The Lecture Contains:

Previous Next

Design of Experiments

Even after the objectives of an experiment are laid out, various possibilites for the route to be adopted can emerge. Thus, two main questions that arise during experimentation are: (1) Among the various options, find the one that is the best, subject to constraints of accuracy and cost; (2) For a given experiment, what is the best action plan, in terms of spacing between data points, duration of the experiment and other such decision variables. These questions are addressed in the theory of design of experiments. The subject is illustrated below through examples.

Example 1: The response of a system $f(t)$ is expected to vary as

$$f = C\,t\exp(1-t)$$

An experiment is designed to determine the constant C by a least squares procedure. It is required that data be collected at equal time intervals with a total of N points.What should be the duration of the test? Let T be the total time needed for the experiment. If T is very small, the decaying feature of the function f is lost. If T is very large, Δt the time step $(= T/(N-1))$ also becomes large and results in large resolution errors.

Previous Next

Clearly an optimum exists. The optimum time duration is found using the following criterion. If σ^2 is the variance in the data of $f(t)$, that in C will be

$$\sigma_c^2 = \frac{\sigma^2}{\Delta}$$

where

$$\Delta = \frac{1}{T}\int_0^T f^2(t)dt$$

Hence, minimizing σ_c^2 is equivalent to maximizing the quantity Δ with respect to experimental test time T and so

$$\frac{d\Delta}{dT} = 0$$

This can be simplified to the integral equation

$$\int_o^T f^2(t)dt = Tf^2(T)$$

and the test duration T can be found by a root-finding procedure.

Module 1: Introduction to Experimental Techniques

Lecture 5: Design of experiments

Example 2: It is required to determine the cooling rates of a heated circular cylinder exposed to an air stream in terms of the heat transfer coefficient h. This coefficient is defined as the energy lost per unit area per unit temperature difference between the cylinder and the air stream. Two experiments are proposed for estimating h. In the first, the cylinder is heated to a temperature T_1 and suddenly placed in the air stream at a temperature T_a. The heat transfer coefficient is then obtained from the cooling curve of the cylinder. In functional form, the cooling curve is given as

$$\frac{T - T_a}{T_1 - T_a} = \exp\left(-\frac{hA}{mC_p}t\right)$$

where T is the cylinder temperature at any time t, A is the surface area, m is the mass of the cylinder and C is its specific heat. The experiment is stopped when the left side of the above equation is $\exp(-1)$ for which $t = \tau$ and

$$\frac{hA}{mC_p} = \frac{1}{\tau}$$

Previous Next

In the second experiment, the measurements are carried out at steady state by continuously heating the cylinder. This can be accomplished by directly passing current through the cylinder, or by using electric rod heaters, held concentrically within the cylinder. Depending on the air velocity the cylinder will attain a temperature T_F. From energy balance, one can write

$$\dot{Q} = hA(T_F - T_a)$$

where $\dot{Q}$ is the electrical heater input to the cylinder.

The question here is: Of the transient and steady state experiments identify the method to be preferred for obtaining the heat transfer coefficient. To answer this question, one needs to realize that the transient method relies on measurement of time (τ)while the steady state method relies on the measurement of electrical power $\dot{Q}$. Assuming fixed uncertainties in time and electrical power ($= \delta\tau$ and $\delta\dot{Q}$ respectively), it is clear that the transient method is preferable if

$$\frac{\delta\tau}{\tau} < \frac{\delta\dot{Q}}{\dot{Q}}$$

and the steady state method should be adopted if

$$\frac{\delta\dot{Q}}{\dot{Q}} < \frac{\delta\tau}{\tau}$$

Previous Next

In words, the transient method is preferable if the measurement time τ large; this will happen if h itself is small, for example at low air speeds. The steady state method is to be preferred if the power input is large; this will happen if h is large, for example at high air speeds. A composite plot of the relative errors in time and power input as a function of the heat transfer coefficient is shown in Figure 1.14.

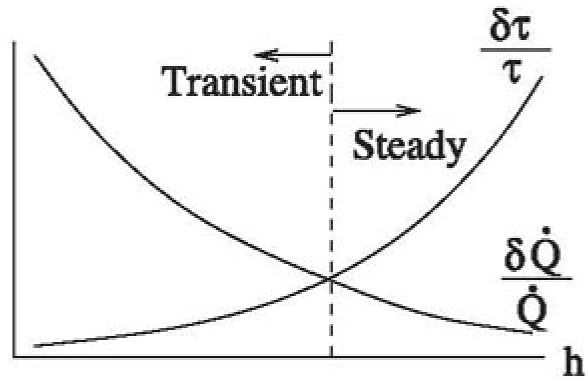

Figure 1.14: Relative Errors as a Function of the Heat Transfer Coefficient

Previous Next