Module 10 : Vibration of Two and Multidegree of freedom systems; Concept of Normal Mode; Free Vibration Problems and Determination of Ntural Frequencies; Forced Vibration Analysis; Vibration Absorbers; Approximate Methods - Dunkerley's Method and Holzer Method

Lecture 30 : Approximate Methods (Dunkerly's Method)

Objectives

In this lecture you will learn the following

- Dunkerly's method of finding natural frequency of multi- degree of freedom system

We observed in the previous lecture that determination of all the natural frequencies of a typical multi d.o.f. system is quite complex. Several approximate methods such as Dunkerly's method enable us to get a reasonably good estimate of the fundamental frequency of a multi d.o.f. system.

Basic idea of Dunkerly's method

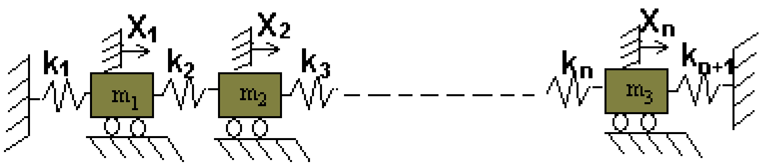

Fig 10.4.1 A typical multi d.o.f. system

Consider a typical multi d.o.f. system as shown in fig 10.4.1

Dunkerly's approximation to the fundamental frequency of this system can be obatined in two steps:

Step1: Calculate natural frequency of all the modified systems shown in Fig 10.4.2. These modified systems are obatined by considering one mass/inertia at a time. Let these frequencies be $\omega_{11}, \omega_{12}, \omega_{13}, \omega_{14}, \ldots\ldots$

Step2: Dunkerly's estimate of fundamental frequency is now given as:

$$\frac{1}{\omega_{1d}^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{12}^2} + \frac{1}{\omega_{13}^2} + \frac{1}{\omega_{14}^2} + \cdots\cdots$$

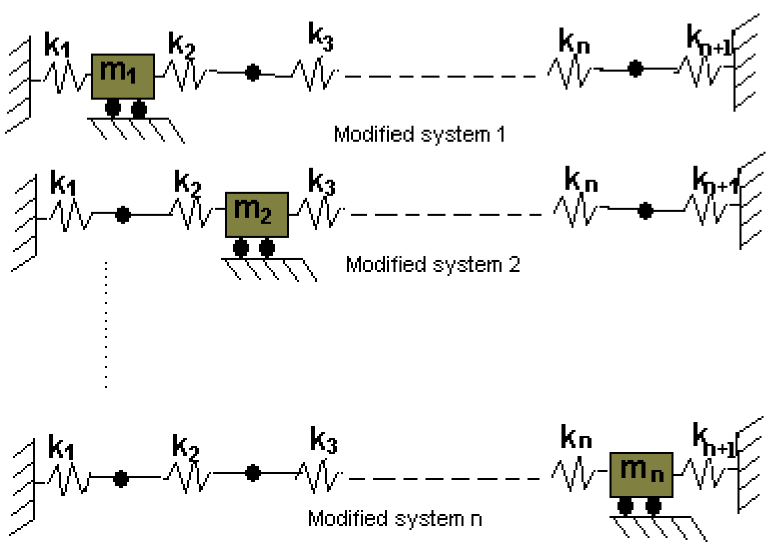

Fig 10.4.2 Modified system considered in Dunkerly's Method

Explanation of Basic Idea

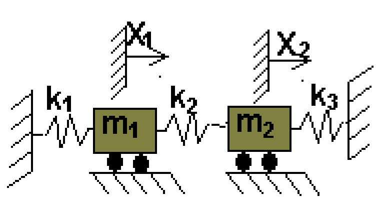

Fig 10.4.3 A Typical two d.o.f. example

Consider a typical two d.o.f. system as shown in Fig 10.4.3 and the equations of motion are given as:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix}+\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad 10.4.2$$

For harmonic vibration, we can write:

$$\begin{aligned} x_1 &= X_1 \sin \omega t \\ x_2 &= X_2 \sin \omega t \end{aligned} \qquad 10.4.3$$

Thus,

$$\begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2+k_3 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}=\omega^2\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \qquad 10.4.4$$

Inverting the stiffness matrix and re-writing the equations

$$\frac{1}{\omega^2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}=\begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} \qquad 10.4.5$$

$$\text{i.e.,}\quad \begin{bmatrix} \frac{1}{\omega^2}-c_{11}m_1 & -c_{12}m_2 \\ -c_{21}m_1 & \frac{1}{\omega^2}-c_{22}m_2 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad 10.4.6$$

The equation characteristic can be readily obtained by expanding the determinant as follows:

$$\left(\frac{1}{\omega^2}\right)^2-\left(\frac{1}{\omega^2}\right)(c_{11}m_1+c_{22}m_2)+(m_1m_2)(c_{11}c_{22}-c_{12}\ c_{21}) = 0 \qquad 10.4.7$$

As this is a two d.o.f. system, it is expected to have two natural frequencies viz ω_1 and ω_2 ., Thus we can write Eq. 10.4.7 as:

$$\left(\frac{1}{\omega^2}-\frac{1}{\omega_1^2}\right)-\left(\frac{1}{\omega^2}-\frac{1}{\omega_2^2}\right)=\left(\frac{1}{\omega^2}\right)^2-\left(\frac{1}{\omega^2}\right)(c_{11}m_1+c_{22}m_2)+(m_1m_2)(c_{11}c_{22}-c_{12}m_{21}) \qquad 10.4.8$$

Comparing coefficients of like terms on both sides, we have:

$$\frac{1}{\omega_1^2}+\frac{1}{\omega_2^2}=(c_{11}m_1+c_{22}m_2) \qquad 10.4.9$$

$$\left(\frac{1}{\omega_1^2}\right)\left(\frac{1}{\omega_2^2}\right) = m_1m_2(c_{11}+c_{22}-c_{12}m_{21}) \qquad 10.4.10$$

It would appear that these two equations (10.4.9-10) can be solved exactly for and . While this is true for this simpleexample, we can't practically implement such a scheme for an n-d.o.f system, as it would mean similar computational effort as solving the original problem itself. However, we could get an approximate estimate for the fundamental frequency.

If $\omega_2 >> \omega_1$, then we can approximately write from Eq. (10.4.9),

$$\frac{1}{\omega_1^2}\approx c_{11}m_1+c_{22}m_2 \qquad 10.4.11$$

Let us now study the meaning of and . It is easily verified that

$$c_{11}m_1 \qquad c_{22}m_2$$

$$c_{11} = \frac{k_2 + k_3}{k_1k_2 + k_2k_3 + k_3k_1} \qquad 10.4.10$$

$$c_{22} = \frac{k_1 + k_2}{k_1k_2 + k_2k_3 + k_3k_1} \qquad 10.4.13$$

These can be readily verified to be the reciprocal of the equivalent stiffness values for the modified systems shown in Fig. 10.4.4.

Thus, we can write:

$$\frac{1}{\omega_1^2} \approx \frac{m_1}{k_{eq1}} + \frac{m_2}{k_{eq2}} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{12}^2} \qquad 10.4.14$$

Practical Implication

The most crucial assumption in Dunkerley's method is when we go from Eq. (10.4.9) to Eq.(10.4.11).

We are assuming that the second frequency and hence other higher frequencies are far higher than the fundamental frequency. In other words, when the system modes are well separated, Dunkerley's method works well.

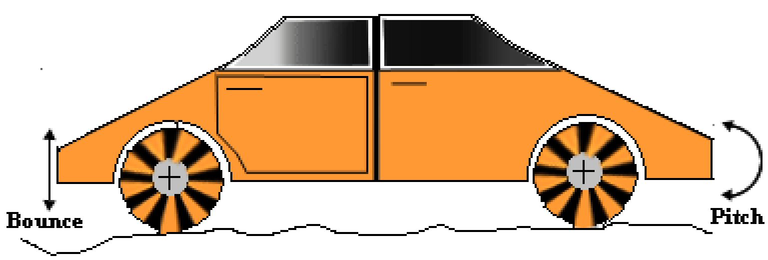

Fig 10.4.5 Bounce and Pitch modes of an automobile

For a typical automobile, for example, the frequencies of bounce and pitch are both in the range of 1-2Hz and the system has several frequencies within the 0-15 Hz range. Hence Dunkerley's approximation is unlikely to give good estimates.

Recap

In this lecture you have learnt the following

Dunkerly's method of determining approximate by the fundamental natural frequencies of system

- $\frac{1}{\omega_{1d}^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{12}^2} + \frac{1}{\omega_{13}^2} + \frac{1}{\omega_{14}^2} + \cdots\cdots$
- Concept of Dunkerly's method.
- Practical implication of Dunkerly's method.

Congratulations, you have finished Lecture 4. To view the next lecture select it from the left hand side menu of the page