Module 6 : Vibration of Mechanical Systems; Types of Vibration; Lumped Parameter Models; Linearization of System

Elements; Degrees of Freedom; Types of Restoration and Dissipation Mechanisms; Types of Excitation

Lecture 10 : Modeling of Mechanical Vibrations.

Objectives

In this module, you will learn the following

- Elements in a Vibrating System
- Degrees of Freedom
- Linearization and Lumping of Elements

Vibration is essentially a to-and-fro motion. Thus there is a force (excitation/disturbance) that initiates the motion. We will learn about different types of excitation forces in the next lecture. Under the influence of the external disturbance excitation, the system masses move i.e., they accelerate and decelerate setting up inertia forces. If external excitation were the only type of force on the system, the system would exhibit a rigid body motion. However since most systems are elastic, the movement of the masses invariably causes stretching or compression of springy elements setting up elastic restoring forces.

For example when an automobile passes on a road, the road roughness is the external excitation. The mass of the vehicle moves up-down (pitch and bounce), left-right (roll) setting up inertia forces. The suspension spring gets stretched and compressed as the vehicle mass moves up and down. When a spring is stretched or compressed from its free length position, it exerts a restoring force on the mass trying to bring it back to its free length position. In the process of course the mass would have gained momentum and continues to travel farther than the static equilibrium, free length position. Once again the spring tries to pull the mass back to its free length position and the cycle repeats.

The cycle of to-and-fro motions would however not repeat forever due to the dissipation present in most systems. For example, an automobile suspension always has a shock absorber i.e., a damper that dissipates the energy of vibration into friction against a moving fluid.

Thus the four fundamental elements of a vibrating system are:

- Mass or Inertia
- Springiness or Restoring element
- Dissipative element (often called damper)
- External excitation

When we refer to modeling a vibrating system, we need to distinguish between two types of models viz., physical models and mathematical models. Physical model of a system is a representation of the physics of the system that we would like to include in our study. For example, we may consider dissipation negligible in a system for a given study and may say that the physical model of the system consists of purely spring-mass systems. Now a mathematical model refers to a mathematical relation that defines the input-output relation of the elements. For example, we could have a simple linear relation between force and deflection for a spring or a more complicated non-linear relation (for example softening or hardening types) as shown in Fig. 6.2.1. For a given problem of engineering, we typically develop first the physical model i.e., decide which physics is important to include in the model and then build the mathematical description of the various elements to develop the mathematical model.

Modeling of mass or inertia of a system seems fairly straight forward, at a first glance. We just need to worry about the total mass/inertia of the system and use it in system models. For example in an automobile, there is a certain mass of the vehicle – chassis, body, engine etc and we can include these masses in the model of the system. Of course the vehicle mass changes with the number of passengers and the luggage but that can also be determined and taken into account.

However as we study the issue deeper we realize some difficulties. For example, we want to develop a mathematical model of the vehicle and study its dynamics (to ensure its satisfactory performance) before the vehicle prototype is actually built. How do we ascertain the system mass moment of inertia at this stage? But we need the inertia numbers for accurate modeling of the roll, yaw, pitch etc. How do we precisely locate the centre of gravity of the vehicle that is yet to be built?! Similarly consider the example of sloshing of a fluid inside a tanker being transported from place to place. Due to road undulations and driving characteristics, the liquid is bound to slosh inside the tanker. When we are modeling the dynamics of the vehicle body subjected to road undulation, how do we exactly model the mass of the fluid? Another interesting example arises in the case of marine structures for examples, ships, submarines etc. When a ship hull vibrates, how much mass of the water adjacent to it participates in the vibration?

Thus we see that while for simple cases it is easy to model the mass of a vibrating system, for complex problems, this could become a big issue and demands deeper study.

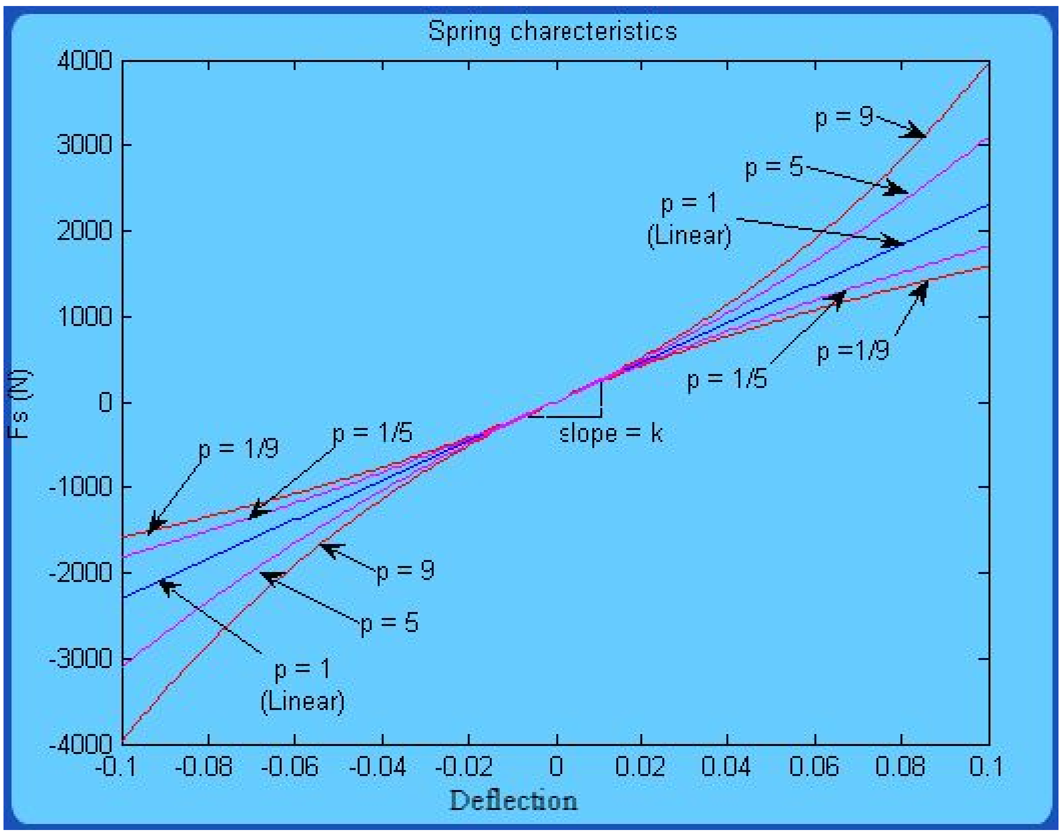

Fig 6.2.1: Nonlinear spring characteristics

The restoring force in a vibrating system may be provided by a real physical spring (say a close coiled helical spring or a leaf spring in an automobile suspension) or by an "effective" spring action. For example in the case of the to-and-fro oscillations of a simple pendulum, the restoring force is actually provided by gravity. It is the weight of the bob that tends to bring it back to its original equilibrium position and thus gravity acts as the effective spring here. There could also be electro-magnetic or such other forces that provide the restoring action. We will consider all of these under the head, springiness in the system.

Dissipation in a system is perhaps the hardest to model mathematically in accurate manner, simply because there can be many sources of damping involving complex physical phenomena. For example, automobile shock absorbers have piston-cylinder kind of fluid friction based dampers normally referred to as viscous dampers. There can also be contact friction between two mating surfaces (such as piston-cylinder interface; cam-follower etc) which is normally considered to be of Coulombic type. Additionally there can be material internal friction (hysteresis) damping. Considering all such various sources of damping present in a system and developing accurate mathematical models for the same is a challenging task. Often, total energy being dissipated is approximately modeled using a simple linear viscous damper.

We will discuss the fourth element of a typical vibrating system namely the external disturbance in the next lecture.

Various elements of a typical vibrating system and the standard configuration for assembly of the vibrating system are shown pictorially in Fig. 6.2.2. The system shown in this figure is what is known as a Single Degree of Freedom system. We use the term degree of freedom to refer to the number of coordinates that are required to specify completely the configuration of the system. Here, if the position of the mass of the system is specified then accordingly the position of the spring and damper are also identified. Thus we need just one coordinate (that of the mass) to specify the system completely and hence it is known as a single degree of freedom system.

What is shown in Fig. 6.2.3 is a two degree of freedom system. With reference to automobile applications, this is referred as “quarter car” model. The bottom mass refers to mass of axle, wheel etc components which are below the suspension spring and the top mass refers to the mass of the portion of the car and passenger. Since we need to specify both the top and bottom mass positions to completely specify the system, this becomes a two degree of freedom system.

A half-car model shown in Fig. 6.2.4 is a four degree of freedom model where G is the centre of mass. A full car model, on the other hand (Fig. 6.2.5) has several degrees of freedom.

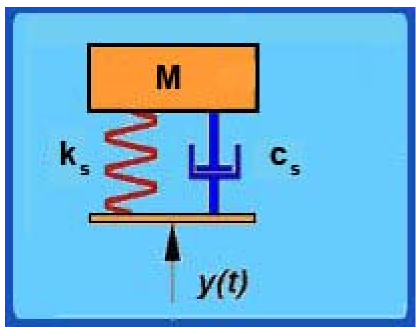

Figure 6.2.2

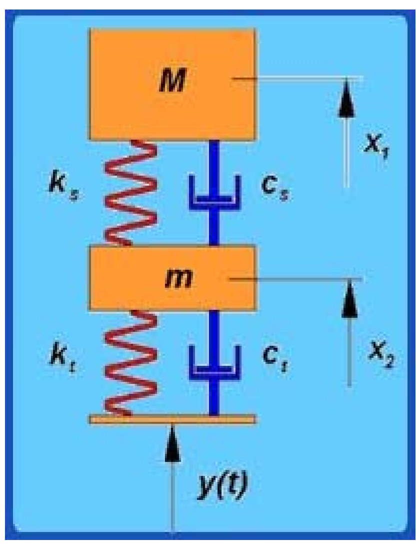

Figure 6.2.3

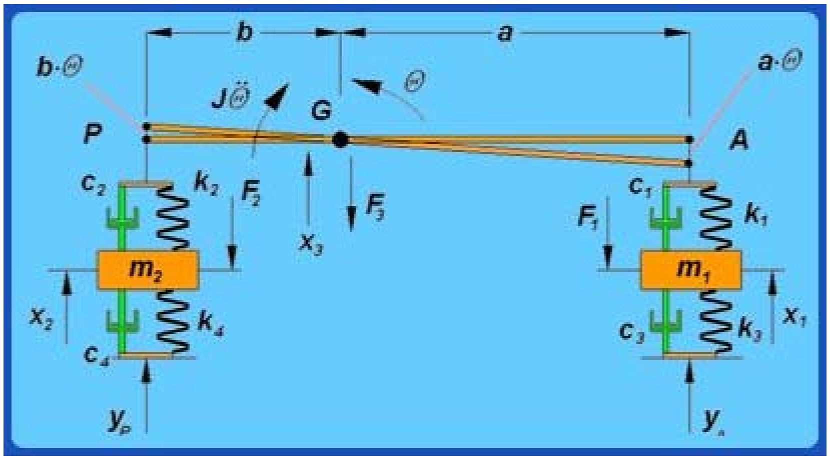

Figure 6.2.4

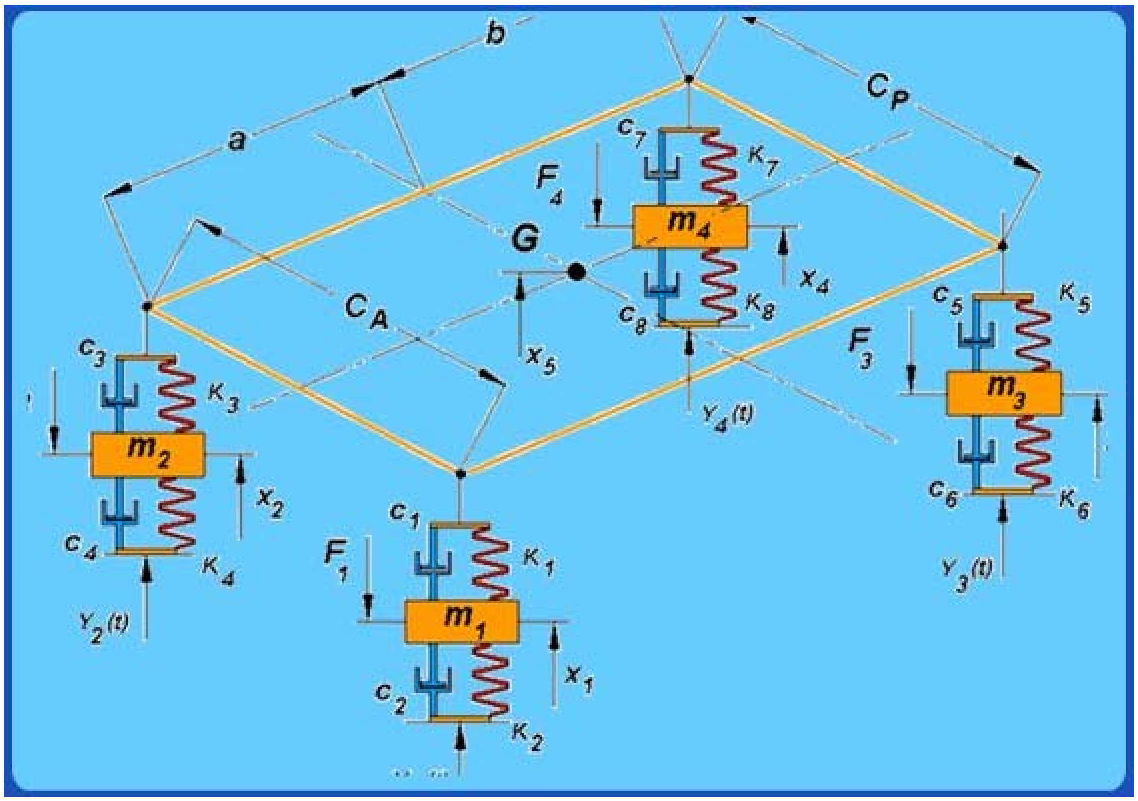

Figure 6.2.5

In all these systems considered as examples so far, the properties of mass/inertia; springiness and dissipation are modeled as "lumped" into certain physical devices. For example the mass does not have any springiness nor dissipation. The spring does not have any mass and does not dissipate either. The mass-less dashpot is a pure viscous damper without any springiness. Thus these are idealized models of typical physical phenomena that are observed in vibrating system namely mass/inertia; restoring force, dissipative force etc. In a building subjected an earthquake, the whole building shakes and it is not possible to isolate which part of the building is a pure mass; which is a pure spring element and which is pure dashpot. These elements are all "distributed" within the complete system and need to be modeled as such.

However modeling as lumped elements eases up the modeling effort because the resulting equations are ordinary differential equations (ODEs), while the distributed parameter models are described by partial differential equations (PDEs). Since ODEs are lot easier to solve than PDEs, it has been common practice to develop simplified lumped parameter models first. Latest computational techniques such as finite element method convert given partial differential equations of distributed parameter models (such as a building etc) into ODEs before solving them on the computer.

A spring or a dashpot is mathematically represented by a force – deformation or force – velocity curve/equation. In general this curve can be straight line or a general curve as shown in Fig. 6.2.1. When the relationship is a straight line i.e., a linear spring or damper model is used, there is direct proportionality between the cause and the effect and the resulting governing equations of motion are also linear. Linear models are extensively used in most engineering analyses, at least as a first approximation in design. The linear equations (differential or algebraic) are lot easier to solve than non-linear equations and hence these models are commonly used eventhrough many times practical systems do exhibit non-linear behaviour. Nonlinear systems exhibit very interesting and sometime unexpected behavior (for example chaos) and do become important in special situations. In our course we will be limiting our attention to only linear systems.

Recap

In this module you have learnt the following

- Spring, damper, inertia and external excitation constitute four primary elements of a Vibrating System;
- Concept of Degrees of Freedom
- Lumped models of springiness, inertia and damping