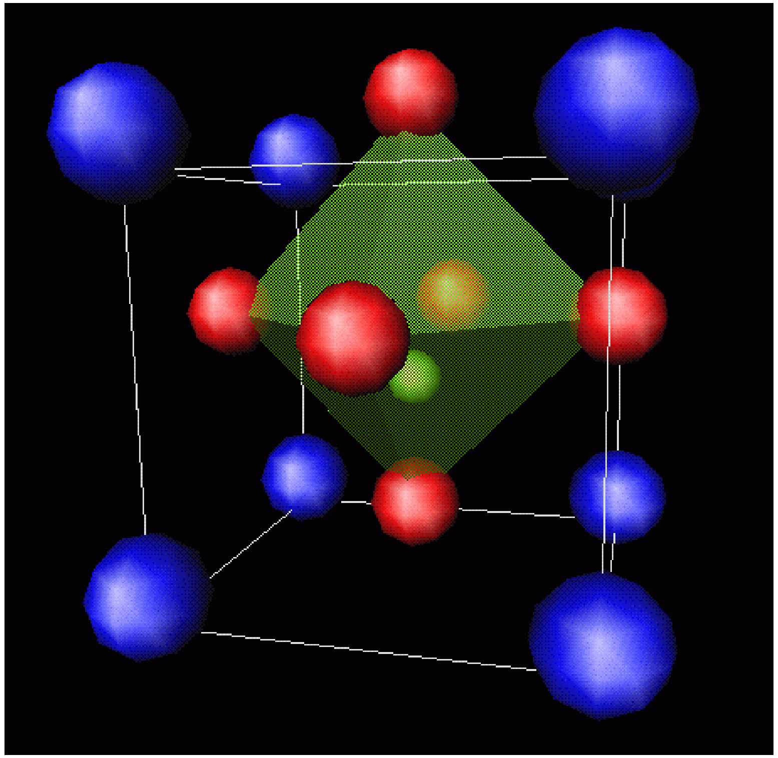
Modelling of Smart Materials



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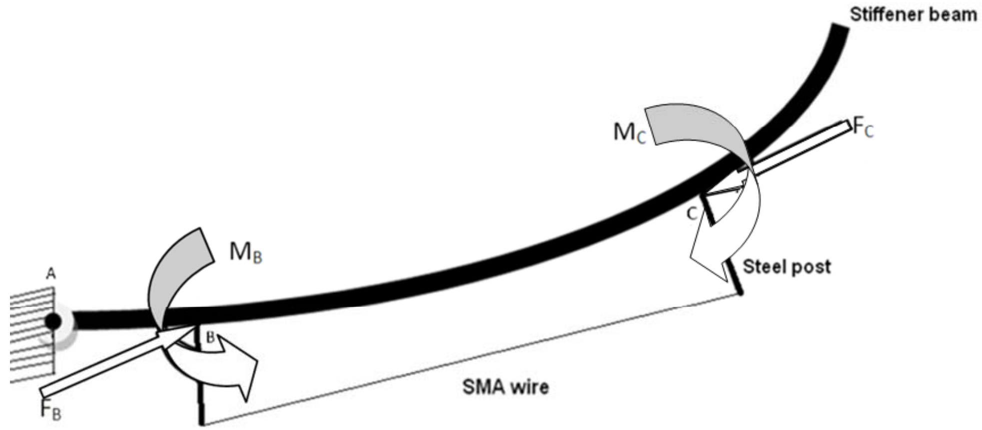
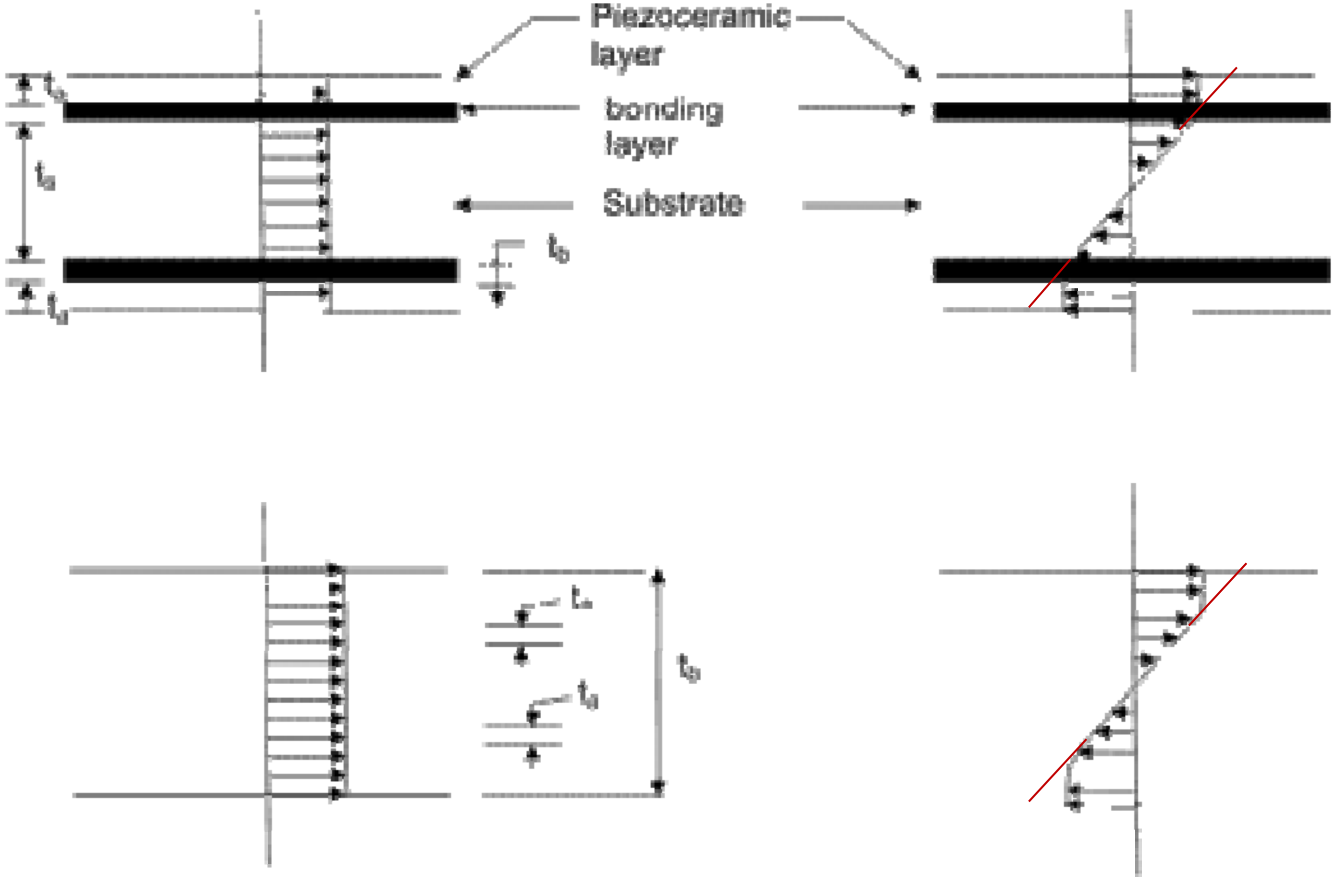


LECTURE 11
Modelling of Induced Strain Actuation
(Part 2)

Organization of this Lecture

- ISA – Euler-Bernoulli Model
- ISA Model for Magnetostrictive Mini Actuator
- Active Fibre Composite Actuation

Another modeling of smart beam for static analysis of induced strain, specifically used for embedded smart patches, is based on the assumption of linear strain variation along the entire cross–section of the beam (Euler-Bernoulli model).



Strain at any point of the Beam CS

- Following Euler-Bernoulli assumptions, the strain at any point at a distance **z** from the neutral axis of the structure can be expressed as

$$S_t = S_s - z\kappa$$

- where S_s is the mid-plane strain and κ is the curvature of the beam.

Total Stress at any point

- Total stress at any point consists of the combination of elastic stress and induced stress and is given by

$$\sigma_t = \sigma_s + E_a \Lambda$$

- where the subscripts *s* and *a* stands for the passive and active substrate, respectively

Using the last two equations for strains and stress and integrating over the cross-section of the smart beam,

the equations of force and moment balance corresponding to perfect bonding may be obtained as shown in the next slide …

Final Equations of Equilibrium

$$\begin{bmatrix} EA & EB \\ EB & EC \end{bmatrix} \begin{Bmatrix} S_s \\ \kappa \end{Bmatrix} = \begin{Bmatrix} P_s + P_\Lambda \\ M_s + M_\Lambda \end{Bmatrix}$$

where

$$(EA) = \int_z E(z)\, dz\,, \qquad (EB) = \int_z E(z) z\, dz\,,$$

$$(EC) = \int_z E(z) z^2\, dz\,, \qquad P_s = \int_z \sigma(z) dz, \qquad M_s = \int_z \sigma(z) z dz\,,$$

$$P_\Lambda = \int_z E_a(z) \Lambda(z) dz \quad \text{and} \quad M_\Lambda = \int_z E_a(z) \Lambda(z) z dz\,.$$

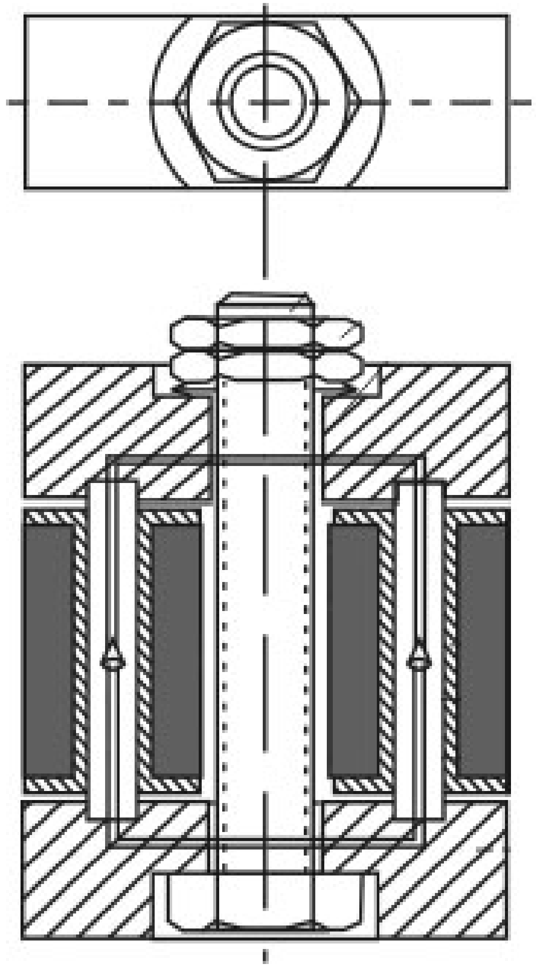
New Stiffness Ratio for axial deformation

parameter Ψ_e now becomes

$$\Psi_e = \frac{\left(E_s A_s - E_s A_a\right)}{E_a A_a}$$

Substitution of passive material by the active piezo-layer has been taken into account in this model

ISA Modelling for Magnetostrictive Actuation



Modelling

- Terfenol-D rods work very much similar to Piezo-stacks
- Governing equation describing the constrained strain is similar
- Unlike piezo-actuators, the effect of thickness of a Magnetostrictive Mini Actuator (MMA) is not negligible

- Induced (constrained) strain related to the free-strain by the following relationship:

$$\boxed{\varepsilon_c = \left(1 + \frac{Ebt^2}{12 A_m E_m (t + t_m)}\right)^{-1} \Lambda}$$

- *b* and *t* are, the width and thickness of the host beam, A_m and t_m are the cross-sectional area and thickness of the MMA. *E*-denotes the modulus of elasticity of the host-beam and E_m denotes the modulus of elasticity of the magnetostrictive material.

- Expression for the free-strain Λ, contains an additional term - the thermal effect due to current passing through the solenoid and hence Λ is expressed as

$$\boxed{\Lambda = dGi(t) + \alpha' K \int_0^t e^{-t/C_2} i^2(t)\,dt}$$

- *G* is a parametric constant, α' is the coefficient of equivalent thermal expansion, *K* and C_2 are two thermal constants which are obtained experimentally and *i* is the current passed through the magnetizing coil

Blocking Force of MMA

- Blocking force attainable is 2 times the blocking force required for each individual Terfenol-D rod to block.

$$\boxed{F = 2A_m\sigma = -\frac{2d}{S} G A_m i(t) - \frac{2\alpha' K A_m}{S} \int_0^t e^{-\frac{t}{c_2}} i^2(t)\, dt}$$

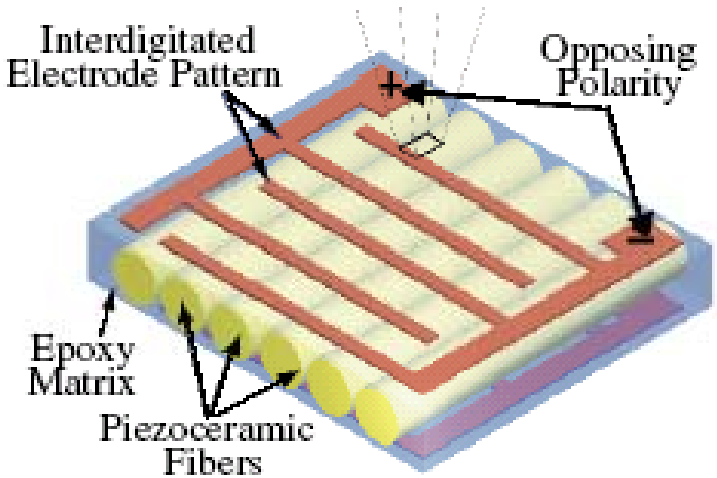
- Force of the Rod is directly proportional to current at the initial phase
- The temperature effect is non-negligible later

Active Fibre Composites

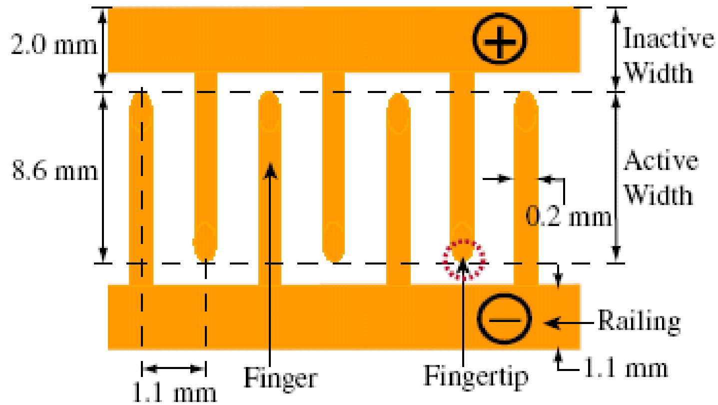
- Magnitude of piezoelectric constant d_{33} is significantly higher than d_{31}. For example, for a soft piezoelectric material d_{31} is about -55 pC/N, whereas d_{33} is around 190 pC/N.
- Tempted material scientists to develop piezoelectric fibres and actuate them axially to generate larger actuation strain.

AFC and MFC

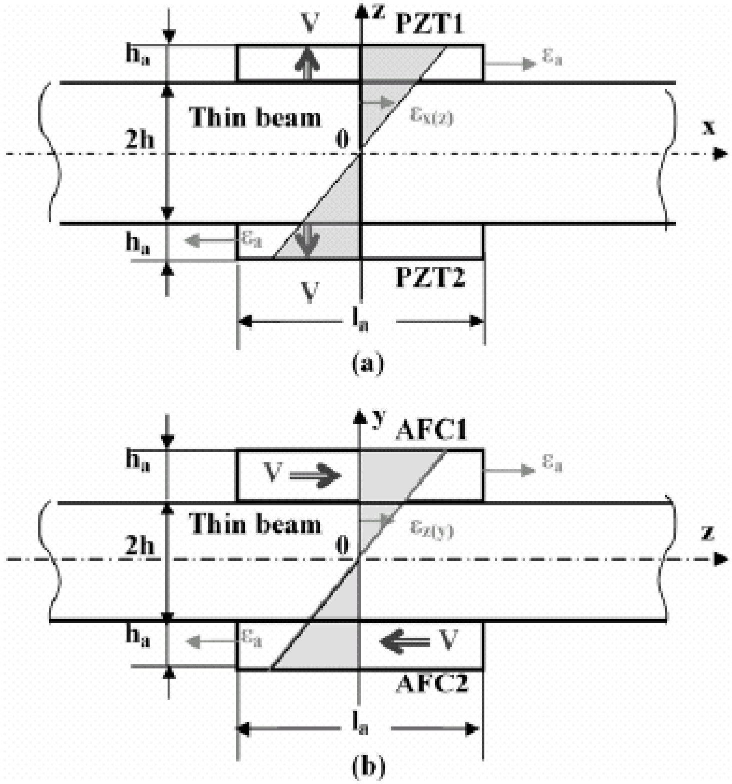
- Two types of smart composites have been developed using piezo-ceramic fibres; these are Active Fibre Composite (AFC) and Macro Fibre Composite (MFC).
- AFC fibres are developed using standard sol-gel technique, the MFC fibres are essentially chopped from PZT blocks.
- MFC fibres are rectangular in cross-section and hence it offers better electrical contact between the fibres.



Nguyen and Kornmann
2006



Euler Bernoulli Model



- Following the balance of active and reactive moments, the active strain at a distance *z* from the neutral axis can be expressed as

$$\varepsilon_x = K \Lambda z,$$

$$K = \frac{3E_a[(h+h_a)^2 - h^2]}{2[E_a((h+h_a)^3 - h^3)) + Eh^3]} \text{ and}$$

$$\Lambda = d_{33}\frac{V}{h_a}.$$

- Voltage is applied along the length of the fibre and hence the free strain Λ is obtained using the piezoelectric constant d_{33}.

References

- Crawley, E.F. and Luis, J.D., Use of Piezoelectric actuators as elements of intelligent structures, *AIAA Journal*, Vol. 25 (10), 1371-1385, 1987
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- Nguyen, C. and Kornmann, X., A comparison of dynamic piezoactuation of fiber-based actuators and conventional piezo patches, *Journal of Intelligent Material Systems and Structures*, Vol. 17, 45-56, 2006

END OF LECTURE 11