Module 7 : Free Undamped Vibration of Single Degree of Freedom Systems; Determination of Natural Frequency ; Equivalent Inertia and Stiffness; Energy Method; Phase Plane Representation.

Lecture 14 : Energy Method

Objectives

In this lecture you will learn the following

- Derivation of equation of motion using energy methods.

Energy Method

Energy methods can also be used to analyse the vibration behaviour of a system, just as we used the Newton's laws of motion.

Law of conservation of energy states that energy can neither be created nor destroyed, but can be only converted from one form to another. In a simple vibrating spring – mass system, the two forms of energy are the kinetic energy and the potential energy. Thus this principle means that the relative contribution of the kinetic and potential energies to the total energy of the system can change from time to time, but the total energy of the system itself has to remain constant.

Rayleigh's method is based on this principle. Let,

- T=Kinetic Energy
- V=Potential Energy
- U=Total Energy

The system shown in fig 7.4.1 has the kinetic energy of the form

$$T = \frac{1}{2} m\dot{x}^2 \qquad 7.4.1$$

And the Potential Energy of the form

$$V = \frac{1}{2}kx^2 \qquad 7.4.2$$

The total Energy is

$$U = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 \qquad 7.4.3$$

Since there is no dissipative element in the model, the total energy in this conservative system must remain constant all the time. Differentiating total energy with respect to time and equating the resultant to zero we get,

$$\frac{dU}{dt} = m\dot{x}\ddot{x} + kx\dot{x} = 0 \qquad 7.4.4$$

So the Equation of motion is,

$$\ddot{x} + \frac{k}{m}x = 0 \qquad 7.4.5$$

which is identical to the equation obtained from Newton 's Laws.(see section 9.1_5)

For the same physical model, the governing equations obtained by any method of formulation must be identical. Application of Newton 's laws will involve drawing free body diagrams and dealing with vector quantities such as forces, accelerations. One has to be extra careful when dealing with moving frames of reference. Energy methods involving quantities such as kinetic and potential energies, on the other hand, deal with scalar quantities. One can choose any method of formulation but the final governing equations will be identical.

Recap

In this lecture you have learnt the following

- Derivation of free undamped vibration system equation using the energy conservation principle.
- Determine the total energy of the system ie sum of kinetic and potential energy of vibrating system.
- Similarity in equations derived using Newton's laws and using energy conservation principle.

Congratulations, you have finished Lecture 4. To view the next lecture select it from the left hand side menu of the page