Lecture 35: The problem of implicitness for Stiff systems

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For many classes of methods (certainly for the class of linear multistep methods). A-stability, and even A(0)-stability, imply implicitness. Thus, for a linear multistep method for example, we must solve, at each integration step, a set of simultaneous non-linear equations of the form

$$y_{n+K} = h\,\beta_K\, f(t_{n+K}, y_{n+K}) + g \qquad (9.3)$$

where g is a known vector.

Predictor-Corrector Techniques

Predictor-Corrector techniques prove to be inadequate when the system is stiff. If we attempt to use a $P(EC)^m E$ or $P(EC)^m$ mode with fixed m, then. The absolute stability region of the method is no longer that of the corrector alone-in general, the A –or $A(0)$ – stability is lost. Nor is the mode of correcting to convergence feasible since is order that the iteration should converge we would require that

$$L|h\,\beta_K| < 1 \qquad (9.4)$$

where L is the Lipschitz constant of $f(t, y)$ with respect to y. We know that when the system is stiff, this Lipschitz constant is very large and consequently (9.4) imposes a severe restriction on step length; in practice it is of the same order of severity as that imposed by stability requirements when a method with a finite region of absolute stability is employed.

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Module 9: Stiff-Initial Value Systems

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Newton's Method

An alternative method for handling (9.3) is the vector form of the well known Newton method

$$y^{[s+1]} = y^{[s]} - F(y^{[s]})/F'(y^{[s]}), \qquad s = 0,1,\ldots$$

for the iterative solution of the scalar equation $F(y) = 0$. If a set of m simultaneous equations is m unknowns, $F_i(y_1, y_2, \ldots y_m) = 0, i = 1,2,\ldots,m$ is written in the vector form $F(y) = 0$, then the Newton method may be written as

$$y^{[s+1]} = y^{[s]} - J^{-1}(y^{[s]})F(y^{[s]}), s = 0,1,\ldots$$

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where $J(y)$ is the Jacobian matrix $\frac{\partial F(y)}{\partial y}$. If this method is applied to (9.3), we obtain

$$y_{n+k}^{[s+1]} = y_{n+K}^{[s]} - \left[I - h\,\beta_K \frac{\partial f}{\partial y}\left(t_{n+K}, y_{n+K}^{[s]}\right)\right]^{-1} \left[y_{n+K} - h\,\beta_K\, f\left(t_{n+K}, y_{n+K}^{[s]}\right) - g\right], s = 0,1,2,\ldots \quad (9.5)$$

sufficient conditions for the convergence of Newton's method for a system are rather complicated. However, when (9.5) is applied to a stiff system, Convergence is usually obtained without a restriction on h of comparable severity to that implied by (9.4), provided that we can supply a sufficiently accurate initial estimate, $y_{n+k}^{(0)}$: a separate predictor can be used for this last purpose. Note, however, that (9.5) calls for the re-evaluation of the Jacobian $\frac{\partial f}{\partial y}$ and the consequent re-inversion of the matrix $I - h\,\beta_K \frac{\partial f}{\partial y}$ at each step of the iteration . This can be very expensive in terms of computing time, and a commonly used device is to hold the value of $\frac{\partial f}{\partial y}$ in (9.5) constant for a number of consecutive iteration steps. If the iteration converges when so modified, then it will converge to the theoretical solution of (9.3); if after a few step (typically three) it appears not to be converging, then the Jacobian is re-evaluated and the corresponding matrix re-inverted. For problems for which $\frac{\partial f}{\partial y}$ varies very slowly, it may even prove possible to hold the same value for $\frac{\partial f}{\partial y}$ for a number of consecutive integration steps.

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