Module 6 : Robot manipulators kinematics

Lecture 18 : Homogeneous coordinate transformation and examples

Objectives

In this course you will learn the following

- What is meant by homogeneous coordinate systems for manipulators.

- Its properties & Example for the same.

Homogenous Coordinates (Refer Figure 18.1)

Let x1,y1,z1 be global ref frame with x2,y2,z2 as local frame for point P. Now homogenous coordinates are represented as 4x4 matrix of position & orientation matrix of this point .

$$\begin{Bmatrix} x_p \\ y_p \\ z_p \end{Bmatrix} = \begin{Bmatrix} x_{o2} \\ y_{o2} \\ z_{o2} \end{Bmatrix} + \left[{}^1R_2 \right] \begin{Bmatrix} xp \\ yp \\ zp \end{Bmatrix}_2$$

is now represented as

$$\begin{bmatrix} x_{p1} \\ y_{p1} \\ z_{p1} \\ 1 \end{bmatrix} = \begin{bmatrix} & & & x_{021} \\ & \left[{}^1R_2 \right] & & y_{021} \\ & & & z_{021} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{p2} \\ y_{p2} \\ z_{p2} \\ 1 \end{bmatrix}$$

$$\{P\}_1 = \left[{}^1T_2 \right]\{P\}_2$$

Also,

$$ {}^iT_k = \left[{}^iT_j \right]\left[{}^jT_k \right]$$

Quiz : Pr *ove that* $\left[{}^iT_k \right] = \left[{}^kT_i \right]^{-1}$

Figure 18.1 Homogeneous coordinates

Theorem : To prove that $[{}^iT_k] = [{}^kT_i]^{-1} = [{}^kT_i]^{T}$

Fig. 18.2 Ref & body attached coordinate systems

Here we would like to find 3x3 transformation matrix R that will transform the coordinates of puvw to the coordinates expressed w.r.t. the OXYZ coordinate system, after the OUVW coordinate system has been rotated. That is

$$p_{xyz} = R p_{uvw}$$

Recalling the definitions of the components of a vector, we have

$$p_{uvw} = p_u i_u + p_v j_v + p_w k_w$$

where p_x, p_y, p_z represent the projections of **p** onto respective axis. Now using definitions of scalar product and above equation, we have

$$p_x = i_x * p = i_x * (p_u i_u + p_v j_v + p_w k_w) = i_x * i_u * p_u + i_x * j_v * p_v + i_x * k_w * p_w$$
$$p_y = j_y * p = j_y * (p_u i_u + p_v j_v + p_w k_w) = j_y * i_u * p_u + j_y * j_v * p_v + j_y * k_w * p_w$$
$$p_z = k_z * p = k_z * (p_u i_u + p_v j_v + p_w k_w) = k_z * i_u * p_u + k_z * j_v * p_v + k_z * k_w * p_w$$

OR expressed in matrix form, it will be as

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} i_x * i_u & i_x * j_v & i_x * k_w \\ j_y * i_u & j_y * j_v & j_y * k_w \\ k_z * i_u & k_z * j_v & k_z * k_w \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}$$

$$p_{xyz} = R * p_{uvw}$$

Similarly one can obtain the coordinates of puvw from coordinates of pxyz as,

$$p_{uvw} = Q * p_{xyz}$$

$$\begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix} = \begin{bmatrix} i_u * i_x & i_u * j_y & i_u * k_z \\ j_y * i_x & j_v * j_y & j_v * k_z \\ k_z * i_x & k_w * j_y & k_w * k_z \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix}$$

from above two equations; since dot product are commutative we can write

$$Q = R^{-1} = R^T$$

Thus we have proved the theorem.

Theorem : To prove that $[{}^i T_k] = [{}^k T_i]^{-1} = [{}^k T_i]^T$

Fig. 18.2 Ref & body attached coordinate systems

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OR expressed in matrix form, it will be as

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Similarly one can obtain the coordinates of puvw from coordinates of pxyz as,

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Example

Find position and orientation matrices (transformation) of end effecter in initial and final position for the system shown below.

Solution :
A object with 3 pegs is placed on a table as shown below Figure 18.3. It need to be picked up & placed on another inclined plane part as shown. Problem is about matching the two coordinate systems viz (x2,y2,z2) to (x5,y5,z5).

Figure 18.3

Data given that,

$$\left\{\begin{matrix} x_2 \\ y_2 \\ z_2 \end{matrix}\right\}_1 = \left\{\begin{matrix} 300 \\ 400 \\ 500 \end{matrix}\right\}; \left\{\begin{matrix} x_4 \\ y_4 \\ z_4 \end{matrix}\right\}_1 = \left\{\begin{matrix} 500 \\ 800 \\ 0 \end{matrix}\right\}; \left\{\begin{matrix} x_5 \\ y_5 \\ z_5 \end{matrix}\right\}_4 = \left\{\begin{matrix} 150 \\ 180 \\ 200 \end{matrix}\right\}; \left\{\begin{matrix} x_6 \\ y_6 \\ z_6 \end{matrix}\right\}_5 = \left\{\begin{matrix} 25 \\ 50 \\ 10 \end{matrix}\right\}$$

The rotation matrix is $^1R_6 = {}^1R_2 \ {}^2R_3 \ {}^3R_4 \ {}^4R_5 \ {}^5R_6$ (All 3x3 matrices) & Transformation matrix is (4x4) $^1T_6 = {}^1T_2 \ {}^2T_3 \ {}^3T_4 \ {}^4T_5 \ {}^5T_6$ which need to be found for initial & final position of part.

Part (A)

We have ,

$$\left[{}^1R_4\right] = \left[\hat{x}_4 \ \hat{y}_4 \ \hat{z}_4\right] = \begin{bmatrix} \cos 30 & \cos 15 * \cos(30+90) & 0.1295 \\ \sin 30 & \cos 15 * \sin(30+90) & -0.2243 \\ 0 & \sin 15 & 0.966 \end{bmatrix}$$

$where \ \hat{z}_4 = \hat{x}4 \ X \ \hat{y}4$

$$\therefore \left[{}^1R_4\right] = \begin{bmatrix} 0.866 & -0.483 & 0.1295 \\ 0.5 & 0.837 & -0.2243 \\ 0 & 0.259 & 0.966 \end{bmatrix}$$

Also, local coordinate systems 4, 5 & 6 are parallel. Therefore their rotation

matrices are Identity matrices.

$$\left[{}^4R_5\right] = \left[I_{3x3}\right]; \left[{}^5R_6\right] = \left[I_{3x3}\right]$$

Thus rotation matrix in global coordinate is known for final position of End effecter (EE).

Also for initial position of part, the coordinate systems are parallel to each other except directions. Therefore rotation matrix is

$$\left[{}^1R_3 \right] = \left[{}^1R_2 \right]\left[{}^2R_3 \right] = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Part(B)

To find the transformation matrices for initial & final position of end

Effecter, we proceed as follows.

For initial position of EE,

$$\left[{}^1T_3 \right] = \left[{}^1T_2 \right]\left[{}^2T_3 \right] = \begin{bmatrix} & & & {}^1x_2 \\ & {}^1R_2 & & {}^1y_2 \\ & & & {}^1z_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} & & & {}^2x_3 \\ & {}^2R_3 & & {}^2y_3 \\ & & & {}^2z_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For final position of EE,

$$\left[{}^1T_6 \right] = \left[{}^1T_4 \right]\left[{}^4T_5 \right]\left[{}^5T_6 \right] = \begin{bmatrix} & & & {}^1x_4 \\ & \left[{}^1R_4 \right] & & {}^1y_4 \\ & & & {}^1z_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} & & & {}^4x_5 \\ & \left[{}^4R_5 \right] & & {}^4y_5 \\ & & & {}^4z_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} & & & {}^5x_6 \\ & \left[{}^5R_6 \right] & & {}^5y_6 \\ & & & {}^5z_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Thus all the matrices are known from above to calculate ${}^1T_6$

Recap

In this course you will learn the following

- Specifying position and orientation of rigid bodies can be done by homogeneous coordinate representation

- How to express end effector coordinate in base reference frame using homogeneous coordinates

Congratulations, you have finished Lecture 18. To view the next lecture select it from the left hand side menu of the page