The Lecture Contains:

Important Definitions :

The boundary layer thickness for laminar flow past a flat plate $\equiv \delta = 5.0x/\sqrt{Re_x}$

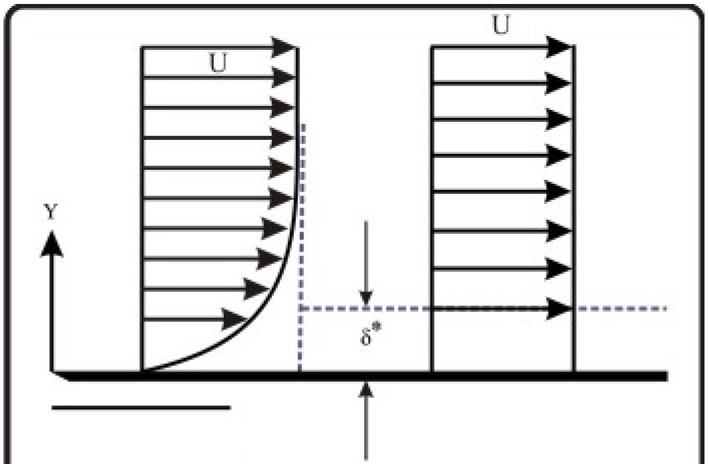

Figure 1.7: Displacement Thickness

Displacement thickness is the distance by which the external potential flow is displaced outwards as a consequence of decrease in velocity in the boundary layer **(Figure 1.7)**

$$U\delta^* = \int_0^\infty (U-u)dy \qquad \textbf{(1.116)}$$

$$\delta^* = \int_0^\infty \left(1-\frac{u}{U}\right)dy \qquad \textbf{(1.117)}$$

Momentum thickness, δ^{**} is defined as the loss of momentum in the boundary layer as compared with that of potential flow:

$$\rho U^2 \delta^{**} = \int_0^\infty \rho u (U - u) dy \quad \textbf{(1.118)}$$

$$\delta^{**} = \int_0^\infty \left(1 - \frac{u}{U}\right) \frac{u}{U} dy \quad \textbf{(1.119)}$$

Laminar Flow : Fluid motion is well ordered, layers slide over each other :

Turbulent Flow : Fluid motion is highly irregular and characterized by velocity fluctuations **(Figure 1.8)**

The critical Reynolds number for external flows is $Re_c = \rho U x_c/\mu \approx \ 5 \times 10^5$

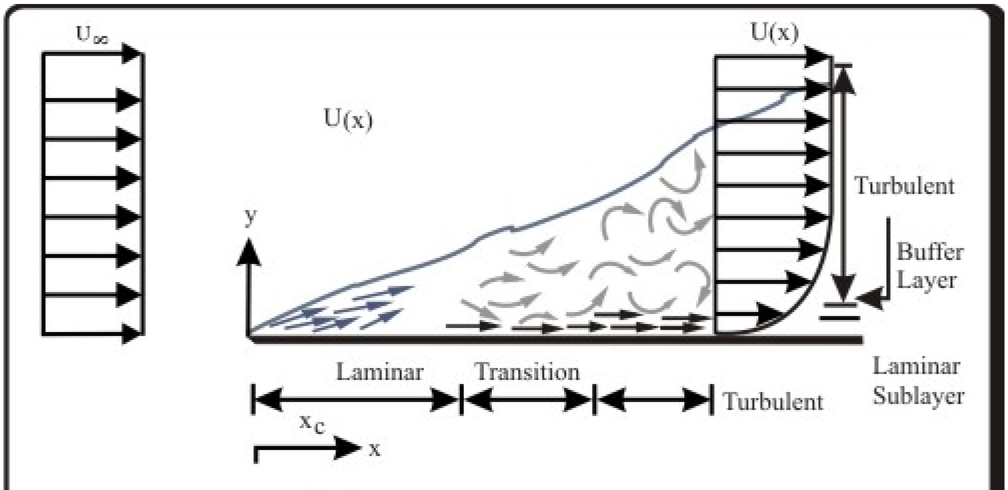

Figure 1.8: Transition in a Flat Plate Flow

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Prandtl number and Ratio of Boundary Layers

Let us consider a case for $\delta_T > \delta$ **(see Figure 1.9).** From the thermal boundary layer consideration, a scale analysis within the boundary layer gives **:**

$$\begin{aligned} u\frac{\partial T}{\partial x} &\sim \alpha\frac{\partial^2 T}{\partial y^2} \\ \text{or, } U_\infty\frac{\Delta T}{L} &\sim \alpha\frac{\Delta T}{(\delta_T)^2} \\ \text{or, } \left(\frac{\delta_T}{L}\right)^2 &\sim \frac{\alpha}{U_\infty L} \\ \text{or, } \left(\frac{\delta_T}{L}\right)^2 &\sim \frac{\nu}{U_\infty L}\cdot\frac{\alpha}{\nu} \end{aligned} \qquad \textbf{(1.120)}$$

From the velocity boundary layer consideration, a scale analysis within the boundary layer gives **:**

$$\begin{aligned} u\frac{\partial u}{\partial x} &\sim \nu\frac{\partial^2 u}{\partial y^2} \\ \text{or, } U_\infty\frac{U_\infty}{L} &\sim \nu\frac{U_\infty}{\delta^2} \\ \text{or, } \left(\frac{\delta}{L}\right)^2 &\sim \frac{\nu}{U_\infty L} \end{aligned} \qquad \textbf{(1.121)}$$

From equations **(1.120) and (1.121)** we get **:**

$$\begin{aligned} \left(\frac{\delta_T}{L}\right)^2 &\sim \left(\frac{\delta}{L}\right)^2\cdot\frac{\alpha}{\nu} \\ \text{or, } \left(\frac{\delta}{\delta_T}\right)^2 &\sim Pr \\ \text{or, } (Pr)^{\frac{1}{2}} &\sim \left(\frac{\delta}{\delta_T}\right) \end{aligned} \qquad \textbf{(1.122)}$$

Through a similar analysis, it can be shown that **:**

$$(Pr)^{\frac{1}{3}} \sim \left(\frac{\delta}{\delta_T}\right) \quad \text{for} \quad \delta > \delta_T \qquad \textbf{(1.123)}$$

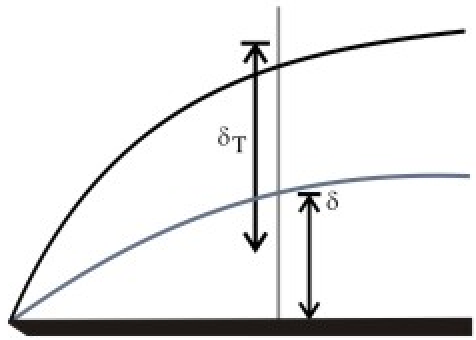

Figure 1.9: The situation

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