

Solidification Time

- Solidification takes time
- Total solidification time TST = time required for casting to solidify after pouring
- TST depends on size and shape of casting by relationship known as *Chvorinov's Rule*



Chvorinov's Rule

$$TST = C_m \left(\frac{V}{A} \right)^n$$

Where, TST = total solidification time; V = volume of the casting; A = surface area of casting; n = exponent usually taken to have a value = 2; and C_m is *mold constant*

Mold Constant in Chvorinov's Rule

56

- C_m depends on mold material, thermal properties of casting metal, and pouring temperature relative to melting point
- Value of C_m for a given casting operation can be based on experimental data from previous operations carried out using same mold material, metal, and pouring temperature, even though the shape of the part may be quite different



What Chvorinov's Rule Tells Us

- A casting with a higher volume-to-surface area ratio cools and solidifies more slowly than one with a lower ratio
- To feed molten metal to main cavity, TST for riser must be greater than TST for main casting
- Since riser and casting mold constants will be equal, design the riser to have a larger volume-to-area ratio so that the main casting solidifies first
- This minimizes the effects of shrinkage



Numerical Examples

58

1. A riser in the shape of a sphere is to be designed for a sand casting mold. The casting is a rectangular plate, with length = 200 mm, width = 100 mm, and thickness = 18 mm. If the total solidification time of the casting itself is known to be 3.5 min, determine the diameter of the riser so that it will take 25% longer for the riser to solidify.

Solution: Casting volume $V = LWt = 200(100)(18) = 360,000 \text{ mm}^3$
Casting area $A = 2(200 \times 100 + 200 \times 18 + 100 \times 18) = 50,800 \text{ mm}^2$
 $V/A = 360,000/50,800 = 7.0866$
Casting TST = $C_m(7.0866)^2 = 3.50 \text{ min}$
 $C_m = 3.5/(7.0866)^2 = 0.0697 \text{ min/mm}^2$
Riser volume $V = \pi D^3/6 = 0.5236D^3$
Riser area $A = \pi D^2 = 3.1416D^2$
 $V/A = 0.5236D^3/3.1416D^2 = 0.1667D$
TST = $1.25(3.5) = 4.375 \text{ min} = 0.0697(0.1667D)^2 = 0.001936D^2$
 $D^2 = 4.375/0.001936 = 2259.7 \text{ mm}^2 \quad D = 47.5 \text{ mm}$

Numerical Examples

59

2. A cylindrical riser is to be used for a sand casting mold. For a given cylinder volume, determine the diameter-to-length ratio that will maximize the time to solidify.

Solution: To maximize TST, the V/A ratio must be maximized.

Cylinder volume $V = \pi D^2 L / 4$. $L = 4V / \pi D^2$

Cylinder area $A = 2\pi D^2 / 4 + \pi D L$

Substitute the expression for L from the volume equation in the area equation:

$$A = \pi D^2 / 2 + \pi D L = \pi D^2 / 2 + \pi D (4V / \pi D^2) = \pi D^2 / 2 + 4V / D$$

Differentiate the area equation with respect to D:

$$dA/dD = \pi D - 4V/D^2 = 0$$

$$\text{Rearranging, } \pi D = 4V/D^2$$

$$D^3 = 4V/\pi$$

$$D = (4V/\pi)^{0.333}$$

From the previous expression for L, substituting in the equation for D that we have developed,

$$L = 4V/\pi D^2 = 4V/\pi (4V/\pi)^{0.667} = (4V/\pi)^{0.333}$$

Thus, optimal values are $D = L = (4V/\pi)^{0.333}$, and therefore the optimal D/L ratio = 1.0

Numerical Examples

60

3. A cylindrical riser of 6 cm diameter and 6 cm height has to be designed for a sand casting mould for producing a steel rectangular plate casting of 7 cm x 10 cm x 2 cm dimensions having the total solidification time of 1.36 min. Determine the total solidification time for the riser.

SOLUTION:

$$V_{plate} = 7 \times 10 \times 2 = 140 \text{ cm}^3$$

$$A_{plate} = 2(7 \times 10 + 7 \times 2 + 10 \times 2) = 2(70 + 14 + 20) = 208 \text{ cm}^2$$

$$(V/A)_{casting} = (140/208) = 0.673$$

$$\text{Mould Constant, } C_m = TST/(V/A)^2 = 1.36/(140/208)^2 = 1.36/0.453 = 3 \text{ min/cm}^2$$

$$\text{For Riser: } V_{riser} = (\pi D^2 h / 4), A_{riser} = \pi D h + (2\pi D^2 / 4)$$

$$D/h = 1, V = (\pi D^3 / 4)$$

$$A_{riser} = \pi D^2 + (2\pi D^2 / 4) = 1.5 \pi D^2$$

$$(V/A)_{Riser} = D/6 = 1 \text{ cm}$$

$$TST_{Riser} = 3 \times (D/6)^2 = 3 \times 1 = 3 \text{ min}$$

Numerical Examples

61

4. Two castings of the same metal have the same surface area. One casting is in the form of a sphere and the other is a cube. What is the ratio of the solidification time for the sphere to that of the cube?

SOLUTION:

$$t_s = B(V/A)^2 ; B - \text{constant}$$

$$\text{For sphere of radius } r, (V_{\text{sph}}/A_{\text{sph}})^2 = (4/3\pi r^3)/(4\pi r^2) = r^2/9$$

$$\text{For cube of side } l, (V_{\text{cube}}/A_{\text{cube}})^2 = (l^3/6l^2)^2 = l^2/36$$

$$\text{From Chrovinov's rule, } (t_{\text{sph}}/t_{\text{cube}}) = (r^2/9)/(l^2/36) = 4r^2/l^2$$

$$\text{Since the surface areas are the same, } 4\pi r^2 = 6l^2$$

$$\text{Therefore, } (4r^2/l^2) = (6/\pi)$$

$$\text{So, } (t_{\text{sph}}/t_{\text{cube}}) = (6/\pi) = 1.91$$

Shrinkage

Shrinkage occurs in three steps:

- (1) liquid contraction during cooling prior to solidification;**
- (2) contraction during the phase change from liquid to solid, called solidification shrinkage; and**
- (3) thermal contraction of the solidified casting during cooling to room temperature.**