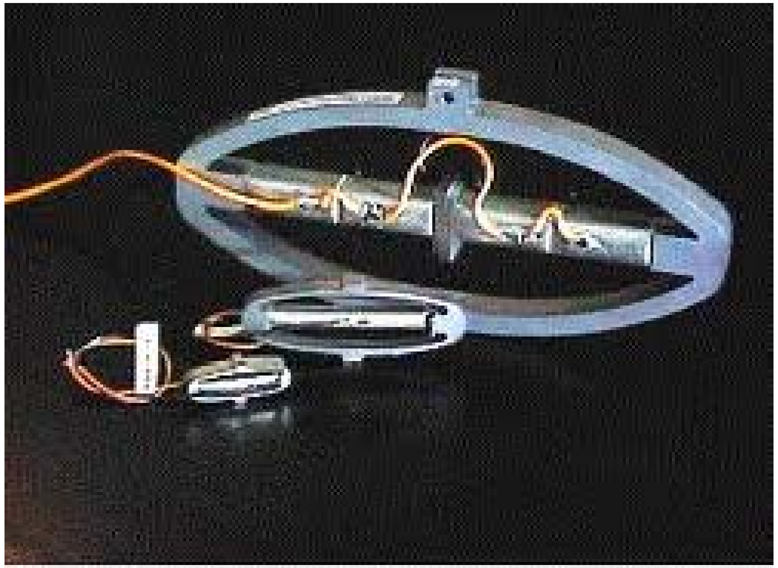
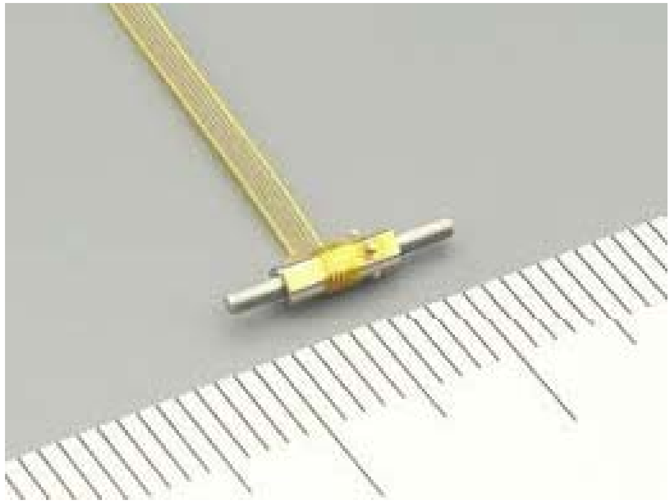


APA230L, APA150M, APA100S

Module 5: Actuators & Sensors based on HBLS Smart Materials



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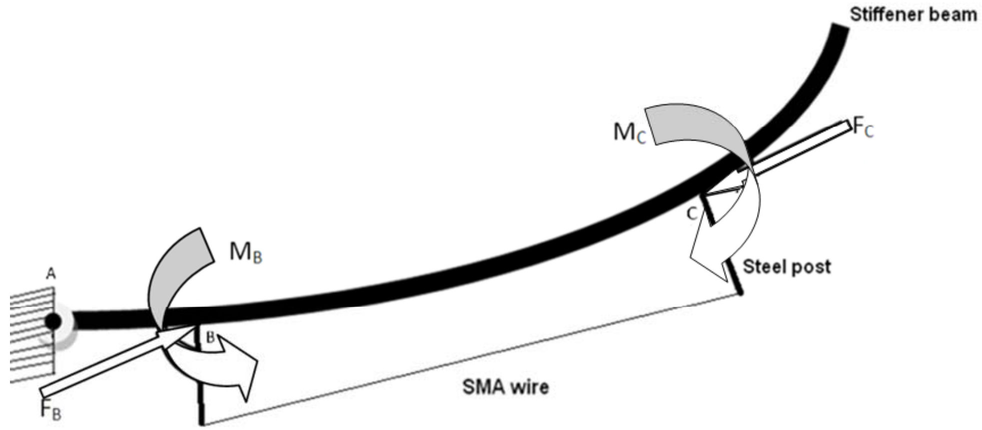
Topics Covered in the Last Lecture

- ❖**Introduction to HBLS Materials**
- ❖**Smart Magnetostrictive Material**
- ❖**Modelling of Smart Laminated Beam**
- ❖**Basic Assumptions**



LECTURE 34
Modelling of Smart Composite Beam
(Part 2)

Organization of this Lecture

- Displacement Field of Smart Composite Beam
- Governing Equation of Motion
- A Distributed Control Model for Vibration Control

Displacements

- $u(x,y,z)=u_0(x,y)-z\phi_x(x,y)$
- $v(x,y,z)=v_0(x,y)-z\phi_y(x,y)$
  - if the plate is thin

$$\phi_x(x,y)=\frac{\partial w_0}{\partial x}$$

$$\phi_y(x,y)=\frac{\partial w_0}{\partial y}$$

- assuming:
  - length of A-D is constant ε_{zz}~0
  - ϕ_x and ϕ_x are very small
- $w(x,y,z)=w_0(x,y)$

Strain-Displacement Relations

The strains at any point in a plate are:

$$\varepsilon_x(x, y, z) = \frac{\partial u}{\partial x}$$

$$\varepsilon_x(x, y, z) = \frac{\partial v}{\partial x}$$

$$\gamma_{xy}(x, y, z) = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

$$\gamma_{yz}(x, y, z) = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$$

$$\gamma_{xz}(x, y, z) = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

Strains and Curvatures

- Plugging plate deformation equations into the strain-displacement relations and simplifying yields:
- <u>Strains in terms of midplane strains and curvatures</u>

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon^0{}_x \\ \varepsilon^0{}_y \\ \gamma^0{}_{xy} \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \text{strains} \\ \text{in} \\ \text{plate} \end{Bmatrix} = \begin{Bmatrix} \text{mid}- \\ \text{surface} \\ \text{strains} \end{Bmatrix} + z \{\text{curvatures}\}$$

Mid-plane Strains and Curvatures

- Midsurface Strains

$$\varepsilon^0{}_x(x,y) = \frac{\partial u_0}{\partial x}$$

$$\varepsilon^0{}_y(x,y) = \frac{\partial v_0}{\partial y}$$

$$\gamma^0{}_{xy}(x,y) = \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x}$$

- Curvatures

$$\kappa_x(x,y) = -\frac{\partial \phi_x}{\partial x} = -\frac{\partial^2 w_0}{\partial x^2}$$

$$\kappa_y(x,y) = -\frac{\partial \phi_y}{\partial y} = -\frac{\partial^2 w_0}{\partial y^2}$$

$$\kappa_{xy}(x,y) = -2\frac{\partial^2 w_0}{\partial x \partial y}$$

$$\boxed{S^{i} = (1/\overline{Q}_{11}^{i})}$$

$$\overline{Q}_{11}^{i} = Q_{11}^{i}\, Cos^{4}\theta_{i} + 2(Q_{12}^{i} + 2Q_{66}^{i})\, Cos^{2}\theta_{i}\, Sin^{2}\theta_{i} + Q_{22}^{i}\, Sin^{4}\theta_{i}$$

$$Q_{11}^{i} = \frac{E_{11}^{i}}{1 - v_{12}^{i}} v_{21}^{i}, \quad Q_{22}^{i} = \frac{E_{22}^{i}}{1 - v_{12}^{i} v_{21}^{i}}$$

$$Q_{12}^{i} = Q_{21}^{i} = \frac{v_{21}^{i} E_{11}^{i}}{1 - v_{12}^{i} v_{21}^{i}}$$

$$Q_{66}^{i} = G_{12}$$

where $E_{11}{}^{i}$, $v_{21}{}^{i}$ etc. are Elastic constants and Poisson's ratio respectively. Following Euler - Bernoulli model, the total strain at any layer may also be expressed as

$$\boxed{\varepsilon_x^i = -z\,w,_{xx}.}$$

Assume velocity proportional control algorithm, the active strain becomes proportional to the transverse velocity of the beam,

$$\varepsilon_x = -C\dot{w}$$

the symbol '$\cdot$' denotes differentiation with respect to time, the constant of proportionality *C* is function of electro/magneto-mechanical constant(*d*), and controller gain (*f*).

In case of magnetostrictive material,

$$C = d_m k_1 \, f$$

where the coil constant k_1 is given by

$$k_1 = \frac{n_c}{\sqrt{w_c^2 + 4r_c^2}}$$

n_c is the total number of coil-turns, r_c – effective length of magnetizing coil and w_c – effective width of magnetizing coil.

Stress at any layer may now be written as

$$\sigma_x^i = \bar{Q}_{11}^i \left[\left(-z\, w_{,xx}\right) - c\, \dot{w}\right.$$

Using the Hamilton's principle, the governing Equation of Motion may be derived as

$$D_{11} w,_{xxxx} + \beta_{11} \dot{w},_{xx} + m\ddot{w} = P(x,t)$$

$$D_{11} = \sum_{i=1}^{n} \int_{z_1}^{z_{1+1}} \overline{Q_{11}}^{\,i} z^2 dz$$

$$\beta_{11} = \sum_{i=1}^{n} \int_{zk}^{zk+1} S^{-1} C \delta_{ia} z \, dz$$

$$m = \sum_{i=1}^{n} \int_{z_i}^{z_{i+1}} \rho \, dz$$

P (x, t) is the generalized distributed load on the beam

For distributed control, there is an interesting difference in forming the equation of motion of the system.

In the usual Hamiltonian, the actuation of the structure is considered as an outside effect and the work done due to actuation is considered separately, whereas in this approach, the effect of actuator is considered implicitly.

Consider a freely vibrating simply supported beam of length L, subjected to distributed control.

The specified initial condition of velocity is 1 unit. Find out the responses of the system and compare it with uncontrolled vibration for a control gain of unity.

Let us consider w, the transverse deflection as the product of the following spatial and temporal functions -

$$\omega(x,t) = T_1(t) Sin \frac{k\pi x}{L}$$

when after variable separation, the governing eqn. reduces to

$$DT_1 + \dot{\beta} T_1 + m\ddot{T} = 0$$

The damped natural frequency and the damping constant *α* of the system may be evaluated as:

$$\omega_d = \omega_n \sqrt{1-\varepsilon^2}, \quad \varsigma = \frac{c}{2m\omega_n} \qquad \omega_n^2 = \frac{D}{m}, c = \beta * f$$

For the case of initial displacement specified as $T_1(0) = 1$;
Displacement:

$$\omega = \frac{\omega_d}{\sqrt{\alpha^2 + \omega_d^2}} e^{-\alpha t} \left[\sin\left(\omega_d t - \tan^{-1}\left(-\frac{\omega_d}{\alpha}\right)\right) \right] \sin\frac{\pi x}{L}$$

Actuation Effort – Voltage or Current:

$$\mathrm{E(t)} = \frac{f\omega_d}{\sqrt{\alpha^2 + \omega_d^2}} \frac{d}{dt} \left[e^{-\alpha t} \sin\left(\omega_d t - \tan^{-1}\left(-\frac{\omega_d}{\alpha}\right)\right) \right] \sin\frac{\pi x}{L}$$

Actuation Stress:

$$\sigma_a = \frac{C\omega_d}{\sqrt{\alpha^2 + \omega_d^2}} \frac{d}{dt} \left[e^{-\alpha t} \sin\left(\omega_d t - \tan^{-1}\left(\frac{\omega_d}{\alpha}\right)\right) \right] \sin\frac{\pi x}{L}$$

The spatial variation for the actuation stress and the applied voltage is the same as that of deflection. However, the variation with respect to time is different for deflection and application of electric / magnetic field.

The present solution is although found out for the first mode of vibration of a simply supported beam, it should be noted that the controller is not designed individually for each mode.

It is shown through the solution of transverse deflection *w*, that the presence of constant gain velocity feedback can suppress any mode of vibration and hence the overall vibration of the beam.

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Thus, the problem of 'spill-over' etc. does not arise here. The general solution of deflection shows that the damping effect of smart actuator is implicitly incorporated through the term α.

$$\alpha = \frac{\beta f}{2m}$$

By controlling the control gain and thereby the term β, one can achieve the required damping. The presence of smart layer also affects the natural frequency of the beam through both damping coefficient ξ and material property constant D_n.

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- Krishna Murty, A. V., Anjanappa, M., Wu, Y., Bhattacharya, B. and Bhat, M.S. (1998): Vibration Suppression in Laminated Composite Beams Using Embedded Magnetostrictive Layers. **Journal of the Institution of Engineers (India)**, Vol 78, 38-44.

- Bhattacharya, B., Krishna Murty, A. V., Bhat, M. S. and Anjanappa, M. (1996): Vibration Suppression in Slender Composite Beams Using Magneto-strictive Actuation, **Journal of Aeronautical Society of India**, Vol.48, No. 2.

END OF LECTURE 34