Module 3: Internal Flows

Lecture 14: Physical Explanation of Surface Conditions

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Thermal Considerations during Internal Flows

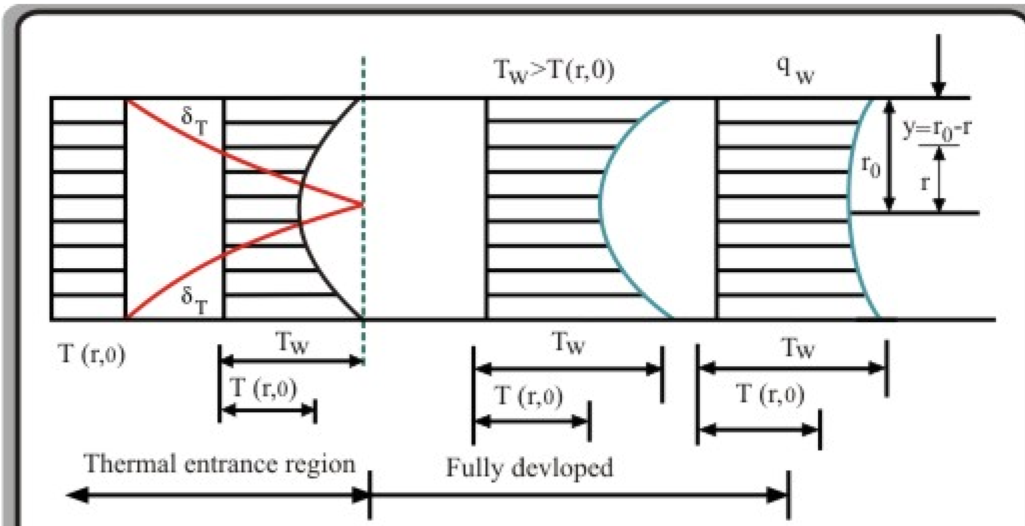

Figure 3.4 Thermally developing flow in a duct

Tube surface condition: either a uniform wall temperature **(UWT)** condition or a uniform wall heat ux condition **(UWH)**. A fully developed temperature prole diers according to the surface conditions. For

both surface conditions, however, the amount by which the fluid temperature exceeds the entrance temperature increases with increasing (x) **(Figure3.4).** In order to study the evelopment of thermal boundary layer,a hydrodynamically fully developed dow may be considered. **(figure3.5)**

1. For laminar flow, the thermal entrance length may be expressed as

$$\left(\frac{x_{e,t}}{D}\right)_{lam} \approx \begin{cases} 0.033 Re_D\ Pr & \text{for UWH} \\ 0.043 Re_D\ Pr & \text{for UWT} \end{cases} \qquad \textbf{(3.20)}$$

we can say,$(x_{e,t/D}) = 0.05\ Re_D\ Pr$

Comparing this with hydrodynamics boundary layer, it can be said that if $Pr > 1$,hydrodynamic boundary layer grows more rapidly.

$Pr > 1,\ x_{e,h} < x_{e,t} \Rightarrow \delta > \delta_T$ at any section; even otherwise $Pr \sim (\delta/\delta_T)^{1/2}$.

Therefore, for $Pr > 1,\ \delta$ has to be $> \delta_T$

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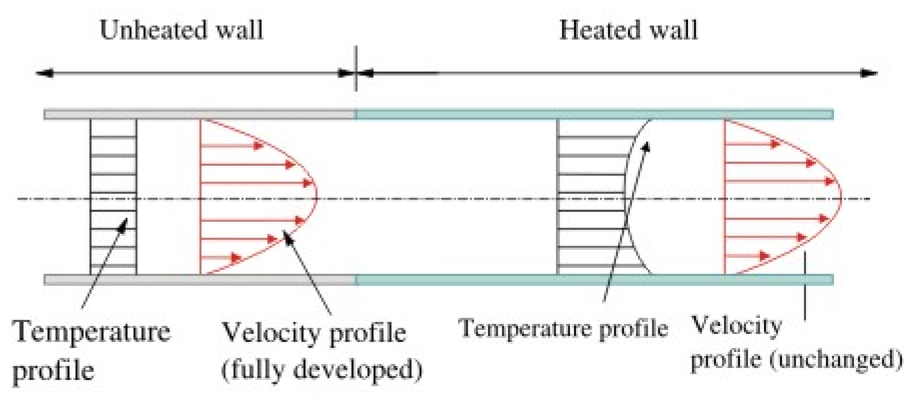

Figure 3.5 : Developing velocity and temperature fields

Inverse is true for $Pr < 1. Pr \sim (\delta/\delta_T)^{1/3}$ and consequently δ_T is greater than δ However,for extremely High Prandtl number fluids (such as oil), $Pr \geq 100$, $x_{e,h} << x_{e,t}$.Throughout the thermal entrance region, hydrodynamically fully developed velocity profile can be assumed **(figure3.6).**

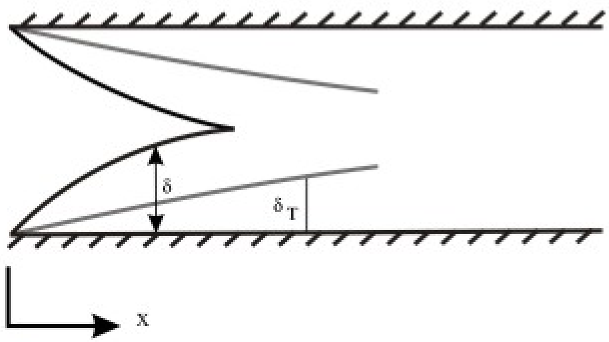

figure 3.6: Hydrodynamic and thermal boundary layer thickness

figure 3.6: Hydrodynamic and thermal boundary layer thickness $Pr >>$ 1 for For turbulent flow, conditions are nearly independent of Prandtl number.It is fair enough to accept $\left(\frac{x_{e,t}}{D}\right)_{tur} = 10$ Bulk mean temperature at any section (x) is given by

$$T_m = \frac{2}{u_m r_0^2} \int_0^{r_0} u(r) T(r) r dr \tag{3.21}$$

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Newton's law of cooling

$$q_w^{''} = h(T_w - T_m) \qquad \textbf{(3.22)}$$

The bulk mean temperature T_m is a convenient reference temperature for internal flows T_m behaves like T_∞ for external flows.Essential difference is T_m varies in the flows direction. T_m is increased in the flows direction if heat transfer is from surface to fluid.

Thermally Fully developed conditions

Since the existence of convective heat transfer between the surface and the fluid dictates that the fluid temperature must continue to change with x, one might question whether fully developed thermal conditions ever can be reached.The situation is certainly dierent from the hydrodynamic case , for which $(\partial u/\partial x) = 0$ in the fully developed region. **In contrast, if there is heat transfer, (dT_m/dx) as well as $(\partial T/\partial x)$ at any radius is not zero.**

Introducing a dimensionless temperature $(T_w - T)/(T_w - T_m)$, condition for which this ratio becomes independent of x are known to exist. Although the temperature profile $T(r)$ continues to change with x, the relative shape of the profile does not change and the flow is said to be fully developed.instead of $(dT_m/dx) = 0$ or $(dT/dx) = 0$, the condition is

$$\frac{d}{dx}\left[\frac{T_w(x) - T(r,x)}{T_w(x) - T_m(x)}\right] = 0 \qquad \textbf{(3.23)}$$

i.e., $(T_w - T)$ changes in the same way as $(T_w - T_m)$

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Now, we can write

$$\frac{(T_w - T_m)\left(\frac{dT_w}{dx} - \frac{dT}{dx}\right) - (T_w - T)\left(\frac{dT_w}{dx} - \frac{dT_m}{dx}\right)}{(T_w - T_m)^2} = 0$$

or

$$\frac{1}{(T_w - T_m)}\frac{dT_w}{dx} - \frac{dT}{dx}\frac{1}{(T_w - T_m)} - \frac{T_w - T}{(T_w - T_m)^2}\frac{dT_w}{dx} + \frac{(T_w - T)}{(T_w - T_m)^2}\frac{dT_m}{dx} = 0$$

or

$$\frac{dT_w}{dx} - \frac{dT}{dx} - \frac{T_w - T}{T_w - T_m}\frac{dT_w}{dx} + \frac{(T_w - T)}{(T_w - T_m)}\frac{dT_m}{dx} = 0$$

or

$$\frac{dT}{dx} = \frac{dT_w}{dx} - \frac{T_w - T}{T_w - T_m}\frac{dT_w}{dx} + \frac{(T_w - T)}{(T_w - T_m)}\frac{dT_m}{dx} \qquad \textbf{(3.24)}$$

Two surface conditions are available.**uniform heat flux,** $(q_w'' = \text{constant})$ and **uniform surface temperature** , $(dT_w/dx = 0)$.

For $(q_w'' = \text{constant})$:If the tube wall were heated electrically or, if the outer surface were uniformly irradiated.

For $(dT_w/dx) = 0$: if the tube were occurring at the outer surface .

However, **for constant heat flux,** we get

$$\frac{dT_w}{dx} = \frac{dT_m}{dx}$$

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Substituting this condition in the above equation for dT/dx, we get

$$\left.\frac{dT}{dx}\right|_{\text{fully developed}} = \left.\frac{dT_m}{dx}\right|_{\text{fully developed}} \quad \textbf{(3.26)}$$

For constant surface temperature, $\frac{dT_w}{dx} = 0$

$$\left.\frac{dT}{dx}\right|_{fd,t} = \frac{(T_w - T)}{(T_w - T_m)} \left.\frac{dT_m}{dx}\right|_{fd,t} \quad \textbf{(3.27)}$$

Several important features of thermally developed flow may be inferred from **equation (3.24)**. Since nondimensional temperature profile is independent of x , derivative of the profile with respect to r (gradient) must also be independent of x . **Evaluating this derivative at the tube surface, we obtain**

$$\frac{\partial}{\partial r}\left[\frac{T_w - T}{T_w - T_m}\right]_{r=r_0} = \frac{-\partial T/\partial r|_{r=r_0}}{(T_w - T_m)} \neq f(x) \quad \textbf{(3.28)}$$

Again we know

$$q_w'' = k\frac{\partial T}{\partial r}|_{r=r_0} = h(T_w - T_m)$$

or,

$$\frac{-\frac{\partial T}{\partial r}|_{r=r_0}}{(T_w - T_m)} = \frac{h}{k} \quad \textbf{(3.29)}$$

From (3.28) and (3.29), we get, $\frac{h}{k} \neq f(x)$.for fully developed flow **(Figure 3.7)**. Herk is assumed to be constant, **, hence h is independent of.**

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Module 3: Internal Flows:

Lecture 14: Physical Explanation of Surface Conditions

Energy Balance in Ducted Flows

Because the flow in a tube is completely enclosed, an energy balance may be applied to determine how the mean temperature (T_m) varies with position along the tube and how the total convective heat transfer is related to the dierence in temperatures at the tube inlet outlet. **Consider Figure 3.8.**

$$dq_{conv} + \dot{m}(c_v T_m + pv) - \left[\dot{m}(c_v T_m + pv) + \dot{m}\frac{d(c_v T_m + pv)}{dx}dx\right] = 0$$

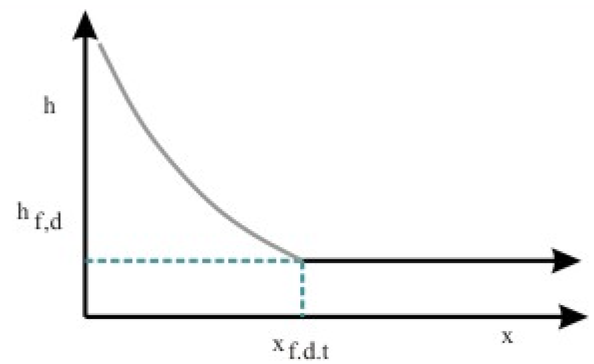

Figure 3.7
Heat transfer coefficient for thermally fully developed flow

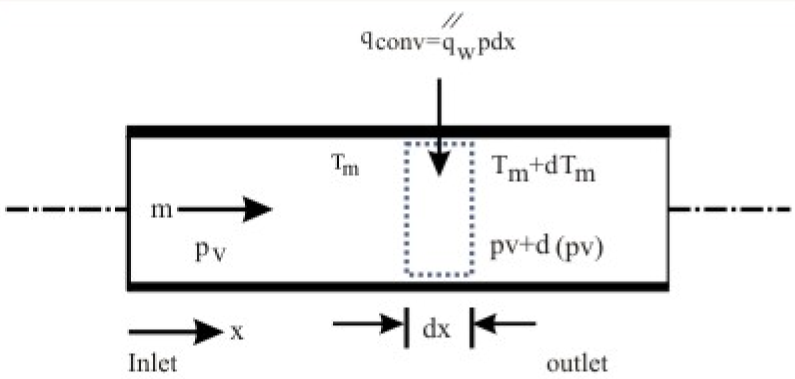

Figure 3.8: Energy balance in apipe flow

or

$$dq_{conv} = \dot{m}\ d(c_v T_m + pv) \qquad \textbf{(3.30)}$$

For compressible flow:

$$dq_{conv} = \dot{m}\ d(c_v T_m + pv)$$

or

$$dq_{conv} = \dot{m}\ d(c_v T_m + RT_m)$$

or,

$$dq_{conv} = \dot{m} c_p dT_m [\text{since } d(pv) = 0 \text{ and } c_p = c_v]$$

or,

$$dq_{conv} = \dot{m}\ c_p dT_m \qquad \textbf{(3.31)}$$

For incompressible flow:

$$dq_{conv} = \dot{m} c_p dT_m [\text{since } d(pv) = 0 \text{ and } c_p = c_v]$$

Therefore, whether the flow is compressible or incompressible, we can write

$$q_{conv} = \dot{m} c_p (T_{m,0} - T_{m,i}) \qquad \textbf{(3.32)}$$

we can also write

$$q_{conv} = \dot{m} c_p (T_{m,0} - T_{m,i})$$

or,

$q_w'' P dx = \dot{m} c_p dT_m$ [P is the surface perimeter = πD]

or,

$$\frac{dT_m}{dx} = \frac{q_w'' P}{\dot{m} c_p} = \frac{Ph(T_w - T_m)}{\dot{m} c_p} \qquad \textbf{(3.33)}$$

So long, $T_w > T_m$, heat is transferred to fluid and T_m increases with x. The manner in which **equation (3.33)** varies should be noted.

Constant surface temperature:

P may vary with x, but it is constant for pipe. $P/\cdot mc_p$ is constant. For fully developed flow, h is constant, although it varies with x, in the entrance region. Finally T_w is constant. So, T_m will vary with x except for the trivial case $(T_w = T_m)$ of no heat transfer **(Figure 3.9).**

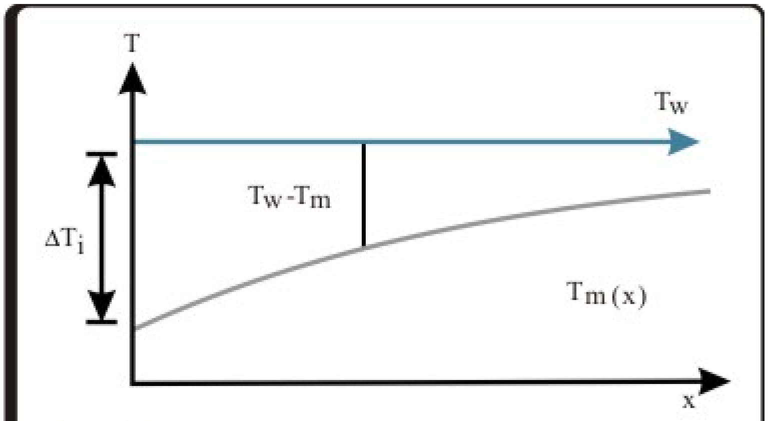

Figure 3.9: Variation of bulk-mean temperature in a tube (Constant Wall Temperature)

Constant heat flux:

q_w'' is independent of x,. Total heat transfer rate $q_{conv}^{total} = q_w'' PL$; again, from equation

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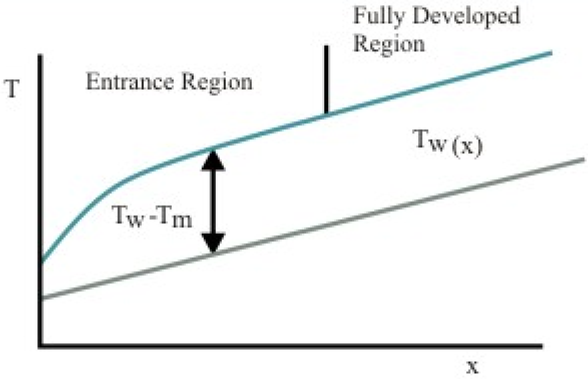

Figure 3.10: Variation of bulk-mean temperature in a tube (Constant wall Heat Flux)

(3.30-3.32) , $q_w^{''} PL = \dot{m} c_p (T_{m,0} - T_{m,i})$. so if $q_w^{''},\ \dot{m},\ c_p$ and the geometry are known, it is possible to calculate the fluid temperature rise $(T_{m,0} - T_{m,i})$

otherwise

$$\frac{dT_m}{dx} = \frac{q_w^{''} P}{\dot{m} c_p} \text{ or } T_m = \frac{q_w^{''} P}{\dot{m} c_p} x + c_1$$

at $x = 0$, $T_m = T_{m,i}$ therefore $c_1 = T_{m,i}$

$$T_m(x) = T_{m,i} + \frac{q_w^{''} P}{\dot{m} c_p} x \tag{3.34}$$

We can see the variation $T_m(x)$ with x, for constant wall heat flux **(Figure 3.10)** .Let us now find out the mathematical relation for variation of $T_m(x)$ with x, for constant wall temperature case.

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Mathematical relation for variation of $T_m(x)$ for constant wall temperature

P may vary with x, but it is constant for pipe. $P/\cdot mc_p$ is constant. For fully developed flow, h is constant, although it varies with x, in the entrance region. Finally T_w is constant. So , T_m will very with x, except for the trivial case $(T_w = T_m)$ of no heat transfer. From **equation (3.33)**, $\frac{dT_m}{dx} = \frac{q_w'' P}{\dot{m} c_p} = \frac{Ph(T_w - T_m)}{\dot{m} c_p}$, defining $\Delta T = T_w - T_m =$ difference between surface temperature and mean temperature.

$$\frac{dT_m}{dx} = -\frac{d(\Delta T)}{dx} = \frac{Ph(\Delta T)}{\dot{m} c_p} \tag{3.35}$$

$$\int_{\Delta T_i}^{\Delta T_0} \frac{d(\Delta T)}{(\Delta T)} = \frac{-P}{\dot{m} c_p} \int_0^L h dx$$

or,

$$\frac{(\Delta T_0)}{(\Delta T_i)} = -\frac{PL}{\dot{m} c_p} \cdot \frac{1}{L} \int_0^L h dx$$

or,

$$\frac{\Delta T_0}{\Delta T_i} = -\frac{PL}{\dot{m} c_p} \overline{h}_L$$

$$\frac{\Delta T_0}{\Delta T_i} = \frac{T_w - T_{m,0}}{T_w - T_{m,i}} = exp\left(-\frac{PL}{\dot{m} c_p} \bar{h}_L\right) \tag{3.36}$$

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Had we integrated from the tube inlet to some axial position $x,$ withthin the tube, we would have obtained the similar, but more general expression.

$$\frac{T_w - T_m(x)}{T_w - T_{m,i}} = \frac{T_w - T_m(x)}{T_w - T_{m,i}} = exp\left(-\frac{Px}{\dot{m}c_p}\bar{h}\right) \quad \textbf{(3.37)}$$

where $\bar{h}$ is now the average value of h from the inlet to the distance $x,$. This result suggests that the temperature difference $(T_w = T_m)$ decays exponentially **(also clear from the figure shown earlier)**. we can also write

$$q_{conv} = \dot{m}\ c_p(T_{m,0} - T_{m,i})$$

or,

$$q_{conv} = \dot{m}\ c_p[(T_w - T_{m,i}) - (T_w - T_{m,0})]$$

or,

$$q_{conv} = \dot{m}\ c_p[(T_w - T_{m,i}) - (T_w - T_{m,0})] \quad \textbf{(3.38)}$$

On the other hand we have seen that

$$\text{In}\ \frac{\Delta T_0}{\Delta T_i} = -\frac{PL}{\dot{m}c_p}\bar{h}_L \quad \textbf{(3.39)}$$

Substituting for $\dot{m}c_p$ in **(3.38) from (3.39) we get**

$$q_{conv} = \frac{\bar{h}_L PL(\Delta T_0 - \Delta T_i)}{\ln(\Delta T_0/\Delta T_i)} = \bar{h}_{av} A_w \Delta T_{l,m} \quad \textbf{(3.40)}$$

This is a form of **Newton's law of cooling for the entire tube**. $\Delta T_{l,m}$ is the appropriate average of temperature difference for the entire tube. The log nature of this average temperature difference [in contrast to an arithmetic mean temperature difference of the form $\Delta T_m = \frac{\Delta T_i + \Delta T_0}{2}$ **is due to exponential nature of the curve.**

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