Smart Materials, Adaptive Structures, and Intelligent Mechanical Systems

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Lecture 16
Behavior of Unidirectional Composites

References for this Lecture

1. Analysis and Performance of Fiber Composites, Agarwal, B.D. and Broutman, L. J., John Wiley & Sons.

2. Mechanics of Composite Materials, Jones, R. M., Mc-Graw Hill

3. Structural Analysis of Laminated Composites, Whitney, J. M., Technomic

4. Nonlinear Analyis of Plates, Chia, C., McGraw-Hill International Book Company

Lecture Overview

- Predictive models for transverse stiffness

- Shear modulus and Poisson's ratio

- Estimates for transverse strength

- Predictive models for coefficient of thermal expansion

- Thermal conductivity

- Failure mechanisms in unidirectional composites

Predicting Transverse Modulus of Unidirectional Lamina

- Figure 16.1 shows a simple model for predicting transverse modulus of unidirectional lamina. Here, the model constitutes of two "slabs" of materials, fiber and matrix, of thicknesses t_f and t_m, respectively. The overall thickness of composite slab is t_c, which is sum of t_f and t_m. It may be noted here that these thicknesses of fiber and matrix are directly proportional to their respective volume fractions.
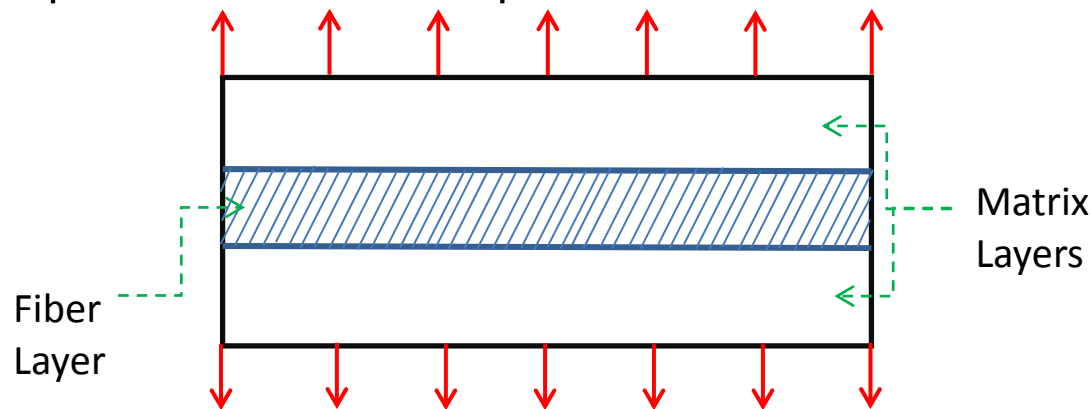
Fiber Layer

Matrix Layers

σ_c

Fig. 16.1: A "Slab" Like Model for Predicting Transverse Properties of Unidirectional Composites.

- In such a system, externally imposed stress on the composite (σ_c) is assumed to be same as that seen by fiber (σ_f) and also by matrix (σ_m).

- This is in contrast to the model developed for predicting longitudinal modulus, where we had assumed that strains, and not stresses, in composite, fiber and matrix are equal.

Predicting Transverse Modulus of Unidirectional Lamina

- Further, in such a model, which is akin to springs in series, the overall displacement in composite (Δ_c) in transverse direction due to external load is a sum of displacement in fiber (Δ_f) and displacement in matrix (Δ_m).

$$\Delta_c = \Delta_f + \Delta_m$$

- Further, recognizing the relation between strains in each constituent, and their thicknesses, above equation can be rewritten as:

$$\varepsilon_c\, t_c = \varepsilon_m\, t_m + \varepsilon_f\, t_f$$

- Dividing above equation by thickness of composite (t_c), and realizing that t_f/t_c, and t_m/t_c equal V_f and V_m, respectively, we get:

$$\varepsilon_c = \varepsilon_m\, V_m + \varepsilon_f\, V_f$$

- In linear-elastic range, strain is a ratio of stress and the modulus. Hence, above equation can be further re-written as:

$$(\sigma_c/E_c) = (\sigma_m/E_m)V_m + (\sigma_f/E_f)V_f$$

Predicting Transverse Modulus of Unidirectional Lamina

- However, we had earlier assumed that externally applied stress on the composite (σ_c) is same as that seen by fiber (σ_f) and also by matrix (σ_m). Thus, previous equation can be rewritten as:

$1/E_c = V_m/E_m + V_f/E_f$ (Eq. 16.1a)

Or alternatively,

$E_c = (E_f E_m)/([(1-V_f)E_f + V_f E_m]$ (Eq. 16.1b)

- Equation 16.1 gives us an estimate for transverse modulus of unidirectional lamina. The relation shows that a significant increase in fiber volume fraction is required to raise overall transverse modulus in moderate amounts. This is in stark contrast with longitudinal modulus, which is linearly dependent on fiber volume fraction.

- Equation 16.1, even though based on a simple model, is not borne out well be experimental data. To address this inconsistency, several alternative models have been developed.

Predicting Transverse Modulus of Unidirectional Lamina

- However, in this lecture we will use simple and generalized expressions for transverse modulus as developed by Halpin and Tsai. These are relatively simple relations, and hence easy to use in design practice. The results from Halpin and Tsai are also quite accurate especially if fiber volume fraction is not too close to unity.

- As per Halpin and Tsai, transverse modulus (E_T) can be written as:

  $E_T/E_m = (1 + \xi\eta V_f)/(1 - \eta V_f)$ (Eq. 16.2)

  where,

  $\eta = [(E_f/E_m) - 1] / [(E_f/E_m) + \xi]$

  Here, ξ is a parameter that accounts for packing and fiber geometry, and loading condition. Its values are given below for different fiber geometries.
  - $\xi = 2$ for fibers with square and round cross-sections.
  - $\xi = 2a/b$ for fibers with rectangular cross-section. Here a is the cross-sectional dimension of fiber in direction of loading, while b is the other dimension of fiber's cross-section.

Shear Modulus and Poisson's Ratio

- A perfectly isotropic material has two fundamental elastic constants, E and v. Its shear modulus and bulk modulus can be expressed in terms of these two elastic constants.

- Likewise, a transversely isotropic composite ply has four elastic constants. These are:
  - E_L, i.e. elastic modulus in longitudinal direction.
  - E_T i.e. elastic modulus in transverse direction.
  - G_{LT} i.e. longitudinal shear modulus.
  - v_{LT} i.e. Poisson's ratio

- A detailed discussion on the mathematical logic underlying existence of these four constants will be conducted in a subsequent lecture.

- Till so far, we have developed relations for E_L, and E_T. Now we will learn about similar relationships for G_{LT} and v_{LT}.

Shear Modulus and Poisson's Ratio

- Halpin and Tsai have developed relations similar to Eq. 11.2 which can be used to predict longitudinal shear modulus, G_{LT}. This is shown below.

  $G_{LT}/G_m = (1 + \eta V_f)/(1 - \eta V_f)$ (Eq. 16.3)

  where,

  $\eta = [(G_f/G_m) - 1] / [(G_f/G_m) + 1]$

- For predicting Poisson's ratio v_{LT}, we exploit the fact that a longitudinal tensile strain in fiber direction, will generate Poisson contraction in transverse direction in both, matrix and fiber materials.

- In this context, we also use the fact that relative strain values for such a contraction will be proportional to each constituent material's volume fraction. Thus, overall Poisson's ratio v_{LT} for the composite can be written as:

  $v_{LT} = v_f V_f + v_f V_m$ (Eq. 16.4)

Transverse Strength

- We have seen that a unidirectional ply, when put to tension in fiber direction tends to break at stress values which exceed matrix tensile strength. This is particularly true when fiber volume fraction exceeds V_{crit}. Similarly, fibers play a central role in significantly enhancing the stiffness of the ply in fiber direction, and the overall stiffness of the system tends to far surpass that of pure matrix.

- This occurs because fibers, which are stronger and stiffer vis-à-vis matrix, carry a major portion of external load, thereby enhancing composite's stiffness and strength.

- However, the same may not be said for a unidirectional ply loaded in tension in the transverse direction. This is because load-sharing between fiber and matrix in a transversely loaded ply is very less. In contrast, the extent of load sharing between fiber and matrix in a longitudinally loaded ply is very significant.

- When a unidirectional load is subjected to transverse tension, fibers which are far more stiff vis-à-vis matrix, act to constrain matrix deformation.

Transverse Strength

- Such a constraint on matrix deformation, tends to increase ply's transverse modulus, though only marginally (unless fiber volume fraction is high).

- However, the story is even more starkly different in case of transverse strength. The deformation constraints imposed on matrix by fibers tend to generate strain and stress concentrations in matrix material.

- These stress and strain concentrations cause the matrix to fail at much lesser values of stress and strain, than a sample of matrix material which has no fibers at all. Thus, unlike longitudinal strength, transverse strength tends to get *reduced* for composites due to presence of fibers.

- This reduction in transverse strength of a unidirectional ply is characterized by a factor, *S*, the *strength-reduction-factor*. The exact value of this factor can be calculated by using a combination of advanced elasticity formulations and numerical solution techniques.

- The strength of unidirectional ply in transverse direction, σ_{uT}, can be written as:

   $\sigma_{uT} = \sigma_{uf}/S$ (Eq. 16.5)

Some Other Properties of Unidirectional Plies

- Using approaches as described earlier, thermal conductivity in L (k_L) direction can be written as:

  $$k_L = V_f k_f + V_m k_m \qquad\qquad\qquad\qquad \text{(Eq. 16.6)}$$

- Similarly, transverse conductivity, k_T, can be written as:

  $$k_T/k_m = (1 + \xi\eta V_f)/(1 - \eta V_f) \qquad\qquad \text{(Eq. 16.7)}$$

  where,

  $$\eta = [(k_f/k_m) - 1] / [(k_f/k_m) + \xi], \qquad \text{where,} \qquad \log\xi = 1.732 \log(a/b)$$

- Finally, longitudinal and transverse thermal expansion coefficients have been shown in engineering literature to be:

  $$\alpha_L = (E_f V_f \alpha_f + E_m V_m \alpha_m)/E_L \qquad\qquad \text{(Eq. 16.8)}$$

  $$\alpha_T = (1+v_f)V_f \alpha_f + (1+v_m)V_m \alpha_m - \alpha_L v_{LT} \qquad\qquad \text{(Eq. 16.9)}$$

What you learnt in this lecture?

- Predictive models for transverse stiffness

- Shear modulus and Poisson's ratio

- Estimates for transverse strength

- Predictive models for coefficient of thermal expansion

- Thermal conductivity

- Failure mechanisms in unidirectional composites