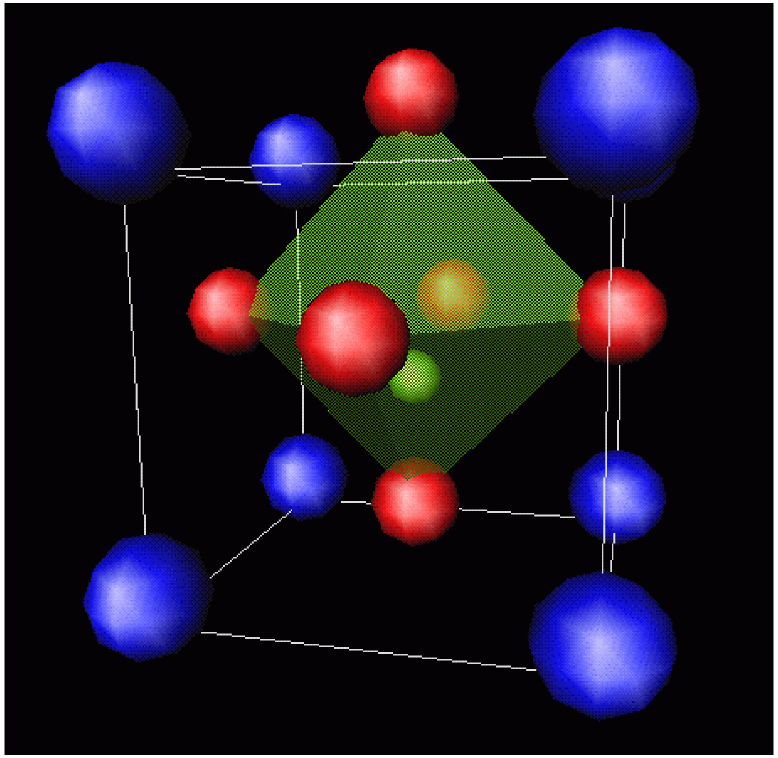
Modelling of Smart Materials



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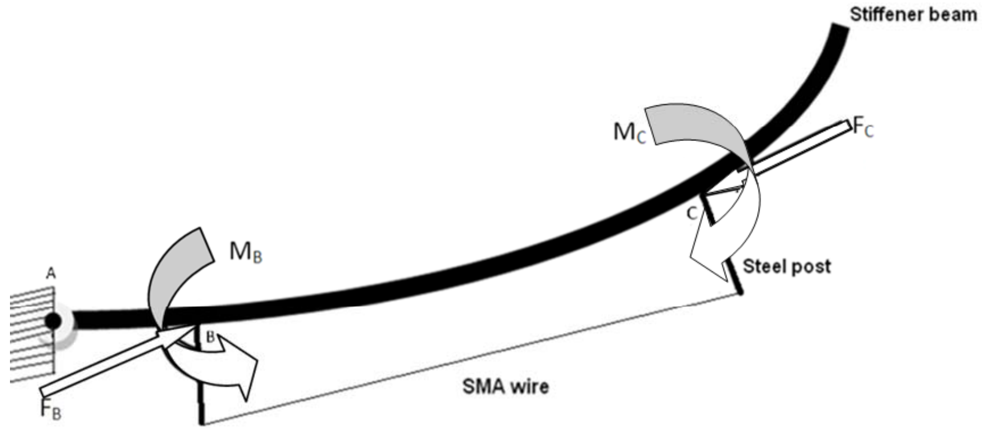


LECTURE 10
Modelling of Induced Strain Actuation
(Part 1)

Topics Covered in the Last two Lectures:

- Constitutive Relationship of Piezoelectric Material
- Simplified Equations for Piezo-patch
- Piezoelectric Constants
- Different Piezoelectric Materials and their properties
- Piezo-actuators – various displacement and force measurement techniques

Organization of this Lecture

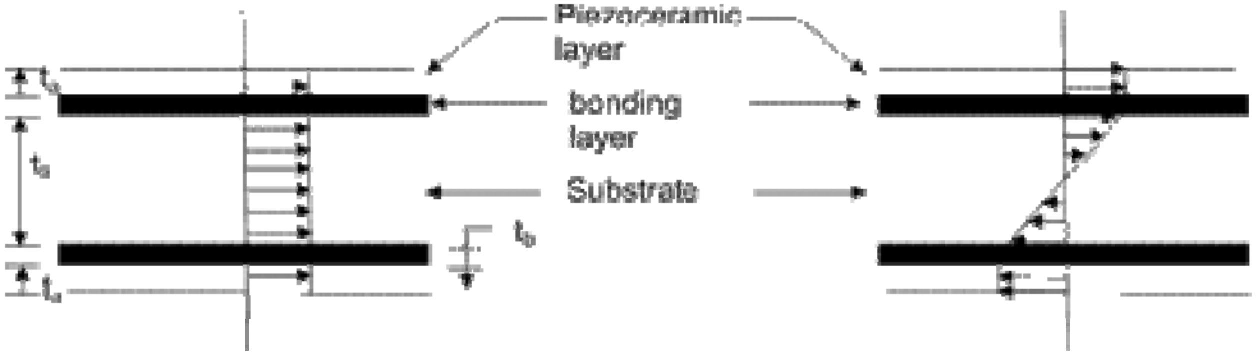
- Induced Strain Actuation (ISA) – Uniform Strain Model
- Static Equilibrium Configuration against Uniform Strain
- Configuration against Bending Strain

Motivation

- Vibration and Shape Control of Flexible Structures and Compliant joints demand development of strain/mechanical force inside such systems.
- Hence, it is important to study how piezoelectric patches would behave while integrated to such systems.
- This is the motivation of modelling induced strain actuation of flexible structures.
- The smart patches could be either surface-bonded or embedded inside the structure.

Induced Strain Actuation – Uniform Strain Model

- Static model was developed by Crawley and Anderson
- Assumed that the strain remains constant across the piezo-patch while it varies linearly inside the substructure.



Induced Strain Actuation – Uniform Strain Model

- The model has been used for surface bonded actuation.
- Actuators embedded/ bonded on top and bottom of the beam are excitable in the same phase to cause uniform extension or contraction.
- Bending also can be generated through out-of-phase excitation of the piezo-ceramics

Static equilibrium corresponding to the induced strain can be expressed as

$2\,F_a + F_s = F_i$,

F_a is the reactive force developed in each active layer, F_s is that in the substrate, and F_i is the total force.

Equilibrium equation in terms of Strain

$$2(E_p S_a A_p) + E_s S_s A_s = 2\Lambda E_p A_p$$

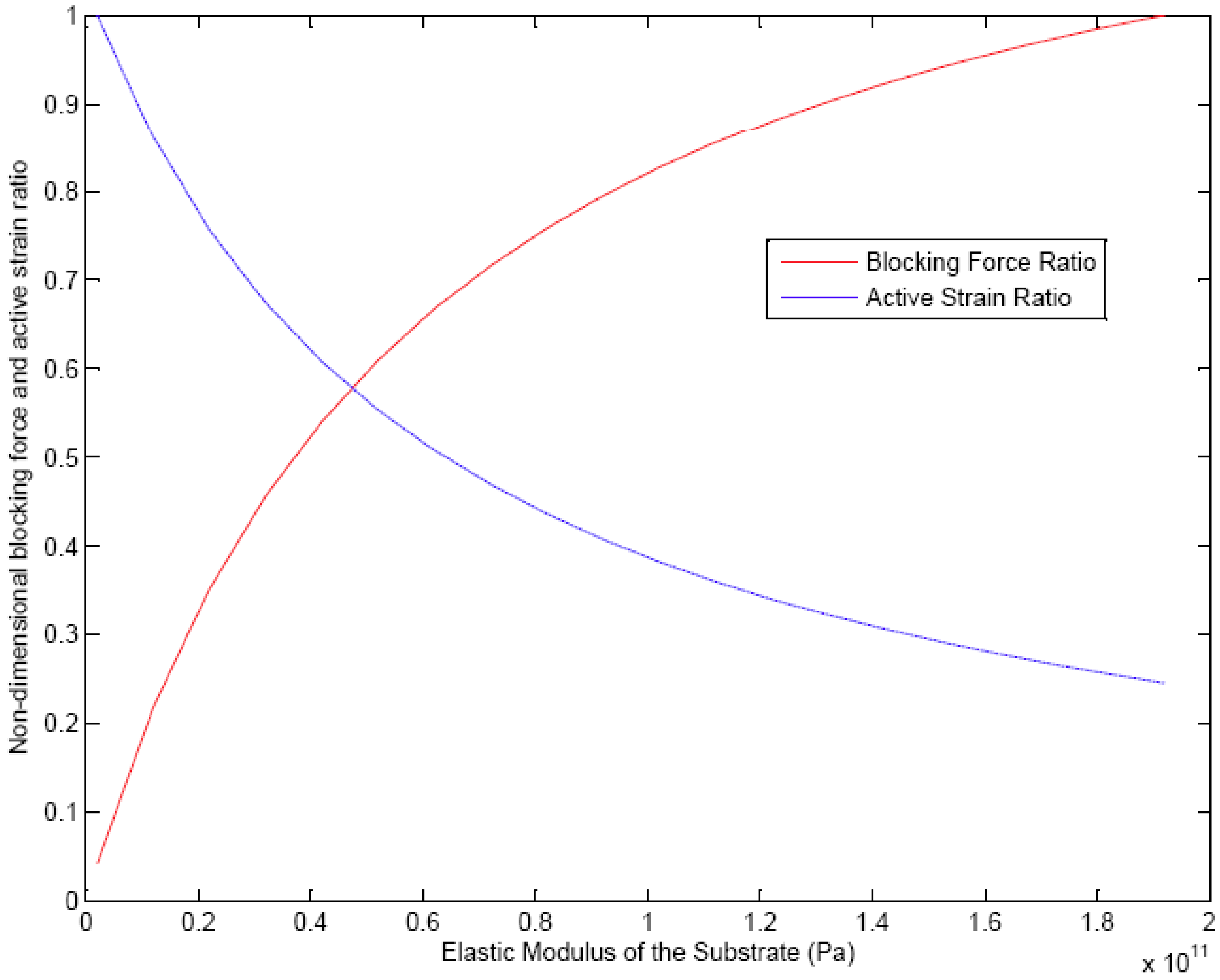
where S denotes the strain, E the modulus of elasticity, A the area of cross-section and Λ the free strain.

The subscript p denotes the piezoelectric material and s denotes the substrate.

It is assumed that near the actuator-substrate interface, the strain remains unchanged, hence, we can write

$$S_a = S_s = \frac{2\Lambda}{2 + \Psi_e}$$

where, the in-plane stiffness ratio ψ_e is $(E_s A_s) / (E_p A_p)$.



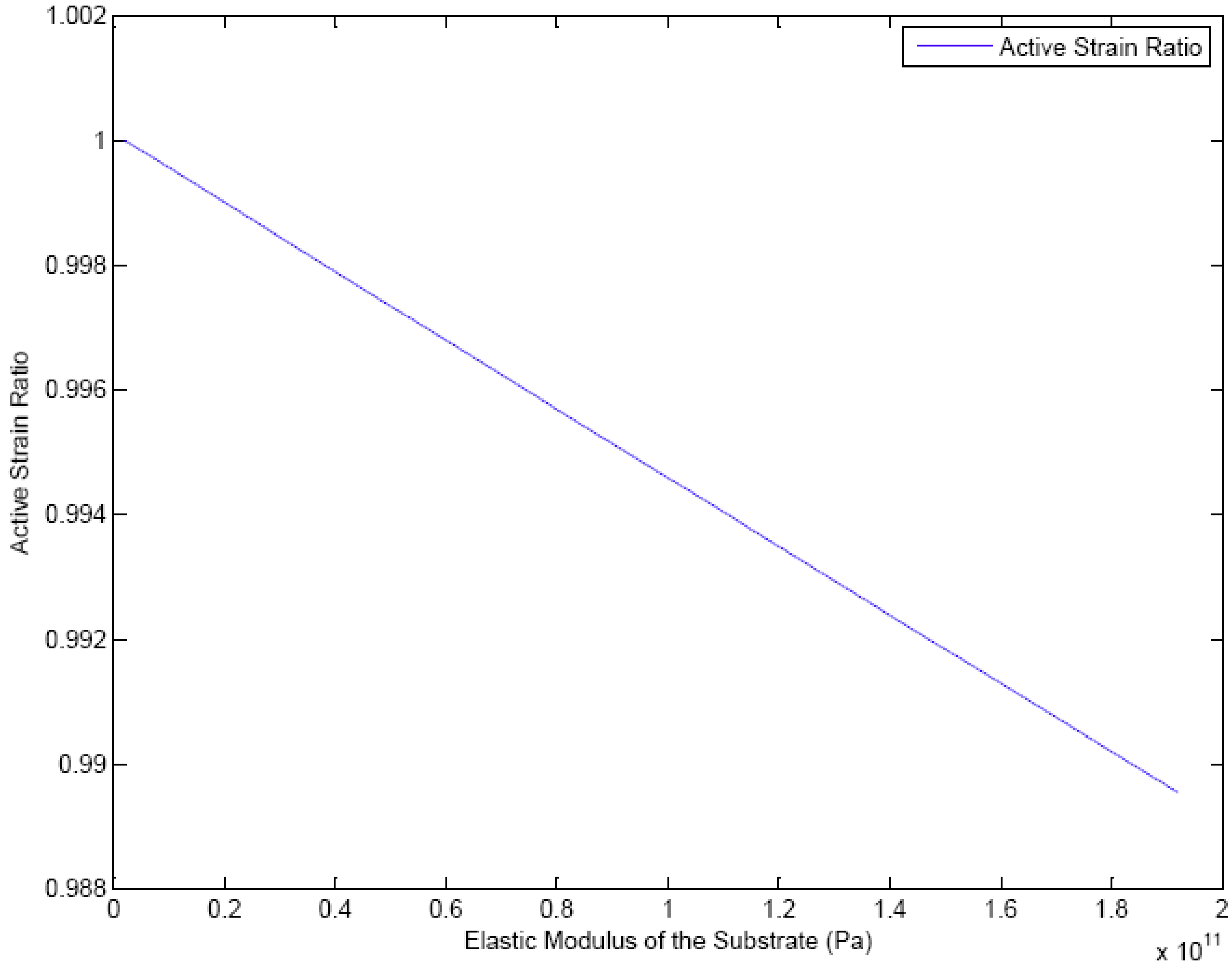
Blocking Force Ratio
Active Strain Ratio
Non-dimensional blocking force and active strain ratio
Elastic Modulus of the Substrate (Pa)
x 10^11
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

For bending, considering equilibrium of active and reactive moments one gets

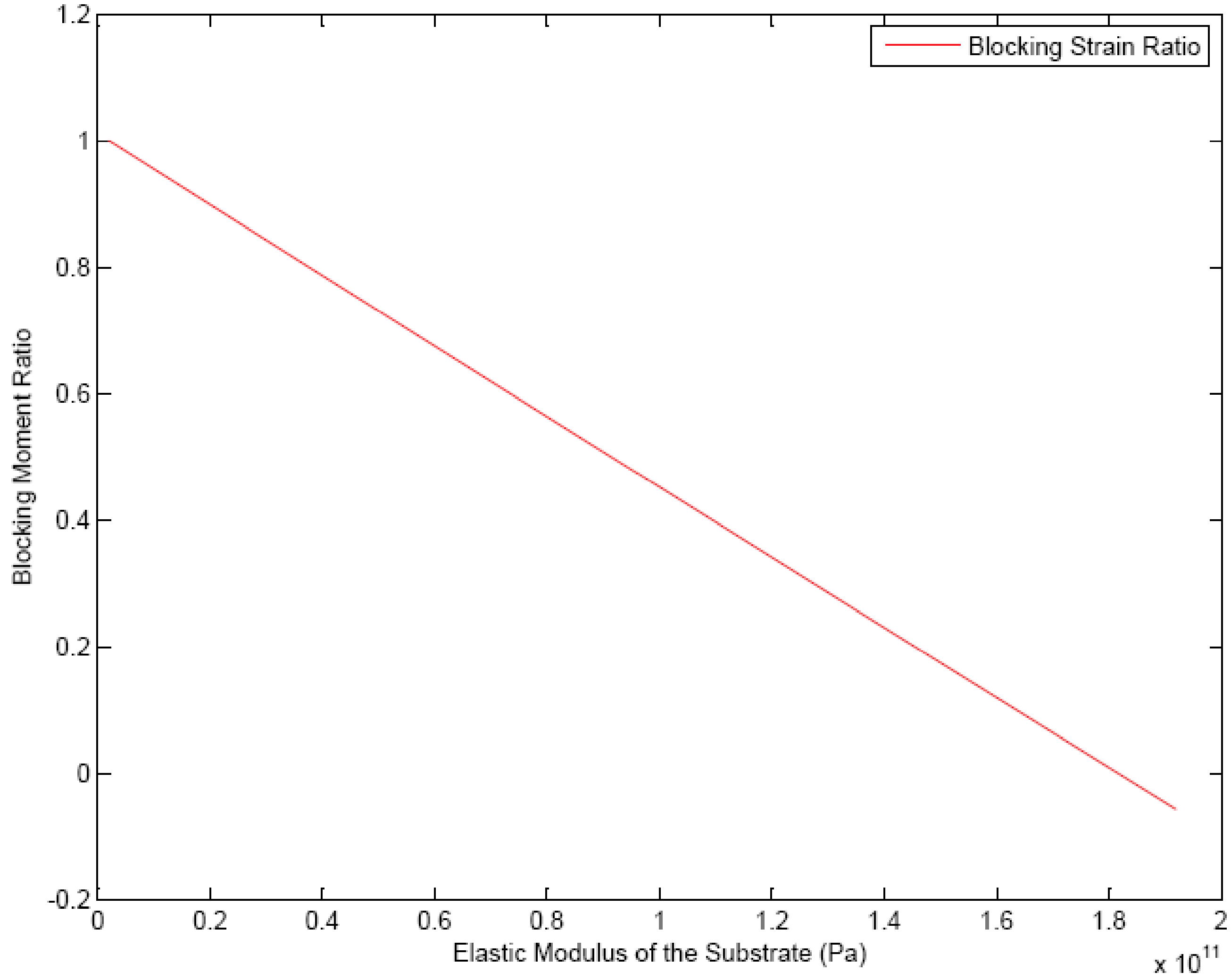
$$E_p S_a A_p t_s + \frac{2E_s I_s S_a}{t_s} = \Lambda E_p A_p t_s$$

$$S_a = S_s = \frac{6\Lambda}{6+\psi_b}$$

with the bending stiffness ratio
$\psi_b = 12\,(EI)_s\, /\, [t_s^2\ (EA)_p]$.



Active Strain Ratio
1.002
1
0.998
0.996
0.994
0.992
0.99
0.988
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Elastic Modulus of the Substrate (Pa)
x 10^11
Active Strain Ratio



Blocking Strain Ratio
Blocking Moment Ratio
Elastic Modulus of the Substrate (Pa)
x 10^11
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2

When the thickness of the bonding layer is finite, the presence of visco-elastic bonding material reduces the transmission of stress from actuator to substrate and the induced strain to actuation strain ratio is shown to be given by the following relationships

$$S_a = \frac{\alpha}{\alpha + \psi} \Lambda \left[1 + \frac{\psi \cosh(\Gamma \bar{x})}{\alpha \cosh(\Gamma)} \right]$$

Strain in the Host Substrate will be

$$S_s = \frac{\alpha}{\alpha + \psi} \Lambda \left[1 - \frac{\cosh(\Gamma \overline{x})}{\alpha \cosh(\Gamma)} \right]$$

$$\Gamma^2 = \overline{D} \, \frac{\alpha + \psi}{\psi}, \qquad \overline{D} = \frac{(G / E_a)(t_b / t_p)}{(t_b / l_p)^2}$$

Γ is the non-dimensional length parameter varying from –1 to +1 (edge to edge of the actuator).

Geometric constant α is 2 for extension and 6 for bending; ψ is the stiffness ratio related to bending or extension based on the appropriate case. G is the shear modulus of the bonding layer and Γ is the shear lag contributed by the bonding layer of thickness t_b

References

- Crawley, E. F., Intelligent Structures for Aerospace: a technology overview and assessment, AIAA, 33 (8), 1994, pp. 1689-1699

END OF LECTURE 10