Module 6: Free Convections

Lecture 26: Evaluation of Nusselt Number

The Lecture Contains:

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Heat transfer coefficient

$$q = -k \left.\frac{\partial T}{\partial y}\right|_{y=0} = h_x (T_w - T_\infty)$$

$$h_x = \frac{-k(\partial T/\partial y)_{y=0}}{T_w - T_\infty}$$

or

$$h_x = -\frac{k}{(T_w - T_\infty)} \frac{\partial}{\partial y} \left[T_\infty + (T_w - T_\infty) \left(1 - \frac{y}{\delta}\right)^2 \right]_{y=0} = \frac{2k}{\delta}$$

or

$$\frac{h_x x}{k} = \frac{2x}{\delta} = \frac{2}{3.93} \left[\frac{Pr^2\, Gr_x}{Pr + \frac{20}{21}} \right]^{1/4} \qquad \textbf{(6.27)}$$

or

$$Nu_x = 0.508 \left[\frac{Pr^2 Gr_x}{(Pr + \frac{20}{21})} \right]^{1/4}$$

Note :

Free convection is negligible if $Gr/Re^2 << 1$

Forced convection is negligible if $Gr/Re^2 >> 1$

Combined free and forced convection,if $Gr/Re^2 \sim 1$

so

$$\frac{g\beta\theta_m L^3}{\nu^2} = \frac{U^2L^2}{\nu^2}$$

an equivalent free convection velocity may be defined as

$$u = \sqrt{g\beta\theta_m L}$$

The above **equation (6.27)** is used for laminar boundary layer. It is customary to correlate the transition to turbulence if the Rayleigh number crosses certain limit.

$$Ra_{x,c} = Gr_{x,c}\ \ Pr = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}\ \frac{\nu}{\alpha}$$

Flow is turbulent for $Ra_{x,c} \geq 10^9$

Correlations:

Vertical plate: $\overline{Nu}_L = 0.68 + \frac{0.67 Ra_L^{1/4}}{\left[1+(0.492/Pr)^{9/16}\right]^{4/9}},\ 10^{-1} < Ra_L < 10^9$

Horizontal plate: $L = \frac{A_s}{P} = \frac{\text{Surface area}}{\text{Perimeter}}$

If upper side of the plate is heated, then $\overline{Nu}_L = 0.54 Ra_L^{1/4}\ 10^4 \leq Ra_L \leq 10^7$

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Long horizontal cylinder:- [due to Churchill and Chu] $\overline{Nu}_D = \left[0.60 + \frac{0.387(Ra_D)^{1/6}}{\left[1+(0.559/Pr)^{9/16}\right]^{8/27}}\right]$

$10^{-5} \le Ra_D \le 10^{12}$

Vertical cylinder $\overline{Nu}_L = 0.13(Ra_L)^{1/3}$ (Warner and Arpaci). However, there are better correlations for vertical cylinder.

Vertical cylinder

The average Nusselt number for free convection on a vertical cylinder is same as that of a vertical plate if the thickness of the thermal boundary layer is much smaller than the cylinder radius namely, if the curvature
effects are neglected. For fluids, having a Prandtl number 0.7 and higher, a vertical cylinder may be treated as a vertical at plate when

$$\frac{L/D}{(Gr_L)^{1/4}} < 0.025$$

The **Figure 6.2** shows a plot of the ratio of the average Nusselt number for a vertical cylinder to that for a vertical plate as a function of the parameter $\xi = \frac{2\sqrt{2}}{(Gr_L)^{1/4}}.\left(\frac{L}{R}\right)$ Here, **R** is the radius of the cylinder.

Plumes.

A plume is the buoyancy induced flow resulting in a fluid, when energy is supplied continuously at just one location in the fluid. In practice free plumes, like free jets are
generally turbulent. An axisymmetric plume is found to be laminar only if the Rayleigh number based on its heat source and height of the plumes is less than 1010.

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Integral Analysis of a Steady Plume

The governing equations for the flow in a plume are the continuity equation,the Navier-Stokes equations and the energy equation. Assuming the mean flow in a plume to be steady and axisymmetric, the Reynolds averaged form of these equations are

$$\frac{1}{r}\frac{\partial}{\partial r}(r\bar{u})+\frac{\partial \bar{v}}{\partial y}=0 \tag{6.28}$$

$$\frac{\partial}{\partial r}(\bar{v}^2)+\frac{1}{r}\frac{\partial}{\partial r}(r\bar{u}\bar{v})=\frac{1}{r}\frac{\partial}{\partial r}\left[r(\nu+\nu_t)\frac{\partial \bar{v}}{\partial r}\right]+g\beta(\bar{T}-T_\infty) \tag{6.29}$$

$$\frac{\partial}{\partial r}(\bar{v}T)+\frac{1}{r}\frac{\partial}{\partial r}(r\bar{u}\bar{T})=\frac{1}{r}\frac{\partial}{\partial r}\left[r(\alpha+\epsilon_t)\frac{\partial \bar{T}}{\partial r}\right] \tag{6.30}$$

Integration of **equation (6.28)** from $r=0$ to $r\rightarrow\infty$ gives

$$(r\bar{u})_\infty+\frac{d}{dy}\int_0^\infty \bar{v}rdr=0 \tag{6.31}$$

$$\frac{d}{dy}\int_0^\infty \bar{v}^2rdr=g\beta\int_0^\infty(\bar{T}-T_\infty)rdr \tag{6.32}$$

$$\frac{d}{dy}\int_0^\infty \bar{v}(\bar{T}-T_\infty)rdr=0$$

$$\text{or,}\quad \int_0^\infty \bar{v}(\bar{T}-T_\infty)rdr=\frac{q}{2\pi\rho c_p} \tag{6.33}$$

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where **q** is the rate of hear release (the strength of the heat source) at the origin of the plume.

For proceeding further with the analysis, it is essential to make an assumption for the velocity and temperature profiles. The experimental observations show a Gaussian profile for both velocity and temperature.

Thus the following profiles are assumed.

$$\bar{v} = \bar{v}_c \, exp\left[-\left(\frac{r}{b}\right)^2\right] \quad \textbf{(6.34)}$$

$$\bar{T} - T_\infty = (\bar{T}_c - T_\infty) \, exp\left[-\left(\frac{r}{b_T}\right)^2\right] \quad \textbf{(6.35)}$$

where b and b_T are characteristic radial dimensions proportional to the plume thickness and the ration of b and b_T is of the order of one. $\bar{v}_c$ and $\bar{T}_c$ Tc are the centerline velocity and temperatures respectively.

To solve the resultant equations for all the unknowns, an additional assumptions is required for the entrainment term, $(r\bar{u})_\infty$. Here, we can make use of another experimental observation that the radius of the plume increases linearly with its height or

$$b \sim y \quad \textbf{(6.36)}$$

Using this in the continuity equation leads to the result

$$\begin{aligned} \bar{v}_c b &\sim (r\bar{u})_\infty \\ \text{or,} \quad (r\bar{u})_\infty &= -\hat{\alpha} b \bar{v}_c \end{aligned} \quad \textbf{(6.37)}$$

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where $\hat{\alpha}$ is the entrainment coefficient to be determined experimentally and has an approximate value of 0.12. Using the profiles **(6.34)** and **(6.35)** and the relation **(6.37)** leads to the following solution of the **equations (6.31-6.33).**

$$b = \frac{6}{5}\hat{\alpha} y \quad \textbf{(6.38)}$$

$$\bar{v}_c = \left[\frac{25}{24\pi\hat{\alpha}^2}\frac{qg\beta}{\rho c_p y}\left(1+\frac{b_T^2}{b^2}\right)\right]^{1/3} \quad \textbf{(6.39)}$$

$$\bar{T}_c - T_\infty = 0.685\left(1+\frac{b^2}{b_T^2}\right)\left(1+\frac{b_T^2}{b^2}\right)^{-1/3}\left(\frac{q}{\pi\rho c_p}\right)^{2/3}\hat{\alpha}^{-4/3}y^{-5/3} \quad \textbf{(6.40)}$$

where $\hat{\alpha}$ is the entrainment coefficient to be determined experimentally and has an approximate value of **0.12.**

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