Module 3 : Radiation

Lecture 31 : Example Problems in Radiation

Objectives
In this class:

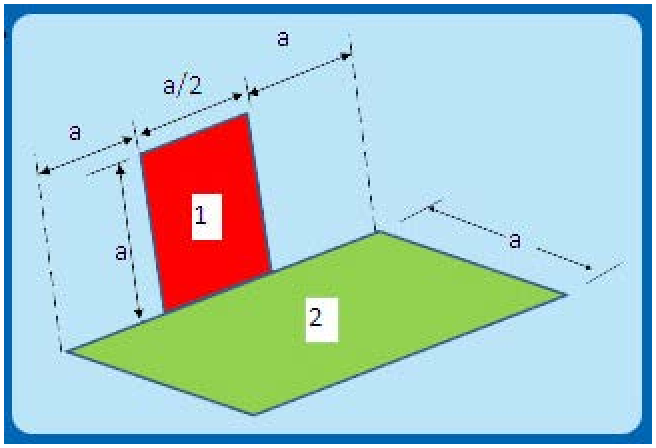
- The detaTwo examples will be taken and the solution procedure outlined.
- First is a problem where the view factor enclosure manipulations are illustrated and the second is a problem where the discrete ordinates method is outlined.

Example Problem 1:1

- When two rectangles meet each other at 90° at a common edge of equal size the view factor can be easily calculated. In this question assume that the view factor is known for such configurations provided the geometry is known. When two rectangles meet each other at 90° at a common edge of different sizes then the calculations are more involved. However, the enclosure relationships can be used to perform the calculations.

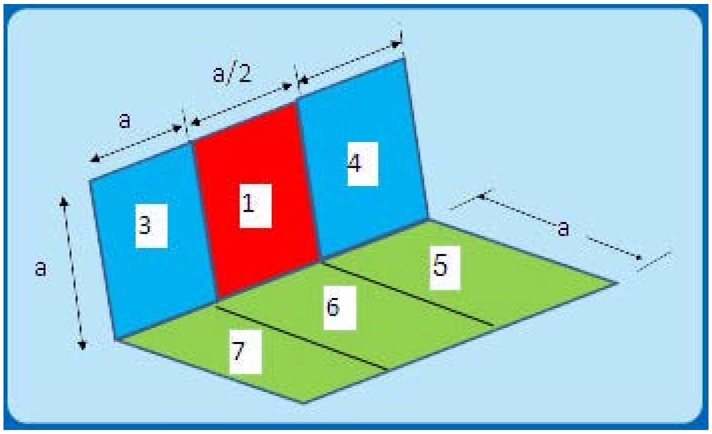
Example Problem 1:2

- Determine the shape factor F_{12} for the red and green rectangles meeting at 90° as shown below.



Solution 1:1

- Two new imaginary surfaces 3 and 4 of blue colour are created. In addition, the surface 2 is divided into three surfaces, 5, 6 and 7.



- From the problem statement, F_{1-6}, F_{3-7}, F_{4-5}, $F_{(31)-(76)}$, $F_{(14)-(65)}$ are known

Solution 1:2

- Notice that from symmetry: $F_{1-7} = F_{6-3}$
- Also from reciprocity:

$$F_{3-6} = F_{6-3}\frac{A_3}{A_3} = F_{1-7}/2$$

- Also:

$$F_{1\text{-}6} + F_{1-7} = F_{1-(6,7)} = 3F_{(6,7)-1} \quad (31.1)$$

$$F_{3-7} + F_{3-6} = F_{3-(6,7)} \Rightarrow F_{3-7} + \frac{1}{2}F_{1-7} = \frac{3}{2}F_{(6,7)-3} \quad (31.2)$$

Solution 1:3

- Add equ^{n}s (31.1) and (31.2) to get:

$$F_{1-6} + 2F_{1-7} + 2F_{3-7} = 3(F_{(6,7)-1} + F_{(6,7)-3})$$
$$\Rightarrow \boxed{F_{1-6}} + 2F_{1-7} + 2\boxed{F_{3-7}} = 3\boxed{F_{(6,7)-(1,3)}}$$

- Notice that all the red terms in the above equation are known and $F_{1\text{-}7}$ can be calculated. Also, from symmetry $F_{1-7} = F_{1-5}$
  $F_{1-2} \equiv F_{1-(5,6,7)} = F_{1-5} + F_{1-6} + F_{1-7} = F_{1-6} + 2F_{1-7}$
- F_{1-6} and F_{1-7} are known, F_{1-2} is calculated.

Example 2

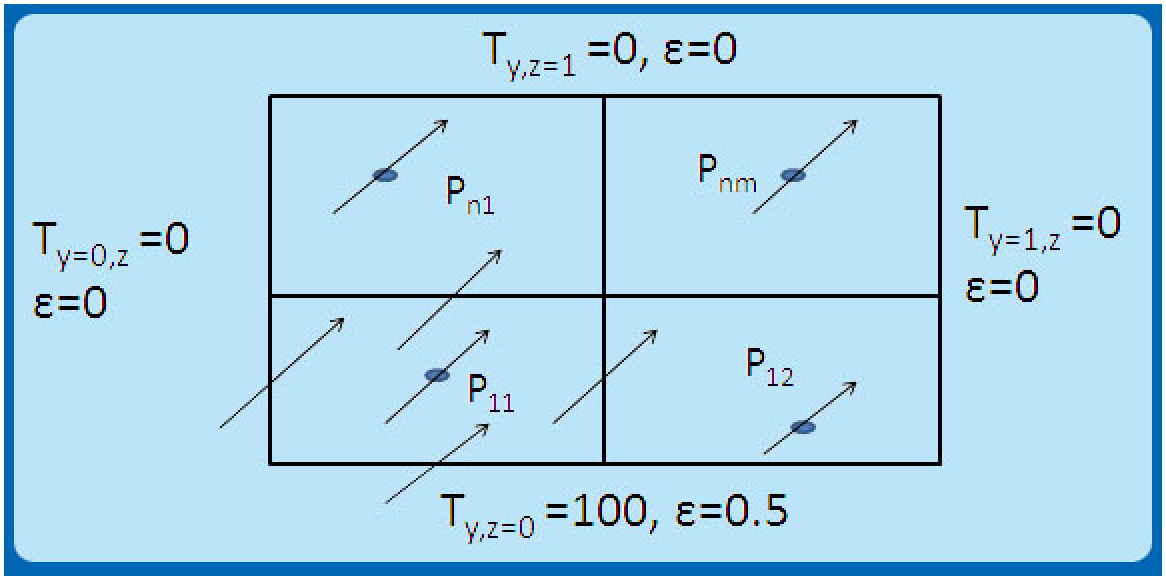
- A square domain with 1m side is considered. The bottom wall is at 100K and all other walls are at 0K. The linear anisotropic scattering model is valid for the medium inside the domain $a_0 = 0.75$. The absorption (α) and scattering coefficient (ρ) for the medium are both assumed equal to 0.5 each. The wall with temperature 100K has ε = 0.5, all other walls are black (i.e. ε = 1).

Solution 2:1

- The domain is divided into **2X2** cells. Each cell has dimension of 0.5m.
- The S_2 method is used for illustration which has 4 directions. The direction cosines are $m = \pm .5$, $n = \pm .5$ and the weights $w_m = \pi$, $w_n = \pi$.
- Bottom wall has emissivity specified and therefore part of the incident energy is reflected. Black walls absorb entire incident energy.
- Assume η = 0.5

Solution 2:2

- The figure of the domain is shown below with only one intensity direction shown:



Solution 2:3

- For node (1,1) the general equation (29.13) becomes:

$$I^{k}_{P,Q}=\frac{\begin{array}{c}0m(0.5L)+n\dfrac{\sigma T_{w1}^{4}}{\pi}(0.5L)+.5\varepsilon 0V_{P,Q}+\\ 0.5*0.75*\dfrac{(1-\varepsilon)}{4\pi}\sum_{j=1}^{n}w_j.\mathrm{I}(s,\theta_j).\hat{\Phi}(s,\theta_j,\theta_e)V_{P,Q}\end{array}}{(m.5L+n.5L+.5(\alpha+\rho)V_{P,Q})} \tag{31.3}$$

- Where $I_w=0;I_s=\dfrac{\sigma T_{w1}^{4}}{\pi}$ is used.

Solution 2:4

- Use equ^{n}s (30.6), (30.7), (30.8) to evaluate the summation terms in (31.1). In particular the equ^{n}s (30.6a), (30.7a), (30.8a) can be used for (31.3):

$$\begin{array}{l}w_1I_{1,1}^{1}+w_2I_{1,1}^{2}+w_3I_{1,1}^{3}+w_4I_{1,1}^{4}+\\ m^{1}(w_1\mathrm{I}_{1,1}^{1}m^{1}+w_2\mathrm{I}_{1,1}^{2}m^{2}+w_3\mathrm{I}_{1,1}^{3}m^{3}+w_4\mathrm{I}_{1,1}^{4}m^{4})+\\ n^{1}(w_1\mathrm{I}_{1,1}^{1}n^{1}+w_2\mathrm{I}_{1,1}^{2}n^{2}+w_3\mathrm{I}_{1,1}^{3}n^{3}+w_4\mathrm{I}_{1,1}^{4}n^{4})\end{array} \tag{31.3a}$$

Solution 2:5

- Notice that in equ^{n} (31.3a) all the intensities at the nodes are unknown. For the first iteration therefore all are assumed to be zero. The entire term therefore becomes zero. Equ^{n} (31.3) therefore becomes:

$$I_{1,1}^{1}=\frac{n\dfrac{\sigma T_{w1}^{4}}{\pi}(0.5L)}{(m(.5L)+n(.5L)+.5(\alpha+\rho)V_{P,Q})} \tag{31.4}$$

$$=\frac{.5*(5.67X10^{-8}*100^{4}/\pi)*.5*1}{.5*.5*1+.5*.5*1+.5*(.5+.5)*.25}=\frac{.451}{.625}=.721$$

Solution 2:6

- Now use equ^{n} (30.9) to calculate the intensities on the faces:

$$I^1_{1,1n} = \frac{I^1_{1,1} - (1-\eta)I^1_{1,1s}}{\eta} = \frac{.721 - .5(1.81)}{0.5} = -.368 \equiv 0$$

$$I^1_{1,1e} = \frac{I^1_{1,1} - (1-\eta)I^1_{1,1w}}{\eta} = \frac{0.721 - 0.5*0}{0.5} = 1.44$$

- Notice that negative intensities are meaningless and are largely due to the numerical procedure adopted and therefore initialized to zero. The problem should go away with a finer grid.

Solution 2:7

- Now move on to the next volume (1,2). The equation (29.13) is again used. However, now:

$$I^1_{w1,2} = I^1_{e1,1} = 1.44; I^1_{s1,2} = \frac{\sigma T_{w1}^4}{\pi} = 1.81$$

- The rest of the calculation procedure remains the same and the following is obtained:

$$I^1_{1,2} = 1.3; I^1_{e1,2} = 1.16; I^1_{n1,2} = 0.795;$$

- Notice that $I_{e1,2}$ that is calculated is different from the specified boundary value – this is only the first iteration and the solution is not the converged one and therefore the anomaly.

Solution 2:8

- Now move on to the next volume (2,1). The equation (29.13) is again used. However, now:

$$I^1_{w2,1} = 0; I^1_{s2,1} = I^1_{n1,1} = 0$$

- The rest of the calculation procedure remains the same and the following is obtained:

$$I^1_{2,1} = 0; I^1_{e2,1} = 0; I^1_{n2,1} = 0;$$

Solution 2:9

- Now move on to the next volume (2,2). The equation (29.13) is again used. However, now:

$$I^1_{w2,2} = I^1_{e1,2} = 0; I^1_{s2,2} = I^1_{n1,2} = .795$$

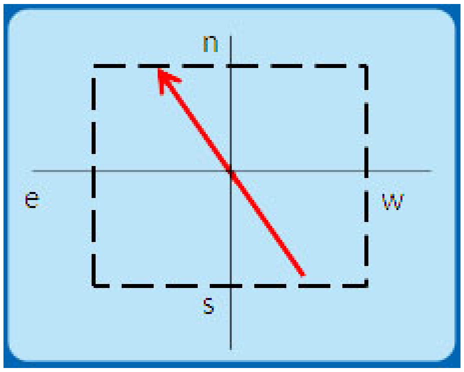
- The rest of the calculation procedure remains the same and the following is obtained:

$$I^1_{2,2} = .318; I^1_{e2,2} = .636; I^1_{n2,2} = -.159 \equiv 0;$$

- Calculations for this direction are now complete for all nodes.

Solution 2:10

- Now consider the next direction (m = −.5, n = .5) Again, the general equation (29.13) needs to be used. This corresponds to the template shown below and discussed earlier. The same expressions can be used except now the 'ewns' directions are as shown



Solution 2:11

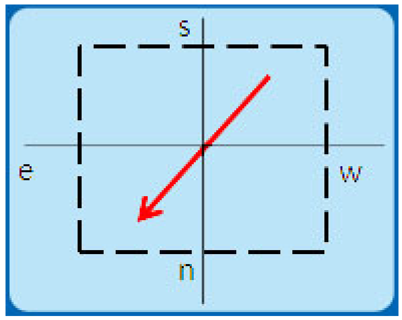
- Since the direction of the discrete ordinate is from left to right, i.e. m = −0.5, n = 0.5, the calculations are started from the right corner i.e. the (1,2) node. The following conditions come from the boundary conditions:

$$I_w = 0; I_s = \frac{\sigma T^4_{\text{bottom wall}}}{\pi}$$

- The numerical values from the calculations are identical to the earlier calculations and we get $I^2_{1,2} = 0.721$; $I^2_{1,1} = 1.3$; $I^2_{2,2} = 0$; $I^2_{2,1} = 0.318$

Solution 2:12

- Now consider the next direction i.e. m = −.5, n = −.5. Now the following template is applicable:

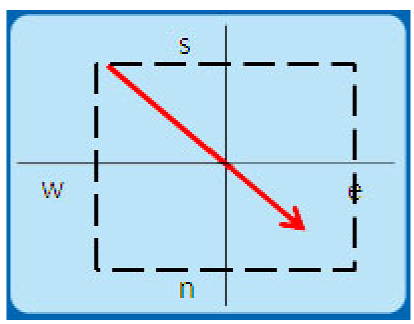


- Calculations now start from the node (2,2) for which: $I_w = 0; I_s = 0$

- Numerical calculations will give: $I^3_{2,2} = 0$; $I^3_{2,1} = 0$; $I^3_{1,2} = 0$; $I^3_{1,1} = 0$.

Solution 2:13

- Now consider the last direction i.e. m = 0.5, n = −0.5. Now the following template is applicable:



- Calculations now start from the node (2,1) for which: $I_w = 0; I_s = 0$

- Numerical calculations will give: $I^4_{2,2} = 0$; $I^4_{2,1} = 0$; $I^4_{1,2} = 0$; $I^4_{1,1} = 0$.

Solution 2:14

- Intensity at all nodes in all the directions have now been calculated for the first iteration. Now the temperatures at the nodes can be calculated using the equ^n (29.19)

$$4\pi\sigma T_{1,1}^{\ 4} = 4\pi I_{b1,1} = \int I d\omega = w_1 I_{1,1}^1 + w_2 I_{1,1}^2 + \ldots\ldots + w_n I_{1,1}^n$$

$$\Rightarrow I_{b1,1} = \frac{w_1 I_{1,1}^1 + w_2 I_{1,1}^2 + \ldots\ldots + w_n I_{1,1}^n}{4\pi} = \frac{\pi.721 + \pi 1.3 + \pi 0 + \pi 0}{4\pi} = .505$$

- Similarly, $I_{b2,2} = 0.0795$; $I_{b2,1} = 0.0795$; $I_{b1,2} = 0.505$;

Solution 2:15

- The first iteration is now complete and we are ready for the next iteration. Recall that all the intensities were assumed to be zero for the first iteration. Now the intensities that have been computed at the nodes will be used for the next iteration.
- Calculations for all the terms except the scattering term have been demonstrated in the first iteration.

Solution 2:16

- Now look at the scattering term:

$$
\begin{aligned}
& w_1 I_{1,1}^1 + w_2 I_{1,1}^2 + w_3 I_{1,1}^3 + w_4 I_{1,1}^4 + \\
& m^1 (w_1 I_{1,1}^1 m^1 + w_2 I_{1,1}^2 m^2 + w_3 I_{1,1}^3 m^3 + w_4 I_{1,1}^4 m^4) + \\
& n^1 (w_1 I_{1,1}^1 n^1 + w_2 I_{1,1}^2 n^2 + w_3 I_{1,1}^3 n^3 + w_4 I_{1,1}^4 n^4) \\
& = \pi * .723 + \pi * 1.3 + \pi * 0 + \pi * 0 \\
& .5 * \pi (.723 * .5 + 1.3 * (-.5) + 0 * (-.5) + 0 * .5) + \\
& .5 * \pi (.723 * .5 + 1.3 * (-.5) + 0 * (-.5) + 0 * .5) + \\
& = 6.355 - .453 - .453 = 5.45
\end{aligned}
$$

Solution 2:17

- The intensity at the point (1,1) is therefore:

$$
I_{\mathrm{P,Q}}^{\mathrm{k}} = \frac{\begin{array}{c} I_w^{\mathrm{k}} m(\eta A_w + (1-\eta) A_{\mathrm{e}}) + n I_s^{\mathrm{k}} (\eta A_s + (1-\eta) A_{\mathrm{n}}) + \\ \eta \varepsilon I_{\mathrm{b}} V_{\mathrm{P,Q}} + \eta \dfrac{\rho(s)}{4\pi} \displaystyle\sum_{j=1}^{\mathrm{n}} w_j . \mathrm{I}(s, \theta_j) . \hat{\Phi}(s, \theta_j, \theta_{\mathrm{e}}) V_{\mathrm{P,Q}} \end{array}}{(m A_{\mathrm{e}} + n A_{\mathrm{n}} + \eta (\alpha + \rho) V_{\mathrm{P,Q}})}
$$

$$
= \frac{0 + .5 * 1.81 * .5 + .5 * .5 * .505 * .25 + .5 * .5 * .25 * 5.45 / 4\pi}{.625}
$$

$$
= .818
$$

Solution 2:18

- The calculations for the faces are identical to those presented earlier:

$$
I_{1,1\mathrm{n}}^1 = \frac{I_{1,1}^1 - (1-\eta) I_{1,1s}^1}{\eta} = \frac{.818 - .5(1.81)}{.5} = -.174 \equiv 0
$$

$$
I_{1,1\mathrm{e}}^1 = \frac{I_{1,1}^1 - (1-\eta) I_{1,1w}^1}{\eta} = \frac{.818 - .5 * 0}{.5} = 1.636
$$

- These intensities are used for the next volume to obtain I_{12} and the calculations proceed in exactly the same fashion as outlined in the previous iteration.

Solution 2:19

- The following are the values that will be obtained upon doing the calculations:
- Node (1,2):
  $I^1 = 1.49$, $I^1_E = 1.337$, $I^1_N = 1.163$
- Node (2,1):
  $I^1 = 0.012$, $I^1_E = 0.024$, $I^1_N = 0.024$
- Node (2,2):
  $I^1 = 0.527$, $I^1_E = -0.190$, $I^1_N = 0.024$

Solution 2:20

- The following are values for second direction:

- Node (1,2):
  $I^2 = 0.825$, $I^2_E = 1.337$, $I^2_N = -0.159$
- Node (1,1):
  $I^2 = 1.48$, $I^2_E = 1.319$, $I^2_N = 1.161$
- Node (2,2):
  $I^2 = 0.0159$, $I^2_E = 0.0318$, $I^2_N = 0.0318$
- Node (2,1):
  $I^2 = 0.493$, $I^2_E = 0.9546$, $I^2_N = -0.1749$

Solution 2:21

- The following are values for third direction:
- Node (2,2):
  $I^3 = 0.012$, $I^3_E = 0.0239$, $I^3_N = 0.0239$
- Node (2,2):
  $I^3 = 0.0295$, $I^3_E = 0.0351$, $I^3_N = 0.0589$
- Node (1,2):
  $I^3 = 0.104$, $I^3_E = 0.02072$, $I^3_N = -0.1834$
- Node (1,1):
  $I^3 = 0.215$, $I^3_E = 0.222$, $I^3_N = 0.371$

Solution 2:22

- The following are values for fourth direction:
- Node (2,1):
  $I^4 = 0.0159$, $I^4_E = 0.0319$, $I^4_N = 0.0319$
- Node (2,2):
  $I^4 = 0.0287$, $I^4_E = 0.0255$, $I^4_N = 0.0573$
- Node (1,1):
  $I^4 = 0.114$, $I^4_E = 0.2281$, $I^4_N = 0.1962$
- Node (1,2):
  $I^4 = 0.215$, $I^4_E = 0.203$, $I^4_N = 0.374$

Solution 2:23

- The second iteration is now complete and the process continues until the intensities at all points do not vary within a given tolerance band in consecutive iterations.

Recap
In this class:

- The detaTwo examples will be taken and the solution procedure outlined.
- First is a problem where the view factor enclosure manipulations are illustrated and the second is a problem where the discrete ordinates method is outlined.