Module 5 : Trajectory Planning of end effectors

Lecture 14 : Example of 2R manipulator trajectory planning

Objectives

In this course you will learn the following

- Demonstrate the calculations using 2R manipulators.

Example 1*:* Trajectory planning for a planar 2R manipulator

Fig 14.1.1: A 2R Manipulator

For a planar 2R manipulator (Fig.14.1.1) with, $l_1 = l_2 = 1m$, move the end-effector from rest at the initial position to rest at final position in 1s. The coordinates of the end-effector in the initial and final positions are:

$$X_{Bi} = 1.366\ m; Y_{Bi} = 1.366\ m;$$
$$X_{Bf} = -1.366\ m; X_{Bf} = 1.366\ m$$

Solution : The problem will be solved in essentially three steps.

Step 1 : Convert given end effector initial and final positions to initial and final joint angles.

Step 2 : Plan for $\theta_1(t), \theta_2(t)$ using one of the strategies discussed

Step 3 : Estimate actual $X_B(t), Y_B(t)$ from the planned $\theta_1(t), \theta_2(t)$ above.

We will now illustrate the detailed computations.

Contd...

Step 1:

$$X_{Bi} = l_1 \cos\theta_{1i} + l_2 \cos(\theta_{1i} + \theta_{2i}) = 1.366$$
$$Y_{Bi} = l_1 \sin\theta_{1i} + l_2 \sin(\theta_{1i} + \theta_{2i}) = 1.366$$

$$\therefore\ \theta_{1i} = 30^0; \theta_{2i} = 30^0$$

Similarly $\theta_{1f} = 150^0\ \theta_{2f} = -30^0$

Since $\dot{X}_{Bi} = \dot{X}_{Bf} = \dot{Y}_{Bi} = \dot{Y}_{Bf} = 0, \quad \dot{\theta}_{1i} = \dot{\theta}_{2i} = \dot{\theta}_{1f} = \dot{\theta}_{2f} = 0$

Contd...

Step 2:

Case (1): Using a simple cubic curve fit

$$\theta_{1i} = 30°; \qquad \dot{\theta}_{1i} = 0$$

$$\theta_{1f} = 150°; \quad \dot{\theta}_{1f} = 0\ T_{if} = 1s$$

$$\therefore \quad \theta_1(t) = c_0 + c_1\tau + c_2\tau^2 + c_3\tau^3$$

$$\theta_1(0) = \theta_{1i} = 30° = \frac{\pi}{6}\text{ Rad}$$

$$\dot{\theta}_1(0) = \dot{\theta}_{1i} = 0$$

$$\theta_{1f}(1) = \theta_{1f} = 150° = \frac{5\pi}{6}\text{ Rad}$$

$$\dot{\theta}_{1f}(1) = \dot{\theta}_{1f} = 0$$

Upon solving

$$c_0 = \frac{\pi}{6}$$
$$c_1 = 1$$
$$c_2 = 2\pi$$
$$c_3 = -\frac{4\pi}{3}$$

$$\therefore \quad \boxed{\theta_1(\tau) = \frac{\pi}{6} + 2\pi\tau^2 - \frac{4\pi}{3}\tau^3} \qquad \tau = 0 \rightarrow 1s$$

Similarly for $\theta_2(\tau)$ we can obtain,

$$\boxed{\theta_2(\tau) = \frac{\pi}{6} - \pi\tau^2 + \frac{2\pi}{3}\tau^3} \qquad \tau = 0 \rightarrow 1s$$

Contd...

Step 3:

We estimate X_A, Y_A, X_B *and* Y_B *using,*

Here,

$$X_A = l_1 \cos\theta_1$$
$$Y_A = l_1 \sin\theta_1$$
$$X_B = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)$$
$$Y_B = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2)$$

Using these equations, X_A, Y_A, X_B, Y_B are computed and plotted in Fig. 14.1.2 and the corresponding joint motions are shown in Fig. 14.1.3.

Fig 14.1.2: Trajectory plot for a 2R Manipulator (Ex. 1) using simple Cubic fit.

Fig 14.1.3: Joint angular displacements for Ex. 1

Contd...

Case (1)

Explained the procedure to fit in a cubic curve via two points through the reachable workspace. This strategy, ensured that the motion of the two links, along with the change in rotation angles was smooth and uniform.

Case (2)

Now we may add one more constraint to the present problem that we want the end-effector to pass through a particular point. In practice, as stated earlier, this need may arise due to some obstruction in the path of the trajectory. We have the starting and the end point as,

$$X_{Bi} = 1.366 \text{ m}; \; Y_{Bi} = 1.366 \text{ m}$$

$$X_{Bf} = -1.366 \text{ m}; \; Y_{Bf} = 1.366 \text{ m}$$

We choose via point as say (0, 1.366) which is to be reached in t=0.5s.

$$Y_{Bv} = 1.366, \; X_{Bv} = 0;$$

We need to first determine the via-points in the joint space. From the simple kinematics (as in step 1) we get,

$$\theta_{1v} = 43.08 \text{ o} = 0.7519 \text{ rad}; \; \theta_{2v} = 93.84 \text{ o} = 1.6378 \text{ rad};$$

Two cubic curve fits for i-v and v-f, which calls for 8 coefficients to be determined following the discussion of sec 1.2.

For first joint

$$\text{I-V} \quad \theta_1(\tau) = \frac{\pi}{6} - 3.5436\tau^2 + 8.9135\tau^3 \qquad \tau = 0 \rightarrow 0.5$$

$$\text{V-F} \quad \theta_1(\tau) = 0.7519 + 9.8267\tau^2 - 17.2911\tau^3 \qquad \tau = 0 \rightarrow 0.5$$

For second joint

$$\theta_{2i} = \frac{\pi}{6}; \quad \theta_{2f} = -\frac{\pi}{6}; \quad \theta_{2v} = 1.6378$$

$$\dot{\theta}_{2i} = 0; \quad \dot{\theta}_{2f} = 0$$

$$\text{I-V} \quad \theta_2(\tau) = \frac{\pi}{6} + 16.512\tau^2 - 24.11\tau^3 \qquad \tau = 0 \rightarrow 0.5$$

$$\text{V-F} \quad \theta_2(\tau) = 1.6378 - 1.5708\tau - 19.6536\tau^2 - 28.30\tau^3 \qquad \tau = 0 \rightarrow 0.5$$

Contd...

Fig 14.1.4: Trajectory plot for a 2R Manipulator by Via point method

Fig 14.1.5: Joint angular displacements for a 2R Manipulator Trajectory obtained by Via point method

Fig 14.1.5 shows the joint angular displacements for both the cases. It is observed that in the duration 0-0.5s, θ_1 changes very little as the via point in joint space is close to the initial position. In the next 0.5s, the joint has to make up by covering the remaining motion at a much higher velocity. Second joint motion trajectory with via point is also significantly different from the case without via point.

The reader is advised to solve the problem using other techniques discussed.

We have so far discussed several strategies to plan the trajectories in joint space. We have studied the use of cubic spline, higher order polynomials and the simple trapezoidal velocity profile strategy. When the joints move through such trajectories, the actual motion of the end effector is some complicated shape in 3-D depending on the kinematics of the manipulator. We may desire that the tool held in the end-effector move through space in a simple straight line or some such regular path. We wish to have a general trajectory planning strategy for Cartesian space where arbitrary functions of Cartesian variables as functions of time could be used to specify a path. In principle, we could extend the techniques we discussed for joint space trajectory planning. However in planning a trajectory in Cartesian space, a few significant problem situations should be borne in mind. One typical problem is illustrated here, again using the planar 2R manipulator where one link is much shorter than the other (unlike in ex 1.).

The reachable workspace for the manipulator tip is the annulus region (shown by hatching lines in Fig. 14.2.1) enclosed by the circles of radii $(l_1 + l_2)\, and\, (l_1 - l_2)$ .If we plan to specify a simple straight line path for the end effector between I and F, there will be some points on the line IF in general which are out of reach. Thus for the trajectory planning in Cartesian space to work, the user must carefully specify the paths.

Fig 14.2.1: Concept of A Reachable workspace

Another problem arises when an end-effector point in Cartesian space corresponds to a near-singular configuration for the manipulator. In such a case user defined velocities for the end - effector may either be unattainable or may require very large velocities at the joints.

Recap

In this course you will learn the following

- We have worked out the trajectory planning for 2R manipulator with & without via points.

Congratulations, you have finished Lecture 14. To view the next lecture select it from the left hand side menu of the page