Module 2 : Convection

Lecture 18 : Developed velocity and Developing temperature in Pipe flow with Constant Wall temperature

Objectives

In this class:

- The developing temperature profile for fully developed velocity profile and uniform circumferential heating with constant wall temperature is obtained. Analytical solutions cannot be obtained for the entire problem and numerical solutions are borrowed from text books where-ever required.

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-1

- Now, look at the developing profile for a constant wall temperature.
- The variables are non-dimensionalized in a manner identical to that for the fully developed case. The only exception is the temperature.

$$r^*=\frac{r}{r_0};\quad u^*=\frac{u}{\bar{u}}=2\left(1-r^{*2}\right)\;;\; z^*=\frac{2z}{\mathrm{Re}\,\mathrm{Pr}\,r_0};\quad \theta=\frac{T-T_w}{T_i-T_w} \tag{18.1}$$

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-2

- The Governing Equation remains the same.

$$2(1-r^{*2})\frac{\partial\theta}{\partial z^*}=\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial\theta}{\partial r^*}\right) \tag{18.2}$$

- The Boundary conditions become

$$\left.\frac{\partial\theta}{\partial r^*}\right|_{r^*=0}=0 \tag{18.3}$$

$$\left.\theta\right|_{r^*=1}=0 \tag{18.4}$$

$$\left.\theta\right|_{z=0}=1 \tag{18.5}$$

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-3

- Again use separation of variables

$$\theta(z^*,r^*)=Z(z^*)R(r^*) \tag{18.6}$$

- The governing equ^n (18.2) therefore becomes:

$$\frac{1}{Z}\frac{dZ}{dz^*}=\frac{\frac{\mathrm{d}}{\mathrm{d}r^*}\left(r^*\frac{dR}{\mathrm{d}r^*}\right)}{2r^*(1-r^{*2})R}=-\beta \tag{18.7}$$

- The 'Z' component of the equ^n (18.7) gives:

$$Z=Ce^{-\beta z^*} \tag{18.8}$$

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-4

- The 'R' component of the equ^n (18.7) gives:

$$\frac{d}{dr^*}\left(r^*\frac{dR}{dr^*}\right)=-\beta 2r^*(1-r^{*2})R \tag{18.9}$$

- The associated boundary conditions are:

$$R'(0) = 0, R(1) = 0 \tag{18.10}$$

- Unlike for the constant wall flux developing case, the boundary conditions provide a unique solution

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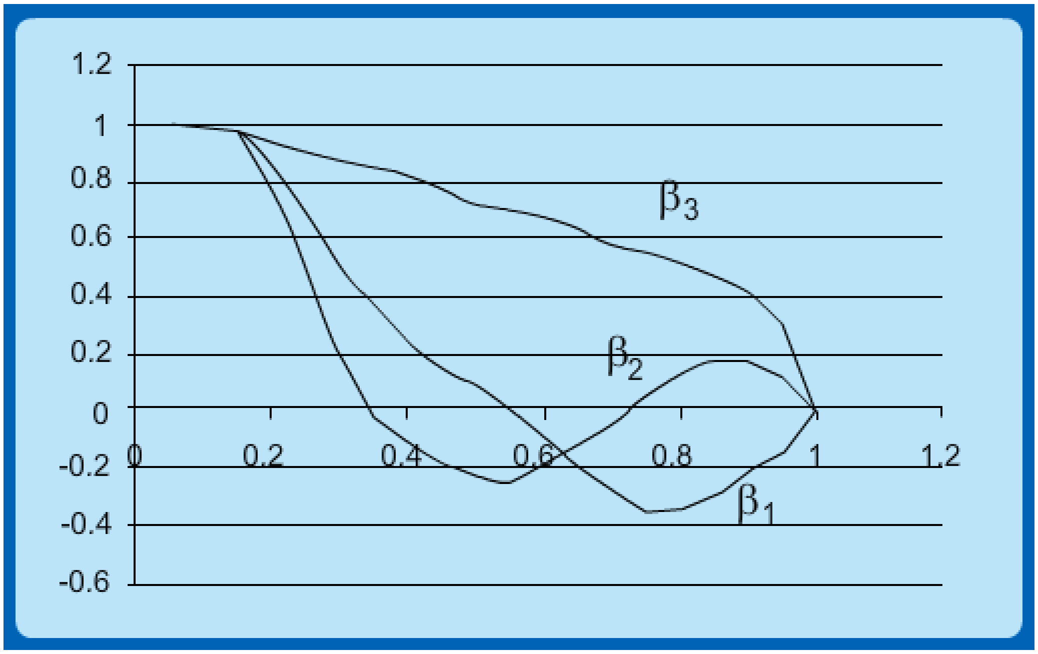
- Need to use numerical methods to obtain the solution. Some qualitative profiles are shown below:



Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-6

- Again the initial condition has to be satisfied and only a summation solution will be suitable

$$\theta(z^*, r^*) = \sum C_n R_n(r^*) e^{-\beta_n z^*} \tag{18.10}$$

- Imposing the initial condition in equ^n (18.10) gives:

$$1 = \sum C_n R_n(r^*) \tag{18.11}$$

- Equ^n (17.28) continues to be valid since the differential equation has remained the same

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-7

- Using equ^n (17.28) in conjunction with equ^n (18.11) the constants for the solution equ^n (18.10) are evaluated as:

$$\int_0^1 2r^*(1-r^{*2}) R_n dr^* = \int C_n R_n^2 2r^*(1-r^{*2}) dr^*$$

$$C_n = \frac{\int_0^1 2r^*(1-r^{*2}) R_n dr^*}{\int R_n^2 2r^*(1-r^{*2}) dr^*} \tag{18.12}$$

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-8

- Again interest is in the wall quantities and since the temperature profile is known these are evaluated:

$$q'' = -k\frac{\partial T}{\partial r}\bigg|_{r=r_0} = -\frac{k}{r_0}(T_i - T_w)\frac{\partial \theta}{\partial r^*}\bigg|_{r^*=1}$$

$$\Rightarrow Nu = \frac{q''}{(T_b - T_w)}\frac{2r_0}{k}$$

$$= -2\frac{\partial \theta}{\partial r^*}\bigg|_{r^*=1}\frac{(T_i - T_w)}{(T_b - T_w)} \qquad (18.12a)$$

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-9

- First evaluate the blue term of equ^n (18.12). Use the definition of bulk temperature

$$T_b = \frac{\int_0^{r_0} \rho c_p u T 2\pi r dr}{\rho c_p \bar{u} \pi r_0^2}$$

- Use manipulations similar to those used in the fully developed case:

$$T_b - T_w = 2\int_0^1 2r^*(1 - r^{*2})(T - T_w)dr^*$$

$$\frac{T_b - T_w}{T_i - T_w} = 4\int_0^1 r^*(1 - r^{*2})\theta dr^* \qquad (18.13)$$

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-10

- Substitute the θ to obtain:

$$\frac{T_b - T_w}{T_i - T_w} = 4\sum C_n e^{-\beta_n z^*}\int_0^1 r^*(1 - r^{*2})R_n dr^* \qquad (18.14)$$

- Use the original differential equ^n (18.9), integrate it and use the boundary condition equ^n (18.10) to get:

$$r^*\frac{dR}{dr^*}\bigg|_1 - r^*\frac{dR}{dr^*}\bigg|_0 = \int_0^1 -\beta 2r^*(1 - r^{*2})Rdr^*$$

$$\Rightarrow \int_0^1 r^*(1 - r^{*2})Rdr^* = -\frac{R'|_{r^*=1}}{2\beta} \qquad (18.15)$$

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-11

- Substitute equ^n (18.15) in equ^n (18.14) to get:

$$\frac{T_b - T_w}{T_i - T_w} = -4\sum C_n e^{-\beta_n z^*}\frac{R_n'|_{r^*=1}}{2\beta_n} \qquad (18.16)$$

- The red term in equ^n (18.12) is now evaluated:

$$-\frac{d\theta}{dr^*}\bigg|_{r^*=1} = -\sum C_n e^{-\beta_n z^*}R_n'|_{r^*=1} \qquad (18.17)$$

- Substitute equ^n (18.16) and (18.17) in (18.12a) to get:

$$Nu = \frac{\sum C_n e^{-\beta_n z^*} R_n'|_{r^*=1}}{\sum C_n e^{-\beta_n z^*} \frac{R_n'|_{r^*=1}}{\beta_n}} \qquad 18.18$$

Pipe - Fully Dev. Vel. Profile, Developing Temp. Profile Constant Wall Temp.-12

- For large values of 'z' only the first term is of importance and Nu = β_1 = 3.66 which is the same as the fully developed situation.
- At large values of 'z' the influence of the developing region diminishes. The nondimensional length at which flow developes is approximately z/D/RePr ~ 0.1

Recap

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- The developing temperature profile for fully developed velocity profile and uniform circumferential heating with constant wall temperature is obtained. Analytical solutions cannot be obtained for the entire problem and numerical solutions are borrowed from text books where-ever required.