Module 9 : Forced Vibration with Harmonic Excitation; Undamped Systems and resonance; Viscously Damped Systems; Frequency Response Characteristics and Phase Lag; Systems with Base Excitation; Transmissibility and Vibration Isolation; Whirling of Shafts and Critical Speed.

Lecture 23 : Response of base excitation systems

Objectives

In this lecture you will learn the following

- Examples of system with base excitation
- Equation of motion of such systems
- Nature of solution

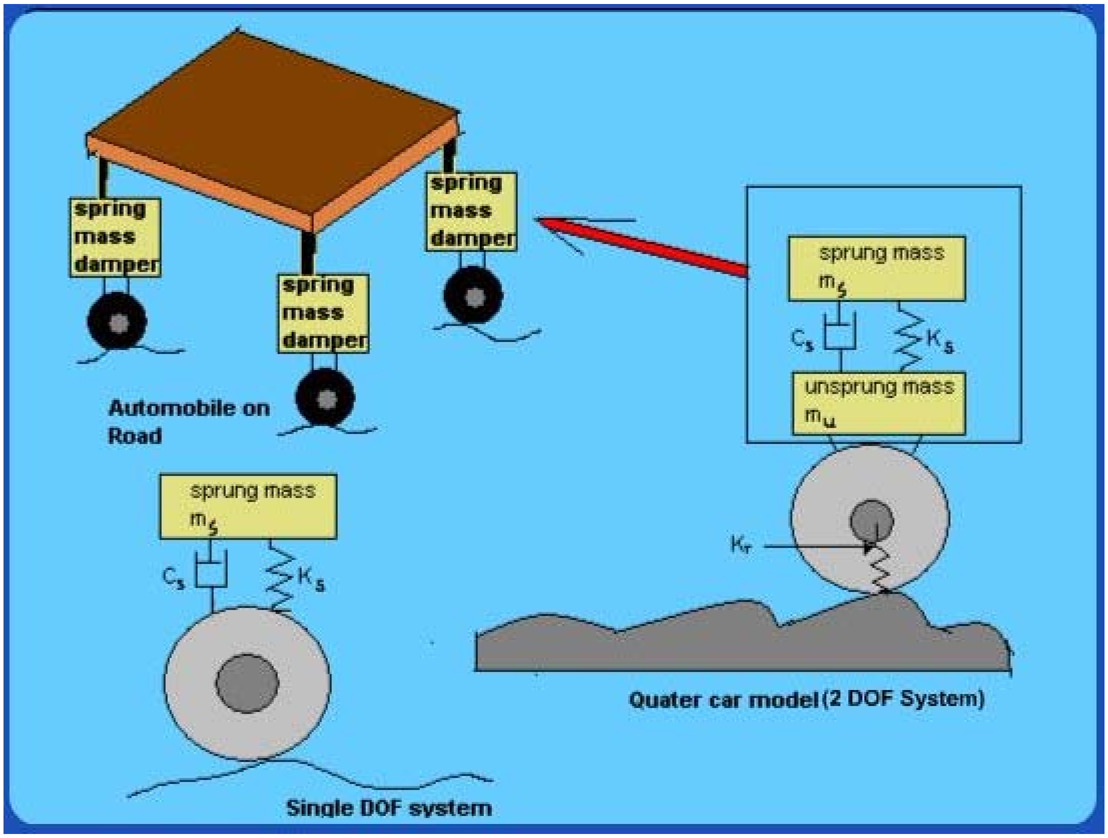

Fig 11.3.1Quater car model

In the previous lecture we discussed the response of the single d.o.f system to a sinusoidal force when the force is acting on the mass, such as in the case of an unbalanced machine operating on its springy foundation. Another important practical situation arises for example in the case of an automobile on the road as shown in Fig. 11.3.1. Typical tire stiffness is in the range of 200 N/mm while that of the suspension is of the order of 20 N/mm. Thus assuming the tire to be practically rigid, we can come up with a simplified single d.o.f model of one-quarter of the car as indicated in the figure. In this case, the excitation of interest for the present discussion is the road undulation. Excitations of course arise from the operation of the engine and hence proper design of the engine mounts becomes important but this will be taken up later. The road undulation is represented as a ground motion disturbance input given to the bottom end of the spring. Let us now investigate the response of such “base-excitation” systems.

Fig 9.3.2

The physical system under consideration and the corresponding free body diagram are given in Fig. 9.3.2. The governing equation of motion can be readily obtained as:

$$\begin{aligned} m\ddot{x} &= -k\left(x - xg\right) \\ &= -c\left(\dot{x} - \dot{x}g\right) \end{aligned} \qquad (9.3.1)$$

$$\therefore m\ddot{x} = +c\dot{x} + kx = kx_g + c\dot{x}_g \qquad (9.3.2)$$

We could choose two sets of coordinates in this problem viz., the absolute motion of the ground and the mass (or) the motion of the mass relative to the ground. Both have practical utility in different contexts and we shall derive the necessary expressions for both at this stage. For example, if we are interested in the ride comfort of a passenger seated in a car, we will be worried about the absolute acceleration felt by the passenger due to road unevenness. On the other hand if we mounted any measuring instrument on a shaking ground, it can measure only the relative motion of the mass with respect to the ground. If we introduce z = x – y as the relative motion coordinate, the governing equation in terms of z can be given as:

$$m\ddot{z} + c\ddot{z} + kx_z = -m\ddot{x}_g \qquad (9.3.3)$$

Assuming that the free vibration transients would have died down due to the damping present in the system, the steady state absolute response of the mass can be shown to be:

$$\text{Given } x_g = X_g \sin \Omega t \qquad (9.3.4)$$

$$\text{Assume } x = X_0 \sin(\Omega t - \phi) \qquad (9.3.5)$$

$$X_0 = X_g \sqrt{\frac{k^2 + (c\Omega)^2}{(k - (m\Omega)^2)^2 + (c\Omega)^2}} \qquad (9.3.6)$$

Variation of the response with forcing frequency is plotted in Fig. 9.3.3 and can be observed to be very similar to that harmonic force excitation acting directly on the mass itself. If we wish the motion of the mass to be less than that of the ground, then the system natural frequency has to be designed in such a manner that

Fig 9.3.3

$$\eta = \frac{\Omega}{\omega_n} \qquad (9.3.7)$$

This is a very important observation and has several practical implications as indicated below:

Practical Implication

When the suspension of a car is designed, it has to be designed such that the natural frequency of the car is less than about ? or ¼ the road disturbance frequency, which itself depends on the waviness profile of the road surface as well as the forward speed of the vehicle. Typical values for suspension stiffness are in the range of 20 N/mm for passenger cars such that the natural frequencies for vertical oscillation are about 1 Hz. Thus we need to design a "soft" suspension for better ride comfort. Of course there are other constraints (such as vehicle handling) that determine the optimal design of the suspension.

Similarly we can obtain the steady state RELATIVE motion from eqn (9.3.3) as:

$$z = Z_0 \sin(\Omega t - \phi) \qquad (9.3.9)$$

where

$$Z_0 = \frac{m\Omega^2 x_9}{\sqrt{(k - m\Omega^2)^2 + (c\Omega)^2}} \qquad (9.3.10)$$

$$\phi = \tan^{-1}\left(\frac{c\Omega}{k - m\Omega^2}\right) \qquad (9.3.9)$$

Variation of z (i.e. motion of the mass relative to the base) with respect to the forcing frequency is plotted in Fig. 9.3.4. It is observed that the asymptotes of the response have interchanged when we compare the response to Fig. 9.3.3 i.e., at lower frequencies the relative motion of the mass tends to zero while at high frequencies the magnification factor tends to unity. We will return to this plot at a later stage when we discuss vibration measuring instruments etc.

Fig 9.3.4

Recap

In this lecture you have learnt the following

- Concept of base excitations and their governing equations
- Conept of relative dispacement plot and variation of the relative displacement with base excitations
- Concept of absolute displacement and relevance to