Module 2 : Convection

Lecture 22 : Rough Surface Turbulent Flows

Objectives

In this class:

- Smooth surface turbulence concepts are extended to rough surfaces
- The development of correlations for heat transfer using experimental data for rough pipe / channel flow is discussed
- Material borrowed from: Han J. C., “Heat transfer and Friction in channels with two opposite rib roughness walls.” ASME Journal of Heat Transfer, Vol. 110, pp 321-328, 1988

Turbulent Flow: Rough pipe-1

- For rough pipes with a mean roughness height ‘k_s’ a roughness Reynolds number or a non-dimensional roughness is defined as $k_s \dfrac{u_\tau}{\nu}$
- When the nondimensional roughness is below 5 the roughness is within the laminar sublayer and therefore considered ‘smooth’ i.e. the friction is a function of Reynolds number only

Turbulent Flow: Rough pipe-2

- For $5 < k_s \dfrac{u_\tau}{\nu} < 70$ the friction is a function of the Reynolds number and the roughness height to pipe diameter ratio k_s/D
- For $k_s \dfrac{u_\tau}{\nu} > 70$ flow is fully turbulent and the friction is a function only of the k_s/D ratio. In this regime form drag becomes important and is much greater than the skin friction values
- Note that $k_s \dfrac{u_\tau}{\nu}$ is referred to as the roughness Reynolds number

Turbulent Flow: Rough pipe-3

- Log-law of the wall is postulated to be valid for the rough pipes also, therefore

$$\frac{u}{u_\tau} = \frac{1}{\kappa}\ln\frac{y}{k_s} + B' \qquad (22.1)$$

- For a smooth pipes this should result in the equation (21.28):

$$\frac{u}{u_\tau} = \frac{1}{\kappa}\ln\frac{y}{Y} + 5.0 \qquad Y = \frac{\nu}{u_\tau} \qquad (22.2)$$

Turbulent Flow: Rough pipe-4

- The parameter B’ is noticed to be equal to 8.5 for fully developed rough flow
- The parameter B’ is therefore chosen in the following manner

$$B' = 5 + 2.5\ln\frac{u_\tau k_s}{\nu} \quad \text{Hydraulically smooth} \qquad (22.3)$$

$$B' = 8.5 \qquad \text{Fully rough region} \qquad (22.4)$$

Turbulent Flow: Rough pipe-5

- In general one can therefore say: $B' = f(\dfrac{u_\tau k_s}{\nu})$ and the functional dependence needs to be determined.
- Overall, for rough pipes, use

$$\frac{u}{u_\tau} = \frac{1}{k}\ln\frac{y}{k_s} + f\left(\frac{u_\tau k_s}{\nu}\right) \qquad (22.5)$$

- The functional dependence for the Roughness Reynolds number in the above equation needs to be determined

Turbulent Flow: Rough pipe-6

- Now, try to relate the global friction factor parameter which is often measured, to the wall parameters

$$f = \frac{\Delta p}{\frac{L}{D}\frac{V^2}{2}\rho} = \frac{8\tau}{\rho V^2} \Rightarrow V^2 = \frac{4\tau}{\rho f/2} \qquad (22.6)$$

$$u_\tau^2 = \frac{\tau}{\rho} \Rightarrow \frac{V^2}{u_\tau^2} = \frac{8}{f}$$

Turbulent Flow: Rough pipe-7

- Note that 'V' is the average velocity of the flow in the pipe and therefore one can define an 'average' 'u^+' and obtain:

$$\overline{u^+}^2 = \frac{8}{f} \text{ where } \overline{u^+} = \frac{V}{u_\tau}$$

$$\Rightarrow \overline{u^+} = \sqrt{\frac{8}{f}}; \qquad (22.7)$$

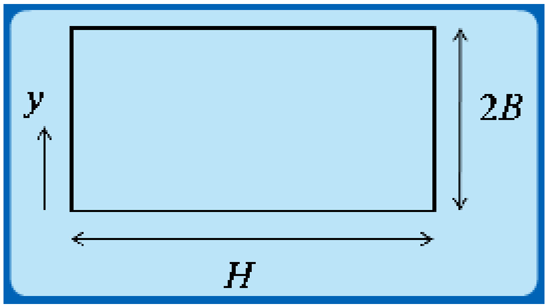
$$\overline{u^+} = \sqrt{\frac{2}{f_F}} \quad ; f_F = \text{Fanning friction factor} \qquad (22.8)$$

Turbulent Flow: Rough pipe-8

- Irrespective of the type of roughness in the pipe the wall shear is related to the mean flow and the friction factor which is of engineering importance.
- This concept is of use in trying to develop generalized correlations for rough pipe flow that are fitted with heat transfer enhancement devices and this is illustrated next for a rectangular channel

Application to rectangular channels-1

- Consider a rectangular channel as shown below. The channel is supposed to have some roughening elements. The roughness elements have a height 'e'.



Application to rectangular channels-2

- Integrate the u^+ profile over entire channel to obtain average value:

$$\overline{u^+} = \frac{1}{A_c}\int_0^{\delta}\left[2.5\ln y/e + R(e^+)\right]dA_c \tag{22.9}$$

$$\overline{u^+} = \frac{2.5}{A_c}\int_0^{\delta}\ln(y/e)\,dA_c + \frac{1}{A}\int_0^{\delta}R(e^+)\,dA_c \tag{22.10}$$

- In the above equation, the unknown function "f" in equ^n (22.5) is denoted as "R" and the roughness Reynolds number is denoted as "e^+"

Application to rectangular channels-3

- Look at the first term in equ^n (22.10). Take the thickness of the boundary layer to be half the height of the channel in the 'y' direction

$$2\,\frac{2.5}{A_c}\int_0^{B}\ln(y/e)\,H\,dy \tag{22.11}$$

- Notice the limit of integration is only till 'B' and therefore the multiplicative factor '2' is introduced to span the entire cross-section since the width of the channel is assumed 2B.

Application to rectangular channels-4

- We simplify the expression (22.11) by putting y/e = p and dy = edp:

$$\frac{2(2.5)He}{A_c}\int_0^{B/e}\ln P\,dp \quad = \frac{2(2.5)He}{A_c}\left[p\ln p - p\right]_0^{B/e}$$

$$\frac{2(2.5)He}{(2BH)}\left[\frac{B}{e}\ln B/e - \frac{B}{e}\right] = \left[2.5\ln B/e - 2.5\right] \tag{22.12}$$

Application to rectangular channels-5

- Substituting equ^n (22.12) for the first term in equ^n (22.10) and integrating the second term of equ^n (22.10) gives:

$$\overline{u^+} = 2.5\ln B/e - 2.5 + R(e^+) \tag{22.13}$$

- This now needs to be related to the friction factor which is the useful quantity

Application to rectangular channels-6

- Using equ^n (22.8) to relate the average u^+ to the friction factor gives:

$$\sqrt{\frac{2}{f}} = 2.5\ln\frac{B}{e} - 2.5 + R(e^+) \tag{22.14}$$

- 'f' is a measured/computed parameter and B and e are geometric parameters and therefore compute $R(e^+)$ for different geometries of a similar roughness configuration

Application to rectangular channels-7

- A similar procedure needs to be evolved for the temperature profile also. The wall heat flux is related to the temperature gradient by the molecular (α) and turbulent thermal (ε_H) diffusivity as follows:

$$\frac{q_w}{\rho C_p} = -(\alpha + \varepsilon_H)\frac{\partial T}{\partial y} \tag{22.15}$$

$$\frac{q_w}{\rho C_p} = -(\alpha + \varepsilon_H)\frac{u_\tau}{\upsilon}\frac{\partial T}{\partial y^+} \tag{22.16}$$

Application to rectangular channels-8

- Integrate over the temperature profile to obtain:

$$\int_{T_w}^{\bar{T}} dT = \frac{q_w}{\rho C_p u_\tau}\int_0^{y^+}\frac{dy^+}{1/\Pr + \varepsilon_H/\upsilon}$$

$$\frac{(T_w - T)u_\tau}{q_w/\rho C_p} = \int_0^{y^+}\frac{dy^+}{1/\Pr + \varepsilon_H/\upsilon} \equiv T^+ \tag{22.17}$$

$$\text{Define } T^+ = \frac{\rho c_p u_\tau}{q''}(T_w - T) \tag{22.18}$$

Application to rectangular channels-9

- Very close to the wall the turbulent component is ignored(consider Pr nearly constant) to get

$$T^+ = \Pr y^+ \tag{22.19}$$

- In the turbulent zone ignore the laminar term. Define a turbulent Prandtl number as the ratio of the turbulent viscosity and diffusivity:

$$\Pr_t = \varepsilon_M/\varepsilon_H \tag{22.20}$$

Application to rectangular channels-10

- In the turbulent zone ignore the laminar term

$$\begin{aligned} T^+ &= \int_0^{y^+}\frac{dy^+}{\varepsilon_H/\upsilon} = \int_0^{y^+}\frac{dy^+}{(\varepsilon_H/\varepsilon_M)(\varepsilon_M/\upsilon)} \\ &= \int_0^{y^+}\frac{\Pr_t dy^+}{\kappa y^+} \\ &= \frac{\Pr_t}{\kappa}\ln y^+ \end{aligned} \tag{22.21}$$

Application to rectangular channels-11

- For the temperature profile borrow concepts from the velocity profile already obtained earlier and write a similar expression:

$$T^+ = 2.5\ln y/e + g(e^+, \Pr) \tag{22.22}$$

- A physical meaning to ‘g(e$^+$,Pr)’ is ascribed as being the T$^+$ value at the tip of the roughness element although the usefulness of this is limited.

Application to rectangular channels-12

- Perform an integration similar to that performed earlier to obtain equ^n (22.12), to obtain an average T$^+$ value:

$$\overline{T^+} = (2.5)\ln B/e - 2.5 + g(e^+.\Pr) \tag{22.23}$$

- Use equ^n (22.14) for friction factor to obtain

$$\overline{T^+} = \sqrt{\frac{2}{f}} - R(e^+) + g(e^+, \Pr) \qquad (22.24)$$

Application to rectangular channels-13

- Again, the average T^+ expression needs to be simplified to obtain a quantity of engineering interest. Therefore the definition is used to simplify this into a quantity that is often measured:

$$\overline{T^+} = \frac{\rho c_p u_\tau (\overline{T_w - T})}{q''} = \frac{\rho c_p \bar{u}_\tau}{h} \frac{u}{\bar{u}} = \sqrt{\frac{f}{2}} \div St \qquad (22.25)$$

Application to rectangular channels-14

- Use equ^n (22.25) in equ^n (22.24) and using equ^n (22.14)

$$g(e^+, \Pr) = \frac{\sqrt{f/2}}{St} - \sqrt{\frac{2}{f}} + R(e^+) \qquad (22.26)$$

- Since $R(e^+)$ is already established using the friction factor information $g(e^+,\Pr)$ can get established

Application to rectangular channels-15

- Overall procedure for the development of correlations is now outlined.
- Initially use the friction factor information to obtain:

$$\mathrm{R}(e^+) = \mathrm{Q}_1(e^+, p/e, \ldots\ldots\ldots\ldots\ldots)$$

- Now use this information to obtain

$$g(e^+) = Q_2(e^+, p/e, \ldots\ldots\ldots\ldots\ldots\ldots)$$

Application to rectangular channels-16

- The expressions for $R(e^+)$ and $g(e^+)$ can be used to evaluate friction and Nusselt numbers for configurations for which the geometry is known
- Note that if friction factor is relatively insensitive to the roughness Reynolds number, e^+, the flow is in the fully rough zone.

Recap

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