Module 8 : Free Vibration with Viscous Damping; Critical Damping and Aperiodic Motion; Logarithmic Decrement;Systems with Coulomb Damping.

Lecture 17 : Free Vibration with Viscous Damping

Objectives

In this lecture you will learn the following

- Solution of equation of motion for damped-single degree of freedom system.
- Concept of damping and damping factor.
- Different types of damped systems (underdamped, overdamped and critically damped systems).

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Free Vibrations with Viscous Damping

Fig 8.1.1 Free Damped Vibrations

Fig 8.1.2 Free body Diagram

A single degree of freedom damped system and its free body diagram are shown in Fig.8.1.1 and 8.1.2. Applying Newton 's second Law,

$$\text{Inertia Force} = \sum \text{External Forces}$$

$$m\ddot{x} = -kx - c\dot{x} \qquad 8.1.1$$

$$\Rightarrow \ddot{x} + \frac{c}{m}\dot{x} + \frac{k}{m}x = 0$$

This is an ordinary linear differential equation of second order.

Assuming the solution to the above equation of the form, $x = e^{st}$ we get

$$s^2 e^{st} + \frac{c}{m} s e^{st} + \frac{k}{m} e^{st} = 0$$

$$s^2 + \frac{c}{m} s + \frac{k}{m} = 0 \qquad 8.1.2$$

This is called the characteristic equation of the system which has two roots,

$$S_{1,2} = \frac{c}{2m}\left\{-1 \pm \sqrt{1 - \frac{4km}{c^2}}\right\} \qquad 8.1.3$$

Therefore the genral solution to the equation of motion is of the form,

$$\boxed{x = Ae^{s_1 t} + Be^{s_2 t}}$$

where A and B are constants to be determined from initial conditions on position and velocity.

We define Damping factor ξ as

$$\xi = \frac{c}{2\sqrt{km}}$$

$$\text{such that } \frac{4km}{c^2} = \frac{1}{\xi^2} \qquad 8.1.4$$

$$\text{And } \quad \xi = \frac{c}{2m\omega_n}$$

$$\therefore \frac{c}{2m} = \xi\omega_n \qquad 8.1.5$$

From equations 8.1.4 and 8.1.5 , we can write the two roots s_1 and s_2 as follows:

$$s_{1,2} = \omega_n\left\{-\xi \pm \sqrt{\xi^2 - 1}\right\}$$

Thus mainly three cases arise depending on the value of ξ

$\xi > 1 \Leftrightarrow$ *Overdamped System*

$\xi = 1 \Leftrightarrow$ *Critically damped System*

$\xi < 1 \Leftrightarrow$ *Underdamped System*

When $\xi \geq 1$ the system undergoes aperiodically decaying motion and hence such systems are said to be Overdamped Systems. An example of such a system is a door damper – when we open a door and enter a room, we want the door to gradually close rather than exhibit oscillatory motion and bang into the person entering the room behind us! So the damper is designed such that $\xi \geq 1$

Critically damped motion ($\xi = 1$ a hypothetical borderline case separating oscillatory decay from aperiodic decay) is the fastest decaying aperiodic motion.

When "ξ < 1", x(t) is a damped sinusoid and the system exhibits a vibratory motion whose amplitude keeps diminishing. This is the most common vibration case and we will spend most of our time studying such systems. These are referred to as **Underdamped systems .**

Fig 8.1.3 Damped vibration at various damping factors

In Fig. 8.1.3 we capture typical motions for overdamped, critically damped and underdamped systems.The initial conditions for the system are:

$$t = 0, \qquad x = 1.3cm$$
$$\dot{x} = 0$$

Phase plot for underdamped oscillations showing the system coming to rest at its equilibrium position is given in Fig. 8.1.4

Fig 8.1.4 Phase plot of damped oscillations for ξ =0.1

Recap

In this lecture you have learnt the following

- Damping and damping factor
- Free vibration response with damping
- Overdamped, critical and underdamped systems

- Governing differential equations and typical solution plots for these systems

Congratulations, you have finished Lecture 1. To view the next lecture select it from the left hand side menu of the page