Module 2 : Dynamics of Rotating Bodies; Unbalance Effects and Balancing of Inertia Forces

Lecture 5 : Two-plane balancing technique

Objectives

In this lecture you will learn the following

- Balancing of rigid rotors

- Two-plane method for balancing

Consider the turbo-machine rotor that was discussed earlier wherein each stage contains several blades around the circumference of a disk. Eventhough typically each stage is balanced in itself to the extent possible, it has a likely net unbalance. When the rotor is set to spin, it will cause dynamic forces and moments on the bearings that support the shaft. Therefore it is of interest to achieve “good balance” of this shaft so that the fluctuating forces on the bearings are reduced. Conceptually our strategy can be simply stated as follows:

Step 1: Consider the shaft supported on its bearings. For each unbalance mass, there will be a centrifugal force set-up when the rotor spins at some speed Ω. This would cause some reactions at the supports. Estimate these support reactions that would come onto the bearings.

Step 2: Estimate the balancing mass that needs to be placed in the plane of bearings, to counter this reaction force due to unbalance mass.

Repeat steps 1 and 2 for each unbalance mass in the system and each time add the balancing masses obtained in step 2 vectorially to determine the resultant balancing mass required.

Let us now understand the details of the technique mentioned earlier. Firstly we choose to place “balancing or correcting” masses on the shaft (rotating along with the shaft) to counter-act the unbalance forces. We understand that this is to be done on the rotor on site, perhaps during a maintenance period. From the point of view of accessibility, we therefore choose the balancing masses to be kept near the bearings.

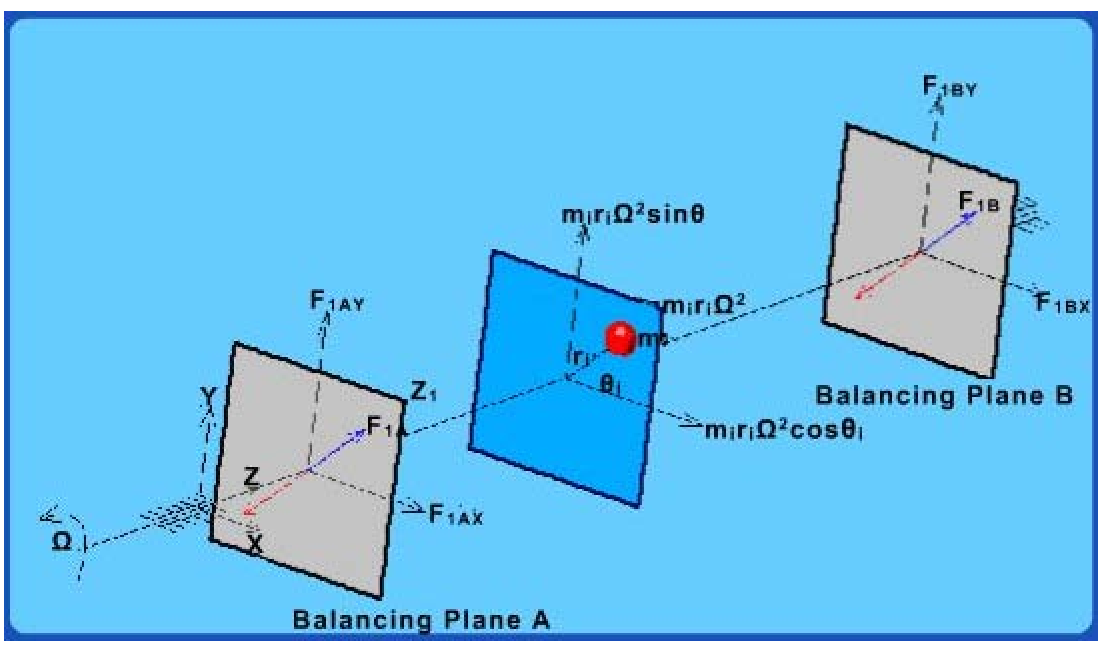


Figure 2.3.1 Two plane balancing technique

The calculations proceed as shown in Fig. 2.3.1. For an unbalance mass m_i situated at an angular location θ_i

in a plane at an axial distance z_i from the left end bearing and rotating at a radius r_i as shown in the figure, the unbalance force is $m_i r_i \Omega^2$. It is resolved into X and Y components as shown in the figure. These forces are represented by EQUIVALENT FORCES in the balancing planes (shown in blue F_{IA}, F_{IB}). These forces can be readily calculated (based on calculations similar to those involved in finding support reactions for a simply supported beam). In order to counterbalance this force, we need to place a balancing mass m_b at a radius r_b in the balancing plane such that it creates an equal and opposite force (shown in red).

Now we need to repeat the calculations for ALL the unbalance masses m_i (i = 1,2,3,.....) and find the resultant equivalent force in the balancing plane as shown in blue in Fig. 2.3.1. This resultant force is balanced out by placing a suitable balancing mass creating an equal and opposite force (shown in red). Since all the masses are rotating at the same speed Ω along with the shaft, we can drop Ω in our calculations – i.e., a rotor balanced at one speed will remain balanced at all speeds or in other words, our technique of balancing is independent of speed. We will review this towards the end of the lecture.

While these calculations can be done in any manner perceived to be convenient, a tabular form (see Table 2.3.1) is commonly employed to organize the computations. While doing this, it is also common practice to include the two balancing masses in the balancing planes as indicated in the table.

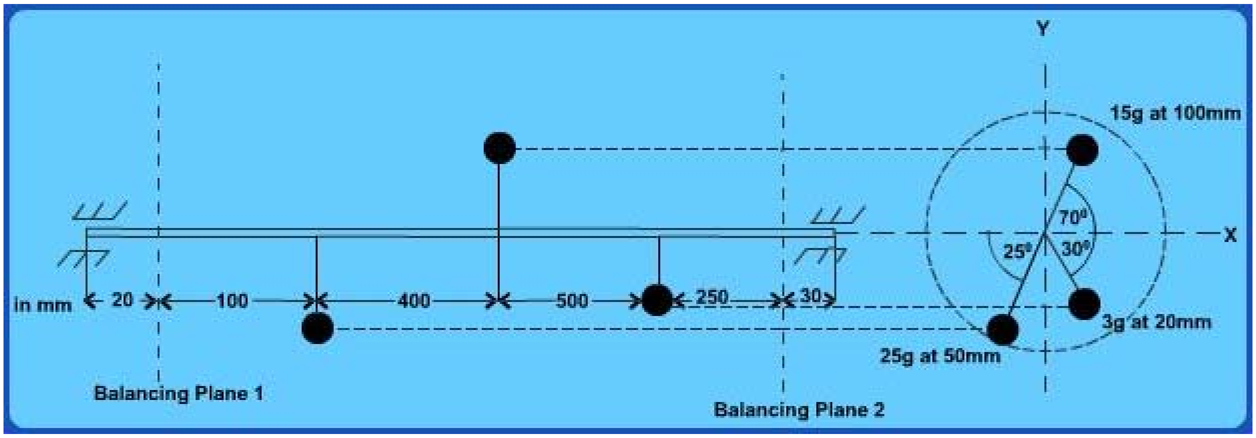
Table 2.3.1 Tabular form of organizing the computations for two-plane balancing technique

Sr. No	Z_i	M_i	R_i	θ_i	$M_i R_i$ Cos(θ_i)	$M_i R_i$ Sin(θ_i)	$M_i R_i Z_i$ Cos(θ_i)	$M_i R_i Z_i$ Sin(θ_i)
1								
2								
3								
….								
…..								
Balancing Plane 1	0	M_{b1}	R_{b1}	θ_{b1}				
Balancing Plane 2	L	M_{b2}	R_{b2}	θ_{b2}				
TOTAL FORCES					**0**	**0**	**0**	**0**

It is observed in Table 2.3.1 that the balancing masses and their locations (radial as well as angular) are unknowns while the location of the balancing plane itself is treated as a known (any accessible location near the bearings etc). The resultant total forces and moments must sum up to ZERO and therefore we have four equations but six unknowns. Thus any two of the six unknowns can be freely chosen and the other four determined from the computations given in the table. This method of balancing is known as the "two-plane balancing technique" since balancing masses are kept in two planes.

We will now work out an example, to illustrate the procedure.

Ex. 2.3.1 For the rotor shown in **Fig. 2.3.2**, find the magnitude and the angular location of the balancing masses.



The calculations are shown in the table below.

Sr. No.	Z_i	M_i	R_i	θ_i	$M_i R_i \cos\theta_i$	$M_i R_i Sin\theta_i$	$M_i R_i Z_i \cos\theta_i$	$M_i R_i Z_i Sin\theta_i$
1	120	25	50	205^0	-1132.88	-528.27	135945.6	-63392
2	520	15	100	70^0	513.03	1409.54	266775.71	732960.24
3	1020	3	20	330^0	51.96	-30.00	53000.75	-30600.00
Balancing Plane1	20	m_{b1}	r_{b1}	θ_{b1}	$m_{b1} r_{b1} \cos\theta_{b1}$	$m_{b1} r_{b1} \sin\theta_{b1}$	$20 m_{b1} r_{b1} \cos\theta_{b1}$	$20 m_{b1} r_{b1} \sin\theta_{b1}$
Balancing Plane2	1270	m_{b2}	r_{b2}	θ_{b2}	$m_{b2} r_2 \cos\theta_{b2}$	$m_{b2} r_2 \sin\theta_{b2}$	$1270 m_{b2} r_2 \cos\theta_{b2}$	$1270 m_{b2} r_2 \sin\theta_{b2}$
Total					**0**	**0**	**0**	**0**

From the table, the final equations are obtained as follows:

$m_{b1} r_{b1} \cos\theta_{b1} + m_{b2} r_2 \cos\theta_{b2} = 567.89$

$m_{b1} r_{b1} \sin\theta_{b1} + m_{b2} r_2 \sin\theta_{b2} = -851.27$

$20 m_{b1} r_{b1} \cos\theta_{b1} + 1270 m_{b2} r_2 \cos\theta_{b2} = -455722.06$

$20 m_{b1} r_{b1} \sin\theta_{b1} + 1270 m_{b2} r_2 \sin\theta_{b2} = -638967.5$

Upon solution, we get

$\theta_{b1} = 340.53^0$;

$m_{b1} r_{b1} = 979.5$ kg-mm

$\theta_{b2} = 236.0^0$

$m_{b2} r_{b2}$ = 634 kg-mm

We have said that the balancing achieved by our method was independent of speed. This is true only up to a certain limit, as we have implicitly assumed the shaft to be perfectly rigid. As the speed increases, the unbalance forces increase and the shaft tends to deform elastically under the action of these forces. Also, as the speed becomes closer to the fundamental transverse bending natural frequency of the shaft in its bearings, chances of resonance are also there. Thus the assumption of shaft being perfectly rigid is no longer valid. A rotor can be assumed to be “rigid” if the speed of rotation is less than about 1/3 rd the fundamental natural frequency. Two plane balancing technique is essentially a rigid rotor balancing method and is useful only within the speed limit mentioned above.

Recap

In this lecture you have learnt the following

- The two plane method of balancing rigid rotors

- The tabular form of organizing the computations