Module 3 : Dynamics of Reciprocating Machines with Single Slider;Unbalance in Single Cylinder Engine Mechanisms

Lecture 6 : Dynamics of Reciprocating Machines with Single Slider;Unbalance in Single Cylinder Engine Mechanisms

Objectives

In this lecture you will learn the following

- Approximate acceleration analysis of an IC Engine mechanism
- Equivalent Link model of a connecting rod
- Estimation of Inertia forces in a crank-slider mechanism

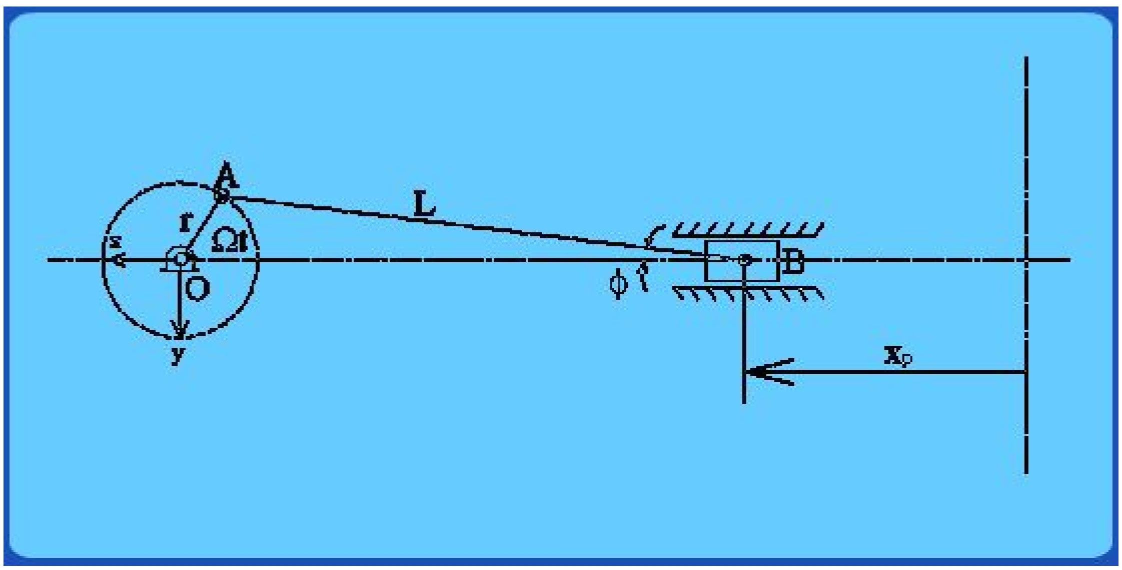

Figure 3.1 Slider-crank Mechanism of IC Engine

A typical crank-slider mechanism as used in an IC Engine is shown in Fig. 3.1. It essentially consists of four different parts viz., frame(i.e., cylinder) ,crank ,connecting rod and reciprocating piston. The frame is supposedly stationary; crank is undergoing purely rotary motion while the piston undergoes to-and-fro rectilinear motion. The connecting rod undergoes complex motion – its one end is connected to the crank (undergoing pure rotation) and the other end is connected to the piston (undergoing pure translation).

We know that the inertia forces are given by mass times acceleration and we shall now estimate the inertia forces (shaking forces and moments) due to the moving parts on the frame (cylinder block).

PISTON

From the geometrical construction shown in Fig 3.1, we have:

$$x_p = r\,(1 - \cos(\Omega t)) + L\,(1 - \cos(\varphi)) \tag{3.1}$$

$$\sin(\varphi) = (r/L)\,\sin(\Omega t) \tag{3.2}$$

(3.3)

$$x_p = r\,(1-\cos(\Omega t)) + L\left[1-\sqrt{1-\left(\frac{r}{L}\right)^2 \sin^2(\Omega t)}\right]$$

It is to be observed that if r > L, the crank wont rotate fully. Even if r = 0.5 L, the transmission angle $(\pi/2 - \phi)$ will still be poor. Therefore, in practical crank-slider mechanisms used in IC Engines, crank radius "r" is less than one-fourth of the length of the connecting rod. Thus

$$\left(\frac{r}{L}\right)^2 \ll 1 \tag{3.4}$$

We can therefore approximately write: $\sqrt{1-\left(\frac{r}{t}\right)^2 \sin^2 \Omega t} \;\approx\; 1-\frac{1}{2}\left(\frac{r}{t}\right)^2 \sin^2 \Omega t$

$$But\ Sin^2 \Omega t = \frac{1-\cos^2 \Omega t}{2}$$

$$\therefore \sqrt{1-\left(\frac{r}{t}\right)^2 \sin^2 \Omega t} \;\approx\; 1-\left(\frac{r}{t}\right)^2 \frac{\left(1-\cos^2 \Omega t\right)}{4}$$

$$\therefore\ x_p \approx \left(r+\frac{r^2}{4L}\right) - r\left(\cos(\Omega t)+\frac{r}{4L}\cos(2\Omega t)\right) \tag{3.5}$$

Note: For r/L = 0.25, the error involved in the above approximation is just 0.05% and thus is negligible for all practical purposes. However, for multi-cylinder engines(high Ω possible because of better balance), the higher harmonics ($\cos^4 \Omega t$ etc.) become significant and are typically included in the dynamic analysis.

Differentiating twice with respect to time yields an approximate expression for the acceleration of the piston as follows,

$$\ddot{x}_p \approx r\Omega^2\left(\cos(\Omega t)+\frac{r}{L}\cos(2\Omega t)\right)$$

(3.6)

When multiplied by the mass of the piston, this gives the inertia force due to the piston.

$$\left(I.F\right)_p \approx m_p r\Omega^2\left(\cos(\Omega t)+\frac{r}{L}\cos(2\Omega t)\right) \tag{3.7}$$

The first term signifies variation at the same frequency as the speed of rotation and hence is known as the PRIMARY force term and the second term is known as the SECONDARY force term. For a typical case, these two terms are pictorially shown in Fig 3.2

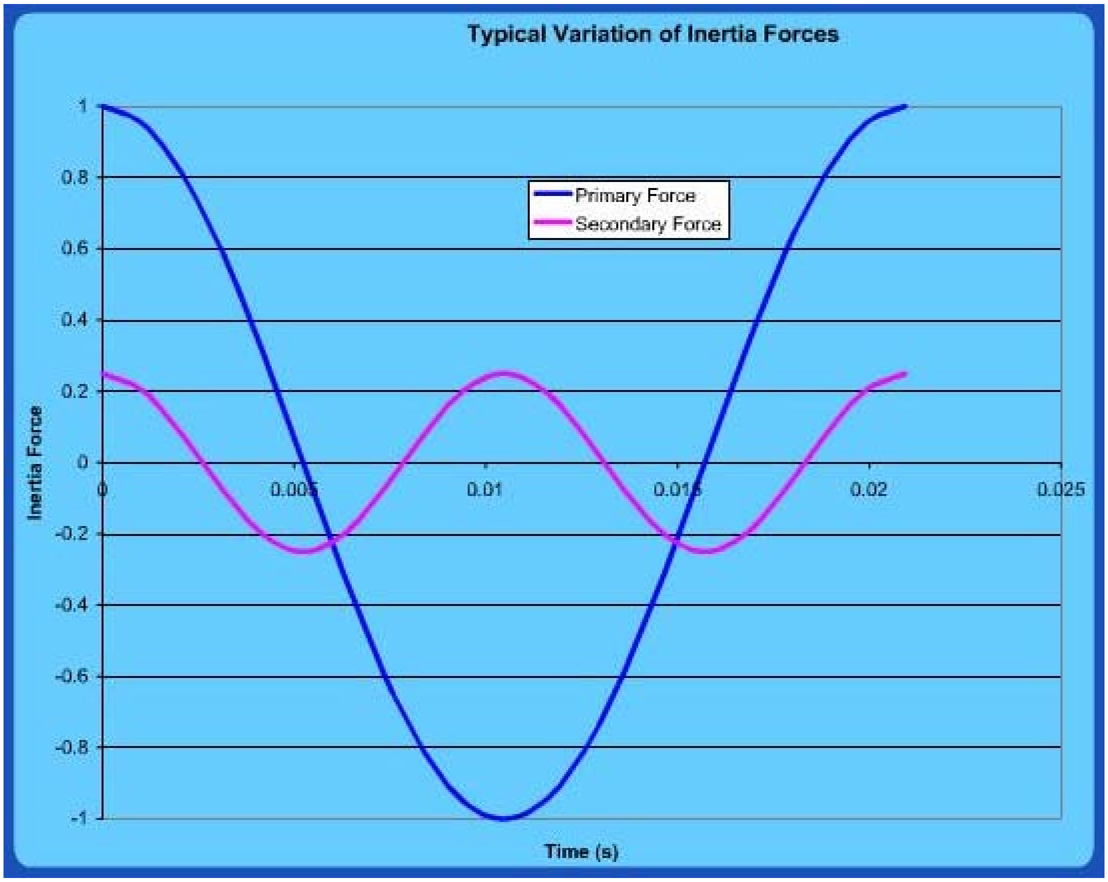

Figure 3.2

CRANK

Crank undergoes pure rotary motion and let us assume that it is rotating at a constant speed Ω. Let G be the center of mass of the crank as shown in Fig 3.3 and so we can write:

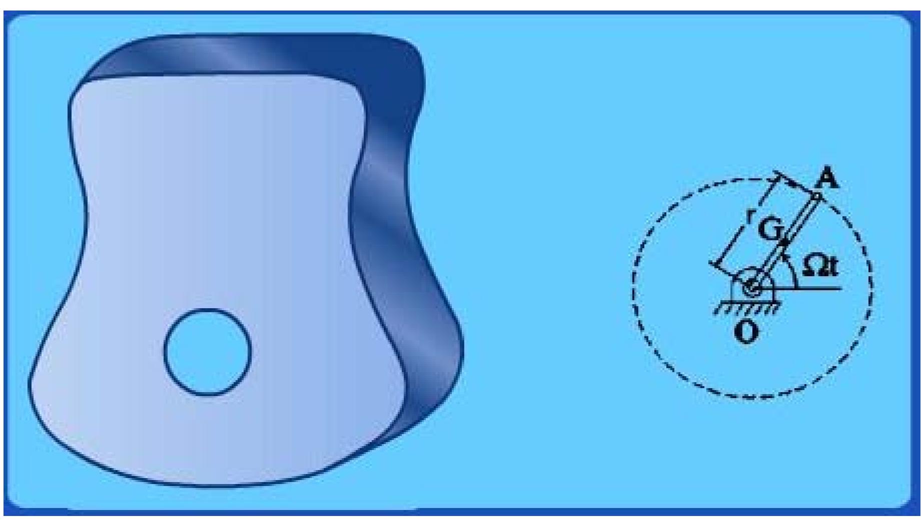

Figure 3. 3 Typical Crank

Centripetal acceleration = (OG) Ω^2 (3.8)

The inertial force (radial) due to crank rotation is given by:

Inertia force = m_c (OG) Ω^2 (3.9)

where m_c is the mass of the crank

This inertia force can be balanced out by the techniques learnt in the previous module and we could make sure that G coincides with O. Then the inertia force would be reduced ideally to zero. If need be, we could always use the above formula to estimate the inertia forces due to crank.

For reasons that will become clear when we discuss the dynamics of connecting rod, it is common to assume that the entire mass of the crank is actually concentrated at the pin A. Thus we can write,

Effective crank mass at A = m_c (OG)/(OA) (3.10)

Comparison of inertia forces due to rotating and reciprocating masses

Now we have seen the inertia forces due to crank which undergoes pure rotation and those due to the piston that undergoes pure translation. We observe that the inertia force due to crank is always of fixed magnitude but ever directed radially i.e., continually changing direction. The inertia force due to a reciprocating mass is always directed along the line of motion but its magnitude is ever changing. This fundamental difference between the two types of inertia forces is captured in Fig. 3.4.

Figure. 3.4

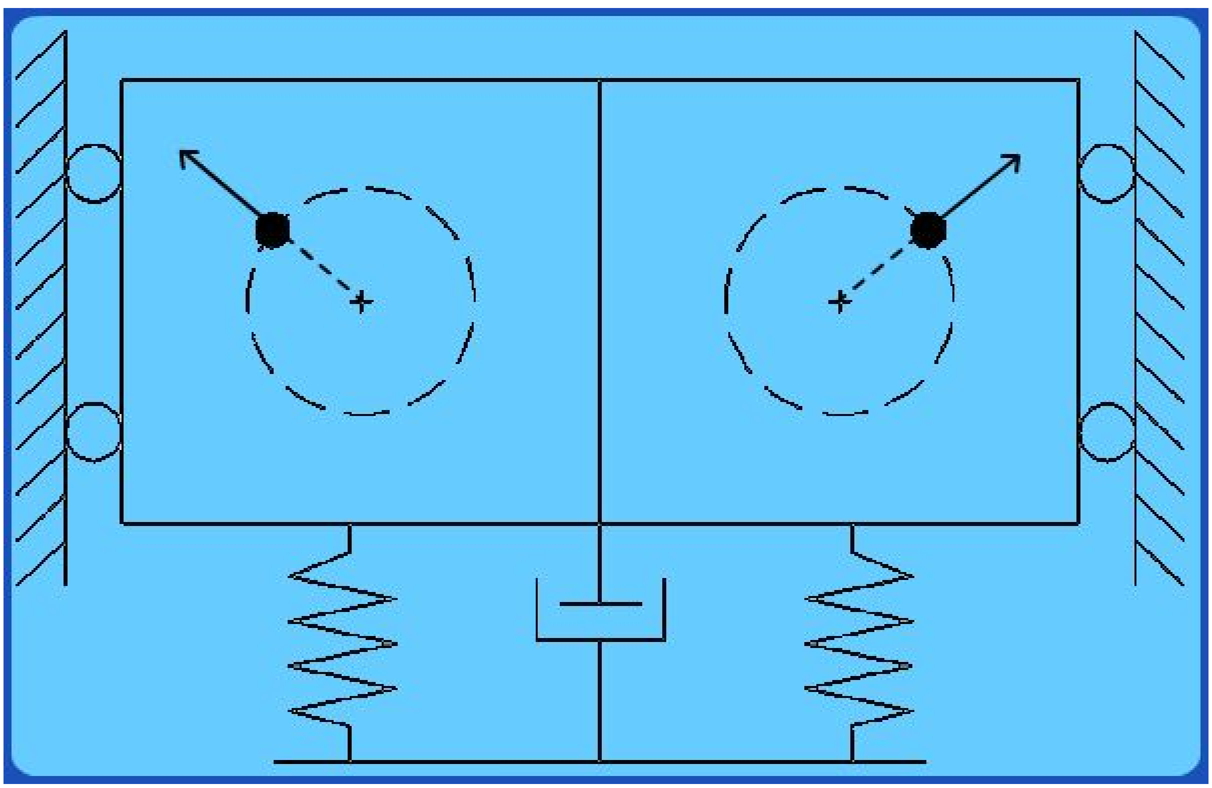

Figure 3.5 Mechanical Shaker

An interesting configuration with two rotating masses (in phase opposition) is shown in Fig. 3.5 which results in a force only along one line/axis. This is effectively used as a mechanical shaker for dynamic testing of structures.

CONNECTING ROD

One end of the connecting rod is circling while the other end is reciprocating and any point in between moves in an ellipse. It is conceivable that we derive a general expression for the acceleration of any point on the connecting rod and hence estimate the inertia forces due to an elemental mass associated with that point. Integration over the whole length of the connecting rod yields the total inertia force due to the entire connecting rod. Instead we try to arrive at a simplified model of the connecting rod by replacing it with a "dynamically equivalent link" as shown in Fig 3.6.

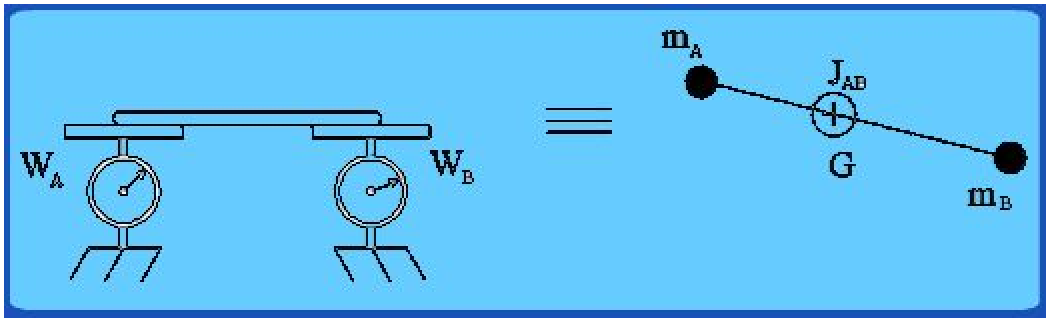

Figure 3.6 Dynamically Equivalent link for a connecting rod .

In order that the two links are dynamically equivalent, it is necessary that:

- Total mass be the same for both the links
- Distribution of the mass be also same i.e., location of CG must be same and the mass moment of inertia also must be same.

Thus we can write three conditions:

$$\begin{aligned} m &= m_A + m_B \\ m_A\,(AG) &= m_B\,(GB) \\ J_G &= m_A\,(AG)^2 + m_B\,(GB)^2 + J_{AB} \end{aligned} \qquad (3.11)$$

For convenience we would like the equivalent link lumped masses to be located at the big and small end of the original connecting rod and if its center of mass (G) location is to remain same as that of original rod, distances AG and GB are fixed. Given the mass m and mass moment of inertia J_G of the original connecting rod, the problem of finding dynamically equivalent link is to determine m_A, m_B and J_{AB}.

An approximate equivalent link can be found by simply ignoring J_{AB} and treating just the two lumped masses m_A and m_B connected by a mass-less link as the equivalent of original connecting rod. In such a case we take:

$m_A = m\,(GB)/L$

$m_B = m\,(AG)/L$ (3.12)

Thus the connecting rod is replaced by two masses at either end (pin joints A and B) of the original rod. m_A rotates along with the crank while m_B purely translates along with the piston. It is for this reason that we proposed use of crank's effective rotating mass located at pin A, which can now be simply added up to part of connecting rod's mass.

On the shop floor m_A, m_B can be immediately determined by mounting the existing connecting rod on two weighing balances located at A and B respectively. The readings of the two balance give m_A and m_B directly

Dynamic Model of a single cylinder IC Engine Mechanism

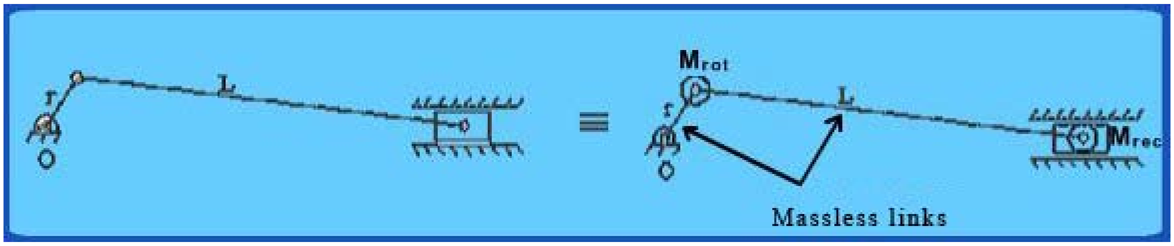

Figure 3.7 Dynamic Model of slider-crank Mechanism

Based on our discussion thus far, we can arrive at a simplified model of the crank-slider mechanism for the purpose of our dynamic analysis as shown in Fig. 3.7. Thus we have either purely rotating masses or purely translating masses and these are given by:

$$M_{rot} = m_c\big|_{at\ A} + m_A$$
$$M_{rec} = m_p + m_B \qquad (3.13)$$

where the first term in the rotating masses is due to the effective crank mass at pin A and the second term is due to the part of equivalent connecting rod mass located at pin A. Similarly the first term in reciprocating masses is due to the mass of the piston and the second is due to the part of equivalent connecting rod mass located at pin B. There are inertia forces due to M_{rot} and M_{rec}.

The inertia forces due to M_{rot} can be nullified by placing appropriate balancing masses as indicated in Fig. 3.9. Thus the effective force transmitted to the frame due to rotating masses can ideally be made zero.

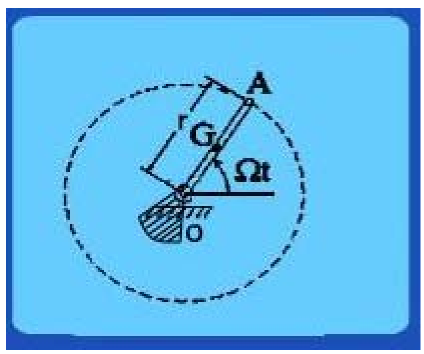

Figure 3.8 Counter balancing rotating masses

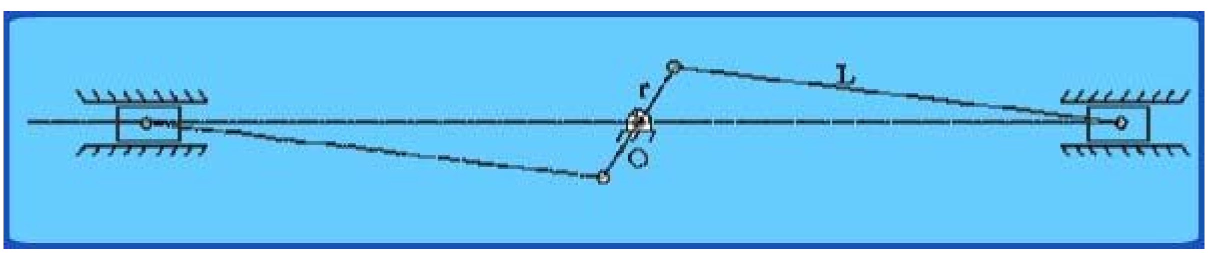

Figure 3.9 Opposed position configuration

However it is not so straight forward to make the unbalanced forces due to reciprocating masses vanish completely. As given in Eq. (3.7) and depicted in Fig. 3.2, there are components of the force which are at the rotational speed and those at twice this speed. It is conceivable to use a configuration as shown in Fig. 3.9 to completely balance out these forces but the mechanism becomes too bulky. Thus a single cylinder engine is inherently unbalanced.

DRIVING TORQUE and INERTIA TORQUE

Gas forces due to the internal combustion of fuel, drive the piston's motion which is transmitted through the connecting rod to the crank. as shown in Fig. 3.10. An expression for the “driving torque” or “gas torque” is given by:

$$T_g = \left(\rho_g A\right) r \, Sin\Omega t\left[1+\frac{r}{L}cos\,\Omega t\right]$$

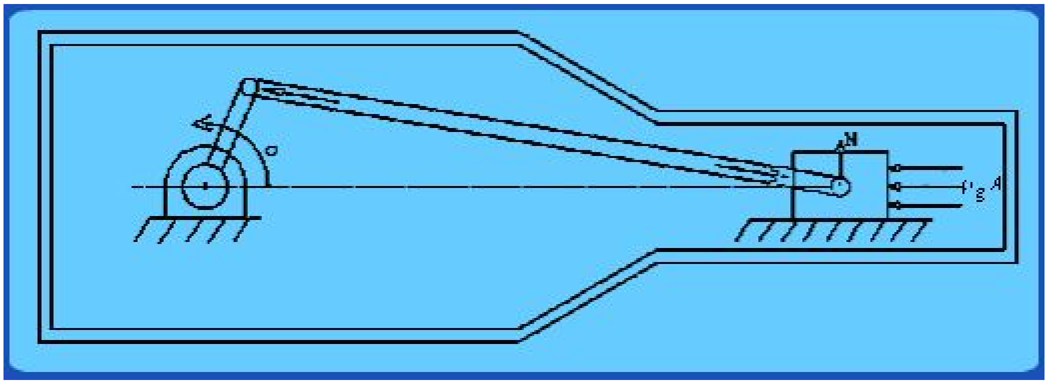

Figure 3.10

For a four-stroke cycle, power is delivered only in one stroke. Thus the gas torque varies with time. When such a source is used as a prime mover, instantaneous speed of the shaft fluctuates from time to time within a cycle of rotation, depending on the “load torque” requirement. If the load torque ideally matches the driving torque at every instant, the speed will be theoretically uniform. Thus in general we will need a device to “iron out” these fluctuations in shaft speed within a cycle. Such a device is known as “flywheel” and we shall discuss it in greater detail in a later module.

By replacing the gas pressure by inertia force due to M_{rec} (i.e., $m_{rec}\,\ddot{x}_p$), we can estimate the “shaking moment” or “inertia torque” as follows:

$$T_{inertia} = \frac{m_{rec}}{2} r^2 \Omega^2 \left[\frac{r}{2L} Sin\Omega t - Sin^2\Omega t - \frac{3r}{2L} Sin^3\Omega t\right]$$

Recap

In this module you have learnt the following

- A single cylinder IC engine mechanism is inherently unbalanced
- Approximate analysis of the dynamics of single cylinder IC engine
- Estimation of the forces and moments felt on the “frame” i.e., cylinder block during the operation of the IC Engine