Module 1 : Conduction

Lecture 1 : Introduction and Heat Diffusion Equation

Objectives

In this class:

- An introduction to the overall heat transfer phenomenon is presented in brief.
- Conduction, convection and radiation are introduced
- The general heat diffusion equation is derived by using energy balance.

Introduction

- A second level course in Heat Transfer
- An essential part of the course is the review of concepts seen in a first level course and some new material will be introduced.
- No numerical methods will be covered in this course. Much of the material will concentrate on convection. Conduction will be covered in reasonable detail. Radiation will not be covered in any great detail.
- Several symbols are used in the slides that are presented for the several ‘classes’. Where-ever the symbols appear for the first time they are defined and therefore a separate nomenclature is not provided
- The material has been taken from several text books that are available on the subject

Conduction(Intro)

- Transfer of energy from more energetic particles to less energetic particles. Energy transfer between the neighboring molecules. Does not require bulk motion of molecules.
  In gases and liquids: conduction is due to collisions and of molecules during their random motion.
  In solid: energy transfer due to vibration of the molecules in a lattice.

Conduction(Contd.)

- Rate equation to compute amount of heat energy transferred per unit time.
- Fourier’s law

$$q_n^{''} = -k \frac{dT}{dn} \tag{1.1}$$

- k, is thermal conductivity (W/m.K) and ‘n’ is the direction normal to the direction of lines of constant temperature

Convection (Intro)

- Transfer of energy due to bulk motion of the fluid and random molecular motion.
- Newton’s law of cooling

$$q^{''} = h(T_s - T_b) \tag{1.2}$$

- Where h, is heat transfer coefficient ($W/m^2.K$), T_s is the surface temperature and T_b is some reference temperature. The reference temperature is chosen so that ‘h’ is constant.

Radiation (Intro.)

- Energy transfer due to electro-magnetic waves.

- Equation often used for radiation calculations:

$$q'' = \varepsilon\sigma(T_s^4 - T_{surr}^4) \tag{1.3}$$

- ε, is emissivity of the surface, σ is Stefan-Boltzmann constant
- $\sigma = 5.67 \times 10^{-8}$ $W/m^2.K^4$), T_s is a surface temperature and T_{surr} is an appropriate surrounding temperature

Conduction-Fourier's Law

- Fourier's law is based on experimental evidence
- Heat flux is a vector quantity. For the 'x' direction:

$$\dot{q}_x = -k\frac{dT}{dx} \tag{1.4}$$

- For all directions in cartesian frame:

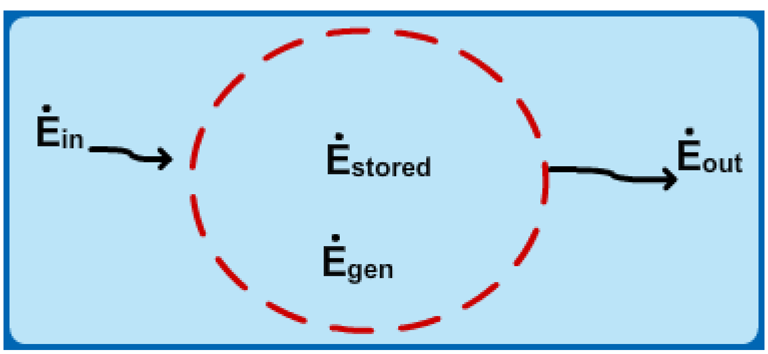
$$q'' = -k\left[\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}\right] = -k\nabla T \tag{1.5}$$

Fourier's Law (contd.)

- Notice that $-k$ T is independent of frame of reference
- Use mathematical transformations to obtain the equation in other frames of reference

Control volume energy conservation

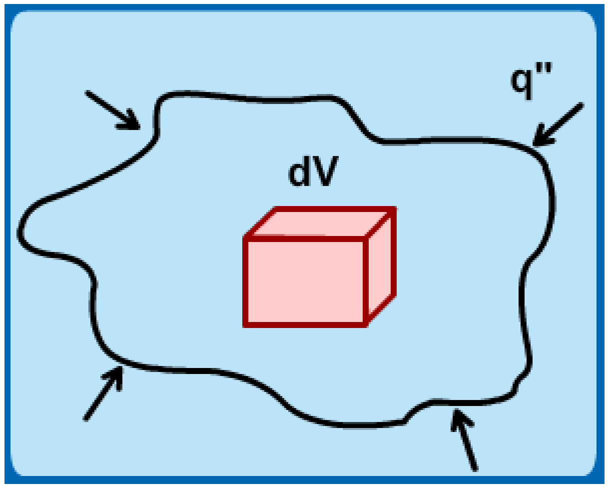
- Consider a control volume and write the energy balance



- $\dot{E}_{in} + \dot{E}_{gen} - \dot{E}_{out} = \dot{E}_{stored}$
- $\dot{E}_{stored} = 0$ ⟹ steady state

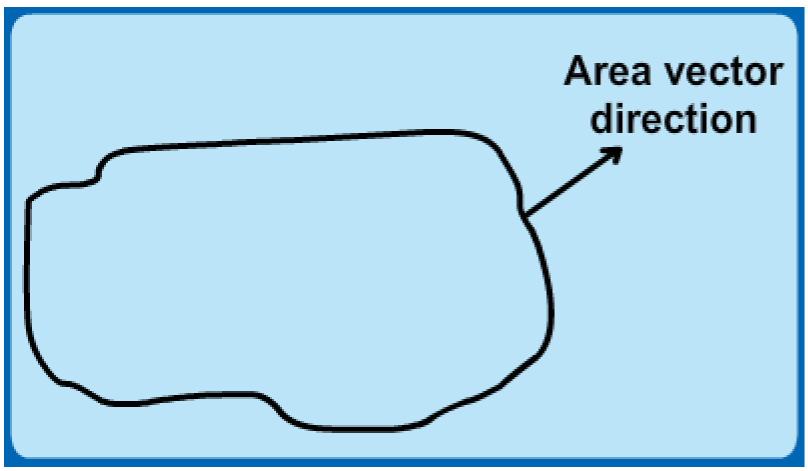
Heat diffusion equation-1

- Consider an arbitrary volume of a given medium.
- Consider a small control volume 'dV'. Energy enters/leaves across the surface. Generation and storage are within the volume



Heat Diffusion Equation-2

- Area is considered a vector
- Outward drawn normal for a given volume is considered direction of area vector



- Flux entering is considered positive

Heat Diffusion Equation-3

- Net heat leaving the control volume

$$q = \iint_A \bar{q}'' \cdot d\bar{A} \tag{1.6}$$

- Heat flux and area are vectors and therefore the dot product in the above expression
- Suppose heat flux is leaving a surface. The two vectors are in the same direction giving positive q i.e. leaving the surface. Heat flux entering the surface means it is opposite to area vector now giving a negative product.

Heat Diffusion Equation-4

- Net heat rate out $= \iint_A q'' . dA$ (1.7)

- Total heat generation rate $= \iiint_V q''' dV$ (1.8)

- Total storage rate $= \iiint_V \frac{\partial}{\partial t}(\rho C T) dV$ (1.9)

- Use these expressions in the energy balance equation

Heat Diffusion Equation-5

- Energy balance equation becomes:

$$-\iint \bar{q}''.d\bar{A} + \iiint q'''dV = \iiint \frac{\partial}{\partial t}(\rho C_p T)dV \tag{1.10}$$

- The above is the integral form of the conduction/Heat Diffusion equation.
- Often it is useful to have the differential form of the equation

Heat Diffusion Equation-6

- The Gauss Divergence theorem and the mean value theorem provide the vehicle for the conversion from the integral to the differential form.
- The theorem is written on the next slide for a vector and a scalar quantity.

Gauss Divergence Theorem

- Consider an arbitrary parameter ‘a’

$$\iint_A \bar{a}.\hat{n}\, dA = \iiint_V \nabla.\bar{a}\, dV \quad \bar{a} \text{ is a vector} \tag{1.11}$$

$$\iint_A a\hat{n}\, dA = \iiint_V \nabla a dV \quad a \text{ is a scalar} \tag{1.12}$$

Mean Value Theorem

- Integration over a finite volume can be represented as the product of the mean value of the integrand and the volume.
- Since volume is not zero the integrand has to be zero if the integral is zero.
- Now take the volume to very small value over which the properties can be assumed constant and equate the integrand to zero. This gives the differential form of the equation

Differential form of equation

- Using the Gauss divergence theorem in (1.10):

$$-\iiint \nabla.q''dV + \iiint q'''\, dV = \iiint \frac{\partial}{\partial t}(\rho C_P T)dV \tag{1.13}$$

- Group the terms to get

$$\iiint_V \left(-\nabla.q'' + q''' - \frac{\partial}{\partial t}(\rho C_P T)\right)dV = 0$$

Heat Diffusion Equation-7

- Use the mean value theorem to get

$$\left[-\nabla . q'' + q''' - \frac{\partial}{\partial t}\left(\rho C_p T\right)\right]_{Mean} V = 0$$

- Since V is finite, the mean value of the integrand has to be zero. When the volume is shrunk to a very small value, the mean value and the local value are the same, giving the differential form of the equation

Heat Diffusion Equation-8

- The differential form of the heat diffusion equation is therefore:

$$-\nabla . q'' + q''' = \frac{\partial}{\partial t} \rho C_p T \tag{1.14}$$

- This form of the equation is not a very useful form since the flux is not an easily measurable quantity