Module 6 : Robot manipulators kinematics

Lecture 17 : Euler's angle and fixed frame rotation for specifying position and orientation

Objectives

In this course you will learn the following

- Euler's angle for specifying orientation of a rigid body

- Fixed frame rotation and rotation about arbitrary fixed axis

Euler Angles

Three independent variables are required for orientation(as shown in Figure 17.1). Euler specified 3-1-3 angles(a,b,c). Let x1,y1,z1 be global vectors. Initially assume local frame coincident with global axis. Now rotate local z1 axis w.r.t global z2 axis. Now expressing in terms of rotation matrices as

$$\left[{}^{1}R_{4}\right]=\left[{}^{1}R_{2}\right]\left[{}^{2}R_{3}\right]\left[{}^{3}R_{4}\right] \textit{where}$$

$$\left[{}^{1}R_{2}\right]=R(\hat{z},a)=\begin{bmatrix}\cos a & -\sin a & 0\\ \sin a & \cos a & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\textit{Similarly}\left[{}^{2}R_{3}\right]=R(\hat{x},b);\left[{}^{3}R_{4}\right]=R(\hat{z},c)$$

$$\therefore\left[{}^{1}R_{4}\right]=\begin{bmatrix}\cos a.\cos c-\sin a.\cos b.\sin c & -\cos a.\sin c.\cos b.\cos c & \sin a.\sin b\\ \sin a.\cos c+\cos a.\cos b.\sin c & -\sin a.\sin c+\cos a.\cos b.\cos c & -\cos a.\sin b\\ \sin b.\sin c & \sin b.\cos c & \cos b\end{bmatrix}$$

Fig 17.1 Euler angles

Inverse Kinematics problem of above will be as, given the [R] matrix, find out the Euler's angles a, b, c. The solution will be as, equating the above matrices elements to the elements of [R] specified and solving for a, b, c.Here we will make a note that rigid body requires only 6 parameters to specify position & orientation (3 each) although R matrix have 9 elements of which 6 are dependent on others. Thus orientation specification requires 3 elements namely a, b, c here.

Fixed frame rotation (Refer Figure 17.2) :- Here we rotate local ref frame w.r.t. global ref frame in the sequence mentioned as 1-2-3 by angle a, b, c. The x_1, y_1, z_1 is local reference frame & initially local &global frames coincides. Now rotated fixed reference frame i.e. x_2, y_2, z_2 is global ref frame. And hence rotation matrix is now (4 being global reference frame & 1 being local)

$$\left[{}^{G}R_{L}\right] = \left[{}^{4}R_{1}\right] = \left[{}^{4}R_{3}\right]\left[{}^{3}R_{2}\right]\left[{}^{2}R_{1}\right]$$
$$= R(\hat{z},c).R(\hat{y},b).R(\hat{x},a)$$

Fig 17.2 Angles for fixed frame rotation

Rotation about fixed vector of obtaining rotation specification by rotating about a particular vector (u) by an angle θ. That is find ${}^{G}R_{L}$, given u & θ.

Fig 17.3 Rotation Angle specification

Solution:

Fig 17.4

from the figure, the sina, sinb, cosa, cosb are given as

$\sin b = r_x$; $\cos b = \text{sqrt}(ry2+rz2)$; $\sin a = ry/(\text{sqrt}(ry2+rz2))$; $\cos a = rz/(\text{sqrt}(ry2+rz2))$......(1)

and the overall rotation matrix is given as $R_{r,\theta} = R_{x,-\alpha}\ R_{y,\beta}\ R_{z,\theta}\ R_{y,-\beta}\ R_{x,\theta}$ .....(2) .

where the typical rotation matrix is 3x3 matrix with sin & cos terms as mentioned earlier.

from 1 & 2 we can write the rotation matrix $R_{r,\theta}$ in terms of a, b, rx, ry, rz.

Recap

In this course you will learn the following

- Euler specified 3 angles for orientation of a rigid bodies
- How end effector rotary motion about an arbitrary axis can be achieved

Congratulations, you have finished Lecture 17. To view the next lecture select it from the left hand side menu of the page