Module 3 : Radiation

Lecture 27 : One Dimensional Radiation Calculations

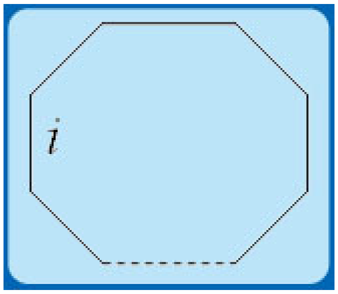
Objectives
In this class:

- Methodology for making radiative heat transfer calculations using the electrical analogy is discussed.
- Using the electrical analogy for calculations in a participating medium are also discussed.

Enclosure View Factor-1

- Consider an enclosure with several surfaces. Consider the i^{th} surface. Energy leaving surface 'i' must be intercepted by the other surfaces

- Let there be a total of 'j' surfaces. i^{th} surface will interact with all these surfaces.



Enclosure View Factor-2

- Let E_i be the emissive power of the i^{th} surface. The total energy emitted by this surface is therefore:

$$A_i E_i \tag{27.1}$$

- Energy received by the j^{th} surface is $A_i F_{ij} E_i$ and Energy received by all surfaces is

$$\sum_{j=1}^{n} A_i F_{ij} E_i \tag{27.2}$$

Enclosure View Factor-3

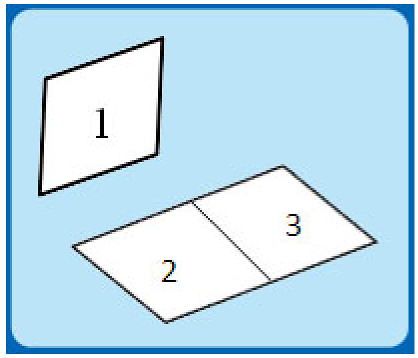
- Equating the energies represented in expressions (27.1) and (27.2):

$$\begin{aligned} A_i E_i &= \sum A_i F_{ij} E_i = A_i E_i \sum F_{ij} \\ \Rightarrow \sum F_{ij} &= 1 \end{aligned} \tag{27.3}$$

- Note the F_{ii} need not necessarily zero i.e. the i^{th} surface can exchange energy with itself also.

Enclosure View Factor-4

- Equ^n (27.3) is very useful for several manipulations. Consider the following geometry. Surface 1 'views' a surface which is a combination of two surfaces: 2 and 3



Enclosure View Factor-5

- Consider an imaginary surface 'e' that will 'close' these two surfaces into a big enclosure. Therefore:

$$F_{11} + F_{12} + F_{13} + F_{1e} = 1 \tag{27.3}$$

- Equ^{n} (27.3) is rewritten as:

$$F_{11} + F_{1\text{-}23} + F_{1e} = 1$$
$$F_{1\text{-}23} = F_{12} + F_{13} \tag{27.4}$$

- This type of approach is useful if e.g. $F_{1\text{-}23}$ and F_{12} are available and F_{13} is required.

Enclosure View Factor-6

- However, note that

$$F_{23\text{-}1} \neq F_{21} + F_{31}$$

- If $F_{23\text{-}1}$ is required and even if F_{21} and F_{31} are available one needs to first evaluate $F_{1\text{-}23}$ and then use the reciprocity and get the desired value for the view factor.

Radiation exchange between nonblack surfaces-1

- Irradiation (G): Total radiation incident on a surface per unit time per unit area
- Radiosity (J): Total radiation that leaves the surface per unit time per unit area
- Assume opaque body i.e. transmittivity = 0

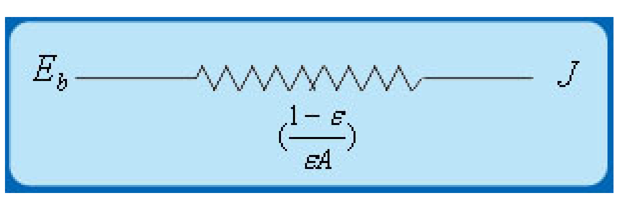
$$J = \varepsilon E_b + \rho G \tag{27.5}$$
$$\tau = 0 \therefore \rho = 1 - \alpha = 1 - \varepsilon \tag{27.6}$$
$$J = \varepsilon E_b + (1-\varepsilon) G \tag{27.7}$$

Radiation exchange between nonblack surfaces-2

- Net energy leaving a surface is the difference between the radiosity and irradiation

$$\frac{Q}{A} = J - G = J - \frac{J - \varepsilon E_b}{1-\varepsilon} = \frac{(1-\varepsilon)J - J + \varepsilon E_b}{1-\varepsilon}$$
$$\therefore Q = \frac{\varepsilon A}{1-\varepsilon}(E_b - J) \quad \text{or} \quad \frac{Q}{A} = \frac{E_b - J}{\left(\frac{1-\varepsilon}{\varepsilon}\right)} \tag{27.8}$$

- Amenable to electrical analogue (Oppenhiem, 1964):



Radiation exchange between nonblack surfaces-3

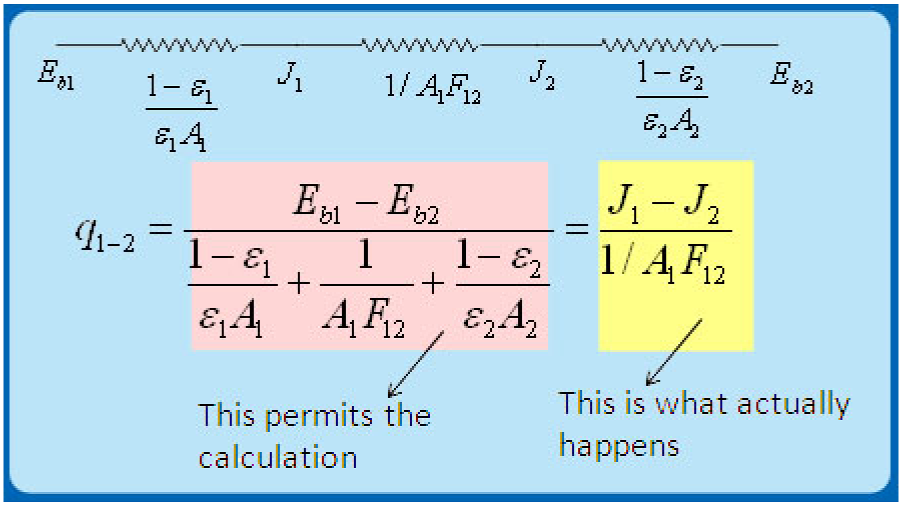
- Two or more surfaces essentially interact through the radiosities
- Energy leaving surface 1or 2 and intercepted by surface 2 or 1 respectively are $J_1A_1F_{12}$, $J_2A_2F_{21}$
- Net exchange is therefore expressed as:

$$q_{1-2} = J_1A_1F_{12} - J_2A_2F_{21} = \left(J_1 - J_2\right) A_2F_{21}$$

$$q_{1-2} = \frac{J_1 - J_2}{1/A_2F_{21}} = \frac{J_1 - J_2}{1/A_1F_{12}} \qquad (27.9)$$

Radiation exchange between nonblack surfaces-4

- Equ^{n}s (27.8) and (27.9) are combined together to obtain the electrical circuit:



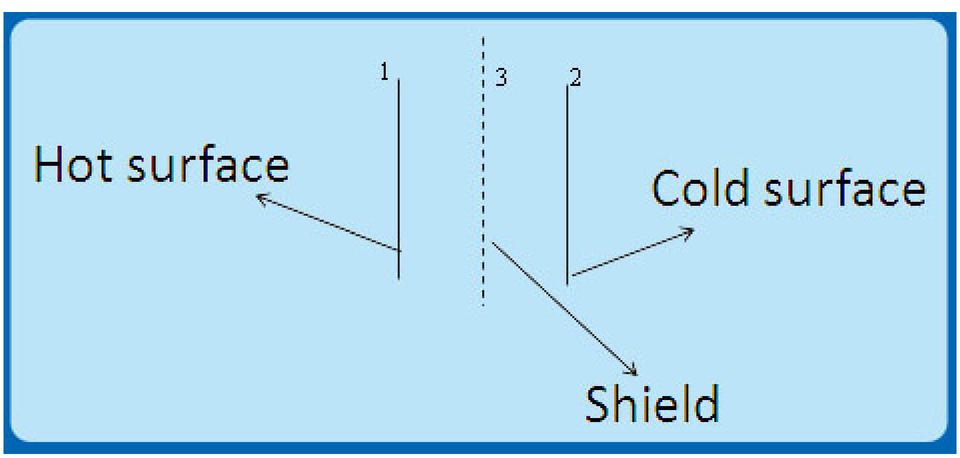
- Easily extendable to multiple surfaces

Radiation exchange between nonblack surfaces-5

- Perfectly insulated surfaces can also participate in the radiation process and are typically called reradiating surfaces. However, $J = E_b$ since 'current' (i.e. heat flux) to the surface is zero i.e. a perfect insulator radiates whatever energy is incident on it.

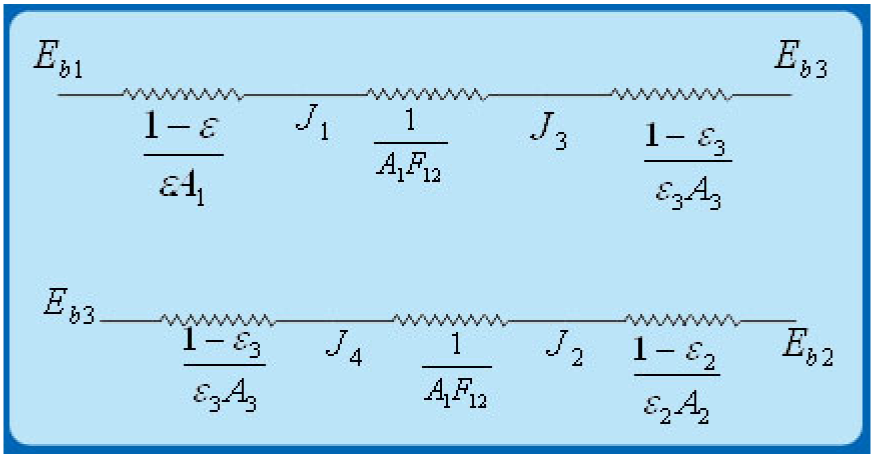
Radiation exchange between nonblack surfaces: Example-1

- It is easy to block thermal radiation. A radiation shield is just a simple obstruction in the path of the radiation and is quite effective.



Radiation exchange between nonblack surfaces: Example-2

- Surfaces 1 and 2 exchange radiation between them and a one dimensional analysis is carried out here. The electrical circuit is:



Radiation exchange between nonblack surfaces: Example-3

- Note that J_3 and J_4 are different . However, E_{b3} is the same since it is a function of temperature of the surface only
- Assume $A_1 = A_2 = A_3$. $F_{12} = F_{13} = F_{23} = 1$. This is good for infinitely large surfaces. The heat transferred between surface 1 and surface 3 is therefore:

$$Q = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1F_{12}} + \frac{1-\varepsilon_3}{\varepsilon_3 A_3}} \qquad (27.10)$$

Radiation exchange between nonblack surfaces: Example-4

- Equ^n (27.9) is simplified to:

$$\frac{Q_{1-3}}{A} = \frac{\sigma(T_1^4 - T_3^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \qquad (27.11)$$

- At steady state the same heat should be transferred to the outer surface also

$$\frac{Q_{3-2}}{A} = \frac{\sigma(T_3^4 - T_2^4)}{\frac{1}{\varepsilon_3} + \frac{1}{\varepsilon_2} - 1} \qquad (27.12)$$

Radiation exchange between nonblack surfaces: Example-5

- Assume $\varepsilon_1 = \varepsilon_2 = \varepsilon_3$ and equating equ^{n}s (27.11) and (27.12) gives the shield temperature:

$$T_3^4 = \frac{(T_1^4 + T_2^4)}{2} \qquad (27.13)$$

- Now compute the heat transferred from one plate to another:

$$Q = \frac{\sigma\left(T_1^4 - \frac{(T_1^4 + T_2^4)}{2}\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} = \frac{1}{2}\,\frac{\sigma\left(T_1^4 - T_2^4\right)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_3} - 1} \qquad (27.14)$$

Radiation exchange between nonblack surfaces: Example-6

- Notice that the 'Q' in equ^n (27.14) is half that without the shields in between the surfaces 1 and 2, i.e. a single shield in between reduces radiation heat loss by 50%. The shields are therefore very effective. Use more shields depending on the need.
- The procedure for calculations for opaque surfaces has been illustrated with the radiation shield

example.

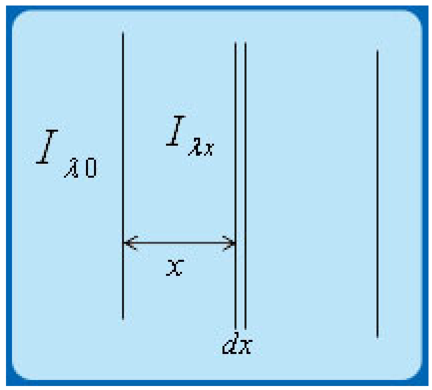
Gas/Participating Medium Radiation - 1

- Several gases are transparent to radiation at most wavelengths of interest e.g. N_2 and O_2 are transparent at low temperatures but H_2O and CO_2 absorb some wavelengths considerably.
- Consider radiation of a wavelength λ and intensity I_λ entering a thick gas layer
- The radiation will get attenuated as it passes through the gas

Gas/Participating Medium Radiation - 2

- Consider a beam of radiation of intensity $I_{\lambda 0}$ passing through an absorbing medium. Change in intensity of the beam as it goes through a small distance 'dx' is given by:

$$dI_\lambda = -a_\lambda I_\lambda dx \qquad (27.15)$$



Gas/Participating Medium Radiation - 3

- a_λ is defined as monochromatic absorption coefficient and has units m^{-1}.
- Equ^n (27.15) is integrated to obtain the variation of intensity as a function of distance:

$$\int \frac{dI_\lambda}{I_\lambda} = \int_0^x -a_\lambda dx \Rightarrow \frac{I_{\lambda x}}{I_{\lambda 0}} = e^{-a_\lambda x} \qquad (27.16)$$

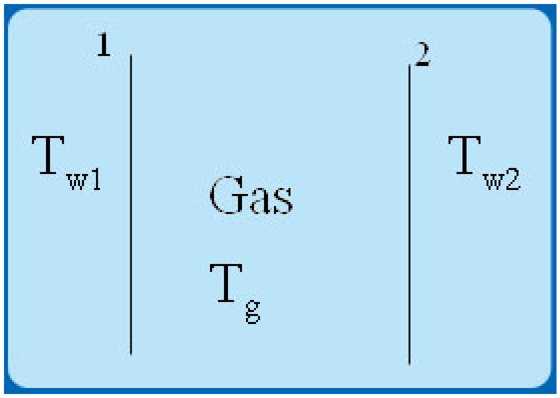
- Equ^n (27.16) is often known as the Beer's law

Gas/Participating Medium Radiation - 4

- Equ^n (27.16) is based on the assumption that the beam passes linearly across a given layer. In reality the beam can originate anywhere and pass through the thickness at various angles. This more complicated case will be handled later.
- A characteristic length called the mean beam length, defined as L_e= 3.6V/A, where V and A are the total volume and surface area which takes care of the problem outlined above in an empirical fashion.

Gas/Participating Medium Radiation - 5

- Consider a volume of gas at temperature T_g enclosed between two surfaces at temperature T_w. Surfaces 1 and 2 will interact through the gas.



Gas/Participating Medium Radiation - 6

- Assume the medium (subscript 'm' for medium) i.e. the gas in between, does not reflect. Then using $\rho = 0$ in equ^n (26.2):

$$\alpha_m + \tau_m = 1 \Rightarrow \varepsilon_m + \tau_m \Rightarrow \tau_m = 1 - \varepsilon_m \tag{27.17}$$

- Therefore, only a part of the energy leaving the surface is transmitted through the medium. Only a portion of this energy is intercepted by another surface due to view factor limitations.

Gas/Participating Medium Radiation - 7

- The energy leaving surface 1 and getting intercepted by surface 2 is:

$$Q_{1\rightarrow 2} = J_1 A_1 F_{12} \tau_m \tag{27.18}$$

- Similarly energy leaving surface 2 and getting intercepted by surface 1 is:

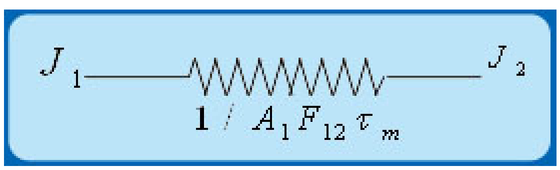
$$Q_{2\rightarrow 1} = J_2 A_2 F_{21} \tau_m \tag{27.19}$$

- Net energy interaction from surface 1 to surface 2 is therefore:

$$J_1 A_1 F_{12} \tau_m - J_2 A_2 F_{21} \tau_m = (J_1 - J_2) A_1 F_{12} (1 - \varepsilon_m) \tag{27.20}$$

Gas/Participating Medium Radiation - 8

- The equ^n (27.20) is representable in the form of an electrical circuit as:



- In addition the medium will radiate to the two walls. Since the medium is assumed to be a non reflecting one it can only emit and therefore (use equ^n (27.5)):

$$J_m = \varepsilon_m E_{bm} \tag{27.20}$$

Gas/Participating Medium Radiation - 9

- The energy received by the surface1 due to the gas would therefore be

$$\varepsilon_m E_{bm} F_{m1} A_m \tag{27.21}$$

- Medium also receives energy from surface 1 since it can participate:

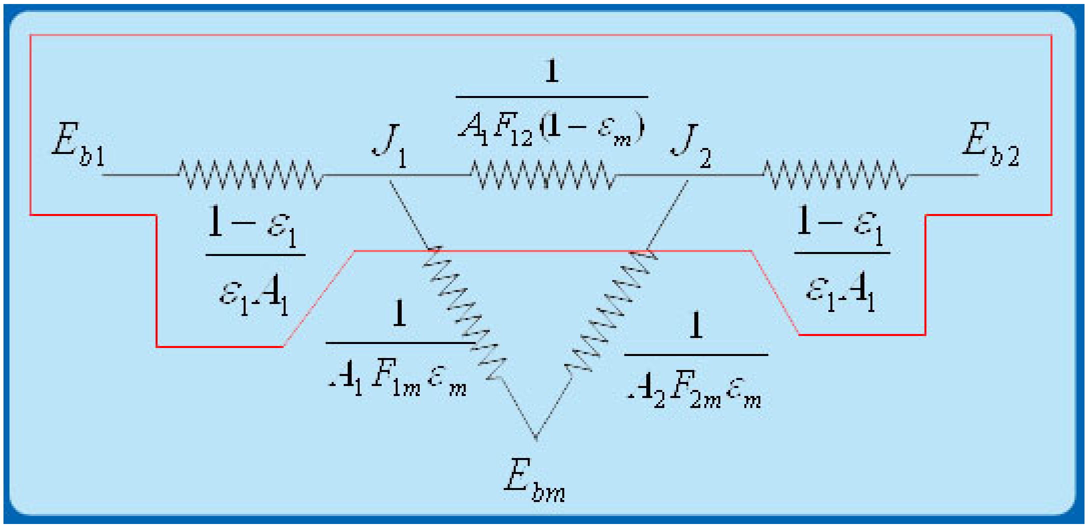
$$J_1 A_1 F_{1m} \alpha_m \tag{27.22}$$

- Net exchange between medium and surface 1 is

$$\varepsilon_m E_{bm} F_{m1} A_m - J_1 A_1 F_{1m} \alpha_m = \frac{E_{bm} - J_1}{1/A_1 F_{1m} \varepsilon_m} \quad (27.23)$$

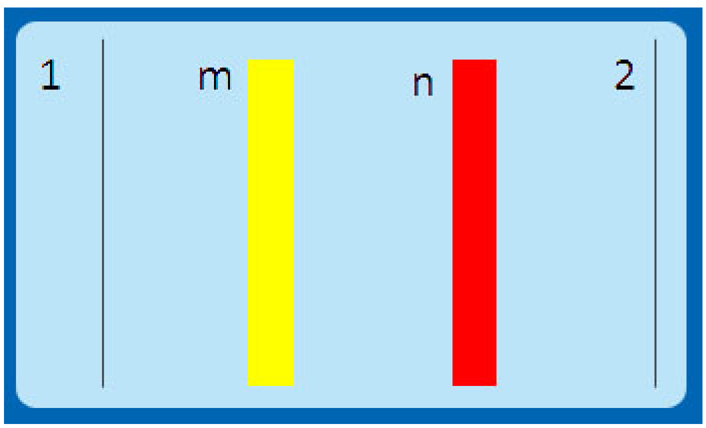
Gas/Participating Medium Radiation - 10

- Equ^{n}s (27.20) and (27.23) are combined together in the form of an electrical circuit. Note that for no gas in between the portion marked red will only exist with $(1 - \varepsilon_m) = 1$



Gas/Participating Medium Radiation - 11

- Consider now two layers of gaseous participating medium, ‘m’ and ‘n’ in between the two surfaces ‘1’ and ‘2’



Gas/Participating Medium Radiation - 12

- Surface 1 will interact with surface ‘2’, medium ‘m’ and medium ‘n’.
- Surface 1 interacts with surface 2 through the media ‘m’ and ‘n’. Radiation leaving surface 1 is attenuated due to the transmittivity of the two media in between. The same is true of surface 2 interacting with surface 1. the net energy interaction between the surfaces is:

$$Q_{1\leftrightarrow 2} = A_1 F_{12} J_1 \tau_m \tau_n - A_2 F_{21} J_2 \tau_m \tau_n \quad (27.24)$$

Gas/Participating Medium Radiation - 13

- Equation (27.24) is rewritten as:

$$Q_{1\leftrightarrow 2} = \frac{J_1 - J_2}{1/(A_1 F_{12} \tau_m \tau_n)} = \frac{J_1 - J_2}{1/(A_1 F_{12}(1-\varepsilon_m)(1-\varepsilon_n))} \quad (27.25)$$

- Surface 1 interacts with medium m. Energy absorbed by medium ‘m’ out of energy incident on it from surface 1 is:

$$A_1 F_{1m} \alpha_m J_1 = A_1 F_{1m} \varepsilon_m J_1 \quad (27.26)$$

- Energy incident on surface 1 from medium 'm' is given by:

$$A_m F_{m1} \varepsilon_m E_{bm} \quad (27.27)$$

Gas/Participating Medium Radiation - 14

- Net energy exchange between 1 and 'm' is obtained by combining equ^{n}s (27.26) and (27.27) together:

$$Q_{1\leftrightarrow m} = A_1 F_{1m} \varepsilon_m J_1 - A_m F_{m1} \varepsilon_m E_{bm} = \frac{J_1 - E_{bm}}{1/A_1 F_{1m} \varepsilon_m} \quad (27.28)$$

- Surface 1 also interacts with medium 'n'. There is the medium 'm' in between which attenuates the amount of energy incident on medium 'm' due to the surface 1. Out of the energy incident on 'n' the part absorbed is:

$$(A_1 F_{1n} J_1 \tau_m) \varepsilon_n = A_1 F_{1n} J_1 (1 - \varepsilon_m) \varepsilon_n \quad (27.29)$$

Gas/Participating Medium Radiation - 15

- Medium 'n' emits energy to surface 1 but again a portion is absorbed by medium 'm' in between. Energy received by surface 1 is:

$$A_n F_{n1} \varepsilon_n E_{bn} \tau_m = A_n F_{n1} \varepsilon_n E_{bn} (1 - \varepsilon_m) \quad (27.30)$$

- Net energy exchange between surface 1 and 'n' is therefore:

$$Q_{1\leftrightarrow n} = (A_1 F_{1n} J_1 \tau_m) \varepsilon_n - A_n F_{n1} \varepsilon_n E_{bn} (1 - \varepsilon_m) = \frac{J_1 - E_{bn}}{1/(A_1 F_{1n} J_1 (1 - \varepsilon_m) \varepsilon_n)} \quad (27.31)$$

Gas/Participating Medium Radiation - 16

- Energy emitted by 'm' received by 'n' is:

$$\varepsilon_m E_{bm} A_m F_{mn} \alpha_n = \varepsilon_m E_{bm} A_m F_{mn} \varepsilon_n \quad (27.32)$$

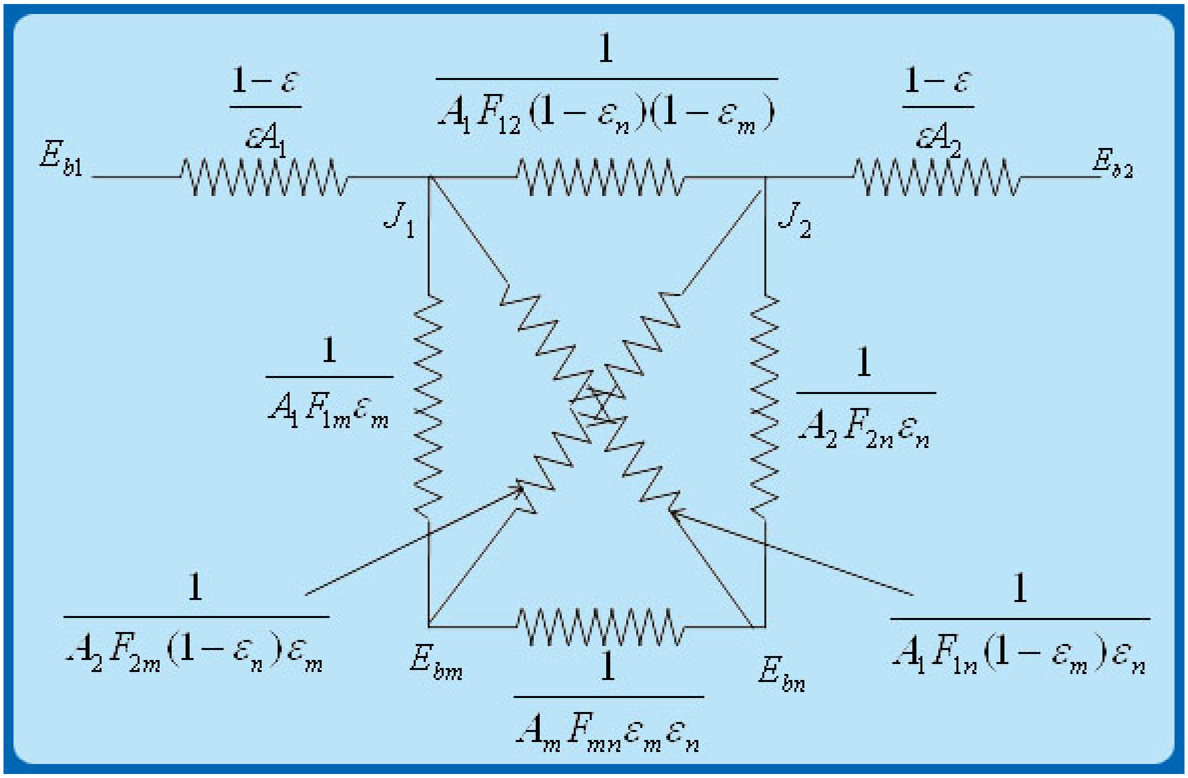
- Net energy exchange between 'm' and 'n' is:

$$Q_{m-n} = E_{bm} A_m F_{mn} \varepsilon_n \varepsilon_m - A_n F_{nm} \varepsilon_n \varepsilon_m E_{bn} = \frac{E_{bm} - E_{bn}}{1/A_m F_{mn} \varepsilon_n \varepsilon_m} \quad (27.33)$$

- Equations (27.25), (27.28), (27.31), (27.33) along with similar equations can be combined to give the electrical circuit for the problem.

Gas/Participating Medium Radiation - 17

- Equivalent electric circuit is:



Recap

In this class:

- Methodology for making radiative heat transfer calculations using the electrical analogy is discussed.
- Using the electrical analogy for calculations in a participating medium are also discussed.