Module 6 : Robot manipulators kinematics

Lecture 15 : Forward & inverse kinematics examples of 2R, 3R & 3P manipulators

Objectives

In this course you will learn the following

- Introduction to robot kinematics
- Examples of simple manipulators

Introduction

Consider a block to be picked and placed by a robot or a manipulator as shown in figure15.1.Here four positions of end effecter are known as position 1,2,3,4. These positions are to be reached by End Effector (EE) within some known intervals of time, without fouling with any object in the path. Here again the joint interpolation motion will be as required to get desired position of end effecter.

Obtaining θs, at each t1, t2, t3, t4 intervals knowing the EE position & Orientation, is a inverse kinematics problem. Another problem would be forward kinematics problem. Here knowing the joint variables as each time interval, find out the position & orientation of all links is a problem. Thus we can conclude as, forward position problem with given data of fixed parameters of linkages and joint variable values, we need to find out position and orientation of all links. Inverse position Problem would be as, given the fixed parameters of mechanism and position & orientation of end effecter, we need to find out the joint variables values.

Figure 15.1. Pick & Place Robot example with joint motion

Example 1: Planer 2R manipulator

Part A : Forward position problem:

Given $\ell_1, \ell_2, \theta_1, \theta_2$ as fixed parameters of links and joint variable angles, find position X_B, Y_B, Xp, Yp and orientation angles α_1, α_2 of the end effectors.

Solution :

From geometry of mechanism(Figure 15.2), we have

$$X_B = \ell_1 \cos\theta_1;\ Y_B = \ell_1 \sin\theta_1$$
$$Xp = \ell_1 \cos\theta_1 + \ell_2 \cos(\theta_1 + \theta_2)$$
$$Yp = \ell_1 \sin\theta_1 + \ell_2 \sin(\theta_1 + \theta_2)$$

And the orientation is $\alpha_1 = \theta_1,\ \alpha_2 = \theta_1 + \theta_2$

Figure 15.2: simple 2 R Manipulator

Part (B): Inverse position calculation

Data given: - ℓ_1, ℓ_2 the linkage parameters and end effecter position as (Xp, Yp) Find Out: θ_1, θ_2 the joint variables.

Solution:

Using the cosine rule,

$$\theta_1 = \tan^{-1}\left(\frac{X_p}{Y_p}\right) +/- \cos^{-1}[(l_1^2 + X_p^2 + Y_p^2 - l_2^2)/(2l_1\sqrt{(X_p^2 + Y_p^2)}$$

Here we observe that 2 solutions exist for the angle with +/- sign in RHS of the equation.

If the EE position is on the end of workspace, only one solution is possible. Also, if the EE position lies outside the work space, then there exists no solution for angles.

Example 2: 3R manipulator

Here given data are fixed link parameters as ℓ_1, ℓ_2, ℓ_3 and EE specification Xp, Yp, α_3.

It is required to find: $\theta_1, \theta_2, \theta_3$

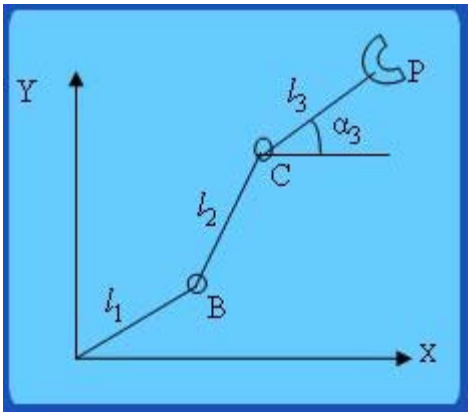

Figure 15.3: 3R Manipulator

Solution :

From Figure 15.3 the point C coordinates are found as

$$XC = Xp - \ell_3 \cos \alpha_3$$
$$Yc = Yp - \ell_3 \sin \alpha_3$$

Now the problem becomes same as 2R manipulator with parameters as X C , Yc, ℓ_1, ℓ_2

Also $\theta_3 = \alpha_3 - \alpha_2 = \alpha_3 - (\theta_1+\theta_2)$

Example 3: 3P Manipulator

Here the inverse kinematics problem will be to find S_1, S_2, S_3 given the EE coordinates as Xp, Yp, Zp

Recap

In this course you will learn the following

- What is meant by forward and inverse kinematics
- Examples of simple 2R, 3R and 3P manipulators

Congratulations, you have finished Lecture 15. To view the next lecture select it from the left hand side menu of the page