Module 6 : Robot manipulators kinematics

Lecture 16 : Specifying position & orientation of rigid bodies

Objectives

In this course you will learn the following

- **How many parameters are required for specifying position and orientation of rigid body**
- **What is meant by Rotation matrices w.r.t. robot kinematics**

Introduction

The joints of a manipulator can be electrically, hydraulically or pneumatically actuated. The number of joints determines the degree of freedom of manipulator. Typically a manipulator should pose 6 DOF; three for positioning and three for orientation. This is explained below.

Specifying position & orientation of rigid body :

It is about specifying a set of variables using the value of which one can determine position of all points on the body and direction of every vector on the body. (Refer Figure 16.1)

Fig 16.1 Specifying position and orientation of a rigid body

Example 16.2 : Given Local coordinates of point & vector ; find its global coordinates (Refer Figure 16.2)

Fig 16.2 Local Vs Global coordinates

$$
\begin{aligned}
&= \hat{x}.x_{\hat{v}} + \hat{y}.y_{\hat{v}} + \hat{z}.z_{\hat{v}} \\
&= [\hat{x}\ \hat{y}\ \hat{z}] * \begin{Bmatrix} x_{\hat{v}} \\ y_{\hat{v}} \\ z_{\hat{v}} \end{Bmatrix} \\
&= \begin{bmatrix} X_{\hat{x}} & X_{\hat{y}} & X_{\hat{z}} \\ X_{\hat{x}} & Y_{\hat{y}} & Y_{\hat{z}} \\ Z_{\hat{x}} & Z_{\hat{y}} & Z_{\hat{z}} \end{bmatrix} \begin{Bmatrix} x_{\hat{v}} \\ y_{\hat{v}} \\ z_{\hat{v}} \end{Bmatrix} \\
&= \left[{}^{G}R_{L} \right] * \{u\}_{L}
\end{aligned}
$$

Out of 9 elements of R, only 3 are genuine variables for orientation as,

$$
\begin{aligned}
&\hat{z} = \hat{x}\ X\ \hat{y} \quad \& \quad \hat{x}\ \&\ \hat{y}\ \textit{are orthogonal} \\
&\therefore\ \| x \| = 1\ \&\ \| y \| = 1\ \ \&\ \hat{x}^{T}\hat{y} = 0
\end{aligned}
$$

Example 16.3 :

A typical rotation matrix is as for the figure shown is

$$
\left[{}^{1}R_{2} \right] = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
$$

Figure 16.3

Example 16.3

Position & orientation of 3 w.r.t 2 is known and Position & Orientation of 2 w.r.t 1 is known, find position & orientation of 3 w.r.t 1?

$$\begin{Bmatrix} x_v \\ y_v \\ z_v \end{Bmatrix} = \left\{\overline{O_1O_2}\right\}_1 + \left\{\overline{O_2P}\right\}_1$$

$$= \{O_2\}_1 + \left[{}^1R_2\right] * \begin{Bmatrix} x_v \\ y_v \\ z_v \end{Bmatrix}_2$$

$$= \{O_2\}_1 + \left[{}^1R_2\right] \left(\{O_3\} + \left[{}^2R_3\right] * \begin{Bmatrix} x_v \\ y_v \\ z_v \end{Bmatrix}_3 \right)$$

$$= \{O_3\}_1 + \left[{}^1R_3\right] * \begin{Bmatrix} x_v \\ y_v \\ z_v \end{Bmatrix}_3$$

Thus we have,

$$\left[{}^1R_3\right] = \left[{}^1R_2\right] * \left[{}^2R_3\right] * \ldots\ldots\ldots\ldots \left[{}^{n-1}R_n\right]$$

and

$$\left[{}^GR_L\right] = \begin{bmatrix} X_x & X_y & X_z \\ X_x & Y_y & Y_z \\ Z_x & Z_y & Z_z \end{bmatrix}$$

Above are three orthonormal column vectors

with property that

$$\left[{}^LR_G\right] = \left[{}^GR_L\right]^{-1} = \left[{}^GR_L\right]^T$$

Figure 16.4

Recap

In this course you will learn the following

- Six parameters or DOF are required for specifying position & orientation

- It is possible to get rotation matrices for two typical coordinate systems

Congratulations, you have finished Lecture 16. To view the next lecture select it from the left hand side menu of the page