Module 10 : Vibration of Two and Multidegree of freedom systems; Concept of Normal Mode; Free Vibration Problems and Determination of Ntural Frequencies; Forced Vibration Analysis; Vibration Absorbers; Approximate Methods - Dunkerley's Method and Holzer Method

Lecture 31 : Approximate Methods (Holzer's Method)

Objectives

In this lecture you will learn the following

- Holzer's method of finding natural frequency of a multi-degree of freedom system

Holzer's Method

This method is an iterative method and can be used to determine any number of frequencies for a multi-d.o.f system. Consider a typical multi-rotor system as shown in Fig. 12.5.1.

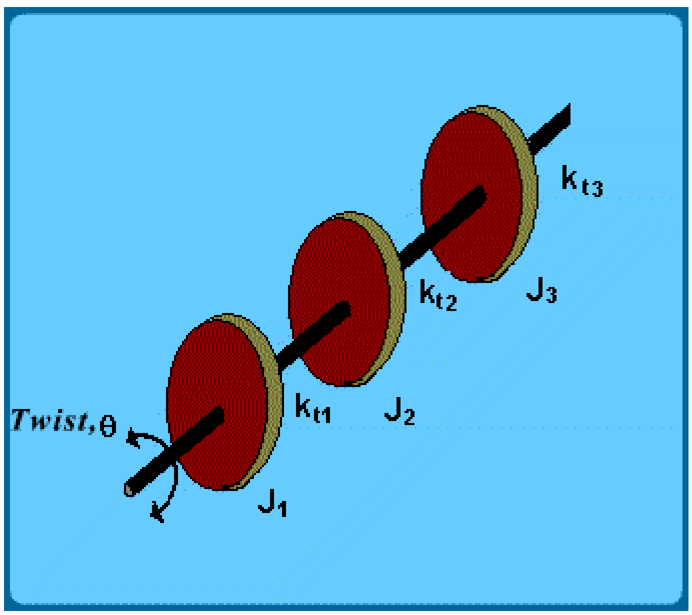

Fig 12.5.1 Typical multi-rotor system

The equations of motion for free vibration can be readily written as follows:

$$
\begin{aligned}
&J_1\ddot{\theta}_1 + k_{t_1}(\theta_1 - \theta_2) = 0\\
&J_2\ddot{\theta}_2 + k_{t_1}(\theta_2 - \theta_1) + k_{t_2}(\theta_2 - \theta_3) = 0\\
&J_3\ddot{\theta}_3 + k_{t_2}(\theta_3 - \theta_2) + k_{t_3}(\theta_3 - \theta_4) = 0
\end{aligned}
\qquad 12.5.1
$$

For harmonic vibration, we assume

$$\theta_i = \Theta_i \sin \omega t \qquad 12.5.2$$

Thus:

$$
\begin{aligned}
&-\omega^2 J_1 \Theta_1 + k_{t_1}(\Theta_1 - \Theta_2) = 0\\
&-\omega^2 J_2 \Theta_2 + k_{t_1}(\Theta_2 - \Theta_1) + k_{t_2}(\Theta_2 - \Theta_3) = 0\\
&-\omega^2 J_3 \Theta_3 + k_{t_2}(\Theta_3 - \Theta_2) + k_{t_3}(\Theta_3 - \Theta_4) = 0
\end{aligned}
\qquad 12.5.3
$$

Summing up all the equations of motion, we get:

12.5.4

$$\sum_{i=1}^{n} J_i \, \Theta_i \omega^2 = 0$$

This is a condition to be satisfied by the natural frequency of the freely vibrating system.

Holzer's method consists of the following iterative steps:

Step 1: Assume a trial frequency $\omega_n \approx \omega_{try}$

Step 2: Assume the first generalized coordinate $\Theta_1 = 1$ say

Step 3: Compute the other d.o.f. using the equations of motion as follows:

$$\Theta_2 = \Theta_1 - \frac{\omega_{try}^2 J_1 \Theta_1}{k_{t_1}} \qquad \textbf{10.5.5}$$

$$\Theta_3 = \Theta_2 - \frac{\omega_{try}^2 (J_1 \Theta_1 + J_2 \Theta_2)}{k_{t_2}}$$

Step 4: Sum up and verify if Eq. (10.5.4) is satisfied to the prescribed degree of accuracy.

If Yes, the trial frequency is a natural frequency of the system. If not, redo the steps with a different trial frequency.

In order to reduce the computations, therefore one needs to start with a good trial frequency and have a good method of choosing the next trial frequency to converge fast.

Two trial frequencies are found by trial and error such that $\sum J_i \Theta_i \omega_{try}^2$ is a small positive and negative number respectively than the mean of these two trial frequencies(i.e. bisection method) will give a good estimate of for which $\sum J_i \Theta_i \omega_{try}^2 \approx 0$.

Holzer's method can be readily programmed for computer based calculations

Recap

In this lecture you have learnt the following

- Holzer method of determining natural frequencies based on $\sum_{i=1}^{n} J_i \, \Theta_i \omega^2 = 0$