# Module 5 : Trajectory Planning of end effectors

## Lecture 12 : Trajectory planning I ( point to point and continuous trajectories)

Objectives

In this course you will learn the following

Point to point and continuous path trajectories

General trajectory planning problem statement

Cubic curve fit strategy

### Introduction

So far, you have studied various types of robots; sensor and actuators used in typical robots etc. You have learnt about the various mechanism elements used in transmitting the motion from the actuators at the joints to the end-effector which performs certain desired tasks. For example, a stationary robot manipulator may be designed to carry out spray painting. On the other hand a mobile robot (such as an AGV) may be required to move around the shop floor autonomously, performing various tasks.

We will now study the nature of such tasks performed by a robot and the motions required to be generated at the joints in order that the end-effector moves as per the requirement of the desired task. Let us look at a typical pick-and-place device as shown in Fig. 12.1.1.

The robot is required to pick objects from point A and place them at point B. The specific path taken by the end-effector is not of much concern except that it should avoid collision with other objects in the environment. Consider another typical task such as welding depicted in Fig. 12.2.2, wherein the end-effector is required to strictly trace the desired path to ensure a proper weld.

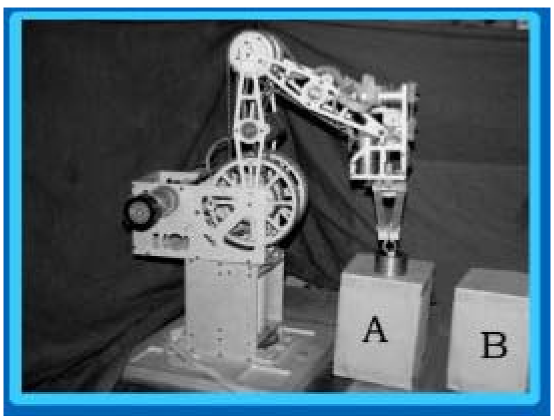


*Fig 12.1.1: Pick and Place Robot*

*Fig12.1.2: A typical Weld*

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Tasks of the first type are referred to as “Point-to-Point” while the second group of tasks is referred to as “continuous-path” type. In either case, the joints have to be moved such that the end-effector moves through the specified points in the workspace positioning the end-effector at appropriate orientation or passing through those points with desired velocity and/or acceleration within the specified time. Since a robot manipulator tip (i.e. end-effector) carries the tool that performs the desired tasks, it is observed that the end-effector, holding the welding gun, is required to follow a specific path strictly, in order to get a proper weld. The specification of the desired motion naturally occurs in terms of the goal position/orientation of the end-effector in space, its velocity etc. One could visualize a robot system with appropriate sensors that continuously track the global position/orientation etc. of the end-effector and feed these into a closed loop feed-back control system. For such a system, in principle, motion planning, generation and control could take place directly in terms of end-effector's desired motion in world coordinates. However, it must be observed that having to continuously measure the motion variables (such as position, orientation, velocity etc.) of the end-effector in space is not a trivial task. Most manipulators, on the other hand, have individual joint motion sensors that feed into a lower level control system at each joint. Thus we wish to find the individual joint motions required to achieve the desired end-effector motion. A trajectory would thus refer to the time history of position, velocity and acceleration of each individual d.o.f. The individual joint motions and the end-effector motion are of course related through the robot kinematics and can be converted from one to the other.

Consider, for example, a simple planar 2R manipulator as shown in Fig.12.1.3, $l_1$ , $l_2$ are the known link lengths of OA and AB respectively.

**Fig 12.1.3: A Planar 2R manipulator**

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The global (X, Y) coordinates of the end-effector are:

$$X_B = l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \qquad (12.1.1)$$

$$Y_B = l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \qquad (12.1.2)$$

Given $\theta_1$ and $\theta_2$ i.e., the joint positions, finding the (X, Y) coordinates of the end-effector involves solution of simple Linear Algebraic Equations (LAE). However, if we are given the desired end-effector positions $X_B$ , $Y_B$ finding $\theta_1$ and $\theta_2$ will require solution of trigonometric (hence nonlinear) equations.

Let us now differentiate the two position equations with respect to time to get the velocities:

$$\dot{X}_B = -\ell_1\dot{\theta}_1 \sin\theta_1 - \ell_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\sin\left(\theta_1 + \theta_2\right) \qquad (12.1.3)$$

$$\dot{Y}_B = \ell_1\dot{\theta}_1 \cos\theta_1 + \ell_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)\cos\left(\theta_1 + \theta_2\right) \qquad (12.1.4)$$

i.e

$$\begin{Bmatrix}\dot{X}_B \\ \dot{Y}_B\end{Bmatrix} = \begin{bmatrix}-\ell_1 \sin\theta_1 - \ell_2 \sin(\theta_1+\theta_2) & -\ell_2 \sin(\theta_1+\theta_2) \\ \ell_1 \cos\theta_1 + \ell_2 \cos(\theta_1+\theta_2) & \ell_2 \cos(\theta_1+\theta_2)\end{bmatrix}\begin{Bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2\end{Bmatrix} \qquad (12.1.5)$$

i.e, $\{\dot{X}\} = [C]\{\dot{\theta}\}$ Or $\{\dot{\theta}\} = [C]^{-1}\{\dot{X}\}$

At a given position ($\dot{\theta}_1$, $\dot{\theta}_2$, $X_B, Y_B$) for known joint velocity $\dot{\theta}_1$ and $\dot{\theta}_2$ finding $\dot{X}_B, \dot{Y}_B$ or vice-versa involves solution of simple LAE. Of course there could be some configurations (i.e $\dot{\theta}_1, \dot{\theta}_2$) where $[C]^{-1}$ may not exist (singularities) and may pose difficulties. Similarly we can relate the joint-space and Cartesian-space accelerations.

Given joint position, velocity and acceleration, finding the Cartesian space position, velocity and acceleration of the end-effector is referred to as the FORWARD KINEMATICS problem. Given end effector Cartesian position, velocity, acceleration, finding the joint coordinate position, velocity and acceleration is termed the INVERSE KINEMATICS problem. Formal description of the kinematics of a robot manipulator in 3-D space and intricacies of FORWARD/INVERSE kinematics would be discussed at a later stage. For the purpose of present discussion, we simply state that the motion of a robot manipulator when specified as desired end effector motion can be converted into joint space motion and vice-versa. Thus, in order to plan the motion of the end-effector we could simply plan the motion of each joint. This is known as Joint Interpolated Motion.

**Cubic Fit for Two Given Positions**

To begin with, let us consider a single revolute joint's motion between any two specified points i and j. At these two points, the position and velocity are prescribed viz., $\theta_i, \dot{\theta}_i, \theta_j, \dot{\theta}_j$ .

These could be the initial and final positions and so we may require starting from rest and coming to rest at the final position i.e. $\dot{\theta}_i = 0 = \dot{\theta}_j$ . On the other hand, both these points could be intermediate or via points and so $\dot{\theta}_i \neq 0;\ \dot{\theta}_j \neq 0$ .

The specified $\theta$ and the tangents $\dot{\theta}$ are indicated in Fig.12.2.1. We consider a local time fame ($\tau$,0 $\rightarrow$ T) for the motion between the points i, j and wish to find a smooth curve $\theta$($\tau$) satisfying these end conditions indicated by the dotted line in the figure. Since there are four constraints, we can fit a cubic spline. Let,

$$\theta(\tau) = c_0 + c_1\tau + c_2\tau^2 + c_3\tau^3 \qquad (12.2.1)$$

$$\therefore\ \dot{\theta}(\tau) = c_1 + 2c_2\tau + 3c_3\tau^2 \qquad (12.2.2)$$

**Fig12.2.1: Cubic Fit for given Two Points**

Substituting the end conditions,

$$\theta_i = c_0 \tag{12.2.3}$$

$$\dot{\theta}_i = c_1 \tag{12.2.4}$$

$$\theta_j = c_0 + c_1 T + c_2 T^2 + c_3 T^3 \tag{12.2.5}$$

$$\dot{\theta}_j = c_1 + 2c_2 T + 3c_3 T^2 \tag{12.2.6}$$

We can solve these four equations,(12.2.3 to 12.2.6) to get the four coefficients $c_0 - c_3$ . Solving, we will obtain the coefficients as,

$$c_0 = \theta_i \tag{12.2.7}$$

$$c_1 = \dot{\theta}_i \tag{12.2.8}$$

$$c_2 = \frac{3}{T^2}(\theta_j - \theta_i) - \frac{2}{T}\dot{\theta}_i - \frac{1}{T}\dot{\theta}_j \tag{12.2.9}$$

$$c_3 = -\frac{2}{T^3}(\theta_j - \theta_i) + \frac{1}{T^2}(\dot{\theta}_i + \dot{\theta}_j) \tag{12.2.10}$$

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Within the interval (0, T), typical variation of position, velocity and acceleration is indicated in Fig.12.2.2, where the constants were chosen as $T = 1s;\ \theta_i = 1; \theta_j = 2$ and $\dot{\theta}_i = \dot{\theta}_j = 0$

Fig 12.2.2: Displacement, Velocity and Acceleration Plots for a Cubic Fit via given Two Points

Recap

In this course you have learnt the following

- The nature of trajectories that a typical end effectors traces
- Viewing the end effectors motion as composite of individual joint motions
- Trajectory planning in joint space
- One strategy for joint motion planning

Congratulations, you have finished Lecture 12. To view the next lecture select it from the left hand side menu of the page