Smart Materials, Adaptive Structures, and Intelligent Mechanical Systems

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Lecture 31

The Role of D_{16} in a Simply-Supported Rectangular Plate

References for this Lecture

1. Analysis and Performance of Fiber Composites, Agarwal, B.D. and Broutman, L. J., John Wiley & Sons.

2. Mechanics of Composite Materials, Jones, R. M., Mc-Graw Hill

3. Structural Analysis of Laminated Composites, Whitney, J. M., Technomic

4. Nonlinear Analyis of Plates, Chia, C., McGraw-Hill International Book Company

Influence of D_{16} on Response of a SS-SS Plate

- Consider a plate of dimensions *a* X *b* in *x* and *y* directions, respectively, and which is simply supported on all four sides.

- Further, we assume that this plate has a symmetric.
  - [B] = [0]   due to symmetry

- Here, we do not assume that the plate is specially orthotropic. Hence terms D_{16}, and D_{26}, are not zero.

- Finally we assume that the plate is normally loaded as shown in Fig. 31.1 with a load intensity of $q(x,y)$.

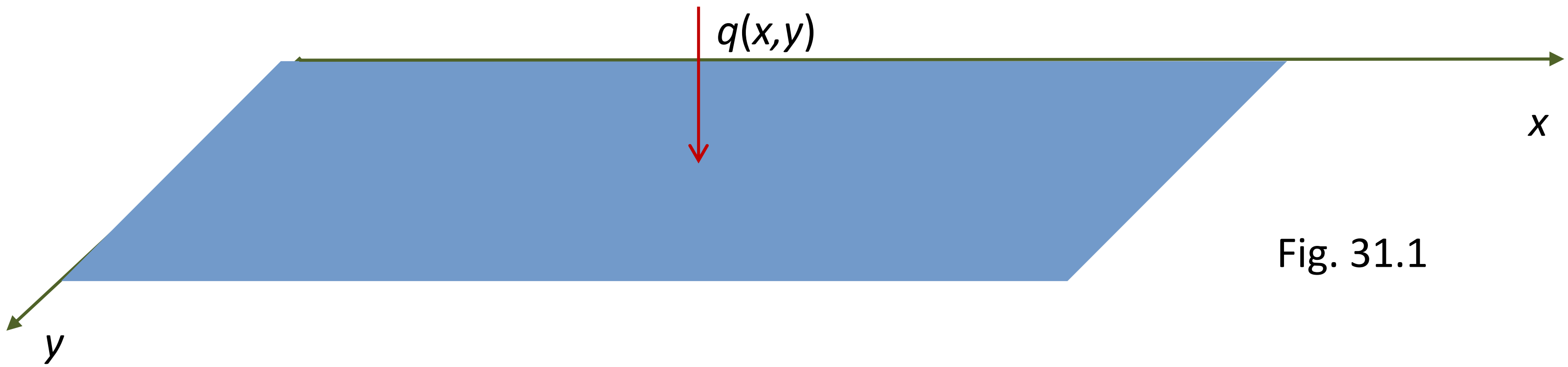

Fig. 31.1

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- For such a plate, the out-of-plane boundary conditions are:
  - $w^{\pm} = 0$ on all four sides.
  - $M_x^{\pm} = 0$ at $x = 0, a$.
  - $M_y^{\pm} = 0$ at $y = 0, b$.

- As explained earlier, the governing equation for equilibrium for out-of-plane direction for such a plate is decoupled with in-plane equations, because the plate's lamination sequence is symmetric. This equation is reproduced below.

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + q = 0 \qquad \text{(Eq. 31.1)}$$

- Also, writing expressions for M_x, M_y, and M_{xy}, we get:

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- Also, writing expressions for M_x, M_y, and M_{xy}, we get:

$$M_x = D_{11}\frac{\partial^2 w^o}{\partial x^2} + D_{12}\frac{\partial^2 w^o}{\partial y^2} + 2D_{16}\frac{\partial^2 w^o}{\partial x \partial y}$$

$$M_y = D_{12}\frac{\partial^2 w^o}{\partial x^2} + D_{22}\frac{\partial^2 w^o}{\partial y^2} + 2D_{26}\frac{\partial^2 w^o}{\partial x \partial y}$$

(Eq. 31.2)

$$M_{xy} = D_{16}\frac{\partial^2 w^o}{\partial x^2} + D_{26}\frac{\partial^2 w^o}{\partial y^2} + 2D_{66}\frac{\partial^2 w^o}{\partial x \partial y}$$

- Substituting these equations in Eq. 31.1 yields:

$$\left[D_{11}\frac{\partial^4 w^o}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w^o}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w^o}{\partial y^4}\right] + 4D_{16}\frac{\partial^4 w^o}{\partial x^3 \partial y} + 4D_{26}\frac{\partial^4 w^o}{\partial x \partial y^3} - q_o = 0$$

(Eq. 31.3)

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- At this stage, we use Galerkin approach to solve this equation. Here we assume that out-of-plane displacement is of the form:

$$w^o(x,y) = \sum\sum w_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \qquad \text{(Eq. 31.4)}$$

- We chose such a form for w^o, because it satisfies all the kinematic boundary conditions of the system. Inserting Eq. 31.4 in Eq. 31.3, we get the error in the equation as:

$$\begin{aligned} E(x,y) = {} & \sum\sum w_{mn}\left[D_{11}\left(\frac{m\pi}{a}\right)^4 + 2(D_{12}+2D_{66})\left(\frac{m\pi}{a}\frac{n\pi}{b}\right)^2 + D_{22}\left(\frac{n\pi}{b}\right)^4\right]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \\ & + \sum\sum w_{mn}\left[-D_{16}\left(\frac{m\pi}{a}\right)^3\left(\frac{n\pi}{b}\right) - D_{26}\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^3\right]\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} - q_o \end{aligned}$$

(Eq. 31.5)

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- As per Galerkin method, the virtual work performed by this force error, when integrated over plate's area, should be zero. This is shown in Eq. 31.6.

$$\int_0^b \int_0^a \left\{ \sum\sum w_{mn} \left[D_{11} \left(\frac{m\pi}{a}\right)^4 + 2(D_{12} + 2D_{66}) \left(\frac{m\pi}{a}\frac{n\pi}{b}\right)^2 + D_{22} \left(\frac{n\pi}{b}\right)^4 \right] \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \right.$$
$$+ \sum\sum w_{mn} \left[-D_{16} \left(\frac{m\pi}{a}\right)^3 \left(\frac{n\pi}{b}\right) - D_{26} \left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)^3 \right] \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$
$$\left. - q_o \right\} \epsilon w_{ij} \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b} \, dx dy = 0$$

(Eq. 31.6)

- Equation 31.6 involves summation over indices *m*, and *n*. In this equation, constants w_{mn}, are unknown. When this equation is integrated over the domain, we will get several terms multiplied by w_{ij} coefficient of virtual work function, where *i* and *j*, are indices with varying integer values.

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- Equation 31.6 gives a set of simultaneous equations which equal m X n in number, where values of w_{mn} are not known. Solving these equations gives us the desired solution.

- We have seen that in Galerkin approach, our choice of displacement function requires only kinematic boundary conditions to be satisfied. However, in this problem, the displacement as well as moment along all four edges of the plate is zero.

- While our choice displacement function (as defined in Eq. 31.4) ensures satisfaction of displacement boundary condition, the same may not be true of the following boundary conditions:
  - $M_x^{\pm} = 0$ at $x = 0, a$.
  - $M_y^{\pm} = 0$ at $y = 0, b$.

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- Consider the expression for M_x.

$$M_x = D_{11}\frac{\partial^2 w^o}{\partial x^2} + D_{12}\frac{\partial^2 w^o}{\partial y^2} + 2D_{16}\frac{\partial^2 w^o}{\partial x \partial y}$$

- Substituting Eq. 31.4 in this equation we get:

$$M_x = \sum\sum w_{mn}\left\{-\left[D_{11}\left(\frac{m\pi}{a}\right)^2 + D_{12}\left(\frac{n\pi}{b}\right)^2\right]\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} + 2D_{16}\left(\frac{mn\pi^2}{ab}\right)\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\right\}$$

- M_x should be zero at along edges x=0,a. At x=0, that is not the case. At that edge, M_x, as calculated from above equation, is:

$$M_x = 2D_{16}\sum\sum w_{mn}\left(\frac{mn\pi^2}{ab}\right)\cos\frac{n\pi y}{b}$$

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- Thus we see that usage of approximate methods such as Galerkin approach, generates moments at edges for certain lamination sequences, even if the edge is simply-supported.

- This phenomenon is called the “Edge Effect”.

- Thus, while using such approaches, care has to be taken that these residual moments due to edge effect are not significantly large. For this the following is recommended.
  - Find the maximum value of M_x over the plate’s area. For a rectangular plate which is simply supported on all edges and which is loaded with uniform load intensity, M_{max}, may typically exist at the center of the plate.
  - Ensure that the ratio of “residual moments” at edges, and M_{max}, is small. Typical upper threshold for this ratio should be 5%.

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- Like D_{16}, D_{26} causes generation of edge effects along edges y=0,b.

- This is a limitation of all approximate methods. The extent of this problem may be minimized by increasing the number of terms in approximate solution.

- Such an approach drives edge-moments closer to zero.

- Similar issues are also encountered in other approximate methods as well, such as the finite element method.

- A reasonably accurate and converged solution requires that these edge moments must be minimized to the extent possible.