The Lecture Contains:

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Introduction

The following are the characteristics of Turbulent motion

Irregularity

Complex variations of velocity, temperature, etc. with space and time **(fluctuations)** are the dominant properties of a turbulent flow. The irregular motion is generated due to random fluctuations **(Figure 5.1)**. It is postulated that the fluctuations inherently come from disturbances (such as, roughness of the solid surface) and they may be either dampened out due to viscous damping or may grow by drawing energy from the free stream. At a Reynolds number less than the critical, the kinetic energy of flow is not enough to sustain the random fluctuations against the viscous damping and in such cases laminar flow continues to exist. At somewhat higher Reynolds number than the critical Reynolds number.

Tthe kinetic energy of flow supports the growth of fluctuations and transition to turbulence is induced.

Strong mixing

High transfer of momentum, heat, mass by fluctuating turbulent motion practically most important feature.

Turbulent motion always 3D

For a parallel flow, it can be written that the axial velocity component is.

$$u(y,t) = \overline{u}(y) + u'(\Gamma, t)$$

y is the normal direction, Γ is any space variable

Even if the bulk motion is parallel, the fluctuation u' being random varies in all directions.
Now let us look at the continuity equation

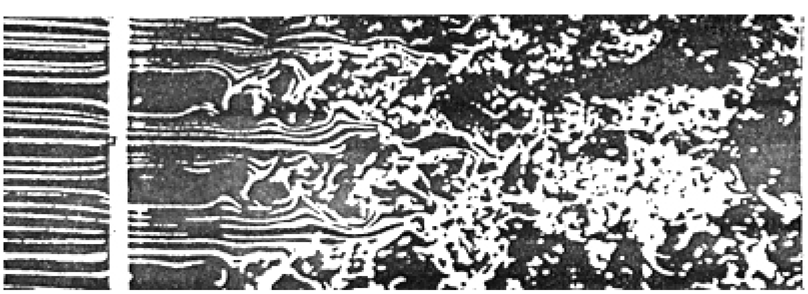

Figure 5.1: Turbulence is generated in the flow field

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$$\frac{\partial \bar{u}}{\partial x}+\frac{\partial u'}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

Since $\frac{\partial u'}{\partial x} \neq 0$, the above equation depicts that y and z components of velocity exist even for the parallel flow if the flow is turbulent .

We can write

$$u(y,t)=\overline{u}(y) \ + \ u^{'} \ (\Gamma,t)$$

$$v=0 \ + \ v^{'} \ (y,t)$$

$$w=0 \ + \ w^{'} \ (y,t)$$

Turbulent motion carries vorticity - is composed of eddies interacting with each other

Wide spectrum of eddy sizes and corresponding fluctuation frequencies are shown **Figure 5.2**

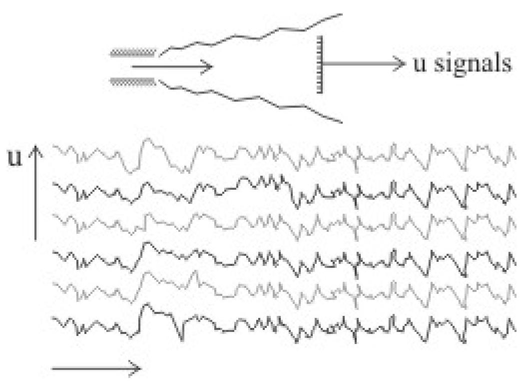

Figure 5.2: Wide spectrum of eddy sizes and corresponding frequencies

- The term homogeneous turbulence implies that the velocity fluctuations in the system are random.

- The average turbulent characteristics are independent of the position of the **fluid, i.e., invariant to axis translation.**

- In addition to its homogeneous nature, if the velocity fluctuations are independent of the axis of reference, i.e., invariant to axis rotation and reaction.

This restriction leads to **isotropic turbulence, which by definition is always homogeneous.**

Classical Idealization of Turbulent Flows

Statistical calculation methods :

Details of turbulent fluctuations usually not of interest to engineers anyway.

Hence statistical approach is taken and turbulence is averaged out. Different trends of variation of the the mean and fluctuating components are shown in **Figures 5.3 and 5.4.**

Statistical quantities:

$$u_i = \overline{u}_i + u_i', \ P = \overline{P} + p', \ \overline{u} = \frac{1}{2\,t}\int_{to}^{to+t} u dt$$

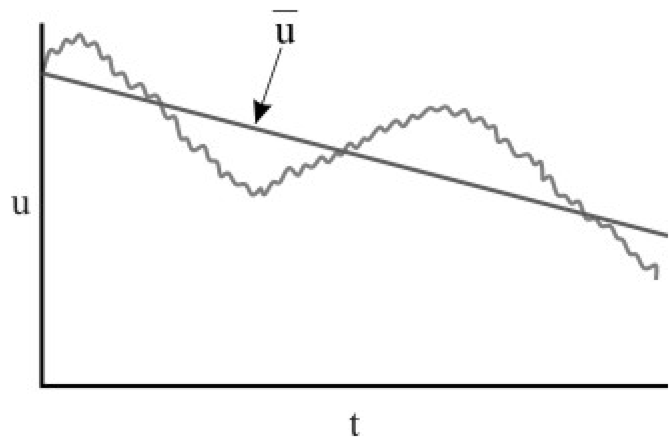

Figure 5.3: Mean motion and fluctuations

However the fluctuating components do not bring about the bulk displacement of a fluid element. The instantaneous displacement is $u'\ dt$ and if that is indeed not responsible for the bulk motion, we can conclude that

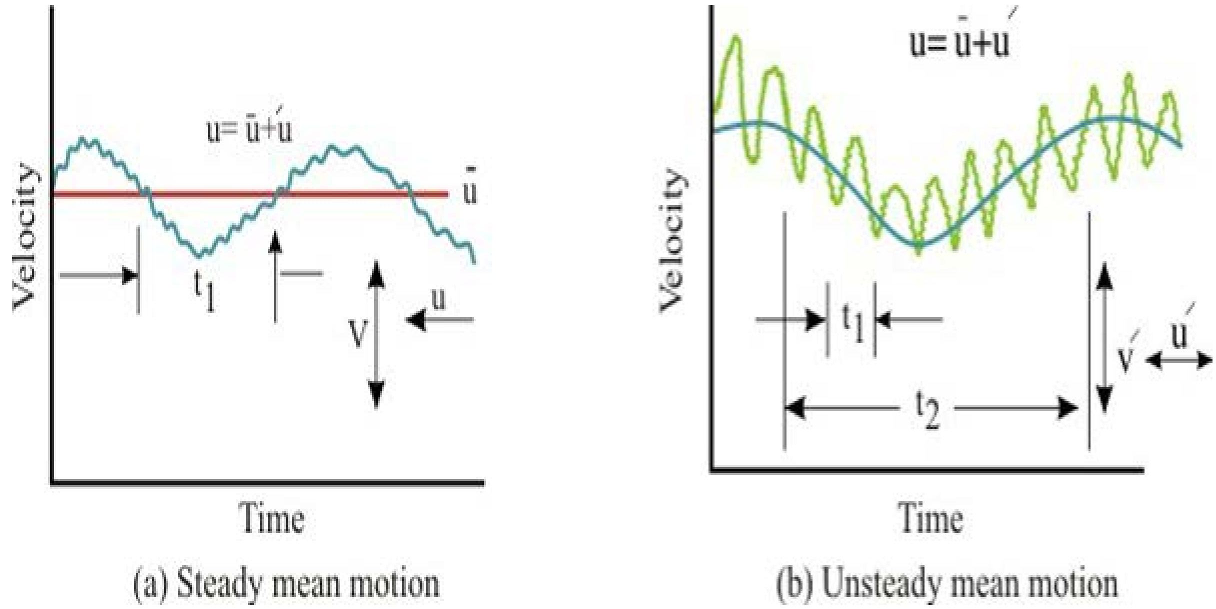

Figure 5.4: Steady and unsteady mean motions in a turbulent flow

$$\int_{-t}^{t} u^{'} dt = 0 \;\; \{t_1 < t < t_2\} \tag{5.2}$$

Due to the interaction of fluctuating components,macroscopic momentum transport takes place. Therefore, interaction effect between two fluctuating components over long period is nonzero and this yields

$$\int_{-t}^{t} u^{'} v^{'} dt \neq 0 \tag{5.3}$$

We take time average of these two integrals and write

$$\overline{u^{'}} = \frac{1}{2t} \int_{-t}^{t} u^{'} dt = 0 \tag{5.4}$$

and

$$\overline{u^{'} v^{'}} = \frac{1}{2t} \int_{-t}^{t} u^{'} v^{'} dt \neq 0 \tag{5.5}$$

Now, we can make a general statement with any two fluctuating parameters, say,with $f^{'}$ and $g^{'}$ as ($f^{'}$ and $g^{'}$ can be vectors or passive scalars)

$$\overline{f^{'}} = \overline{g^{'}} = 0 \qquad \overline{\frac{\partial f^{'}}{\partial s}} = \overline{\frac{\partial^2 f^{'}}{\partial s^2}} = 0 \tag{5.6}$$

and

$$\overline{f^{'} g^{'}} \neq 0 \qquad \text{and} \qquad \frac{\overline{\partial (f^{'} g^{'})}}{\partial s} \neq 0 \tag{5.7}$$

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We shall state some rules of operating on mean time-averages herein. If f and g are two dependent variables and if s denotes any one of the independent variables x , y ,z , t then

$$\overline{\frac{\partial f}{\partial s}} = \frac{\partial \overline{f}}{\partial s}\ ; \quad \overline{\int f\,ds} = \int \overline{f}\,ds \qquad \textbf{(5.8)}$$

$$u = \overline{u} + u^{'},\ \ v = \overline{v} + v^{'},\ \ w = \overline{w} + w^{'},\ \ p = \overline{p} + p^{'} \qquad \textbf{(5.9)}$$

Plug in continuity

$$\nabla \cdot \overline{u}_i = 0 \qquad \textbf{(5.10)}$$

and

$$\nabla \cdot u_i^{'} = 0 \qquad \textbf{(5.11)}$$

Introduction of this separation into the **Navier-Stokes equations** and subsequent averaging leads to the appearance of turbulence correlations **(turbulent - or Reynolds stresses)**

For example, if we perform the aforesaid exercise on the **x momentum equation**, we obtain

$$\rho\left\{\frac{\partial \bar{u}}{\partial t} + \frac{\partial(\bar{u}\bar{u})}{\partial x} + \frac{\partial(\bar{u}\bar{v})}{\partial y} + \frac{\partial \bar{u}\bar{w}}{\partial z}\right\} = -\frac{\partial \bar{p}}{\partial x} + \mu\nabla^2\bar{u} - \rho\left[\frac{\partial \overline{u'^2}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z}\right] \qquad \textbf{(5.12)}$$

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Introducing simplifications arising out of continuity equation we shall obtain

$$\rho\frac{\partial\overline{u}}{\partial t}+\rho\left[\overline{u}\frac{\partial\overline{u}}{\partial x}+\overline{v}\frac{\partial\overline{u}}{\partial y}+\overline{w}\frac{\partial\overline{u}}{\partial z}\right]=-\frac{\partial\overline{p}}{\partial x}+\mu\nabla^2\overline{u}-\rho\left[\frac{\partial\overline{u'^2}}{\partial x}+\frac{\partial\overline{u'v'}}{\partial y}+\frac{\partial\overline{u'w'}}{\partial z}\right]$$

performing a similar treatment on **y and z momentum equations**, finally we obtain the momentum equations in the form

$$\rho\frac{\partial\overline{u}}{\partial t}+\rho\left[\overline{u}\frac{\partial\overline{u}}{\partial x}+\overline{v}\frac{\partial\overline{u}}{\partial y}+\overline{w}\frac{\partial\overline{u}}{\partial z}\right]=-\frac{\partial\overline{p}}{\partial x}+\mu\nabla^2\overline{u}$$
$$-\rho\left[\frac{\partial\overline{u'^2}}{\partial x}+\frac{\partial\overline{u'v'}}{\partial y}+\frac{\partial\overline{u'w'}}{\partial z}\right]\qquad\textbf{(5.13)}$$

similarly,

$$\rho\frac{\partial\overline{v}}{\partial t}+\rho\left[\overline{u}\frac{\partial\overline{v}}{\partial x}+\overline{v}\frac{\partial\overline{v}}{\partial y}+\overline{w}\frac{\partial\overline{v}}{\partial z}\right]=-\frac{\partial\overline{p}}{\partial y}+\mu\nabla^2\overline{v}$$
$$-\rho\left[\frac{\partial\overline{u'v'}}{\partial x}+\frac{\partial\overline{v'^2}}{\partial y}+\frac{\partial\overline{v'w'}}{\partial z}\right]\qquad\textbf{(5.14)}$$

$$\rho\frac{\partial\overline{w}}{\partial t}+\rho\left[\overline{u}\frac{\partial\overline{w}}{\partial x}+\overline{v}\frac{\partial\overline{w}}{\partial y}+\overline{w}\frac{\partial\overline{w}}{\partial z}\right]=-\frac{\partial\overline{p}}{\partial z}+\mu\nabla^2\overline{w}$$
$$-\rho\left[\frac{\partial\overline{u'w'}}{\partial x}+\frac{\partial\overline{v'w'}}{\partial y}+\frac{\partial\overline{w'^2}}{\partial z}\right]\qquad\textbf{(5.15)}$$

It is to be noted that the terms containing prime were not there in original **NS equations.**

$$\sigma_T=\begin{bmatrix}\sigma'_{xx} & \tau'_{xy} & \tau'_{xz}\\ \tau'_{xy} & \sigma'_{yy} & \tau'_{yz}\\ \tau'_{xz} & \tau'_{yz} & \sigma'_{zz}\end{bmatrix}=-\rho\begin{bmatrix}\overline{u'^2} & \overline{u'v'} & \overline{u'\ w'}\\ \overline{u'v'} & \overline{v'^2} & \overline{v'\ w'}\\ \overline{u'w'} & \overline{v'w'} & \overline{w'^2}\end{bmatrix}\qquad\textbf{(5.16)}$$

σ_T is the Reynolds stress tensor and written in compact form as $-\rho\overline{u'_i u'_j}$

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$$\sigma_{xx} = -p + 2\mu\frac{\partial \overline{u}}{\partial x} - \rho\overline{u'^2} \quad \textbf{(5.17)}$$

$$\tau_{xy} = \mu\left(\frac{\partial \overline{u}}{\partial y} + \frac{\partial \overline{v}}{\partial x}\right) - \rho\overline{u'v'} \quad \textbf{(5.18)}$$

Averaged equations:

$$\frac{\partial \overline{u}_i}{\partial t} + u_j\,\frac{\partial \overline{u}_i}{\partial x_j} = -\frac{1}{\rho}\,\frac{\partial \overline{P}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu\,\frac{\partial \overline{u}_i}{\partial x_j} - \overline{u_i'u_j'}\right) \quad \textbf{(5.19)}$$

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \quad \textbf{(5.20)}$$

We have more unknowns than number of available equations .
The modified system of equations cannot be closed within itself unless empirical relations are supplied from experiments to correlate the fluctuating components with the mean motion.This is termed as the closure problem.

Closure problem

The turbulent stresses need to be determined with the aid of a turbulence model.

Here only models for all turbulent motions discussed,
not subgrid-scale models for large-eddy simulations.

$$-\overline{u_i'u_j'} = \nu_t\,\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) - \frac{2}{3}\,k\delta_{ij} \quad \textbf{(5.21)}$$

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The term ν_t is turbulent (eddy) viscosity. The term involving the Kronecker delta δ_{ij} in **equation (5.21)** is perhaps a somewhat unfamiliar addition to the eddy-viscosity expression. It is necessary to make the expression applicable also to normal stresses **(when i = j).** The first part of (5.21) involving the velocity gradients would yield the normal stresses.

$$\overline{u'^2} = -2\nu_t\frac{\partial \overline{u}}{\partial x}, \quad \overline{v'^2} = -2\nu_t\frac{\partial \overline{v}}{\partial y}, \quad \overline{w'^2} = -2\nu_t\frac{\partial \overline{w}}{\partial z} \qquad \textbf{(5.22)}$$

Whose sum is **zero** because of the continuity equation.
However, all normal stresses are by definition positive quantities, and their sum is twice the kinetic energy k of the fluctuating motion:

$$k = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) \qquad \textbf{(5.23)}$$

Inclusion of the second part of the eddy viscosity expression (5.21) assures that the sum of the normal stresses is equal to 2k .

The normal stresses act like pressure forces **(i.e.perpendicular to the faces of a control volume),** and because like the pressure itself , the energy k is a scalar quantity, the second part of **(5.21)** constitutes a pressure. Therefore, when **equation (5.21)** is used to eliminate $\overline{u_i' u_j'}$ in the momentum equation and this second part can be absorbed by the pressure-gradient term so that in effect the static pressure is replaced as unknown quantity by the pressure $\overline{P} + \frac{2}{3}k$. Therefore the appearance of k in **equation (5.21)** does not necessitate the determination of k, it is the distribution of the eddy viscosity ν_t only that has to be determined. Here it may be mentioned that k can be linked to the intensity of turbulence which is given by.

$$I = \sqrt{\frac{1}{3}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})} \;/U_\infty \qquad \textbf{(5.24)}$$

Reynolds Averaged form of Energy Equation

Let us consider a two-dimensional situation,

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2} + \alpha\frac{\partial^2 T}{\partial y^2} \qquad \textbf{(5.25)}$$

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Use Reynolds' decomposition of velocity and temperature

$$u = \overline{u} + u', \quad v = \overline{v} + v', \quad T = \overline{T} + T' \qquad \textbf{(5.26)}$$

we get ,

$$\overline{u}\frac{\partial \overline{T}}{\partial x} + \overline{v}\frac{\partial \overline{T}}{\partial y} = \frac{\partial}{\partial x}\left(\alpha\frac{\partial \overline{T}}{\partial x} - \overline{u'T'}\right) + \frac{\partial}{\partial y}\left(\alpha\frac{\partial \overline{T}}{\partial y} - \overline{v'T'}\right) \qquad \textbf{(5.27)}$$

The terms $\overline{u'T'}$ and $\overline{v'T'}$, thus, cause additional heat ux in the x and y directions respectively, due to turbulent motion. The total heat flux in the two directions will therefore be given by.

$$q_x'' = -\rho c_p\left(\alpha\frac{\partial \overline{T}}{\partial x} - \overline{u'T'}\right); \quad q_y'' = -\rho c_p\left(\alpha\frac{\partial \overline{T}}{\partial y} - \overline{v'T'}\right) \qquad \textbf{(5.28)}$$

As in case of turbulent transport of momentum , it is convenient to define an eddy viscosity or turbulent viscosity , to study the turbulent transport of thermal energy , a turbulent thermal diffusivity can be defined . If x is the main flow direction , generally in practical problems the gradient of mean temperature in the y direction will be more significant than that in the x-direction . The turbulent diffusivity may be therefore defined as

$$\alpha_t = -\frac{\overline{v'T'}}{\partial \overline{T}/\partial y} \qquad \textbf{(5.29)}$$

The total heat flux in the y direction will therefore be given as

$$\frac{q_y''}{\rho c_p} = -\left(\alpha + \alpha_t\right)\frac{\partial \overline{T}}{\partial y} \qquad \textbf{(5.30)}$$

Like eddy viscosity, **t** is not a fluid property but depends on the state of turbulence. In fact the **Reynolds analogy** between heat and momentum transport suggests

$$\alpha_t = \frac{\nu_t}{\sigma_t} \qquad \textbf{(5.31)}$$

The quantity σ_t called **turbulent Prandtl number**.