Module 2: External Flows

Lecture 10: Heat Transfer Reversal

The Lecture Contains:

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Viscous Dissipation Effects on Laminar Boundary Layer Flow Over a Flat Plate

For the high speed flows, the effects due to viscous dissipation have to be considered, Now $u = U\ f'(\eta)$ and $\eta = y/\sqrt{(\nu x)/U}$, and so the viscous dissipation term.

$$\mu\left[\left(\frac{\partial u}{\partial y}\right)^2 + ...\right]$$

$$= \mu\left[\left(\frac{\partial u}{\partial \eta}\frac{\partial \eta}{\partial y}\right)^2 + ...\right]$$

$$= \mu\left[\left(U\ f''\sqrt{\frac{U}{\nu x}}\right)^2 + ...\right]$$

The general energy equation is simplified as

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left[\frac{\partial^2 T}{\partial y^2}\right] + \frac{\mu}{\rho c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2 + ...\right] \qquad \textbf{(2.104)}$$

$$U\ f'\left[\theta\frac{dT_w}{dx} + (T_w - T_\infty)\left(\frac{\partial \theta}{\partial \eta}\right)\left(-\frac{y}{2x}\right)\sqrt{\frac{U}{\nu x}}\right]$$

$$-\frac{1}{2}\sqrt{\frac{\nu\ U}{x}}(f - \eta f')\left\{(T_w - T_\infty)\frac{\partial \theta}{\partial \eta}\sqrt{\frac{U}{\nu x}}\right\}$$

$$= \frac{k}{\rho c_p}\frac{\partial^2 \theta}{\partial \eta^2}(T_w - T_\infty)\frac{U}{\nu x} + \frac{\mu}{\rho c_p}U^2(f'')^2\frac{U}{\nu x}$$

Multiply by $\frac{x}{U(T_w - T_\infty)}$ to obtain :

$$\frac{f'\theta x}{(T_w - T_\infty)}\frac{dT_w}{dx} - \frac{f\theta'}{2} = \frac{\theta''}{Pr} + Ec(f'')^2 \qquad \textbf{(2.105)}$$

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If we assume $(dT_w/dx)0$, **the General equation becomes**

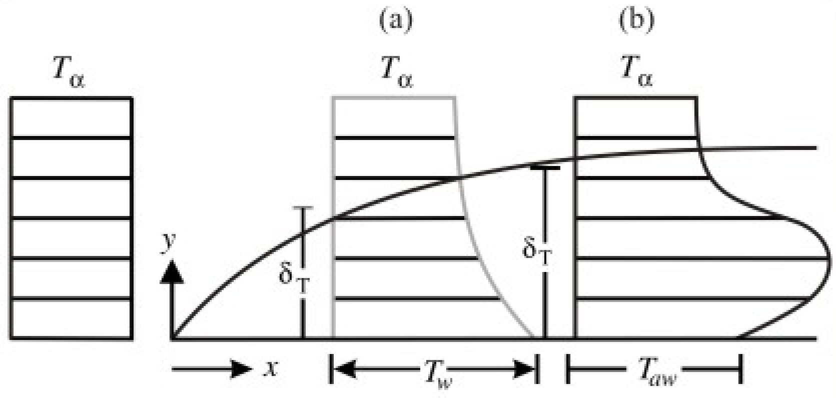

Figure 2.7:
Temperature profile within the boundary layer
(a) for low speed flows (b) for high speed flows

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$$\frac{\theta''}{Pr} + f\frac{\theta'}{2} + Ec(f'')^2 = 0 \qquad \textbf{(2.106)}$$

As defined earlier, $\varnothing = \dfrac{T - T_\infty}{T_\omega - T_\infty}$ and the boundary conditions are $\theta(0) = 1,\ \theta(\infty) = 0$ with $T_w =$ wall temperature. If the heat flux at the wall is assumed to be zero, i.e., $q_w = 0$, the wall temperature becomes adiabatic wall temperature. T_{aw} In other words, $T_{aw} =$ Adiabatic wall temperature, if the surface temperature is the temperature of a perfectly insulated surface. As a consequence of the dissipation in the boundary layer, even if there is no heat transfer to a body in a flow, a thermal boundary layer forms at the body. If the surface of the body is impermeable to heat, i.e., adiabatic, the dissipation means that the distribution of the wall temperature is such that it is above the surrounding temperature.For the temperature field at an adiabatic body, if $\phi \sim \frac{\theta}{Ec/2}$, the general equation reduces to the form :

$$\frac{Ec}{2}\frac{1}{Pr}\phi'' + \frac{Ec}{2}\frac{f}{2}\phi' + 2Ec(f'')^2 = 0 \qquad \textbf{(2.107)}$$

with the boundary conditions $\phi'(0) = 0, \phi(\infty) = 0$. As defined.

$$\phi \sim \frac{\theta}{Ec/2} = \frac{T - T_\infty}{T_w - T_\infty}\frac{2c_p(T_w - T_\infty)}{U^2} = \frac{T - T_\infty}{U^2/2c_p}$$

Thus we get,

(2.108)

$$T = T_\infty + \frac{U^2}{2c_p}\phi(\eta)$$

Which becomes :

$$T_{aw} = T_\infty + \frac{U^2}{2c_p}\phi(0) \qquad \textbf{(2.109)}$$

T_{aw} can be determined if **φ(0)** is known, where **φ(0)** is also called recovery factor, r and

$$T_{aw} = T_\infty + r\frac{U^2}{2c_p} \text{ to find } \emptyset\,(0) \text{ we use } \emptyset(\mathrm{n}) = \frac{\emptyset\,(\mathrm{n})}{\mathrm{E}_{\frac{c}{2}}} \text{ which means } \emptyset\,(0) = \frac{\emptyset\,(0)}{\mathrm{E}_{\frac{c}{2}}} = \frac{1}{\mathrm{E}_{\frac{c}{2}}}$$

$\phi(0)$ is found from experiments:

$$\phi(0) = \mathrm{fn}(Pr), \quad \phi(0) = \sqrt{Pr} \quad \text{(see Figure 2.8)}$$

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For the low speed flows $\phi(0) = r = 0$ as no kinetic energy is recovered as thermal energy.

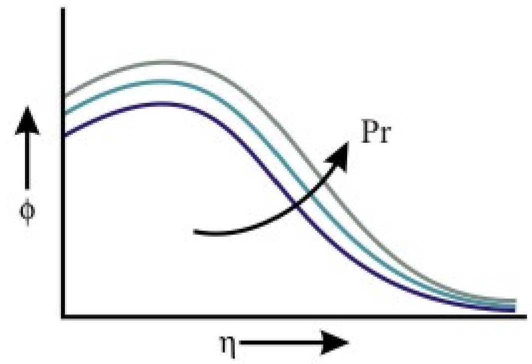

Figure 2.8: φ as a function of Pr and η

For ideal gases: i.e., i = e + p v = e + RT
or,

$$\frac{di}{dT} = \frac{de}{dT} + R$$

or,

$$c_p = c_v + R$$

which gives on simplification

$$c_p = \frac{\gamma R}{\gamma - 1}$$

Now ,

$$T_{aw} = T_\infty\left[1 + \frac{rU^2(\gamma-1)}{2\gamma R T_\infty}\right] \qquad \textbf{(2.110)}$$

i.e,

$$T_{aw} = T_\infty\left[1 + \frac{r(\gamma-1)}{2}M_\infty^2\right] \qquad \textbf{(2.111)}$$

For air, $Pr = 0.72,\ \phi(0) = \sqrt{0.72} = \ 0.85 = r$.Putting $\gamma = 1.4$ we get $T_{aw} = T_\infty[1 + 0.17M_\infty^2]$

Consider the general solution for **:**

$$\frac{\theta''}{Pr} + \frac{1}{2}f\theta' + Ec(f'')^2 = 0$$

with $\theta(0) = 1,\quad \theta_\infty = 0$

Now, let us propose **:** $\theta \ =[$

$$\theta(\eta) \sim \frac{E_c}{2}\phi(\eta)$$

at **n=0** we get ,

$$\theta(0) \sim \frac{E_c}{2}\phi(0)$$

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$$[\theta(\eta) - \theta(0)] \sim \left[\frac{E_c}{2}\phi(\eta) - \frac{E_c}{2}\phi(0)\right]$$

$$\theta(\eta) \sim \left[1 - \frac{E_c}{2}\phi(0)\right] + \frac{E_c}{2}\phi(\eta)$$

$$\theta = \left[1 - \frac{Ec}{2}\phi(0)\right]\tilde{\theta} + \frac{1}{2}Ec\ \phi(\eta)$$

The derivatives terms can be written :

$$\theta'' = \left[1 - \frac{Ec}{2}\phi(0)\right]\tilde{\theta}'' + \frac{1}{2}Ec\ \phi''(\eta) \qquad \textbf{(2.113)}$$

$$\theta' = \left[1 - \frac{Ec}{2}\phi(0)\right]\tilde{\theta}' + \frac{1}{2}Ec\ \phi'(\eta) \qquad \textbf{(2.114)}$$

On substitution general equation we get :

$$\left[1 - \frac{Ec}{2}\phi(0)\right]\frac{\tilde{\theta}''}{Pr} + \frac{1}{2}f\left[1 - \frac{Ec}{2}\phi(0)\right]\tilde{\theta}' + \frac{1}{2}Ec\left[\frac{\phi''(\eta)}{Pr} + \frac{f}{2}\phi'(\eta)\right] + Ec(f'')^2 = 0$$

or,

$$\left[1 - \frac{Ec}{2}\phi(0)\right]\left[\frac{\tilde{\theta}''}{Pr} + \frac{1}{2}f\tilde{\theta}'\right] + \frac{Ec}{2}\left[2(f'')^2 + \frac{\phi''}{Pr} + \frac{f}{2}\phi'\right] = 0$$

we know

$$\frac{\phi''}{Pr} + \frac{f}{2}\phi' + 2(f'')^2 = 0$$

with $\phi'(0) = 0$ and $\phi(\infty) = 0$ After simplification and reduction we get,

$$\frac{\tilde{\theta}''}{Pr} + \frac{1}{2}f\tilde{\theta}' = 0 \qquad \textbf{(2.115)}$$

Known conditions are θ(0) = 1and θ(∞) = 0.Let us revisit the relationship between θ and ˜θ, equation (2.112). We shall obtain the following θ(0) = 1 means ˜θ (0) = 1; and θ (∞) = 0 means ˜θ(∞) = 0 [since φ(∞) = 0] The solution of this equation that results finally is of the form.

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$$\tilde{\theta}(\eta) = \frac{\int_0^\eta exp\left(-\frac{Pr}{2}\int_0^\eta f d\eta\right) d\eta}{\int_0^\infty exp\left(-\frac{Pr}{2}\int_0^\eta f(\eta)d\eta\right) d\eta} \quad \textbf{(2.116)}$$

Now the heat transfer for the wall :

$$q_w = -k\frac{\partial T}{\partial y} = -k(T_w - T_\infty)\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = -k(T_w - T_\infty)\frac{\partial \theta}{\partial \eta}\left.\frac{\partial \eta}{\partial y}\right|_{\eta=0}$$

$$q_w = -k(T_w - T_\infty)\sqrt{\frac{U}{\nu x}}\theta'(0) \quad \textbf{(2.117)}$$

since $\phi'(0) = 0$, we have $\theta'(0) = \left[1 - \frac{Ec}{2}\phi(0)\right]\tilde{\theta}'(0)$ Substituting $\theta'(0)$ and Ec, we get

$$q_w = -k(T_w - T_\infty)\sqrt{\frac{U}{\nu x}}\left[1 - \frac{U^2}{2c_p(T_w - T_\infty)}\phi(0)\right]\tilde{\theta}'(0)$$

After rearranging and replacing appropriately using value of T_{aw} from before, we get

$$\frac{q_w}{(T_w - T_{aw})}\frac{x}{k} = -\tilde{\theta}'\sqrt{Re_x} = Nu_x \quad \textbf{(2.118)}$$

This expression is same as for the case without viscous dissipation, but the only differenceis that here we have $(T_w - T_{aw})$ in place of $(T_w - T_\infty)$