Lecture 17: Wave optics and interference phenomenon

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Introduction

Optical methods of measurement are known to have specific advantages in terms of spanning a field-of-view and being inertia-free. Though in use for over half a century, optical methods have seen a resurgence over the past decade. The main factors responsible are the twin developments in the availability of cost-effective lasers along with high performance computers. Laser measurements in thermal sciences have been facilitated additionally by the fact that fluid media are transparent and heat transfer applications in fluids are abundant. Whole-field laser measurements of flow and heat transfer in fluids can be carried out with a variety of configurations: shadowgraph, schlieren, interferometry, speckle and PIV, to name a few. In the present module, temperature field measurements in fluids by laser interferometry has been addressed.

The ability to record interferograms on a PC using CCD cameras has greatly simplified image analysis. It is possible to enhance image quality and perform operations such as edge detection and fringe thinning by manipulating the numbers representing the image. Image analysis techniques have also been discussed in the present module. When combined with holography, laser interferometry can be extended to map three dimensional fields. Holographic interferometry can be cumbersome in some applications due to the need of holographic plates, particularly when large regions have to be scanned. This difficulty is circumvented by using an analytical technique called tomography. Here the interferograms are viewed as projection data of the thermal field. The three dimensional field is then reconstructed by suitable algorithms. In principle, tomography can be applied to a set of projection data recorded by shadowgraph, schlieren, interferometry or any of the other configurations. The present module covers tomography applied to interferograms recorded with a Mach-Zehnder interferometer. These comments carryover to schlieren and shadowgraph methods as well.

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It is important to comment on the physical phenomenon called scattering, the interaction of light with matter. When a beam of light $(\text{Wavelength}\,\lambda)$ falls on a particle of size d_p, the characteristic dimension, the scattered energy will show changes with respect to intensity, directionality, wavelength, phase, and other properties of the wave. The property that shows the most pronounced change depends on the ratio of the wavelength and the particle diameter d_p Broadly speaking, we have the following limits:

1. Ray Optics $\quad \frac{\lambda}{d_p} \ll 1$

2. Wave Optics $\quad \frac{\lambda}{d_p} 1$

3. Quantum Optics $\quad \frac{\lambda}{d_p} \gg 1$

In the context of interferometry, schlieren and shadowgraph, the medium is taken be transparent and hence non-scattering. On the other hand, the medium contributes to wave propagation by altering the wave speed (the speed of light). The material property of relevance is the refractive index n defined as

$$n = \frac{c}{c_o}$$

Here, c_o is the speed of light in vacuum and c is the speed of light in the transparent medium. It can be shown that the refractive index satisfies the inequality $n \geq 1$. The utility of refractive index in measurements arises from the fact that, for transparent media, it is a unique function of material density. Since density, in turn, will depend on temperature and species concentration, refractive index fields carry information related to heat and mass transfer processes.

For a medium that is partly absorbing and partly transparent, the refractive index is a complex quantity. The discussions in modules 4 and 5 deal with transparent media and images originate from the distribution of refractive index in the field of interest. The present module contains discussions on interferometry.

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Optical methods

Techniques that exploit the wave nature of light for measuring temperature are described here. Radiation that belongs to the visible range of the electromagnetic spectrum (wavelength $\lambda = 400 - 700 \text{ nm}$) is referred to light. Optical effects are associated primarily with the electrical field rather than the magnetic field. Hence the propagation of light originating from a conventional light source is expressible as

$$E = \sum_j A_j \sin\left(\frac{2\pi}{\lambda_j}(ct - x) + \phi_j\right) \quad (1)$$

Here E is the electrical field, A_j is its amplitude corresponding to a wavelength $\lambda_{j,} \phi_j$ is the phase associated with the j^{th} harmonic, c is the speed of light and t and x are time and distance respectively. For a monochromatic source, only one wavelength is significant, phase may be set to zero, and

$$E = A \sin\left(\frac{2\pi}{\lambda}(ct - x)\right) \quad (2)$$

E is, in general, a vector but in measurements one works with light beams that are nearly parallel. Hence it is sufficient to consider the magnitude of E but not its direction. Two monochromatic wave fronts arising from a single light source and having a phase difference are represented by the equations,

$$E_1 = A\, sin\left(\frac{2_\pi}{\lambda}(ct - x)\right)$$

$$E_2 = A\, sin\left(\frac{2_\pi}{\lambda}(ct - x) + \phi\right) \quad (3)$$

Interferometric measurements are based on information contained in above equation. The phase difference ϕ is equivalent to a path difference δ where

$$\delta = \frac{\lambda}{2_\pi}\phi$$

δ is called as the optical path length between a pair of points, in contrast to the geometric path length represented by the x-coordinate. Geometric path length forms the basis of distance measurement while the phase difference forms the basis of distance, speed, density and temperature measurements. These measurements are, however, possible only if the phase difference is stable and independent of time. This required the light source to be coherent. The laser is a high quality monochromatic coherent light source and is hence suitable for optical instrumentation.

Interference

Consider the superposition of two nearly parallel waves that originate from the same monochromatic light source; the waves have a phase difference ϕ. Their amplitudes are taken to be equal. Superposition results in the following development:

$$E_1 + E_2 = A\left[sin\left(\frac{2_\pi}{\lambda}(ct - x)\right) + \; sin\left(\frac{2_\pi}{\lambda}(ct - x) + \phi\right)\right]$$

$$= 2A\, cos\frac{\phi}{2}\, sin\left(\frac{2_\pi}{\lambda}(ct - x) - \frac{\phi}{2}\right) \quad (4)$$

The intensity I of the combined beam is $4A^2\, cos^2\, \phi/2$ and is plotted in Figure 4.1. To the human eye, intensities below a certain threshold would be seen as dark while intensities above would be bright. Light sensors can, of course, detect small changes in intensity. To an observer, the intensity distribution of Figure 4.1 is a sequence of dark and bright patches, called fringes. With reference to Figure 4.1, the superposition of two light beams with uniform intensity but a phase difference results in an interference pattern consisting of alternately dark and bright regions, the fringes. The spacing between two lines corresponding to the highest intensity is called as a fringe shift and is marked ϵ in the figure. This fringe shift is also obtained as the spacing between adjacent lines of minimum intensity.

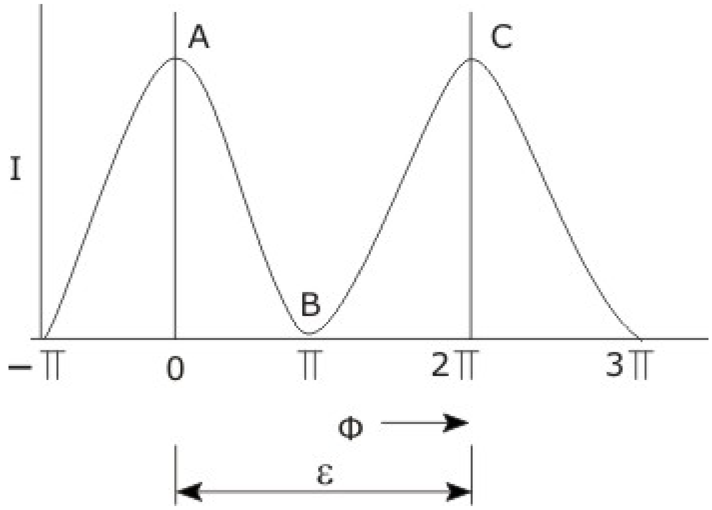

Figure 4.1: Intensity as a Function of Phase Difference between Interfering Light Beams.

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Since intensity varies as $cos^2\ \phi/2,$, the corresponding phase difference for a fringe shift is 2π. The equivalent optical path difference is λ. The interference fringes are ordered i.e. the n^{th} fringe counted for example from the left will represent a phase difference of $2n\pi$ with respect to the reference wave. In distance measurements an integer number of fringe shifts is used and hence the resolution in distance measurement is λ.

In a given problem, individual portions of the light beam traverse distinct optical paths and the phase difference is a spatially distributed variable. In an interferometric measurement, this beam combined with one that has a constant phase. In regions where the phase difference between the two is $(2n-1)\pi, n = 1,2\ldots,$ the intensity of the combined beam is zero and we get destructive interference. When the phase difference is $2n\pi$ we get constructive interference. The corresponding path differences are $n\lambda$ and $(2n-1)\ \lambda/2$ respectively. The phase field in the form of a fringe pattern can be recorded, say by a camera, to extract information about the primary variables of the problem being studied. In applications, quantitative measurements are possible if lines of constant phase exist so that fringes form, and a fringe shift can be identified from one fringe to another.

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Consider the propagation of light in a homogeneous medium as sketched in Figure 4.2. For the interference pattern to be stable in time, we require $\phi_3 = \phi_4$ and $\emptyset_2 - \emptyset_1$ to be independent of time. These conditions are called spatial and temporal coherence respectively. Recall that conventional light sources such as a tungsten filament emit sporadically and the phase is a random variable. Phase quality is a special feature of lasers. A stable interference pattern in which the fringes are ordered is a pre-requisite in all interferometry measurements. Since fringes improve signal-to-noise ratio in LDV measurements, coherence is an essential requirement here as well.

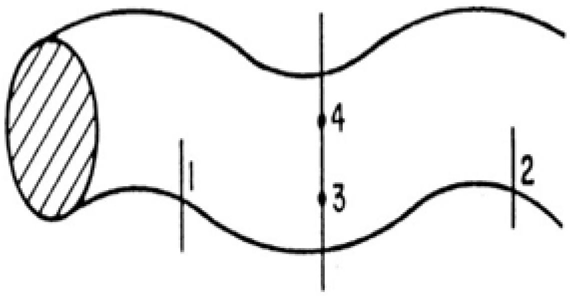

Figure 4.2: Definition of Temporal (between points 3 and 4) and Spatial Coherence (between points 1 and 2).

However, no light source is perfectly coherent and the quality of a light source is judged by its coherent length CL, i.e., the maximum distance between points 1 and 2 in Figure 4.2, over which $\emptyset_2 - \emptyset_1$ is time independent. If T_d is the time duration over which the source emits continuously without interruption then $CL = c \times T_d$ where c is the speed of light $(= 3 \times 10^8\, m/s)$ Helium-Neon lasers have a coherence length of 100-200 mm and are considered most suitable for interferometry.

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Interferometry

The electric field of plane wave propagates as $A\,sin\frac{2\pi}{\lambda}(ct - x)$ and differences in the path traversed by two different beams will produce an interference pattern on recombination. The following two results are important for calculation of phase differences between two light beams.

Consider reflection of a ray of light as it approaches a denser material from a lighter material. We have, $\rho_1 < \rho_2$ and $n_1 < n_2$ where ρ is density and n is the refractive index. Then $\phi_b - \phi_a = \pi$, where ϕ is the phase angle. (Figure 4.3)

Consider reflection of light as it approaches a lighter material from the denser side. We have, $\rho_1 > \rho_2$ and $n_1 > n_2$. Then $\emptyset_a = \emptyset$, i.e. the reflected ray has the same phase as the incident ray. These results can be proved by considering reflection of a transverse wave at a fixed end and a free end respectively.

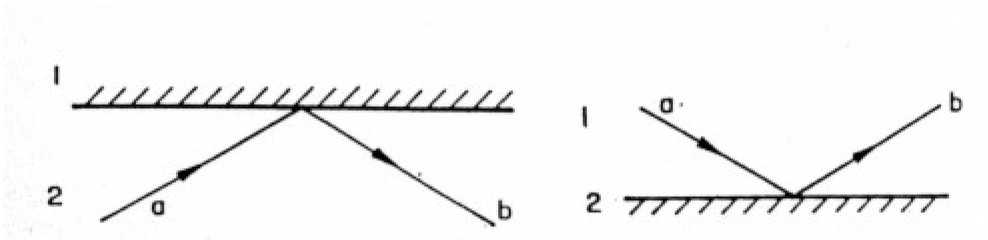

Figure 4.3: Phase Differences Owing to Reflection at Material Interfaces.

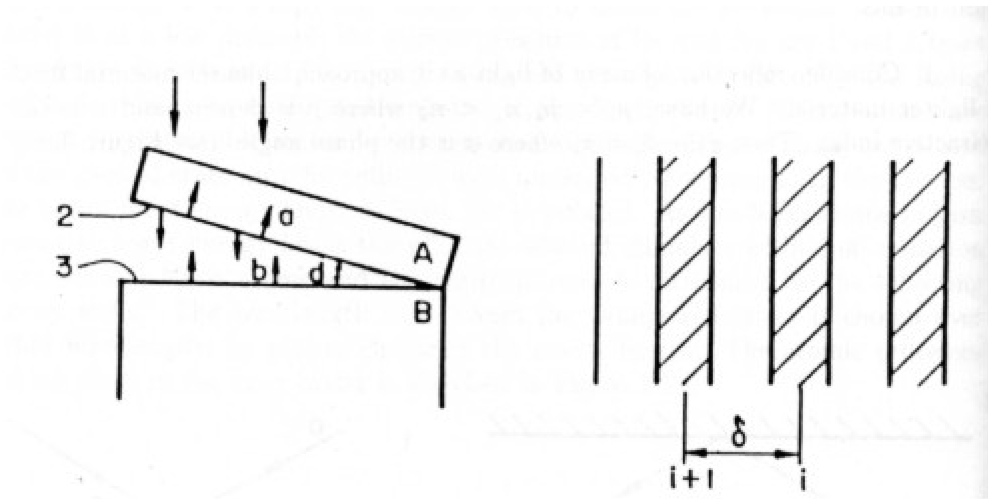

Figure 4.4: Wedge Fringes (right) created by an Air Gap of Varying Thickness (left)

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As an example of the interference phenomena, consider the formation of fringes in the arrangement shown in Figure 4.4. A transparent block A rests on another block B and the spacing d between them varies with position. Surface 2 is partly silvered and surface 3 is fully silvered and these surfaces are exposed to light at normal incidence. The interference pattern arising from the superposition of rays a (leaving A after reflection) and and b (leaving B after reflection) is shown in Figure 4.4. The pattern see in Figure 4.4 is an interference pattern of wedge fringes since they originate from a wedge-shaped all gap.

If the incident light has a phase angle ϕ then $\emptyset_a = \emptyset$ and

$$\phi_b = \phi + \pi + 4\frac{\pi d}{\lambda}$$

Here, the phase difference arising from the block A can be ignored because its contribution is common to both rays a and b. Hence

$$\phi_a - \phi_b = 4\frac{\pi d}{\lambda} + \pi$$

When $d = 0$ the phase difference between the two light beams is π and the first line of the interference pattern is a dark line rising from destructive interference. As we go from the i^{th} to $(i+1)^{th}$ fringe $d$changes from and so $d_i\ to\ d_{i+1,}\ \emptyset_a - \emptyset_b = 2\pi$

$$d_{i+1} = d_i + \frac{\lambda}{4}$$

The factor $1/4$ is related to the fact that ray b traverses the distance d twice. The above method permits measurement of d starting from the first dark fringe where $d = 0$.

Fringes will follow lines of constant d and hence represent contour lines on an uneven surface. This method can be used to measure d or the flatness of a surface, the latter being related to the straightness of the wedge fringes (Figure 4.4).

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Temperature measurement

In fluid flow and temperature measurements, changes in the phase of a light beam originate from the variation in the refractive index of the medium itself. The refractive index of a transparent material is defined as

$$n = \frac{c(vacuum)}{c(material)}$$

Where c is the speed of light. The value of n is greater than or equal to unity. The reduction in the speed of light in matter can also be viewed as an increase in the optical path length to be covered by the electromagnetic waves and hence a source of phase difference. The density and refractive index of a transparent material are uniquely related, and to a leading order, they are connected by the Lorenz-Lorentz formula

$$\frac{n^2-1}{\rho(n^2+2)} = constant \qquad (5)$$

In gases $n \approx 1$ and the relationship reduces to

$$\frac{n-1}{\rho} = constant$$

Hence, in gases, the derivatives

$$\frac{dn}{d\rho} = \text{constant}$$

In liquids, the derivative is nearly constant if the bulk changes in density are small.

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For small changes in temperature (say, up to, around $20°C$ in air), density and temperature T are linearly related as

$$\rho = \rho_o\left(1-\beta(T-T_\circ)\right), \beta > 0$$

It then follows that

$$\frac{dn}{dT} = constant$$

and changes in temperature T will simultaneously manifest as changes in the refractive index n.

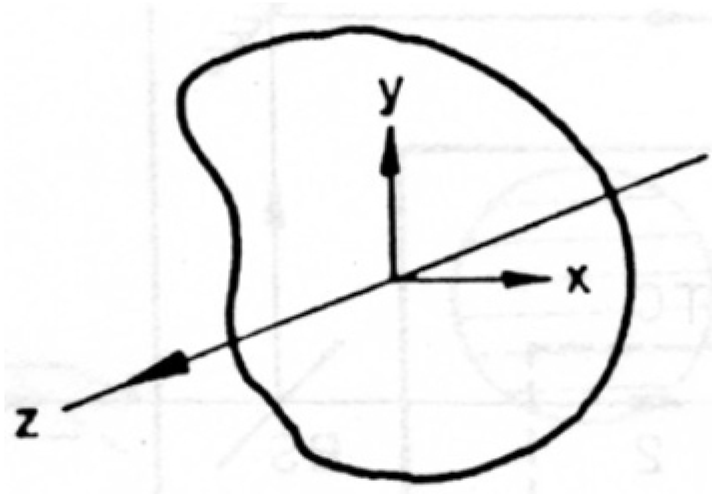

Figure 4.5: Coordinate System with x-y as the Cross-sectional Plane and z, the Direction of Propagation of Light.

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In a general setting, the material density will depend on pressure, temperature, and species concentration. Interferometric measurements of temperature are possible when pressure and concentration are either fully known, or, purely constant.

Consider a beam of light moving through a gaseous medium of varying temperature and of total length L. L is also equal to the geometric path length traversed by the light beam. The optical path length traversed by the light beam in the z -direction, corrected for changes in the light speed is

$$PL = \int c_0 \, dt = \int \frac{c_0}{c} cdt = \int_0^L ndz$$

Here, c_o is the speed of light in vacuum, and n , the refractive index is defined as

$$n = \frac{c_0}{c}$$

The integral is greater than L since $n > 1$ (except in absolute vacuum where $n = 1$). The applicable coordinate system is shown in Figure 4.5.

Let beam 1 propagate through a region of variable density and hence refractive index, n_1 and beam 2 through a region constant density (and n_2). Then, the difference in path lengths between 1 and 2 can be calculated as

$$\Delta PL = PL_1 - PL_2 = \int_O^L (n_1 - n_2) \, d_z$$
$$= \frac{dn}{d\rho} \int_O^L (\rho_1 - \rho_2) \, dz = \frac{dn}{dT} \int_O^L (T_1 - T_2) \, dz \qquad (6)$$

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In problems where T_1 is a two dimensional temperature field $T_1(x,y)$, the integral simplifies to

$$\Delta PL = (T_1 - T_2)\frac{d_n}{dT}L \quad (7)$$

Since a path length difference of λ will generate one fringe shift, the temperature difference ΔT required for this purpose is

$$\Delta T = \lambda/\left(L\frac{dn}{dT}\right) \quad (8)$$

In air $dn/dT = 0.927 \times 10^{-6}/°C$ and in water, it is $0.88 \times 10^{-4}/°C$. Since density decreases with increasing temperature at constant pressure, both derivatives are negative under normal conditions.

For $H_e - N_e$ laser $\lambda = 632.8\ \mathrm{nm}$. Hence $L \times \Delta T$ per fringe shift for a $H_e - N_e$ laser is $0.682°C - \mathrm{m}$ in air and $0.0072^0C - m$ in water. Note that the value ΔT itself decreases with increasing geometric path length L. Hence the sensitivity of measurements can be adjusted by designing apparatus of varying dimensions in the direction of propagation of light. In high temperature application L is made small while in problems involving small temperature differences L can be large. As discussed later, the largest value of L is, limited by refraction errors.

In summary, each fringe of an interferometer represents a line of constant phase, constant refractive index, constant density and hence constant temperature, namely an isotherm. This aspect is useful in qualitative interpretation of interference patterns.

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Remarks:

- The relationship between the optical path length and refractive index can be derived from the consideration of the speed of light as follow. For a light ray (2) traveling in the medium compared to one traveling in vacuum (1), one can write

$$\Delta PL = \int (c_0 - c)dt = \int c(n-1)\,dt = \int (n-1)dz$$

where $dz = cdt$ and z is a coordinate along the direction of propagation of light.

- The expression for the optical path length assumes that rays travel is straight lines. In a more general setting, light rays could refract and the path of light propagation would be a curve $s = s\,(x,y,z)$. The optical path length is then evaluated along this curve as

$$PL = \int (n-1) - \ ds$$

with ds evaluated from Figure 4.5 as:

$$ds = \sqrt{dx^2 + \ dy^2 + dz^2}$$

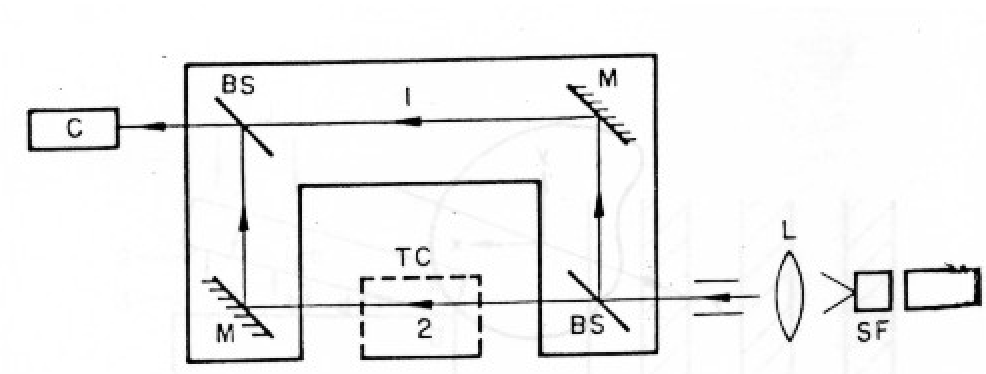

Figure 4.6: Two-beam Mach-Zehnder Interferometer with adjustable Optical Components. BS is beam splitter, M is mirror, L is a plano-convex lens, SF is spatial filter, C is a CCD camera connected to a computer, and TC is the test cell. Ray 1 is the reference beam that passes through vacuum while ray 2 is the test beam that asses through the thermally disturbed zone.

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Mach-Zehnder Interferometer

The Mach-Zehnder interferometer is a popular instrument used in experimental studies of heat transfer in fluids. The heated test cell is located in the path of the test beam 2 shown in the Figure 4.6. Quantitative experiments are possible when the temperature field is two dimensional in the cross-sectional plane and uniform parallel to the beam direction. Otherwise the test beam averages the temperature field as it traverses the test cell and only qualitative information can be obtained. The reference beam 1 passes through a region identical to the test cell except that the fluid here is at a uniform temperature. The spatial filter expands the laser beam which is subsequently made parallel using a convex lens. The spatial filter along with the lens constitutes the collimating arrangement of the interferometer.

The initial (geometric) path lengths of beams 1 and 2 are nearly equal except for possible angular misalignment of the mirrors and the beam splitters. As these are rotated the interferometer approaches a state of complete alignment. The initial fringe pattern corresponds to wedge fringes whose spacing increases as the alignment of the interferometer is improved (Figure 4.7). The alignment referred here is the parallelism of the optical components - two mirrors and two beam splitters. The best initial setting corresponds to the optical components being strictly parallel when the image is a uniformly bright field. In practice, two fringes will span the full field-or-view. This position of the interferometer is called the infinite-fringe setting. Every point in the reference beam and the test beam have the same path length when measured from the pinhole of the spatial filter (the virtual origin of the coordinate system). A thermal disturbance now placed in the path of the test beam will produce a fringe pattern where each fringe is an isotherm.

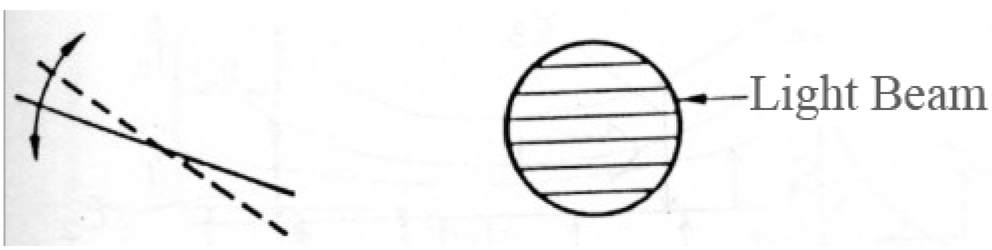

Figure 4.7: Fringes due to Misalignment of the Optics in a M-Z Interferometer.

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Temperature calculation

Each pair of fringes represents a temperature shift of ΔT_{ϵ}. In a region subjected to an overall temperature difference of ΔT, the number of fringes to appear in the field of view can be estimated as $\Delta T/\Delta T_{\epsilon}$. A fringe may be lost in the rounding process and is to be connected to the fact that walls may be isotherms but not necessarily a site for fringe formation where the phase difference must satisfy the condition of destructive interference. The spacing between fringes will depend on the temperature gradient prevailing at that location. The entire information about temperature values, localized at the fringes and the boundary of the material domain can be mapped on to a grid by a suitable interpolation procedure.

Let T_w be the wall temperature and T_1, the temperature of the fringe next to it. Let δ represent the distance between the fringe and the wall; in most applications, δ will be spatially distributed and not be a constant. Near a wall, the heat flux exchanged by the surface with the fluid can be calculated as

$$q_{\omega} = -k_f \frac{\partial T}{\partial_y} \approx -k_f \frac{T_w - T_1}{\delta}$$

Fringe patterns recorded by a Mach-Zehnder interferometer are shown in figure 4.8. Wedge fringes and Michelson interferometry are discussed in Lecture 25.

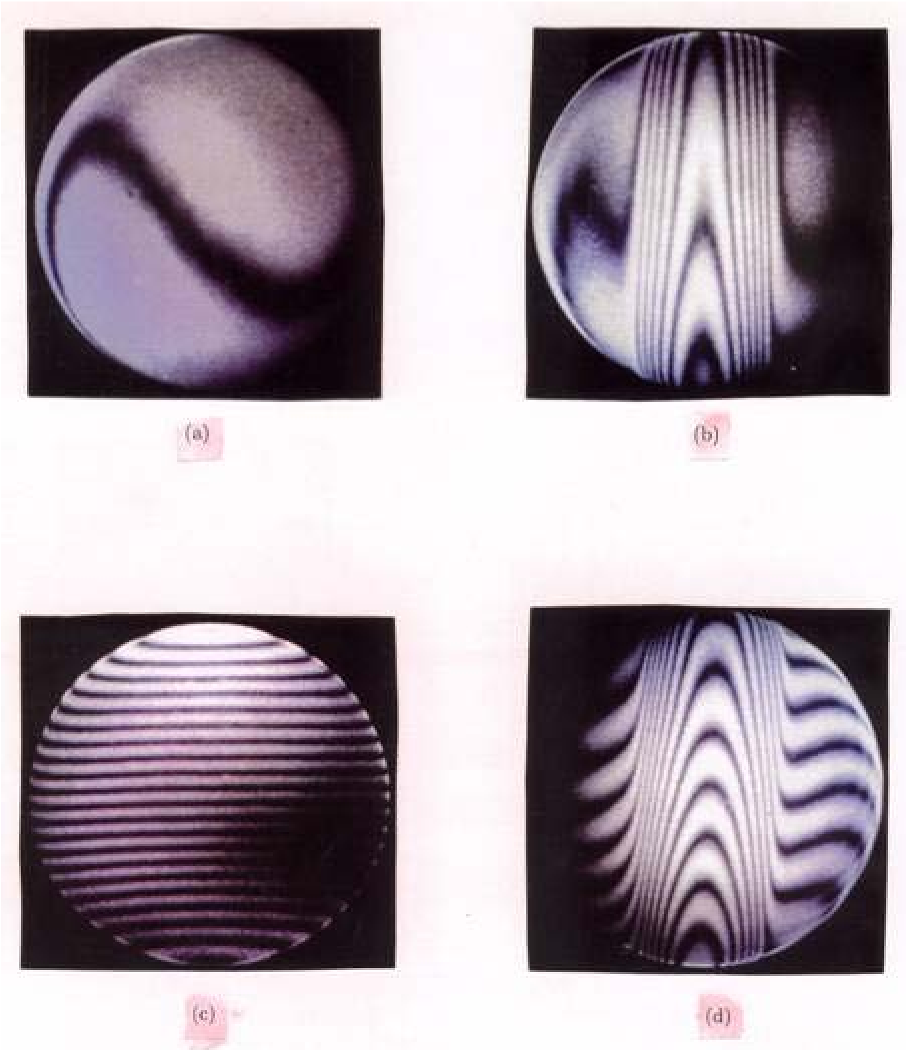

Figure 4.8: Fringes recorded above a Candle Flame in an Infinite Fringe Setting (left) and the Wedge Fringe Setting (right)

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