Module 2 : Convection

Lecture 20a : Illustrative examples

Objectives

In this class:

- Examples will be taken where the concepts discussed for heat transfer for tubular geometries in earlier classes will be used. An example of the performance evaluation of a roughened tube is also discussed.

Example 1 (Problem statement-1)

- You have come up with a new method of determining the Prandtl number of a fluid. You take a fluid with Pr = 10, allow it to flow through a pipe and conduct steady heat transfer experiments at several laminar Reynolds numbers at constant wall flux conditions and determine the Nusselt number at a given location in the pipe. This is your master file. Now you conduct experiments for the fluid for which the Prandtl no. is unknown and determine the

Example 1 (Problem statement-2)

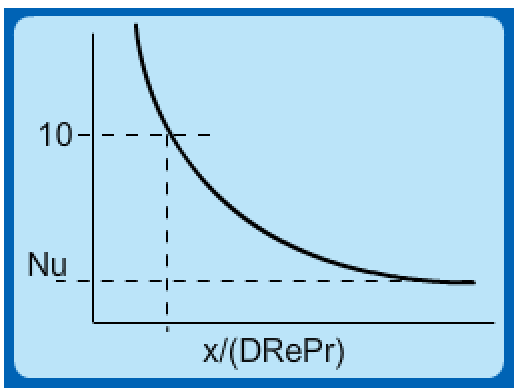
- Nusselt no. at the same location. Now using your master chart you calculate the required Pr number. Assume you did an experiment and found a Nu = 10 for an Re = 100. Your master file shows that for Nu = 10, Re = 10. What is the Prandtl number for this new fluid?

Example 1 (Solution-1)

- The flow is laminar and surface is maintained at constant wall flux. Fully developed Nusselt number is therefore 4.36. Here the value is higher and therefore the flow is developing.
- In the developing region, Nusselt number is a function of Graetz number.
- In the non-dimensional plane the relationship remains the same

Example 1 (Solution-2)

- As already discussed earlier the variation is as shown below:



- At the given location Nusselt number = constant
- Therefore: 100 (Pr) = 10 (10) and Pr = 1 is the unknown value.

Example 2 (Problem statement-1)

- A fluid is heated by passing it through a circular tube (k = 15W/mK) of diameter D_i = 50 mm and D_o = 60 mm and length L = 10 m. Outer surface D_o is maintained at 150 degrees. If the Reynolds No. and inlet temperature are 100000 and 30°, find outlet temperature T_{mo} and total heat transfer rate to the tube. Assume $\mu = 2.5\times10^{-5}$ Pa-s, density = 1 kg/m^3
- Cp = 1000 J/kg-K, k = 0.02W/mK, Pr = 0.7.
- Assume flow to be fully developed in the pipe.

Example 2 (Solution-1)

- Flow Reynolds number is very large and therefore assume it is fully turbulent. Since flow is fully developed, heat transfer coefficient is constant. Use a standard correlation to evaluate the Nusselt number

$$Nu = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4} = 0.023 \times (10000)^{0.8}\,0.7^{0.4}$$
$$= 199.42 = hD/k \Rightarrow h = 79.8 W/m^2K$$

- This heat transfer coefficient is the one inside the tube and heat flux is outside the tube.

Example 2 (Solution-2)

- Using the 1-D approach for calculation of overall heat transfer coefficient gives:

$$\frac{1}{hA} = \frac{1}{h_i A_i} + \frac{\ln(r_o/r_i)}{2\pi l d}$$
$$1/hA = 0.1\left[\frac{1}{79.8\pi(50\times10^{-3})} + \ln\frac{60/50}{2\pi\times15}\right] = .0082$$
$$\Rightarrow hA = 121.95$$

Example 2 (Solution-3)

- Now use the expression for temperature increase in tube for fully developed flow:

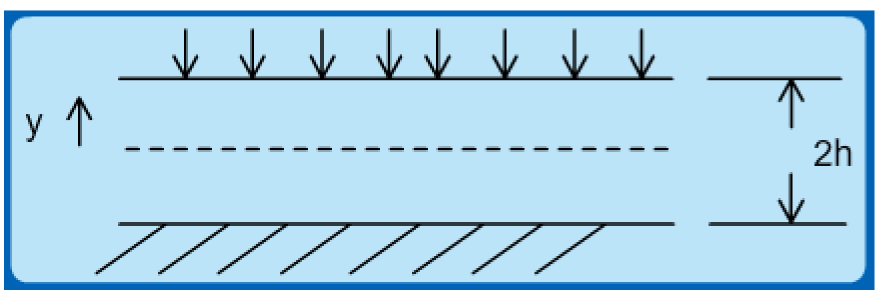
$$\frac{T_\infty - T_o}{T_\infty - T_i} = \exp\left(\frac{-hA}{\dot{m}C_p}\right)$$
$$\frac{150 - T_o}{150 - 30} = \exp\left(\frac{-121.95}{0.098(1000)}\right) \Rightarrow T_o = 115.4$$

Example 3 (Problem statement-1)

- Consider fully developed laminar flow between parallel plates. A constant heat flux q is applied on top plate and constant heat flux 2q on the bottom plate. Determine the temperature distribution for thermally and hydrodynamically fully developed flow. Assume properties are constant. Determine the Nusselt No. for each wall. Use principle of superposition using known solutions.

Example 3 (Solution-1)

- Consider the following problem for fully developed flow where a heat flux is applied on the top wall and the bottom is insulated:



- Show that (this is an exercise problem) the temperature within the fluid 'T' is:

$$T - T_w = \frac{qh}{2k}\left[\frac{3y^{*2}}{4} - \frac{y^{*4}}{8} + y^* - \frac{13}{8}\right]$$
$$where: y^* = \frac{y}{h}; T|_{y=h} = T_w;$$

Example 3 (Solution-2)

- Show now, that the current problem is a superposition of two problems of the above nature – the procedure is identical to what was done in 'conduction'. Write the equations and split into two problems.
- The two problems are:

(a) top wall with heat flux ‘q’ and bottom wall insulated (denote temperature profile by T_1) (b) top wall insulated and bottom wall with heat flux ‘2q’ (denote temperature profile by T_2)

Example 3 (Solution-3)

- Therefore the two temperature profiles are:

$$T_1 - T_{w_1} = \frac{qh}{2k}\left[\frac{3y^{*2}}{4} - \frac{y^{*4}}{8} + y^* - \frac{13}{8}\right]$$

$$T_2 - T_{w_2} = \frac{qh}{k}\left[\frac{3y^{*2}}{4} - \frac{y^{*4}}{8} - y^* - \frac{13}{8}\right]$$

- The combined profile is therefore:

$$T_1 + T_2 = T_{w_1} + T_{w_2} + \frac{qh}{2k}\left[\frac{9y^{*2}}{4} - \frac{3y^{*4}}{8} - y^* - \frac{39}{8}\right]$$

Example 3 (Solution-4)

- Note however that the wall temperatures T_{w1} and T_{w2} are the wall temperatures of the problems corresponding to the individual problems. The wall temperature corresponding to the top wall for the combined problem is therefore:

$$(T_1 + T_2)_{y^*=1} = (T_{w_1} + T_{w_2}) + \frac{qh}{2k}\left[\frac{9}{4} - \frac{3}{8} - 1 - \frac{39}{8}\right]$$

$$\Rightarrow (T_{w_1} + T_{w_2}) = (T_1 + T_2)_{y^*=1} - \frac{qh}{2k}(-4)$$

Example 3 (Solution-5)

- The solution to the problem at hand is therefore:

$$T_1 + T_2 = T_{w_{\text{top}}} + \frac{qh}{2k}\left[\frac{9y^{*2}}{4} - \frac{3y^{*4}}{8} - y^* - \frac{39}{8} + 4\right]$$

- Now compute the bulk temperature:

$$T_{\text{bulk}} - T_{w_{\text{top}}} = \frac{\int_{-1}^{1} \frac{3}{2}\bar{u}(1-y^{*2})\frac{qh}{2k}\left[\frac{9y^{*2}}{4} - \frac{3y^{*4}}{8} - y^* - \frac{39}{8} + 4\right] h\,dy^*}{2h(\bar{u})}$$

$$= \frac{3qh}{8k}\left[\frac{9}{4}\left(\frac{y^{*3}}{3} - \frac{y^{*5}}{5}\right) - \frac{3}{8}\left(\frac{y^{*5}}{5} - \frac{y^{*7}}{7}\right) - 1\left(\frac{y^{*2}}{2} - \frac{y^{*4}}{4}\right) - 0.875\left(y^* - \frac{y^{*3}}{3}\right)\right]_{-1}^{+1}$$

Example 3 (Solution-6)

- Therefore:

$$T_{w_{\text{top}}} - T_{\text{b}} = -\frac{3qh}{8k}(0.5 - 0.21 - 0.583)(2) = \frac{qh}{k}(0.228)$$

- Now introduce the heat transfer coefficient

$$q = \hat{h}(T_{w_{\text{top}}} - T_{\text{b}}) = \hat{h}\frac{qh}{k}(0.228)$$

- Compute the Nusselt number for the top wall:

$$Nu_{top} = \frac{4\hat{h}h}{k} = 4/0.228 = 17.54$$

- Nusselt number for the bottom wall can be similarly computed

Example 4 (Problem statement-1)

- You are working for an industry which is in the business of manufacturing tubes with enhancement devices to obtain high heat transfer coefficients. The following data was obtained using two enhancement techniques on channels with the same heat transfer area:

Technique No.	Friction Factor	Nusselt No.	Reynolds no.	Prandtl No.
1	0.3	150	20000	0.7
2	0.5	160	16874	0.7

Example 4 (Problem statement-2)

- The following is useful information for smooth tube:

$$f = 0.314/Re^{0.25}, \; Nu = 0.023Re^{0.8} \, Pr^{0.33}$$

- Using the constant pumping power and constant surface area criterion, determine which of the enhancement techniques is superior.

Example 4 (Solution-1)

- The criterion to check is

$$\frac{k}{k_s} = \frac{St}{St_s(f/f_s)^{1/3}}$$

- If the above ratio is greater than unity then the enhancement technique is better than the smooth tube at constant pumping power
- Constant pumping power gives:

$$\frac{Gr}{Grs} = \left(\frac{fs}{fr}\right)^{\frac{1}{3}}$$

Example 4 (Solution-2)

- Manipulate the constant pumping criterion a little to get:

$$Re_s = \left(\frac{fr}{f_s}\right)^{\frac{1}{3}} Re_r = 20000(0.3)^{1/3}\left(\frac{Re_s^{\;0.25}}{0.314}\right)^{1/3} \Rightarrow Re_s = 48398$$

- Note that Re_s obtained is the corresponding Reynolds number for which the smooth and rough tubes would have the same pumping power at the given Reynolds number Re_r for the roughened tube.

Example 4 (Solution-3)

- Compute $Nu_s = .023Re^{.8}\,Pr^{.33} = 114.3$

- Now compute the conductance ratio:

$$\frac{k}{k_s} = \frac{Nu(Re_s)(Pr_s)}{(Re_r)(Pr_r)(Nu_s)(f/f_s)^{1/3}} = 1.31$$

- Similarly compute for the second tube:

$$\mathrm{Re}_s = \left[\frac{16874(0.5)^{1/3}}{(0.314)^{1/3}} \right]^{1/0.9167} = 48396; Nu_s = 114.3$$

$$\frac{k}{ks} = \frac{Nur(\mathrm{Re})}{Nus(\mathrm{Re})(f / fs)^{1/3}} = \frac{Nur}{Nus} = \frac{160}{114.3} = 1.40$$

Example 4 (Solution-4)

- Therefore the ratio of the conductances is higher for the tube 2 which is therefore better
- Note that both the rough tubes had to be compared with a smooth tube for taking a decision since the complete information about the tubes was not available.
- Here, the smooth tube Reynolds number turned out to be identical. In reality, compute the conductance ratio at several smooth tube Reynolds numbers for both the situations and compare at a given smooth Reynolds number.

Recap

In this class:

- Examples will be taken where the concepts discussed for heat transfer for tubular geometries in earlier classes will be used. An example of the performance evaluation of a roughened tube is also discussed.