Module 8 : Free Vibration with Viscous Damping; Critical Damping and Aperiodic Motion; Logarithmic Decrement;Systems with Coulomb Damping.

Lecture 20 : Systems with Coulomb Damping.

Objectives

In this lecture you will learn the following

- Damping effect due to friction and its characteristics.
- Solution of equation of motion for frictional damping.
- Frictionally damped natural frequency.

Dry friction or Coloumb Damping:This type of damping occurs when two machine parts rub against each other, dry or unlubricated. The dry friction is a complex phenomenon and there are several theories about the mechanism of dry friction. A typical dry friction characteristic is as shown in Fig 8.4.1. This form of damping brings in non-linearities and the mathematical model of the system becomes non-linear . Non-linear dynamical systems can exhibit very complex behavior and detailed analysis of such systems is beyond the scope of our present discussion. Interested readers can refer to advanced reference material .

Fig 8.4.1 Coulomb Friction

Consider two dry sliding surfaces with a normal reaction R between them. Then the force of friction acting on each of the two mating surfaces is given by:

$$F = \mu R$$

where μ is defined as the coefficient of friction between the two mating surfaces. For ideally smooth surfaces, dry friction coefficient μ is independent of velocity. For rough surfaces, dry friction cofficient decreases somewhat initially with increase in velocity and finally is constant through out. For lubricated surfaces, μ is approximately proportional to velocity giving approxiamtely viscous damping.

The aspects we are intrested here regarding free vibration with Coulomb damping are:

- The frequency of damped oscillations.
- The rate of decay of these oscillations.

FREQUENCY OF DAMPED OSCILLATIONS

Consider a spring mass system shown in fig 8.4.2 with mass sliding on the dry surface , μ being the coefficient of friction between the two surfaces. At equillibrium position, spring is unstretched and no friction force acts on the mass.

Fig 8.4.2 Equilibrium Position

Mass displaced to right and moving towards right:

Fig 8.4.2 Mass displaced and moving to right

Frictional force as well as the spring force in this case are acting towards left.The equation of motion of mass for this part of motion is as follows:

$$m\ddot{x} = -kx - F \qquad 8.4.1$$

$$\text{or} \qquad \ddot{x} + \frac{k}{m}\left(x + \frac{F}{k}\right) = 0 \qquad 8.4.2$$

Let us now introduce a new variable "y" as follows:

$$x + \frac{F}{k} = y$$

$$\therefore \quad \ddot{x} = \ddot{y}$$

Substituting for x and $\ddot{x}$ we have

$$\ddot{y} = -\frac{k}{m}y$$

This is simple harmonic motion about y=0 and is true for that quater of cycle when mass is displaced to right and moving to the right.

Natural frequency for this part of the motion is :

$$\omega_n = \sqrt{\frac{k}{m}} \qquad 8.4.5$$

Mass displaced to right and moving towards left:

Fig 8.4.4

When the mass is under the condition as in fig 8.4.4 displaced to right and moving towards left, equation of motion changes as follows:

$$m\ddot{x} = -kx + F \qquad 8.4.6$$

$$\text{or} \qquad \ddot{x} + \frac{k}{m}\left(x - \frac{F}{k}\right) = 0 \qquad 8.4.7$$

$$x - \frac{F}{k} = y \qquad 8.4.8$$

$$\therefore \quad \ddot{x} = \ddot{y}$$

Substituting for x and $\ddot{x}$ we have

$$\ddot{y} + \frac{k}{m}y = 0 \qquad 8.4.9$$

The natural frequency of this part of motion is also:

$$\omega_n = \sqrt{\frac{k}{m}} \qquad 8.4.10$$

The displacement time plot of such system is shown in Fig. 8.4.5:

Fig. 8.4.5

It is to be observed that the amplitude envelope is exponentially decaying for viscous damped system studies earlier, while in this case, it falls off linearly.

Recap

In this lecture you have learnt the following

- Damping effect due to friction
- Coloumb damped natural frequency
- Rate of decay due to friction

Congratulations, you have finished Lecture 4. To view the next lecture select it from the left hand side menu of the page