Smart Materials, Adaptive Structures, and Intelligent Mechanical Systems

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Lecture 15

Behavior of Unidirectional Composites

References for this Lecture

1. Analysis and Performance of Fiber Composites, Agarwal, B.D. and Broutman, L. J., John Wiley & Sons.

2. Mechanics of Composite Materials, Jones, R. M., Mc-Graw Hill

3. Structural Analysis of Laminated Composites, Whitney, J. M., Technomic

4. Nonlinear Analyis of Plates, Chia, C., McGraw-Hill International Book Company

Lecture Overview

- Material axes in unidirectional composites
- Constituent volume fraction and its relationship with composite density
- Importance of predictive methodologies used for composite material properties
- Predictive model for longitudinal stiffness
- Predictive model for longitudinal strength

The Need for Predictive Models for Determining Composite Properties

- Mechanical properties of a composite material depend on:
  - Properties of constituent materials
  - Orientations of each layer
  - Volume fractions of each constituent
  - Thickness of each layer
  - Nature of bonding between adjacent layers

- These properties may be determined by conducting suitable experiments as per industry standards.

- However, a specific set of experiments can only inform us about *a* specific fiber-matrix system. Hence, if we want to design a composite system by tuning its volume fraction, or fiber-matrix combination, or orientation, then a very large number of experiments may have to be conducted.

The Need for Predictive Models for Determining Composite Properties

- Such a process for material property determination is extremely tedious, prohibitively expensive, and time consuming.

- Still further, exact fiber-matrix combinations may not be always available for testing.

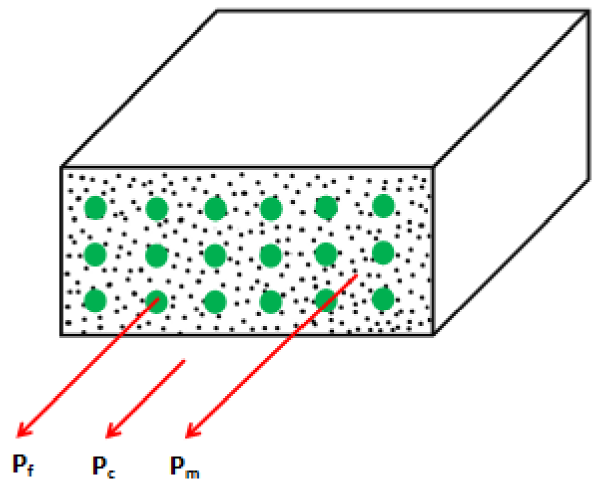
- Hence, there is a need for developing mathematical models, which can reliably predict different thermo-mechanical properties of composite materials.

- Such approaches are very useful for engineers since they provide significant savings in time and cost.

Predicting Longitudinal Modulus of Unidirectional Lamina

- Consider a unidirectional composite lamina with fibers which are continuous, uniform in geometric and mechanical properties, and mutually parallel throughout the length of the lamina. We also assume that the bonding between fiber and matrix is perfect, and thus strains experienced by fiber (ε_f), matrix (ε_f) and composite (ε_c)are same in longitudinal direction (1-direction). For such a composite, when loaded in 1-direction, the total external load P_c will be shared partly by fibers, P_f, and partly by matrix, P_m. This is shown in Fig. 15.1.

Fig. 15.1: Loads on Composite, Fibers, & in a Unidirectional Lamina.



Predicting Longitudinal Modulus Unidirectional Lamina

- We further assume that fibers and matrix behave elastically. Thus, the expression for stress in fibers, and matrix can be written in terms of their moduli (E_f, and E_m) and strains as:

  $\sigma_f = E_f \, \varepsilon_f$

  and

  $\sigma_m = E_m \, \varepsilon_m$

- Further, if total cross-sectional areas of fibers and matrix are A_f and A_m, respectively, then:

  $P_f = A_f \sigma_f = A_f E_f \, \varepsilon_f$,

  and

  $P_m = A_m \sigma_m = A_m E_m$

Predicting Longitudinal Modulus Unidirectional Lamina

- Further, we know that load on composite, P_c, is sum of P_f and P_m. Thus,

  $P_c = A_c\sigma_c = A_f\sigma_f + A_m\sigma_m$,

  or

  $\sigma_c = (A_f/A_c)\sigma_f + (A_m/A_c)\sigma_m$

- However, for a unidirectional composite, A_f/A_c and A_m/A_c are volume fractions for fiber and matrix, respectively. Hence,

  $\sigma_c = V_f\sigma_f + V_m\sigma_m = (V_f E_f + V_m E_m) \cdot \varepsilon$ (Eq. 15.2)

- And, if Eq. 15.2 is differentiated with respect to strain (which is same in fiber and matrix) then.

  $d\sigma_c /d\varepsilon = V_f(d\sigma_f /d\varepsilon) + V_m(d\sigma_m/d\varepsilon)$, or

  $E_c = V_f E_f + V_m E_m$ (Eq. 15.3)

Predicting Longitudinal Modulus of Unidirectional Lamina

- Equations 15.2 and 15.3 show that contributions of fibers and matrix to average composite tensile modulus and stress are proportionately dependent on their respective volume fractions.

- In general, matrix material has a nonlinear stress-strain response curve. For unidirectional composites having such nonlinear matrix materials, Eq. 15.2 works well in terms of predicting their stress-strain. However, the stress-strain response curve in such materials may not show up as strongly nonlinear, since fibers, especially when their volume fractions are high, dominate their stress-strain response.

- The higher the fiber volume fraction, the closer is the stress-strain curve for a unidirectional lamina to that for the fiber.

Predicting Longitudinal Modulus of Unidirectional Lamina

- Experimental data pertaining to tensile test specimens of lamina agree very well with Eqs. 15.2 and 15.3. However, the results for compressive tests are not all that agreeable.

- This is because fibers under compression tend to buckle, and this tendency is resisted by matrix material. This is analogous to a structure with several columns on an elastic foundation. For a unidirectional composite, the compressive response is strongly dependent on shear stiffness of matrix material.

- Further, Eq. 15.2 shows us that load shared by fibers may be increased either by increasing fiber stiffness or by increasing its volume fraction. However, experimental data show that it becomes impractical to aim for fiber volume fractions in excess of 80% due to issues of poor fiber wetting and insufficient matrix impregnation between fibers.

Predicting Longitudinal Strength of Unidirectional Lamina

- To predict longitudinal strength of a unidirectional ply, requires one to understand the nature of deformation of such a ply as load increases. In general, the stress-strain response of unidirectional plies under tension undergoes four stages of change.
  - In first stage, when stresses are small, fiber as well as matrix materials exhibit elastic behavior.
  - Subsequently, matrix starts becoming plastic, while most of the fibers continue to extend elastically.
  - In the third stage, both fibers and matrix deform plastically. This may not happen in case of glass or graphite fibers, as they are brittle in nature.
  - Finally, the fibers fracture leading to sudden rise in matrix stress, which in turn leads to overall composite failure.

- A unidirectional lamina *starts to fail* in tension, when its fibers are stretched to their ultimate fracture strain. Here it is assumed that all of its fiber fail at the same strain level. If at this stage, the volume fraction of matrix is below a certain threshold, the it will not be able to absorb extra stresses transferred to it due to breaking of fibers. In such a scenario, the entire composite lamina will fail.

Predicting Longitudinal Strength of Unidirectional Lamina

- Thus, ultimate tensile strength of a unidirectional ply can be calculated as:

  $\sigma_{uc} = V_f\sigma_{uf} + (1\text{-}V_f)\underline{\sigma}_m$ (Eq. 15.4)

  where σ_{uc} and σ_{uf} are ultimate tensile strengths of ply and fiber, respectively, and $\underline{\sigma}_m$ is stress in matrix at a strain level equaling fracture strain in fiber.

- If the fiber volume fraction does not exceed a certain threshold (V_{min}), then even if all the fibers break, the matrix the total load on the composite. In such a condition, the ultimate tensile strength of composite may be written as:

  $\sigma_{uc} = V_m\sigma_{um} = (1\text{-}V_f)\sigma_{um}$ (Eq. 15.5)

- The relation for V_{min}, can be developed by equating Eqs. 15.4 and 15.5, replacing V_f by V_{min}, and solving for the latter. This is shown in Eq. 15.6.

  $V_{min} = (\sigma_{um} - \underline{\sigma}_m)/(\sigma_{uf} + \sigma_{um} - \underline{\sigma}_m)$ (Eq. 15.6)

Predicting Longitudinal Strength of Unidirectional Lamina

- Further, a well designed unidirectional lamina requires that its ultimate tensile strength should exceed that of matrix. This can happen only when;

  $\sigma_{uc} = V_f\sigma_{uf} + (1-V_f)\underline{\sigma}_m \geq \sigma_{um}$, where σ_{um} is ultimate tensile strength of matrix.

- This equation is satisfied only if fiber volume fraction exceeds a certain critical value, which is defined as:

  $$V_{crit} = (\sigma_{um} - \underline{\sigma}_m)/(\sigma_{uf} - \underline{\sigma}_m) \qquad \text{(Eq. 15.7)}$$

- Thus, if:
  - $V_f < V_{min}$, then failure of matrix will coincide with failure of composite, while fibers will fail prior to failure of matrix.
  - $V_f = V_{min,}$ then failure of matrix, fiber and composite will happen at the same time.
  - $V_f > V_{min}$, then failure of fiber, will immediately lead to failure of matrix as well as of the composite.
  - $V_f > V_{crit}$, then failure of fiber will immediately lead to failure of matrix and also the composite. In such a case the strength of unidirectional composite will exceed that of matrix.

What you learnt in this lecture?

- Material axes of a lamina, and the notion of transverse isotropy
- How to calculate density of a composite, using volume and mass fraction values.
- The rationale underlying development of predictive methodologies for estimating composite material properties
- Relations to predict longitudinal modulus of a unidirectional lamina
- Relations to predict strength of unidirectional lamina
- Different failure criterion for a unidirectional lamina which undergoes pure uniform tension