Module 9 : Robot Dynamics & controls

Lecture 36 : Lyapunovs theorems application continued and force control in robots

Objectives

In this course you will learn the following

- In last lecture we have done Stability analysis of Robot Manipulator by using Lyapunov's theorem ,Lasalle's theorem but we have discussed only Regulation Problem. Now we will study stability analysis of Controller for Trajectory tracking. Till the last lecture we were dealing with Position control system but there are certain areas of Robot appplication where we need to control Force alongwith position. So we will also study Force control in this Lecture. There is definite relationship between V , $\dot{V}$ & stability of system.

Li-Slotine Controller (High Performance Tracking Controller):

In case of Trajectory tracking as we have to move End-effector along a predefined path, first task is to find out desired joint angle trajectory at each joint as a function of time. If all the joint angles are at predefined value for particular instant of time then it is automatically ensured that end-effector is at desired position at that instant of time.

Application of Lyapunov's asymptotic stability theorems for Trajectory Tracking Problem:

Let $\tau = Da + Cv + Bv - K_d r$ Control Strategy

Where
$$a = \dot{v}$$
$$e = q - q^d$$
$$v = \dot{q}^d - \wedge e$$
$$r = \dot{q} - v = \dot{e} + \wedge e$$

K_d , $\wedge$ are positive definite matrices.

Recall Robot Dynamics Equation in tems of joint variables $\underbrace{\left(D_1(q) + J\right)}_{D}\ddot{q} + C(q,\dot{q})\dot{q} + B\dot{q} + g(q) = u$

$$D\ddot{q} + C(q,\dot{q})\dot{q} + B\dot{q} + g(q) = u$$

Note that the property of $\dot{D} - 2C$ being a skew symmetric is still preserved as J is a constant inertia matrix.

substituting control law in equation of Dynamics

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + B\dot{q} =$$
$$D\left(\ddot{q}^d - \wedge\dot{e}\right) + (C + B)\left(\dot{q}^d - \wedge e\right) - K_d\left(\dot{e} + \wedge e\right)$$

simplifying

$$D\left(\ddot{q} - \ddot{q}^d + \wedge\dot{e}\right) + (C + B)\left(\dot{q} - \dot{q}^d + \wedge e\right) + K_d\left(\dot{e} + \wedge e\right) = 0$$
$$\left(\ddot{e} + \wedge\dot{e}\right) + (C + B)\left(\dot{e} + \wedge e\right) + K_d\left(\dot{e} + \wedge e\right) =$$

$$\therefore D\dot{r} + (C+B)r + K_d r = 0$$

where r = $(\dot{e} + \wedge e)$ is representing error dynamics. it has both error & its derivative.We have to prove that e & edot tents to zero as t tents to infinity. To prove that we will consider Lyapunov function candidate energy based.

Lyapunov function candidate $$V(t,q,\dot{q}) = \frac{1}{2} r^T D r + \wedge e^T K_d e$$

Contd....

First we will prove that V is a pdf. for that

$$r^T D r = (\dot{e} + \Lambda e)^T D(\dot{e} + \Lambda e)$$
$$= (\dot{e}^T + e^T \Lambda^T)(D\dot{e} + D\Lambda e)$$
$$= \dot{e}^T D\dot{e} + e^T \Lambda^T D\Lambda e + \dot{e}^T D\Lambda e + e^T \Lambda^T D\dot{e}$$

substituting in V we will get

$$= \frac{1}{2}\left(\dot{e}^T D\dot{e} + e^T \Lambda^T D\Lambda e + \dot{e}^T D\Lambda e + e^T \Lambda^T D\dot{e}\right) + e^T \Lambda K_d e$$
$$= \frac{1}{2}\dot{e}^T D\dot{e} + e^T (\frac{1}{2}\Lambda^T D\Lambda + \Lambda K_d)e + e^T \Lambda^T D\dot{e}$$

Now we will put this quadratic function in matrix form

$$= \begin{bmatrix} e^T & \dot{e}^T \end{bmatrix} \underbrace{\begin{bmatrix} \frac{1}{2}\Lambda^T D\Lambda + \Lambda K_d & \frac{1}{2}\Lambda^T D \\ \frac{1}{2}\Lambda^T D & \frac{1}{2}D \end{bmatrix}}_{A} \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$$

Quadratic functions are positive definite functions if determinants of successive minors of *A* are all positive.In this case it is positive definite as all minors of A are positive.So the function candidate V satisfies first condition,

Now we will take derivative of V. As we know $V(t,q,\dot{q}) = \frac{1}{2} r^T D r + \wedge e^T K_d e$

$$\dot{V} = \frac{1}{2}\left(2r^T D\dot{r} + r^T \dot{D} r\right) + 2e^T \Lambda K_d \dot{e}$$

substituting for $D\dot{r}$ in above equation (Recall that $D\dot{r} + (C+B)r + K_d r = 0$)

$$\dot{V} = \frac{1}{2}\left[-2r^T \left(C + B + K_d\right)r + r^T \dot{D} r\right] + 2e^T \Lambda K_d \dot{e}$$
$$= -r^T \left(B + K_d\right)r + 1/2 r^T \left(\dot{D} - 2C\right)r + 2e^T \Lambda K_d \dot{e}$$

Expanding r in terms of error & its derivative

$$= -(\dot{e}^T + e^T \Lambda^T)\left(B + K_d\right)(\dot{e} + \Lambda e) + 2e^T \Lambda K_d \dot{e}$$
$$= -\dot{e}^T \left(B + K_d\right)\dot{e} - \dot{e}^T \left(B + K_d\right)\Lambda e - e^T \Lambda^T \left(B + K_d\right)\dot{e}$$
$$-e^T \Lambda^T \left(B + K_d\right)\Lambda e + 2e^T \Lambda K_d \dot{e}$$

Contd....

Note that K_d & Λ are positive definite & symmetric matrices.

$$\dot{V} = \frac{1}{2}\left(2r^T D\dot{r} + r^T \dot{D} r\right) + e^T \Lambda K_d \dot{e}$$
$$= -e^T \Lambda^T \left(B + K_d\right)\Lambda e - 2e^T B \wedge \dot{e} - \dot{e}^T \left(B + K_d\right)\dot{e}$$

writing it in a quadratic form, we will have

$$= -\begin{bmatrix} e^T & \dot{e}^T \end{bmatrix} \underbrace{\begin{bmatrix} \Lambda^T \left(B + K_d\right)\Lambda & B\Lambda \\ B\Lambda & \left(B + K_d\right) \end{bmatrix}}_{A} \begin{bmatrix} e \\ \dot{e} \end{bmatrix}$$

To prove asymptotic stability, we have to prove that all sucessive minors of A are positive.

Consider first minor as it contain all positive definite matrices its determinant is positive.In the same manner it can be proved for second minor.

Note that we are having V as a C^1decrescent lpdf such that $-\dot{V}$ is an lpdf.Thus according to Lyapunov's theorem system is Asymptotically stable.

What is mean by asymptotic stability?

$e, \dot{e} \to 0$ as $t \to \infty$.which implies that $q \to q^d, \dot{q} \to \dot{q}^d$. means that actual joint angle tents to desired joint angle as time tents to infinity. So the end effector will be moving along a predefined path. In this way the trajectory to be tracked can be controlled. The controller discussed above is a very high performance tracking controller.

Till now we have discussed application which require control system for movement of Robot End-effecor from one point to other which may be subjected to constraint of predefined path like Trajectory tracking but there are also certain applications which require to control force that Robot applies at end-effector while handling . We will discuss the need of force control & actually how to control it in next part of lecture.

Need of Force Control:

Whenever the robot has to interact with its environment, it needs force control. Position control can only be used in case, where the manipulator is not interacting significantly with objects in the work space. If manipulator is interacting with the objects in the work space it can be better handled by force control. We need to control the force applied by Robot. Suppose if we apply trajectory tracking control alone for a situation where Robot is interacting with environment just like Robot painting a car or washing windows of car. What will happenif there is slight deviation from predefined path. That may cause lose excessive contact with object surface which may damage to end-effector or object. So we must modify the control law whenever the Robot is interacting with objects in workspace.

Type of Force Control:

- Stiffness Control
- Impedence Control

Consider a example of human being picking up a egg from a basket. How he achieve force control while doing this? Think over it. He took some force feedback from skin which acts as a tactile sensor & from that feedback he controls what force should be applied further.?But how this can be incorporated in case of Robot? One may easily say that use of force or tactile sensor. But these sensors are very costly to implement.

Stiffness Control:

This idea is based on stiffness & compliance. Robot manipulator are mechanically very rigid by design.One solution is to provide a device consisting of virtual spring. By controlling the end-effector position we will have control on the force exreted by end effector.Force control in this fashion can be accomplished by position control only. Following example will make it clear.

Figure 36.1 Force Control

Suppose end effector has to move along trajectory shown in fig. But suppose there is slight deviation of end effector from this trajectory little inside.It will deform the object in environment if it is non-rigid.But suppose if it is rigid then there will be some force exterted by end-effector on object & end effector will experience reaction force

This same thing will be incorporated in force control strategy. We will use parallel trajectory which is inside environment & we will use proportional control strategy with desired position of end effector as inside environment. As the desired point will be inside environment, it is just like introducing springing effect which will exert a restoring force which will depend upon location of desired point inside object. Thus problem of force control is converted to position control problem.If environmental stiffness k_e is very high then force is given by

$$f_e = k_p \left(X_d - X_e \right)$$

In order to exert a given force f_e on an object, we can command a desired position slightly inside the object and use a position control scheme.

Contd....

In case of n-degree of freedom manipulator an analogous stiffness control scheme can be implemented. End effector has six degrees of freedom.Suppose desired stiffness is chosen across each degree of freedom, then in matrix form which can be represented by a 6X6 matrix k_x. We want to find out what torque must be applied at each joint in order to exert a desired amount of force at end-effector to produce the desired stiffness characteristics for end-effector.

Desired restoring force for a virtual displacement of δx is $F = K_x \delta x$.

If δq denotes the resulting joint displacement for the small displacement δx ,

$$\delta x = J(q)\,\delta q$$

Where *J(q)* is the manipulator jacobian Now

$$\tau = J(q)^T F = J(q)^T K_x \delta x = \underbrace{J(q)^T K_x J(q)}_{K_q(q)} \delta q$$

Control law to be implemented is given as

$$\tau = J^T K_x J \left(q^d - q \right) + \tau_b$$

τ b is term taken into account to represent system damping which may be zero.

Impedence Control:

Defination: A mechanical impedence is defined as a ratio of force to velocity. $Z(s) = \dfrac{F_e(s)}{V(s)}$

specify desired impedence as

$$sZ(s) = -(Ms^2 + Bs + K)$$

$$M\ddot{x} + B\dot{x} + Kx = -F_e$$

Dynamics of Robot interacting with environment changes as the extra term which is force of interaction with environment is present in the equation.

$$M(q)\ddot{q} + h(q,\dot{q}) + J^T(q) F_e = U$$

Stiffness control is a special case of impedence control considering steady state force displacement

- relationship.
- When end effector is moving freely impedence is zero. This is a Pure position control case.

Thus Pure position control & stiffness control can be considered as a special case of impedence control.

Implementation issues:

- Stiffness control
- Similar to PD control

simple to implement, but more calculations than simple PD control. $\tau = J^T K_x J \left(q^d - q\right) + \tau_b$

Impedence control

- More complex control law.
- Control is not robust.

Recap

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