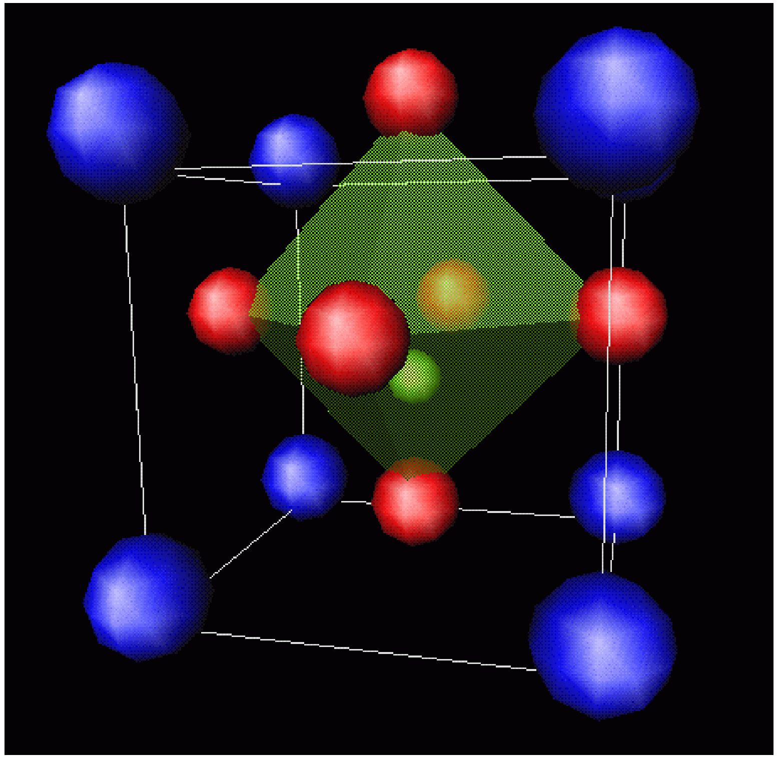
Modelling of Smart Materials



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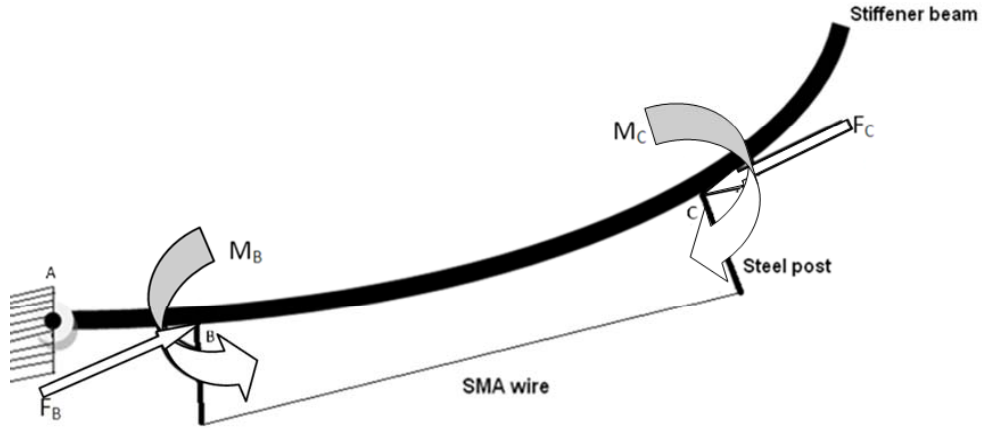


LECTURE 8
Modelling of Piezoelectric Material (Part 1)

Organization

- Piezoelectric Property
- Crystal Structure
- Constitutive Relationship
- Active Strain Evaluation

Output Input	Current/Charge	Magnetization	Strain	Temperature	Light
Electric Field	Conductivity Permittivity	Electromagnetic Effect	Reverse Piezoelectric Effect SA	Ohmic Resistance	Electro-Optic effect
Magnetic Field	Eddy Current Effect	Permeability	Joule Effect Magnetostriction SA	Magneto-caloric Effect	Magneto-Optic effect
Stress	Direct Piezoelectric Effect SS	Villary Effect SS	Elastic Modulus	Thermo-Mechanical Effect SS	Photo-elastic Effect SS
Heat	Pyroelectric Effect	Thermo-magnetization	Thermal Expansion/Phase Transition SA	Specific Heat	Thermo-luminescence
Light	Photo-voltaic Effect	Photo-magnetization	Photostriction SA	Photo-thermal effect	Refractive Index

Properties important for Actuation	Piezoelectric Material		Magnetostrictive Material	Phase-transition dependent Material	
	Piezo-ceramic	PVDF	Terfenol-D	Nitinol	FSMA
Maximum free strain (Λ) in microns	2000	700	2000	20,000	30,000
Young's Modulus (GPa)	60-70	2-3	48	27.5 M-phase, 90 A-phase	0.45 – 0.82
Bandwidth	0.1 Hz-GHz	0.1 Hz-GHz	0.1 Hz-10KHz	0-10 Hz	100 Hz

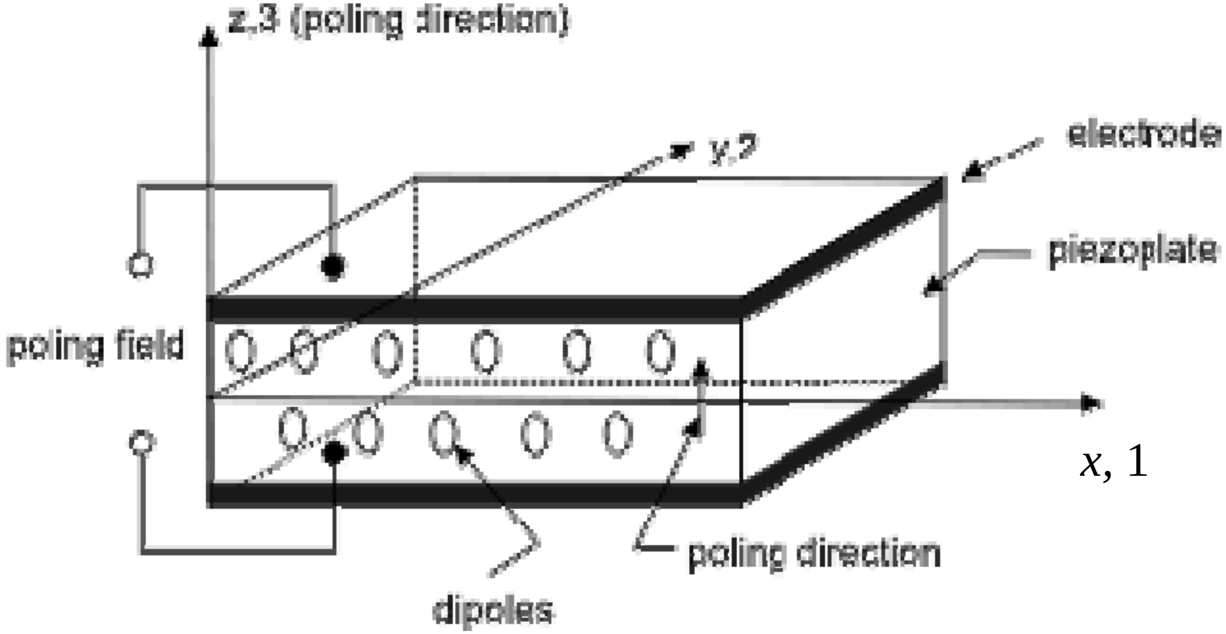
Fundamental equations of piezoelectricity

$$\sigma_{ij} = C^{E}_{ijkl} S_{kl} - e_{k\,ij} E_{k}$$

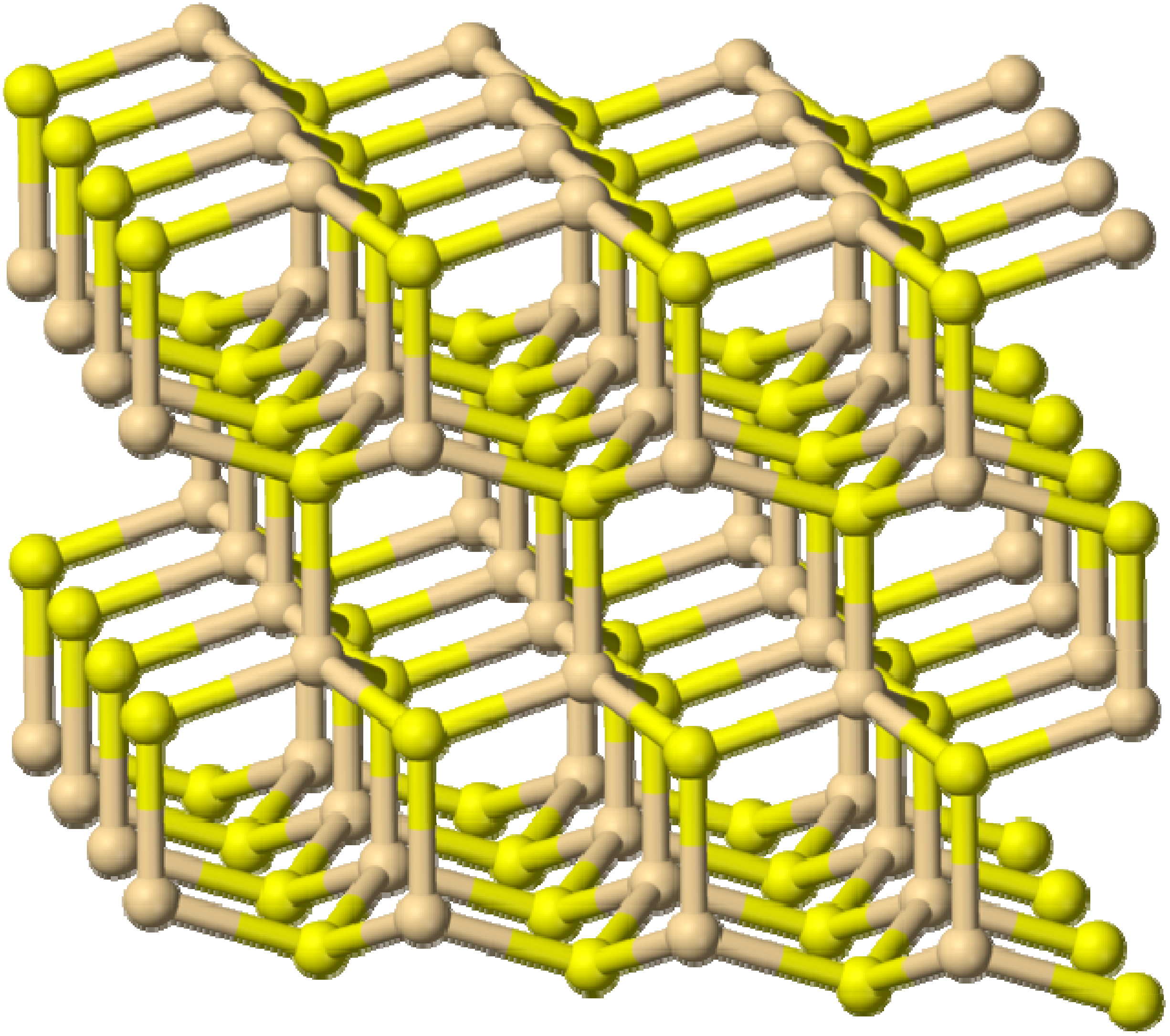
$$D_{i} = e_{i\,kl} S_{kl} + \varepsilon^{S}_{i\;j} E_{j}$$

where, the subscripts *i*, *j*, *k*, *l* = 1, 2, 3 denotes tensorial indices.
The stress tensor is represented by σ, *S* is the strain tensor,
E is the electric field intensity and *D* is the electric displacement field.
Elastic stiffness matrix is denoted by the symbol C^{E}, where
the superscript *E* denotes that the elastic constant is measured
under constant electric field;
e is the piezoelectric stress-charge matrix and ϵ the permittivity matrix,
similar to *C*, is measured under constant strain-condition.

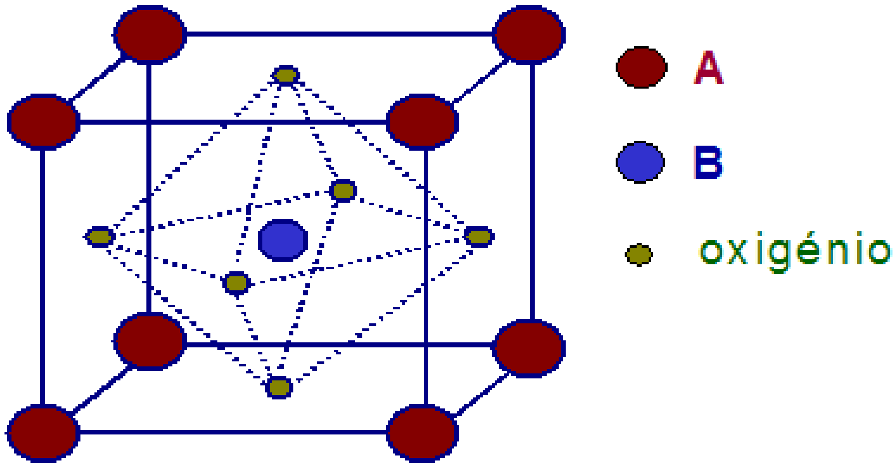
Different Axes



Di Hexagonal Crystal Symmetry

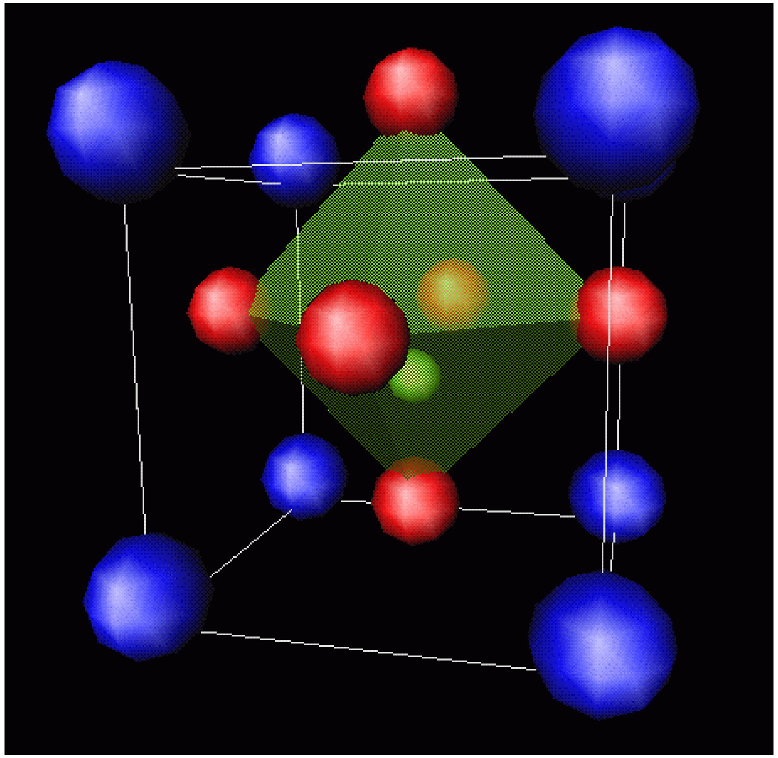


Greenockite Crystal



Tetragonal Wulfenite

Perovskite Structure with 4mm crystal symmetry



Outcome of Symmetry

The crystal structure of common piezoelectric materials shows 4mm or 6mm symmetry. Following material symmetry conditions could be applied to the constitutive relationship

$$C_{ijkl} = C_{jikl} = C_{klij}$$

$$e_{kij} = e_{kji}$$

$$\varepsilon_{ij} = \varepsilon_{ji}.$$

$$
\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \\ D_1 \\ D_2 \\ D_3 \end{Bmatrix}
=
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 & 0 & 0 & -e_{31} \\
C_{12} & C_{11} & C_{13} & 0 & 0 & 0 & 0 & 0 & -e_{31} \\
C_{13} & C_{13} & C_{33} & 0 & 0 & 0 & 0 & 0 & -e_{33} \\
0 & 0 & 0 & C_{44} & 0 & 0 & 0 & -e_{15} & 0 \\
0 & 0 & 0 & 0 & C_{44} & 0 & -e_{15} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & e_{15} & 0 & \varepsilon_1 & 0 & 0 \\
0 & 0 & 0 & e_{15} & 0 & 0 & 0 & \varepsilon_2 & 0 \\
e_{31} & e_{31} & e_{33} & 0 & 0 & 0 & 0 & 0 & \varepsilon_3
\end{bmatrix}
\begin{Bmatrix} S_x \\ S_y \\ S_z \\ S_{yz} \\ S_{xz} \\ S_{xy} \\ E_1 \\ E_2 \\ E_3 \end{Bmatrix}
$$

The electro-mechanical coupling is shown inside the bordered boxes . Axes 1, 2 and 3 used for the electrical system are identical with *x*, *y* and *z*, corresponding to the mechanical system.

Simplified Equation for Piezo-patch

- Ignoring the normal stress σ_z and the shear stresses σ_{xz} and σ_{yz} for plane stress assumption:

$$\begin{Bmatrix} S_x \\ S_y \\ S_{xy} \\ D_3 \end{Bmatrix} = \begin{bmatrix} 1/E_p & -\upsilon/E_p & 0 & -d_{31} \\ -\upsilon/E_p & 1/E_p & 0 & -d_{32} \\ 0 & 0 & 2(1+\upsilon)/E_p & 0 \\ d_{31} & d_{32} & 0 & \varepsilon_{33} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \\ E_3 \end{Bmatrix}$$

E_p is the modulus of elasticity of the piezoelectric material, ν is the Poisson's ratio and d_{ij} are the piezoelectric strain-charge constants

Active Strain Expression

If a piezoelectric thin slab is subjected to mechanical load, the total strain S developed in an active layer, would consist of two parts – the structural or elastic strain S_s and the piezoelectric strain S_a such that

$$S = S_s + S_a$$

where, $S_a = [-d_{31}E_3, \; -d_{32}E_3, \; 0]^T$.

To generate strains along the direction of the thickness of the specimen, ceramics with different crystal-cuts are used which are commonly known as Piezo-stacks. The electro elastic coupling components in the 3-3 directions, like d_{33} or e_{33}, become important in such cases.

References

- Gauenzi, P., Smart Structures, Wiley, 2009
- Cady, W. G., Piezoelectricity, Dover Publication, 1950
- Crawley, E. F., Intelligent Structures for Aerospace: a technology overview and assessment, AIAA, 33 (8), 1994, pp. 1689-1699

END OF LECTURE 8