Module 2: External Flows

Lecture 12: Flow Over Curved Surfaces

The Lecture Contains:

- Description of Flow past a Circular Cylinder
- Experimental Results for Circular Cylinder Flow

Previous Next

Description of Flow past a Circular Cylinder :-

Let us start with a consideration of the creeping flow around a circular cylinder, characterized by Re < 1.**(Here we shall define Re = U∞d/v, based on the upstream velocity and the cylinder diameter)** Vorticity is generated close to the surface because of the no slip boundary condition. In the Stokes approximation this vorticity is simply diffused, not advected, which results in a fore and aft symmetry.

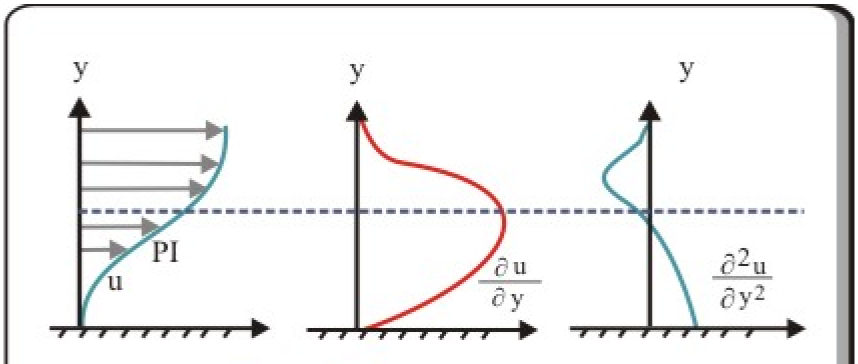

Figure 2.17: Profiles dp /d x > 0

As Re is increased beyond1,the vorticity is increasingly confined behind the cylinder because of advection.For **Re > 4** two small attached or **"standing"** eddies appear behind the cylinder.The wake is completely laminar and the vortices act like **"rollers"** over which the main stream flows **(Figure 2.18)**. The eddies get larger as Re in increased. A very

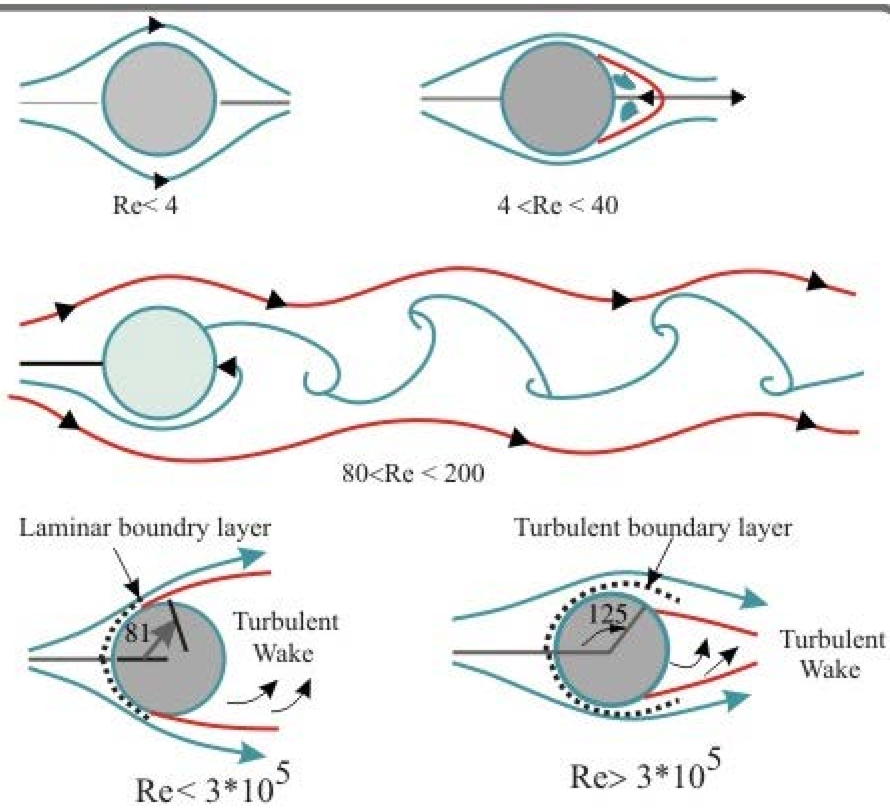

Figure 2.18: Influence of Reynolds number on the wake of a circular cylinder

interesting sequence of events begins to develop when the Reynolds number is increased beyond 40, at which point the wake behind the cylinder becomes unstable.Photographs show that the wake develops a slow oscillation in which the velocity is periodic in time and downstream distance, with the amplitude of the oscillation increasing downstream.The oscillating wake rolls up into two staggered rows of vortices with opposite sense of rotation.**Karman investigated the phenomena as a problem of superposition of irrotational vortices.** He concluded that a nonstaggered row of vortices is unstable, and a staggered row is stable only if the ratio of lateral distance between the vortices to their longitudinal distance is 0.28. Because of the similarity of the wake with foot prints in a street, **the staggered row of vortices behind a blunt body is called a von Karman vortex street**. The vortices move downstream at a speed smaller than the upstream velocity U.This means that the vortex pattern slowly follows the cylinder if it is pulled through a stationary fluid.

Previous Next

In the range $40 < Re < 80$, the vortex street does not interact with the pair of attached vortices. As Re is increased beyond 80, the vortex street advances closer to the cylinder, and the attached eddies **(whose downstream length has now grown to be about twice the diameter of the cylinder)** themselves begin to oscillate. Finally the attached eddies periodically break off alternately from the two sides of the cylinder. While an eddy on one side is shed, that on the other side forms, resulting in an unsteady flow near the cylinder.As vortices of opposite circulations are shed off alternately from the two sides, the circulation around the cylinder changes sign, resulting in an oscillating **"lift"** or lateral force. If the frequency of vortex shedding is close to the natural frequency of some mode of vibration of the cylinder body, then an appreciable lateral vibration has been observed to result. Engineering structures such as suspension bridges and oil drilling platforms are designed so as to break up a coherent shedding of vortices from the cylindrical structures.**This is done by including spiral blades protruding out the cylinder surface, which break up the spanwise coherence of shedding.**

The passage of regular vortices causes velocity in the wake to have a dominant periodicity.**The frequency n is expressed as a nondimensional parameter known as the Strouhal number,defined as.**

$$S \equiv \frac{nd}{U} \qquad \textbf{(2.139)}$$

Experiments show that for a circular cylinder the value of S remains close to 0.21 for a large range of Reynolds numbers.For small values of cylinder diameter and moderate values of U,the resulting frequencies of the vortex shedding and oscillating lift lie in the acoustic range. **For example, at U = 10 m/ s and a wire diameter of 2 mm, the frequency corresponding to a Strouhal number of 0.21 is n = 1050 cycles per second.The "singing"** of telephone and transmission lines has been attributed to this phenomena.

Previous Next

Experimental Results for Circular Cylinder Flow

In order to understand the heat transfer results for the case of ow over a heated circular cylinder, refer to **Figure 2.19**.Consider the results for $Re_D < 10^5$ (Figure 2.19). Starting at the stagnation point, Nu_θ decreases with increasing θ as a result of laminar boundary layer development. However, a minimum is reached at $\theta = 80^o$ At this point separation occurs, and Nu_θ increases with θ because of the mixing associated with the vortex formation in the wake. In contrast, for ReD > 105 the variation of $Re_D < 10^5$ with θ characterized by two minima. The decline in Nu θ from the value at the stagnation point.

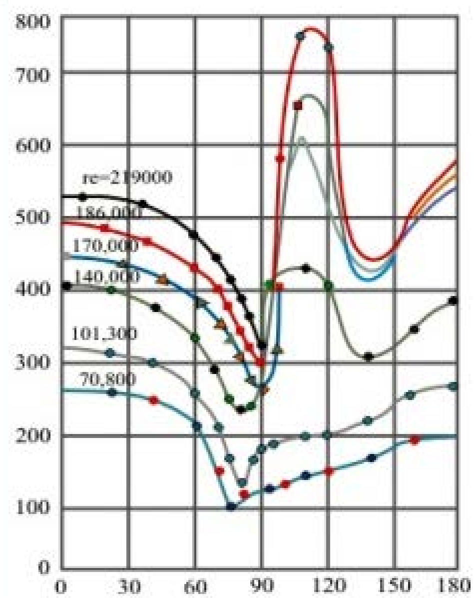

Figure 2.19:

Local Nusselt number distribution for flow over a circular cylinder

is again due to laminar boundary layer development, but the sharp increase that occurs between 80^o and 100^o is now due to boundary layer transition to turbulence. With further development of the turbulent boundary layer, Nuθ again begins to decline. Eventually turbulent boundary layer separation occurs at $\theta \approx 140^o$ and than Nuθ increase as a result of considerable mixing associated with wake region.Over all average condition.

$$\overline{Nu}_D = \frac{\overline{h}D}{k} = CRe^m\ Pr^{1/3} \qquad \textbf{(due to R.Hilpert)} \qquad \textbf{(2.140)}$$

$$\overline{Nu}_D = CRe^m \left(\frac{Pr_\infty}{Pr_w}\right) \qquad \textbf{(by A. Zhukauskas)} \qquad \textbf{(2.141)}$$

Re	C	m
1 - 40	0.75	0.4
40 - 1000	0.51	0.5
$10^3 - 2 \times 10^5$	0.26	0.6
2×10^6 - 10^6	0.076	0.7

Table 2.3

- All properties are evaluated at except T_∞ except Pr_w which is evaluated at T_w.
- C and m are given in Table 2.3
- For $Pr < 10,\ n = 0.37$ and for $Pr > 10 - n = 0.36$

2.12 Some Other Correlations :-

For small Prandtl number fluids **(liquid metals)** the local heat transfer coefficient on flat plate is given by

$$Nu_x = 0.565\ Pe_x^{1/2}, \quad \text{for} \quad Pe_x \leq 0.01 \qquad \textbf{(2.142)}$$

From experiments **(see Schlichting)** the local friction coefficients for turbulent flow over flat plate is given by.

$$C_{f\ x} = 0.0592\ Re_x^{-1/5} \qquad 5 \times 10^5 < Re_x < 10^7 \qquad \textbf{(2.143)}$$

To a reasonable approximation, the velocity boundary layer thickness for turbulent flow over a flat plate may be **expressed as:**

$$\frac{\delta}{x} = \frac{0.37}{(Re_x)^{-1/5}} \qquad \textbf{(2.144)}$$

(Observe that δ varies as $x^{4/5}$ in contrast to $x^{1/2}$ for laminar flow) **The local Nusselt number for turbulent flow over a flat plate is:**

$$Nu_x = 0.0296\ Re_x^{4/5}\ Pr^{1/3} \qquad \textbf{(2.145)}$$

In **mixed boundary layer** situation the average heat transfer coefficient for the flow over.

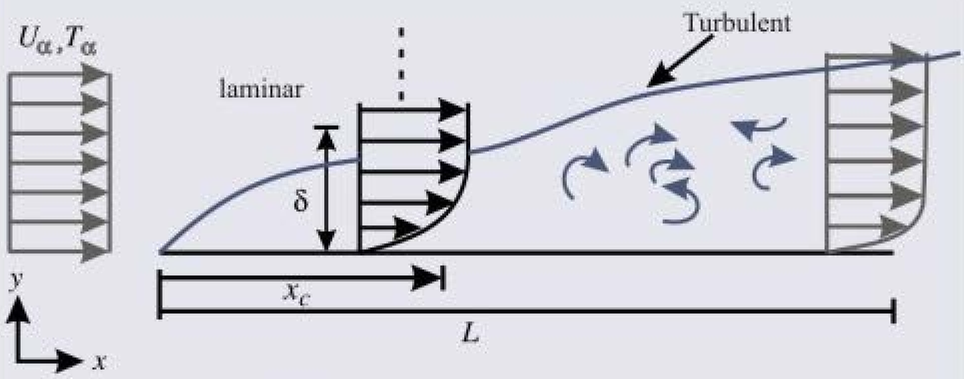

Figure 2.20: Transitional flow over a flat plate

A flat plate **(Figure 2.20)** is given by:

$$\overline{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{lam}\, dx + \int_{x_c}^{L} h_{turb}\, dx\right) \qquad \textbf{(2.146)}$$

or ,

$$\overline{h}_L = \frac{k}{L}\left[0.332\left(\frac{U_\infty}{\nu}\right)^{1/2}\int_0^{x_c}\frac{dx}{x^{1/2}} + 0.0296\left(\frac{U_\infty}{\nu}\right)^{4/5}\int_{x_c}^{L}\frac{dx}{x^{1/5}}\right] Pr^{1/3}$$

or

$$\overline{Nu}_L = \left[0.664\ Re_{x,c}^{1/2} + 0.037\ \left(Re_L^{4/5} - Re_{x,c}^{4/5}\right)\right]\ Pr^{1/3} \qquad \textbf{(2.147)}$$

If the typical,transition Reynolds number is $Re_{x,c} = 5\times 10^5$

$$\overline{Nu}_L = \left(0.037\ Re_L^{4/5} - 871\right)\ Pr^{1/3} \qquad \textbf{(2.148)}$$

$$\overline{C}_{f,L} = \frac{0.074}{Re_L^{1/5}} - \frac{A}{Re_L} \qquad \textbf{(2.149)}$$

- Where **A=1742**

If $L >> x_c \quad (Re_L >> Re_{x,c})$

$$\overline{Nu}_L = 0.037\ Re_L^{4/5}\ Pr^{1/3} \quad \text{and} \quad \overline{C}_{f,L} = 0.074\ Re_L^{-1/5}$$

We know that the fluid properties vary with temperature across the boundary layer and that this variation can certainly influence heat transfer. This influence may be handled by one of the following two ways:In one method all the properties evaluated at a mean boundary layer temperature T_f, termed the **FILM TEMPERATURE.**

$$T_f = \frac{T_w + T_\infty}{2}$$

In the alternate method, evaluate all the properties at T_∞ and multiply the right hand sideof the expression for Nusselt number by an additional parameter. This parameter is commonly of the form $(Pr_\infty/Pr_w)^r$ or $(\mu_\infty/\mu_w)^r$where *w* **designate properties at surface temperature**

Previous Next