Module 7 : Free Undamped Vibration of Single Degree of Freedom Systems; Determination of Natural Frequency ; Equivalent Inertia and Stiffness; Energy Method; Phase Plane Representation.

Lecture 12 : Determination of Natural Frequency

Objectives

In this lecture you will learn the following

- Solution of equqtion of motion for undamped-single degree of freedom system.
- Concept of natural frequency.
- Translational and torsional spring mass systems.
- The Equation of motion found earlier for the spring mass system is reproduced below.

$$\ddot{x} = -\frac{k}{m}x \qquad 7.2.1$$

Equation (7.2.1) is called the equation of motion.

It represents a motion wherein the acceleration (second derivative) is proportional to the negative of displacement. This is characteristic of a simple harmonic motion and the most general solution to this equation can be represented as follows:

$$x = A\sin(\sqrt{\frac{k}{m}}t) + B\cos(\sqrt{\frac{k}{m}}t) \qquad 7.2.2$$

wherein the coefficients A and B can be found using the initial conditions – it is a second order system and hence needs two initial conditions viz., initial position and velocity .Let these be [?] and [?] respectively. Thus we have:

$$x(t) = \frac{\dot{x}_0}{\sqrt{\frac{k}{m}}}\sin\left(\sqrt{\frac{k}{m}}t\right) + x_0\cos\left(\sqrt{\frac{k}{m}}t\right) \qquad 7.2.3$$

The above equation forms the complete soultion to the equation of motion of a simple spring mass system. For the initial conditions as follows:

$$x = X_0 \quad \text{at t=0}$$

$$\dot{x} = 0 \quad \text{at t=0}$$

we get the solution as:

$$x(t) = x_0\cos\left(\sqrt{\frac{k}{m}}t\right) \qquad 7.2.4$$

This is a non-decaying sinusoidal vibration of frequency $\sqrt{k/m}$, so that,

$$\omega_n = \sqrt{\frac{k}{m}} \qquad 7.2.5$$

This frequency is referred to as the natural frequency of the system since it represents the natural motion of the system viz., the way the system vibrates when given an initial disturbance [?] and left free to vibrate on its own.