Module 2 : Dynamics of Rotating Bodies; Unbalance Effects and Balancing of Inertia Forces

Lecture 3 : Concept of unbalance; effect of unbalance

Objectives

In this lecture you will learn the following

- Unbalance in rotating machinery
- Causes and effects of unbalance
- Response of a simple rotor

Consider a shaft mounted in its bearings as shown in Fig. 2.1.1

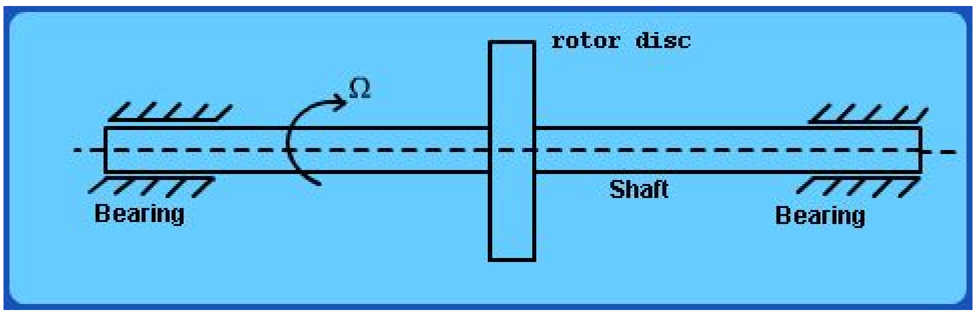

Fig 2.1.1 A simple Rotor

In the idealized situation shown in the figure, the shaft may be assumed mass-less and the rotor is representative of a thin disc with several blades attached around its circumference (such as in a single stage of turbo-machine). The armature of a typical motor can also be represented in this fashion.

It is easy to see that the center of mass of the rotor disc may or may not lie on the geometric axis of the shaft in its bearings. Consider the side-view of the system as shown in Fig. 2.1.2 below. If the center of mass is NOT on the axis of rotation (i.e., the system is "eccentric"), then as the shaft rotates, a centrifugal force will be set-up. The shaft will then be bent away from the line of bearings.

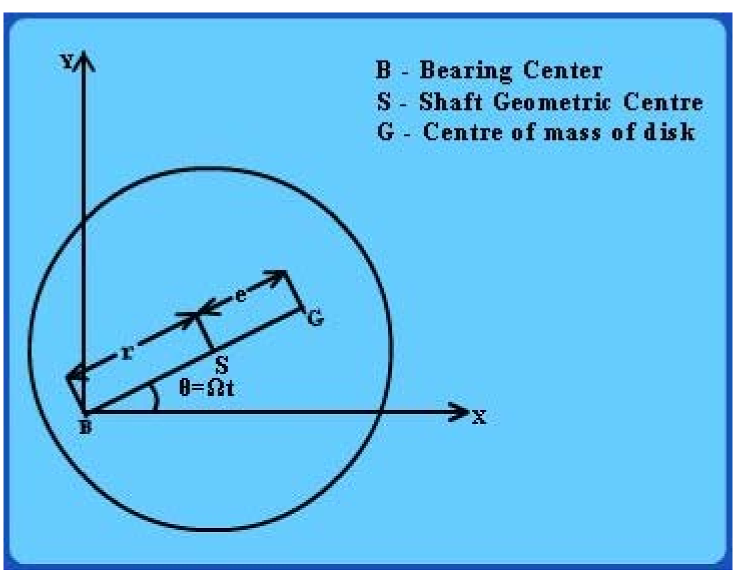

Fig 2.1.2 Side View of The Rotor

Such an unbalance could be normally caused due to manufacturing limitations. For a turbo-machinery bladed disc assembly, there could be as many as 50-100 blades around the disc. Even small fluctuations in the masses of individual blades attached around the periphery of the disk could cause eccentricity of the effective center of mass. Similarly uneven windings in an armature could lead to eccentricity.

For steady state rotation (whirling), from the equilibrium of forces we have,

$$k\,r = m\,\Omega^2\,(r+e) \tag{2.1.1}$$

where,

“k” stands for the equivalent stiffness offered by the shaft to deflection at the location of the disc; If the shaft were simply supported in its bearings at either end and the disc were centrally located, the stiffness of the shaft “k” would be given by $\left(\frac{48EI}{L^3}\right)$.

“r” is the deflection of the shaft's geometric center at the location of the disc;

“m” is the mass of the disc;

“ Ω ” is the speed of rotation;

“e” is the eccentricity i.e., constant distance between the geometric center (S) and center of mass (G).

Thus we have:

$$r = e\,\frac{m\Omega^2}{\left(km\Omega^2\right)} = e\,\frac{\left(\frac{\Omega}{\omega_n}\right)}{1-\left(\frac{\Omega}{\omega_n}\right)^2} \tag{2.1.2}$$

where w_n is the natural frequency of the system given by

$$\omega_n = \sqrt{\frac{k}{m}} \tag{2.1.3}$$

Typical variation of the deflection of the shaft is given in Fig. 2.1.3 below. It is deserved that for very low speeds of operation, (i.e $\Omega \ll w_n$) $r \approx 0$. The unbalanced forces do not cause significant deflection in the shaft. Towards the high speed end(i.e $\Omega \ll w_n$), $r \approx e$. Thus S and G interchange their relative locations and G tends to coincide with the line of bearings.

In between, when $\Omega \ll w_n$, shaft deflection r becomes excessively large

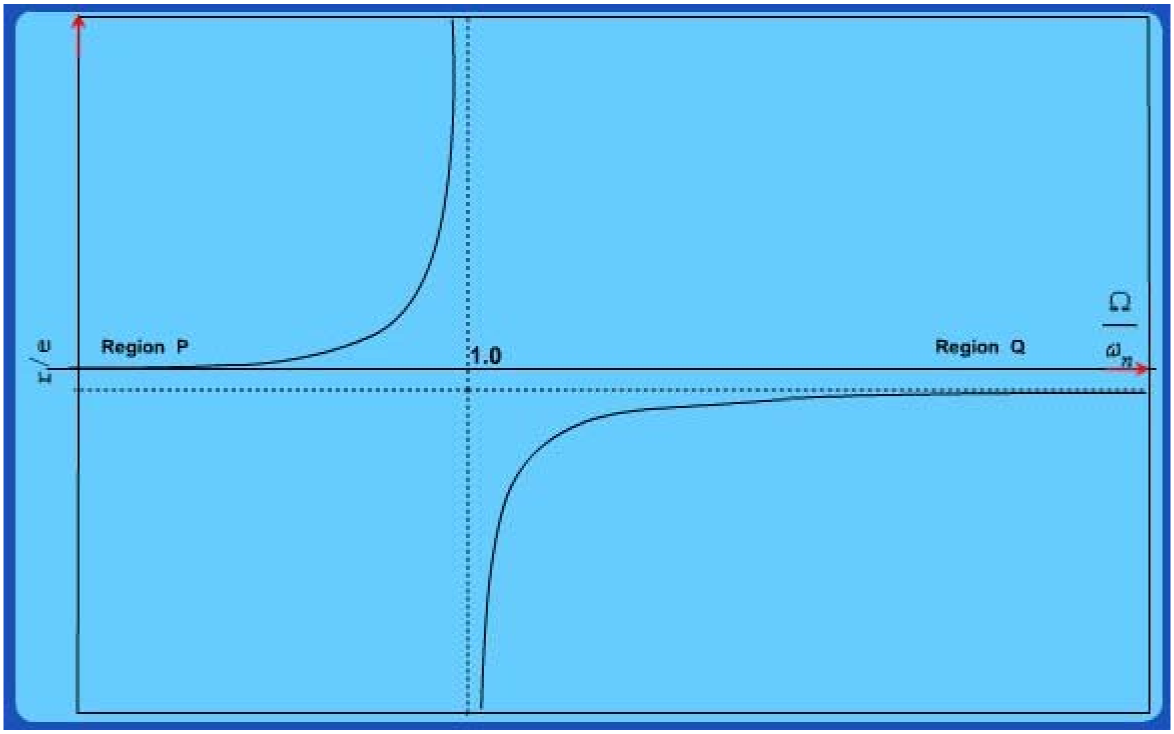

Fig 2.1.3a Variation of the Deflection of the Shaft

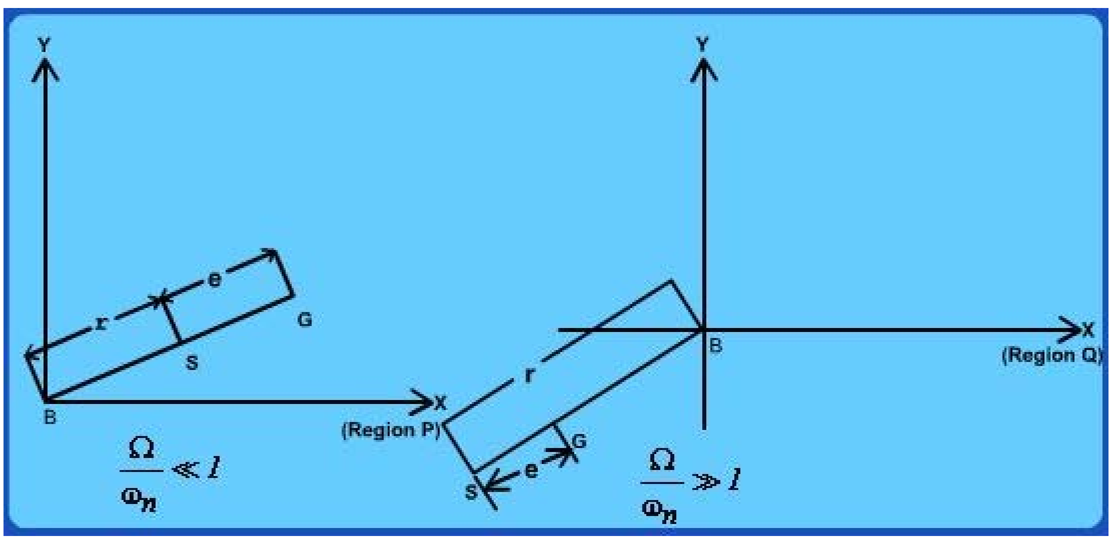

Fig 2.1.3b

From Fig. 2.1.3 it is observed that any eccentricity in the system leads to unbalanced forces and when frequencies match $\left(i.e \frac{\Omega}{w_n} \approx 1\right)$, this could lead to excessive deflections. It is to be observed that because of these forces, the shaft centerline is pulled away from line of bearings and the axis of rotation is thus bent. Please see Fig. 2.1.4. The system continues to rotate with a certain fixed deformed shape, the amount of deflection being given by equation(2.1.2).

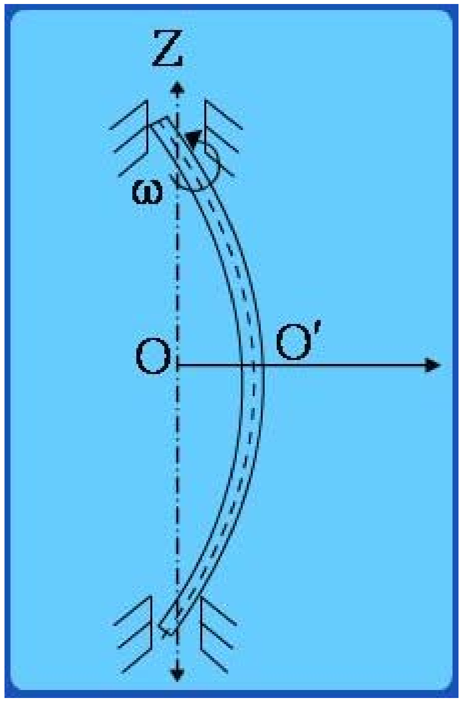

Figure 2.1.4

When the speed of rotation of the shaft about its axis and the speed of rotation of the plane containing the bent shaft about the line of bearings is the same, it is known as "synchronous whirl". In a synchronous whirl, Fig. 2.1.5 shows the side-view of the deformed rotor-shaft system at four different positions in one rotation. Thus the "heavier" fibers (indicated in the figure by the shaded portion) always remain outermost and thus in tension and the inner fibers are always in compression. Thus there is NO cyclic variation of stress from tension-compression for any given fiber. Thus synchronous whirl could lead to excessive amplitudes and possible failure but NOT to fatigue due to alternating stresses.

Fig 2.1.5 Position of Rotor-Disk at 0°, 90°, 180°, 270° in one rotation

In order to reduce the amplitudes of deflection, one could try and reduce the eccentricity or the unbalanced forces (known as the problem of "balancing") or design the system such that the frequencies w_n and Ω do not match. Our present discussion will be focusing on the techniques of balancing out the unbalanced forces.

The "balance grade" suitable for an application is typically prescribed in Indian or International Standards (for example ISO-1940). The balancing grade is usually specified as "G n", where "n" is a number representing the permissible peripheral velocity in mm/s. Thus G100 implies, for a rotational speed of 1000 RPM, a permissible eccentricity of the center of mass from the axis of rotation within 0.1mm. This can be readily obtained from the following formula:

(Eccentricity permitted “e” mm)(speed of rotation Ω rad/s) < 100 mm/s (in general "n" in Gn)

Typical balancing grades for a few applications are listed in the Table below to give you an idea of the task of balancing.

Balance Quality Grade “G”	Rotor Type (application)
G 4000	Crankshaft-drives of rigidly mounted slow marine diesel engines
G 100 - 250	Crankshaft drives of rigidly mounted fast diesel or gasoline engines for cars, trucks, locomotives
G 16	Rotating parts of agricultural or crushing machinery
G 2.5	Gas and steam turbine rotors
G 0.4	Spindles of precision grinders and gyroscopes

Recap

In this lecture you have learnt the following

- Concept of unbalance in rotating machinery
- Causes and effects of unbalance
- Response of a simple rotor to unbalance
- Typical balancing grades as per standards