Module 2 : Convection

Lecture 19 : Heat Transfer Enhancement Evaluation in Roughened Tubes

Objectives

In this class:

- Methodologies for evaluation of the performance of heat transfer enhancement devices in tubes are discussed. The tube wall is directly supplied a heat flux.
- Material essentially borrowed from “Application of Rough Surfaces to Heat Exchanger Design”, International Journal of Heat and Mass Transfer pg. 1647, 1972 by R.L. Webb & E.R.G.Eckert]

Rough Surface heat Transfer in tubes-1

- Rough tubes enhance heat transfer coefficients. Friction also increases. Tradeoff is required. Three major cases:
    - Reduced ht. area for equal heat exchange and friction power.
    - Increased heat exchange capability for equal surface area and friction power.
    - Reduce friction power expenditure for equal transfer of heat.

Rough Surface heat Transfer in tubes-2

- The ratios of the area, conductance and pumping power of the rough and smooth surfaces A_r/A_s , k_r/k_s and P_r/P_s respectively are parameters of interest.
- Here: P = Pumping power, A = heat transfer area, k = hA overall heat conductance. Recall that 1/hA is the resistance to heat transfer
- Use these parameters to judge the ‘usefulness’ of the enhancement device

Rough Surface heat Transfer in tubes-3

- Assume all properties are constant
- The ratios of the conductivities of a rough and smooth wall are a good measure of the effectiveness of the enhancement device.

$$\frac{k_r}{k_s} = \frac{(hA)_r}{(hA)_s} \tag{19.1}$$

- Manipulate the above expression algebraically to generate non-dimensional numbers that can be used usefully

Rough Surface heat Transfer in tubes-4

- Therefore:

$$\begin{aligned}\frac{k_r}{k_s} &= \frac{(hA)_r}{(hA)_s} = \frac{(hA)_r}{(hA)_s}\frac{(\rho c_p)}{(\rho c_p)}\frac{V_r}{V_r}\frac{V_s}{V_s} \\ &= \frac{St_r}{St_s}\frac{g_r}{g_s}\frac{A_r}{A_s}\end{aligned} \tag{19.2}$$

- Where Stanton number $St = \dfrac{h}{\rho c_p V}$
- Mass flux $g = \rho V$

Rough Surface heat Transfer in tubes-5

- Pumping power is the power required to force fluid with volume flow rate ‘Q’ against a pressure drop Δp is:

$$\mathrm{P} = Q\,\Delta p \tag{19.3}$$

- For flow through tubular devices the concept of friction factor can be used with definition of hydraulic diameter.

Rough Surface heat Transfer in tubes-6

- For flow through a rough pipe:

$$Q = V_r A_c \qquad \Delta p = f_r \frac{L}{D_{hy}} \frac{V_r^2}{2} \rho \tag{19.4}$$

- Where:

$$\begin{aligned}
&\text{Hydraulic Diameter } D_{hy} = \frac{4A_c}{P} \\
&\text{Rough pipe friction factor} = f \\
&\text{Area of cross-section} = A_c \\
&\text{Pipe surface area} = A_s
\end{aligned}$$

Rough Surface heat Transfer in tubes-7

- Substituting equ^n (19.4) in equ^n (19.3):

$$P_r = V_r A_c f \frac{L}{D_{hy}} \frac{V_r^2}{2} \rho \tag{19.5}$$

- Using the definition of hydraulic diameter:

$$\frac{L}{D_{hy}} = \frac{L\hat{P}}{4A_c} = \frac{A_r}{4A_c} \tag{19.6}$$

- Substituting equ^n (19.6) in equ^n (19.5):

$$P_r = V_r A_c f_r \frac{A_r}{4A_c} \frac{V_r^2}{2} \rho \frac{\rho^2}{\rho^2} = \frac{g_r^{\,3} f_r A_r}{8\rho^2} \tag{19.7}$$

Rough Surface heat Transfer in tubes-8

- Comparison of enhancement is usually for the same fluid and therefore fluid properties can be considered to be the same for rough and smooth pipes
- Ratio of the pumping powers therefore becomes:

$$\frac{(P)_r}{(P)_s} = \frac{f_r}{f_s} \frac{A_r}{A_s} \left(\frac{g_r}{g_s}\right)^3 \tag{19.8}$$

Rough Surface heat Transfer in tubes-9

- Rewrite (19.8) as:

$$\left(\frac{g_r}{g_s}\right) = \left(\frac{P_r}{P_s} \frac{f_s}{f_r} \frac{A_s}{A_r}\right)^{1/3} \tag{19.9}$$

- Replace mass flux ratio in equ^n (19.2) with equ^n (19.9) to get a relationship between the quantities of interest:

$$\frac{k_r / k_s}{\left(\frac{P_r}{P_s}\right)^{1/3} \left(\frac{A_r}{A_s}\right)^{2/3}} = \frac{St_r / St_s}{\left(f_r/f_s\right)^{1/3}} \tag{19.10}$$

Rough Surface heat Transfer in tubes-10
Constant Pumping Power, Area

- Often, either experimentally or numerically the Nusselt numbers for a particular set of Reynolds and Prandtl numbers are established for a rough pipe
- Similarly the friction factor as a function of Reynolds number is established
- Let us say $Nu_r = f(Re, Pr)$ and $f_r = f(Re)$ is known
- Relations for Nu_s and f_s for the smooth pipe are well known.

Rough Surface heat Transfer in tubes-11
Constant Pumping Power, Area

- Most common expressions for fully developed turbulent flow in smooth tubes:
  - Dittus-Boelter: $Nu = .023\, Re^{.8}\, Pr^{.4}$ (19.10a)
  - Blassius: $f = .3146/Re^{.25}$ (19.10b)
- Assume that a fixed pumping power is available.
- In addition assume that the surface area of the smooth and rough surfaces are identical

Rough Surface heat Transfer in tubes-12
Constant Pumping Power, Area

- From equation (19.8) for constant pumping power:

$$\frac{(P)_r}{(P)_s} = 1 = \frac{f_r}{f_s}\frac{A_r}{A_s}\left(\frac{g_r}{g_s}\right)^3 \qquad (19.11)$$

- Since areas are also same:

$$f_r g_r^3 = f_s g_s^3 \qquad (19.12)$$

- Suppose the characteristics for a tube with a particular roughness are known. For a given Reynolds number, the friction factor is therefore known

Rough Surface heat Transfer in tubes-13
Constant Pumping Power, Area

- Use equ^n (19.10b) in equ^n (19.12). Only unknown to obtain is g_s (or f_s)
- Use now equ^n (19.10) to get

$$\frac{k_r}{k_s} = \frac{St_r / St_s}{(f_r / f_s)^{1/3}} \qquad (19.13)$$

- The above expression is used with St_s evaluated at the corresponding value of g_s obtained from (19.12)

Rough Surface heat Transfer in tubes-14
Constant Pumping Power, Area

- If k/k_s is >1 then the conductance for the rough tube is higher and therefore useful to incorporate this roughness element.
- Manipulate equ^n (19.13) to get:

$$\frac{k_r}{k_s} = \frac{h_r}{\rho C_p u_r}\frac{\rho C_p u_s}{h_s}\left(\frac{u_r^3}{u_s^3}\right)^{1/3} = \frac{h_r}{h_s} \qquad (19.14)$$

- h_r and h_s are the heat transfer coefficients corresponding to g_r and g_s.

Rough Surface heat Transfer in tubes-15
Constant Pumping Power, Area

- Overall procedure for constant pumping power and constant surface area is:
  - Select a g_r and obtain the corresponding Nu_r
  - Use equ^n (19.12) to evaluate corresponding g_s
  - Use an appropriate relationship and obtain the Nu_s corresponding to the g_s calculated in the previous step.

Evaluate if Nu_r/Nu_s is > 1 and if it is, the enhancement technique worth incorporating.

Rough Surface heat Transfer in tubes-16
Constant Pumping Power, Conductance

- The next case is where the k/k_s and P/P_s the same.
- It is required to be investigated if the enhancement mechanism provided can produce a reduction in the surface area required
- Equ^n (19.10) gives:

$$\frac{A_s}{A_r} = \frac{(St_r/St_s)^{3/2}}{(f_r/f_s)^{1/2}} \tag{19.15}$$

Rough Surface heat Transfer in tubes-17
Constant Pumping Power, Conductance

- Pumping power is constant. Therefore using equ^n (19.8):

$$\frac{g_r}{g_s} = \frac{1}{\left(\frac{f_r}{f_s}\right)^{1/3}\left(\frac{A_r}{A_s}\right)^{1/3}} \tag{19.16}$$

- In addition the heat transfer conductance is the same – that is:

$$k_r/k_s = 1 \Rightarrow \frac{h_r}{h_s} = \frac{A_s}{A_r} \tag{19.17}$$

Rough Surface heat Transfer in tubes-18
Constant Pumping Power, Conductance

- Substituting equ^n (19.17) in equ^n (19.16) and simplifying gives:

$$\frac{g_r}{g_s} = \frac{1}{\left(\frac{f_r}{f_s}\right)^{1/3}\left(\frac{h_s}{h_r}\right)^{1/3}} \tag{19.18}$$

$$= \frac{1}{\left(\frac{f_r}{f_s}\right)^{1/3}\left(\frac{\rho u_s}{\rho u_r}\right)^{1/3}\left(\frac{h_s u_r \rho C_p}{h_r u_s \rho C_p}\right)^{1/3}} = \frac{1}{\left(\frac{f_r}{f_s}\right)^{1/3}\left(\frac{g_s}{g_r}\right)^{1/3}\left(\frac{St_s}{St_r}\right)^{1/3}}$$

$$\Rightarrow \frac{g_r}{g_s} = \frac{\left(\frac{St_r}{St_s}\right)^{1/2}}{\left(\frac{f_r}{f_s}\right)^{1/2}} \tag{19.19}$$

Rough Surface heat Transfer in tubes-19
Constant Pumping Power, Conductance

- Assume a g_r and the corresponding f_r and h_r are known. The relationship between g_s, f_s and h_s is also known using e.g. equ^n (19.10a) and equ^n (19.10b). Evaluate the g_s (or f_s or h_s) using the equ^n (19.18).
- For this value of g_s evaluate the corresponding h_s and use either equ^n(19.17) or equ^n (19.15) to find the A_r/A_s expression

Rough Surface heat Transfer in tubes-20
Constant Surface Area, Conductance

- Last case is one where heat transfer resistance and area ratio are same. It is required to investigate whether pumping power is reduced.

$$\frac{k_r}{k_s} = 1; \quad \frac{A_r}{A_s} = 1 \tag{19.20}$$

- Substituting equ^n (19.20) in equ^n (19.1) gives:

$$\frac{h_r}{h_s} = 1 \tag{19.21}$$

Rough Surface heat Transfer in tubes-21
Constant Surface Area, Conductance

- Use the equation (19.10) to get

$$\left(\frac{P_r}{P_s}\right)^{1/3} = \frac{(f_r / f_s)^{1/3}}{St_r / St_s} \tag{19.22}$$

- Use the constant properties to get:

$$\begin{aligned} &\frac{h_r}{h_s} = 1 = \frac{St_r}{St_s}\frac{u_s}{u_r} \\ &\Rightarrow \frac{g_r}{g_s} = \frac{St_s}{St_r} \end{aligned} \tag{19.23}$$

Rough Surface heat Transfer in tubes-22
Constant Surface Area, Conductance

- Choose a g_r. Obtain the corresponding h_r. hs is the same as this value.
- Obtain g_s corresponding to the hs obtained using equ^n (19.10a). Use equ^n (19.8) to evaluate if pumping power ratio is greater or lesser than unity
- Alternately: Choose g_r and the corresponding St_r. Use equn (19.23) to evaluate gs since relation between g_s and St_s is known. Use equ^n (19.22) to evaluate power ratio.

Rough Surface heat Transfer in tubes-23
Constant Surface Area, Conductance

- The usefulness of an enhancement device is determined by either of the three conditions outlined.
- Depending on the condition chosen the parameters for comparison are evaluated

Recap

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