Module 9 : Forced Vibration with Harmonic Excitation; Undamped Systems and resonance; Viscously Damped

Systems; Frequency Response Characteristics and Phase Lag; Systems with Base Excitation;

Transmissibility and Vibration Isolation; Whirling of Shafts and Critical Speed.

Lecture 25 : Whirling and critical speed

Objectives

In this lecture you will learn the following

- Whirl of shaft with a rotor having some eccentricity
- Synchronous whirl
- Critical speed
- Rayleigh's and Dunkerley's formulae

WHIRLING OF SHAFTS – CRITICAL SPEED

Fig 9.5.1

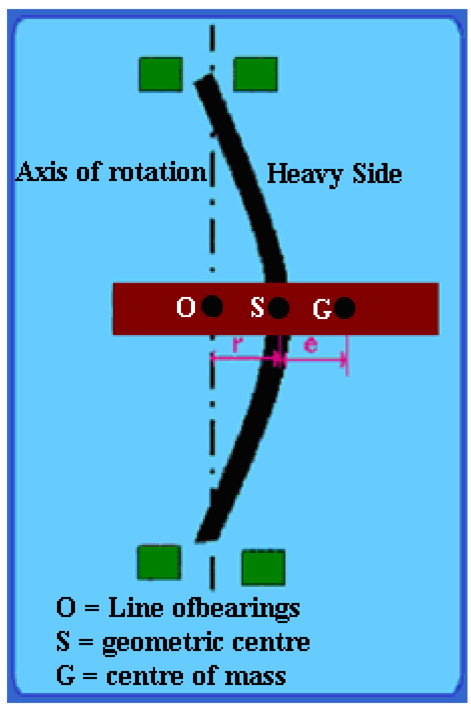

Fig 9.5.2

Consider a typical shaft, carrying a rotor (disk) mounted between two bearings as shown in Fig. 9. 5.1. Let us assume that the overall mass of the shaft is negligible compared to that of the rotor (disk) and hence we can consider it as a simple torsional spring. The rotor (disk) section has a geometric centre i.e., the centre of the circular cross-section and the mass centre due to the material distribution. These two may or may not coincide in general, leading to eccentricity as indicated in Fig. 9.5.2. The eccentricity could be due to internal material defects, manufacturing errors etc. As the shaft rotates, the eccentricity implies that the mass of the rotor rotating with some eccentricity will cause in-plane centrifugal force. Due to the flexibility of the shaft, the shaft will be pulled away from its central line as indicated in the figure. Let us assume that the air-friction damping force is negligible. The centrifugal force for a given speed is thus balanced by the internal resistance force in the shaft-spring and the system comes to an equilibrium position with the shaft in a bent configuration as indicated in the figure. Thus the shaft is rotating about its own axis and the plane containing the bent shaft and the line of bearings rotates about an axis coinciding with the line of bearings. We consider here only the case, wherein these two rotational speeds are identical, called the synchronous whirl.

Fig 9.5.3

Fig. 9.5.3 shows the shaft and rotor system undergoing synchronous whirl in four different positions in a single revolution. Let us write down the force equilibrium equation as follows, following the notation shown in Fig. 9.5.2:

$$\text{Centrifugal force } = m\Omega^2(r+e) \qquad (9.5.1)$$

$$\text{shaft resistance force } = Kr \qquad (9.5.2)$$

wherein, the shaft stiffness *k* is the lateral stiffness of a shaft in its bearings i.e., considering the rotor at mid-span, this is the force required to cause a unit lateral displacement at mid-span of the simply supported shaft. Thus

$$K = 48EI/L^3 \qquad (9.5.3)$$

Where E is the Young's modulus, I is the second moment of area, and L is the length between the supports.

Thus, for equilibrium,

$$Kr = m\Omega^2(r+e) \qquad (9.5.4)$$

$$r = \left(\frac{m\Omega^2 e}{k - m\Omega^2}\right)e$$

ie

$$= \left(\frac{\Omega^2}{\omega_n^2 - \Omega^2}\right)e \qquad (9.5.5)$$

where we have used $\omega_n = \sqrt{\frac{k}{m}}$ to represent the natural frequency of the lateral vibration of the springy-shaft-rotor system. Thus when the rotational speed of the system coincides with the natural frequency of

Ω

lateral vibrations, the shaft tends to bow out with a large amplitude. This speed is known as the critical speed and it is necessary that such a resonance situation is avoided in actual practice. As discussed earlier in the case of resonance, it takes some time for the amplitude to build up to a large value. Some of the turbine rotors whose operating speeds go beyond the critical speed are able to use this fact and rush-through the critical speed. It is necessary to observe from Fig.9.5.3 that, in synchronous whirl. the heavier side remains all the time on the outer side. Thus when the shaft bends, an inner fibre is under compressive stress and outer fibre is under tensible stress but there is NO REVERSAL of stress.

Rayleighs Method

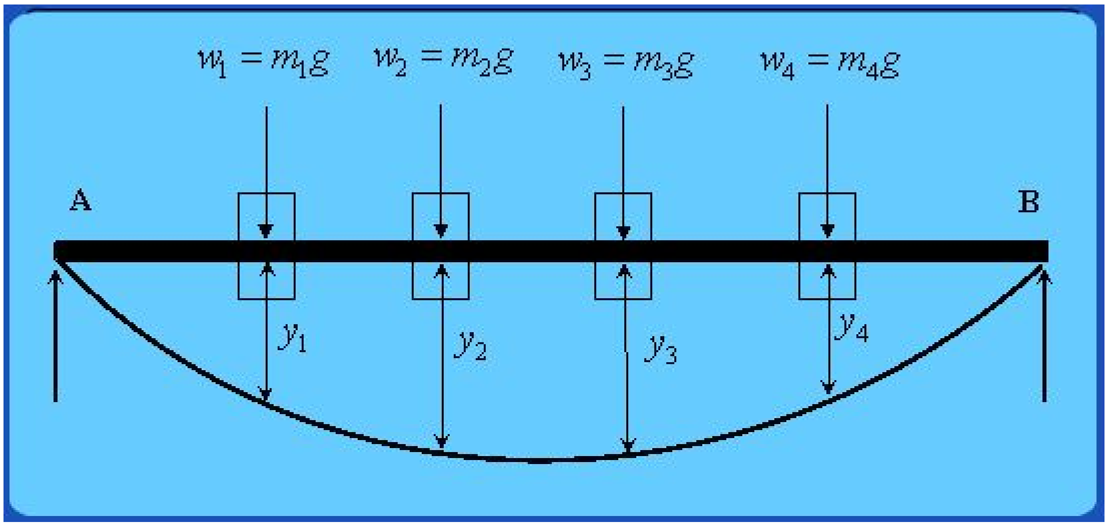

Fig 9.5.3 Multiple disks on a shaft

Rayleigh's method is based on the principle of conservation of energy. The energy in an undamped system consists of the kinetic energy and the potential energy. The kinetic energy T is stored in the mass and is proportional to the square of the velocity. The potential energy U includes strain energy that is proportional to elastic deformations and the potential of the applied forces. For a conservative system, the total energy must remain constant. That is

$$T + U = \text{constant} \tag{9.5.6}$$

Differentiating this expression, we get the equation of motion as follows.

$$\frac{d}{dx}(T+U) = 0 \tag{9.5.7}$$

Note that the amounts of kinetic and potential energy in the system may change with time but their sum must remain constant. Thus if T_1 and U_1 are energies at time t_1 and T_2 and U_2 are energies at time t_2 , then

$$T_1 + U_1 = T_2 + U_2 \tag{9.5.8}$$

For a shaftas shown in Fig.9.5.4, the potential energy is zero at the specific instant of time when the mass is passing through its static equilibrium position and kinetic energy is at its maximum T_{max}. Similarly at the instant when the mass is at its extreme position the kinetic energy is zero and the potential energy is at its maximum U_{max} . Thus we have the following relationship.

$$\begin{aligned} T_{max} &= U_{max} \\ T_{max} &= \frac{1}{2}mv^2 = \frac{1}{2}m(y\omega_n)^2 \\ U_{max} &= \frac{1}{2}Wy = \frac{1}{2}mgy \end{aligned} \tag{9.5.9}$$

Therefore we have, considering all the disks on the shaft,

$$\omega_n^2 \sum_{i=1}^{n} m_i y_i^2 = g\sum_{i=1}^{n} m_i y_i \tag{9.5.10}$$

Where i=1, n represents summation over all the "n" disks.

So we get the frequency of natural vibration as,

$$\omega_n = \sqrt{\frac{g\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i y_i^2}} \qquad (9.5.9)$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i y_i^2}} \qquad (9.5.12)$$

Dunkerley's Empirical Method

When a shaft carries multiple disks it is always efficient to use this method.

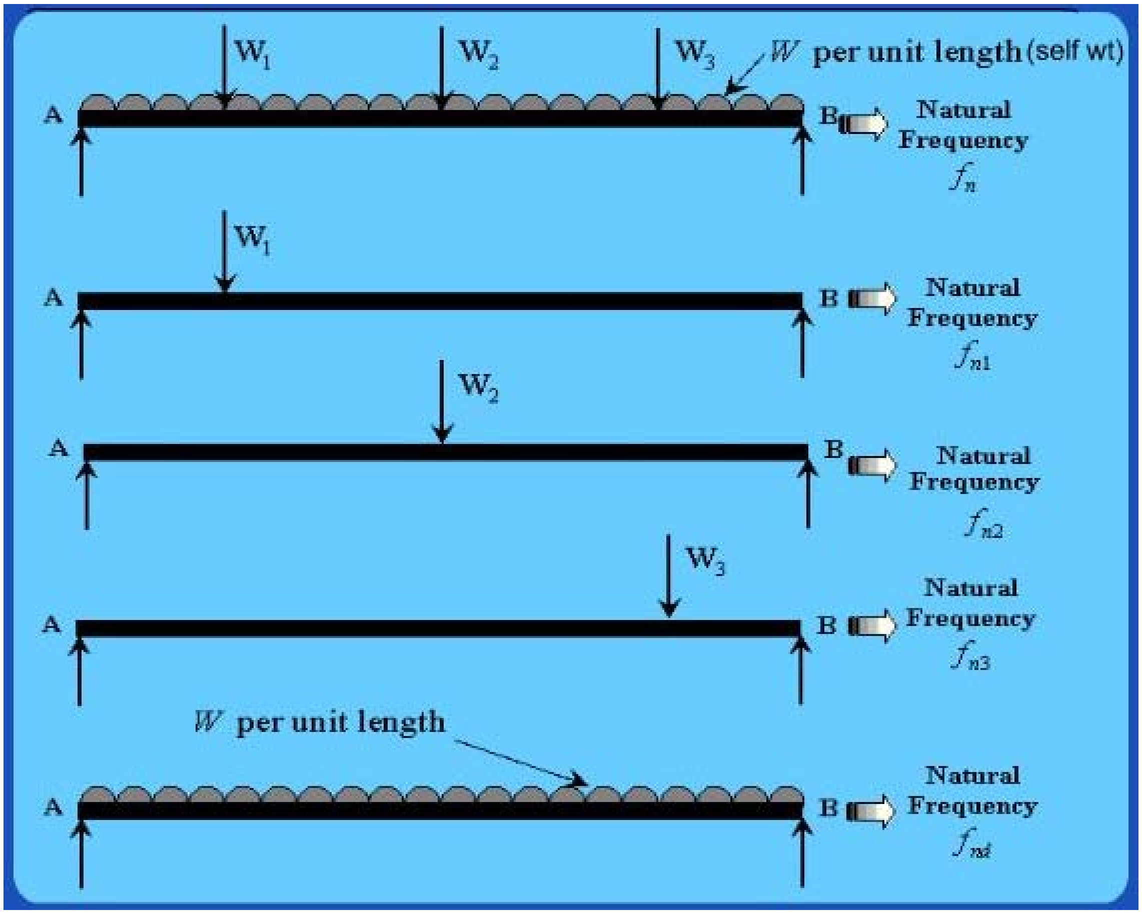

Fig 9.5.4 Dunkerly's approximation for shaft

We consider only one force (wt of disk) acting on the shaft at a time. For each disk, we find the corresponding natural frequency as $f_{n1}, f_{n2}, f_{n3} and\ f_{n4}$. The Nutural frequency of the shaft f_n when all the loads(disks) act on the shaft simultaneously can be found out by the using the formulae:

$$\frac{1}{f_n^2} = \frac{1}{f_{n1}^2} + \frac{1}{f_{n2}^2} + \frac{1}{f_{n3}^2} + \frac{1}{f_{nd}^2} \qquad (9.5.13)$$

For each of the dub-systems ie shaft with only one disk, natural frequency is obtained as $\sqrt{k/m}$ where k is the lateral stiffness of the shaft in its bearings and m is the mass of the disk.

To understand the basis of this method, we need to appreciate multi-d.o.f system vibrations.(Please refer lecture 4, module 10)

Recap

In this lecture you have learnt the following

- Critical speed of rotating shafts

- Use of energy method (Rayliegh's method)and Dunkerly's method for finding the critical speed of shafts.

Congratulations, you have finished Lecture 5. To view the next lecture select it from the left hand side menu of the page.