

Module 5 : Trajectory Planning of end effectors

Lecture 13 : Trajectory planning II (4-3-4 & trapezoidal velocity strategy for robots)

Objectives

In this course you will learn the following

- Use of higher order polynomial strategies
- Use of simple trapezoidal profile

Use of Higher Degree Polynomials

We have so far discussed use of cubic splines. It is the lowest degree polynomial function that allows velocity and acceleration continuity. It is easy to work with and hence well suited for trajectory planning. However we could very well use higher degree polynomials. We will now describe one such strategy. Consider a typical pick-and-place type of task.

Fig 13.1.1: Pick and place Gripper

When the object is being picked-up (Fig. 13.1.1), we wish that the motion of the end-effector (gripper) be such that it does not press into or crash into the supporting structure. We could thus specify a position slightly separated from the initial position, along the normal to the support surface at the initial position and require that the gripper pass through this. We could also specify the time duration for the motion between these two points. In this manner, we can control the take-off process. Similarly when the object is placed on another support surface, we would like to approach a position slightly away from the surface along its outward normal before settling down into the final position. A joint motion in such a situation could be visualized as shown in Fig. 13.1.2:

Fig 13.1.2: Typical Joint Motion in Pick-and-Place

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A 4-3-4 strategy commonly employed for such situations uses a 4 th degree polynomial for I-L; cubic spline from L-S; another 4 th degree polynomial for S-F i.e.,

$$\theta_{il}(\tau) = a_0 + a_1\tau + a_2\tau^2 + a_3\tau^3 + a_4\tau^4 \quad (\tau: 0 \rightarrow t_{il})$$

$$\theta_{ls}(\tau) = b_0 + b_1\tau + b_2\tau^2 + b_3\tau^3 \quad (\tau: 0 \rightarrow t_{ls})$$

$$\theta_{sf}(\tau) = c_0 + c_1\tau + c_2\tau^2 + c_3\tau^3 + c_4\tau^4 \quad (\tau: 0 \rightarrow t_{sf})$$

where, t_{il} , t_{ls} and t_{sf} are the time durations for the motion (I-L), (L-S) and (S-F) respectively. Thus there are 14 coefficients to be determined ($a_0 - a_4; b_0 - b_3; c_0 - c_4$). The 14 equations required can be generated based on the following conditions.

Sr. No	Equation	Condition
1	$\theta_{il}(0) = \theta_i$	Initial position
2	$\dot{\theta}_{il}(0) = \dot{\theta}_i$	Initial velocity
3	$\ddot{\theta}_{il}(0) = \ddot{\theta}_i$	Initial acceleration
4	$\theta_{il}(t_{il}) = \theta_l$	Position at lift-off
5	$\theta_{ls}(0) = \theta_l$	Position at lift-off
6	$\dot{\theta}_{il}(t_{il}) = \dot{\theta}_{ls}(0)$	Continuity of velocity at lift-off
7	$\ddot{\theta}_{il}(t_{il}) = \ddot{\theta}_{ls}(0)$	Continuity of acceleration at lift-off
8	$\theta_{ls}(t_{ls}) = \theta_s$	Position at set-down
9	$\theta_{sf}(0) = \theta_s$	Position at set-down
10	$\dot{\theta}_{ls}(t_{ls}) = \dot{\theta}_{sf}(0)$	Continuity of velocity
11	$\ddot{\theta}_{ls}(t_{ls}) = \ddot{\theta}_{sf}(0)$	Continuity of acceleration
12	$\theta_{sf}(t_{sf}) = \theta_f$	Final position
13	$\dot{\theta}_{sf}(t_{sf}) = \dot{\theta}_f$	Final velocity
14	$\ddot{\theta}_{sf}(t_{sf}) = \ddot{\theta}_f$	Final acceleration

Thus the 14 coefficients can be solved for and we can fit a 4-3-4 joint trajectory. However as the degree of the polynomial increases, so does the number of coefficients. Hence for multi-d.o.f. robotic manipulator we may have to determine quite a large set of polynomial function coefficients. In the next section, we discuss an effective yet simple strategy using linear segments.

Another simple intuitive strategy to plan the trajectory from an initial position to a final position for a joint is depicted in Fig.13.2.1, wherein we use the simple trapezoidal velocity profile i.e. uniform acceleration (rev-up), cruise at constant velocity and decelerate (rev-down).

Fig 13.2.2: Displacement and Acceleration Plots for a Trajectory by use of a Trapezoidal Velocity Profile.

The corresponding displacement and acceleration profiles are shown in Fig. 13.2.2. To simplify the computations, we have assumed that the acceleration and deceleration periods are same ($t_a = t_d$) and so the same magnitude of acceleration/ deceleration can be used. With such a strategy, the (θ_1 vs. t) curve would be parabolic at the beginning and ending and linear in the middle. The motion is always symmetric about the half-way point in time and position. We could accelerate fast and cruise along for a long time or we could accelerate gradually leaving very little time for linear profile. If the uniform acceleration is $\ddot{\theta}$,

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$$\dot{\theta}_A = \ddot{\theta} t_a$$

(Assuming $\dot{\theta}|_{t=0} = 0$)

$$\theta_A = \theta_i + \frac{1}{2} \ddot{\theta} t_a^2$$

$$\theta_B = \theta_A + (\dot{\theta}_A)(t_u)$$

$$\theta_C = \theta_f = \theta_B + \dot{\theta}_A t_a - \frac{1}{2} \ddot{\theta} t_a^2$$

$$\therefore \theta_f = \theta_A + \ddot{\theta} t_a t_u - \ddot{\theta} t_a t_a - \frac{1}{2} \ddot{\theta} t_a^2$$

$$\therefore \boxed{\ddot{\theta} t_a^2 - \ddot{\theta} t_a T + (\theta_f - \theta_i) = 0}$$

$$\therefore t_a = \frac{\ddot{\theta} T \pm \sqrt{(\ddot{\theta})^2 T^2 - 4\ddot{\theta}(\theta_f - \theta_i)}}{2\ddot{\theta}} = \frac{T}{2} \pm \frac{\sqrt{\ddot{\theta}^2 T^2 - 4\ddot{\theta}(\theta_f - \theta_i)}}{2\ddot{\theta}}$$

Since $t_a \leq \frac{T}{2}$, we chose the minus sign

$$\therefore t_a = \frac{T}{2} - \frac{\sqrt{\ddot{\theta}^2 T^2 - 4\ddot{\theta}(\theta_f - \theta_i)}}{2\ddot{\theta}}$$

For real t_a , $\ddot{\theta}^2 T^2 \geq 4\ddot{\theta}(\theta_f - \theta_i)$

$$\text{Or } \boxed{\ddot{\theta} \geq \frac{4(\theta_f - \theta_i)}{T^2}}$$

So for given θ_i , θ_f , T to fit this type of trapezoidal velocity profile, we chose appropriate $\ddot{\theta}$ and evaluate t_a . Choice of $\ddot{\theta}$ may be guided by the joint actuator's characteristics. However when we consider multiple joints, the total time duration T is same for all joints (i.e. all joints start and stop together in a coordinated motion). However in general, the range of displacements ($\theta_f - \theta_i$) may be different. Thus in order to have coordinated motion amongst the various joints, $t_a (= t_d)$ is selected based on the slowest joint and $\ddot{\theta}$ for all other joints are appropriately adjusted.

It must be observed that in this strategy, while each joint moves through an essentially linear trajectory there is no guarantee that the end effector moves through a straight line path in 3-D space as this will depend on the manipulator kinematics. If we have a few via points between the initial and final positions we can plan a similar essentially linear trajectory with parabolic ends (i.e. uniform acceleration/deceleration) as indicated in Fig. 13.2.3. In using such linear-parabolic-blend spline, it must be observed that the via-points are not actually reached unless the joint comes to a stop at these points.

Figure 13.2.3

Recap

In this course you will learn the following

- Typical 4-3-4 (quadric –cubic- quadric) strategy for achieving smooth picks & place operations
- Use of trapezoidal velocity profile (rev-up – cruise – rev down)

Congratulations, you have finished Lecture 13. To view the next lecture select it from the left hand side menu of the page