Module 9: Mass Transfer

Lecture 36: Preliminaries

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Introduction:-

When a system contains one or more components whose concentration vary from point to point, there is a natural tendency for mass to be transported or diffused minimizing the concentration difference within the system.

The mechanism of mass transfer can be understood by drawing an analogy to heat transfer as the following:

1. Heat is transferred towards the lower temperature **(decreasing temperature gradient).** Mass is transferred towards the lower concentration **(decreasing the concentration gradient).**

2. Heat transfer ceases when there is no longer a temperature difference. Mass transfer ceases when concentration gradient is reduced to zero.

3. The rates of heat and mass transfer depend on the driving potential and resistance.

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Fick's Law of Diffusion

The rate equation of mass diffusion is known as **Fick's law.** So the transfer of species A in a binary mixture of **A and B,** it may be expressed as.

Fick's Law of diffusion states that the mass crossing a unit area per unit time is proportional to the gradient of conentartion.

$$\text{Mass flux of species } A = j_A = -\rho D_{AB} \nabla m_A \qquad \textbf{(9.1)}$$

$$j_A \text{ is in } \left[\frac{kg}{sm^2}\right], \rho = \text{mass density} = \rho_A + \rho_B, m_A = \text{mass fraction } \frac{\rho_A}{\rho}$$

$$\text{Molar flux of species } A = j_A^* = -C D_{AB} \nabla x_A \qquad \textbf{(9.2)}$$

$$j_A^* \text{ is in } \left[\frac{k\,mol}{s\,m^2}\right], C = \text{molar concentration } C_A + C_B\ ,\ x_A = \frac{C_A}{C}$$

$$D_{AB} = \text{mass diffusivity}$$

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Restrictive conditions

Mass diffusion is more complicated than heat diffusion. Complications are associated with two restrictive conditions:

1. Mass diffusion may result from a temperature gradient, a pressure gradient or an external force. Here we are assuming that these are not present or are negligible. If this is the case and the dominant driving potential is the species concentration gradient, the condition is referred to as ordinary diffusion.

2. The second restrictive condition is that the fluxes are measured relative to the coordinates that move with an average velocity of the mixture. If the mass flux of a species is expressed relative to a fixed set of coordinates, **equations (9.1) and (9.2)** are not generally valid.

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To obtain an expression for the mass flux relative to a fixed coordinates system, consider a species **A** in a binary mixture of **A** and **B**. The mass flux n_A'' relative to a fixed coordinate system is related to the species absolute velocity V_A by.

$$\text{mass flux} \rightarrow n_A'' = \rho_A V_A \qquad \textbf{(9.3)}$$

The value V_A may be associated with any point in the mixture and it is interpreted as the average velocity of all the particles in a small volume element about the point. An average or aggregate velocity may also be associated with the particles of species **B**, in which case .

$$n_B'' = \rho_B V_B \qquad \textbf{(9.4)}$$

A mass average velocity for the mixture may then be obtained from the requirement that

$$\rho V = n'' = n_A'' + n_B'' = \rho_A V_A + \rho_B V_B \qquad \textbf{(9.5)}$$

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The above equation may also be written as

$$V = m_A V_A + m_B V_B \qquad \textbf{(9.6)}$$

We may now define the mass flux of species **A** relative to mixture mass average velocity as

$$j_A = \rho_A (V_A - V) \qquad \textbf{(9.7)}$$

Whereas n''_A is absolute flux of species A, j_A is the relative or diffusive flux of the species. It represents the motion of the species relative to the average motion of mixture **.**

$$n''_A = j_A + \rho_A V \qquad \textbf{(9.8)}$$

This expression indicates that there are two distinct contributions to the absolute flux of the species A: a contribution due to diffusion **(due to the motion of A relative to the mass-average motion of the mixture)** and a contribution due to motion of A with the mass-average motion of the mixture

$$n''_A = -\rho D_{AB} \nabla m_A + m_A \left(n''_A + n''_B\right) \qquad \textbf{(9.9)}$$

Compare (9.1) and (9.9)

The form given by **(9.1)** determines the transport of A relative to the mixture mass - average velocity, whereas the form given by **(9.9)** determines the absolute transport of A. The foregoing considerations may be extended to species B. **The mass flux of B relative to the mixture mass-average velocity (the diffusive flux) is**

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$$j_B = \rho_B(V_B - V) \qquad \textbf{(9.10)}$$

Where , $j_B = -\rho D_{BA} \nabla m_B$ **(9.11)**

It follows from equations $(9.5)\ [\rho V = n''_A + n''_B = \rho_A V_A + \rho_B V_B]$, $(9.7)\ [j_A = \rho_A(V_A - V)]$ and $(9.10)\ [j_B = \rho_B\ (V_B - V)]$ that the diffusive fluxes in a binary mixture can be related by **.**

$$j_A + j_B = 0 \qquad \textbf{(9.12)}$$

If equations (9.1) $[j_A = -\rho D_{AB} \nabla m_A]$ and (9.11) $(j_B = -\rho D_{AB}\ \nabla m_B)$ are substituted into (9.12) and it is recognized that $\nabla m_A = -\nabla m_B$, since $m_A + m_B = 1$ for a binary mixture, it follows that

$$D_{BA} = D_{AB} \qquad \textbf{(9.13)}$$

Hence , as in equation (9.9) $[n''_A = -\rho D_{AB} \nabla m_A + m_A(n''_A + n''_B)]$, the absolute flux of species B may be expressed as

$$n''_B = -\rho . D_{AB} \nabla m_B + m_B(n''_A + n''_B) \qquad \textbf{(9.14)}$$

Although the foregoing expressions correspond to **mass fluxes** , the same procedures can be used to obtain results in a **molar basis** . The **absolute molar fluxes** of **species A and B** may be expressed as

$$N''_A = C_A V_A \ and \ N''_B = C_B V_B \qquad \textbf{(9.15)}$$

and a molar average velocity ,V^* , is obtained from the requirement that

$$N'' = N''_A + N''_B = CV^* = C_A V_A + C_B V_B \qquad \textbf{(9.16)}$$

The quantity V^* in (9.16) is given by

$$V^* = x_A v_A + x_B v_B \qquad \textbf{(9.17)}$$

$x_A \ and \ x_B$ are called mole fractions.

The significance of molar-average velocity is that when multiplied by the total molar concentration C , it provides the total molar flux with respect to a fixed coordinate system.Equation (9.15) $[N''_A = C_A V_A \ and \ N''_B = C_B V_B]$ provides the absolute molar flux of species A and B. In contrast , the molar flux of A relative to the mixture molar average velocity , j^*_A , termed as the diffusive flux , may be obtained from equation (9.2) $[j_A^* = -CD_{AB}\nabla x_A]$ as

$$j_A^* = C_A(V_A - V^*) \qquad \textbf{(9.18)}$$

To obtain an expression similar in form to (9.9) , we combine (9.15) and (9.18) to yield

$$N_A'' = j_A^* + C_A V^* \qquad \textbf{(9.19)}$$

Invoking (9.2) $[j_A^* = -CD_{AB}\nabla x_A]$ and (9.16) $[N'' = N_A'' + N_B'' = CV^*]$ in (9.19) , we get

$$N_A'' = -CD_{AB}\nabla x_A + x_A(N_A'' + N_B'') \qquad \textbf{(9.20)}$$

Compare the molar fluxes given by (9.2) and (9.20) . In the first case the molar flux is relative to the mixture molar-average velocity and in the second case it is the absolute molar flux. Also note that (9.20) represents the absolute molar flux as the sum of a diffusive flux and a flux due to the bulk motion of the mixture. For the binary mixture it also follows that

$$j_A^* + j_B^* = 0 \qquad \textbf{(9.21)}$$

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