The Lecture Contains:

The concentration boundary layer

In a flowing system, there will be a relative transport of species and species conservation must be satisfied at each point in the concentration boundary layer. The pertinent form of the conservation equation may be obatined by identifying the processes that affect the **transport** and **generation** of species A for a differential control volume in the boundary layer.

[Derivation is similar to thermal boundary layer equation; - carry out. Hints: Species A may be transported by advection **(with the mean velocity of the mixture)** and by diffusion **(relative to the mean motion)** in each of coordinate directions.**]**

The rate at which the mass of species **A** is generated per unit volume due to such reactions as $\dot{n}_A$. If the total mass density ρ is assumed to be constant we get

$$u\frac{\partial \rho_A}{\partial x} + v\frac{\partial \rho_A}{\partial y} = \frac{\partial}{\partial x}\left(D_{AB}\frac{\partial \rho_A}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_{AB}\frac{\partial \rho_A}{\partial y}\right) + \dot{n}_A \qquad \textbf{(9.71)}$$

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in the moral form

$$u\frac{\partial C_A}{\partial x}+v\frac{\partial C_A}{\partial y}=\frac{\partial}{\partial x}\left(D_{AB}\frac{\partial C_A}{\partial x}\right)+\frac{\partial}{\partial y}\left(D_{AB}\frac{\partial C_A}{\partial y}\right)+\dot{N}_A \qquad \textbf{(9.72)}$$

For mass transfer problems, v can no longer be zero at the surface. However, it will be reasonable to assume $v=0$ which is equivalent to assuming that mass transfer has a negligible effect on velocity boundary layer. We note that with mass transfer, the boundary layer fluid is a binary mixture of species A and B In all problems of interest $C_A \ll C_B$ and it is reasonable to assume that the boundary layer properties $(k,\mu,C_p\ etc)$ are those of species C_B

Because boundary layer thickness are typically very small, the following inequality will apply.

$$\frac{\partial C_A}{\partial y} >> \frac{\partial C_A}{\partial x}$$

Using the following non-dimensional variables.

$X=x/L\ ,Y=y/L\ ,U=u/U_{\alpha v}\ ,V=v/U_{\alpha v}$

and $C_A^*=\dfrac{C_A-C_{A,W}}{C_{A,\infty}-C_{A,W}}$

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Eqn (9.72) becomes

$$U\frac{\partial C_A^*}{\partial X} + V\frac{\partial C_A^*}{\partial Y} = \frac{1}{Re_L S_c}\frac{\partial^2 C_A}{\partial y^2} \qquad \textbf{(9.73)}$$

where , S_c is **Schmidt Number** = nu/D_{AB}

S_c is the ratio of momentum and mass diffusivities. For mass transfer in a gas flow over an evaporating liquid or sublimating solid, the convection mass transfer coefficient h_m depends on. L, D_{AB}, ρ, μ and U_{av}.

$$C_A^* = f(X, Y, Re_L, S_c) \qquad \textbf{(9.74)}$$

Again,

$$h_m = -\frac{D_{AB}}{L}\frac{(C_{A,\infty} - C_{A,w})}{(C_{A,w} - C_{A,\infty})}\left.\frac{\partial C_A^*}{\partial y}\right|_{y=0}$$

$$h_m = \frac{D_{AB}}{L}\left.\frac{\partial C_A^*}{\partial y}\right|_{y=0}$$

Which is dimensionless concentration gradient at the surface.

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Sh = Sherwood number $= \frac{h_m L}{D_{AB}} = \frac{\partial C_A^*}{\partial y}\Big|_{y=0}$

$$Nu = f(X, Y, Re_L, Pr)$$
$$Sh = f(X, Y, Re_L, Sc) \qquad \textbf{(9.75)}$$

We have already seen that the significance of **Prandtl number**

$$\frac{\delta}{\delta_t} \approx Pr^n \qquad \textbf{(9.76)}$$

where **n** is a positive exponent. Hence for a gas $\delta_t \approx \delta$; for a liquid metal $\delta_t >> \delta$; for oil $\delta_t << \delta$.

Similarly, the **Schmidt number,** $\left(\frac{\nu}{D_{AB}}\right)$ provides a relative measure of the relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration boundary layers, respectively.

$$\frac{\delta}{\delta_c} = Sc^n \qquad \textbf{(9.77)}$$

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Another parameter which is related to **Pr and Sc**, is the Lewis number, It is defined as**.**

$$Le = \frac{\alpha}{D_{AB}} = \frac{Sc}{P_r} \qquad \textbf{(9.78)}$$

and is relevant to any solution involving simultaneous heat and mass transfer by convection, **From (9.76) (9.78),** it then follows that

$$\frac{\delta_t}{\delta_c} = Le^n \qquad \textbf{(9.79)}$$

The Lewis number is therefore a measure of the relative thermal and concentration boundary layer thicknesses.

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Heat and Mass Transfer Analogy

The analogy may be used to directly relate the two convection coeffiecients.

Nu and Sh are generally proportional to Pr^n and Sc^n repectively ,

Here n is a positive exponent and the value is less than 1. Anticipating this dependence, we may then use **(9.75)** to obtain

$$Nu = f(X,Y,Re_L)Pr^n \quad and \quad Sh = f(X,Y,Re_L)Sc^n$$

in which case

$$\frac{Nu}{Pr^n} = f_1(X,Y,Re_L) = f_2(X,Y,Re_L) = \frac{Sh}{Sc^n}$$

or,

$$\frac{hL/k}{Pr^n} = \frac{h_m L/D_{AB}}{Sc^n}$$

or,

$$\frac{h}{h_m} = \frac{k}{D_{AB}Le^n} = \rho C_p Le^{1-n} \qquad \textbf{(9.80)}$$

For a wide range of application , n = $\frac{1}{3}$

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9.16 Evaporate Cooling

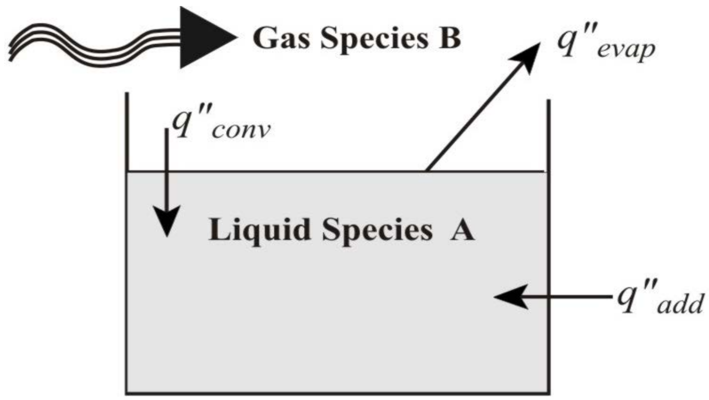

Figure 9.11: Evaporation from a liquid surface

Evaporation must occur from the liquid surface.
Energy associated with the phase change is **LATENT HEAT OF VAPORIZATION** of the liquid.

Evaporation occurs when liquid molecules near the surface experience collisions that increase their energy above that needed to overcome the surface binding energy.

Energy required to sustain the evaporation must come from the internal energy of the liquid, which then must experience a reduction in temperature yielding **cooling effect**.

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If steady state conditions are to be maintanied, the latent energy lost by the liquid because of evaporation must be replenished by energy transfer to the liquid from its surroundings .

This transfer may be due to the convection of sensible energy from the gas or to the heat addition by other means.

$$q^{"}_{conv} + q^{"}_{add} = q^{"}_{evap} \qquad \textbf{(9.81)}$$

$$q_{evap} = n^{''}_{A} h_{fg} = \mu_A N^{''}_{A} h_{fg} \qquad \textbf{(9.82)}$$

If there is no heat addition by other means, **equation (9.81)** will reduce to

$$h(T_\infty - T_w) = h_{fg} h_m [\rho_{A,sat}(T_w) - \rho_{A,\infty}] \qquad \textbf{(9.83)}$$

The magnitude of cooling effect

$$T_\infty - T_w = h_{fg} \left(\frac{h_m}{h}\right) [\rho_{A,sat} - \rho_{A,\infty}] \qquad \textbf{(9.84)}$$

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$$T_\infty - T_w = \frac{h_{fg} M_A}{\Re\, \rho C_p Le^{2/3}} \left[\frac{\rho_{A,sat}}{T_w} - \frac{\rho_{A,\infty}}{T_\infty} \right] \qquad \textbf{(9.85)}$$

The gas **(species B)** properties ρ, c_p and Le should be evaluated at the arithmatic mean temperature of the thermal boundary layer , $T_{am} = (T_w + T_\infty)/2$.

Equation (9.85) may generally be applied to a good approximation.
A somewhat less accurate, but more convenient, form may be obatined by assuming that T_W **and** T_∞ are approximately equal to T_{am} . Accordingly.

$$(T_\infty - T_w) \approx \frac{M_A h_{fg}}{\Re C_p\, L_e^{2/3} \rho T_{am}} [p_{A,sat} - p_{A,\infty}] \qquad \textbf{(9.86)}$$

Recongnizing that $m_A << m_B$, we may introduce the expression $\rho T_{am} = p/(\Re/M_B)$ from the perfect gas law to obtain

$$(T_\infty - T_w) = \frac{M_A/M_B h_{fg}}{C_p\, L_e^{2/3}} \left[\frac{p_{A,sat}}{p} - \frac{p_{A,\infty}}{p} \right] \qquad \textbf{(9.87)}$$

In numerous environmental and industrial applications, Eqn. **(9.87)** is used with a good degree of confidence.

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External Flows:

(a) Laminar Flows

For convection heat and mass transfer at a surface of uniform temperature or species concentration, respectively the $\mathrm{Nu_x}$ and $\mathrm{Sh_x}$ are of the form.

$$Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3}$$

$$Sh_x = \frac{h_{m,x} x}{k} = 0.332\, Re_x^{1/2}\, Sc^{1/3} \qquad \textbf{(9.88)}$$

From the above, the average coeffiecients are calculated as:

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = 0.664\, Re_L^{1/2}\, Pr^{1/3}$$

$$\overline{Sh}_L = \frac{\bar{h}_m L}{k} = 0.664\, Re_L^{1/2}\, Sc^{1/3} \qquad \textbf{(9.89)}$$

(b) Turbulent Flows

$$\frac{\delta}{x} = 0.37 Re_x^{-1/5}$$

$$Nu_x = 0.296\ Re_x^{4/5}\ Pr^{1/3} \qquad \textbf{(9.90)}$$

$$Sh_x = 0.296\ Re_x^{4/5}\ Sc^{1/3}$$

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however $\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{lam}\, dx + \int_{x_c}^{L} h_{turb}\, dx\right), \bar{h}_{m,L} = \frac{1}{L}\left(\int_0^{x_c} h_{m,lam}\, dx + \int_{x_c}^{L} h_{m,turb}\, dx\right)$

finally we obtain **(for plate with large length)**

$$Nu_L = (0.037\ Re_L^{4/5} - 871)\ Pr^{1/3}$$

$$\left\{\begin{array}{l} 0.6 < Pr < 60 \\ 5 \times 10^5 < Re_L \leq 10^8 \\ Re_{x,c} = 5 \times 10^5 \end{array}\right\} \qquad \textbf{(9.91)}$$

$$Sh_L = \left(0.037 Re_L^{4/5} - 871\right) Sc^{1/3}$$
$$\left\{\begin{array}{c} 0.6 < Sc < 3000 \\ 5\ X\ 10^5 < Re_L \leq 10^8 \\ Re_{x,c} = 5\ X\ 10^5 \end{array}\right\} \qquad \textbf{(9.92)}$$

Finally for $L \gg x_c$,

$Nu_L = 0.037\ Re_L^{4/5}\ Pr^{1/3}$

$Sh_L = 0.037\ Re_L^{4/5}\ Sc^{1/3}$

$\bar{C}_{f,L} = 0.074\ Re_L^{4/5}$