Module 3 : Radiation

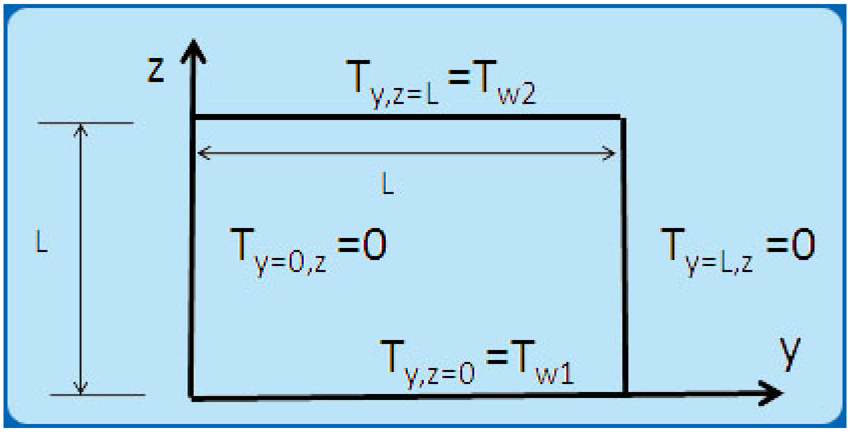
Lecture 30 : Discrete Ordinates Methodology (Contd.)

Objectives
In this class:

- The details of the solution for the RTE using the Discrete Ordinates Methodology are discussed for a general situation. A linear anisotropic medium is chosen.

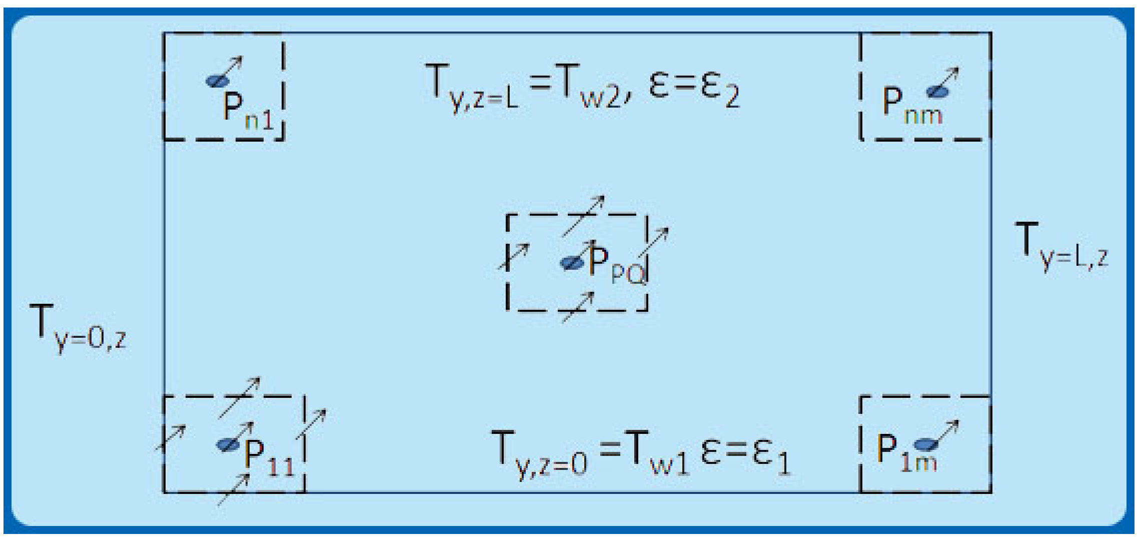
DOM Details-Geometry

- All the required expressions have now been developed for the solution of the Radiation Transport Equation. Now consider square geometry with length **L** and all the wall temperatures specified as shown:



DOM Details-Discretization

- Let the domain be divided into **nXm** cells. One intensity direction among the several possible is shown. Nodes are at the center of the cells and P_{PQ} is a typical node.



DOM Details-Medium

- Let the medium be one with a specified linear anisotropic scattering characteristic:

$$\hat{\Phi}(s, \theta_j, \theta_e) = 1 + a_0 m^k m^{k'} + a_0 n^k n^{k'} \tag{30.1}$$

- In this equation **k** is the direction of propagation of the intensity and **k'** are the inscattered directions.
- Either the wall temperatures or the wall flux can be specified.

- First step is to calculate the intensity at different nodes $\mathbf{I_{PQ}}$ in the chosen direction.
- $A_E = L/(N-1)$; $A_W = L/(N-1)$; $A_N = L/(M-1)$;

$$A_S = L/(M-1) \tag{30.2}$$

- To start with assume all $\mathbf{I_b}$ are zero
- Equ^n (29.13) must be solved to get the result
- $I_{S1,1}$ = known; This is easily calculated from the boundary condition of temperature on this wall if the wall is black.

DOM Details: Calculations -2

- However, if the wall has a given emissivity value, the calculation is started by assuming the wall to be black in the first iteration. This is necessary since the wall reflection can be calculated only if the incident intensity is available which is assumed zero for the first iteration. Therefore:

$$I_{S1,1}^{k} = \frac{\sigma T^4}{\pi} \tag{30.3}$$

DOM Details: Calculations -3

- The last term in the numerator of equ^n (29.13) is now simplified:

$$\eta \frac{\rho(s)}{4\pi} \sum_{j=1}^{n} w_j.\mathrm{I}(\mathrm{s},\theta_j).\hat{\Phi}(s,\theta_j,\theta_e)V_{P,Q} \tag{30.4}$$

$$= \eta \frac{\rho(s)}{4\pi} \sum_{j=1}^{n} w_j.\mathrm{I}(\mathrm{s},\theta_j).\left(a_0(1+mm'+nn'\right)V_{P,Q}$$

$$= \eta a_0 \frac{\rho(s)}{4\pi} \left(\sum_{j=1}^{n} w_j.\mathrm{I}(\mathrm{s},\theta_j).V_{P,Q} + \sum_{j=1}^{n} w_j.\mathrm{I}(\mathrm{s},\theta_j)mm'.V_{P,Q} + \sum_{j=1}^{n} w_j.\mathrm{I}(\mathrm{s},\theta_j)nn'.V_{P,Q} \right) \tag{30.5}$$

DOM Details: Calculations -4

- Now look at the red coloured term in equ^n (30.5)

$$\sum_{j=1}^{n} w_j.\mathrm{I}(s,\theta_j).V_{P,Q} = \sum_{j=1}^{n} w_j.\mathrm{I}_{P,Q}^{j}.V_{P,Q} \tag{30.6}$$

- The expanded version for the first direction for the node **(1,1)** for S_n method is:

$$w_1 I_{1,1}^{1} + w_2 I_{1,1}^{2} \ldots\ldots\ldots\ldots + w_n I_{1,1}^{n} \tag{30.6a}$$

- Note that in the above equ^n (30.6a) $I_{1,1}$ has superscripts and not powers

DOM Details: Calculations -5

- Now look at the yellow coloured term in equ^n (30.5):

$$\sum_{j=1}^{n} w_j . \mathrm{I}(s,\theta_j) m m' . V_{\mathrm{P,Q}} = \sum_{j=1}^{n} w_j . \mathrm{I}^{j}_{\mathrm{P,Q}} m^{\mathrm{k}} m^{j} . V_{\mathrm{P,Q}} \tag{30.7}$$

- The expanded version of the above equation for the first direction for the **(1,1)** node is:

$$m^1 (w_1 \mathrm{I}^1_{1,1} m^1 + w_2 \mathrm{I}^2_{1,1} m^2 + \ldots\ldots\ldots\ldots + w_{\mathrm{n}} \mathrm{I}^4_{1,1} m^{\mathrm{n}}) V_{\mathrm{P,Q}} \tag{30.7a}$$

DOM Details: Calculations -6

- Similarly the green term in equ$^{\mathrm{n}}$ (30.5) becomes:

$$\sum_{j=1}^{n} w_j . \mathrm{I}(s,\theta_j) n n' . V_{\mathrm{P,Q}} = \sum_{j=1}^{n} w_j . \mathrm{I}^{j}_{\mathrm{P,Q}} n^{k} n^{j} . V_{\mathrm{P,Q}} \tag{30.8}$$

- The expanded version of the above equation for the first direction for the **(1,1)** node is:

$$n^1 (w_1 \mathrm{I}^1_{1,1} n^1 + w_2 \mathrm{I}^2_{1,1} n^2 + \ldots\ldots\ldots + w_{\mathrm{n}} \mathrm{I}^4_{1,1} n^{\mathrm{n}}) \tag{30.8a}$$

DOM Details: Calculations -7

- The intensity at the point has been computed and now use equ$^{\mathrm{n}}$ (29.10) to compute the intensities at the faces:

$$I^{\mathrm{k}}_{1,1\mathrm{n}} = \frac{I^{\mathrm{k}}_{1,1} - (1-\eta) I^{\mathrm{k}}_{1,1s}}{\eta}$$
$$I^{\mathrm{k}}_{1,1\mathrm{e}} = \frac{I^{\mathrm{k}}_{1,1} - (1-\eta) I^{\mathrm{k}}_{1,1w}}{\eta} \tag{30.9}$$

- All the intensity values for the volume **(1,1)** have now been computed.

DOM Details: Calculations -8

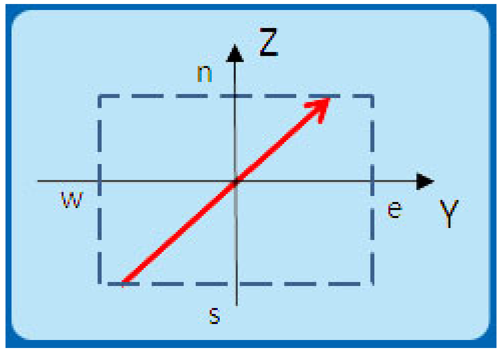
- The calculations for the volume (2,1) are now made till you reach the last control volume in the **z direction**. Here it is possible that the condition at the boundary that forms the face of the control volume is given and will not match with the calculated value. This is natural since this is the first iteration and several others are required before the final solution. Calculations are now made in the y direction by starting with control volume (1,2).

DOM Details: Calculations -9

- This process is repeated till the computations are done on all the control volumes. The sweep can be first in the **y direction** and then in the **z direction** also.
- The calculations have all been performed for the '1' direction. The methodology needs calculations in several directions. The same calculations can be performed in the other directions also using the same expressions that have already been obtained but a little care has to be exercised.

DOM Details: Calculations -10

- Now formalize the nomenclature already used. The direction chosen is given by the red arrow and the faces are labeled as **e w n s**:

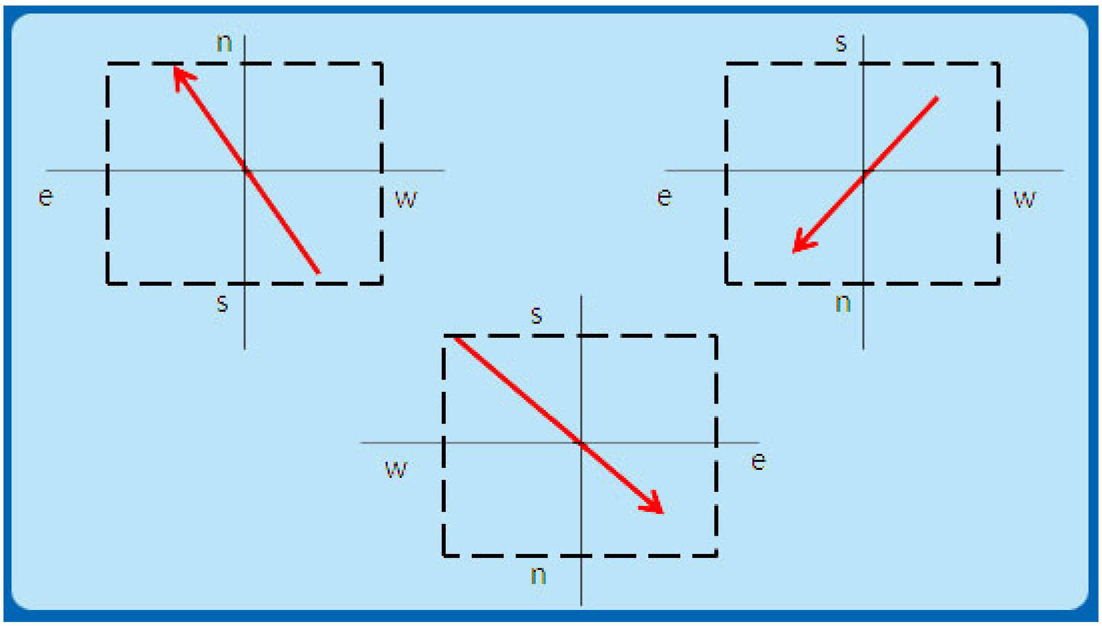


- Direction of projection of chosen direction on

  +Z-axis: Always **n**   −Z axis: Always **s**
  +Y-axis: Always **e**   −Y axis: Always **w**

DOM Details: Calculations -11

- Therefore now the **e, w, n, s** for the different directions become:



DOM Details: Calculations -12

- We have performed the first iteration by assuming all unknown values of $\mathbf{I}^{1}_{1,1}$ as zero. All these values for **I** are calculated except $\mathbf{I_b}$ which is now calculated to complete the methodology. Use the equ^n (29.19)

$$4\pi\sigma T^4 = 4\pi I_b = \int I d\omega = w_1 I_{1,1}^1 + w_2 I_{1,1}^2 + \ldots\ldots + w_n I_{1,1}^n \tag{30.10}$$

- Once I_b is evaluated the entire process is repeated again till difference between successive computations is less than a given tolerance value

Recap
In this class:

- The details of the solution for the RTE using the Discrete Ordinates Methodology are discussed for a general situation. A linear anisotropic medium is chosen.