Module 2 : Convection

Lecture 25a : Illustrative examples

Objectives

In this class:

- Examples will be taken where the boundary layer concepts discussed in earlier classes will be used.
- Examples of the use of correlations for making calculations will also be discussed.

Question 1: Problem statement-1

- Consider the flow of a fluid with velocity U_∞ and temperature T_∞ over a flat surface at constant temperature T_w. The thermal boundary layer is much thicker than the momentum boundary layer. Use the cubic profile for the temperature as obtained in class, in the thermal boundary layer integral equation and determine the variation of the Nusselt no. as a function of the distance along the length of the surface. Make suitable assumptions

Question 1: Solution-1

- Since the thermal boundary layer is much thicker than the momentum boundary layer assume that the velocity is constant within the thermal boundary layer and equal to U_∞.

- The momentum integral equation is:

$$\frac{d}{dx}\int_0^{\delta_t} U_\infty\left(3/2(y/\delta)-1/2(y/\delta)^3\right)(T_w-T_\infty)=\frac{q_w''}{\rho C_p}$$

Question 1: Solution-2

- This simplifies to

$$U_\infty\frac{d}{dx}\int_0^{\delta_t}\left(3/2(y/\delta)-1/2(y/\delta)^3\right)dy=\frac{3k(T_w-T_\infty)}{2\rho C_p\delta_t}$$

$$U_\infty\frac{d}{dx}\left(3/2(y^2/2\delta_t)-1/2(y^4/4\delta^3)\right)\Big|_0^{\delta_t}=\frac{3k}{2\delta_t}$$

$$\Rightarrow U_\infty\frac{5}{8}\frac{d\delta_t}{dx}=\frac{3\alpha}{2\delta_t}\Rightarrow\frac{5}{12}\frac{U_\infty\delta_t}{\alpha}\frac{d\delta_t}{dx}=1$$

Question 1: Solution-3

- Integrating and assuming that $\delta_t = 0$ at $x = 0$ gives:

$$\delta_t^2=\frac{12\alpha x}{5U_\infty}$$

- Use the same procedure adopted in class:

$$h(T_w-T_\infty)=\frac{3}{2}\frac{\alpha(T_w-T_\infty)}{\delta_t}=\frac{3kx}{2\sqrt{\dfrac{12\alpha x}{5U\infty}}}$$

$$Nu_x=hx/k=\frac{3kx}{2\sqrt{\dfrac{12\alpha x}{5U\infty}}}=\sqrt{\frac{5}{12}}3/2\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/2}$$

Question 2: Problem statement-1

- Start with the 2-D continuity equation for flow over a flat plate with free stream velocity U_∞. Use Leibnitz rule and show that the velocity magnitude at the edge of the boundary layer at a given specified location x can be obtained if (1) the cubic polynomial velocity profile is available and (2) the solution of the momentum integral equation is available.

Question 2: Solution-1

- Start with equ^n (25.2) and use Leibnitz rule to get the vertical velocity component:

$$v|_\delta = -\left[\frac{d}{dx}\int_0^\delta u dy + U_\infty \frac{d\delta}{dx}\right]$$
$$= \frac{d}{dx}\int_0^\delta U_\infty\left[\frac{3y}{2\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3 dy\right] + U_\infty \frac{d\delta}{dx}$$
$$= \frac{5U_\infty}{8}\frac{d\delta}{dx} + U_\infty \frac{d\delta}{dx} = \frac{13U_\infty}{8}\frac{d\delta}{dx}$$

Question 2: Solution-2

- From the momentum boundary layer:

$$\delta\frac{d\delta}{dx} = \frac{140}{13}\frac{v}{U_\infty}$$

- Substitute in the expression for velocity to get:

$$v|_\delta = \frac{140}{8}\frac{v}{\delta}$$

The horizontal component is the free stream velocity. The magnitude is therefore:

$$\sqrt{v|_\delta^{\,2} + U_\infty^{\,2}}$$

Question 3: Problem Statement - 1

- A method to obtain the similarity solution for the incompressible 2-D not very high speed boundary layer flow is to use the variables:

$$\eta = C_1 y x^a, \psi = C_2 x^b F(\eta), F(\eta) = \frac{u}{U_\infty}$$

- After making these substitutions and some manipulations the following intermediate equation results for momentum balance:

$$F'\left[C_3^{\,2}(a+b)x^{b-a-1}F' + C_4 C_3 x^{b-a-1}F''\frac{\eta}{C_1}\right] - C_1^2 C_2^{\,2}\left[Fbx^{b-a-1} + C_1 F' x^{b-a-1}\frac{\eta}{C_1}\right]F''$$
$$= C_3^{\,2}(a+b)x^{b-a-1} + vF'''C_1^2 C_3.$$

Question 3: Solution-1

- In the equation given, the following conditions are valid:

$$C_1 C_2 = C_3; C_1 C_3 = C_4; u = C_3 x^{a+b} F'$$

- Assume C_1, C_2, a, b, v are constants. Show that applying the condition that the above equation is amenable to a similarity solution with (a+b) =m transforms the above equation to the standard Falkner-Skan wedge flow equation with $U_\infty = C_3 x^m$

Question 3: Solution-2

- Start with the conditions that are given:

$$C_1C_2 = C_3; C_1C_3 = C_4; \Rightarrow C_1C_2C_1C_3 = C_3C_4$$

$$C_1C_2C_1C_1C_2 = C_3C_4 \Rightarrow \frac{C_3C_4}{C_1} = C_1^2C_2^2$$

- The two terms marked yellow in the expression given in the question therefore cancel out.

- For a similarity solution to exist, the exponent of the 'x' term must equal zero i.e.

$$b - a - 1 = 0$$

Question 3: Solution-3

- The given expression therefore becomes:

$$C_3^2(a+b)F'^2 - C_1^2C_2^2FF''b = C_3^2(a+b) + \nu F'''C_1^2C_3$$
$$\Rightarrow C_3^2(a+b)(1-F'^2) + C_1^2C_2^2FF''b + \nu F'''C_1^2C_3 = 0$$
$$\Rightarrow \frac{C_3\ (a+b)(1-F'^2)}{C_1^2} + \frac{C_3FF''b}{C_1^2} + \nu F''' = 0$$

- C_3 and C_1 are arbitrary constants and therefore chose them such that C_3/C_1^2 is v. It is already given that a + b = m and since b - a = 1 from similarity, b = (m + 1)/2

Question 3: Solution-4

- Now substituting the constants the equation becomes the previously derived Falkner-Skan equation:

$$m(1-F'^2) + \frac{m+1}{2}FF'' + F''' = 0,$$

- Using the definition for stream function:

$$u = \frac{\partial\psi}{\partial y} = C_2x^bF'\frac{\partial\eta}{\partial y} = C_2x^bF'C_1x^a = C_3F'x^{a+b}$$

- Now using the definition for 'u':

$$u/u_\infty = F', u = C_3x^mF' \Rightarrow u_\infty = Ax^m$$

Use of Correlations - 1

- A reasonable amount of analysis has been done to obtain solutions to very simple situations

- Many practical situations are not amenable to analytical solutions – even for laminar flows. A few situations will be discussed here.

- Computations can be done for laminar flows since the equations are well defined and do not need simplifying assumptions made to obtain analytical solutions

Use of Correlations - 2

- Analytical solutions are still very useful since they give a lot of insight into the 'physics' of the situation. Moreover, they are often validation tools for numerical calculations.

- For turbulent flows, correlations are necessary even for simple situations since modeling assumptions are often involved in getting either 'analytical' or numerical solutions.

- We shall look at some simple examples for the use of correlations.

Question 4: Problem Statement - 1

- An electrical bus bar in the shape of a rod of diameter 1 cm generates heat due to passage of current and is cooled by a fan which blows air over it at 8m/s at ambient temperature of 30^0 C. Find the maximum heat dissipation permissible if the surface temperature of the bus bar should not exceed 82^0C.

- Properties are to be evaluated at the mean film temperature i.e. $(T_w+T_\infty)/2 = 57^0$C.
- $\nu = \mu/\rho = 1.86\times10^{-5} m^2/s$, k = 0.0283W/mK, Pr = 0.7

Question 4: Solution 1

- Flow over a cylinder - External flow situation. First compute Reynolds number:

$$\mathrm{Re} = \frac{vD}{\nu} = \frac{8(1\times10^{-2})}{1.86\times10^{-5}} = 4301.1$$

- Use correlation to get:

$$Nu = 0.3 + \frac{0.62\,\mathrm{Re}_D^{1/2}\,\mathrm{Pr}^{1/3}}{\left[1+\left(0.4/\mathrm{Pr}\right)^{2/3}\right]^{1/4}}\left[1+\left(\frac{\mathrm{Re}_D}{282,000}\right)^{5/8}\right]^{4/5} = 39.7$$

Question 4: Solution 2

- Compute heat transfer coefficient:

$$h = \frac{kNu}{D} = \frac{0.0283(0.397)}{1\times10^{-2}} = 112.3\, W/m^2K$$

- The maximum temperature is given as 82^0C and therefore the heat generation per unit length is calculated:

$$\frac{Q}{L} = 112.3\,\pi(1\times10^{-2})(82-30) = 183\ W/m$$

- If the center temperature were the limit on temperature, the conduction equation would have been used with the heat transfer coefficient as the boundary condition.

Question 5: Problem Statement - 1

- Water is to be heated from 15^0C to 65^0C as it flows through a 3cm internal diameter 5m long tube. An electric resistance heater provides uniform heating throughout the surface of the tube. The outer surface of the heater is well insulated so that in steady operation all the heat generated in the heater is transferred to the water in the tube. If the system is to provide hot water at a rate of 10lit/min, determine the power rating of the resistance heater.

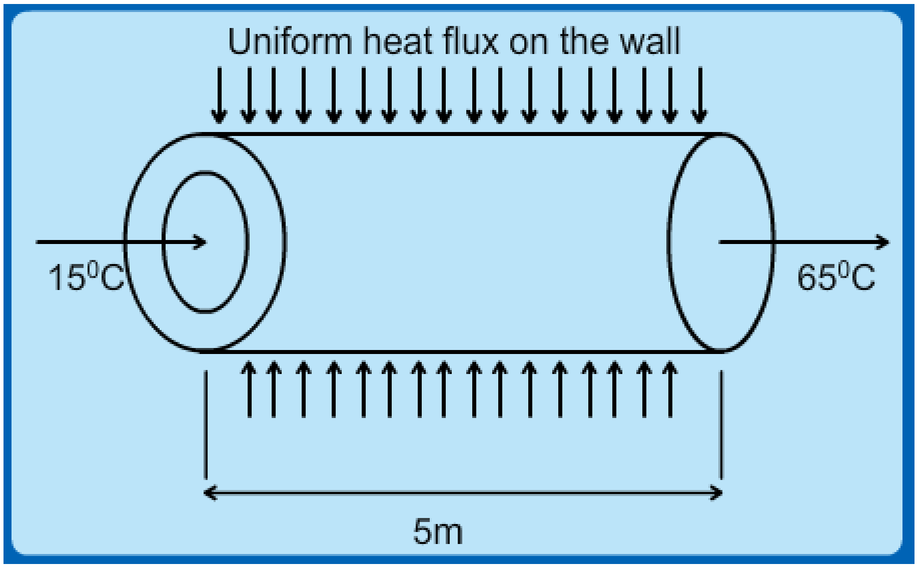
Question 5: Problem Statement - 2

- In addition, estimate the inner surface temperature of the pipe at exit. The properties of water at the bulk mean temperature that are to be used for all calculations are given as:

  $\rho = 992.1 kg/m^3$, $\nu = \mu/\rho = 0.658\times10^{-6} m^2/s$,
  k = 0.631W/mK, Cp = 4179 J/kgK, Pr = 4.32

Question 5: Problem Statement - 3

- A schematic is show below:



Question 5: Solution 1

- Volume flow rate = 10 lit/min = $10 \times 10^{-3}/60$ m^3/sec
- Mass flow rate m = $992(10 \times 10^{-3})/60$ = 0.165kg/s
- Q supplied = mCpΔT = 0.165 (4179) (50)
  = 34546.4W
- Power rating of heater is therefore 34.6kW
- Mean velocity = Q/A = $\dfrac{10\times10^{-3}}{60\dfrac{\pi}{4}(3\times10^{-2})^2} = 0.236 m/s$

Question 5: Solution 2

- Compute Reynolds number:

$$\mathrm{Re} = \frac{0.236(3\times10^{-2})}{0.658\times10^{-6}} = 10759.9$$

- Re > 2000 so flow is turbulent. Entry length is nearly 10 D which is about 30 cm. The pipe is 5m >> 30cm. The developing effects can be ignored and so use the fully developed flow correlation. For real accurate calculations use developing flow relations.

Question 5: Solution 3

- Calculate heat transfer coefficient using known correlation for Nusselt number:

$$\mathrm{Nu} = 0.023(10759.9)^{0.8}(4.32)^{0.4} = 69.4$$
$$\Rightarrow h = 1459.7\,\frac{W}{m^2K}$$

- Now compute the heat transfer coefficient and the wall temperature:

$$Q = hA(T_w - T_b)$$
$$\Rightarrow T_w = \frac{34546.4}{\pi(3\times10^{-2})(5)(1459)} + 65 = 115.2^0\,C$$

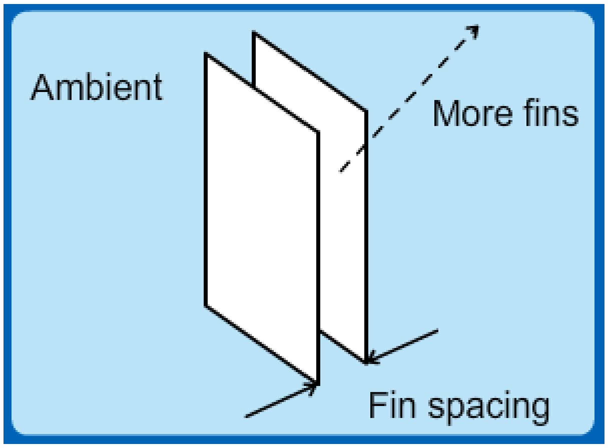
Question 5: Solution 4

- This is the wall temperature at the exit of the channel.

- Assume system pressure is sufficiently above atmospheric to prevent boiling, so that the single phase relationships hold.

Question 6: Problem Statement - 1

- The outermost fin of radiator of a small transformer measures 60 cm x 60 cm. It is placed in the ambient where the temperature is 35^0 C. One face of the radiator will see the neighbouring fin while the other face will see the ambient.



Question 6: Problem Statement - 2

- Assume that calculations are to be performed for the surface that faces the ambient, which must be maintained at 69^0 C. Determine the heat transferred to the ambient by this surface.

- Now perform the calculation for the fin surface that is in adjacent to the neighbouring fin. Assume fins are spaced optimally and the fin temperature is 69^0 C here also.

Question 6: Problem Statement - 3

- Properties of air are evaluated at mean film temperature i.e. (69 + 35) / 2 = 52^0 C and are given to be as follows:
  $v = \mu/\rho = 1.815\times10^{-5}$ m^2/s, k = 0.0279 W/mK,
  Pr = 0.708,
  β = volume coeff of exp. = 1/325 = 0.00308 K^{-1}

- Assume emissivity ε = 0.8

Question 6: Solution-1

- This is a problem of natural convection. Assume the wall temperature is constant although it really is not. First compute the Rayleigh number:

$$Ra = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}\Pr$$
$$= \frac{9.8(0.00308)(69-35)(0.6)^3(0.708)}{(1.815\times10^{-5})^2} = 4.764\times10^8$$

- Use this to evaluate Nusselt number using a correlation:

$$Nu = 0.59(Ra)^{1/4} = 87.17$$

Question 6: Solution-2

- Now compute the heat transfer coefficient:

$$h = kNu/L = 0.0279(87.17)/0.6 = 4.05 \text{ W/m}^2\text{K}$$

- Use this to compute the heat transferred:

$$Q = 4.05(0.6 \times 0.6)(69 - 35) = 49.6W$$

- Compute also the radiative component

$$Q = \varepsilon\sigma(T^4_{surface} - T^4_{amb}) = 76.5W$$

- Notice that radiative loss is higher than convective. This is not surprising for natural convection at low temperatures.

Question 6: Solution-3

- Calculations when a neighbouring surface is present are more involved. Another correlation is required.

- If fins are closely packed, total surface area can be increased since more fins per unit distance can be accommodated therefore increasing the heat transfer. More fins imply larger flow resistance resulting in lower heat transfer coefficient values . Some optimum therefore exists.

Question 6: Solution-4

- A correlation available in the literature for optimum fin spacing is:

  S_{opt} = 2.714L/$(Ra)_{4=}$ = 11mm

- Now calculate the heat transfer coefficient and heat transferred:

  h = 1.31K/S_{opt} = 3.32 W/m^2 K
  Q = 3.32 (0.6 x 0.6) (69.35) = 40.67 W

Recap

In this class:

- Examples will be taken where the boundary layer concepts discussed in earlier classes will be used.
- Examples of the use of correlations for making calculations will also be discussed.