Module 3: Velocity Measurement

Lecture 15: Processing velocity vectors

The Lecture Contains:

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Data Analysis from Velocity Vectors

In applications, the velocity information is often necessary but not sufficient and other quantities will be of interest as well. The velocity field obtained from PIV measurements can be used to estimate relevant quantities by means of differentiation and integration. The vorticity field is of special interest because, unlike the velocity field it is independent of the frame of reference. In particular, if it is resolved temporally, the vorticity field can be much more useful in the study of flow phenomena than the velocity field. This is particularly true in highly vortical flow such as turbulent shear layers, wake vortices and complex vortical flows. Integral quantities can also be obtained from the velocity data. The instantaneous velocity field obtained by PIV can be integrated, yielding either a single path integrated value or another field such as the stream function. Analogous to the vorticity field, the circulation obtained through path integration is also of special interest in the study of vortex dynamics, mainly because it is also independent of the reference frame. In the following section, data analysis for calculation of various derived quantities from PIV measurements are presented.

Velocity differentials

The differential terms are estimated from the velocity vectors obtained from PIV. Since PIV provides the velocity vector field sampled on a two dimensional evenly spaced grid specified as $(\Delta x, \Delta y)$, finite differencing can be employed to get the spatial derivatives. There are a number of finite difference schemes that can be used to obtain the derivatives. The truncation error associated with each operator is estimated by means of a Taylor series expansion. The actual uncertainty in differentiation is due to that in the uncertainty of the velocity estimate ϵ_U. It can be obtained using standard error propagation methods assuming individual data to be independent of the other. There are two schemes that reduce the error associated with differentiation: Richardson extrapolation and least squares approach. The former minimizes the truncation error while the least squares approach reduces the effect of random error, i.e. the measurement uncertainty, ϵ_U. These approaches are briefly discussed bellow.

Least squares estimate of the first derivative:

$$\left(\frac{df}{dx}\right)_i \approx \frac{2f_{i+2} + f_{i+1} - f_{i-1} - 2f_{i-2}}{10\Delta x}$$

Here, the accuracy is of the order of Δx^2 and the associated uncertainty is $\epsilon_U/\Delta x$.

The derivative using Richardson extrapolation is calculated as:

$$\left(\frac{\mathrm{df}}{\mathrm{dx}}\right)_\mathrm{i} \approx \frac{\mathrm{f}_{\mathrm{i}-2} - 8\mathrm{f}_{\mathrm{i}-1} + 8\mathrm{f}_{\mathrm{i}+1} - \mathrm{f}_{\mathrm{i}+2}}{12\Delta \mathrm{x}}$$

The accuracy of the above approximation is of order Δx^3 and the uncertainty associated with the expression is $0.95\epsilon_U/\Delta x$.

Vorticity and circulation

The vorticity components in the x, y and z directions can be calculated from the partial derivatives of velocity using:

$$\omega_x = \partial w/\partial y - \partial v/\partial z$$

$$\omega_y = \partial u/\partial z - \partial w/\partial x$$

$$\omega_z = \partial v/\partial x - \partial u/\partial y$$

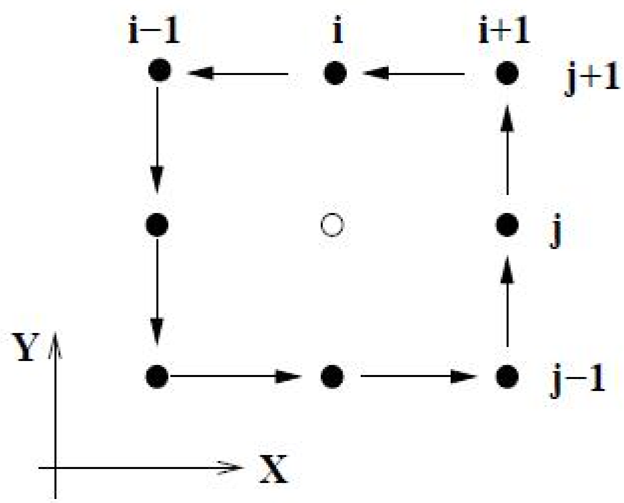

Figure 3.33: Contour for the calculation of circulation for the estimation of vorticity at a point (i; j).

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The expression of mass continuity for an interrogation spot on the $x - y$ plane can be written as

$$\nabla.\mathbf{u} \approx \frac{\partial u}{\partial x} + \partial v/\partial y$$

Circulation can be computed from either velocity using

$$\Gamma = \oint \mathbf{u}.\,dl$$

or vorticity as

$$\Gamma = \int \omega\,.\,dA$$

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In the present work, vorticity has been calculated by choosing a small rectangular contour around which the circulation is calculated from the velocity field using a numerical integration scheme, such as trapezoidal rule. The local circulation is then divided by the enclosed area to arrive at an average vorticity for the sub-domain. The following formula provides a vorticity estimate at a point (i; j) based on circulation using eight neighboring points (see Figure 3.32):

$$(\omega_z)_{i,j} \cong \frac{\Gamma_{i,j}}{4\Delta X \Delta Y}$$

with

$$\Gamma_{i,j} = \frac{1}{2}\Delta x\left(U_{i-1,j-1} + 2U_{i,j-1} + U_{i+1,j-1}\right)$$

$$+\frac{1}{2}\Delta y\left(V_{i+1,j-1} + 2V_{i+1,j} + V_{i+1,j+1}\right)$$

$$-\frac{1}{2}\Delta x\left(U_{i-1,j+1} + 2U_{i,j+1} + U_{i-1,j+1}\right)$$

$$-\frac{1}{2}\Delta x\left(V_{i-1,j+1} + 2V_{i-1,j} + V_{i-1,j-1}\right)$$

It has been observed from experiments that a circulation calculation via the velocity field yields better estimates of vorticity, and in particular, the peak vorticity that is otherwise under-predicted. At other locations, the vorticity field determined by the two approaches are practically identical. The accuracy of the vorticity measurement from PIV data depends on the spatial resolution of the velocity sampling and the accuracy of the velocity measurements. Therefore, the vorticity error can be associated with calculation scheme and the grid size used for velocity sampling. Another source of uncertainty is that propagated from the velocity measurements. PIV velocity measurements are the local averages of the actual velocity in the sense that it represents a low pass filtered version of the actual velocity field. Thus, vorticity from PIV data is only a local average of an already averaged velocity field and not a point measurement.

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RMS velocity

Though the PIV image sequence is collected with a time spacing of 0.25 seconds, RMS velocity fluctuations in the near wake can still be obtained from them. The reasoning is that the near wake is dominated by large structures that in turn are associated with large time scales. A validation of PIV measurement of RMS velocity with hotwire data is reported in the doctoral thesis of Sunshanta Dutta available in this website.

The root mean square velocity (for a generic component u) at a location in the wake is calculated from the formula

$$u_{\mathrm{rms}} = \sqrt{\frac{1}{N-1}\sum (u')^2}$$

The summation extends over Nsamples, N being the number of acquired velocity fields in time. Further, u' is the unsteady part of the velocity field that is given as

$$u' = u(x,t) - U(x)$$

The time averaged velocity field U is determined by averaging the instantaneous velocity vectors using the formula

$$U(x) = \frac{1}{N}\sum_{i=1}^{N} u\,(x, t_i)$$

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Drag coefficient

Two important parameters in the study of flow past bluff bodies are drag coefficient and Strouhal number. Drag coefficient is the dimensionless form of the force acting on the body in the direction of flow. Strouhal number is the non-dimensional vortex shedding frequency. It is also indicative of the time scale of the unsteady forces and determines the nature of flow induced vibrations of the body in both stream-wise and transverse directions.

In the present study, drag coefficient has been calculated by a momentum integration approach over a control volume. It is also called the wake survey method, and has been extensively discussed (Schlichting, 1979). Generally, the method is used to calculate drag coefficient from velocity probe in the intermediate and far wakes, where there is no static pressure variation across the flow. In the present experiments of flow past a square cylinder in a closed channel, static pressure variation has been observed under some conditions even at twenty cylinder widths downstream. Therefore, it is necessary to consider the static pressure variation in the calculations. The drag coefficient is given by the formula (White, 1991)

$$C_d = \frac{\int_{-H}^{H} \rho u(y)\left(U_a - u(y)\right) + \Delta p)\, dy}{0.5_\rho U_a^2 D}$$

Here $u(y)$ is the velocity prole in the wake where x and y are coordinates parallel and perpendicular to the main flow direction. Additionally, U_a is the approach velocity, D is the projected area of the cylinder normal to the flow direction (per unit length along the cylinder axis) and Δp is the static pressure drop between the free stream and the point under consideration. To maintain uniformity with the nomenclature in the published literature, the projected dimension has been taken to be equal to the edge of the cylinder for straight as well as inclined cylinders.

It is clear from the above expression that the drag coefficient cannot be determined exclusively from PIV images. Most experiments in which the pressure drop was measured using a static probe showed that the correction was to the extent of $\pm 5\%$. Hence, results can also be presented without the pressure correction term. For such data, the drag coefficient can be interpreted simply as a momentum loss coefficient.

The symbol H in the formula for the drag coefficient is the width of the control volume over which the local velocity attains the free stream value. In external flow measurements, this location coincides with a boundary that has zero shear. Accordingly, the external force calculated from the momentum balance formula can be attributed entirely to drag on the cylinder. In channel flow, the asymptotic limit is not reached unambiguously; hence the value of H has been selected by first examining the velocity vectors. A second approach employed was to set H equal to the channel half-width and make corrections for the wall shear. Both of these approaches were found to give very similar drag coeffients.

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Streamlines

The flow pattern is visually brought out in terms of streamlines. For this purpose, the stream function is first calculated by integrating the velocity field as per the formula

$$\psi = \int_x u(x,y)\, dy$$

starting from an exterior point where $v = 0$. Here, u and v indicate the time-averaged velocity components of the flow field. The streamlines are then plotted simply as contours of constant stream function in the flow domain.

Turbulent kinetic energy budget

The turbulent kinetic energy budget helps in examining how turbulence is spatially distributed after being produced in the flow field. It also explores the development of the cascade process that directs the kinetic energy of velocity fluctuations from large scales towards the small scales, ultimately to be dissipated by molecular-level processes. Under unique circumstances, reverse cascading, namely energy transfer towards the large scale structures is also possible.

The equation describing the transport of turbulent kinetic energy is

$$\frac{\partial k_t}{\partial t} = -U_j \frac{\partial k_t}{\partial x_j} - \langle u'_i u'_j \rangle S_{ij} - \frac{\partial}{\partial x_j}\langle u u'_i u'_j \rangle - \nu \langle s_{ij} s_{ij} \rangle - \frac{\partial}{\partial x_j}\langle 2\nu u'_i s_{ij} \rangle - \frac{\partial}{\partial x_j}\frac{1}{\rho}\langle u' j^{p'} \rangle$$

The symbols used to follow the meaning employed in the literature on turbulent flows. Lower case symbols (say, u) indicate velocity fluctuations, while the upper case symbol is for the time-averaged velocity $(\text{say } U)$. In the above equation, the turbulent kinetic energy K_t is a time-averaged quantity

$$K_t = \frac{1}{2}\langle u'u' + v'v' + w'w' \rangle$$

The time- averaged rate of strain (S_{ij}) and fluctuating rate of strain (S_{ij}) are

$$S_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right), s_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

The first term on the right hand side of the turbulent kinetic energy equation is advection. Under the assumption of a negligible *w-velocity (w=0)* it reduces to:

$$\text{Advection} = U\frac{\partial K_t}{\partial x} + V\frac{\partial K_t}{\partial y}$$

The second term in the kinetic energy equationis the production of turbulent kinetic energy. With the symmetry assumption at the mid-plane of the test cell $(\partial/\partial z = 0)$ it becomes:

$$\text{Production} = \langle u'u' \rangle \frac{\partial U}{\partial x} + \langle v'v' \rangle \frac{\partial V}{\partial x} + \langle u'v' \rangle \frac{\partial U}{\partial y} + \langle u'v' \rangle \frac{\partial V}{\partial x}$$

The third term indicates diffusion; under the assumption $(< \omega'^2 v' > = < v'^3 > \; and < \omega'^2 u' > = < v'^2 u' >)$. It can be expressed as:

$$\text{Diffusion} = \frac{\partial}{\partial y}\frac{1}{2} < u'^2 v' > + \frac{\partial}{\partial y} < v'^3 > + \frac{\partial}{\partial x}\frac{1}{2} < u'^3 > + \frac{\partial}{\partial x} < u'v'^2 >$$

The fourth term is viscous dissipation. If local isotropy in velocity fluctuations is assumed, we get

$$\text{Isotropic Dissipation} = 15v \langle \frac{\partial u'}{\partial x} \rangle^2$$

Dissipation can also be calculated based on the assumption of locally axisymmetric turbulence and is equal to

$$\text{Axisymmetric dissipation} = v \left[-\langle \left(\frac{\partial u'}{\partial x}\right)^2 \rangle + \langle 2 \left(\frac{\partial u'}{\partial y}\right)^2 \rangle + \langle 2 \left(\frac{\partial v'}{\partial x}\right)^2 \rangle + \langle 8 \left(\frac{\partial v'}{\partial y}\right)^2 \rangle \right]$$

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The 5^{th} and 6^{th} terms in the turbulent kinetic energy equation are termed as viscous diffusion and pressure diffusion respectively. The sum of these two terms has been obtained as the residual of the turbulent kinetic energy equation. It should be noted that the viscous diffusion can be expected to be insignificant at intermediate and high Reynolds number. Therefore the residual term is primarily due to the pressure transport.

The individual budget terms of the turbulent kinetic energy equation can be calculated from PIV measurements. It may be assumed that the total kinetic energy is 1.33 times that of that measured from two dimensions. Panigrahi et al., (2005) have validated the above assumptions from an experiment on flow past a rib using 2-component and stereo-PIV.

The dissipation term of the kinetic energy budget equation has been calculated based on the assumption of both local isotropy and axisymmetry. The assumption of local isotropy requires the eight ratios of velocity fluctuations to be equal to unity. Schenck and Jovanovic (2002) reported the eight isotropic ratios to be in the range of 0.4 to 1.7 from hotwire measurements of the plane wake of a circular cylinder and axisymmetric wake of a sphere. The diffusion term in the turbulent kinetic energy equation is calculated using the assumptions: $(\langle \omega'^2 v' \rangle = \langle v'^3 \rangle \; and \; \langle \omega'^2 u' \rangle = \langle v'^2 u' \rangle)$. The above assumption has also been justified by Panigrahi et al. (2005) through experiments involving flow past a rib.

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