NPTEL Course on

Mathematics in India: From Vedic period to Modern times

Lecture 4

Pāṇini's *Aṣṭādhyāyī*

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Outline

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- Pāṇini and Euclid
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Development of *Vyākaraṇa* or *Śabdaśāstra*

- Pre-Pāṇinian: Yāska's *Nirukta*, *Prātiśākhya* Texts, *Āpiśali*, *Indra*, *Kāśakṛtsna*, *Śākaṭāyana*, *Vyādi*, etc
- Pāṇini (c.500 BCE): *Aṣṭādhyāyī Sūtrapāṭha*, *Dhātupāṭha*, *Gaṇapāṭha*
- Kātyāyana: *Vārttika*, *Pāli-vyākaraṇa*
- Patañjali (c.100 BCE): *Mahābhāṣya*
- Śarvavarman: *Kātantra-vyākaraṇa*
- Candragomin (c.450 CE): *Cāndra-vyākaraṇa*
- Devanandin (c.450): *Jainendra-vyākaraṇa*
- Bhartṛhari (c.450): *Vākyapadīya*, *Mahābhāṣya-dīpikā*

Development of *Vyākaraṇa* or *Śabdaśāstra*

- Jayāditya, Vāmana (c.600): *Kāśikāvṛtti*
- Jinendrabuddhi (c.900): *Kāśikāvivaraṇa-pañjikā* or *Nyāsa*
- Kaiyaṭa (c.900): *Mahābhāṣya-pradīpa*
- Haradatta (c.1000): *Padamañjarī*
- Dharmakīrti (c.1000): *Rūpāvatāra*
- Hemacandra (c.1100): *Siddhahaimacandra*, etc
- Vopadeva (c.1250): *Mugdhabodha*

Development of *Vyākaraṇa* or *Śabdaśāstra*

- Rāmacandra (c.1350): *Prakriyākaumudī*
- Nārāyaṇā Bhaṭṭātiri (c.1600): *Prakriyāsarvasva*
- Bhaṭṭoji Dīkṣita (c.1625): *Siddhāntakaumudī*, *Prauḍhamanoramā*, *Śabdakaustubha*
- Kauṇḍabhaṭṭa (c.1650): *Vaiyākaraṇabhūṣaṇa*
- Varadarāja (c.1650): *Laghu-siddhāntakaumudī*, *Sāra-siddhāntakaumudī*
- Nāgeśabhaṭṭa (c.1700): *Mahābhāṣya-pradīpoddyota*, *Bṛhacchabdenduśekhara*, *Vaiyākaraṇa-siddhāntamañjūṣā*, *Paramalaghumañjūṣā*, *Paribhāṣenduśekhara*

Development of *Vyākaraṇa* or *Śabdaśāstra*

Grammars of Other Languages

- Tamil: *Tolkāppiyam* (c.200 BCE), *Vīrasolīyam* (c.1200), *Nannūl* (c.1300)
- Kannada: *Karnāṭaka-bhāṣābhūṣaṇa* (c.1100), *Śabdamaṇidarpaṇa* (c.1200), *Karnāṭaka-śabdānuśāsana* (c.1600)
- Telugu: *Āndhra-śabdacintāmaṇi* (c.1100), *Āndhrabhāṣābhūṣana* (c.1250), *Triliṅga-śabdānuśāsana* (c.1300)
- Pali: *Kaccāyana-vyākaraṇa, Saddalakkhaṇa* (c.1150)
- Prakrita: *Prākṛta-prakāśa, Prākṛta-śabdānuśāsana* (c.1200)
- Persian: *Pārasīprakāśa* (c.1575)

Śāstras: Present Systematic Procedures

Most of the canonical texts on different disciplines (*śāstras*) in Indian tradition do not present a series of propositions; instead they present a series of rules, which serve to characterize and carry out systematic procedures to accomplish various ends.

These systematic procedures are generally referred to as *vidhi*, *kriyā* or *prakriyā*, *sādhana*, *karma* or *parikarma*, *karaṇa*, *upāya* etc., in different disciplines.

The rules are often formulated in the form of *sūtras*.

Śāstras: Present Systematic Procedures

According to *Viṣṇudharmottarapurāṇa* (3.5.1): A *sūtra* has to be concise, unambiguous, pithy, comprehensive, shorn of irrelevancies and blemish-less.

अल्पाक्षरमसन्दिग्धं सारवद् विश्वतोमुखम्।
अस्तोभमनवद्यञ्च सूत्रं सूत्रविदो विदुः॥

Pāṇini's *Aṣṭādhyāyī* is acknowledged to be the paradigmatic example of a canonical text in Indian tradition. All other disciplines, especially mathematics, have been deeply influenced by its ingenious symbolic and technical devices, recursive and generative formalism and the system of conventions governing rule application and rule interaction.

Pāṇini and Euclid

"In Euclid's geometry, propositions are derived from axioms with the help of logical rules which are accepted as true. In Pāṇini's grammar, linguistic forms are derived from grammatical elements with the help of rules which were framed ad hoc (i.e. *sūtras*)....

Historically speaking, Pāṇini's method has occupied a place comparable to that held by Euclid's method in Western thought. Scientific developments have therefore taken different directions in India and in the West....

In India, Pāṇini's perfection and ingenuity have rarely been matched outside the realm of linguistics. Just as Plato reserved admission to his Academy for geometricians, Indian scholars and philosophers are expected to have first undergone a training in scientific linguistics...."[1]

Note: The word "derived" means "demonstrated" in the case of Euclidean Geometry; it means "generated" in the case of Pāṇini's Grammar (*upapatti* and *niṣpatti*)

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[1] J. F. Staal, Euclid and Pāṇini, Philosophy East and West, 15, 1965, 99-116.

अथ शब्दानुशासनम्।

अनुशासनं प्रकृतिप्रत्ययविभागेन व्युत्पादनं तद्व्याकरणेन साक्षात्क्रियत इति साक्षात्प्रयोजनम्।

[अन्नम्भट्टीय-प्रदीपोद्द्योतव्याख्या]

अथैतस्मिञ्शब्दोपदेशे सति किं शब्दानां प्रतिपत्तौ प्रतिपदपाठः कर्तव्यः गौरश्वः पुरुषो हस्ती शकुनिर्मृगो ब्राह्मण इत्येवमादयः शब्दाः पठितव्याः। नेत्याह। अनभ्युपाय एव शब्दानां प्रतिपत्तौ प्रतिपदपाठः। एवं हि श्रूयते बृहस्पतिरिन्द्राय दिव्यं वर्षसहस्रं प्रतिपदोक्तानां शब्दानां शब्दपारायणं प्रोवाच नान्तं जगाम।

[पातञ्जलमहाभाष्यम् पस्पशाह्निकम्]

Śabdānuśāsana: Pāṇini's *Aṣṭādhyāyī*

Now, the instruction of utterances

> Instruction, namely generation (of utterances) by using *prakṛti, pratyaya* and other components, this is done by grammar, and that it is its direct purpose.
>
> [*Annambhaṭṭīya-Pradīpoddyotavyākhyā*]

> Valid utterances cannot be taught by *pratipada-pāṭha* (stating each of them individually). Bṛhaspati tried to teach Indra valid utterances by *pratipada-pāṭha* for thousand divine years, but reached nowhere near the end.
>
> [*Mahābhāṣya* of Patañjali, *Paspaśāhnika*]

कथं तर्हीमे शब्दाः प्रतिपत्तव्याः। किञ्चित्सामान्यविशेषवल्लक्षणं प्रवर्त्यम्। येनाल्पेन यत्नेन महतो महतः शब्दौघान् प्रतिपद्येरन्।

किं पुनस्तत्। उत्सर्गापवादौ। कश्चिदुत्सर्गः कर्तव्यः कश्चिदपवादः। कथंजातीयकः पुनरुत्सर्गः कर्तव्यः कथंजातीयकोऽपवादः। सामान्येनोत्सर्गः कर्तव्यः। तद्यथा। कर्मण्यण् (३.२.१)। तस्य विशेषेणापवादः। तद्यथा। आतोऽनुपसर्गे कः (३.२.३)।

[पातञ्जलमहाभाष्यम् पस्पशाह्निकम्]

Śabdānuśāsana: Pāṇini's *Aṣṭādhyāyī*

How are these utterances to be known?

Some characterisation with what is general and particular is to be provided, by which, with little effort, great amount of utterances are known.

What is that characterisation? *Utsarga* (general) and *Apavāda* (special/exceptional) rules...

[*Mahābhāṣya* of Patañjali, *Paspaśāhnika*]

Modern Scholarship on Pāṇini's *Aṣṭādhyāyī*

"Of particular interest is the stress laid on the 'small number of primitive elements', themselves not used (i.e., themselves abstract) from which the Sanskrit grammarians are said to derive 'the infinite variety of actual forms in use.' "[2]

"The Descriptive Grammar of Sanskrit, which Pāṇini brought to its perfection, is one of the greatest monuments of human intelligence and an indispensible model for the description of languages."[3]

[2] J. F. Staal on Francois Pons' letter of 1740 (published 1743) in, *A Reader on the Sanskrit Grammarians,* MIT Press, 1972, p.30.

[3] L. Bloomfield, Review of Liebich, Konkordanz das Pāṇini-Candra, *Language,* 5, 267-276, 1929.

Modern Scholarship on Pāṇini's *Aṣṭādhyāyī*

"The idea that a language is based on a system of rules determining the interpretation of its infinitely many sentences is by no means novel. Well over a century ago, it was expressed with reasonable clarity by Wilhelm von Humboldt in his famous but rarely studied introduction to general linguistics (Humboldt 1836). His view that a language 'makes infinite use of finite means' and that a grammar must describe the process that makes this possible.. Pāṇini's grammar can be interpreted as a fragment of such a 'generative grammar' in essentially the contemporary sense of this term. " [4]

"Modern linguistics acknowledges it as the most complete generative grammar of any language yet written and continues to adopt technical ideas from it".[5]

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[4] N. Chomsky, *Aspects of the Theory of Syntax,* MIT Press, 1964, p.v.

[5] P. Kiparsky, Pāṇinian Linguistics, *in Encyclopaedia of Language and Linguistics, VI,* 1994.

Modern Scholarship on Pāṇini's *Aṣṭādhyāyī*

The algebraic formulation of Pāṇini's rules was not appreciated by the first Western students; they regarded the work as abstruse or artificial. ... The Western critique was muted and eventually turned into praise when modern schools of linguistics developed sophisticated notation systems of their own. Grammars that derive words and sentences from basic elements by a string of rules are obviously in greater need of symbolic code than paradigmatic or direct method practical grammars....

It is a sad observation that we did not learn more from Pāṇini than we did, that we recognised the value and the spirit of his "artificial" and "abstruse" formulations only when we had independently constructed comparable systems. The Indian New Logic (*navya-nyāya*) had the same fate: only after Western mathematicians had developed a formal logic of their own and after this knowledge had reached a few Indologists, did the attitude towards the *navya-nyāya* school change from ridicule to respect.[6]

[6] H. Scharfe, *Grammatical Literature*, Wiesbaden 1977, pp.112, 115.

Modern Scholarship on Pāṇini's *Aṣṭādhyāyī*

"Pāṇini has composed a list of formulae called *sūtra*...serving to form words and sentences from a given material of minimal elements...It comprises both lists of primary elements, and a program for the combination of these elements. These elements are the phonemes, the roots, group of words sharing a grammatical feature, morphemes (suffixes) having a meaning ...

The program is made up of operating rules as well as conventions necessary for the application of the rules. It is composed in a true meta-language very apt to its purpose, achieving the maximum brevity, which makes it easy to memorize, and is the first and foremost example of the formalization of the technical exposition in the universal history of sciences. Because of its practical objective and form, it cannot be compared with a systematic grammar of a European type. By contrast, its resemblance to a modern computer program is striking."[7]

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[7] P. S. Filliozat: *The Sanskrit Language: An Overview,* Indica Books, Varanasi 2000 (French Edition 1992), p.24.

Modern Scholarship on Pāṇini's *Aṣṭādhyāyī*

"Pāṇini's grammar is universally admired for its insightful analysis of Sanskrit...Generative linguists for their part have marvelled especially at its ingenious technical devices, and at intricate system of conventions governing rule application and rule interaction that it presupposes, which seem to uncannily anticipate ideas of modern linguistic theory (if only because many of them were originally borrowed from Pāṇini in the first place.)...

The grammar has four distinct components:

1. *Aṣṭādhyāyī*: a system of about 4,000 grammatical rules
2. *Śivasūtras*: the inventory of phonological segments
3. *Dhātupāṭha*: a list of about 2,000 verbal roots...
4. *Gaṇapāṭha*: a list of 261 lists of lexical items...

The grammar is a device that starts from meaning information... and incrementally builds up a completely interpreted sentence."[8]

[8] P. Kiparsky, On the Architecture of Pāṇini's Grammar, 2002.

Śiva-Sūtras and *Pratyāhāras*

१ अ इ उ ण् । २ ऋ ऌ क् । ३ ए ओ ङ् ।
४ ऐ औ च् । ५ ह य व र ट् । ६ ल ण् ।
७ ञ म ङ ण न म् । ८ झ भ ञ् । ९ घ ढ ध ष् ।
१० ज ब ग ड द श् । ११ ख फ छ ठ थ च ट त व् ।
१२ क प य् । १३ श ष स र् । १४ ह ल् ॥

Each *sūtra* has a set of *varṇas* followed by a marker (ण्, क्, ङ्, च्, ट्, etc) called the इत् *varṇa*

एषाम् अन्त्या इतः

Śiva-Sūtras and *Pratyāhāras*

Pratyāhāras are formed by any of the *varṇas* and an इत् which follows it. The *pratyāhāra* then stands for the class of *varṇas* enclosed by them except for the intervening इत् *varṇas*.

अक् stands for {अ, इ, उ, ऋ, ऌ}. इक् stands for {इ, उ, ऋ, ऌ}

अच् stands for all the vowels. हल् stands for all the consonants.

In this way about 300 *pratyāhāras* are possible; Pāṇini uses 42 of them.

Recent studies show that the *Śiva-sūtras* give an optimal encoding for these 42 partially ordered subsets of Sanskrit sounds.

Method of *Aṣṭādhyāyī*

Pāṇini's *Sūtras* are mainly of the following types:

- *Vidhi-sūtra*: Operational rules
- *Saṃjñā-sūtra*: Rules which introduce class names and establish conventions regarding the use of terms
- *Adhikāra-sūtra*: Headings
- *Paribhāṣā-sūtra*: Metarules, which serve to interpret and regulate other rules. They regulate the operations specified in the *vidhi-sūtras*

Method of *Aṣṭādhyāyī*

Examples of *Paribhāṣā-sūtras*

- **Ṣaṣṭhī sthāne-yogā** (1.1.49): Genitive designates 'in place of'.
- **Tasminnitinirdiṣṭe pūrvasya** (1.1.66): Locative defines the right context.
- **Tasmādityuttarasya** (1.1.67): Ablative defines the left context.
- **Yathāsaṃkhyamanudeśaḥ samānām** (1.3.10): For groups with the same number of elements, the corresponding elements are to be related in order.
- **Pūrvatrāsiddham** (8.2.1): (From now on every rule is regarded as) not having taken effect with reference to preceding ones.

Context Sensitive Rules of *Aṣṭādhyāyi*

Phonological rules are typically of the form "sounds of class *A* are replaced by sounds of class *B* if they are preceded by sounds of class *C* and followed by sounds of class *D*", which in modern phonology is usually denoted as

$$A \rightarrow B / C\_D$$

Pāṇini formulates the above rule as follows:

$A +$ genitive, $B +$ nominative, $C +$ ablative, $D +$ locative.

Example: **Ikoyaṇaci** (6.1.77)

Context Sensitive Rules of $Aṣṭādhyāyī$

ikoyaṇaci (6.1.77)

iK stands for { *i*, *u*, ṛ, ḷ },

yaṆ stands for { y, v, r, l }

aC stands for all the vowels.

From 6.1.72, **saṃhitāyām** is carried forward. Thus the $sūtra$ provides that:

i, u, ṛ, ḷ →y, v, r, l before a vowel, in close contact

This gives

$$\mathbf{i} + \mathbf{a} \rightarrow \mathbf{y} + \mathbf{a}, \mathbf{u} + \mathbf{a} \rightarrow \mathbf{v} + \mathbf{a} \text{ and so on.}$$

Akaḥ savarṇe dīrghaḥ (6.1.101) is an $apāvāda$-$sūtra$ to the above, and gives:

$$\mathbf{i} + \mathbf{i} = \bar{\mathbf{i}}, \mathbf{u} + \mathbf{u} = \bar{\mathbf{u}} \text{ and so on.}$$

Pāṇini and Zero

Pāṇini introduces the notion of zero-replacement (zero-phoneme, zero-morpheme etc)

Adarśanaṃ lopaḥ (1.1.60) Non-appearance is zero.

There are about fifty *sūtras* where *lopa* appears explicitly and more than hundred if we take into account *anuvṛtti*.

There are several other kinds of zeroes in Pāṇini.

For instance, there are the इत् *varṇas* in *pratyāhāras*. **Tasya lopaḥ** (1.3.39)

There are also *luk*, *ślu* and *lup* which correspond to non-appearance of a *pratyaya* or suffix.

Vākyāpadīya on *Śāstra* as *Upāya*

भिन्नं दर्शनमाश्रित्य व्यवहारोऽनुगम्यते।
तत्रयन्मुख्यमेकेषां तत्रान्येषां विपर्ययः॥

(वाक्यपदीयम् १.७४)

Worldly activities are accomplished on the basis of different theories and philosophies. What is important in one theory may not be so in another.

Vākyāpadīya on *Śāstra* as *Upāya*

उपादायापि ये हेया तानुपायान् प्रचक्षते।
उपायानाञ्च नियमो नावश्यमवतिष्ठते॥
अर्थं कथञ्चिद् पुरुषः कथञ्चित्प्रतिपद्यते।

(वाक्यपदीयम् २.३८-३९)

Upāyas (procedures taught in *śāstras*) are to be discarded, even though they are to be used for accomplishing an objective. There is no necessary limitation on such means. One accomplishes objectives by one means or the other.

As noted by the commentator Puṇyarāja :

कश्चिदाचार्यः पाणिनिविरचितेन लक्षणशास्त्रेण शब्दानधिगच्छति कश्चिदन्येनेति न नियमः।

References

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2. G. Cardona, *Recent Research in Pāṇinian Studies,* Motilal Banarsidass, Delhi 1999.
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5. G. Huet, A. Kulkarni and P. Scharf Eds., *Sanskrit Computational Linguistics,* Springer, New York 2009.

Thanks!

Thank You