Module 8: Boiling

Lecture 34: Analysis of Bubble Formation on a Heated Wall

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## Bubble Growth with Heat and Mass Transfer :-

The analysis for the growth of a bubble attached to a heated wall can be extremely involved because of the continuous change in the shape of the bubble, the convective motion around the bubble and the effect of the neighboring bubbles. In the literature several attempts to model the growth of bubbles attached to a wall have been made. All of these efforts have tended to be modifications of the model for growth of a bubble in a uniform temperature field. Correction factors are generally incorporated to account for the shape of a bubble being different than that of a sphere. One such an attempt is by **Mikic, Rohsenow and Griffth (1970)** and will be followed here.

### Simple Analysis of Bubble Growth in a Uniform Temperature Field

Assuming that bubble growth rate is slow enough such that the momentum equation can be written as

$$P_{v_0} - P_l = \frac{3}{2}\rho_l \dot{R}^2 \qquad \textbf{(8.60)}$$

Iift is assumed that the contribution of surface tension to the pressure in the bubble is small, the above equation can be written as

$$P_v - P_l = \frac{3}{2}\rho_l \dot{R}^2 \qquad \textbf{(8.61)}$$

where $P_v$ is the pressure of vapor in the bubble. Introducing a constant to account for the shape of the bubble being dierent than that for a sphere, **equation (8.61)** is rewritten as**.**

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$$\dot{R}^2 = \frac{b(P_v - P_l)}{\rho_l} \tag{8.62}$$

where b = 2/3 for spherical bubble. A value of 1/3 for b has been suggested by **Mikic et al.** for a bubble attached to a wall. Using **Clausius Clapeyron relationship,** **equation (8.62)** can be written as

$$\dot{R}^2 = \frac{b(T_v - T_{sat})h_{fg}\rho_v}{\rho_l T_{sat}} \tag{8.63}$$

Defining a temperature difference ΔT as

$$\Delta T = T_\infty - T_{sat} \tag{8.64}$$

**Equation (8.63)** is obtained as

$$R^2 = \frac{A^2(T_v - T_{sat})}{\Delta T} \tag{8.65}$$

where

$$A^2 = \left(\frac{b\rho_v h_{fg}\Delta T}{\rho_l T_{sat}}\right) \tag{8.66}$$

and it has dimensions of square of velocity. In a thermally controlled situation, the solution for the growth rate of a bubble using a plane interface analysis is obtained as

$$\frac{d}{dt}\ (4/3\ \pi\ R^3\ \rho_v\ h_{fg}) = \frac{k_l}{\sqrt{\pi \alpha_l t}}\ (T_\infty - T_v)\ \ 4\pi R^2 \qquad \textbf{(8.67)}$$

$$\frac{dR}{dt} = \frac{k_l}{\sqrt{\pi \alpha_l t}} \frac{(T_\infty - T_v)}{\rho_v h_{fg}} \qquad \textbf{(8.68)}$$

$$\frac{dR}{dt} = \left(\frac{\alpha_l}{\pi t}\right)^{1/2} \frac{\rho_l c_{pl} (T_\infty - T_v)}{\rho_v h_{fg}} \qquad \textbf{(8.69)}$$

Nearly exact solution of **Plesset and Zwick** suggests that to account for sphericity of the interface the right hand side of the **equation (8.69)** should be multiplied by $\sqrt{3}$. After this correction is applied, **equation (8.69)** can be written as

$$\frac{dR}{dt} = \frac{1}{2} \frac{B}{\sqrt{t}} \frac{(T_\infty - T_v)}{\Delta T} \qquad \textbf{(8.70)}$$

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where

$$B = \left(\frac{12\alpha_l}{\pi}\right)^{1/2}\left(\frac{c_{pl}\Delta T p_l}{h_{fg}\rho_v}\right) \qquad \textbf{(8.71)}$$

The second term on the right hand side of **equation (8.71)** is called Jakob number.

$$\mathcal{J}_a = \left(\frac{C_{pl}\Delta T \rho_l}{h_{fg}\rho_v}\right) \qquad \textbf{(8.72)}$$

and parameter **B** has dimensions of $m/(sec)^{1/2}$. **Equation (8.70)** can be written as

$$\frac{dR}{dt} = \frac{B}{2\sqrt{t}}\left(1 - \frac{(T_v - T_{sat})}{\Delta T}\right) \qquad \textbf{(8.73)}$$

Eliminating $\frac{T_v - T_{sat}}{\Delta T}$ between **equations (8.65)** and **(8.73)** yields

$$\frac{dR}{dt} = \frac{B}{2\sqrt{t}}\left(1 - \frac{\dot{R}^2}{A^2}\right) \qquad \textbf{(8.74)}$$

or,

$$\frac{1}{A^2}\left(\dot{R}^2\right) + \frac{2\sqrt{t}}{B}R - 1 = 0$$

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With dimensionless radius and dimensionless time defined as

$$R^* = \frac{AR}{B^2} \quad \textbf{(8.76)}$$

and

$$t^* = \frac{A^2 t}{B^2} \quad \textbf{(8.77)}$$

**equation (8.75)** becomes

$$\left(\frac{dR^*}{dt^*}\right)^2 + 2\sqrt{t^*}\frac{dR^*}{dt^*} - 1 = 0 \quad \textbf{(8.78)}$$

$$\left(\frac{dR^*}{dt^*}\right) = \frac{-2\sqrt{t^*} + \sqrt{4t^* + 4}}{2} = (t^* + 1)^{1/2} - (t^*)^{1/2} \quad \textbf{(8.79)}$$

If we use the initial condition that $R^* = 0 \text{ at } t^* = 0,$ , the integration of the above **equation yields**

$$R^* = \frac{2}{3}\left[(t^* + 1)^{3/2} - (t^*)^{3/2} - 1\right] \quad \textbf{(8.80)}$$

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If $t^* < 1,$ the above equation reduces to the form

$$R^* = t^* \qquad \textbf{(8.81)}$$

On the other hand, for $t^* < 1,$, **equation (8.87)** yields.

$$R^* = t^{*1/2} \qquad \textbf{(8.82)}$$

Thus for early times , the bubble will grow linearly with time and in the limit of thermally controlled growth the **radius of the bubble will increase as square root of time**

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## Simple Analysis of Bubble Growth on a Wall

After bubble departure on a given site, one must wait for a certain period of time before a new bubble is formed at the same site. The time that elapses between departure of a bubble and inception of a new bubble is called the waiting period. During the waiting period, the liquid next to the wall is heated and a bubble embryo develops into a bubble when Hsu's criterion is satisfied. Thereafter as the bubble grows, the superheated liquid around the bubble is also pushed outward along with the bubble. Now the growth rate of the bubble is dictated by the rate at which thermal energy can be transferred from the superheated liquid to the interface. **Mikic, Rohsenow and Griffth (1990)** also obtained a solution for growth of such a bubble. **In carrying out the analysis, they assumed that**

**1**. Liquid layer adjacent to the wall is semi-infinite.

**2.** During bubble growth no relative motion exists between the bubble and the surrounding liquid.

**3.** Liquid vapor interface is plane.

**4.** Interfacial heat flux can be obtained by assuming that vapor temperature remains constant.
The transient diffusion of heat into the liquid during the waiting period and out of the superheated liquid during the growth of the bubble could be broken up into two one dimensional transient conduction problems as.

$$\frac{\partial T}{\partial t} = \alpha_l \frac{\partial^2 T}{\partial y^2} \quad \textbf{(8.83)}$$

$$\begin{gathered} for - t_w \le t \le 0 \\ T = T_l \; for \; all \; y \; and \; t \le -t_w \\ T = T_w \; at \; y = 0 \; for \; 0 \ge t \ge -t_w \\ T = T_l \; as \; y \to \infty \; for \; t \ge -t_w \end{gathered}$$

$$\begin{gathered} for \; 0 \le t \\ T = T_l \; for \; all \; y \; and \; t = 0 \\ T = T_l - (T_w - T_v) \; at \; y = 0 \; for \; t \ge 0 \\ T = T_l \; as \; y \to \infty \; for \; t \ge 0 \end{gathered}$$

Superimposing the solutions for the **two cases**, we obtain

$$T - T_l = (T_w - T_l)\, erfc\left[\frac{y}{2\sqrt{\alpha_l (t + t_w)}}\right] - (T_w - T_v)\, erfc\left[\frac{y}{2\sqrt{\alpha_l t}}\right] \quad \textbf{(8.84)}$$

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$$q_l = -\left[-k_l \frac{\partial T}{\partial y}\bigg| y=0\right] = -k_l (T_w - T_l)\frac{2}{\sqrt{\pi}}\frac{1}{2\sqrt{\alpha_l (t+t_w)}} + (T_w - T_v)\frac{2}{\sqrt{\pi}}\frac{1}{2\sqrt{\alpha_l t}} \quad \textbf{(8.85)}$$

$$= k_l \left[\frac{(T_w - T_v)}{\sqrt{\pi\ \alpha_l\ t}} - \frac{(T_w - T_l)}{\sqrt{\pi\ \alpha_l\ (t+t_w)}}\right] \quad \textbf{(8.86)}$$

To account for the geometry of the bubble, we multiply $q_l$ by $\sqrt{3}$. The energy balance at the bubble liquid interface yields

$$\frac{d}{dt}\left(\rho_v h_{fg} 4/3\, \pi R^3\right) = \frac{\sqrt{3}\, k_l}{\sqrt{\pi \alpha_l t}}\left[(T_w - T_v) - (T_w - T_l)\sqrt{\frac{t}{t+t_w}}\right] 4\pi R^2$$

$$\frac{dR}{dt} = \frac{\sqrt{3}\, k_l}{\rho_v h_{fg}\sqrt{\pi \alpha_l t}}\left[(T_w - T_v) - (T_w - T_l)\sqrt{\frac{t}{t+t_w}}\right] \quad \textbf{(8.87)}$$

$$= \frac{\rho_l c_{pl}}{\rho_v h_{fg}}\Delta T\sqrt{\frac{\alpha_l}{\pi t}}\sqrt{3}\left[\frac{T_w - T_v}{\Delta T} - \frac{T_w - T_l}{\Delta T}\sqrt{\frac{t}{t+t_w}}\right]$$

$$= \left(\frac{12\alpha_l}{\pi t}\right)^{1/2}\frac{Ja}{2}\left[\frac{T_w - T_v}{\Delta T} - \theta\sqrt{\frac{t}{t+t_w}}\right] \quad \textbf{(8.88)}$$

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where $\Delta T = T_W - T_{sat}$

and $\theta = \dfrac{T_w - T_l}{\Delta T}$

In terms of the parameter **B** defined earlier **equation (8.88)** can be written as

$$\frac{dR}{dt} = \frac{B}{2\sqrt{t}}\left[\frac{T_w - T_v}{\Delta T} - \theta\sqrt{\frac{t}{t+t_w}}\right] \qquad \textbf{(8.89)}$$

$$\begin{aligned} 1 - \frac{2\sqrt{t}}{B}\frac{dR}{dt} &= 1 - \frac{T_w - T_v}{\Delta T} + \theta\sqrt{\frac{t}{t+t_w}} \\ &= \frac{T_w - T_{sat} - T_w + T_v}{T_w - T_{sat}} + \theta\sqrt{\frac{t}{t+t_w}} \end{aligned} \qquad \textbf{(8.90)}$$

$$1 - \frac{2\sqrt{t}}{B}\frac{dR}{dt} = \frac{T_v - T_{sat}}{\Delta T} + \theta\sqrt{\frac{t}{t+t_w}} \qquad \textbf{(8.91)}$$

**Rayleigh** equation in an approximate form is written as

$$\begin{aligned} \left(\frac{dR}{dt}\right)^2 &= b\left(\frac{\rho_v - \rho_l}{\rho_l}\right) \\ &= \frac{b(T_v - T_{sat})h_{fg}\rho_v}{T_{sat}\rho_l\Delta T}\Delta T \end{aligned} \qquad \textbf{(8.92)}$$

In terms of parameter **A** defined earlier, **equation (8.92)** can be written as

$$\left(\frac{dR}{dt}\right)^2 = A^2\frac{T_v - T_{sat}}{\Delta T} \qquad \textbf{(8.93)}$$

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Substituting for $T_v - T_{sat}/\Delta T$ from **equation (8.93)** into **equation (8.91)** we get

$$1-\frac{2\sqrt{t}}{B}\frac{dR}{dt}=\frac{1}{A^2}\left(\frac{dR}{dt}\right)^2+\theta\sqrt{\frac{t}{t+t_w}} \qquad \textbf{(8.94)}$$

In dimensionless form,the above equation becomes

$$\left(\frac{dR^*}{dt^*}\right)^2+2\sqrt{t^*}\frac{dR^*}{dt^*}-\left[1-\theta\left(\frac{t^*}{t^*+t_w^*}\right)^{1/2}\right]=0 \qquad \textbf{(8.95)}$$

Solution of **(8.95)** yields

$$\frac{dR^*}{dt^*}=\left[t^*+1-\theta\left(\frac{t^*}{t^*+t_w^*}\right)^{1/2}\right]^{1/2}-(t^*)^{1/2} \qquad \textbf{(8.96)}$$

where we have used the **+** sign in the solution of the **quadratic equation (8.95).**

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