Module 10 : Vibration of Two and Multidegree of freedom systems; Concept of Normal Mode; Free Vibration Problems and Determination of Ntural Frequencies; Forced Vibration Analysis; Vibration Absorbers; Approximate Methods - Dunkerley's Method and Holzer Method

Lecture 29 : Free Vibration of Multi-d.o.f systems.

Objectives

In this lecture you will learn the following

- Equation of motion of multi degree of freedom system.
- Matrix representation of the equation of motion.
- Appreciation of Solution methods.

It must be appreciated that any real life system is actually a continuous or distributed parameter system (i.e. infinitely many d.o.f). Hence to derive its equation of motion we need to consider a small (i.e., differential) element and draw the free body diagram and apply Newton 's second Law. The resulting equations of motion are partial differential equations and more complex than the simple ordinary differential equations we have been dealing with so far. Thus we are interested in modeling the real life system using lumped parameter models and ordinary differential equations. The accuracy of such models (i.e. how well they can model the behavior of the infinitely many d.o.f. real life system) improves as we increase the number of d.o.f. Thus we would like to develop mult-d.o.f lumped parameter models which still yield ordinary differential equations of motion – as many equations as the d.o.f . We would discuss these aspects in this lecture.

Derivation of Equations of Motion

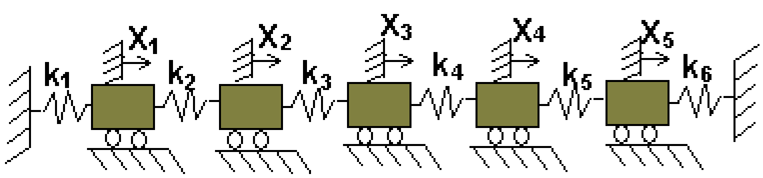

Fig 10.3.1 A Typical multi-d.o.f. system

Consider a typical multi-d.o.f system as shown in Fig. 10.3.1. As mentioned earlier our procedure to determine the equations of motion remains the same irrespective of the number of d.o.f of the system and is recalled to be:

Step 1 : Consider the system in a displaced Configuration

Step 2 : Draw Free Body diagrams

and Step 3 : Use Newton 's second Law to write the equation of motion

From the free body diagrams shown in Fig. 10.3.1, we get the equations of motion as follows:

$$m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = F_1(t)$$
$$m_2\ddot{x}_2 + (k_2 + k_3)x_2 - k_1 x_1 = F_2(t) \qquad 10.3.1$$

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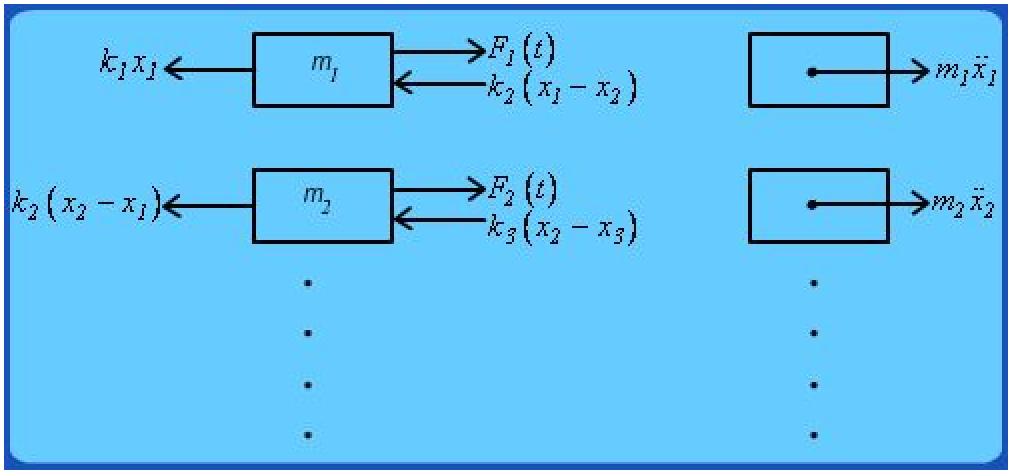

Rewriting the equations of motion in matrix notation, we get:

$$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \\ \ddot{x}_5 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ F_4(t) \\ F_5(t) \end{Bmatrix} \qquad 10.3.2$$

Or in compact form,

$$[M]\{\ddot{X}\} + [K]\{X\} = \{F\} \qquad 10.3.3$$

There are "n" equations of motion for an "n" d.o.f system. Correspondingly the mass and stiffness matrices ([M] and [K] respectively) are square matrices of size (n x n).

If we consider free vibrations and search for harmonic oscillations, $\{F\} = \{0\}$ *and* $\{X\} = \{X_0\}\ Sin\ \Omega t$. Substituting these in eqn 10.3.3 , we get,

$$-\omega^2[M]\{X\} + [K]\{X_0\} = \{0\} \qquad 10.3.4$$

ie

$$[[K] - \omega^2[M]]\ \{X_0\} = \{0\} \qquad 10.3.5$$

For a non-trivial solution to exist, we have the condition that the determinant of the coefficient matrix must vanish. Thus, we can write,

$$\left| [K] - \omega^2[M] \right| = 0 \qquad 10.3.6$$

In principle this (n x n) determinant can be expanded by row or column method and we can write the characteristic equation (or frequency equation) in terms of ω^2 , solution of which yields the "n" natural frequencies of the "n" d.o.f. system just as we did for the two d.o.f system case.

We can substitute the values of ω_n in eqn 10.3. and derive a relation between the amplitudes of various masses yielding us the corresponding normal mode shape. Typical mode shapes are schematically depicted in

Fig. 10.3.2 for a d.o.f system.

Recap

In this lecture you have learnt the following

- Development of equation of motion for forced excitation of multi degree of freedom system.

- Bandedness of the stiffness matrix $$\begin{bmatrix} m_1 & 0 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 & 0 \\ 0 & 0 & m_3 & 0 & 0 \\ 0 & 0 & 0 & m_4 & 0 \\ 0 & 0 & 0 & 0 & m_5 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \\ \ddot{x}_4 \\ \ddot{x}_5 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \\ F_4(t) \\ F_5(t) \end{Bmatrix}$$

- Solution of $\left|[K] - \omega^2 [M]\right| = 0$ gives us the eigen values of the system which are nothing but the natural frequencies of the system and finding the eigen vecors gives us the mode shapes of the system.

Congratulations, you have finished Lecture 3 To view the next lecture select it from the left hand side menu of the page.