Module 6: Free Convections

Lecture 27: Combined Convection

The Lecture Contains:

Mixed convection

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Mixed convection

The reference temperature be T_R and reference pressure P_R.

Thermal expansion number $= K_\rho = -\beta_R T_R$

Archimedes number $= \frac{K_\rho(1-T_w^*)}{F_R^2} = Ar$

Froude number $= \frac{V}{\sqrt{gl}} = F_R$

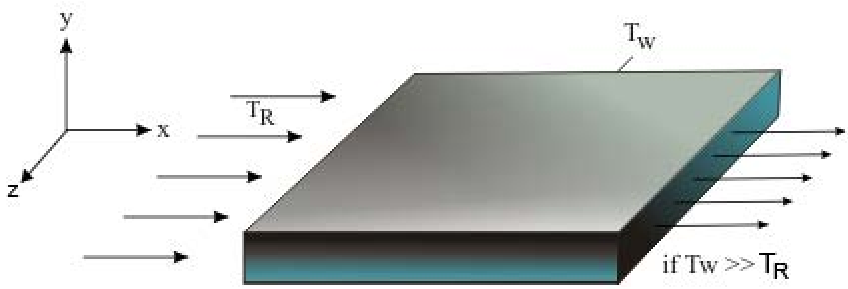

Figure 6.3: Buoyancy induced flow through horizontal channel

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$$Ar = \frac{\beta_R(T_w - T_R)gl}{V^2}$$

$$Gr = Ar * Re^2 = \frac{\beta_R(T_w - T_R)gl}{V^2}\frac{V^2 l^2}{\nu^2}$$

$$= \frac{\beta_R(T_w - T_R)gl^3}{\nu^2}$$

Figure 6.3 shows buoyancy induced flow through a horizontal channel.

The following are the governing equations

Continuity

$$\nabla . u_i = 0$$

x momentum:

$$\frac{\partial u}{\partial t} + (\mathbf{V}.\nabla)u = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

y momentum:

$$\frac{\partial v}{\partial t} + (\mathbf{V}.\nabla)v = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v + g\beta(T - T_\infty)$$

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z momentum:

$$\frac{\partial w}{\partial t} + (\mathbf{V}.\nabla)w = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w$$

Energy:

$$\frac{\partial T}{\partial t} + (\mathbf{V}.\nabla)T = \alpha[\nabla^2 T]$$

After non-dimensionalization:

$$\nabla \cdot u_i = 0$$

$$\frac{Du}{Dt} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\nabla^2 u$$

$$\frac{Dv}{Dt} = -\frac{\partial p}{\partial y} + \frac{1}{Re}\nabla^2 v + \frac{Gr}{Re^2}\theta$$

$$\frac{Dw}{Dt} = -\frac{\partial p}{\partial z} + \frac{1}{Re}\nabla^2 w$$

$$\frac{D\theta}{Dt} = \frac{1}{Re\ .\ Pr}\nabla^2\theta$$

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Depending on the problem **(heating/ cooling),** Ri can be positive and negative. Consider the following problem

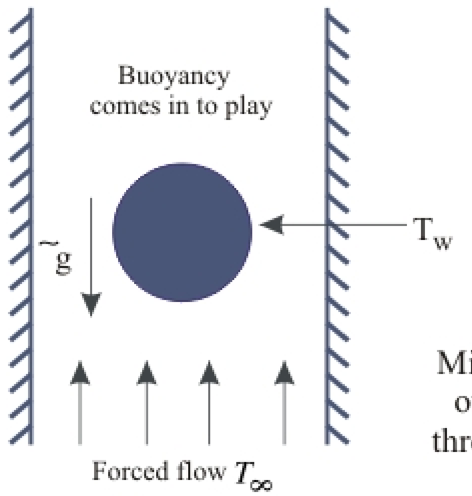

Figure 6.4
Mixed convection over a cylinder through parallel plate

$$T_w > T_\infty, R_i + ve$$

$$T_w < T_\infty, R_i - ve$$

In the mixed convection problem shown in **Figure 6.4** vortex shedding is found to stop completely at a critical Richardson number. The forced flow Reynolds number is 100. Explain why?

The following reference may be used

S.Singh, G.Biswas and A.Mukhopadhyay, Effeect of Thermal Buoyancy on the Flow through a Vertical Channel with a Built-in Circular Cylinder, Numerical Heat Transfer, **Part-A, Vol. 34, pp. 769-789, 1998.**

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6.5.1 Exercise:

Start with a control volume approach **(Figure 6.5)** and nd out an expression for the modified y-momentum equation for a mixed convection flow

Newton's Second Law

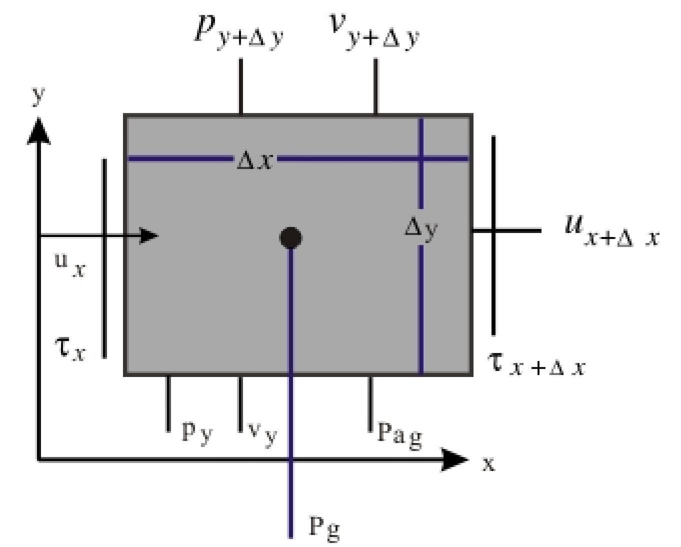

Figure 6.5 : Force balance in a control volume

$$\sum F_y = (\ y\text{ - momentum outflux}) - (y\text{ - momentum influx})$$

Influx of y momentum through bottom face $(\rho v_y\ \Delta x)\ v_y$

Outflux of y momentum through top face $(\rho v_{y+\Delta y}\ \Delta x)\ v_{y+\Delta y}$

Influx of y momentum through left face $(\rho u_x\ \Delta y)\ v_x$

Outflux of y momentum through right face $(\rho u_{x+\Delta x}\ \Delta y)\ v_{x+\Delta x}$

So,

$$p_y \, \Delta x \; - \; p_{y+\Delta y} \, \Delta x \; + \; \tau_{x \, + \, \Delta x} \, \Delta y - \tau_x \, \Delta y \; + \; \rho_\infty g \, \Delta x \, \Delta y \; - \; \rho g \, \Delta x \, \Delta y$$

$$= \rho \; [(vv)_{y+\Delta y} \, \Delta x \; + \; (uv)_{x+\Delta x} \, \Delta y]$$

$$-\rho \; [(vv)_y \, \Delta x \; + \; (uv)_x \, \Delta y \,]$$

or

$$p_y \, \Delta x \; - \; \left(p_y + \frac{\partial p_y}{\partial y} \, \Delta y\right) \, \Delta x \; + \; \left(\tau_x + \frac{\partial \tau_x}{\partial x} \, \Delta x\right) \, \Delta y - \tau_x \, \Delta y + \rho_\infty g \Delta x \, \Delta y - \rho g \, \Delta x \, \Delta y$$
$$= \rho \; \left[\left\{(vv)_y + \frac{\partial (vv)_y}{\partial y} \, \Delta y \right\} \Delta x + \left\{(uv)_x + \frac{\partial (uv)_x}{\partial x} \, \Delta x \right\} \, \Delta y\right] - \rho \; [(vv)_y \, \Delta x \; + \; (uv)_x \, \Delta y]$$

or

$$-\frac{\partial p}{\partial y} \, \Delta x \, \Delta y + \frac{\partial \tau_x}{\partial x} \, \Delta x \, \Delta y \; + \; \rho_\infty g \, \Delta x \, \Delta y - \rho \; g \; \Delta x \, \Delta y$$
$$= \rho \, \Delta x \, \Delta y \; \left[\frac{\partial}{\partial y}(vv)_y + \frac{\partial}{\partial x}(uv)_x\right]$$

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or,

$$-\frac{\partial p}{\partial y}+\frac{\partial \tau_x}{\partial x}+\rho_\infty g-\rho g=\rho\left[v\frac{\partial v}{\partial y}+u\frac{\partial v}{\partial x}+v\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right]$$

or, $$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+(\rho_\infty-\rho)g+\frac{\partial \tau_x}{\partial x}$$

References

B.Gebhart, Y. Jaluria **,** R . L . Mahajan and B Sammakia , **1988** , Buoyance Induced Flows.
Transport,Hemisphere Publishing Corporation Washington.