**Module 7 : Free Undamped Vibration of Single Degree of Freedom Systems; Determination of Natural Frequency ; Equivalent Inertia and Stiffness; Energy Method; Phase Plane Representation.**

## Lecture 11 : Free Undamped Vibration of SDOF systems

### Objectives

In this lecture you will learn the following

- Physical Model / Mathematical model
- Lumped and Distributed parameter systems
- Derivation of Equation of motion

  Free Body Diagram

  Use of Newton's Laws of motion

**Fig. 7.1_2 Simple Spring Mass System**

In the previous module, we learnt of how typical engineering systems experience vibrations and how these can be modeled. We will now begin our detailed study of vibration problems – beginning with a simple single degree of freedom system as shown in Fig. 7.1_2. Our simple system has just a spring and a mass and intentionally we have not included any form of dissipation. We call this a single d.o.f system because it requires just one coordinate (viz., position of mass) to completely specify the configuration of the system.

This is an example of what is known as a Lumped Parameter System i.e., the spring is assumed not to have any mass and the mass has no springiness. The system springiness is lumped into the spring and the inertia is lumped into the mass. Of course this is an idealization – real springs do have mass and no system is perfectly rigid. However this idealization renders the system easy to analyze and we also get very useful insights into the characteristics of typical vibration problems and therefore, we begin our study of vibrations with such lumped parameter models. When we have gained enough insight into the physics and mathematics of such vibrating systems, we will then be ready to study distributed parameter systems in a later module.

The spring mass system can be referred to as a physical model and we need to develop the corresponding mathematical model (governing equations) for our further analysis. The physical model of an engineering system encapsulates the various physical phenomena we wish to model in our study. The physical model, in this example, has provided for some source of springiness and inertia in the system, where as the specific load-deflection characteristics of the spring for example are as yet unspecified. It could be a simple linear function or a complex non-linear relation and will be reflected in the mathematical model that we develop. However, the physical model does not have any element of damping and hence automatically the mathematical model (governing equations) will also not have any damping terms. Thus development of physical model of the complex, real-life engineering systems represents the first source of approximations and will contribute to the deviations between the behavior predicted by the model and the real system behavior. Considering the complexity of the governing equations in the mathematical model, we may further introduce approximations (read errors!) in the solution of these governing equations. Finally every numerical computation involves truncation and round-off errors. Keeping in mind these sources of error, the role of the vibration analyst is to build a model that is easy to handle, yet accurate!

To develop the mathematical model (in the form of governing equation of motion) of a physical model, we rely on the application of laws of physics such as:

Newton 's laws of motion

Energy principles such as conservation of energy.

### Mathematical Model by application of Newton's 2$^{nd}$ Law

You may recall Newton 's 2$^{nd}$ Law of motion which can be loosely stated as follows

"The net force on a body in a given direction is equal to the product of the body mass and its acceleration along that direction."

In order to be able to derive the governing equation of motion for the spring-mass system using Newton 's law, we need to draw a free body diagram for the system. We begin our study of vibration with what is known as free vibrations i.e., the body is given an initial disturbance (such as pulling the mass through a certain displacement) and released to vibrate with no external force acting on it. This is also referred to as Natural vibration as it depicts how the system vibrates when left to itself with no external force. After having understood the system behavior under free vibration, we will then take up the study of forced vibration i.e. response of the system to an external, time varying force. Fig. 9.1.2 shows the forces acting on the mass for an assumed displacement x, $\Delta$ corresponds to static deflection i.e., deflection of the spring under the weight.

**Fig. 7.1.2 Free-body diagram of a spring-mass system**

We next equate the inertia force to the sum of all the external force.

$$\text{Inertia Force} = \sum \text{External Forces}$$

$$m\ddot{x} = mg - k(x+\Delta) \qquad 7.1.1$$

$$\text{But } mg = k\Delta$$

$$\therefore m\ddot{x} = -kx$$

$$\boxed{\therefore \ddot{x} + \frac{k}{m}x = 0} \qquad 7.1.2$$

This is the governing equation of motion.

It is the solution to this differential equation of motion along with its appropriate interpretation which constitutes a major part of vibration analysis. The governing equation as represented by eqn (7.1.2) is a second order, ordinary differential equation. This is characteristic of lumped parameter models. Distributed parameter system models result in partial differential equations, which are definitely more difficult to solve.

## DIFFERENTIAL EQUATIONS

We have already studied the algebric equations in our basic mathematics courses.

$$y = mx + c$$

Above equation represents an equation of a line in a two dimensional space. Differential equations in general represent the equation of the tangent for a given system represented by a curve.

$$\frac{dy}{dx} = 2x$$

represents the equation of the tangent of a parabola $y = x^2 + c$ in general.

Above equation could be termed as first order linear differential equation.

Lets first refresh the understanding the concept of order of differential equation.

$\frac{d^2y}{dx^2} = x$ --2nd order differential equation.

$\frac{d^3y}{dx^3} = x$ --3rd order differential equation.

All the above equations are linear differential equations.

All the above mentioned differential equations represent one single dependent and one independent variable.

But there is every possibility of a system to be dependent on two variables.

For example Heat flux propagation through cooling fins depends on distance from source as well as time. Such systems are represented by differential equations termed as partial differential equations.

$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} = c$ --1st Order partial differential equation

where $f$ is a function of x and y

$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$ --2nd order partial differential equation

**Recap**

In this lecture you have learnt the following

- Concept of degree of freedom of a system

Different types of systems in general

- Lumped parameter system ( finite degrees of freedom represented by ordinary differential equation)
- Distributed parameter system ( infinte degrees of freedom, represented by partial differential equations)

- Laws used in mathematical modelling of physical systems viz

-
    - Newton's laws of motion

- Free body diagram and its role in mathematical modelling

- Derivation of equation of motion for vibration viz

$$\therefore \ddot{x} + \frac{k}{m} x = 0$$

Congratulations, you have finished Lecture 7.1. To view the next lecture select it from the left hand side menu of the page