Module 9 : Robot Dynamics & controls

Lecture 32 : General procedure for dynamics equation forming and introduction to control

Objectives

In this course you will learn the following

- Lagrangian Formulation for 2-R Manipulator
- Proper Characteristics of Dynamical equation of Robot Manipulato
- Newton's Method
- Terms in Robot Dynamic Equations
- Centripetal force
- Typical Control system Configuration for Robot manipulator
- Introduction to Control

We clearly see that to use Lagrange method to get dynamical equations for a robotic manipulator first we need to get expression for K.E. & P.E. of Robot manipulator.

To get the enrgy we need to know linear velocity of c.g. & angular velocity of each link of Robotic manipulator. we have seen in the modules *Kinematics of Robot* how to get these velocities expressed in the global co-ordinate reference frame. Explained below is general preocedure to be followed to carry out Lagrange formulation for a Robotic manipulator.

Following that we will see use of Lagrange method to obtain equations of dynamics for 2-R manipulator.

General procedure for formulation of robot dynamics

STEP 1

Perform kinematic analysis to. find out velocities of c.g.'s of robot links.

STEP 2

Find the kinetic energy

$$KE = \sum_i \frac{1}{2} m_i v_{ci}^T v_{ci} + \frac{1}{2} \omega_i^T I_i \omega_i$$

Note I_i is in the link reference frame & $v_{ci} = J_{vci}(q)\dot{q}$, $\omega_i = R_i^T(q) J_{\omega i}(q)\dot{q}$

STEP 3

Now we will express K.E. as $KE = \frac{1}{2}\dot{q}^T D(q)\dot{q} = \frac{1}{2}\sum_{i,j}^{n} d_{ij}(q)\dot{q}_i\dot{q}_j$ so that we will get D(q) matrix. Refer kinematic analysis modules for details.

STEP 4

The Potential energy is $PE = \sum_i g^T r_{c_i} m_i$ where g = gravity vector in base coordinate system.

STEP 5

Now we will apply Lagrange equation (simplified version) to get the robot equation.

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

$$\text{where,} \quad c_{kj} = \sum_{i=1}^{n} \frac{1}{2}\left\{\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right\}\dot{q}_i$$

$$g(q) = \frac{\partial PE}{\partial q}$$

Note that generalized force may contain damping terms.

The equation is 2nd order ordinary differential equation in general nonlinear . It can be solved for **q'**s, given the torques applied. Its numerical solution can be found out by using MATLAB.

Lagrangian Formulation for 2-R Manipulator :

Figure 32.1 2R Manipulator

STEP 1

velocity of c.g. of link 1 in terms of joint velocities& joint angle q (generalised co-ordinate)

$$v_{c1} = J_{v_{c1}} \dot{q}$$

where $$J_{v_{c1}} = \begin{pmatrix} -l_{c1}\sin q_1 & 0 \\ l_{c1}\cos q_1 & 0 \end{pmatrix}$$

$$v_{c2} = J_{v_{c2}} \dot{q}$$

similarly $$J_{v_{c2}} = \begin{pmatrix} -l_1 \sin q_1 - l_{c2}\sin(q_1+q_2) & -l_{c2}\sin(q_1+q_2) \\ l_1\cos q_1 + l_{c2}\cos(q_1+q_2) & l_{c2}\cos(q_1+q_2) \end{pmatrix}$$

STEP 2 & 3

Using velocities of c.g.'s of links

Translational part of kinetic energy

$$\frac{1}{2}m_1 v_{c1}^T v_{c1} + \frac{1}{2}m_2 v_{c2}^T v_{c2} = \frac{1}{2}\dot{q}\left\{m_1 J_{v_{c1}}^T J_{v_{c1}} + m_2 J_{v_{c2}}^T J_{v_{c2}}\right\}\dot{q}$$

To get the rotational part of K.E.

Angular velocities of link 1 and 2 are

$$\omega_1 = \dot{q}_1$$
$$\omega_2 = \dot{q}_1 + \dot{q}_2$$

Rotational Kinetic energy $= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$

$$= \frac{1}{2}\dot{q}^T\left\{I_1\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix} + I_2\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\right\}\dot{q}$$

Contd...

STEP 4

Potential energy of system

$$V = \left(m_1 l_{c1} + m_2 l_1\right) g \sin q_1 + m_2 l_{c2} g \sin\left(q_1 + q_2\right)$$

STEP 5

$$d_{11} = m_1 l_{c1}^2 + m_2\left(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos q_2\right) + I_1 + I_2$$

$$d_{12} = d_{21} = m_2\left(l_{c2}^2 + 2l_1 l_{c2}\cos q_2\right) + I_2$$

$$d_{22} = m_2 l_{c2}^2 + I_2$$

Element of matrix C

$$C_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -m_2 l_1 l_{c2}\sin q_2 = h$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = h$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = -h$$

$$C_{122} = C_{212} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

$$\frac{\partial V}{\partial q_1} = \left(m_1 l_{c1} + m_2 l_1\right) g\cos q_1 + m_2 l_{c2} g\cos\left(q_1 + q_2\right)$$

$$\frac{\partial V}{\partial q_2} = m_2 l_{c2}\cos\left(q_1 + q_2\right)$$

Final dynamical equations of the system

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_1\dot{q}_2 + c_{221}\dot{q}_2^2 + \frac{\partial V}{\partial q_1} = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \frac{\partial V}{\partial q_2} = \tau_2$$

Contd...

Proper Characteristics of Dynamical equation of Robot Manipulator:

we know that $D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$

for 2R manipulator

$$D = \begin{bmatrix} m_1 l_{c1}^2 + m_2\left(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos q_2\right) + I_1 + I_2 & m_2\left(l_{c2}^2 + l_1 l_{c2}\cos q_2\right) + I_2 \\ m_2\left(l_{c2}^2 + l_1 l_{c2}\cos q_2\right) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$$

$$C = \begin{bmatrix} h.\dot{q}_2 & h.\dot{q}_2 + h.\dot{q}_1 \\ -h.\dot{q}_2 & 0 \end{bmatrix}$$

$$\text{where } h = -m_2 l_1 l_{c2} \left(\sin q_2\right)$$

$$g_1 = (m_1 l_{c1} + m_2 l_1) g \cos q_1 + m_2 l_{c2} g \cos(q_1 + q_2)$$
$$g_2 = m_2 l_{c2} g \cos(q_1 + q_2)$$

D Matrix will be $D = \begin{bmatrix} m_1 l_{c1}^2 + m_2\left(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos q_2\right) + I_1 + I_2 & m_2\left(l_{c2}^2 + l_1 l_{c2} \cos q_2\right) + I_2 \\ m_2\left(l_{c2}^2 + l_1 l_{c2} \cos q_2\right) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$

then

$$\dot{D} = \begin{bmatrix} -2 m_2 l_1 l_{c2} \left(\sin q_2\right).\dot{q}_2 & -m_2 l_1 l_{c2} \left(\sin q_2\right).\dot{q}_2 \\ -m_2 l_1 l_{c2} \left(\sin q_2\right).\dot{q}_2 & 0 \end{bmatrix}$$

taking $-m_2 l_1 l_{c2} \left(\sin q_2\right) = h$

Now the matrix become $\dot{D} = \begin{bmatrix} 2h.\dot{q}_2 & h.\dot{q}_2 \\ h.\dot{q}_2 & 0 \end{bmatrix}$

Matrix $C = \begin{bmatrix} h.\dot{q}_2 & h.\dot{q}_2 + h.\dot{q}_1 \\ -h.\dot{q}_2 & 0 \end{bmatrix}$

$$\dot{D} - 2C = \begin{bmatrix} 0 & -h.\dot{q}_2 - 2h.\dot{q}_1 \\ +h.\dot{q}_2 + 2h.\dot{q}_1 & 0 \end{bmatrix}$$

Figure 32.2 : 2-R MANIPULATOR

Newton's Method :

We will see its application to robot

- First we have to carry out kinematic analysis of the robot to find out accelerations.
- Then we have to draw draw free body diagrams of robot links.
- After applying Newton's second law of motion we can find out dynamical equations of system.
- For complex 3D problems one can write down the vector equations of force summation for each link and use them recursively to eliminate the internal coupling forces .

Following example of NE method will explain the procedure.

Terms in Robot Dynamic Equations

This system is similar to a 2-R manipulator except the mass of each link is assumed to be concentrated at the end of each link.

Figure 32.3 2R Manipulator

Centripetal force:

Suppose link 1 is rotated with $\dot{\theta}_1$ velocity & link 2 is rotated with $\dot{\theta}_2$ velocity.

Now considering link 2,

Centripetal force for link 2 = $m_2 \dot{\theta}_2^{\,2} l_2$

Centripetal force will be provided by link 1. So its reaction on link 1 will be in the direction opposite to the centripetal force . Resolving this reaction component along & perpendicular to link 1

The component perpendicular to link will exert a torque at joint 1 Whose magnitude is given by

$$\text{Torque} = m_2 \dot{\theta}_2^{\,2} l_2 l_1 \sin\theta_2 \quad (\mathbf{A})$$

WEIGHT OF LINKS :

Figure 32.4 2R Manipulator

Now looking at figure given below ,

torue exerted at joint 1 due to wt. of links =

$$m_1 g l_1 \cos\theta_1 + m_2 g[l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)] \quad \textbf{(B)}$$

CORIOLIS FORCE:-

Figure 32.4 Corlolis Force

Contd...

Considering motion of link 2 only , tangential velocity of mass M2 will be = $l_2\dot{\theta}_2$

Now consider an imaginary link AB as shown in fig. which will be always parallel to tangential velocity of link 2. This imaginary link is assumed to be fixed rigidly to 1st link. So it will be rotating with the same angular velocity & in the same sense as that of 1st link.

Now the imaginary link AB alongwith mass M_2 will be analogous to our standard rotary crank & slider system.

$$a_{cor} = 2.\omega.V_{rel.}$$
$$a_{cor} = 2.(\dot{\theta}_1).(l_2\dot{\theta}_2)$$
$$F_{cor} = m_2.a_{cor}$$
$$F_{cor} = 2m_2(\dot{\theta}_1).(l_2\dot{\theta}_2)$$
$$T_{cor} = F_{cor}\sin\theta_2.l_1$$

$$T_{cor} = 2m_2(\dot{\theta}_1).(l_2\dot{\theta}_2).\sin\theta_2.l_1 \quad \textbf{(C)}$$

Eqn of standard 2-R manipulator. for link 1

$$T_1 = \underbrace{(\frac{1}{3}m_1 l_1^2 + m_2 l_1^2 + \frac{1}{3}m_2 l_2^2 + m_2 l_1 l_2 \cos\theta_2)}_{IV}\ddot{\theta}_1 + \underbrace{(\frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 \cos\theta_2)}_{V}\ddot{\theta}_2$$

$$- \underbrace{(m_2 l_1 l_2 \sin\theta_2)\dot{\theta}_1\dot{\theta}_2}_{III} - \underbrace{(\frac{1}{2}m_2 l_1 l_2 \sin\theta_2)\dot{\theta}_2^2}_{II}$$

$$+ \underbrace{(\frac{1}{2}m_1 + m_2)g l_1 \cos\theta_1 + \frac{1}{2}m_2 g l_2 \cos(\theta_1 + \theta_2)}_{I}$$

terms given by equations (**A**), (**B**), & (**C**) if compared with the respective terms in standard 2-R manipulator equation, we will see that they are same the only difference is due to the distributed mass. If that taken into account as effective mass they will represent same equation.

INTRODUCTION TO CONTROL

Control System :

Simply speaking, a control system provides an output or response for a given input or stimulus.

Figure 32.6 Block diagram of an industrial control system

An industrial control system typically consists of an automatic controller, an actuator, a plant and a sensor (measuring element).

Controller:
It is an important element of Control system.It compares the actual value of the plant output with the reference input (desired value), determines the deviation and produces a control signal that will reduce the deviation to zero or to a small value. The manner in which the controller produces the control signal is called the *control action*. Wise design of controller lead to substantial cost savings and performance improvement. It detects the actuating error signal, which is usually at a very low power level, and amplifies it to a sufficiently high level. The output of an automatic controller is fed to an actuator, such as an electric motor, a hydraulic motor or a pneumatic motor or a valve. We will see various types of controller & certain

controllers in detail but later on.

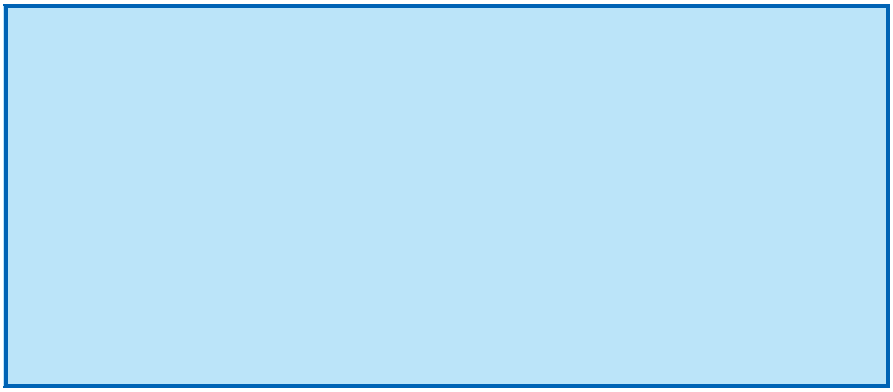
Actuator:
It is a power device that produces the input to the plant according to the control signal so that the output of plant will approach the refernce input.In case of Robot control system , actuator is generally electric motor.

Sensor or measuring element:
It is a device that converts the output variable into another suitable variable such as a displacement pressure or voltage that can be used to compare the output to the refernce input signal. This element is in the feedback path of the closed loop system.

Plant:
It is the one whose output is to be controlled. e.g. In case of Robot control system, plant is the Robot Manipulator.

Contd...

Typical Control system Configuration for Robot manipulator:



Excluding Higher level controller, the rest of system is same as that of typical industrial control system.Each axis of Robot will have this part of control sustem excluding higher level controller.Higher level controller generates commands and send it to lower level controllers. Higher level controller has to co-ordinate between various axes of robots.Now we will see controller design issues.

Controller Design issues:

- Stability of controllers :
  It should have stabilty both in numerical implementation & actual performance.

- Performance of controllers :
  As per application need, performance requirement should found out & depending on that decision on control strategy should be taken.

- Energy required to achieve high performance:
  It is the most important issue as there is upper limit on energy input to controller.

Types of Controller:

- Proportional Derivative Integral (PID)**,** Proportional & Derivative **(PD),** or Proportional & Integral (PI) used in many industries (not suitable for high performance applications).

- **Nonlinear** (Most of systems in nature are non-linear).

- **Robust** (to external/internal disturbances).

- Adaptive (adapt to system changes).

- Neural network.

- Fuzzy logic (example: washing machine).

- Optimal (minimization of cost function).

- Passivity based

First two are widely used in Robotics application which we will see in coming lectures.

Controller Design Steps:

- Mathematical modeling of system.

A mathamatical model of system is defined as a set of equations that represents the dynamics of the system accurately or, at least fairly well. e.g. By using Lagrange's formulation we have derived mathmatical model for n-link Robot manipulator. Good understanding of Dynamics is needed to carry out this step. We will see different ways of mathamatical representation of the system in detail in coming part of this lecture.

  - Selection of control strategy.

  - Design of control parameters.

  - Simulation and experimental verification on actual system.

2nd & 3rd step & simulation part of 4th step we will see in detail in coming lectures.

Mathamatical Modelling of physical system:

We must keep in mind that deriving reasonable mathematical models is the most important part of the entire analysis of control systems.Mathamatical representation of given system may assume any one of the form among the following.

1. Differential Equation form:

It represents differential equations governing system dynamics. Such differential equations may be obtained by using physical laws governing a particular system e.g. Newton's laws for mechanical systems and Kirchoff's laws for electrical systems.

2. **Transfer Function:**

The model is derived from linear time invariant differential equation using the Laplace Transform. Though transfer function can be used only for linear systems, it yield more intuitive information than the differential equation. In this the input, output & system are distinct & separate parts.

With this we are in position to see the effect of changing system parameters on the system response.

The transfer function is also useful in modeling the interconnection of subsystems by forming block diagram but with a mathematical function inside each block.

Transfer function is obtained by using Laplace transforms of differential equations (linear systems).

$$G(s) = \frac{N(s)}{D(s)}$$

3. State-Space Representation:

This can be used for that systems that can not be described by linear differential equations.

State space methods are used to model systems for simulation on digital computer.

This representation turns an nth order differential equation into n simultaneous first order differential equation.

Issues in Design and Analysis of control system:

Major objectives of Control system analysis & Design:-

1. Producing desired **transient response**.
2. Reducing **steady state error.**
3. Achieving stability.

Consider a example of block sliding on frictionless floor.

We want to move block from AA to BB'. i.e Move to BB' is our command.

Figure 32.7 Response

Physical systems can't change position instantaneously. They require some time to change position. So it will take some time to attain the desired position. So our desired response & actual response will differ from each other. The graph of displacemant versus time will be as follows which is actual response of block. Block is at rest initially & finally it should be at rest.

See that slope of OA = +ve & increasing i.e. Velocity is increasing (acceleration).

slope of AB= + ve & constant.

Slope of BC = -ve & decreasing i.e. Velocity is decreasing (Retardation).

Transient Response:

Steady state error:

Now suppose the task of moving is done but if there is difference between final desired position & actual position. That difference $X_D - X_a$ is known as steady state error. This should be as minimum as possible.

Stability:

Discussion on Transient response & Steady state error is immaterial if the system is not stable. Then what is meant by Stability?

For that consider a standard spring-mass-damper system.

Contd...

Its response to given input consist of two parts

1. Natural Response: (Complementary Function)

$$m\ddot{x} + c\dot{x} + kx = 0$$

It will die out after some time.

2. Forced response: (Particular Integral)

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

It will last as long as there is forcing function & dependent on same.

Now total response= Forced response + Natural response

Transient response can be directly related to Natural Response & steady state response can be directly related to Forced response. Initially the response will be mixed one & after some time natural response will die out & only forced response of system will be there which will be dependent upon input.

Consider a standard spring-mass system & suppose there is no damper in the system & system is disturbed from mean position & released, system will never return to its mean position. It will perform vibrations about mean position. System is said to be unstable in this case. Now suppose there is damper in the system, the system will return to mean position after some time which will be dependent on amount of damping.

So for system to be stable, transient response should die out after some time.Once the transient response dies out, the systems response will depend on input only.

If the input is bounded one then output will be bounded one.This leads to another definition of stability. System is said to be *stable* if bounded input to system gives bounded output(BIBO).

Non-linear systems have Asymptotic stability means their output reaches to equilibrium as $t \rightarrow \infty$

Linear systems are only exponentially stable means their output reaches to equilibrium in exponential fashion.

Contd...

Important notions :

One of them is stability which we have seen .Others are controllability & observability.

Controllability:

System is said to be Controllable if robot can be taken from one to another state in finite time .

Linear systems:

Conditions on matrix A,B,C,D for controllability. Matrix A, B, C, D are the coefficient matrix in state space representation.

Observability :

It checks whether feedback taken from given sensors is sufficient to define state of system or not. Conditions on A, B, C, D for observability.

Classical Controller Design

Figure 32.8

Control Startegy

$$H(s) = k_p + k_d s + \frac{k_i}{s}$$

PID control action :

We will see some features of PID controller which is widely used in practice.

- It is widely used for linear and nonlinear systems.
- Contol action is combination of Proportional, Integral & Derivative action .
- It May not always lead to stabilizing controller.

Controller Design Techniques :

For linear systems:

- Root locus .
- Bode plot.
- Nyquist plot.

(All the three above mentioned are graphical techniques)

- State space design tools.

- For Non-linear systems:

- Lyapunov method.

- Singular perturbation.

- Feedback linearization

Controller Implementation:

Analog domain :

- Use of Electronic circuits (main element is op-amp.)

Digital domain:

- In this computer or micro-controller is used as a controller..

- In case of Computer as a controller, sensors & actuators handle analog signals while Computer handles digital signals. Conversion from Analog to Digital and Digital to Analog is necessary.It is done by using Analog to digital & digital to analog converter.

Contd...

Computer as controller:

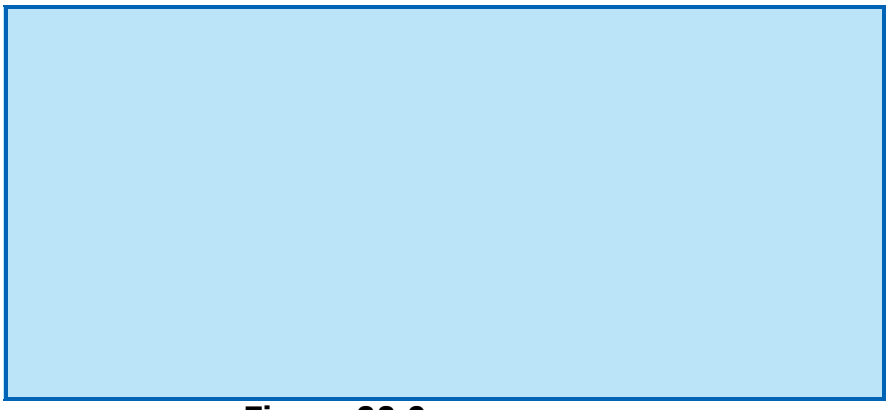


Figure 32.9 **Block Diagram**

Microcontroller as controller:

Figure 32.10 Block Diagram

Issues in digital Control implementation:

- Sampling time: Sampling time should be such that there should not be any kind of distortion of information carried by signal.

- Effect on system due to sampling.
- Filters: necessary for different computations ex. Derivative computation of PD control.
- Speed of computation : Whatever processor we are using it should compute as per control law in given sapling time.
- Speed of A/D and D/A conversion.
- Number of sensors and actuators.
- Noise coming from various sources.
- Cost : Being important it should be considered alongwith performance.

Recap

In this course you have learnt the following

- Lagrangian Formulation for 2-R Manipulator
- Proper Characteristics of Dynamical equation of Robot Manipulato
- Newton's Method
- Terms in Robot Dynamic Equations
- Centripetal force
- Typical Control system Configuration for Robot manipulator
- Introduction to Control

Congratulations, you have finished Lecture 32. To view the next lecture select it from the left hand side menu of the page