Module 6 : Robot manipulators kinematics

Lecture 24 : Introduction to dynamics of robots and Jacobian for velocity analysis

Objectives

In this course you will learn the following

- Velocity analysis of general manipulator
- Static force analysis of manipulators
- Finding acceralation of links of manipoulator
- Finding relation between forces and acceralations of the links

Fig 24.1 Velocity Analysis for manipulator

Velocity Analysis: We have end effector velocity as

$$\begin{Bmatrix} v_{P_kP_0} \\ \omega_k \end{Bmatrix} = \begin{bmatrix} J_{C_1} & J_{C_2} & \cdots & J_{C_k} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_k \end{Bmatrix}$$

$$\dot{q}_i = \dot{\theta}_i \text{ if } i \text{ is revolute}$$

$$J_{C_i} = \begin{Bmatrix} \hat{z}_{i-1} \times r_{P_iP_0} \\ \hat{z}_{i-1} \end{Bmatrix}$$

$$\dot{q}_i = \dot{d}_i \text{ if } i \text{ is prismatic}$$

$$J_{C_i} = \begin{Bmatrix} \hat{z}_{i-1} \\ 0 \end{Bmatrix}$$

With forward kinematics problem as, given θ˙ , find v & w and the inverse kinematics as given v & w, find θ˙ .Solution is possible only when [J] is non singular. In short any velocity that cannot be generated by EE is singular condition. Consider following example of 2R manipulator. Figure 24.2 indicates the only possible velocity that can be generated.

Figure 24.2 Illustation of singularity

Now consider example of PUMA 6R Manipulator. (refer previous figure). In fully stretched condition PUMA can give in plane velocities & w as in unique way and other v & w are not possible. This is singular condition .In PUMA if n>6 , inverse condition will give infinite solutions. As we know 6 parameters are required here for defining v & w (3 for each) & if n>6 , the condition is rank deficiency of corresponding matrices. Consider example of pick & place a block by Robot. (see figure 24.3)Here initial starting and ending velocities are zero. Forward and inverse solution can be found out.

Figure 24.3 Pick & Place operation using PUMA

Static force analysis is about finding relationship between End effector Force F (force & couple) and joint motors forces (torques or linear force). For this Principle of Virtual Work in context of manipulator is being used. Within admissible configuration of manipulator, the total virtual work done is zero. i.e

Relation between F and T

F=End effector force; T=joint force

$$\begin{Bmatrix} v \\ w \end{Bmatrix}^T \{F\} + \{\dot{q}\}^T \{T\} = 0$$

$$\begin{Bmatrix} v \\ w \end{Bmatrix} = [J]\{\dot{q}\}$$

$$\{\dot{q}\}^T \left([J]^T\{F\} + \{T\}\right) = 0 \Rightarrow \{T\} = -[J]^T\{F\}$$

Thus the end effector force F that to be balanced by joint torque T can be found. Now static force analysis uses the matrix [J] which we have derived earlier velocity analysis. Thus if we do velocity analysis which means we have done static force analysis. The acceralation relationship for a rigid body shown below (and 2 nd equation is for serial manipulator)

Acceralations Analysis:

Figure24.4 : Acceralation relationship for rigid bodies.

$$a_Q = a_P + \omega \times \left(\omega \times r_{QP}\right) + \alpha \times r_{QP}$$

$$\begin{Bmatrix} a_{P_k P_0} \\ \alpha_k \end{Bmatrix} = [J_k] \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \vdots \\ \ddot{q}_k \end{Bmatrix} + f_k\left(q_1, \ldots, q_k, \dot{q}_1, \ldots, \dot{q}_k\right)$$

For parallel manipulator like Stewart platform, consider velocity equation for each loop of links.

Dynamic equations for manipulator

It is about finding relationship between forces and accelerations of links. First case is forward dynamics problem where knowing joint motion & link parameters we need to find end effectors motion in terms of displacement, velocity & forces. The 2 nd is inverse dynamics problem where we know end effectors motion and need to find joint actuators motion. The generalized dynamics equation is as follows where the terms have meaning as defined earlier.

$$\tau = [M]\ddot{q} + h(q, \dot{q}) + b(q, \dot{q}) + g(q) + [\bar{J}]F$$

$$\ddot{q} \leftarrow \tau, q, \dot{q}, \text{ inertia parameters, friction,}$$
$$\text{external forces}$$

$$\tau \leftarrow q, \dot{q}, \ddot{q}, \ldots$$

Recap

In this course you will learn the following

- How Jacobian matrix relates the joint actuator velocities to manipulator link velocities

How it means that we have done velocity analysis, implies we have done static force analysis.

- General theory of finding acceralations of links of manipulator

- Dynamics equations of manipulator (Forward and Inverse Dynamics)

Congratulations, you have finished Lecture 24. To view the next lecture select it from the left hand side menu of the page.