

Module 7 : Free Undamped Vibration of Single Degree of Freedom Systems; Determination of Natural

Frequency ; Equivalent Inertia and Stiffness; Energy Method; Phase Plane Representation.

Lecture 15 : Representation of Vibratory Behaviour

Objectives

In this lecture you will learn the following

Representation of Vibratory motion

- Phase plane plots

Phase Plane Method

The vibratory motion of a spring mass system with initial conditions x_0 and \dot{x}_0 has been obtained earlier give reference to that section and is reproduced here

$$x(t) = \frac{\dot{x}_0}{\sqrt{\frac{k}{m}}} \sin\left(\sqrt{\frac{k}{m}}t\right) + x_0 \cos\left(\sqrt{\frac{k}{m}}t\right) \quad 7.5.1$$

We depict the vibratory motion in the form of a chart showing displacement, [x vs time, t](#). While this is one common way of plotting the vibration response, we will now discuss another very useful method of depicting the response viz., the phase-plane plot.

Eq. 7.5.1 may be re-written as

$$x = A \sin(\omega_n t + \phi) \quad 7.5.2$$

$$\text{where } A = \sqrt{x_0^2 + \frac{\dot{x}_0^2}{\omega_n^2}}$$

and

$$\phi = \tan^{-1}\left(\frac{\omega_n x_0}{\dot{x}_0}\right)$$

Differentiating the equation for displacement, we have

$$\dot{x} = A \omega_n \cos(\omega_n t + \phi) \quad 7.5.3$$

or

$$\frac{\dot{x}}{\omega_n} = A \cos(\omega_n t + \phi) \quad 7.5.4$$

Squaring and adding 7.5.3 and 7.5.4 , we have

$$x^2 + \left(\frac{\dot{x}}{\omega_n}\right)^2 = A^2 \quad 7.5.5$$

- Eqn 7.5.5 represents a circle with coordinate axes x and \dot{x}/ω_n .
- Radius of the circle is the amplitude of oscillations and centre is at the origin. This is shown in Fig. 7.5.1 below.

Fig 7.5.1 Phase-Plane Plot

Time is implicit in this plot and from this diagram, displacement and velocity of motion are available from single point which corresponds to a particular time instant. This is called the phase-plane plot. The horizontal projection of the phase trajectory on a time base gives the displacement-time plot of the motion and similarly the vertical projection on time base gives velocity-time plot of the motion. Let us see the correlation of phase plane and regular displacement – time plots .

Fig 7.5.2 Phase and Displacement time plots

As can be seen in Fig. 7.5.2 above, the starting point (with finite displacement and velocity at time $t=0$) is marked P_1 . After t_1 seconds, we reach P_2 where $\angle P_1OP_2 = \omega_n t_1$ radians. There are many other interesting forms of graphical representation of dynamic response of a system. Since it is an undamped system, when started with some initial conditions, it continues to move forever. Starting point P_1 is reached after every cycle (time period).

If the system is damped, then the mass gradually dissipates away energy and comes to rest. Please see section for the motion of such systems and their phase plane plots.

Recap

In this lecture you have learnt the following

- Phase plane representation of the simple harmonic motion with axes as velocity and displacement.
- Essentially the phase plot of a simple harmonic motion is a circle.

Congratulations, you have finished Lecture 5. To view the next lecture select it from the left hand side menu of the page

