Module 1: Introduction to Experimental Techniques

Lecture 4: Similarity principles

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Similarity Principles and Dimensional Analysis

An experiment that is carried out in a laboratory under reference conditions will invariably be a scaled version of the prototype for which a mathematical model is to be developed. Data obtained from scaled experiments will be useful for the prototype only if certain conditions (called similarity principles) are satisfied. These principles have been developed for systems operating under deterministic conditions, presumably under continuous operation. These cannot be readily extended to stochastic problems, boundary-layer transition, for example. When similarity principles are followed while setting up the experiment, the data recorded in the experiment will be distortion-free . The measured data can then be linearly scaled from the experiment to the real-life process.

The principles of similarity that relate the experimental setup with the prototype are stated as:

1. Geometric similarity should be observed.

It requires that the experiments preserve the shape of the prototype, while the linear dimensions be scaled in proportion.

2. Dynamic similarity should be enforced.

It is a statement that the ratio of the relevant forces in the prototype be preserved in the model as well. This rule leads to the matching of certain dimensionless parameters.

3. Kinematic similarity should be realized.

It requires that the constant flux lines (streamlines, for example) in the experiment match those in the prototype. This requirement is most appropriate for steady state patterns since constant flux lines in unsteady problems may not be physically meaningful. If geometric and dynamic similarity are realized, kinematic similarity will be automatically satisfied.

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In practice, none of the similarity principles can be strictly applied. For example, if an object has a large aspect ratio, scaling will produce a model, one of whose dimensions will be excessive small (or large) for the purpose of fabrication. Consider, as an example the motion of a submarine in sea water. It is of interest to determine the drag force acting on it. It will be shown below that dynamic similarity leads to a requirement

$$V \times d = \ constant$$

where V is the characteristic speed of the submarine and d is its transverse length scale. It is assumed here that the laboratory experiment is also carried out in sea water. For a prototype, one can expect V=20 m/s and d= 10 m. Under experimental conditions, one would like to specify d= 0.4 m leading to V= 400 m/s. Producing such high speeds of water may be impossible under laboratory conditions; if it were to be possible free surface and compressibility effects would drastically corrupt the force data. Hence, the extension of laboratory data to the field scale is a daunting task and must be carried out with care.

The second issue that is addressed in the present section is dimensional analysis. It is advisable to list all the relevant quantities that play a role in the experiment and subsequently reduce them to dimensionless form. Several advantages can be derived from this step. These are:

1. A reduction in the number of variables,
2. Development of a statement of the similarity principles
3. Independent variables such as length and time adopt values close to unity.

Reconsider the example of measurement of drag. To cover all possible cases one must write drag D in functional form as

$$D = function\ (V, \rho, \mu, d, L)$$

where V is the vehicle speed, d and L are its transverse and longitudinal length scales, and ρ and μ are density and viscosity of seawater, respectively. Various strategies are available for casting equations such as the one for drag in dimensionless form.

The Buckingham- π theorem is useful in this context. The dimensionless form of the drag equation can be easily derived as

$$C_D = function\left(Re, \frac{L}{D}\right)$$

where C_D is the drag coefficient, and Re is Reynolds number based on the velocity V and dimension d. The statements of principles of similarity can now be stated as

$$kinematic : \left.\frac{L}{d}\right|_{model} = \left.\frac{L}{d}\right|_{prototype}$$

$$dynamic \; : Re(model) = Re(prototype)$$

If conditions of similarity are satisfied, the dimensionless drag coefficient recorded in experiments will be equal to one for the prototype at every Reynolds number.

The dimensionless expression in terms of the drag coefficient shows the following: If fluid properties in the laboratory and field experiments are identical, the similarity principle requires the respective Reynolds numbers to be equal. Equivalently, we require the product $V \times d$ for the model and the prototype to be equal, as stated in the numerical example above.

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