Module 10 : Vibration of Two and Multidegree of freedom systems; Concept of Normal Mode; Free Vibration Problems and Determination of Ntural Frequencies; Forced Vibration Analysis; Vibration Absorbers; Approximate Methods - Dunkerley's Method and Holzer Method

Lecture 28 : Forced Vibration of two d.o.f. systems

Objectives

In this lecture you will learn the following

- Dynamic response of two degree of freedom system to sinusoidal exciation.
- Multiple Resonances.
- Concept of Vibration Absorber.

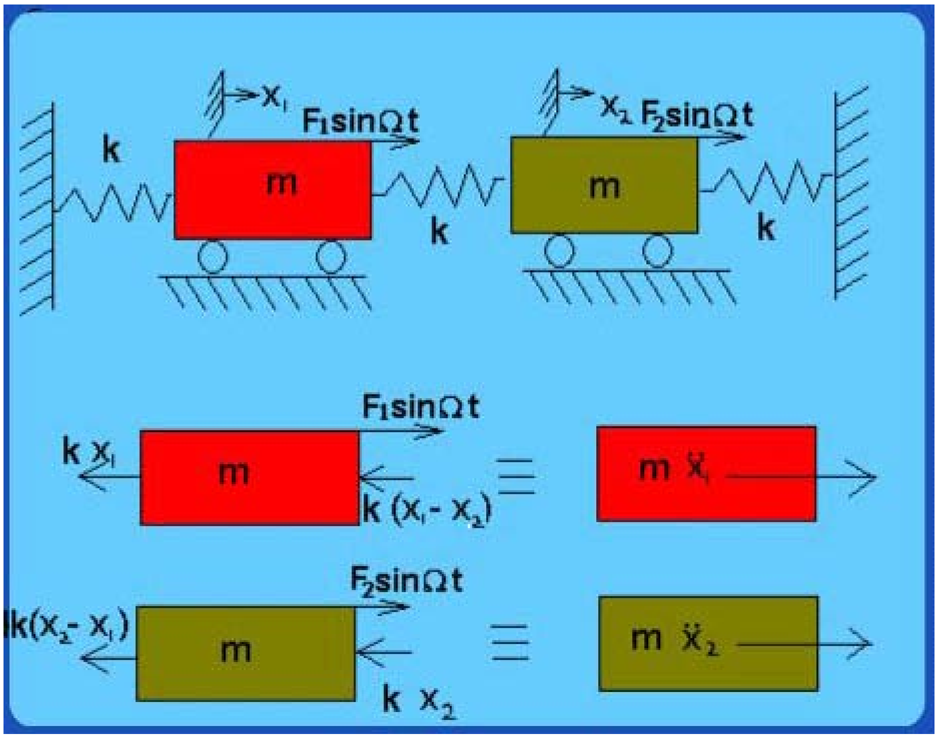

Figure 10.2.1

Consider the undamped two d.o.f system as we discussed in the previous lecture, shown in Fig. 10.2.1. We now study its response to harmonic excitation. Accordingly we have shown two sinusoidal forces acting on the masses. The free body diagrams are shown in the figure and the equations of motion can be readily written down based on Newton 's second Law as follows:

$$\begin{aligned} m\ddot{x}_1 + kx_1 + k(x_1 - x_2) &= F_1 \sin \Omega t \\ m\ddot{x}_2 + k(x_2 - x_1) + kx_2 &= F_1 \sin \Omega t \end{aligned} \qquad 10.2.1$$

Re-writing in matrix notation, we get,

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2k & -k_2 \\ -k & 2k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \sin \Omega t \qquad 10.2.2$$

Under sinusoidal excitation, the response is also sinusoidal at the same frequency as the excitation (as we discussed for single d.o.f case) and so we assume the vibratory movements of the two masses to be as follows:

$$x_1 = X_1 \sin \Omega t$$
$$x_2 = x_2 \sin \Omega t \qquad 10.2.3$$

Substituting in eqn 10.2.2 , we get,

$$\begin{bmatrix} 2k - m\Omega^2 & -k \\ -k & 2k - m\Omega^2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad 10.2.4$$

The solution is readily obtained as follows:

$$x = X_0 \sin(\Omega t - \phi) \qquad 10.2.5$$

Typical response of a two d.o.f system to a sinusoidal excitation is shown in Fig.10.2.2

Fig 10.2.2 Typical Forced vibration response of a two d.o.f. system

As can be seen from eqn 10.2.5 as well as from Fig. 10.2.2 , we notice two resonance peaks i.e. when the forcing (i.e., driving or excitation) frequency matches with either of the natural frequencies, the amplitude of vibration of both the masses shoots upto infinit large values. Thus a two d.o.f. system exhibits two resonant frequencies and our design should ensure that the operating frequency is not near either of the resonant frequencies.

Undamped Vibration Absorber

An interesting practical application situation of a two d.o.f system is when one spring-mass sub-system is designed and used so as to absorb (i.e. suppress) the vibration of the main spring-mass system. Such a system is known as “Vibration Absorber” and we will now discuss this interesting example.

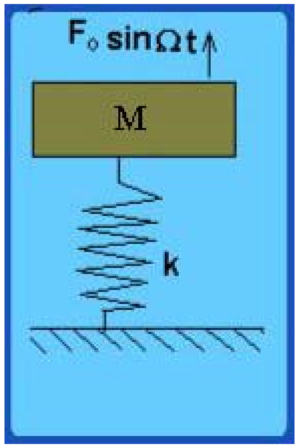

Fig 10.2.3 Undamped Main Spring-mass System

Consider a main spring-mass (K, M respectively) single d.o.f system (such as a machine on its foundation etc) as shown in Fig. 10.2.3 subjected to a forcing function F_0 Sin Ω . We have already studied such a system extensively and we know that it exhibits large amplitude vibrations near its resonance . We now attach another spring-mass system (k_a and m_a respectively) as shown in Fig. 10.2.4. The primary issue of interest is the following:

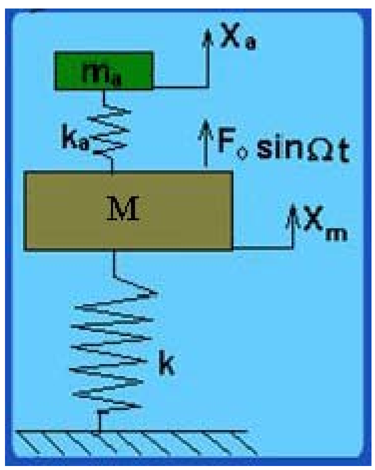

Fig 10.2.4 Main system with an undamped vibration absorber

Can we choose appropriate values for k_a and m_a such that the amplitude of vibration of the main mass, under this excitation, can be made zero?

The equation of motion is given by:

$$\begin{bmatrix} k+k_a & -k_a \\ -k_a & k_a \end{bmatrix} \begin{Bmatrix} x_M \\ x_a \end{Bmatrix} + \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \ddot{x}_M \\ \ddot{x}_a \end{Bmatrix} = \begin{Bmatrix} F_0 \sin \Omega t \\ 0 \end{Bmatrix} \quad 10.2.6$$

$$\text{Let} \quad x_M = X_M \sin \Omega t$$
$$x_a = X_a \sin \Omega t \quad 10.2.7$$

We get the vibratory response of the two masses to be:

$$\begin{Bmatrix} X_M \\ X_a \end{Bmatrix} = \frac{1}{[(k+k_a-\Omega^2 M)(k_a-\Omega^2 m_a)-k_a^2]} \begin{bmatrix} k_a-\Omega^2 m_a & k_a \\ k_a & k+k_a-\Omega^2 M \end{bmatrix} \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix} \qquad 10.2.9$$

In order to make $X_M = 0$ for all time, we have,

$$k_a - \Omega^2 m_a = 0$$

$$ie \quad \sqrt{\frac{k_a}{m_a}} = \Omega$$

Thus k_a and m_a are to be chosen such that the natural frequency of the absorber system is same as the excitation frequency i.e.

$$\omega_a = \sqrt{\frac{k_a}{m_a}} = \Omega$$

It is important to realize that the absorber system will work **ONLY WHEN** $\sqrt{\frac{k_a}{m_2}} = \Omega$

Since k_a and m_a are fixed values for a given absorber, it is therefore imperative that the excitation frequency does not change. So this type of absorber can work only for one frequency. Since the amplitude of vibration for the parent system is likely to be large only within the vicinity of its own natural frequency, we are usually interested in using the absorber when $\omega_n \approx \Omega$. This should not lead to the misconception that this is the governing equation for an absorber design. It must be clearly be remembered that, for the absorber to work, the only condition is that the absirber natual frequency must be equal to the excitation frequency.
If we have put an absorber and the excitation frequency changes even slightly, we could be in real big trouble as evident from the response of the system given in Fig. 10.2.5 below. Instead of the earlier one resonance peak we now have two frequencies (since it has now become a two d.o.f system) when the amplitude will be very large. Therefore it is imperative to provide sufficient amount of damping to limit the peak amplitudes.

Fig 10.2.5 Forced Vibration response of a system with vibration absorber

Recap

In this lecture you have learnt the following

- Developement of equation of motion for forced excitation of two degree of freedom system.
- Solution for the dynamic response.
- Criterion for the design of vibration absorber. $\omega_a = \sqrt{\frac{k_a}{m_a}} = \Omega$

Congratulations, you have finished Lecture 2. To view the next lecture select it from the left hand side menu of the page