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Module 9 : Robot Dynamics & controls

Lecture 31 : Robot dynamics equation (LE & NE methods) and examples

Objectives

In this course you will learn the following

- Fundamentals of Dynamics
- Coriolis component of acceleration:
- Why to study Robot Dynamics & Control?
- Methods to derive Dynamical equations
- Lagrange Method
- Lagrangian formulation for Robot arm
- Theorem
- Bio-MEMS Sensor

Fundamentals of Dynamics:

While Kinematics deals with finding position, velocity & acceleration based on geometrical constraints, dynamics is concerned with solving for these when an external force acts on the system or the system is released to evolve from some initial position(e.g. Pendulum).

Now we will consider some simple examples to make clear understanding of dynamics.

Consider a block sliding on the frictionless floor. A force of constant magnitude F is applied to block. Dynamical equation can be easily found out in this case.Basically dynamical equations are mathamatical model governing dynamic behaviour of system.These equations are the force-mass-acceleration or the torque-inertia-angular acceleration relationships. By knowing the magntude of applied force & mass of block, velocity & acceleration can be easily found out at every instant of time if we know initial conditions such as whether body is at rest or moving with certain velocity.

$$F = ma$$

$$F = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} = \frac{F}{m} = constant.$$

Dynamical equation

Figure 31.1 Sliding Block

Suppose the task is to move the block from one position to other with condition on its velocity plot w.r.t time. (While moving from one position to other, it's velocity plot w.r.t time should be as shown in the fig.) We will divide this task into two steps. First step is related with finding out force requirement & second step related with application of requisite amount of force. Reason for dividing the task into two steps will be made clear in later part of this lecture.

First step:

Force requirement can be easily found out using dynamical equation.

$$F=m.a$$

(Acceleration can be found out from velocity profile w.r.t. time.)

For moving from O to A, a constant magnitude accelerating force should be applied. For moving from A to B , no force should be applied & for moving from B to C a constant magnitude retarding force should be applied. How to apply this kind of force? Think over it. Answer to this question will be given in end of this section.

Figure 31.2 Velocity profile w.r.t. time

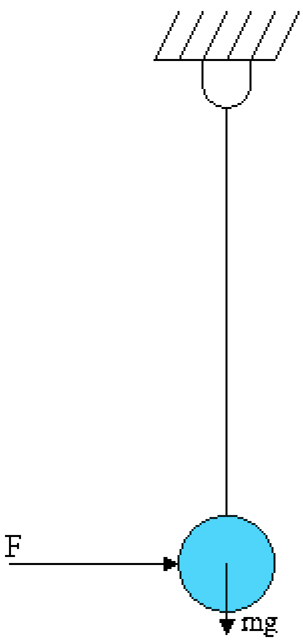


Figure 31.3 Simple Pendulum

Now consider the case of simple pendulum of mass m. Let us apply a force F whose line of action will be always perpendicular to string & whose magnitude is less than weight of Bob of pendulum .

What will happen? As there is no resisting force in the mean position, pendulum starts rotating about hinge. But as soon as it leaves its mean position, resisting force which is a component of self weight of bob comes into picture. As the angle increases its magnitude increases. At some angle let say θ , these two forces will balance each other.

$$F=mg\sin\theta$$

Suppose the task is to move the pendulum by an angle θ from mean position , then with the help of above equation we can find out what should be the magnitude of applied force. (Step 1)

How to apply that amount of force accurately? Think over it. Answer to this question will be given in end of this section.

In between these two equilibrium positions, if one wishes to find out velocity & acceleration of bob, then one can use

$$Fl - mg\sin\theta l = I\alpha \quad \text{or}$$

$$I\frac{d^2\theta}{dt^2} + mg\sin\theta l = F.l$$

Above equations are called **dynamical equations.** These are ordinary differential equations. By solving them for given initial conditions one can find out velocities, accelerations at various positions. One thing to mention is that the term ma or $I\alpha$ appearing in above equation is inertia force or inertia torque about which we will have some insight in coming part of this lecture.

One point to mention is that the bob of pendulum is performing circular motion with l as radius of rotation. Centripetal force is provided by tension in string. Magnitude of centripetal force is

$$\text{Centripetal force} = \frac{mv^2}{r} = m\omega^2 r$$

Inertia:

As per Newton 's first law of motion, every body has a tendency to resist the change in its state of motion. Means a body at rest will resist to move or a moving body will resist to stop or resist to move with different velocity. This tendency is known as inertia. In linear motion, mass represents linear inertia while in angular motion mass moment of inertia represents angular inertia. Means a body having large mass or large mass moment of inertia will have more tendency to resist the change in its state of motion. Means we need more force or torque to change its state of motion.

$$\text{Inertia force} = m.a$$

$$\text{Inertia torque} = I\,\alpha$$

Contd...

Coriolis component of acceleration:

Till now we have discussed simple examples of linear motion (i.e sliding block) & angular motion(i.e Pendulum). Now we will consider one simple example where both linear & angular motions are involved. Consider a link OB hinged at O rotating in clockwise direction with angular velocity ω . Consider a block sliding relative to the link with constant speed V_{rel} . Sliding block experiences an acceleration whose direction is given by rotating the relative sliding velocity vector by 90 ° in the sense given by angular velocity of link. This acceleration is known as Coriolis component of acceleration.

Its magnitude is given by

$$a_{cor} = 2\omega V_{rel}$$

Figure 31.4 Coriolis Acceleration

WHY CORIOLIS COMPONENT OF ACCELERATION EXIST ?

Once again consider above example. Link OB is performing pure rotation. Anything attached to it will have velocity corresponding to tangential velocity at that attached point. Block can be considered as attached to link & its attachment point varies as it moves along link. Block is having two velocity components one is V_{rel} along link which is constant & other is perpendicular to link which is a function of block position along the link. As block moves along link ,its velocity component perpendicular to link changes.This corresponds to Coriolis component of acceleration.

Velocity change in limiting case will be perpendicular to the link. Hence Coriolis component of acceleration will be always perpendicular to link.

Why to study Robot Dynamics & Control?

Basically a Robot manipulator is a positioning device. .Any task specified to it is finally converted to Regulation problem or Trajectory tracking problem.

Suppose the task is to pick an object from one point & place it to another place, this is a regulation problem. (i. e. point-to-point motion problem) & if there is a constraint that while moving from one point to another, it should follow a particular path then it will be trajectory tracking problem(continuous path).

Whatever may be the type of problem, this can be further divided into two steps. First step is related with finding out force/torque requirement which needs dynamical equations of Robot manipulator & second step is related with supplying proper amount of force/torque at joint which needs to control input to actuator.

Study of Robot Dynamics gives dynamical equations of Robot manipulator which will help in finding out these torque requirements. It is the actuator which is going to satisfy the requirements imposed by Robot manipulator.Hence these equations are coupled with Actuator dynamics.

By studying Robot Control one can develop a control system & algorithm as per performance requirement. With the help of that we can control input to actuator as per requirement.

Hence study of Robot Dynamics & Control module is very important.

To derive robot dynamical equation first kinematic analysis has to be carried out. Then we can use two methods to get the governing equations of dynamics which are explained in next part of this lecture.

Methods to derive Dynamical equations:

- Formulation using Lagrange equation

- Newton's method

Lagrange Method:-

Features:

- This Method is based on energy.

- Equations are obtained without considering the internal reaction forces.

- It is ideal for more complex robotic manipulator configurations. e.g. Complex 3D robot, flexible link robot .

- It is better than Newton's method for robotic applications.

It is based on differentiation of energy terms with respect to the systems variables & time. In this method we have to form the Lagrangian of the system, which is the difference of kinetic & potential energy of the system.

L = K.E.- P.E.
L= Lagrangian , K.E.= Kinetic Energy, P.E.= Potential Energy.

$$\tau_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

$\tau_1 =$ external force or torque applied to the system at joint i to drive link i in direction of generalised co-ordinate q_i .

q_i =generalised co-ordinate which may be joint angle θ_i for revolute joint or offset distance d_i

(Please refer D-H representation.)

1. Now we will apply this method to simple systems like spring-mass system.

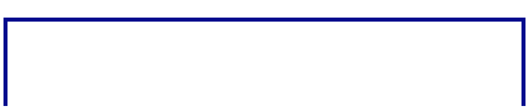


Figure 31.5 Spring- mass system

Kinetic energy of the system KE = $\frac{1}{2}m\dot{x}^2$

Potential energy of the system PE = $\frac{1}{2}kx^2$

$$L=KE-PE= \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

$$\frac{dL}{d\dot{x}} = m\dot{x}$$

From Lagrange equation

$$\frac{d}{dt}(m\dot{x}) + kx = F$$

$$m\ddot{x} + kx = F$$

This is the standard 2nd order ordinary differential equation governing dynamics of spring-mass system. Note that here we have not considered internal reaction forces. In above example there is no term involving gravitational acceleration.Now we will consider example of single link. In this example term containing g will come into picture.

Consider a single link alongwith a motor. Gear ratio between motor & manipulator is n:1

2.

$$K.E. = \tfrac{1}{2} J_m \dot{\theta}_m^2 + \tfrac{1}{2} J_l \dot{\theta}_l^2$$

$$= \tfrac{1}{2}(J_m + J_l / n^2)\dot{\theta}_m^2$$

$$P.E. = Mgl\left(1 - Cos(\theta_l)\right)$$

Lagrangian

$$L = \frac{1}{2}(J_m + J_l / n^2)\dot{\theta}_m^2 - Mgl\left(1 - \cos(\theta_l)\right)$$

From Lagrange equation

$$\left(J_m + \frac{J_l}{n^2}\right)\ddot{\theta}_m + \frac{Mgl}{n}\sin\frac{\theta_m}{n} = \tau$$

τ Consists of motor torque input u

and damping torque

$$\tau = u - \left(B_m + B_l / n^2\right)\dot{\theta}_m$$

Figure 31.5 Single link manipulation

thus equation governing system dynamics is

$$J\ddot{\theta}_m + B\dot{\theta}_m + C\sin(\theta_m / n) = u$$

Where

$$J = (J_m + J_l / n^2)$$

$$B = \left(B_m + B_l / n^2\right)$$

$$C = \frac{Mgl}{n}$$

In this example link is performing pure rotation. Hence it is very easy to find out its K.E.

3. Lagrangian formulation for Robot arm:

In general Robot link is performing general plane motion. In this case K.E. is sum of Rotational & Translational kinetic energies. Therefore

$$\text{K.E.} = \frac{1}{2} m \mathbf{v}_{\mathbf{c}}^{T} \mathbf{v}_{\mathbf{c}} + \frac{1}{2} \omega^{\mathbf{T}} I \omega$$

$$\text{where } I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

Inertia matrix should be computed in the same reference frame w.r.t. which angular velocity of link is defined.

Total kinetic energy of one robot link i is

$$KE_i = \frac{1}{2} m_i v_{ci}^{T} v_{ci} + \frac{1}{2} \omega_i^{T} I_i \omega_i$$

Figure 31.7 Robot link performing general plane motion

summing for all joints the sum can be written as

$$KE = \frac{1}{2} \dot{q}^{T} D(q) \dot{q} = \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q) \dot{q}_i \dot{q}_j$$

where D(q) is a symmetric, positive definite matrix that is in general configuration dependent.The matrix D(q) is called the inertia matrix.We will see its significance later in the dynamical equation.

Hence Lagrangian L = K.E. - P.E.

$$= \frac{1}{2}\sum_{i,j}^{n} d_{ij}(q)\dot{q}_i\dot{q}_j - PE(q)$$

$$\frac{\partial L}{\partial \dot{q}_k} = \sum_j d_{kj}(q)\dot{q}_j$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_j d_{kj}(q)\ddot{q}_j + \sum_j \frac{d}{dt} d_{kj}(q)\dot{q}_j$$

$$= \sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i}\dot{q}_i\dot{q}_j$$

$$\frac{\partial L}{\partial q_k} = \frac{1}{2}\sum_{i,j} \frac{\partial d_{ij}}{\partial q_k}\dot{q}_i\dot{q}_j - \frac{\partial PE}{\partial q_k}$$

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i\dot{q}_j + \underbrace{\frac{\partial PE}{\partial q_k}}_{g(q)} = \tau_k$$

where k= 1,2,.....,n.

Interchanging order of summation & taking advantage of symmetry

hence,

$$\sum_{i,j}\left\{\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2}\frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i\dot{q}_j = \sum_{i,j}\frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i\dot{q}_j$$

Let $C_{ijk} = \sum_{i,j}\frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}$ substituting that

$$\sum_j d_{kj}(q)\ddot{q}_j + \sum_{i,j} C_{ijk}(q)\dot{q}_i\dot{q}_j + g(q) = \tau_k$$

Contd...

Theorem:

Define the matrix $N(q,\dot{q}) = \dot{D}(q) - 2C(q,\dot{q})$ then $N(q,\dot{q})$ is skew symmetric matrix, that is,
The component n_{jk} of N satisfy $n_{jk} = -n_{kj}$

Proof:

we have $c_{kj} = \sum_{i=1}^{n}\frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i$

For a given D matrix kjth component is given by $\dot{d}_{kj} = \sum_{i=1}^{n} \frac{\partial d_{kj}}{\partial q_i}\dot{q}_i$

kjth component of $N = \dot{D} - 2C$ is

$$n_{kj} = \dot{d}_{kj} - 2c_{kj}$$

$$= \sum_{i=1}^{n}\left[\frac{\partial d_{kj}}{\partial q_i} - \left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}\right]\dot{q}_1$$

$$= \sum_{i=1}^{n}\left[\frac{\partial d_{ij}}{\partial q_k} - \frac{\partial d_{ki}}{\partial q_j}\right]\dot{q}_1$$

Since inertia matrix D(q) is symmetric, i.e. $d_{ij} = d_{ji}$

It follows that by interchanging the indices k and j $n_{jk} = -n_{kj}$.

This skew symmetry property is very important from control perspective.Once again note that D(q) is symmetric , positive definite & non singular matrix.

Micro-sensors

- ## Accelerometer:

Device design has sensing method of strain gauge type. This is as shown below. Design calculations involved in these are, Finding dimensions and stresses by FEM (finite element method) analysis, Finding frequencies by Modal FEM, the consideration for thermal expansion and damping can be done by FEM and experimental verification.

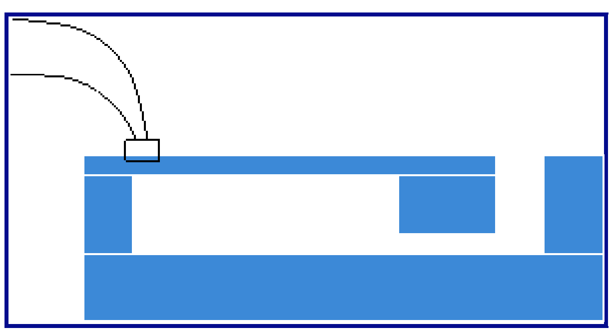


Figure 31.8 Accelerometer

Sensing Methods :

- Resistive sensing
    - Based on strain: accelerometers where resistance changes with strain

    - Based on temperature: where resistance changes with temperature
- Capacitive sensing: accelerometers, pressure sensors
- Bimetallic strips : which has strains as per the temperature changes
- Thermocouple effect: This has voltage developed as per temperature effect.
- Piezoelectric effect : This has charge developed as per strains in piezo
- Using optics laser source and detectors
    - Diffraction effects: where the light diverges into wider angle
    - Interference effects : where the interference occurs with other waves.
    - Quadrature photo diodes: where the photodiodes output is fed to quadrature decoding arrangement.

More MEMS devices

- Other micro-sensors:
    - Pressure sensor
    - Vibrating gyroscope
    - Bio-MEMS sensors: DNA chips, “lab on chip”
- Micro actuators
    - Comb actuators, micro-motors
    - Thermal actuators
    - Piezo-actuators
- Micro-gears, micro-engines
- Micro-fluidic systems: Inkjet printers, drug delivery

- Grating light valve (GLV display)
- Digital optical switches
- And many more Yet to come...

Bio-MEMS Sensor

Cantilever based

- Concept: cantilever structure with antigen
- Effects:
  - Change in mass
  - Deflection due to repulsive forces
- Detection of bacteria by
  - Optical
  - Capacitive
  - Other techniques like resonance freq.

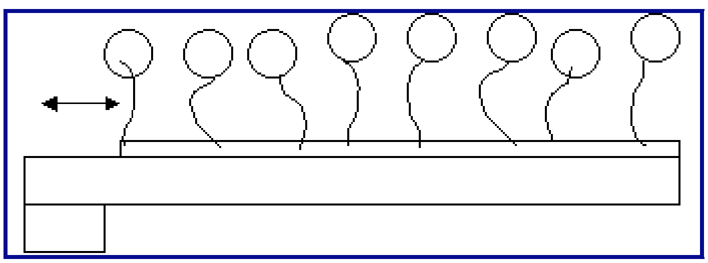


Figure 31.9 Bio MEMS Sensor.

Micro-actuators example

Comb Drive

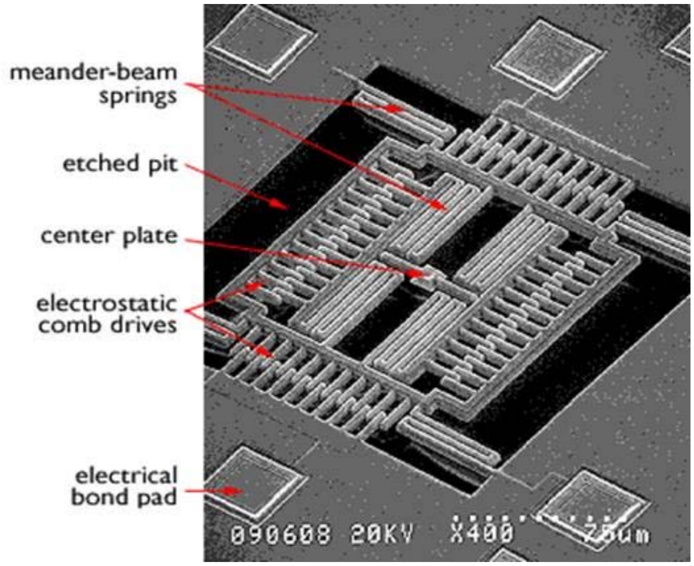


Figure 31.10 Comb Drive

Micro-actuators

Comb drive operating gears

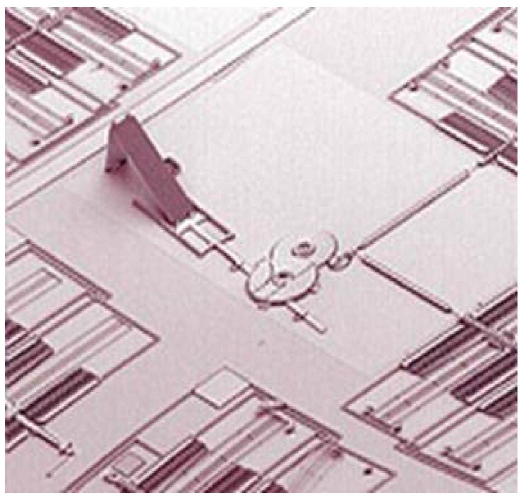


Figure 31.11

Micro-actuators example

Micro-motor at Sandia Labs

- High speeds
- Electrostatic force pulses

Figure 31.12 Micromotor

Micromotor Working Principle

Similar to working of stepper motor.

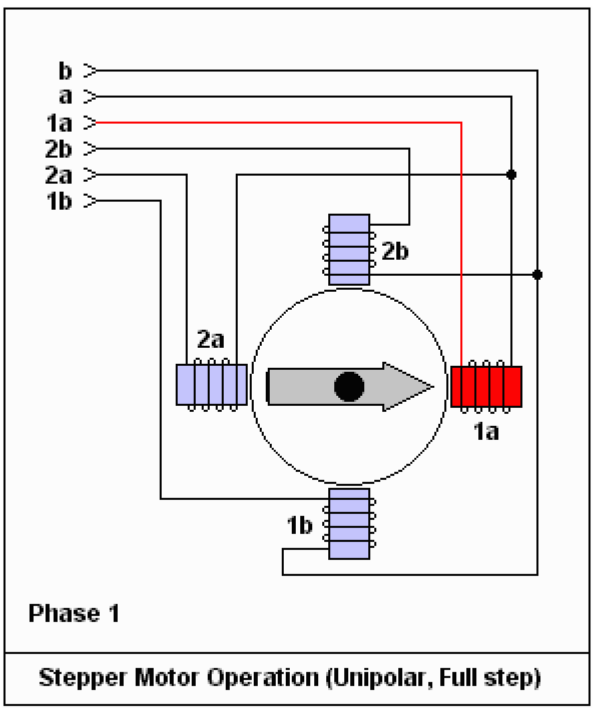
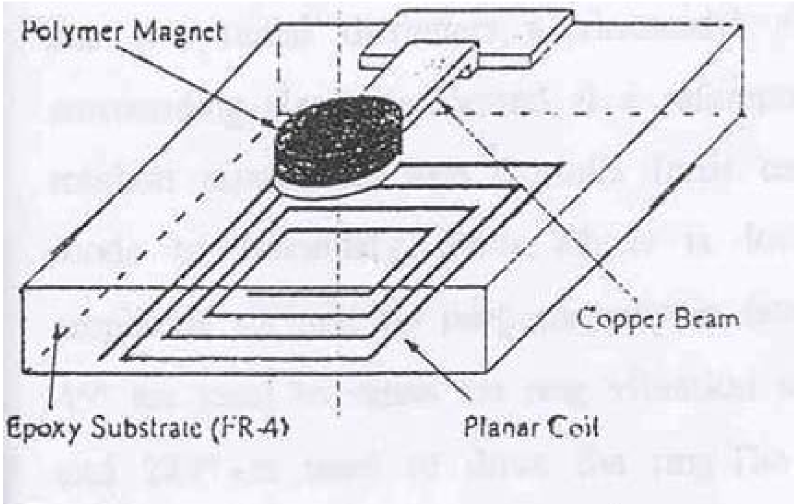


Figure 31.2 Working Principle of Micro motor

Micro-actuators

Cantilever beam magnetic micro-actuator:



Schematic view of the cantilever beam microactuator

Figure 31.3

FABRICATION SEQUENCE OF MAGNETIC MICROACTUATOR (animation)

Figure 31.4

Summery

Fabrication processes

- Lithography
  - PPR (positive photo-resist)
  - NPR (negative photo-resist)
  - E-BEAM lithography
- Material removal processes
  - Chemical etching

        - Isotropic
        - Anisotropic
    - Plasma etching: RIE
- Material Deposition processes
    - Oxidation
    - Sputtering
    - Chemical vapor deposition (CVD)
    - Electroplating
- Surface micromachining
- LIGA
- Micro sensors and actuators

MEMS is inevitable in the future consumer and industrial mechatronic systems because of their higher quality and accuracy at the reduced cost

Recap

In this course you have learnt the following

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- Coriolis component of acceleration:
- Why to study Robot Dynamics & Control?
- Methods to derive Dynamical equations
- Lagrange Method
- Lagrangian formulation for Robot arm
- Theorem
- Bio-MEMS Sensor

Congratulations, you have finished Lecture 31. To view the next lecture select it from the left hand side menu of the page