Module 8 : Free Vibration with Viscous Damping; Critical Damping and Aperiodic Motion; Logarithmic Decrement; Systems with Coulomb Damping.

Lecture 19 : Logarithmic Decrement

Objectives

In this lecture you will learn the following.

- Concept of logarithmic decrement
- Formulation of equation for logarthmic decrement.
- Application of the concept.

When "$\xi < 1$", x(t) is a damped sinusoid and the system exhibits a vibratory motion whose amplitude keeps diminishing as shown in fig 10.3.1. This is the most common vibration case and we will spend most of our time studying such systems. These are referred to as Underdamped systems

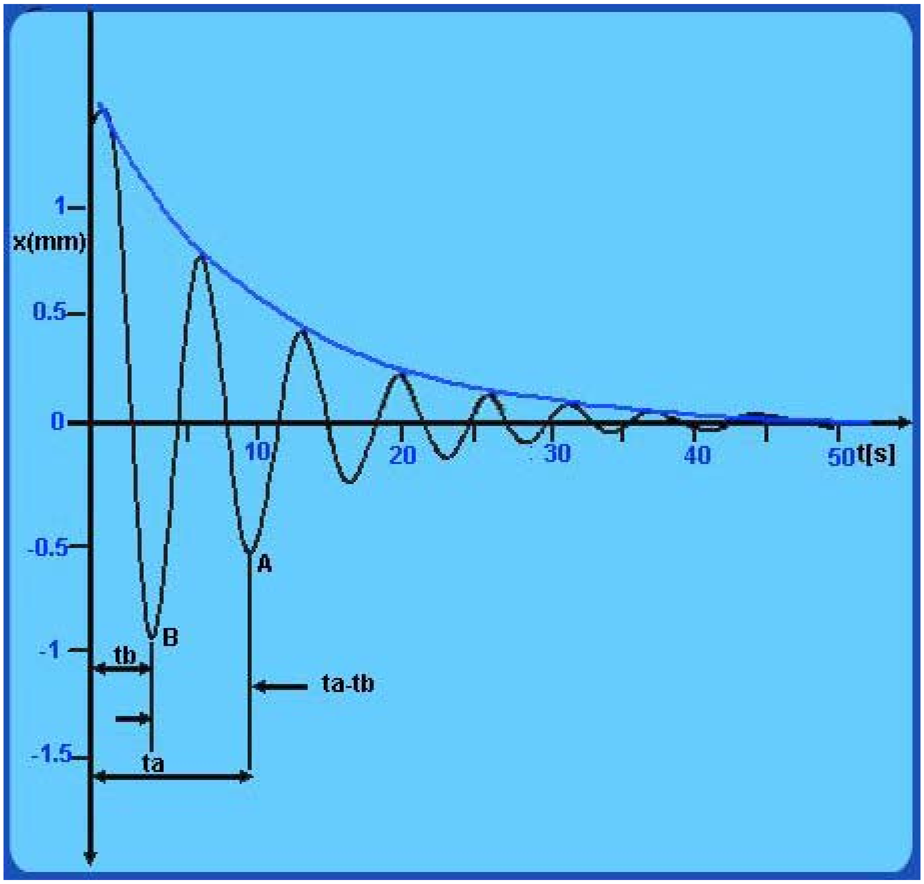

Fig 8.3.1 Underdamped Oscillations

The decrease in amplitude from one cycle to the next depends on the extent of damping in the system. The successive peak amplitudes bear a certain specific relationship involving the damping of the system, leading us to the concept of "logarithmic decrement" which we will now discuss.

It is often necessary to estimate the extent of damping present in a given system. Essentially the experimental techniques to determine damping in a system fall into two categories -- those based on free vibration tests and secondly those based on forced vibration tests. The latter require more sophisticated equipment/instruments, while the former is a relatively simple test. In a free vibration test, based on the measured peak amplitudes over several cycles (and thus estimating the "logarithmic decrement"), one can readily find the damping factor for the given system. We will now discuss these aspects.

Lograthmic decrement comes as an accurate and practically feasible tool to determine the damping in the system.

Logarithmic Decrement:

Consider the two peaks A and B as shown in fig 8.3.1. The amplitude at A and B are x_A and x_B at time t_A and t_B respectively.The periodic displacement from x_A to x_B represents a cycle. The time period for this complete cycle is given by:

$$t_A - t_B = \frac{2\pi}{\omega_d} = \frac{2\pi}{\sqrt{1-\xi^2}\,\omega_n} \qquad 8.3.1$$

This is the time period of damped oscillations and ω_d is damped natural frequency.

The amplitude of damped oscillations is given by the expression:

$$x = \frac{X_0}{\sqrt{1-\xi^2}\,\omega_n} e^{-\xi\omega_n t} \qquad 8.3.2$$

which is the envelope of maximum of displacement -time curve. Therefore at t= t_A and t_B the amplitudes are given by:

$$x_A = \frac{X_0}{\sqrt{1-\xi^2}\,\omega_n} e^{-\xi\omega_n t_A} \qquad 8.3.3$$

$$x_B = \frac{X_0}{\sqrt{1-\xi^2}\,\omega_n} e^{-\xi\omega_n t_B} \qquad 8.3.4$$

Therefore

$$\frac{x_B}{x_A} = e^{\xi\omega_n(t_A - t_B)}$$

$$\text{But from equation } 8.3.1 \qquad t_A - t_B = \frac{2\pi}{\sqrt{1-\xi^2}\,\omega_n} \qquad 8.3.5$$

Therefore $$\frac{x_B}{x_A} = e^{2\pi\xi / \sqrt{1-\xi^2}} \qquad 8.3.6$$

$$\log_e \frac{x_B}{x_A} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

This is called the Logarithmic Decrement denoted by

$$\delta = \log_e \frac{x_B}{x_A} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \qquad 8.3.7$$

if $\xi << 1$ $\delta \approx 2\pi\xi$

This shows that the ratio of any two succesive amplitudes for an underdamped system, vibrating freely , is constant and is a function of the damping only.

Sometimes, in experiments, it is more convenient/accurate to measure the amplitudes after say "n" peaks rather than two succesive peaks (because if the damping is very small, the difference between the succesive peaks may not be significant). The lograthmic decrement can then be given by the equation

$$\delta = \frac{1}{n}\log_e \frac{x_0}{x_n} \tag{8.3.8}$$

Recap

In this lecture you have learnt the following

- Concept of logarithmic decrement and its application as a tool to determine the system damping
- Derivation of formula for lograthmic decrement $\log_e \frac{x_B}{x_A} = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$

Congratulations, you have finished Lecture 3. To view the next lecture select it from the left hand side menu of the page