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Module 4: Systems of Equations and Equations of Order Greater Than One

Lecture 12: Systems of Equations

We shall now discuss the manner in which the earlier techniques can be extended in two ways: to systems of equations and to equations of higher order. The order of an equation is the order of the highest derivative appearing in it. We will assume that it can be written in the form

$$y^{(p)} = f\left(y,\ y',\ldots\ldots,\ y^{(p-1)},\ t\right) \qquad (4.1)$$

where f satisfies a Lipschitz constant in each of the $y^{(i)}$.

A system of equations involves are independent variable t and more than one dependent variable. Thus,

$$y' = z$$

$$z' = -y \qquad (4.2)$$

is a system of two first order equations.

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Application of One-step Techniques to Systems of Equations:

All of the methods discussed earlier are directly applicable to systems of equations. Each member of the system is treated separately but simultaneously. Thus, the Euler method for

$$y' = f(y, z)$$

$$z' = g(y, z) \qquad (4.3)$$

is

$$y_{n+1} = y_n + h\, f(y_n,\ z_n)$$

$$z_{n+1} = z_n + h\, g(y_n,\ z_n)$$

The classical Runge-Kutta method requires that a set of $K's$ be calculated for each dependent variable. Thus, the four step Runge-Kutta process for (4.3), can be given by

$$K_1^1 = h\, f\,(y_n, z_n)$$

$$K_1^2 = h\, g\,(y_n, z_n)$$

$$K_2^1 = h\, f\left(y_n + \frac{1}{2}\, K_1^1, z_n + \frac{1}{2}\, K_1^2\right)$$

$$K_2^2 = h\, g\left(y_n + \frac{1}{2}\, K_1^1, z_n + \frac{1}{2}\, K_1^2\right)$$

$$K_3^1 = h\, f\left(y_n + \frac{1}{2}\, K_2^1, z_n + \frac{1}{2}\, K_2^2\right)$$

$$K_3^2 = h\, g\left(y_n + \frac{1}{2}\, K_2^1, z_n + \frac{1}{2}\, K_2^2\right)$$

$$K_4^1 = h\, f\,(y_n + K_3^1, z_n + K_3^2)$$

$$K_4^2 = h\, g\,(y_n + K_3^1, z_n + K_3^2)$$

$$y_{n+1} = y_n + \frac{1}{6}(K_1^1 + 2\, K_2^1 + 2\, K_3^1 + K_4^1)$$

$$z_{n+1} = z_n + \frac{1}{6}(K_1^2 + 2\, K_2^2 + 2\, K_3^2 + K_4^2)$$

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Reduction of a Higher Order Equation to a System of First Order Equations:

One common technique for handling equations of the form (4.1) is to transform them into an equivalent first order system. If we define the variables

$$y_i = y^{(i-1)} \qquad i = 1, 2, \dots, p \tag{4.4}$$

we can write (4.1) as

$$(y_p)' = f(y_1, y_2, \dots \dots, y_p, t) \tag{4.5a}$$

while by differentiation of (4.4) for $i = 1,2, \dots.., p-1,$ we get

$$(y_i)' = y^{(i)} = y_{i+1} \tag{4.5b}$$

(4.5) is a system of p first order equations which can be handled by the methods already discussed. The original problem (4.1) will have initial values specified for $y^{(i-1)}(0), i = 1,2, \dots., p,$ so (4.5) will have initial values specified for $y_i(0)$ as required.

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