Module 1 : Dynamics of Rigid Bodies in Plane Motion; Dynamic Force Analysis of Machines

Lecture 1 : Dynamics of Rigid Bodies in Plane Motion; Dynamic Force Analysis of Machines.

Objectives

In this lecture you will learn the following

- Introduction to dynamics of machines
- Equations of motion for a planar body
- Equations for a mechanism
- Joint reactions
- Different types of forces

Introduction

The main problems addressed in dynamics of machines can be summarized as:

- Determine forces required to generate given accelerations of a mechanism.
- Determine the acceleration and motion resulting from given forces on a mechanism.

In both cases, internal joint reactions and frictional forces are also determined. We focus mainly on rigid body planar mechanisms.

In the first lecture, we derive equations of motion for planar mechanisms with different types of joints.

In the second and third lectures of this module, we use the above equations to solve the problems of determining forces, accelerations and motions in various types of mechanisms.

Equations of Motion of a Planar Rigid Body

We can use Newton 's equations for particles to derive the equations of motion of a rigid body. A rigid body, which is constrained to move in a plane, has three degrees of freedom, and hence it has three independent equations of motion which relate the forces in the plane to accelerations. Consider the body shown in Fig. 1.1 (a). A set of forces $\vec{F}_k$ and couples $\vec{C}_k$ act on it, with the force $\vec{F}_k$ acting at the point $(x_k,\ y_k)$. The position coordinates are with respect to an inertial frame. The rigid body can be considered to be made of elemental masses which can be regarded as particles. Newton 's equation for the i^{th} elemental mass Δm_i (Fig. 1.1 (b)) at the location $(x_i,\ y_i)$ is

$$\Delta m_i \vec{a}_i = \vec{F}i + \sum_j \vec{R}_{ij} \qquad (1)$$

Where a i is the acceleration of the elemental mass and $\vec{F}_i$ is the external force acting on it. Note that every elemental mass may not have an external force. R_{ij} is the force on the i^{th} elemental mass from the j^{th} elemental mass. This arises due to the constraint that the two masses have to remain at a fixed distance from each other.

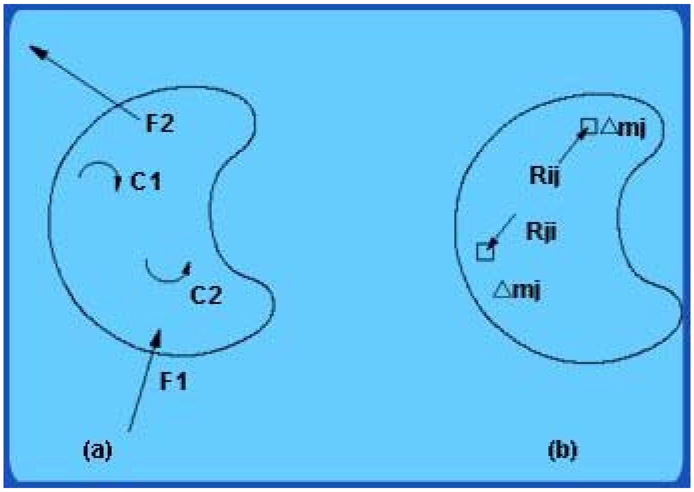

Fig. 1.1.1 A rigid body under planar forces and couples

Adding up the equations of all the elemental masses (keeping in mind that the internal reaction forces occur in equal but opposite pairs), we get the equations of motion for the body, relating linear accelerations to linear forces

$$m\vec{a}_c = \sum_k \vec{F}_k \tag{2}$$

Where a c is the acceleration of the centre of mass. Note that the force and acceleration vectors, though three dimensional, lie in the plane of motion of the rigid body. Hence, resolving the forces along the x and y directions in the plane, we get two scalar equations of motion.

$$\begin{aligned} m\ddot{x}_c &= \sum_k F_{xk} \\ m\ddot{y}_c &= \sum_k F_{yk} \end{aligned} \tag{3}$$

The third equation in planar motion, for the rotation of the body is obtained as:

$$I_c\alpha = \sum_k \vec{r}_{ck} \times \vec{F}_k + \sum_k \vec{C}_k \tag{4}$$

where I_c , the moment of inertia about the center of mass is given by

$$I_c = \sum_i \Delta m_i r_{ic}^2 \tag{5}$$

Equations (3) and (4) are the fundamental equations of motion of a rigid body in planar motion.

Joint Reactions

A mechanism consists of bodies which are connected together by kinematic pairs. One approach to derive the equations of motion for mechanisms (called Newton-Euler approach) considers each body as a free body, along with the forces due to the constraints, called joint reactions. We use this approach here. First, let us consider the nature of joint reactions for different types of joints.

Revolute Joint:

Two bodies connected by a revolute joint are shown in Fig. 1.2(a) and the free body diagrams of the two bodies are shown in Fig. 1.2(b). The revolute joint prevents the two bodies from undergoing relative translational motion along say x and y. Hence, the reaction force is represented by two components, R_x and R_y. The reaction forces on the two bodies are equal and opposite, as required by Newton 's third law.

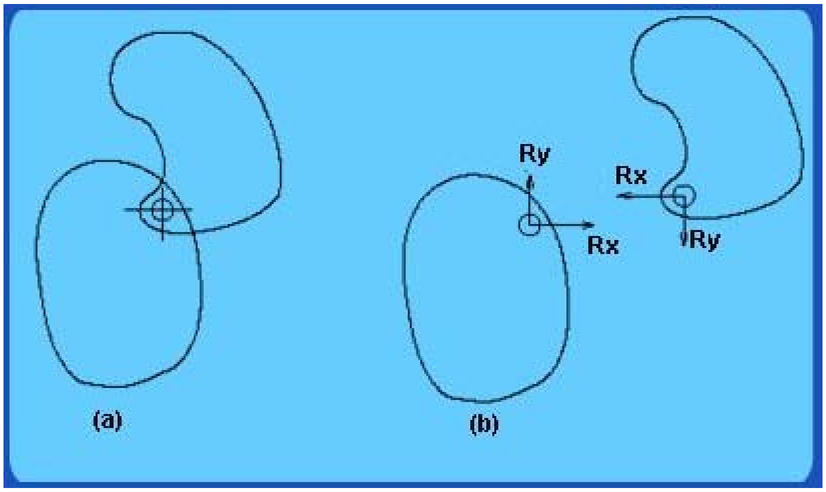

Fig. 1.1.2 Revolute Joint

Prismatic Joint:

Two bodies connected by a prismatic joint are shown in Fig. 1.3(a) and the free body diagrams of the two bodies are shown in Fig. 1.3(b). The prismatic joint prevents relative translation in a direction normal to the line of the joint and also relative rotation. Hence, the reaction forces are represented by normal reaction force R_n and couple R_c. The point at which R_n acts is usually fixed to the piston and moves with the piston. The position of R_n is not important, however, the direction of R_n has to be normal to the direction of relative translation between the two bodies. The reaction forces and couples on the two bodies are equal and opposite, as required by Newton 's third law.

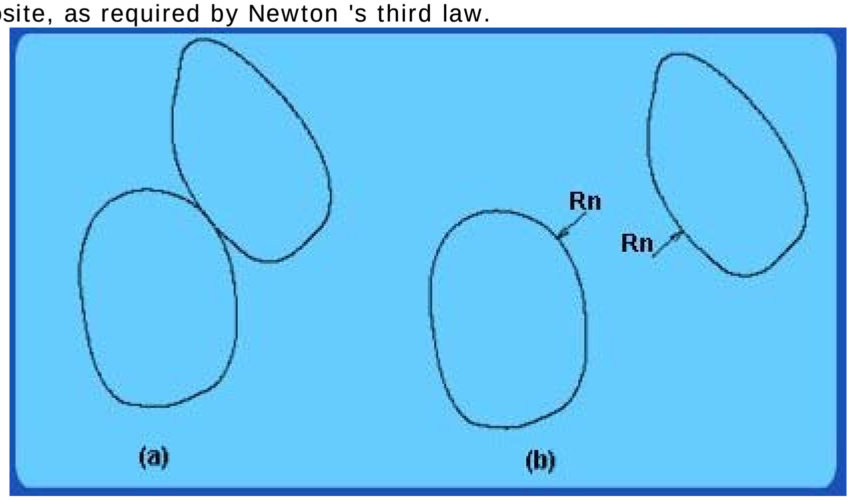

Fig. 1.1.3 Prismatic Joint

Cam-Follower Joint with Sliding:

Two bodies with a cam follower joint which allows sliding are shown in Fig. 1.4(a) and the free body diagrams of the two bodies are shown in Fig. 1.4(b). There is only one reaction force R_n which acts at the point of contact, and is normal to the surfaces at that point. The reaction force is a pushing force as shown and it cannot be negative. Negative value indicates separation of the two bodies. The reaction forces on the two bodies are equal and opposite, as required by Newton 's third law.

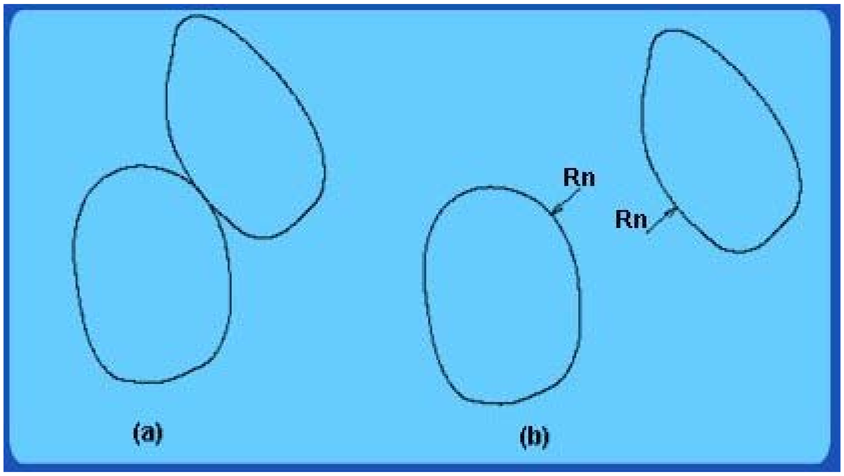

Fig. 1.1.4 Cam-follower-with-sliding

Cam-Follower Joint with Pure Rolling:

Two bodies with a cam follower joint which allows only pure rolling are shown in Fig. 1.5(a) and the free body diagrams of the two bodies are shown in Fig. 1.5(b). There are two reaction forces, both acting at the point of contact. R_n is normal to the surfaces, while R_t is tangential. R_n has the same characteristics (only pushing) as the corresponding force in the case of cam follower joints with sliding. R_t can be in any one of the two tangential directions. The reaction forces on the two bodies are equal and opposite, as required by Newton 's third law.

The joints considered above are those which commonly occur in planar mechanisms. We now show how the equations of motion of the mechanism can be derived using the free body (Newton-Euler) approach.

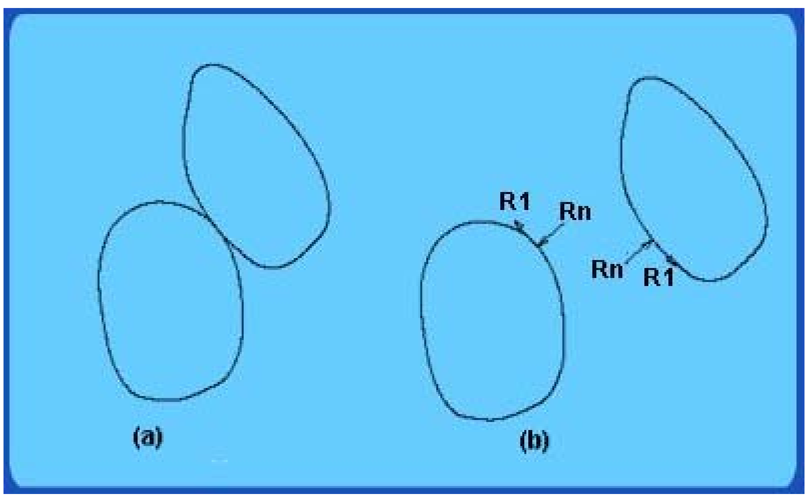

Fig. 1.1.5 Cam-follower-pure-rolling

Equations of Motion of Mechanisms

Consider the mechanism shown in fig. 1.6. It is a 4 link, 2 degree of freedom mechanism with two revolute joints, one prismatic joint, and one cam-follower joint which allows sliding. The link numbered 1 is the frame. It has a revolute joint A with link 2 and a cam-follower joint with link 4, with the contact being at E . Links 2 and 3 are connected by a prismatic joint, while links 3 and 4 are connected by a revolute joint D . The point B is on link 3. The centers of mass of link i is denoted by C i . Note that the center of mass of link 3 is outside the physical bounds of the link, a possibility with links with concave portions in their profile. A torque _ 2 acts on link 2, while a force F 4 acts on link 4 at point G as shown. The direction of gravity is as shown in figure.

The free body diagrams of the moving links are shown in Fig. 1.7, along with the reaction and applied forces on them.

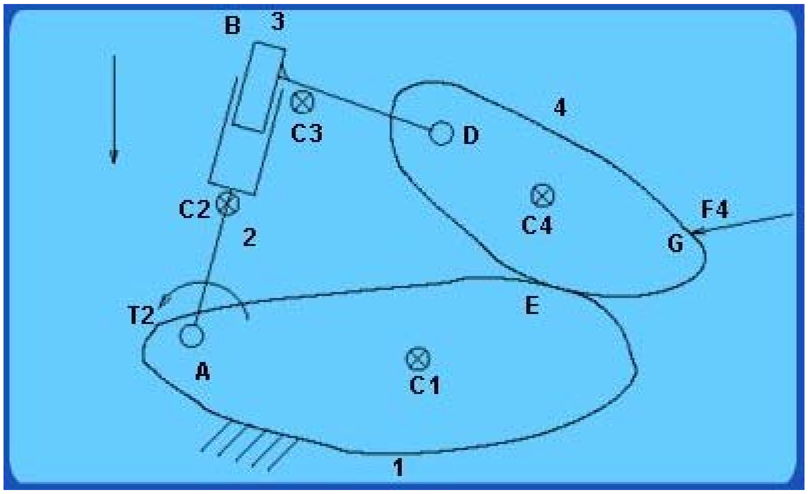

Fig. 1.1.6 Typical planar mechanism

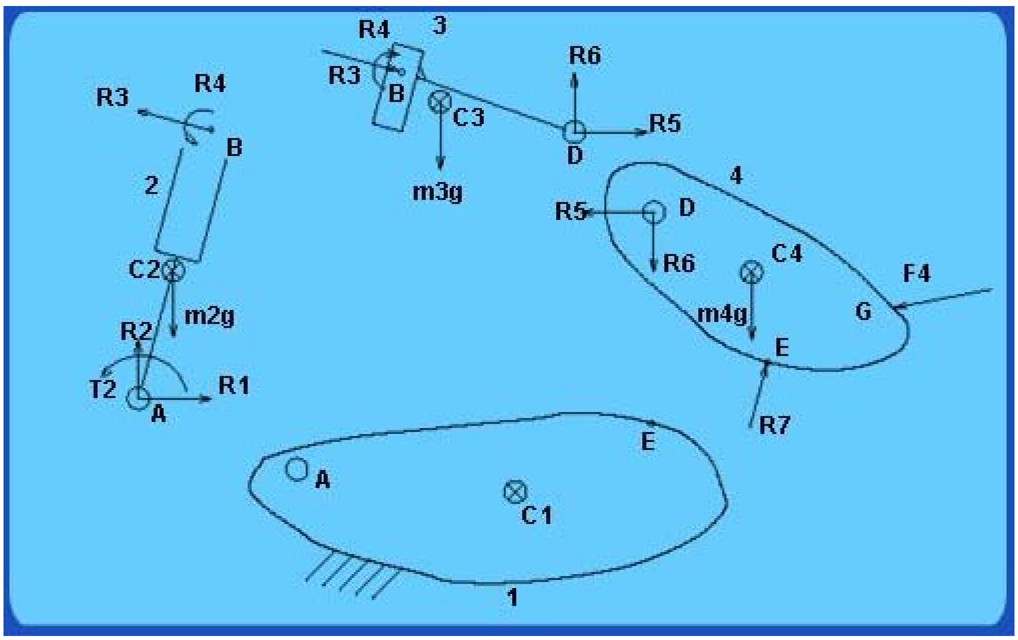

Fig. 1.1.7 Free Body Diagrams for Links of mechanism in Fig. 1.1.6

Recap

In this lecture you have learnt the following

- Developing equations of motion for a planar body/mechanism
- Different types of joints and their reaction forces

Congratulations, you have finished Lecture 1. To view the next lecture select it from the left hand side menu of the page