Module 6 : Robot manipulators kinematics

Lecture 22 & 23 : Velocity Analysis of Manipulators And Example for velocity analysis of Manipulators

Objectives

In this course you will learn the following

- Velocity analysis for robots links (forward & Inverse).

Velocity Analysis: Forward kinematics for Velocity analysis

Here the following parameters for mechanism are given mechanism fixed parameters, position & orientation of each link of mechanism and first derivative of joint variables. And then find the velocity of every link. The velocity of a link has 6 parameters again as 3 linear velocities and 3 angular velocities as $v_B = v_A + \omega \times r_{BA}$.

Example : Consider the 6 revolute joints (6R) PUMA serial manipulator.Schematic diagram of PUMA coordinate frames is shown Figure 22.1

Figure 22.1 Schematic diagram of PUMA coordinate frames

Here

$$
\begin{aligned}
v_6 &= \frac{d}{dt}\left(r_{P_6P_0}\right) \\
&= \frac{d}{dt}\left(r_{P_1P_0} + r_{P_2P_1} + \ldots + r_{P_6P_5}\right) \\
&= \frac{d}{dt}\left(\left[{}^0R_1\right]\left(r_{P_1P_0}\right)_1 + \left[{}^0R_2\right]\left(r_{P_2P_1}\right)_2 + \ldots + \left[{}^0R_6\right]\left(r_{P_6P_5}\right)_6\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{d}{dt}\left[{}^0R_k\right] &= \frac{d}{dt}\left\{{}^0R_1\,{}^1R_2 \ldots {}^{k-1}R_k\right\} \\
&= \sum_{i=0}^{k} {}^0\dot{R}_1 \ldots {}^{i-1}\dot{R}_i \ldots {}^{k-1}\dot{R}_k
\end{aligned}
$$

Where

$$
\begin{aligned}
{}^{i-1}\dot{R}_i &= \begin{bmatrix} -\sin\theta_i & -\cos\alpha_i\cos\theta_i & \sin\alpha_i\cos\theta_i \\ \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i \\ 0 & 0 & 0 \end{bmatrix}\dot{\theta}_i \\
&= \left[\hat{z}_{i-1}\times(\hat{x}_i)_{i-1} \quad \hat{z}_{i-1}\times(\hat{y}_i)_{i-1} \quad \hat{z}_{i-1}\times(\hat{z}_i)_{i-1}\right]\dot{\theta}_i \\
&= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\left[{}^{i-1}R_i\right]\dot{\theta}_i \\
&= \left[\hat{z}_{i-1}\right]\left[{}^{i-1}R_i\right]\dot{\theta}_i
\end{aligned}
$$

Using the property of derivative of multiplication;

$$
\begin{aligned}
\frac{d}{dt}\left[{}^{0}R_k\right]\left(r_{P_kP_{k-1}}\right)_k &= \sum_{i=1}^{k}\left({}^{0}\dot{R}_1\ldots{}^{i-1}\dot{R}_i\ldots{}^{k-1}\dot{R}_k\right)\left(r_{P_kP_{k-1}}\right)_k \\
&= \sum_{i=1}^{k}\left({}^{0}R_1\ldots\left[\hat{z}_{i-1}\right]{}^{i-1}R_i\ldots{}^{k-1}R_k\dot{\theta}_i\right)\left(r_{P_kP_{k-1}}\right)_k \\
&= \sum_{i=1}^{k}\left({}^{0}R_{i-1}\left[\hat{z}_{i-1}\right]\right)\left(r_{P_kP_{k-1}}\right)_{i-1}\dot{\theta}_i \\
&= \sum_{i=1}^{k}{}^{0}R_{i-1}\left(\left[\hat{z}_{i-1}\right]\times r_{P_kP_{k-1}}\right)_{i-1}\dot{\theta}_i \\
&= \sum_{i=1}^{k}\left[\hat{z}_{i-1}\right]_o\left(r_{P_kP_{k-1}}\right)_o\dot{\theta}_i
\end{aligned}
$$

Contd...

Substituting back in velocity equation,

$$
\begin{aligned}
\left(v_{P_6P_0}\right)_0 &= \dot{\theta}_1\hat{z}_0\times r_{P_1P_0} + \left(\dot{\theta}_1\hat{z}_0+\dot{\theta}_2\hat{z}_1\right)\times r_{P_2P_1} \\
&\quad +\ldots+\left(\dot{\theta}_1\hat{z}_0+\dot{\theta}_2\hat{z}_1+\ldots+\dot{\theta}_6\hat{z}_5\right)\times r_{P_6P_5} \\
&= \left(v_{P_5P_0}\right)_0 + \left(\dot{\theta}_1\hat{z}_0+\dot{\theta}_2\hat{z}_1+\ldots+\dot{\theta}_6\hat{z}_5\right)\times r_{P_6P_5} \\
&= \left(v_{P_5P_0}\right)_0 + \omega_6\times r_{P_6P_5}
\end{aligned}
$$

$$
\omega_k = \left(\dot{\theta}_1\hat{z}_0+\dot{\theta}_2\hat{z}_1+\ldots+\dot{\theta}_k\hat{z}_{k-1}\right)
$$

In above equation, first term indicates the linear velocity of point P1. From above equation,

$$\left(v_{P_6P_0}\right)_0 = \dot{\theta}_1\hat{z}_0 \times \left(r_{P_1P_0} + r_{P_2P_1} + \ldots + r_{P_6P_5}\right) + \dot{\theta}_2\hat{z}_1 \times \left(r_{P_2P_1} + \ldots + r_{P_6P_5}\right) + \ldots + \dot{\theta}_6\hat{z}_5 \times \left(r_{P_6P_5}\right)$$

$$\left(v_{P_6P_0}\right)_0 = \dot{\theta}_1\hat{z}_0 \times r_{P_6P_0} + \dot{\theta}_2\hat{z}_1 \times r_{P_6P_1} + \ldots + \dot{\theta}_6\hat{z}_5 \times r_{P_6P_5}$$

Expression for velocity of kth link of serial chain with revolute joints

$$\begin{Bmatrix} v_{P_kP_0} \\ \omega_k \end{Bmatrix} = \begin{bmatrix} \hat{z}_0 \times r_{P_kP_0} & \hat{z}_1 \times r_{P_kP_1} & & \hat{z}_{k-1} \times r_{P_kP_{k-1}} \\ & & \cdots & \\ \hat{z}_0 & \hat{z}_1 & & \hat{z}_{k-1} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_k \end{Bmatrix}$$

$$= \left[J_k\right] \begin{Bmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_k \end{Bmatrix} \qquad f(x): m \text{ from } n \text{ row} \quad \text{and} \quad \nabla_X f(x): J$$

Thus forward and inverse kinematics is defined with in terms of Jacobian Matrix.

$$\begin{Bmatrix} v_{P_kP_0} \\ \omega_k \end{Bmatrix} = \begin{bmatrix} J_{C_1} & J_{C_2} & \cdots & J_{C_k} \end{bmatrix} \begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_k \end{Bmatrix}$$

$\dot{q}_i = \dot{\theta}_i$ if i is revolute

$$J_{C_i} = \begin{Bmatrix} \hat{z}_{i-1} \times r_{P_iP_0} \\ \hat{z}_{i-1} \end{Bmatrix}$$

$\dot{q}_i = \dot{d}_i$ if i is prismatic

$$J_{C_i} = \begin{Bmatrix} \hat{z}_{i-1} \\ 0 \end{Bmatrix}$$

Where the symbols and terms have meanings defined earlier.

Recap

In this course you will learn the following

- How to obtain forward and inverse velocity of links of 6R Puma robot.

Congratulations, you have finished Lecture 22. To view the next lecture select it from the left hand side menu of the page.