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Dynamics of Bubble Growth on a Wall

The size to which a bubble attached to the wall will grow before detaching depends on balance between several forces acting on the bubble. In this section we identify these forces. We will also determine the forces which determine the terminal velocity of a bubble rising through a pool of liquid.

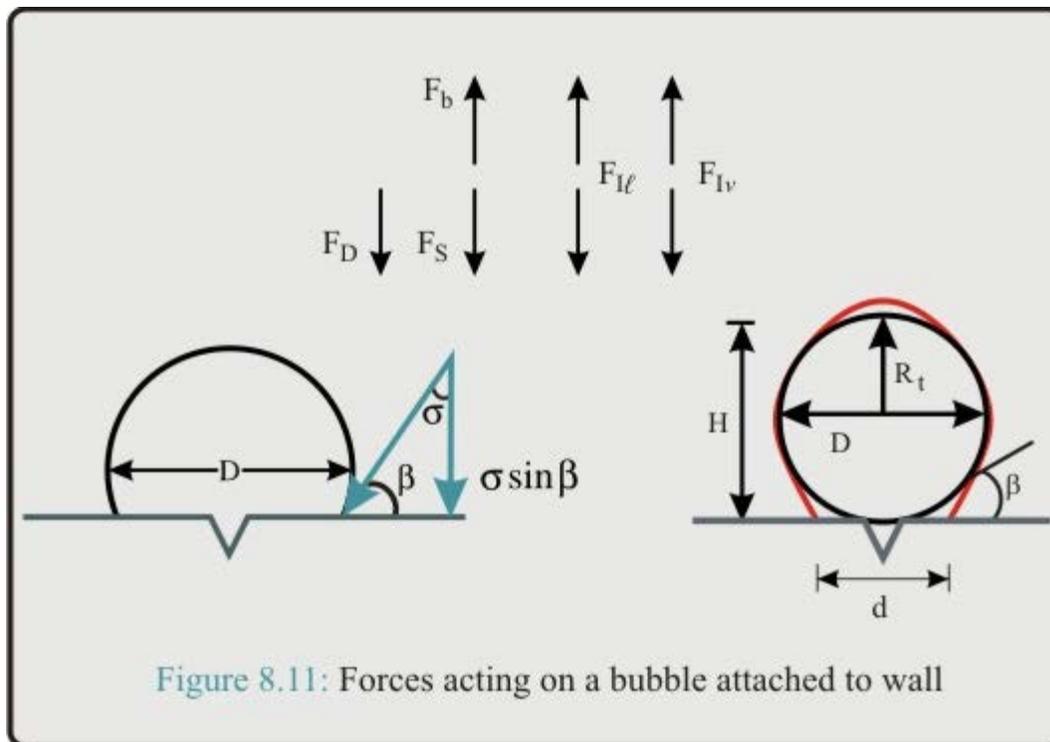


Figure 8.11: Forces acting on a bubble attached to wall

Forces Acting on a Bubble on a wall

In 1935, Fritz proposed that main forces acting on a bubble were the surface tension and buoyancy. The surface tension, tends to hold the bubble to the wall while buoyancy tries to lift it away. Referring to the spherical bubble shown in the left hand sketch in Fig. 8.11, the surface tension force is

$$F_s = \pi D \sigma \sin^2 \beta$$

(8.49)

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The buoyancy force is

$$F_B = \frac{\pi D^3}{6}(\rho_l - \rho_v)g \left[\frac{1 + \cos\beta}{2} \right]^2 [2\cos\beta] \quad (8.50)$$

Equating the surface tension and buoyancy force from **equations (8.48) and (8.49)** we get

$$D_d = f(\beta) \sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}} \quad (8.51)$$

where

$$f(\beta) = \left[\frac{24\sin^2\beta}{2 + \cos\beta (2 - \cos\beta)} \right] \quad (8.52)$$

Fritz correlated his experimentally observed bubble diameters at departure as

$$D = 0.0208\beta \sqrt{\frac{\sigma}{g(\rho_l - \rho_v)}} \quad (8.53)$$

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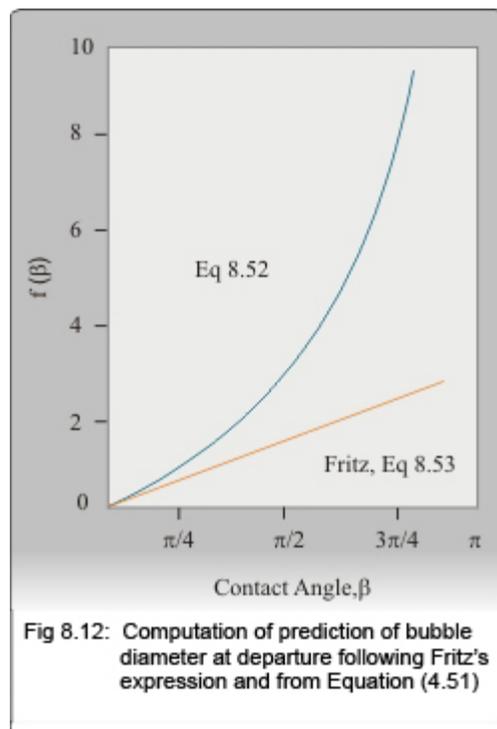
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where β now is the contact angle measured in degrees.

Figure 8.12 shows the dependence on β of function $f(\beta)$ obtained from **equation (8.52)** and the equivalent function obtained by **Fritz** and given in **equation (8.53)**. The difference between the two expressions is probably reflective of the non-sphericity of the bubbles and non inclusion of other forces which act on the bubble. However, for contact angles smaller than $\pi/2$, the difference between the two expressions is rather small. As pointed out earlier, for wettable surfaces the contact angle is smaller than $\pi/2$.

Bubbles are generally not totally spherical and as many investigations such as those of



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Siegel and Keshock (1964), and Cochran and Aydelot (1966) have shown, the bubbles not only experience surface tension and buoyancy forces during their growth but several other forces come into play. These additional forces are those of inertia and drag. Since these forces act both on volume and surface, it is difficult to determine a common equivalent diameter of a non spherical bubble.

If we define an equivalent diameter based on bubble **volume** as

$$D = \left(\frac{6V}{\pi} \right)^{1/3}$$

The various forces acting on a bubble such as that shown in the right hand sketch of **Fig.8.11**.

(i) Buoyancy Forces,

$$F_b = \frac{\pi}{6} D^3 (\rho_l - \rho_v) g \quad (8.54)$$

(ii) Surface Tension Force,

$$F_s = \pi d \sigma \sin \beta \quad (8.55)$$

(iii) Liquid Inertia Force,

$$F_{II} = K_I \rho_l \frac{d}{dt} \left(\frac{\pi}{12} D^3 \frac{dD}{dt} \right) \quad (8.56)$$

In the above equation, K_I is the coefficient defining the virtual mass of the liquid surrounding the bubble and $\frac{1}{2} \frac{dD}{dt}$ is the velocity of center of the bubble.

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(iv) Bubble Inertia Force,

$$F_{Iv} = \rho_v \frac{d}{dt} \left(\frac{\pi}{12} D^3 \frac{dD}{dt} \right) \quad (8.57)$$

At low pressure the vapor density ρ_v is much smaller than the liquid density ρ_l . Thus the inertia of the bubble in comparison to that of the liquid can be ignored.

(v) Viscous Drag Force,

$$F_D = C_D \frac{\rho_l}{2} \frac{\pi}{4} D^2 \left(\frac{dD}{dt} \right)^2 \quad (8.58)$$

In writing **equation (8.58)**, the velocity of the top surface of the bubble is used instead of the velocity of the center of the bubble. The constant C_D is the drag coefficient. The buoyancy force written as **equation (8.54)** is for a body fully submerged in a liquid. Since the bubble is attached to the wall, the buoyancy force as has been suggested by **Cochran and Aydelot** should be corrected such that.

$$F_B = (\rho_l - \rho_v) g \left(\frac{\pi}{6} D^3 - \frac{\pi}{4} d^2 H \right) + \frac{\sigma}{R_t} \frac{\pi}{4} d^2 \quad (8.59)$$

In the above equation the second term accounts for excess pressure in the bubble and is written by assuming that the bubble top has no curvature in a plane normal to the plane of the Figure.

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