Module 2: External Flows

Lecture 8: Numerical Solution of Flow and Heat Transfer on a Flat Plate

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- Devlop a computer code to calculate Thermal Boundry Layer

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Devlop a computer code to calculate thermal boundry layer

The computer program can be devloped in the following way. Assumes a slightly different temperature profile. Because the wall temperature is uniform in the situation being considered, it is logical to assume that the temperature profiles are similar in the same sense as the velocity profiles.

For this reason, the following dimensionless temperature is may be introduced.

$$\theta = \frac{T_w - T}{T_w - T_\infty} \qquad \textbf{(2.58)}$$

The assumption that the temperature profiles are similar is equivalent to assuming that θ depends only on the similarity variables, η. Since both T_w and T_∞ are constant, the energy equation can be **written in terms of θ as**

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad \textbf{(2.59)}$$

The boundary conditions are **:**

$$\begin{aligned} y = 0 &: \theta = 0 \\ y \to \infty &: \theta \to 1 \end{aligned} \qquad \textbf{(2.60)}$$

Because the temperature profiles are being assumed to be similar, i.e., it is being assumed that θ is a function of η, Tthe relations for the velocity components have been previously derived, **Eqn. (2.59) gives**

$$f'\frac{d\theta}{d\eta}\frac{\partial \eta}{\partial x} + \left[\frac{1}{2}\sqrt{\frac{\nu}{U_\infty x}}(\eta f' - f)\right]\frac{d\theta}{d\eta}\frac{\partial \eta}{\partial y} = \frac{1}{Pr}\frac{\nu}{U_\infty}\frac{d^2\theta}{d\eta^2}\left[\frac{\partial \eta}{\partial y}\right]^2 \qquad \textbf{(2.61)}$$

On rearrangement this equation becomes**:**

$$\theta'' + \frac{Pr}{2}\theta' f = 0 \qquad \textbf{(2.62)}$$

While boundary conditions on the solution given in **eqn. (2.60)** can be written as:

$$\eta = 0 : \theta = 0$$
$$\eta \rightarrow \infty : \theta \rightarrow 1 \qquad \textbf{(2.63)}$$

The partial differential equation governing the temperature distribution has been reduced to an ordinary differential equation. This confirms the assumption that the temperature profiles are similar. A computer program that implements the above procedures for obtaining the similarity profiles for velocity and temperature can be devloped in this program, the solution to the set of equations defining the velocity profile function is obtained using the basic Runge-Kutta procedure. Some typical variation of θ with η for various values of **Pr (0.7, 1.0, 3.0, 6.5)** can be obtained using this may be plotted. The curve for a Prandtl number of 1 is identical to that giving the variation of *f* with η. This is the check.

The heat transfer rate at the wall is given by **:**

$$q_w = -k \cdot \frac{\partial T}{\partial y}\Big|_{y=0} \qquad \textbf{(2.64)}$$

Hence **,**

$$\frac{q_w}{k(T_w - T_\infty)} = \frac{\partial \theta}{\partial y}\Big|_{y=0} = \frac{\partial \theta}{\partial \eta}\frac{\partial \eta}{\partial y}\Big|_{y=0} \qquad \textbf{(2.65)}$$

i.e,

$$\frac{q_w x}{k(T_w - T_\infty)} = \theta'\big|_{\eta=0}\sqrt{Re_x} \qquad \textbf{(2.66)}$$

i.e,

$$Nu_x = \theta'\big|_{\eta=0}\sqrt{Re_x} \qquad \textbf{(2.67)}$$

Nu_x and Re_x, are, **the local Nusselt and Reynolds numbers** based on **x**. Because θ depends only on η for a given $Pr, \theta'|_{\eta=0}$ depends only Pr and its values can be obtained from the solution for the variation θ with η for any value of Pr. It is convenient to define.

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$$A(Pr) = \theta'|_{\eta=0} \quad \textbf{(2.68)}$$

In terms of this function A, **eqn (2.67)** can be written as :

$$Nu_x = A\sqrt{Re_x} \quad \textbf{(2.69)}$$

Values of A for various values of Pr are shown in Table 2.2.

Values of A for various of pr

pr	$\mathrm{A} = \theta'\|_{\eta=0}$	$0.332\mathrm{Pr}^{1/3}$
0.6	0.276	0.280
0.8	0.307	0.308
0.9	0.320	0.321
1.0	0.332	0.332
1.1	0.344	0.343
7.0	0.645	0.635
10.0	0.730	0.715
15.0	0.835	0.819

Over the range of Prandtl numbers covered in the table, it has been found that A varies very nearly as $Pr^{1/3}$ and, as will be seen from the results given in the table, is quite closely represented by the approximate relation.

$$A = 0.332Pr^{1/3}$$

Now, in practical situations, the concern is more likely to be with the total heat transfer rate from the entire surface than with the local heat transfer rate. Consideration is, therefore, now given to the total heat transfer rate from a plate of length,L.Unit width of the plate is considered because the flow is, by assumption, two-dimensional. The total heat transfer rate per unit width Qw will, of course, be related to the local heat transfer rate, q_w by:

$$Q_w = \int_0^L q_w dx \quad \textbf{(2.71)}$$

But **eqn. (2.66)** gives the local heat transfer rate as:

$$q_w = Ak(T_w - T_\infty)\sqrt{\frac{U_\infty}{x\nu}} \quad \textbf{(2.72)}$$

Substituting this result **eqn. (2.71)** then gives on carrying the integration :

$$Q_w = 2Ak(T_w - T_\infty)\sqrt{\frac{UL}{\nu}} \qquad \textbf{(2.73)}$$

If a mean heat transfer coefficient for the whole plate, is defined such that:

$$\bar{h} = \frac{Q_w}{L(T_w - T_\infty)} \qquad \textbf{(2.74)}$$

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Then, since unit width of the plate is being considered, **eqn. (2.73)** gives:

$$\bar{h} = \frac{2Ak}{L}\sqrt{\frac{U_\infty L}{\nu}} \tag{2.75}$$

Now, consider the case where:

$$\begin{aligned} T_w &= T_\infty + Cx^\lambda \\ T_w - T_\infty &= Cx^\lambda \end{aligned} \tag{2.76}$$

C and λ being constant :

The following dimensionless temperature is again introduced:

$$\theta = \frac{T_w - T}{T_w - T_\infty} = 1 - \frac{T - T_\infty}{T_w - T_\infty} \tag{2.77}$$

And it again assumed that θ depends only on the similarity variable, η . In the present case is an function of x because the energy equation can be written in terms of θ as :

$$-u\frac{\partial}{\partial x}[(1-\theta)(T_w - T_\infty)] + v\frac{\partial \theta}{\partial y}(T_w - T_\infty) = \left[\frac{\nu}{Pr}\right]\frac{\partial^2\theta}{\partial y^2}(T_w - T_\infty) \tag{2.78}$$

In terms of θ, the boundary conditions again are:

$$\begin{aligned} y = 0 &: \theta = 0 \\ y \to \infty &: \theta \to 1 \end{aligned} \tag{2.79}$$

Using the relations for the velocity components previously derived, **eqn. (2.78)** gives:

$$f'\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial x} - \frac{\lambda f'(1-\theta)}{x} + \left[\frac{1}{2}\sqrt{\frac{v}{U_\infty x}}(\eta f' - f)\right]\frac{d\theta}{d\eta}\frac{\partial\eta}{\partial y} = \frac{1}{Pr}\frac{\nu}{U_\infty}\frac{d^2\theta}{d\eta^2}\left[\frac{\partial\eta}{\partial y}\right]^2 \tag{2.80}$$

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On rearrangement, this equation becomes:

$$\theta'' + \lambda Pr f'(1-\theta) + \frac{Pr}{2}\theta' f = 0 \quad \textbf{(2.81)}$$

While the boundary conditions on the solution given in **eqn. (2.79)** can be written as:

$$\begin{array}{c} \eta = 0 : \theta = 0 \\ \eta \to \infty : \theta \to 1 \end{array} \quad \textbf{(2.82)}$$

Thus,as was the case with uniform plate temperate, the partial diffrential equation governing the temperate distribution has been reduced to an ordinary diffrential equation.This confirms the assumption that the temperature profiles are similar.For any prescribed values of $Pr \text{ and } \lambda$, the variation of θ with η can be obtained by solving eqn. (2.81). A computer program, **FLAT**, which is an extension of that for the uniform temperature surface case obtains this solution.The program, first obtains the velocity profile solution and then uses the same procedure to solve eqn. (2.81). The program is available in the way discussed in the text. **The heat transfer rate at the wall is as before given by :**

$$q_w = -k\frac{\partial T}{\partial y}|_{y=0} \quad \textbf{(2.83)}$$

Hence,**using (2.77),** it again follows that:

$$\frac{q_w x}{k(T_w - T_\infty)} = \theta'|_{\eta=0}\sqrt{Re_x} \quad \textbf{(2.84)}$$

For any prescribed values of $Pr \text{ and } \lambda, \theta'|_{\eta=0}$ will have a specific values. It therefore follows that qw will be proportional to $(T_w - T_\infty)/x^{0.5}$ Hence, the case where the heat ux at the surface of the plate is uniform corresponds to the case where $\lambda = 0.5,$ a similarity solution exists for flow over a at plate with a uniform surface heat ux. Some typical variations of $\theta'|_{\eta=0}$ with for various values of *Pr* can be obtained using the above program.