Module 9 : Robot Dynamics & controls

Lecture 34 : Trajectory tracking control (feed forward, computed torque and inverse dynamics approach)

Objectives

In this course you will learn the following

- Trajectory tracking control
- Trajectory Tracking using PD+Feedforward
- Trajectory Tracking
- Positive Definite Function (pdf)
- Locally Positive Definite Function (lpdf)
- Lyapunov's Notions of Stability
- Derivative along Trajectories
- Quadratic Functions
- Theorem

Trajectory tracking control:

Feed forward.

Computed torque

Linear form with disturbance $J_{eff}\ddot{\theta}_{mk} + B_{eff}\dot{\theta}_{mk} = KV_k - d_k r_k$ **Robot & actuator Dynamics**

Here J_{eff} is effective inertia which is assumed to be constant over a range of motion & is function of generalised co-ordinates.

B_{eff} - is constant damping term.

d_k - disturbance term(which accounts for coriolis, centripetal, inertia & coupling forces.)

In Trajectory tracking problem we have to move end effector along predefined path. By using inverse kinematic analysis we can find out desired joint angles which are function of time. From these we can find out desired angles through which the motor should rotate as we know gear reduction ratio for each actuator.

.i.e. $\theta_i^d = \theta_i^d(t) \Rightarrow \theta_{mk}^d = \theta_{mk}^d(t)$.

Feed forward+PD Control:

As its name suggests , there is some information which is feeded in forward direction in this control strategy alongwoth conventional PD control.It is basically used for trajectory tracking problem in which we have to move end-effector along a predefined path.

PD or PID control are suitable for slow point to point motion problem(regulation problem). In case of trajectory tracking problem the end-effector is moving along predefined path. In case of tracking problem desired position is a function of time. So it continuously changes with respect to time. Motor response will never catchup the desired position within shortest time by using PD or PID techniques only. So we cannot use PD or PID control strategy for trajectory tracking problem.

In this example we have to move end effecor from Ps to Pf along a straight line.

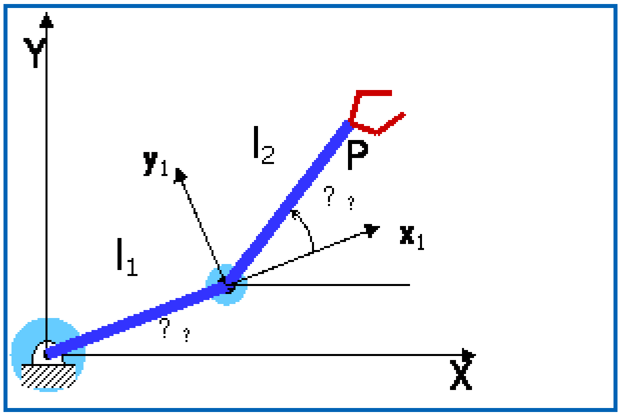

Figure 34.1

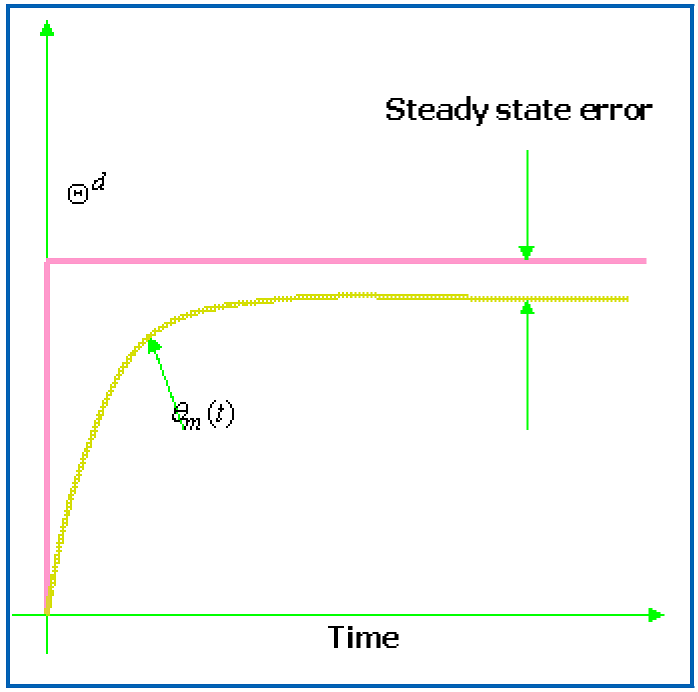

Figure 34.2

In trajectory tracking problem this θ^d is not constant but a function of time.

Contd...

Trajectory Tracking using PD+Feedforward :

In this J_{eff} & B_{eff} are computed along the trajectory at various instants of time & these terms are feed forwarded . This is the basis of Feed forward algorithm. Along predefined path we can find out θ^d , $\dot{\theta}_m^d$, $\ddot{\theta}_m^d$. We can also estimate J_{eff} & B_{eff} at various points along trajectory. So along given trajectory, actuator(i.e. motor) have to overcome following torques.

$J_{eff} \cdot \ddot{\theta}_m^d$ = Effective inertia force.

$B_{eff} \cdot \dot{\theta}_m^d$ = Effective damping force.

This information is feed forwarded in this technique.

Figure 34.3 PP +Feedforward control

Now Linear form with disturbance

$$J_{eff}\ddot{\theta}_{mk} + B_{eff}\dot{\theta}_{mk} = KV_k - d_k r_k$$

but

$KV_k = J_{eff}\ddot{\theta}^d + B_{eff}\dot{\theta}^d + K_P(\theta^d - \theta_m) + K_D(\dot{\theta}^d - \dot{\theta}_m)$ Controller Equation

Substituting this controller equation , system dynamics becomes

$$J_{eff}\ddot{\theta}_m + B_{eff}\dot{\theta}_m = J_{eff}\ddot{\theta}^d + B_{eff}\dot{\theta}^d + K_P(\theta^d - \theta_m) + K_D(\dot{\theta}^d - \dot{\theta}_m) - d_k r_k$$

Define $e(t) = \theta_m - \theta^d$

we get $J_{eff}\ddot{e} + (B_{eff} + K_D)\dot{e} + K_P e = -d_k r_k$

For zero disturbance by choosing K_p & K_D such that whatever settling time chosen,

$e, \dot{e} \rightarrow 0$ as $t \rightarrow \infty$

One can now easily say that the tracking error is due to d_k term.

Substituting this controller equation , system dynamics becomes

$$J_{eff}\ \ddot{e} + (B_{eff} + k_D)\dot{e} + k_p . e = -(d_k - d_k^d)r_k$$

$$e(t) = \theta_m - \theta^d$$

Here the trackting error is further reduced,

- Nonlinear Coriolis, centripetal gravitational forces have to be computed, hence more comutational load.

- Again, so far we have not seen any non-linear analysis or design of controllers .

- Non-linear analysis & design necessary to further improve the performance.

Trajectory Tracking:

PDF Computed torque Technique :

Linear form with disturbance $J_{eff}\ddot{\theta}_{mk} + B_{eff}\dot{\theta}_{mk} = KV_k - d_k r_k$

In this method alongwith feed forwarding the information calculation of d_k term in above equation is done which contains a lot of terms representing Coriolis, Centripetal, Gravity & coupling.

$$d_k = \sum_{j \neq k} d_{kj}(\mathbf{q})\ddot{q}_j + \sum_{i,j} c_{ijk}(\mathbf{q})\dot{q}_i\dot{q}_j + g_k$$

Block diagram:

Figure 34.4 Trajectory Tracing control

$KV_k = J_{eff}\ddot{\theta}^d + B_{eff}\dot{\theta}^d + K_P(\theta^d - \theta_m) + K_D(\dot{\theta}^d - \dot{\theta}_m) + d_k^d r_k$ Controller Equation

Substituting this controller equation , system dynamics becomes

$$J_{eff}\ddot{e} + (B_{eff} + K_D)\dot{e} + K_P e = -(d_k - d_k^d)r_k$$

where $e(t) = \theta_m - \theta^d$

The tracking error is further reduced but at the cost of that we have to compute Nonlinear terms representing coriolis, centripetal, gravitational & coupling forces which will increase complexity.This may demand faster microcontroller or microprocessor which may increase cost.

Recall Robot & Actuator dynamics equation. $J_{eff}\ddot{\theta}_{mk} + B_{eff}\dot{\theta}_{mk} = KV_k - d_k r_k$

We have assumed this seconnd order differemtial equation linear by combining all the non-linear terms & treating them as disturbance. But this is true in small range of operation. But as speed of motion increases, non-linear terms containing vary at faster rate. So there will be error. Therefore there is upper limit on speed when using linear controller for robot. Hence productivity will be very low. Instead of this if we use non-linear controller which you will see in next lectures,we will have highly accurate control of Robot & there will be large range of speed.

In case of linear control system high quality actuators & sensors are used to produce linear behaviour in specified operation range which increases cost. But less expensive components with non-linear characteristics can be used in case of non-linear control system.

Any real world system is highly non-linear in nature. Non-linear system is better approximation to real world system than linear one.

Qualititatively a system is said to be stable if starting the system somewhere near its operating point implies that it will be around the point ever after.

Consider a simple pendulum. In one case bob is vertically down & in one case it is vertically up. Now see if we perturbed the system slightly , what will happen in both cases. In one case the system will return back to its original position after some time or it will be around that point ever after but in other case it will not be around the point if we perturbed it. This can clear that the given equilibrium is stable only when if we perturb the system from equilibrium position slightly & if it is around that equilibrium point ever after then it is said to be stable.
e.g. Consider a missile moving along a particular trajectory & if there is slight perturbation from desired tarjectory due to any disturbances & if the control system is stable then it will return back to or move along desired path.

Enables analysis of nonlinear controllers

Synthesis of nonlinear, adaptive, robust controllers possible for both regulation and trajectory tracking applications

Study of some mathematical preliminaries is needed before studying Lyapunov's theorem of stability:

Locally Positive Definite Function (lpdf) :

A function $V : R_+ \times R^n \to R$ is said to be a locally positive definite function(lpdf) if

- It is continuous.
- $V(t,0) = 0 \;\; \forall t \geq 0$.

- There exist a constant r>0 and a function α of class K such that $\alpha(\|\mathbf{x}\|) \le V(t,\mathbf{x}), \forall t \ge 0, \forall \mathbf{x} \in B_r$

  There is a ball of radius r & x should lie in that ball. In 2-dimensional this ball wll be circle , in three dimensional will be sphere & in n-dimensional it will be hypersphere.

Figure 34.5

Norm of x (||x||) is similar to distance in geometry but it is in n-dimensional space.

$$\|\mathbf{x}\|_2 = \sqrt{x_1^2 + x_2^2 + x_3^2 \ldots + x_n^2}$$

Positive Definite Function (pdf):

A function $V : R_+ \times R^n \to R$ is said to be positive definite function (pdf) if

- It is continuous.
- $V(t,0) = 0 \ \ \forall t \ge 0$.

There exist a function α of class K such that $\alpha(\|x\|) \le V(t,x), \forall t \ge 0$, hold for all $X \in R^n$

. Examples:

$$1.\ V(t,x) = (t+1)(x_1^2 + x_2^2) \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\left.\begin{array}{l} 2.\ V(t,x) = x_1^4 + x_2^2 + x_3^2 \\ 3.\ V(t,x) = x_1^4 + x_2^2 \end{array}\right\} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

For the function to be positive definite function it must satisfy following conditions.

(i) Continuous.

(ii) V(t,0) = 0 for all t.

(iii)a function condition on V(t,x) for nonzero x .

Function 1 & 2 satisfy all three conditions but function 3 violates condition no.2 because as it does not contain X_3 so function will be zero for $X_1 = X_2 = 0 \quad \& \ X_3 \ne 0$. Basically PDF functions are used in Lyapunov theory analysis. This is very important consideration while dealing with Lyapunov theory..A function is said to to be PDF if it contain all the components of vector X.

Other Definitions:

A function V is decrescent if there exist a contstant r > 0 and a function β of class K such that

$$V(t,x) \le \beta(\|x\|), \forall t \ge 0, \forall x \in B_r \ - locally\ decrescent.$$

$$\forall x \in R^n \ - globally\ decrescent.$$

V is radially unbounded if $\alpha(\|x\|) \le V(t,x), \forall t \ge 0,$ for all and for some continuous function α with additional property that $\alpha(r) \to \infty \ as\ r \to \infty$.

V is a locally negative definite function if –V is an lpdf and a negative definite function if –V is a positive definite function.

Quadratic Functions :

- Very popular function used in Lyapunov theory applied to robots.

Quadratic functions can be written in the form

$$\begin{aligned} V(x) &= a_{11}x_1^2 + a_{22}x_2^2 + \ldots + a_{nn}x_n^2 \\ &+2a_{12}x_1x_2 + 2a_{13}x_1x_3 + \ldots + 2a_{1n}x_1x_n \\ &+2a_{23}x_2x_3 + \ldots + 2a_{2n}x_2x_n \\ &+\ldots.. + 2a_{(n-1)n}x_{n-1}x_n \end{aligned}$$

in matrix form $V(x) = x^T A x$ where

$$A = \begin{bmatrix} a_{11} & a_{12} & & a_{1n} \\ a_{12} & a_{22} & & \\ . & . & . & a_{(n-1)n} \\ a_{1n} & & & a_{nn} \end{bmatrix}$$ & X is a state vecor.

Theorem:

1. Quadratic functions are positive definite functions if determinants of successive minors of *A* are all positive.

$$A = \begin{bmatrix} \boxed{a_{11}} & a_{12} & & a_{1n} \\ a_{12} & a_{22} & & \\ . & . & . & a_{(n-1)n} \\ a_{1n} & & & a_{nn} \end{bmatrix} \text{e.g } |a_{11}|, \begin{vmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{vmatrix}$$ & so on should be positive.

2. Quadratic functions are positive definite functions if eigenvalues of ***A*** are all positive.
eigen(A) >0, i=1,2,...n

Contd...

Lyapunov's Notions of Stability:

Stability is the Characteristic of equilibrium of $\dot{x} = f(t,x)$ Equilibrium is obtained by solving for x in $f(t,x) = 0$

- Equilibrium **0** is said to be stable if for each $\varepsilon > 0$, for each $t_0 \in R_+$ there exists a $\delta = \delta(\varepsilon, t_0)$ such that

$$\| x_0 \| < \delta(\varepsilon, t_0) \Rightarrow \| s(t, t_0, x_0) < \varepsilon, \forall t \geq t_0$$

Figure 34.6(a)

Figure 34.6(b)

Equilibrium **0** is attractive if for each $t_0 \in R_+$ there is an such that $\eta(t_0) > 0$.

$$\| \mathbf{x}_0 \| < \eta(t_0) \Rightarrow \| \mathbf{s}(t, t_0, \mathbf{x}_0) \| \mapsto 0 \text{ as t} \rightarrow \infty$$

Equilibrium is said to be asymptotically stable if it is stable and attractive.

Figure 34.7(a)

Figure 34.7(b)

Derivative along Trajectories:

Definition: Let $V : R_+ \times R^n \rightarrow R$ continuously differentiable with respect to all of the arguments and let $\nabla V : R_+ \times R^n \rightarrow R$ denote the gradient of V with respect to x (written as a row vector). Then the function $\dot{V} : R_+ \times R^n \rightarrow R$ is defined by

$$\dot{V}(t,x) = \frac{\partial V}{\partial t}(t,x) + \nabla V(t,x)\dot{X}$$

but $\dot{x} = f(t,x)$

$$\dot{V}(t,x) = \frac{\partial V}{\partial t}(t,x) + \nabla V(t,x) f(t,x)$$

And is called the derivative of V along the trajectories of $\dot{x}(t) = f[t, x(t)], \forall t \geq 0$.

Figure 34.8 Lyapunov function

Summary

- Trajectory Tracking control : Feed forward and C T Control
- Maths prelimi. for Lyapunov theory
- Lyapunov stability Theorms.

Recap

In this course you have learnt the following

- Trajectory tracking control
- Trajectory Tracking using PD+Feedforward
- Trajectory Tracking
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- Lyapunov's Notions of Stability
- Derivative along Trajectories
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- Theorem