Module 3 : Radiation

Lecture 28 : Radiative Transport Equation

Objectives
In this class:

- The radiative transport equation will be derived.

Radiative Transport Equation (RTE)-1

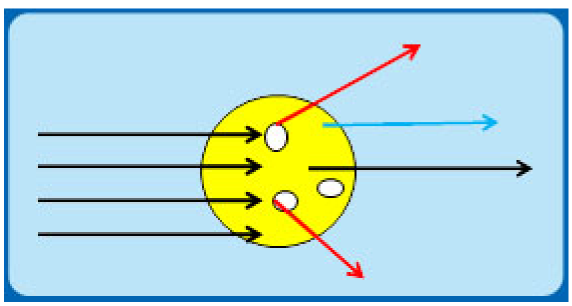
- When scattering exists then the one dimensional approach is no longer valid and another more convenient equation called the radiative transport equation is used.
- Consider a ray moving in a particular direction. Its attenuation and augmentation due to various source/sink terms in that direction are incorporated to obtain a transport equation for intensity of radiation.

Radiative Transport Equation (RTE)-2

- The intensity of a ray of radiation moving through a region in a given direction can either:
  - Increase due to emission
  - Decrease due to absorption
  - Decrease due to scattering from the region
  - Increase due to scattering from neighboring regions
- This is the basis of the derivation of the radiative transport equation

RTE: Different terms-1

- Consider a region (yellow here) where particles are present. A bundle of rays (four black rays in this case), entering the region can either get deflected by the particles (the red rays in this case), or exit without changing path, black ray here, or get absorbed, the bottom black incident ray. In addition the region can emit a ray (blue here)

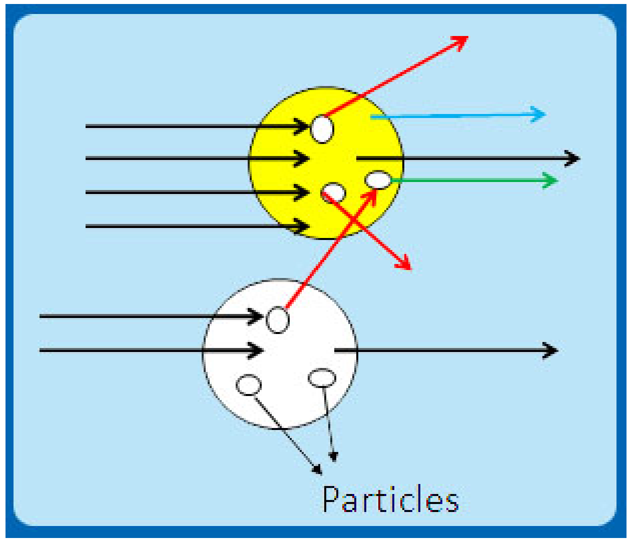


RTE: Different terms-2

- Now consider a neighbouring region also. Here too particles are present and again a bundle of rays entering this region can either be scattered, absorbed or transmitted. The scattered rays from a given region can enter the neighbouring region and again get scattered in a particular direction.

RTE: Different terms-3

- Consider the earlier yellow region. In addition to rays in the given direction an added ray is that due to scattering of a ray from the nearby white region and scattered (green here) in the given direction



RTE: Different terms-4

- Therefore expressing this mathematically:

$$\frac{dI}{ds} = \varepsilon I_b - \alpha I - \rho I \qquad (28.1)$$

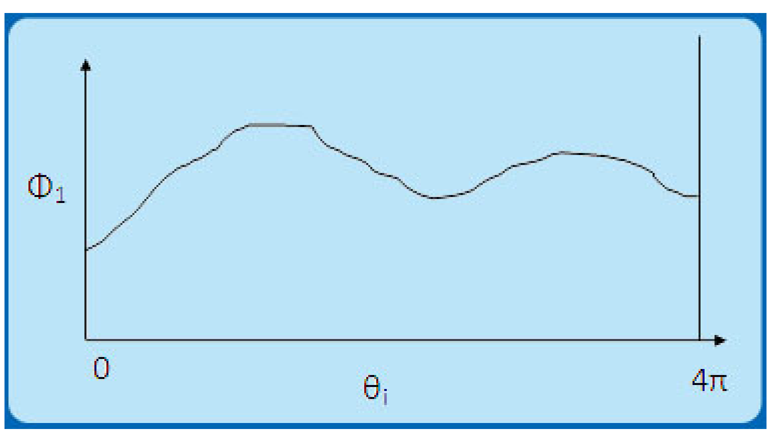
- ε, α, ρ are the emission, absorption and scattering coefficients respectively.
- Use of Kirchoffs laws can give ε = α. Both absorption and scattering attenuates the intensity in the given direction and therefore (α+ρ) is known as the extinction coefficient

RTE: Scattering term-1

- Consider now the mathematical representation of the scattering term due to radiation entering the control volume. The particle distribution within the control volume determines the scattering characteristic.
- Consider a ray in a direction 'θ_i' incident into the region. The probability that it will be scattered in the direction 'θ' which will depend on the probability density function.

RTE: Scattering term-2

- Let Φ_1 be a function such that it is the fraction of intensity of a ray travelling in a direction 'θ_i' that is deflected in the direction 'θ_e'. Therefore Φ_1 is a function of 'θ_i' and could be in general represented as: (Notice that the angle 'θ_i' can vary from 0 to 4π)



RTE: Scattering term-3

- The intensity of rays travelling between θ_i and $\theta_i + d\theta_i$ being deflected in a direction 'θ_{e1}' is given by:

$$\int_{\theta_i}^{\theta_i+d\theta_i} I(\theta_i)\Phi_1(\theta_i)d\theta_i \qquad (28.2)$$

- Similarly there could be another function Φ_2 which could represent the fraction of intensity of a ray travelling in a direction 'θ_i' that is deflected in the direction 'θ_{e2}'

RTE: Scattering term-4

- The intensity of a rays travelling between θ_i and $\theta_i + d\theta_i$ being deflected in a direction 'θ_{e2}' is given by:

$$\int_{\theta_i}^{\theta_i+d\theta_i} I(\theta_i)\Phi_2(\theta_i)d\theta_i \qquad (28.3)$$

- Therefore generalizing the above, intensity of a ray travelling in a direction θ_i that gets deflected in a direction θ_e is given by

$$I(\theta_i)\Phi(\theta_i,\theta_e) \qquad (28.4)$$

RTE: Scattering term-5

- The fraction of intensity of a ray travelling between θ_i and $\theta_i + d\theta_i$ being deflected in a direction 'θ_e' is given by:

$$\int_{\theta_i}^{\theta_i+d\theta_i} \Phi(\theta_i,\theta_e)d\theta_i \qquad (28.5)$$

- The intensity of rays travelling in the direction between 0 and 4π being deflected in the direction 'θ_e' is therefore

$$\int_0^{4\pi} I(\theta_i)\Phi(\theta_i,\theta_e)d\theta_i \qquad (28.6)$$

RTE: Scattering term-6

- Let there be equal intensities of the ray being deflected in all directions. The incident ray can be deflected in any of the 4π angles. Therefore the intensity deflected in any particular angle is $I(\theta_i)\,\Phi(\theta_i, \theta_e) = I(\theta_i)/4\pi$. The fraction of rays incident in the angle range between 0 and 4π that get deflected in any given direction is unity. That is:

$$\int_0^{4\pi} \Phi(\theta_i,\theta_e)\big|_{\text{equal intensity}} d\theta_i = 1 \qquad (28.7)$$

RTE: Scattering term-7

- Therefore since $\Phi(\theta_i,\theta_e)\big|_{\text{equal intensity}}$ is constant:

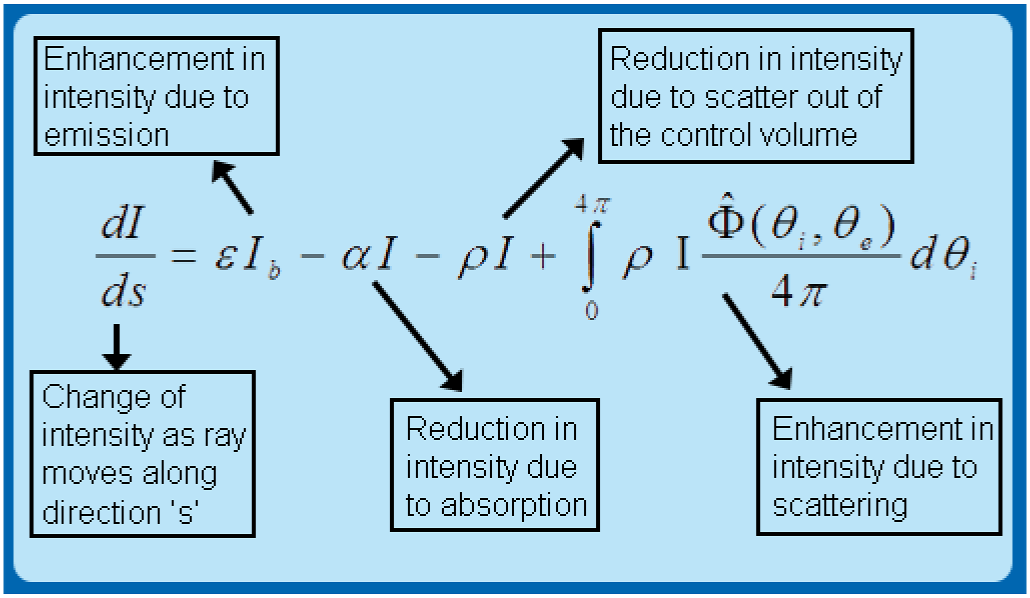
$$\Phi(\theta_i,\theta_e) = \frac{1}{4\pi} \Rightarrow 4\pi\Phi(\theta_i,\theta_e) = 1 \qquad (28.8)$$

$$\text{Define: } 4\pi\Phi(\theta_i,\theta_e) = \hat{\hat{\Phi}}(\theta_i,\theta_e) \qquad (28.9)$$

- The variable $\hat{\hat{\Phi}}(\theta_i,\theta_e)$ is often termed as the scattering phase function.
- The energy scattered from surrounding volumes into a given volume and scattered in a particular direction contributes to radiation intensity in that direction

RTE: Final Equation-1

- Equ^n (28.1) is now written as:



$$\frac{dI}{ds} = \varepsilon I_b - \alpha I - \rho I + \int_0^{4\pi} \rho\, I \frac{\hat{\Phi}(\theta_i, \theta_e)}{4\pi} d\theta_i \tag{28.10}$$

RTE: Final Equation-2

- Equ^n (28.10) is called the radiative transport equation. It is an integro-differential equation.
- To determine the intensity field the equ^n (28.10) needs to be solved. However, the temperature field within the domain must be known to solve the intensity since the emission term I_b can be calculated only if the intensity field is known

RTE: Final Equation-3

- We assume in this course that the influence of conduction and convection are negligible and radiation is the dominant mode of heat transfer.
- An energy balance is derived later on which relates the intensity to the temperature within a given volume of participating medium.

Recap

In this class:

- The radiative transport equation will be derived.