$U_c = \frac{F_c V}{V_f B d} \quad (\text{as mentioned earlier})$$

$\underbrace{\hspace{10em}}_{MRR}$

$$P = U_c \times MRR, \quad \therefore U_c = \frac{F_c V_c}{MRR} = \frac{F_c V_c}{V_f B d}$$

- This specific energy in grinding has been found to be an order of magnitude higher than that for single point cutting such as turning.
- It has shown experimentally that it increases rapidly when the chip thickness t_{av} is decreased as shown below:



Reason:

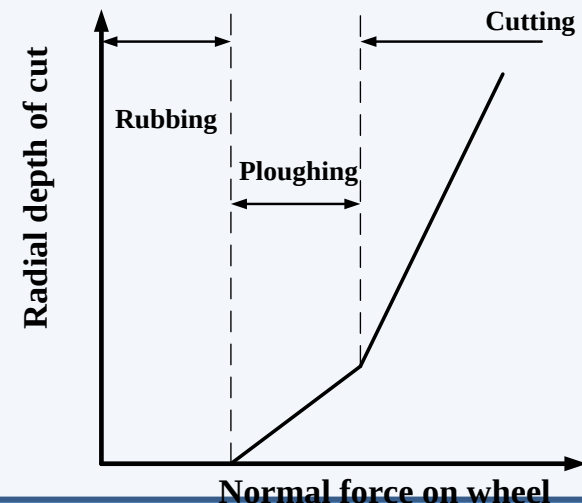
- At very small chip thickness involved in grinding, the plastic deformation occurs over an area having very few or no **imperfection** and the material tends to behave like an ideal material with no inhomogeneties.
- Under these conditions the shear stress required to cause plastic deformation is very much higher and therefore the specific energy is high.

- The increase in U_c with decreasing chip thickness is called **size effect**

Forces in Grinding

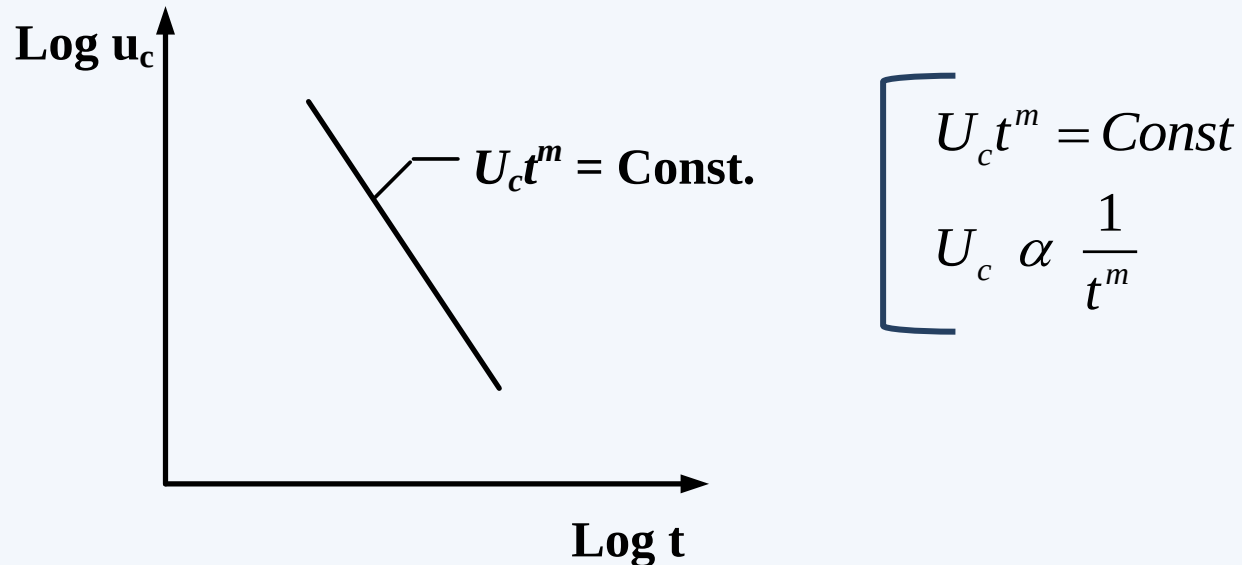
- While size effect is an important factor contributing to high specific energy in grinding, evidence of excessive **rubbing** and **ploughing** during grinding have also been presented
 - During controlled force plunge grinding or during cutting experiments with single abrasive grains, it has been found that under light loads there is no material removal and all the energy is consumed in rubbing.
 - When the load is further increased, ploughing occurs where the material is simply pushed to the sides of the grooves generated by the abrasive grains on the wheel.
 - Material removal or cutting occurs when the load is increased beyond a certain value. These three regions are shown below:

Thus both size effects as well as rubbing and ploughing are significant factors contributing to high specific energy in grinding.



Specific energy vs Chip thickness in Grinding

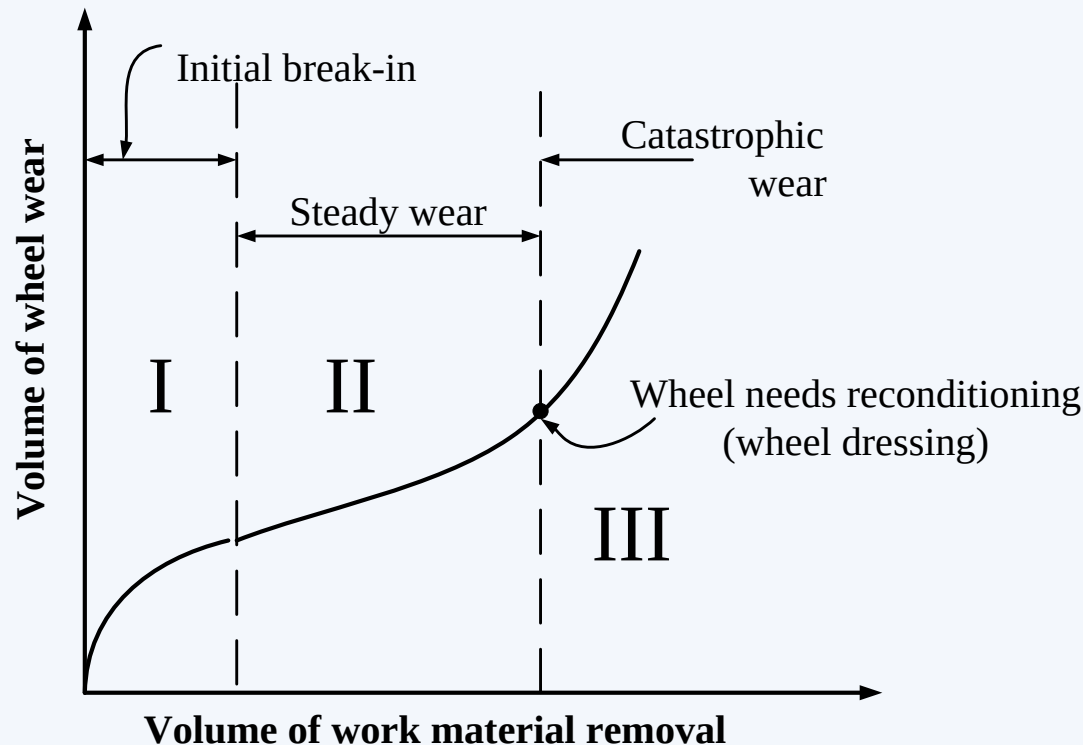
Plot of specific Energy (U_c) against chip thickness:



- In the fine grinding range, the value of exponent $m = 0.9 \approx 1$
- In the rough grinding range, the value of exponent $m = 0.6$
- For single point cutting range, the value of exponent $m = 0.2$ only

Wheel wear and Grinding Performance

Wheel wear:



Phase I – Rapid wear due to wearing of high spots, on the wheel surface

Phase II – Rate of wheel is constant

Phase III – Wheel dressing is necessary

➤ **Phase III** – Wheel becomes dull due to loading and developed a glazed appearance

➤ **Phase III** – Material removal rate is reduced and grinding forces are increased

● During stock removal (rough grinding) such as vertical surface grinding, abrasive cut-off etc. phase II almost extends infinitely due to the fact that

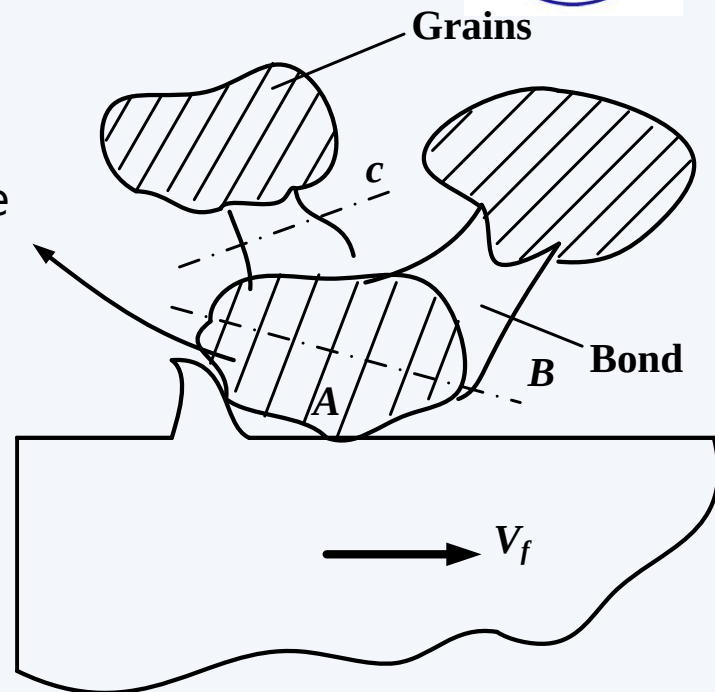
- there is continuous self-sharpening action and hence,
- there is practically no wheel dulling which obviates the need for dressing.

Wheel wear and Grinding Performance

Wheel wear phenomenon:

❑ **Attritious wear:** Attritious wear occurs at the grain-workpiece contact surface

- Gradual wearing away of the sharpness of the cutting points resulting in the formation of wear flats on the grain tips
 - *It is due to microfracture of cutting points, plastic flow at the grain tips and*
 - *Crumbling of sharp edges.*
- These get aggravated due to high temperature obtained in grinding



❑ **Fracture wear:**

- When the stress on the grain increase to a sufficiently high value, the grain fractures into fragments and fresh cutting edges are exposed. This is called the **fracture wear**.

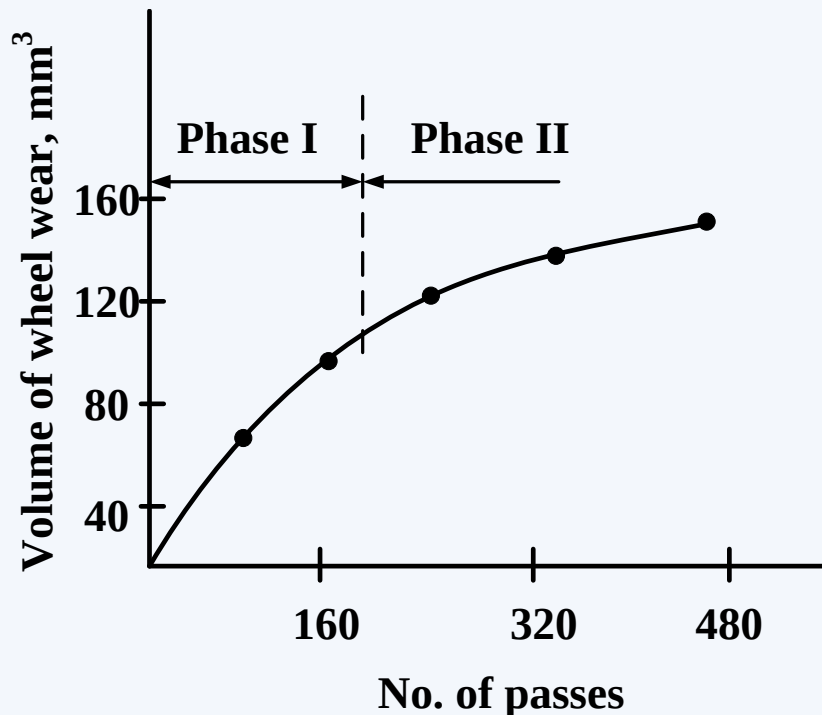
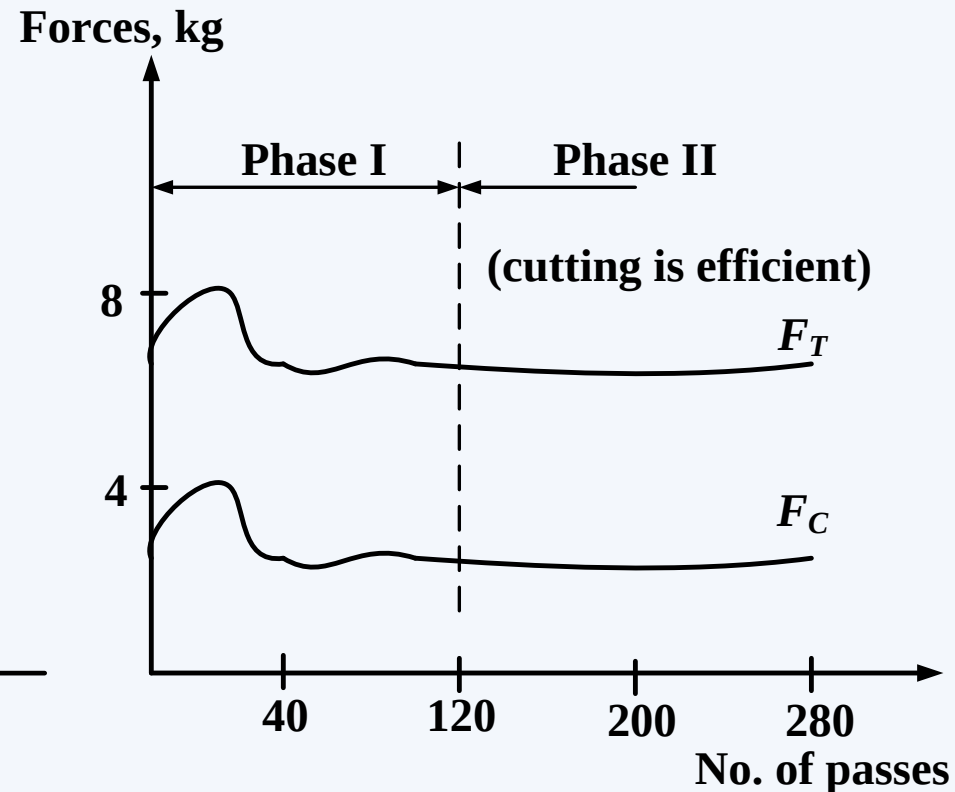
❑ **Bond-post fracture:**

- When the grain is stronger than the bond post (as in the case of softer wheels) the grains get pulled out of the bond-post when sufficient force is applied at the grain tip.

Wheel wear and Grinding Performance

- Generally the interface between the grain and the bond post is quite strong, hence the pulling out of the grain would be due to fracture of the bond post. This is called **Bond-post fracture wear**.
- **Glazing occurs** when worn abrasive grains are not fractured away or dislodged from the wheel.

Experimental Results:



Grinding Temperature

Thermal Analysis:

- It is extremely difficult to develop a satisfactory analytical model for calculating the surface temperature during grinding.
- We shall therefore adopt an indirect approach for yielding the approximate results.
- It is not very illogical to assume, that the grinding temperature depends directly on the energy spent per unit surface area ground.

$temp. \theta \propto \text{energy/unit time}$

$\theta \propto \text{energy/unit surface}$

So,

$$\theta \propto \frac{P}{BV_f} = \frac{BV_f dU_c}{60 \cdot BV_f}$$

$$\Rightarrow \theta \propto dU_c$$

$$\Rightarrow \theta \propto dU_o \left(\frac{1}{2} t_{1\max} \right)^{-0.4}$$

$$\Rightarrow \theta \propto dU_o \left(\frac{V_f^{0.5} d^{0.5}}{N^{0.5} D^{0.5} C^{0.5} D^{0.25} \gamma_g^{0.5}} \right)^{-0.4}$$

So,

$$\theta \propto \frac{U_o d^{0.9} N^{0.2} D^{0.3} C^{0.2} \gamma_g^{0.2}}{V_f^{0.2}}$$

Because,

$$t_{1\max} = \sqrt{\frac{8V_f \sqrt{d}}{\pi DNC \gamma_g \sqrt{D}}}$$

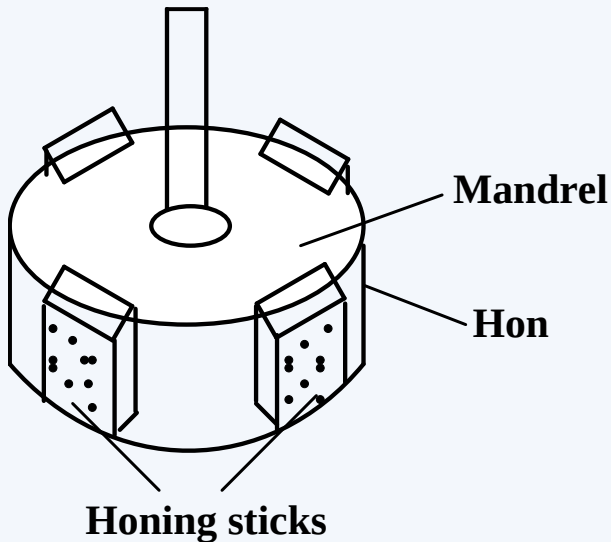
and power, $P = \frac{BV_f d}{60} \cdot U_c$

- So, the temperature and also the defects caused by higher temperature can be reduced either by decreasing d , D , C , or N or by increasing the table feed V_f .
- Temperature in tool-chip interface can go beyond 1500°C , chip do not melt because it is flown away at a high speed (1/1000) sec.

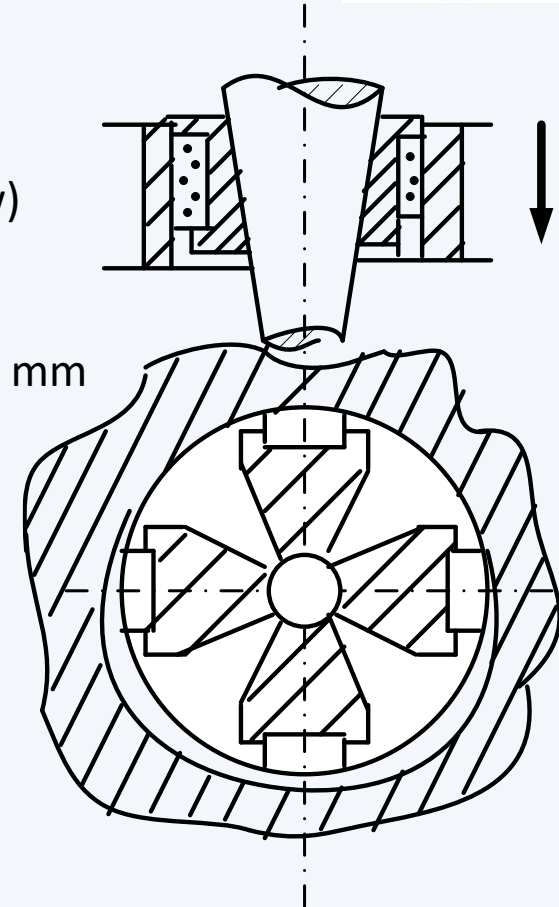
Polishing

Honing:

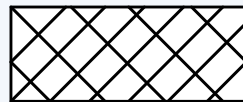
Used for proper finish of the internal surface of a hole



Cutting speed – 10 – 30 m/min (low)
Grit size – 80 – 600 mesh
Pressure – 1 – 3 N/mm²
Max. mat. removal – 0.25 mm – 0.5 mm



- Honing is carried out at low speeds
- Microchips can be entangled with the honing stone to form excessive heat if the process is performed at high speed.



Surface obtained



- No repetition of grain marks.
- Retains lubricant well.

Polishing

lapping:

- Lapping is an abrading process used for improving the accuracy and finish.
- Laps are made of material softer than workpiece material.
- Straight narrow grooves are cut at 90° on the lap surface and this surface is charged by sprinkling the abrasive powder (200 – 1200 grit) and a suitable fluid (often mineral oil).
- The workpiece is then held against the lap and moved in an unrepeatable path, in the form of eight '8'.

lapping:

- Hand lapping
- Machine lapping
- Hole lapping
- shaft lapping

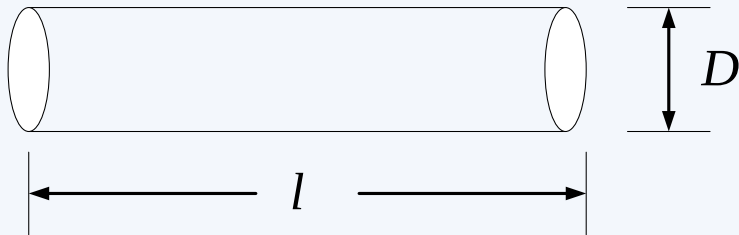
- In machine lapping, lapping plate or lap becomes a rotating table.
- The part to be lapped are confined into cages that impart rotary and gyratory motion at the same time. Covering the entire surface of the lap. Parallelism is maintained by having a stationary lapping plate on the top of the workpieces.
- Hole lapping is similar to honing, consisting of sectors ranging from 2 to 6, mounted on the arbor and expanded by springs. Hole are rarely lapped slow production.

Economics of Machining

- The objective of machining is to produce a component of required dimensions and surface finish at minimal possible cost
- The factors that can be varied during machining operation to change the cost are:
 - Speed ▪ Feed ▪ Depth of cut ▪ Tool and work materials
 - Tool geometry ▪ Cutting fluid
- Technical data regarding speed, feed and depth of cut are usually not available and their optimum selection can not be made from cost consideration
- An optimum cutting speed always exists for each application which would mean a compromise between production rate and tool cost
- The cost of machining is minimum at a particular set of cutting conditions (speed, feed, depth of cut).
- For simplicity we will only consider the case of a single pass straight turning operation and obtain the value of optimum cutting speed.

Economics of Machining

Let us assume simple turning operation of cylindrical bars



Important cutting parameters

- Cutting speed
- Feed
- Depth of cut

➤ Technological aspects include:

- Surface finish
- Power requirement
- Force

➤ Economical aspects:

- Minimum production cost criterion
- Maximum production rate
- Maximum profit rate criterion

Minimum Production Cost Criteria: Cost analysis

Designation:

- R = cost / piece ▪ R_1 = material cost / piece ▪ R_2 = settling + ideal time cost
- R_3 = machining cost / piece ▪ R_4 = Cost of the tool changing / piece
- R_5 = tool cost / piece ▪ t_s = (settling + ideal time) / piece ▪ T = tool life
- T_{ct} = tool changing time ▪ t_m = machining time / piece
- λ_1 = overhead + labor cost / min ▪ λ_2 = tool cost / grinding
- Frequency of tool changing (in producing one piece) = (t_m / T)

Minimum Production Cost Criteria: Cost analysis

- **Cost/piece**, $R = R_1 + R_2 + R_3 + R_4 + R_5$
- **Material cost (R_1)**: Does not depend on cutting conditions and remains as a constant
- **Set-up and idle time cost (R_2)**: Independent of cutting conditions f and V

$$\Rightarrow R_2 = \lambda_1 \times t_s$$

- **Machining cost (R_3)**:

$$\Rightarrow R_3 = t_m \times \lambda_1 = \frac{L}{fN} \times \lambda_1 = R_3 = \lambda_1 \frac{\pi DL}{1000Vf}$$


 (machining time)

$$t_m = \frac{L}{fN} = \frac{L\pi D}{f \cdot 1000V}$$

- **Tool changing cost (R_4)**:

$$R_4 = \frac{t_m}{T} \times t_{ct} \times \lambda_1$$

From the tool life equation: $VT^n = K^n \quad T = \frac{K}{V^{1/n}}$

$$\Rightarrow R_4 = \lambda_1 \cdot t_{ct} \cdot \frac{\pi DL}{1000Vf} \cdot \frac{V^{1/n}}{K}$$

Minimum Production Cost Criteria: Cost analysis

➤ Tool cost / piece (R_5):

$$R_5 = \frac{t_m}{T} \times \lambda_2$$

$$\text{So, } R_5 = \lambda_2 \cdot \frac{\pi DL}{1000Vf} \cdot \frac{V^{1/n}}{K}$$

○ Since, the variable at our disposal is V , we write:

$$f(V) = R_1 + \lambda_1 t_s + \lambda_1 \frac{\pi DL}{1000Vf} + \lambda_1 \cdot t_{ct} \cdot \frac{\pi DL}{1000Vf} \cdot \frac{V^{1/n}}{K} + \lambda_2 \cdot \frac{\pi DL}{1000Vf} \cdot \frac{V^{1/n}}{K}$$

$$\Rightarrow f(V) = R_1 + \lambda_1 t_s + \lambda_1 \frac{\pi DL}{1000f} V^{-1} + \lambda_1 \cdot \frac{\pi DL}{1000fK} \cdot V^{\left(\frac{1}{n}-1\right)} t_{ct} + \lambda_2 \cdot \frac{\pi DL}{1000fK} \cdot V^{\left(\frac{1}{n}-1\right)}$$

$$\text{When } V = V_{opt} \Rightarrow \frac{\partial f}{\partial V} = 0$$

$$\Rightarrow \frac{\partial f}{\partial V} = -\lambda_1 \frac{\pi DL}{1000f} V^{-2} + \lambda_1 \cdot \frac{\pi DL t_{ct}}{1000fK} \left(\frac{1}{n}-1\right) V^{\left(\frac{1}{n}-2\right)} + \lambda_2 \cdot \frac{\pi DL}{1000fK} \left(\frac{1}{n}-1\right) V^{\left(\frac{1}{n}-2\right)}$$

Minimum Production Cost Criteria: Cost analysis

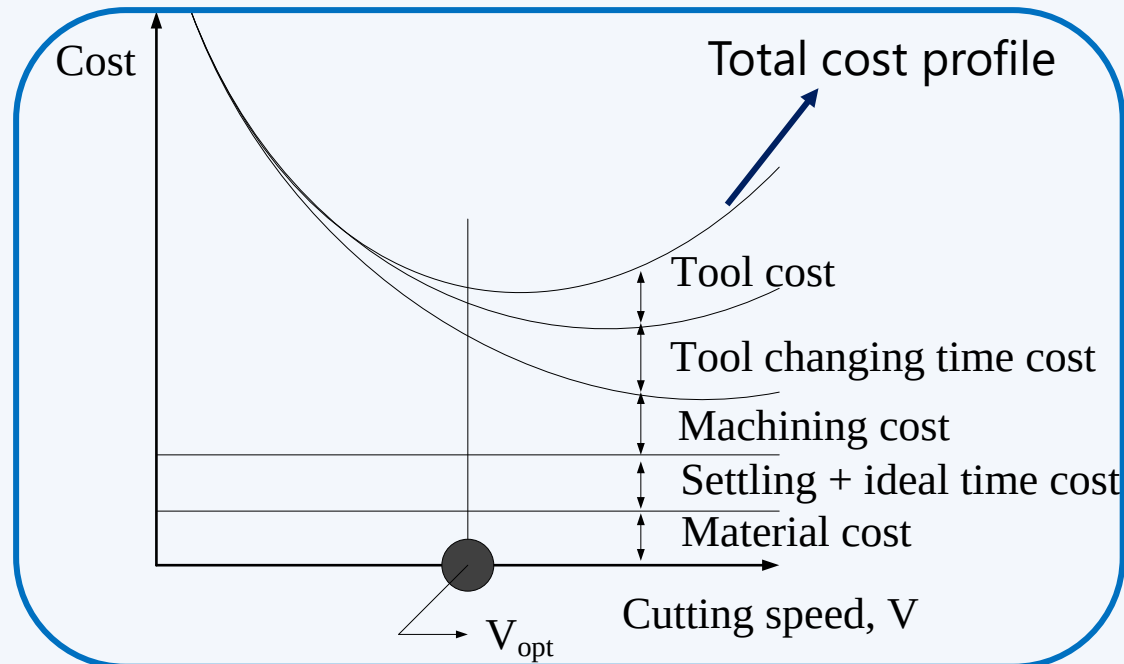
Hence,

$$-\lambda_1 \frac{\pi DL}{1000 f} V_{opt}^{-2} + \lambda_1 \cdot \frac{\pi DL t_{ct}}{1000 f K} \left(\frac{1}{n} - 1 \right) V_{opt}^{\left(\frac{1}{n} - 2 \right)} + \lambda_2 \cdot \frac{\pi DL}{1000 f K} \left(\frac{1}{n} - 1 \right) V_{opt}^{\left(\frac{1}{n} - 2 \right)} = 0$$

$$\Rightarrow -\lambda_1 + \lambda_1 \cdot \frac{t_{ct}}{K} \left(\frac{1}{n} - 1 \right) V_{opt}^{\frac{1}{n}} + \lambda_2 \cdot \frac{1}{K} \left(\frac{1}{n} - 1 \right) V_{opt}^{\frac{1}{n}} = 0$$

$$\Rightarrow V_{opt} = \left[\frac{Kn\lambda_1}{(1-n)(\lambda_1 t_{ct} + \lambda_2)} \right]^n$$

Minimum cost criterion



Maximum Production Rate Criterion:

Total time = t / piece

- The maximum Production rate can be achieved if the total time required per piece is reduced to minimum.

t = (settling and idle time /piece + machining time /piece + tool changing time /piece).

$$\begin{aligned} \Rightarrow t &= t_s + t_m + t_{ct} \cdot \frac{t_m}{T} \quad [\text{min}] \\ &= t_s + \frac{\pi DL}{1000 f \cdot V} + \frac{\pi DL}{1000 f V} \cdot \frac{V^{1/N}}{K} \cdot t_{ct} \end{aligned}$$

For optimum speed, $\left. \frac{\partial t}{\partial V} \right|_{V=opt} = 0$

$$\frac{\partial t}{\partial V} = -\frac{\pi DL}{1000 f} V_{opt}^{-2} + \frac{\pi DL}{1000 f} \frac{1}{K} \left(\frac{1}{n} - 1 \right) V_{opt}^{\left(\frac{1}{n} - 2 \right)} \cdot t_{ct} = 0$$

$$V_{opt} = \left[\frac{Kn}{(1-n)t_{ct}} \right]^n$$

$$\begin{aligned} \left[-1 + \left(\frac{1-n}{Kn} \right) V_{opt}^{\frac{1}{n}} \cdot t_{ct} \right] &= 0 \\ V_{opt} &= \left[\frac{Kn}{(1-n)t_{ct}} \right]^n \end{aligned}$$

Maximum Profit Rate Criterion:

Profit rate, $P = \frac{S - R}{t}$

Where,

S = amount received per piece (excluding material cost and cost per piece)

R = cost per piece

t = Total time per piece

V_{opt} can be found from $\left. \frac{\partial P}{\partial V} \right|_{V_{\text{opt}}=V} = 0$

- In order to simplify the analysis, the cutting condition is generally optimized without considering any increase in the income per piece or the selling price of the commodity

Thus, S = Constant

Restrictions on Cutting Conditions:

- The final choice of the optimum value of cutting speed will have to satisfy a number of restrictions.
- Maximum Power Restriction
- Force and Vibration Restriction
- Speed Restriction
- Surface Finish Restriction

Maximum power restriction:

Power $P_w = B_w V f^{m_1} d^{m_2} \Rightarrow P_w = B'_w \cdot V$ Where, $B'_w = B_w f^{m_1} d^{m_2} = \text{const.}$

m_1 and m_2 are constants for a given tool-work combination

- The max. power available in machine is limited by the power capacity of the main drive.
- If the V_{opt} is such that the available power is exceeded, the speed must be adjusted to meet this restriction.

Restrictions on Cutting Conditions:

Speed restriction:

- Most machine tools have cutting speeds in steps over a certain range.
- Step closest to the optimum value should be used. If optimum value is beyond the range cutting speed must not exceed the upper limit.
- Lower limit of the speed is generally limited by formulation of Built-up-edge

Force and vibration restriction:

- Machine components are designed for a maximum permissible load beyond which the tool-work deflections are excessive
 - Resulting in dimensional inaccuracies

The cutting force can be expressed as:

$$F_c = B_p f^{m_1} d^{m_2} \quad ; B_p = \text{const.}$$

Restrictions on Cutting Conditions:

- Cutting speed has very little effect on cutting force and for all practical purposes its effects can be neglected for all practical purposes
- There is no restriction on cutting speed in terms of cutting force but this will limit the max. value of feed and depth of cut that can be used
- Excessive vibration and chatter also put restriction on cutting conditions

Surface finish restriction:

- Surface finish during cutting operation depends on
 - Tool-work material
 - Tool geometry
 - Cutting conditions
 - Coolant

Surface Finish Restriction can be expressed as:

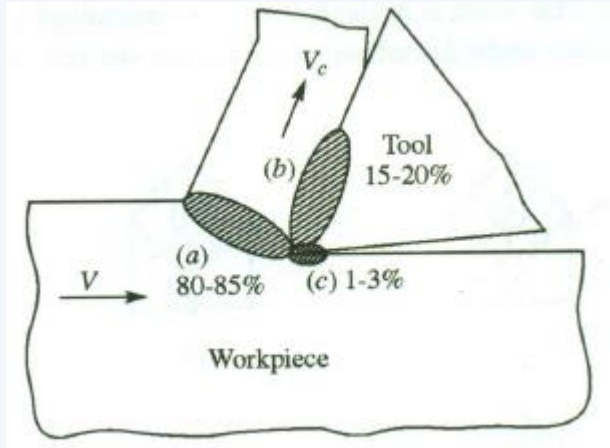
$$h_m \leq B_s f^{a_1} V^{a_2}$$

Where, h_m = Max. permissible surface roughness, B_s , a_1 , a_2 = constants

Restrictions on Cutting Conditions:

- From the point of view of surface finish feed marks are the predominant phenomenon and This puts a serious restriction on feed, f
- Cutting speed affects the surface finish by the formation of built-up-edge Which gets smaller and smaller and disappears at sufficiently high speed, improving the surface finish

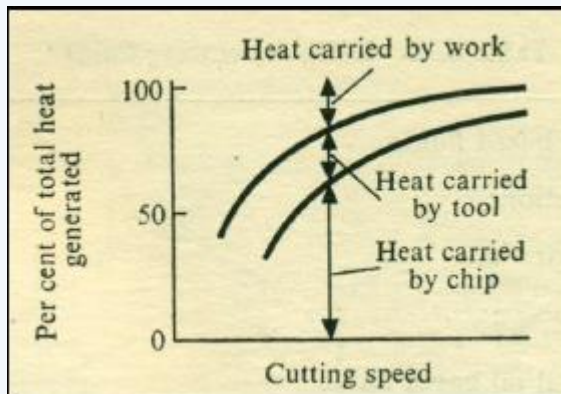
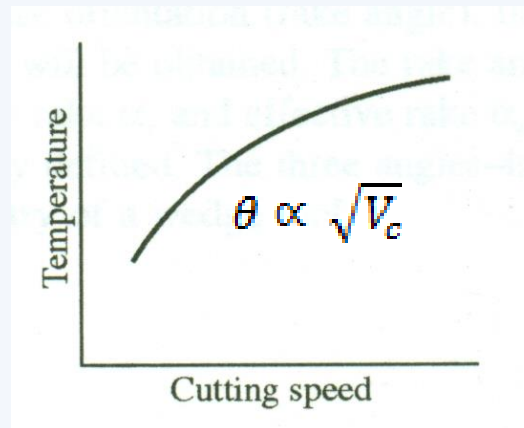
Thermal Aspects of machining



Heat Sources:

- (a) **The Shear Zone** – heat generated due to plastic deformation of work material
- (b) **The chip-Tool Interface** - heat generated due to frictional rubbing between the rake face of the tool and the chip
- (c) **The work-tool interface** - heat generated due to frictional rubbing between the flank face of the tool and the workpiece

Distribution of Heat



Surface Finish

Roughness in the produced surfaces is caused because of two reasons:

- Inherent geometrical configuration involved in the process

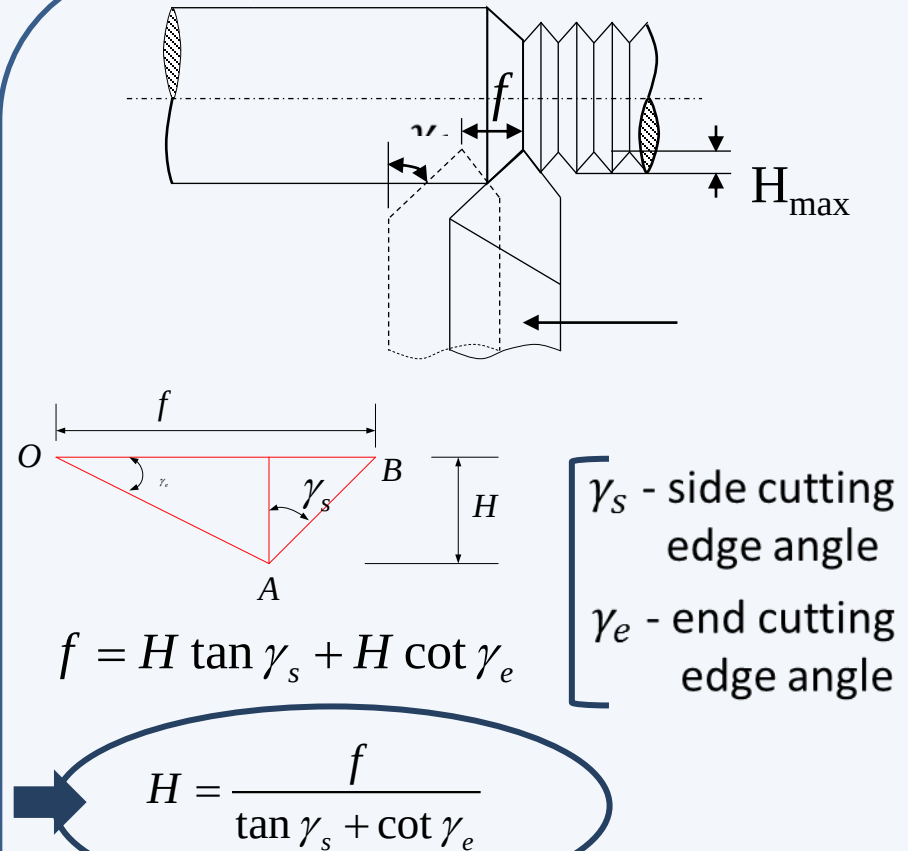
➡ Ideal Roughness

- Due to built-up-edge, vibration etc.

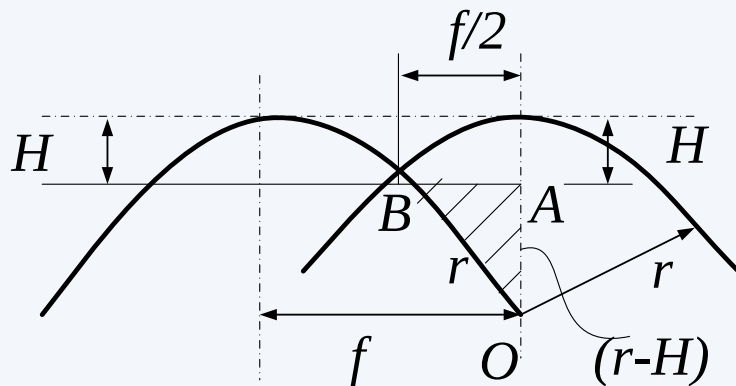
➡ Natural Roughness

- **Total roughness** = Ideal roughness + Natural roughness

- **Ideal roughness:** Indicates the best possible finish that can be obtained by a given operation



Therefore, surface roughness (H) in turning operation depends on feed (f) when γ_s and γ_e are constant for a certain tool

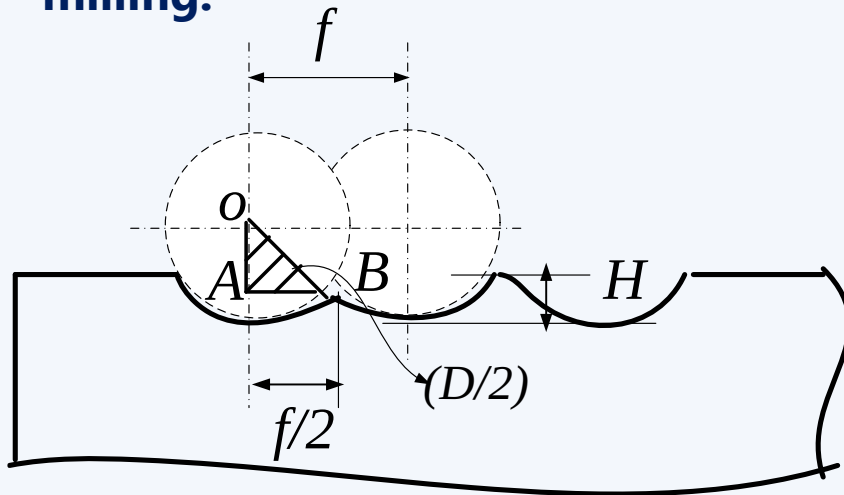

$$r^2 = \frac{f^2}{4} + (r - H)^2$$
$$\Rightarrow \frac{f^2}{4} + \cancel{H^2}^0 - 2rH = 0$$

$$\Rightarrow \frac{f^2}{4} = 2rH$$

➔ $H = \frac{f^2}{8r}$

Surface Finish

During milling:



Feed = advance / tooth; f = feed / cutting tooth

ΔOAB

From

$$OB = \frac{D}{2} ; AB = \frac{f}{2} ; OA = \left(\frac{D}{2} - H \right)$$

$$OB^2 = OA^2 + AB^2$$

$$\Rightarrow \left(\frac{D}{2} \right)^2 = \left(\frac{f}{2} \right)^2 + \left(\frac{D}{2} - H \right)^2$$

$$\Rightarrow \frac{D^2}{4} = \frac{f^2}{4} + \frac{D^2}{4} + H^2 - DH$$

$$\Rightarrow DH = \frac{f^2}{4} + H^2 \Rightarrow \frac{f^2}{4} = DH \Rightarrow H = \frac{f^2}{4D}$$

Since, $f = \frac{V_f}{NZ}$

$$H = \frac{V_f^2}{4DN^2Z^2}$$

Surface Finish

Conclusion

- We can change the H (Surface roughness) by either changing
 - Feed
 - Speed of cutter, N
- If we change the feed, then the power consumption will be changed since the MRR will be changed

$$\left[\text{Power} = \text{MRR} \times U_c \quad \text{and} \quad \text{MRR} = B d V_f \right]$$

- If we change 'N' keeping other conditions constant then we can reduce the surface unevenness

Example:

mm

$$\left. \begin{aligned} V_f &= 5 \text{ mm/min}, D = 100 \\ Z &= 20, N = 100 \text{ rpm} \end{aligned} \right\}$$



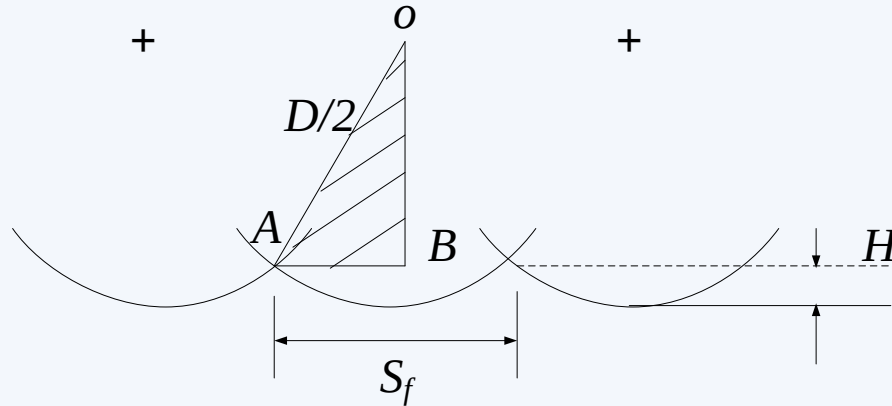
$$H = \frac{25}{4 \times 100 \times 100^2 \times 20^2} = 1.56 \times 10^{-8}$$

Small value

Therefore, better surface is produced in milling

Surface Finish

During Grinding:



- If there are 'C' number of cutting points per unit area on the wheel surface and the average chip has a width b

➤ distance between two successive points (grain spacing) on the wheel surface will be $(1/bc)$

➤ The corresponding distance on the workpiece surface will be (V_f / V_c) times the distance between the two successive points $(1/bc)$

➤ The distance (S_f) , moved by the workpiece between passage of successive grains can be evaluated as:

$$S_f = \frac{V_f}{V_c C b_{\max}}$$

$$\left[\begin{array}{l} V_f = \text{work speed} \\ V_c = \text{wheel speed} \end{array} \right.$$

$$\gamma_g = \frac{b_{\max}}{\frac{1}{2} t_{1\max}}$$



$$b_{\max} = \frac{1}{2} t_{1\max} \gamma_g$$

Also, from $\triangle OAB$

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Thank you for your attention

