Module 9 : Forced Vibration with Harmonic Excitation; Undamped Systems and resonance; Viscously Damped

Systems; Frequency Response Characteristics and Phase Lag; Systems with Base Excitation;

Transmissibility and Vibration Isolation; Whirling of Shafts and Critical Speed.

Lecture 22 : Phase and Phasor representation

Objectives

In this lecture you will learn the following

- Phase between excitation and response.
- Phasor analysis of resonance behaviour.

So far we focussed our attention on the amplitude of vibration and let us now look at the phase as given below:

$$\tan\phi = \frac{2\xi\eta}{1-\eta^2} \qquad (9.2.1)$$

Fig 9.2.1 Variation of Phase angle with frequency ratio

The variation of the phase with the frequency of excitation is shown in Fig. 9.2.1

For an undamped system, as seen in the figure,

Phase = 0 degree for $\Omega < \omega_n$

Phase = 90 degree for $\Omega = \omega_n$

Phase = 180 degree for $\Omega > \omega_n$

even if there is damping in the system, the phase will be 90 degree for $\Omega = \omega_n$. This condition is therefore often used to identify the occurrence of resonance in an experiment.

Let us now discuss and understand the physical meaning of phase between the excitation and the response. The excitation and response of undamped system have zero phase difference for all values of forcing frequency less than the natural frequency of the system i.e., response is said to be in phase with the excitation. When the forcing frequency is greater than the natural frequency of the system, the phase difference becomes exactly 180 degree i.e., the response is in phase-opposition to the excitation. In a phasor diagram, the excitation and response can be plotted as shown in Fig. 9.2.2. In an undamped system as we are discussing here, there are essentially three forces viz., externally applied excitation force, force representing spring resistance, inertia force. These three force vectors are also plotted in this figure for all the cases. Of particular interest is the case of resonance. At resonance($\Omega = \omega_n$), if $x = X_0$ (Sin ω_n t) then the acceleration =- ω_n^2 Sin ω_n t. Hence the spring resistance force and the inertia force magnitudes are given by:

$$\text{Spring force} = kx = kX_0 \sin \omega_n t \qquad (9.2.2)$$

$$\text{Interia force} = m\ddot{x} = -m\omega_n^2 X_0 \sin \omega_n t \qquad (9.2.3)$$

Fig 9.2.2 Phasor representation of forces in an undamped system

Since the natural frequency of the system is given by ω_n^2 = k/m, we see that the spring and inertia forces are exactly equal and opposite as indicated in the figure. In view of the 90 degree phase difference between the excitation and response, the excitation goes unbalanced and keeps pumping energy into the system in each cycle thus leading to a build-up of the amplitudes .Fig. 9.2.1 shows the variation of the phase with respect to the forcing frequency. Due to the presence of viscous damping, the response always lags the excitation. Irrespective of the value of damping, the phase is always 90 degree when $\Omega = \omega_n$ i.e., $\eta = 1$. This is in fact used to identify the occurrence of resonance condition in an experiment. Fig. 9.2.3 (similar to Fig. 9.2.2) shows the phasor representation of spring resistance force, viscous damping force, inertia force and the externally applied excitation force for the cases when $\eta < 1$, $\eta = 1$ and $\eta > 1$. It is observed that at resonance the spring force and inertia force match each other exactly (like in the case of undamped system) while the damping force balances the external excitation force. This is another way of interpreting how the damping limits the vibratory response at resonance, while the vibration amplitudes build-up to infinitely large values for undamped systems.

In the case of a damped system, we observe that the viscous damping force (being proportional to velocity and hence 90 degree phase to the displacement response) balances the external disturbance force and limits the displacement to a finite value.

A pertinent question to ask at this stage is as follows – what happens if the system is excited at resonance, will the amplitude immediately shoot up to infinity and the system fails? Fortunately the answer is no – in all practical systems, there is damping present (however small) and hence the response will not immediately shoot up to infinity. Secondly, it can be readily shown that at resonance the amplitude of vibration keeps building up with time approximately in a linear fashion i.e., it surely increases with time but it does take finite time to build up to a dangerously huge value. So what does this mean – it implies that when operating a machine, if a resonance condition needs to be crossed, we should “rush through” the critical speed without letting the system build up enough vibration. This is followed in many practical system e.g., steam turbines where the operating rotational speeds are beyond the fundamental critical speed of the shaft.

Fig 9.2.3 Phasor representation of forces in a viscously damped system

Practical Implication

Spring and Mass (Inertia) control the value of the natural frequency but have no role to play in deciding the amplitude of vibration near resonance. Design of the damper critically affects the dynamics of the system near resonance and could potentially determine whether or not the system fails.

As the mass is subjected to the excitation force and undergoes vibratory motion, some of the force is transmitted to the ground through the spring and the damper. This force can be represented in a graphical manner as shown in Fig. 11.2.4 and the magnitude can be readily computed as follows:

Fig 11.2.4 Force transmitted to foundation

$$F_T = \sqrt{(kX_0)^2 + (c\Omega X_0)^2} \qquad (11.2.4)$$

While mounting many machines, we wish to ensure that F_T is as small as possible.

Recap

In this lecture you have learnt the following

- Relation between phase angle and frequency ratio $\eta = \dfrac{\Omega}{\omega_n}$
- Phasor representation of vibration systems in damping.
- Role of damping in supressing the vibrations.
- Concept of force transmissibility.

Congratulations, you have finished Lecture 2 To view the next lecture select it from the left hand side menu of the page.