Module 3 : Radiation

Lecture 26 : Introduction to Radiation

Objectives
In this class:

- The definitions relevant to radiative heat transfer will introduced.
- Concept of view factor for making heat transfer calculations is discussed.

Radiation: Introduction-1

- Typically thermal radiation is assumed to be electromagnetic with wavelength range from 0.1 – 100 µm; note that visible light range lies in between 0.35 and 0.75 µm.
- A black surface is one which absorbs all incident radiation.
- Optically black surfaces are not necessarily thermally black.

Radiation: Introduction-2

- Emissive power (energy emitted per unit area per unit time per unit wavelength) of a black body emitting into vacuum can be derived using quantum electromagnetic concepts as:

$$E_{\lambda b} = \frac{2\pi C_1}{\lambda^5 (e^{\frac{C_2}{\lambda T}} - 1)};\ \text{where } C_1 = hc_0^{\ 2}, C_2 = \frac{hc_0}{k} \qquad (26.1)$$

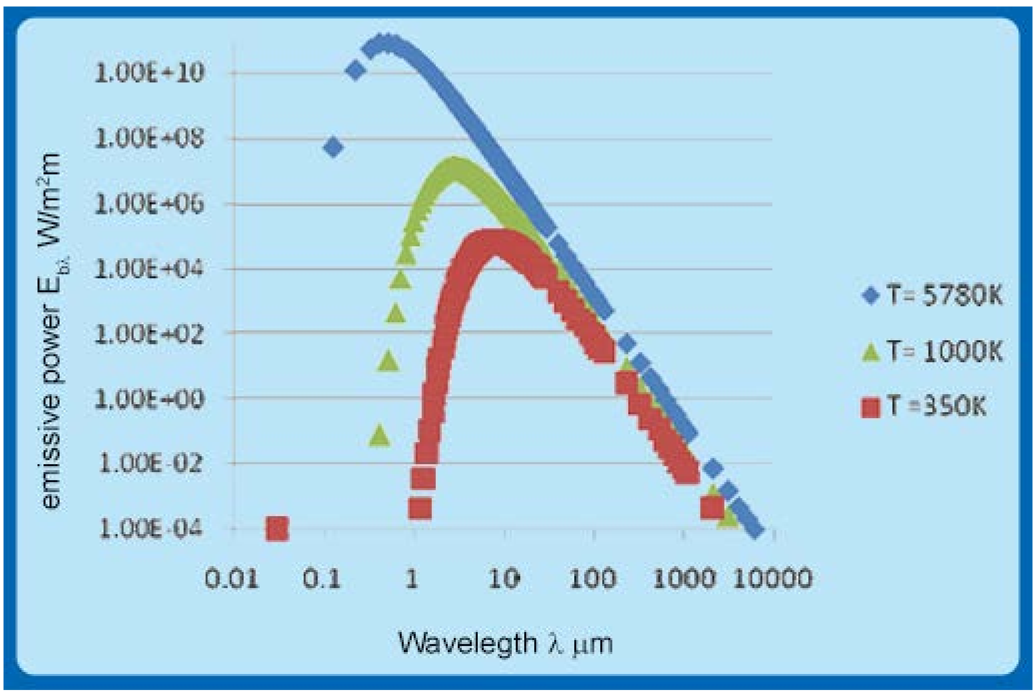
$$h = \text{ Planck's Constant} = 6.62X10^{-34}\ J/s$$

$$c_0 = \text{ Speed of light in vacuum} = 2.99X10^{8}\ m/s$$

$$k = \text{ Boltzmann Constant} = 1.38X10^{-23}\ J/K$$

Radiation: Introduction-3

- Emissive powers at three arbitrary temperatures are plotted below:



Radiation: Introduction-4

- Sun is often modeled as a black body at 5780 K. Notice that very high emissive powers exist in the visible range at this temperature.
- Often very hot bodies have high emissive powers in the visible range (e.g. at 1000 K) making them visible due to their temperatures.
- Low temperature bodies (e.g. at 350 K) can also be 'seen' by devices that can detect radiation in the high wavelength range.

Radiation: Introduction-5

- Total emissive power (energy emitted per unit area per unit time) of a black body, in vacuum, E_b can be obtained from equ^n (26.1) by integrating over all the wavelengths:

$$E_b = \int_0^\infty E_{\lambda b} d\lambda = \int_0^\infty \frac{2\pi C_1}{\lambda^5 (e^{\frac{C_2}{\lambda T}} - 1)} d\lambda = \sigma T^4 \tag{26.2}$$

- 'T' is the absolute temperature in Kelvin and σ = 5.667 x 10^{-8} W/m^2K^4 is the Stefan-Boltzmann constant. When the medium has a refractive index then:

$$E_{\lambda b} = \mu^2 E_{\lambda b}$$

Radiation: Introduction-6

- A general surface will either absorb, transmit or reflect incident radiation.
- The ratio of energy absorbed, reflected or transmitted to the incident energy is defined as absorptivity (α) or reflectivity (ρ) or transmittivity (τ) respectively. Thus:

$$\rho = \frac{E_{\text{reflected}}}{E_{\text{incident}}}; \alpha = \frac{E_{\text{absorbed}}}{E_{\text{incident}}}; \tau = \frac{E_{\text{transmitted}}}{E_{\text{incident}}} \tag{26.3}$$

$$\rho + \alpha + \tau = 1$$

Radiation: Introduction-7

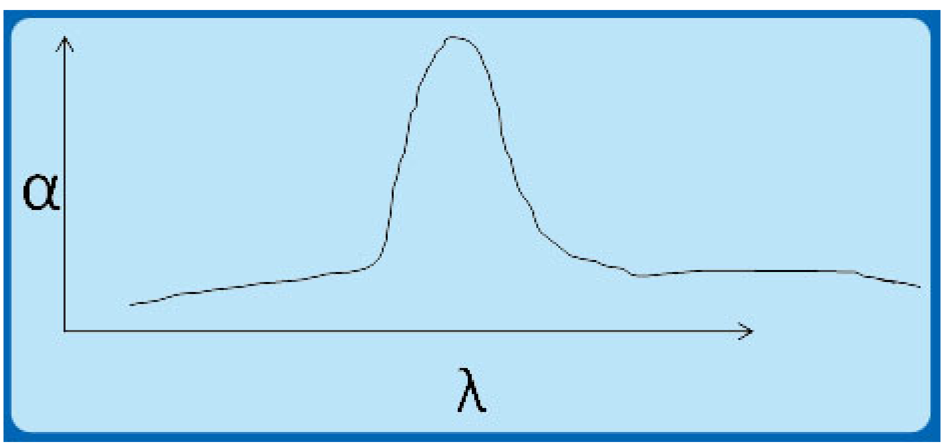
- Reflectivity and emissivity can be specular or diffuse – in general smooth surfaces are specular and rough surfaces are diffuse.
- Materials like glass, Carbon dioxide display strong dependence of emissivity and absorptivity with wavelength. Such materials may let energy contained in radiation at one wavelength value pass through but will block energy contained in radiation of another wavelength.

Radiation: Introduction-8

- A typical example is the green house where the glass walls let energy from the sun that is contained at small wavelengths to pass through. This energy is absorbed by the bodies inside which are at a much lower temperature. Much of the energy radiated by the bodies inside corresponds to larger wavelengths which are not allowed to pass through, heating up the enclosure inside.

Radiation: Introduction-9

- The carbon dioxide in the earth's atmosphere has this very same behaviour and therefore contributes is termed as a 'greenhouse gas'.
- A likely distribution for absorptivity of incident radiation for a material could be the following:



Radiation: Introduction-10

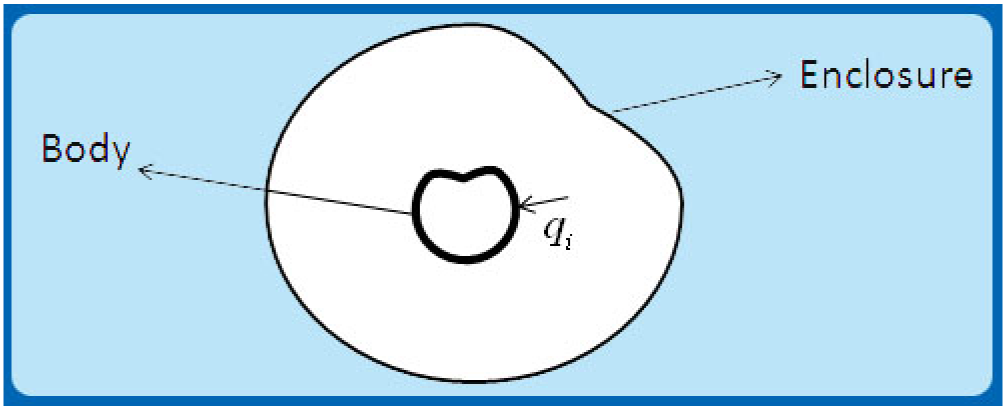
- Emissivity of a surface is the ratio of the energy emitted by a surface and the energy emitted by it if it were a perfect black surface.

$$\varepsilon = \frac{E_{\text{emitted}}}{E_{\text{black}}} \tag{26.4}$$

- Emissivity is often a function of wavelength. Emissivity is unity for black surfaces.
- It is often useful to have a relationship between α and ε. This is the Kirchoff's law which is discussed next.

Kirchoff's law (Thought expt.)-1

- Consider a black enclosure. A body kept in the enclosure is in equilibrium with the enclosure



- Energy absorbed by the body = energy emitted since otherwise its temperature will change

Kirchoff's law (Thought expt.)-2

- Energy emitted by the body is absorbed completely by the black enclosure. The enclosure is assumed to be a very large one.
- Incident energy on body is equal to emitted energy. The body absorbs only part of what is incident on it. Let q_i be incident flux and E_{body} be emitted flux for the body

$$q_i \, A_{\text{body}} \alpha = E_{\text{body}} \cdot A_{\text{body}} \tag{26.5}$$

Kirchoff's law (Thought expt.)-3

- Now put a black body in the enclosure of the same shape and size. Therefore:

$$q_i A_{\text{body}} (1) = E_{\text{blackbody}} \, A_{\text{blackbody}} \tag{26.6}$$

- The temperature of the enclosure is the same and therefore q_i is the same. Divide equ^n (26.5) by equ^n (26.6) to get:

$$\alpha = \frac{E_{\text{body}}}{E_{\text{blackbody}}} = \varepsilon \Rightarrow \alpha = \varepsilon \tag{26.7}$$

- Emissivity = Absorptivity is the Kirchoff's law

Kirchoff's law (Thought expt.)-4

- In reality the emissivity of a body is a function of wavelength, temperature and surface texture and therefore the emissivity is defined for a single wavelength called the spectral monochromatic emissivity which is defined as:

$$\varepsilon_\lambda = \frac{E_\lambda}{E_{b\lambda}} \tag{26.8}$$

- Total energy emitted by the body is therefore:

$$E = \int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda \tag{26.9}$$

Emissivity- Wavelength dependence

- For a black body:

$$E_b = \int_0^\infty \varepsilon_{\lambda b}(=1) E_{b\lambda} d\lambda \tag{26.10}$$

- Therefore using the definition of emissivity in equ^n (26.4):

$$\varepsilon = \frac{E}{E_b} = \frac{\int_0^\infty \varepsilon_\lambda E_{b\lambda} d\lambda}{\sigma T^4} \tag{26.11}$$

- Although Kirchoff's law is derived for bodies thermal equilibrium, it is assumed for cases where equilibrium conditions do not exist.

Gray Body

- A gray body is one whose emissivity is independent of wavelength. Therefore for a gray body:

$$\varepsilon = \varepsilon_\lambda \tag{26.12}$$

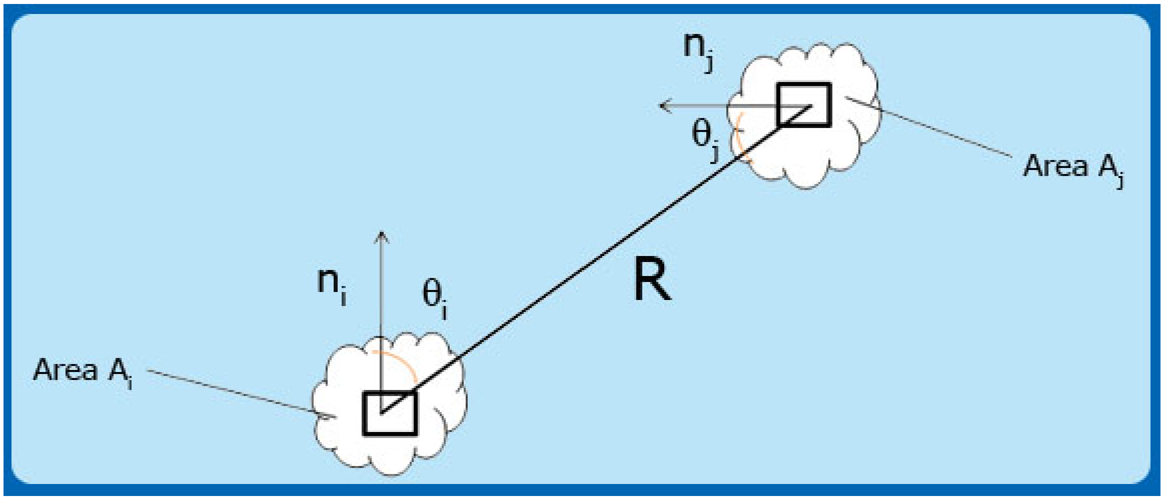
- A diffuse surface emits radiation equally in all directions. Therefore for a diffuse surface the angular dependence of emissivity is absent
- For a gray diffuse surface therefore only one value of emissivity needs to be specified.

Radiation Shape Factor-1

- Usually there are two or more surfaces at different temperatures and the energy exchange between them is desirable
- Consider two surfaces 'i' and 'j' between which energy exchange is to be calculated.
- Consider two infinitesimally small areas 'dA_i' and 'dA_j' on these surfaces separated by a distance 'R'. The unit normals to these areas make angles 'θ_i ' and 'θ_j' respectively.

Radiation Shape Factor-2

- The pictorial view is shown below:



Radiation Shape Factor-3

- Define F_{ij} as the fraction of energy leaving 'i' that is intercepted by 'j'.
- Assume that the surfaces are black
- Energy leaving surface 'i' and reaching 'j' is given by $E_{bi}\ A_i\ F_{ij}$

Similarly energy leaving ‘j’ and reaching ‘i’ is given by $E_{bj}\,A_j\,F_{ji}$

- Net exchange between the surfaces is:

$$E_{bi}\,A_i\,F_{ij} - E_{bj}\,A_j\,F_{ji} \tag{26.13}$$

Radiation Shape Factor-4

- If the two bodies are assumed to be at the same temperature there is no net energy exchange and therefore

$$A_i\,F_{ij} = A_j\,F_{ji} \tag{26.14}$$

- This is the reciprocity relation and is valid even when the bodies are not at identical temperatures.
- Derivation above used black surfaces. The relation is valid for non black diffuse surfaces also.

Radiation intensity-1

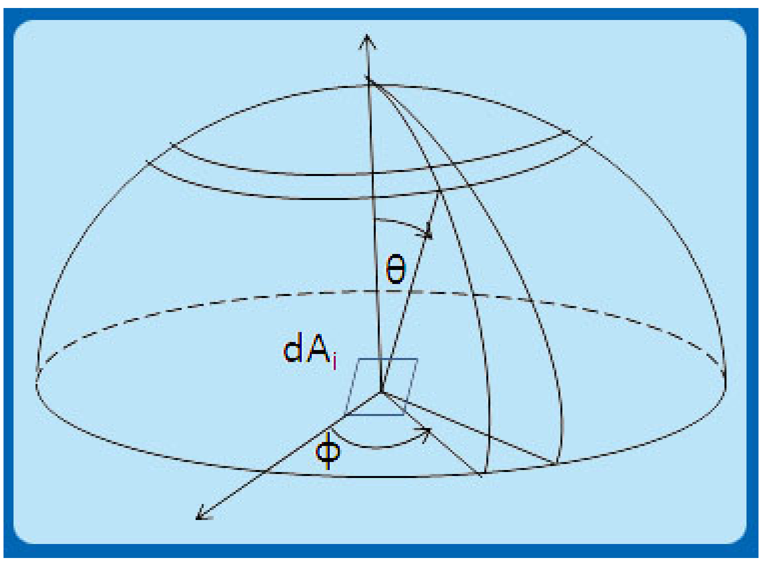
- For a diffuse surface the intensity of radiation is same in all directions.
- Intensity in a particular direction is defined as the radiation per unit area normal to the surface per unit solid angle per unit wavelength.
- Planar angle: Draw two rays at an angle ‘θ’ with a separation ‘dθ’ at the centre of a circle of radius‘r’.

Radiation intensity-2

- The two rays intersect the circumference and include a distance ‘dl’ between them. The planar angle is defined as dl/r.
- Similarly define a solid angle. Consider a sphere and at the center of the sphere, draw rays with an angle dω and they will include an area dA at the circumference and the solid angle is calculated as dA/r^2.

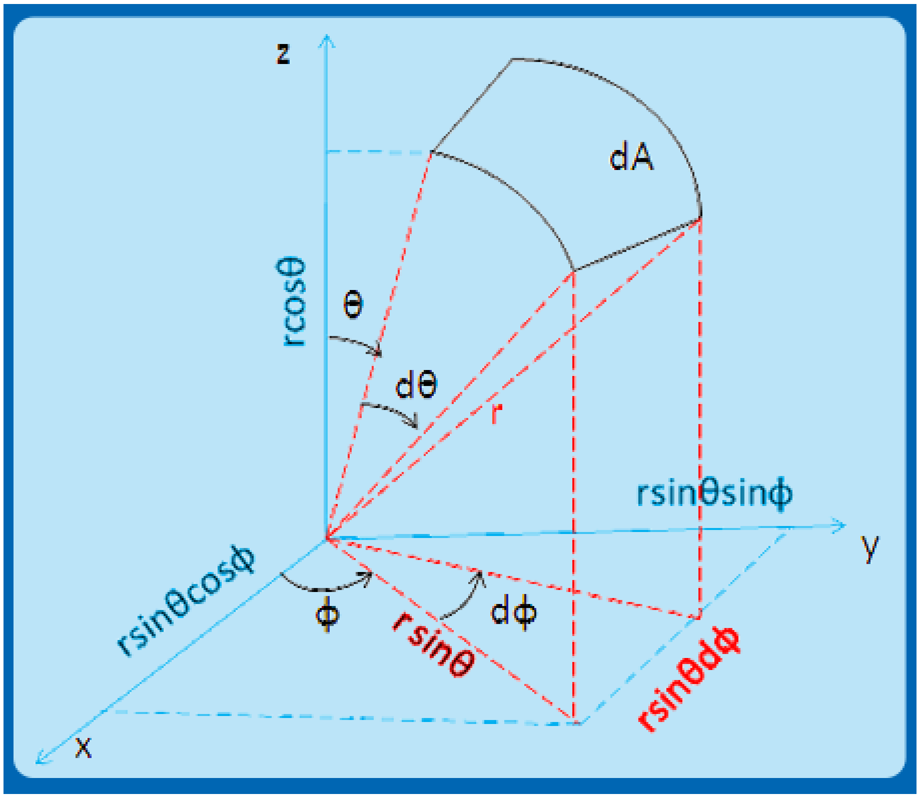
Radiation intensity-3

- Consider a diffuse surface ‘dA_i’ emitting radiation and surround it with a hemisphere. All energy emitted has to be incident on the hemisphere. Consider a small area ‘dA’ on the surface at an angle ‘θ’ from the ‘z’ and ‘φ’ from the ‘x’ axes respectively.



Radiation intensity-4

- An expanded view of the area ‘dA’ is shown below:



$$dA = r d\theta (r \sin\theta d\phi)$$

$$d\omega = \frac{dA}{r^2}$$

Radiation intensity-5

- Intensity is defined as the energy per unit area normal to the emitting surface per unit wavelength per unit solid angle in the given direction:

$$I = \frac{E / A_{\text{emitter}}}{d\omega \, d\lambda} \tag{26.15}$$

- For a diffuse surface the intensity is equal in all directions. Note that $A_{emitter}$ in the above expression is the area of the emitting surface normal to the intensity direction

Radiation intensity-6

- Radiation intercepted from the surface ‘dA_i’ by a small area ‘dA_n’ on the hemispherical surface is

$$I_i dA_i \cos\theta \frac{dA_n}{r^2} \tag{26.16}$$

- The total energy intercepted by the hemisphere per unit area of the emitting surface is therefore:

$$\frac{E_{\lambda \text{total}}}{dA_i} = \int I_\lambda \cos\theta \frac{(r d\theta)(r \sin\theta d\varphi)}{r^2} \tag{26.17}$$

Radiation intensity-7

- Since by assumption intensity I_λ is a constant, it can be taken out of the integral sign in equ^n(26.17) which is simplified as follows:

$$\frac{E_\lambda}{dA_i} = I_\lambda \int_0^{2\pi}\int_0^{\pi/2} \cos\theta \sin\theta d\theta d\varphi$$
$$= 2\pi I_\lambda \int_0^{\pi/2} \cos\theta \sin\theta d\theta$$
$$= 2\pi I_\lambda \left[-\frac{\cos 2\theta}{4}\right]_0^{\pi/2} = \pi I_\lambda \qquad (26.18)$$

Radiation intensity-8

- Equ^n (26.18) is for a given wavelength. Now extend the same to all wavelengths by using

$$E = \int E_\lambda d\lambda = \int \pi I_\lambda d\lambda = \pi I \qquad (26.19)$$

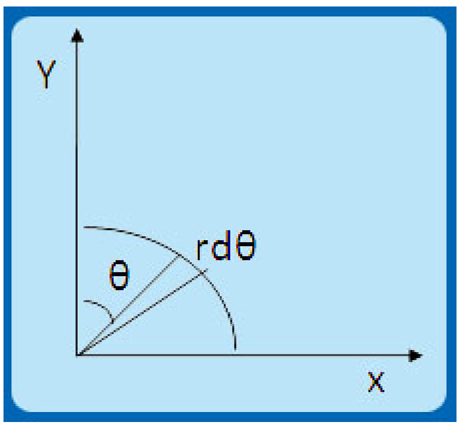
- The hemispherical total (over all wavelengths) emissive power is therefore E = πI
- $E_{blackbody}$ = σT^4 and therefore the intensity for a blackbody is $\sigma T^4/\pi$.

Radiation intensity-9

- Suppose the body is not three dimensional but two dimensional. The relationship between intensity and emissive power will have to be re-derived.
- Intensity can again be defined as the energy per unit length normal to emitting line per unit linear angle per unit wavelength.
- For a diffuse surface assume this is constant and not a variable with respect to angle.

Radiation intensity-10

- Consider a line 'dl' emitting at all angles



- Consider a circle encompassing the linear source. Source emits in the upward direction only and all the energy emitted is intercepted by the circular envelope

Radiation intensity-11

- The energy in the direction 'θ' is given by:

$$dE_\lambda = (I_\lambda)(dl_i \cos\theta)(d\theta) \qquad (26.19)$$

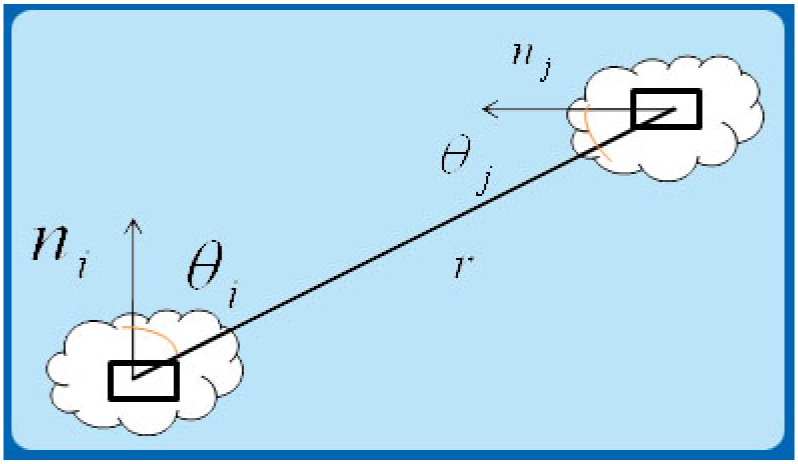
- Integrating the equation gives the total energy per unit length of the emitting surface:

$$\frac{dE_\lambda}{dl_i} = I_\lambda \cos\theta d\theta$$
$$\Rightarrow E_{\lambda 2D} = \int_{-\pi/2}^{\pi/2} (I_\lambda \cos\theta)(d\theta) = 2I_\lambda \qquad (26.20)$$

- Compare (26.18) and (26.20) and notice the totally different relationship between the emissivity and intensity

View Factor Calculation-1

- Now consider the radiation exchange between two surfaces 'i' and 'j'. The distance between elemental areas is 'r' and the outward drawn normals are 'n_i' and 'n_j'



View Factor Calculation-2

- Energy leaving surface 'i' that is intercepted by surface 'j' is given by:

$$
\begin{aligned}
dq_{i-j} &= I_i(\cos\theta_i dA_i)\frac{dA_n}{r^2}\cos\theta_j \\
&= I_i(\cos\theta_i dA_i)\frac{dA_j}{r^2}\cos\theta_j
\end{aligned}
\tag{26.21}
$$

- For a diffuse surface intensity is independent of direction i.e. $I_i = E_i / \pi$. Therefore:

$$dq_{i-j} = E_i \frac{\cos\theta_i \cos\theta_j}{\pi r^2} dA_i dA_j \tag{26.22}$$

View Factor Calculation-3

- Equ^n (26.22) was for an small area on the surfaces. For the entire surface:

$$q_{i-j} = E_i \iint \frac{\cos\theta_i \cos\theta_j}{\pi r^2} dA_i dA_j$$

- Define a geometric factor called the view factor F_{ij}

$$
\begin{aligned}
q_{i-j} &= F_{i-j} E_i A_i \\
F_{i-j} &= \frac{1}{A_i} \iint \frac{\cos\theta_i \cos\theta_j}{\pi r^2} dA_i dA_j
\end{aligned}
\tag{26.23}
$$

Recap
In this class:

- The definitions relevant to radiative heat transfer will introduced.
- Concept of view factor for making heat transfer calculations is discussed.