NPTEL Course on

Mathematics in India: From Vedic period to Modern times

Lecture 39

Mathematics in Modern India 1

M. D. Srinivas
Centre for Policy Studies, Chennai

Outline

- Continuing tradition of Indian Astronomy and Mathematics (1770-1870)
- Surveys of indigenous education in India (1825-1835)
- Survival of indigenous education system till 1880

- Modern Scholarship on Indian Mathematics and Astronomy (1700-1900)
- Rediscovering the Tradition (1850-1900)

- Development of Higher Education and Modern Mathematics in India (1850-1910)

Outline

- Srinivasa Ramanujan (1887-1920)
  - Brief outline of the life and mathematical career of Ramanujan
  - Hardy's assessment of Ramanujan and his Mathematics (1922, 1940)
  - Some highlights of the published work of Ramanujan and its impact
  - Selberg's assessment of Ramanujan's work (1988)
  - The saga of Ramanujan's Notebooks
  - Ongoing work on Ramanujan's Notebooks
  - The enigma of Ramanujan's Mathematics

Background: Continuted Development of Mathematics in Medieval India

- *Gaṇitasārakaumudī* (in Prakrita) of Ṭhakkura Pherū (c.1300) and other works in regional languages such as *Vyavahāragaṇita* (Kannada) of Rājāditya, *Pāvulūri-gaṇitamu* of Pāvulūri Mallaṇa in Telugu.
- *Gaṇitakaumudī* and *Bījagaṇitāvataṃsa* of Nārāyaṇa Paṇḍita (c. 1350)
- Mādhava (c.1350): Founder of the Kerala School. Infinite series for π, Sine and cosine functions and fast convergent approximations to them.
- Works of Parameśvara (c.1380-1460)
- Works of Nīlakaṇṭha Somayājī (c.1444-1540): Revised planetary model.

Background: Continuted Development of Mathematics in Medieval India

- Systematic exposition of Mathematics and Astronomy with proofs in *Yuktibhāṣā* (in Malayalam) of Jyeṣṭhadeva (c.1530) and commentaries *Kriyākramakarī* and *Yuktidīpikā* of Śaṅkara Vāriyar (c.1540).
- Works of Jñānarāja (c.1500), Gaṇeśa Daivajña (b.1507), Sūryadāsa (c.1541) and Kṛṣṇa Daivajña (c.1600): Commentaries with *upapattis*.
- Works of Munīśvara (b.1603) and Kamalākara (b.1616).
- Mathematics and Astronomy in the Court of Savai Jayasiṃha (1700-1743). Translations from Persian of Euclid and Ptolemy.
- Works of later Kerala astronomers Acyuta Piṣāraṭi (c.1550-1621), Putumana Somayājī (c.1700).

A European Account of Indian Astronomy (c.1770)

“While waiting in Pondicherry for the Transit of 1769, Le Gentil tried to gather information about native astronomy...

Le Gentil eventually contacted a Tamil who was versed in the astronomical methods of his people. With the help of an interpreter he succeeded in having computed for him the circumstances of the lunar eclipse of 1765 August 30, which he himself had observed and checked against the best tables of his times, the tables of Tobias Mayer (1752).

The Tamil Method gave the duration of the Eclipse 41 second too short, the tables of Mayer 1 minute 8 seconds too long; for the totality the Tamil was 7 minutes 48 seconds too short, Mayer 25 seconds too long.

These results of the Tamil astronomer were even more amazing as they were obtained by computing with shells on the basis of memorised tables and without any aid of theory.

A European Account of Indian Astronomy (c.1770)

"Le Gentil says about these computations: 'They did their astronomical calculations with swiftness and remarkable ease without pen and pencil; their only accessories were cauries... This method of calculation appears to me to be more advantageous in that it is faster and more expeditious than ours.' "[1]

Note: What Neugebauer is referring to as "Tamil method" is nothing but the $V\bar{a}kya$ method developed in south India, especially by Kerala Astronomers. Neugebauer also refers to the report of John Warren (in his $K\bar{a}lasa\dot{n}kalita$) about the calculation of a lunar eclipse in 1825 June 1, where the Tamil method predicted midpoint of the eclipse with an error of about 23 minutes.

[1] Neugebauer, *A History of Ancient Mathematical Astronomy,* Vol. III, Springer , 1975, p.20, (Le Gentil's quote translated from French).

Continuing Tradition of Indian Astronomy (c.1820)

Śaṅkaravarman (1784-1839): Raja of Kaḍattanāḍ in Malabar. Due to the wars with Hyder and Tipu, he is supposed to have spent his early years with Mahārāja Svāti Tirunāl at Tiruvanantapuram.

In 1819, He wrote *Sadratnamālā* (one of the four works mentioned by Whish in 1835), an Astronomical manual following largely the *Parahita* system. He also wrote his own Malayalam commentary, perhaps a few years later (published along with the text in Kozhikode in 1899).

Chapter I has interesting algorithms for calculation of square and cube roots. Chapter IV deals with computation of sines.

Śaṅkaravarman also gives the following value of π which is accurate to 17 decimal places: $\pi \approx 3.14159265358979324$

Continuing Tradition of Indian Astronomy (c.1870)

Candraśekhara Sāmanta (1835-1904): Popularly known as Paṭhāni Sāmanta, he had traditional Sanskrit education. Starting from around 1858, he carried out extensive observations for over eleven years, with his own versatile instruments, with a view to to improve the almanac of Puri Temple.

He wrote his *Siddhāntadarpaṇa* with nearly 2500 verses in 1869 (published at Calcutta 1899). Based on his observations, Sāmanta improved the parameters of the traditional works, he detected and included all the three major irregularities of lunar motion, and improved the traditional estimates of the Sun-Earth distance.

In Chapter V of his work, Sāmanta has presented his planetary model where all the planets move around th Sun, which moves around the Earth.

Mahatma Gandhi on Indigenous Education in the 19th Century

"We have the education of this future State. I say without fear of my figures being challenged successfully, that **today India is more illiterate than it was fifty or a hundred years ago, and so is Burma,** because the British administrators, when they came to India, instead of taking hold of things as they were, began to root them out. They scratched the soil and began to look at the root, and left the root like that, and the beautiful tree perished. The village schools were not good enough for the British administrator, so he came out with his programme.... I defy anybody to fulfil a programme of compulsory primary education of these masses inside of a century. This very poor country of mine is ill able to sustain such an expensive method of education. Our State would revive the old village schoolmaster and dot every village with a school both for boys and girls."[2]

[2] Mahatma Gandhi, Speech at Chatham House, London, October 30, 1931.

"If a good system of agriculture, unrivalled manufacturing skill, a capacity to produce whatever can contribute to convenience or luxury; **schools established in every village, for teaching reading, writing, and arithmetic;** the general practice of hospitality and charity among each other; and above all a treatment of the female sex, full of confidence, respect and delicacy, are among the signs which denote a civilised people, then the Hindus are not inferior to the nations of Europe; and if civilisation is to become an article of trade between the two countries, I am convinced that this country [England] will gain by the import cargo."[3]

"We refer with particular satisfaction upon this occasion to that distinguished feature of internal polity which prevails in some parts of India, and by which the **instruction of the people is provided for by a certain charge upon the produce of the soil, and other endowments in favour of the village teachers,** who are thereby rendered public servants of the community."[4]

[3] Thomas Munro's Testimony before a Committee of House of Commons, April 12, 1813.

[4] Public Despatch from London to Bengal, June 3, 1814.

Reports on Indigenous Education in 19th Century

"There are probably as great a proportion of persons in India who can read, write and keep simple accounts as are to be found in European countries...."[5]

"I need hardly mention what every member of the Board knows as well as I do, that there is hardly a village, great or small, throughout our territories, in which there is not at least one school, and in larger villages more; many in every town, and in large cities in every division; where young natives are taught reading, writing and arithmetic, upon a system so economical, from a handful or two of grain, to perhaps a rupee per month to the school master, according to the ability of the parents, and at the same time so simple and effectual, that there is hardly a cultivator or petty dealer who is not competent to keep his own accounts with a degree of accuracy, in my opinion, beyond what we meet with amongst the lower orders in our own country; whilst the more splendid dealers and bankers keep their books with a degree of ease, conciseness, and clearness I rather think fully equal to those of any British merchants."[6]

[5] Annual Report of Bombay Education Society, 1819.

[6] Minute of G. Prendargast, Member Bombay Governor's Council, 1821.

Indiginous Education in Madras Presidency (c.1825)

The British Government conducted a detailed survey of the indigenous system of education covering all the Districts of the Madras Presidency during 1822-25. **The Survey found 11,575 schools and 1094 "colleges" in the Presidency.** Summarising the survey information the then Governor Thomas Munro wrote in his Minute of March 10, 1826:

"It is remarked by the Board of Revenue, that of a population of $12\frac{1}{2}$ millions, there are only 188,000, or 1 in 67 receiving education. This is true of the whole population, but not as regards the male part of it, of which the proportion educated is much greater than is here estimated... if we reckon the male population between the ages of five and ten years, which is the period which boys in general remain at school, at one-ninth... the number actually attending the schools [and colleges] is only 184,110, or little more than one-fourth of that number. ... **I am, however, inclined to estimate the portion of the male population who receive school education to be nearer to one-third** than one-fourth of the whole, because we have no returns from the provinces of the numbers taught at home...."

Indiginous Education in Madras Presidency (c.1825)

District	Brahmin	Kshatriya	Vaisya	Sudra	Other Castes	Muslims	Total	Total Population
Telugu Districts	13,893	121	7,676	10,076	4,755	1,639	38,160	4,029,408
% Total	*36.41*	*0.32*	*20.12*	*26.40*	*12.46*	*4.30*		
Boys of Sch-going age	9,111	2,507	7,387	134,896	59,479	10,387	223,856	
% of Community	*152.49*	*4.83*	*103.91*	*7.47*	*7.99*	*15.78*	*17.05*	
Malabar	2,230		84	3,697	2,756	3,196	11,963	907,575
% of Total	*18.64*		*0.70*	*30.90*	*23.04*	*26.72*		
Boys of Sch-going age	953	15	620	25,447	9,893	13,286	50,421	
% of Community	*234.01*	*0.00*	*13.54*	*14.53*	*27.86*	*24.06*	*23.73*	
Tamil Districts	11,557	369	4,442	57,873	13,196	5,453	92,890	6,622,474
% of Total	*12.44*	*0.40*	*4.78*	*62.30*	*14.21*	*5.87*		
Boys of Sch-going age	10,191	1,619	7,910	255,260	77,373	14,901	367,915	
% of Community	*113.40*	*22.79*	*56.16*	*22.67*	*17.06*	*36.60*	*25.25*	
TOTAL	29,721	490	13,449	75,943	22,925	10,644	153,172	12,850,941
% of Total	***19.40***	***0.32***	***8.78***	***49.58***	***14.97***	***6.95***		
Boys of Sch-going age	23,203	4,212	16,778	457,279	169,275	42,051	713,941	
% of the Community	***128.09***	***11.63***	***80.16***	***16.61***	***13.54***	***25.31***	***21.45***	

Source: Data from Dharampal, *The Beautiful Tree, Impex India,* Delhi 1983, pp.21-22.

Boys of school going age in each community, estimated by using the community profile of the total population as per 1871 Census, and by estimating the boys of school-going age (5-10 years) as one-ninth of total population, following Munro.

Indiginous Education in Madras Presidency (c.1825)

The languages of instruction in most of the 11,575 schools were the regional languages. The average period of instruction was around 5-7 years. The subjects taught were reading writing and arithmetic.

The instruction in most of the 1,094 “colleges” or institutions of higher learning was in Sanskrit. Details of the subjects taught are available for the 618 colleges in four districts: 418 taught *Vedas*, 198 Law, 34 Astronomy and *Gaṇita* and 8 taught *Āndhra Śāstram*.

Further, in Malabar, 1594 scholars were receiving higher instruction privately, of whom 808 studied Astronomy (of whom 96 were *dvijas*) and 154 Medicine (of whom 31 were *dvijas*).

Indiginous Education in Madras Presidency (c.1825)

As regards the financial support received by the indigenous schools and colleges the situation was clearly stated by the Collector of Bellary:

> "Of the 533 institutions for education, now existing in this district, I am ashamed to say not one now derives any support from the state... There is no doubt that in former times, especially under the Hindoo Governments very large grants, both in money, and in land, were issued for the support of learning."

Indiginous Education in Bengal Presidency (c.1835)

William Adam's survey (1835) of indigenous education in selected districts of Bengal and Bihar showed the following subject-wise distribution of institutions of higher learning.

Institutions of Sanscritic learning In some districts of Bengal & Bihar

	Murshidabad	Beerbhoom	Burdwan	South-Bihar	Tirhoot	Total
Number of Institutions	24	56	190	27	56	353
Number of Students (Subject wise)						
Grammar	23	274	644	356	127	1,424
Logic	52	27	277	6	16	378
Law	64	24	238	2	8	336
Literature	2	8	90	16	4	120
Mythology	8	8	43	22	1	82
Astrology	—	5	7	13	53	78
Lexicology	4	2	31	8	3	48
Rhetoric	—	9	8	2	—	19
Medicine	—	1	15	2	—	18
Vedum	—	3	3	5	2	13
Tantra	—	1	2	2	—	5
Mimansa	—	—	—	2	—	2
Sankhya	—	—	—	1	—	1
Total Number of Students	153	362	1,358	437	214	2,524

Indiginous Education in Bengal Presidency (c.1835)

Adam's survey also showed that textbooks used in these institutions of higher learning included, apart from the ancient canonical texts of the various disciplines, many of the important advanced treatises commentaries and monographs composed during the late medieval period.

These included the works of Bhaṭṭoji Dīkṣita (1625), Kauṇḍabhaṭṭa (c.1650), Hari Dīkṣita and Nāgeśa Bhaṭṭa (c.1700) in *Vyākaraṇa*, the works of Raghunātha (c.1500), Mathurānātha (c.1570), Viśvanātha (c.1650), Jagadīśa (c.1650) and Gadādhara (c.1650) in *Navya-nyāya*, the works of Raghunandana (c.1550) in *Dharmaśāstra* and the works *Vedāntasāra* (c.1450) and *Vedāntaparibhāṣā* (c.1650) in *Vedānta*.

The period of study in these institutions of higher learning was between ten and twenty-five years. In many of these centres of higher learning a large part of the students came from outside, many from even different regions of India. All the students were taught gratis and outside students were provided in addition free food and lodging.

The University of Navadvīpa

On visiting Navadvīpa or Nuddeah in 1787, William Jones wrote to Earl of Spencer that "This is the third University of which I am a member". An account of Navadvīpa published in Calcutta Monthly Register in January 1791 noted:

> "The grandeur of the foundation of the Nuddeah University is generally acknowledged. It consists of three colleges Nuddeah, Santipore and Gopulparra. Each is endowed with lands for maintaining masters in every science....in the college of Nuddeah alone, there are at present about eleven hundred students and one hundred and fifty masters. Their numbers, it is true, fall very short of those in former days. **In Rajah Roodre's time (circa 1680) there were at Nuddaeah no less than four thousand students** and masters in proportion.

According to Adam, in 1829 there were reported to be 25 schools of learning in Navadvīpa with 500 to 600 students. Some of these schools were supported by a small allowance from British Government.

Indigenous Education in Madras Presidency (1855-1880)

At the time when the Department of Education waas established in 1855, there were only 83 schools under it, while nearly 12,500 indigenous schools were still functioning with a total of 1.6 lakh students.

The situation was similar even till 1870-1, except that about 3,000 schools had been brought under the scheme of Aided Schools.

It was only during the decade 1870-1880, that the Education Department seems to have managed to bring nearly 10,000 indigenous schools under the aided scheme. Only around 1875, did the number of students studying under the aegis of the Department of Education become comparable to the number who studied in the indigenous schools fifty years earlier in 1825.

Emergence of Modern European Scholarship on Indian Astronomy and Mathematics (c.1700-1800)

In 1687, Simon de La Loubere (1642-1729) the French Ambassador to Siam, brought to Paris manuscripts describing Indian methods of astronomical computations.

In the book *Du Royaume de Siam* (1691), he discussed these methods, along with comments by Cassini. He also gave the Indian methods of construction of magic squares. The methods of calculation in the Siamese tables were commented upon in a book by Bayer (1738) along with a note by Leonard Euler on the Indian Solar year.

Emergence of Modern European Scholarship on Indian Astronomy and Mathematics (c.1700-1800)

Le Gentil (1725-1792) who visited India during 1761 and 1769, to observe ther transit of Venus, gave a detailed account of Indian Astronomy in 1770s based on Tables and Texts obtained in Pondicherry.

This led to the treatise *Traite de l'Astronomie Indienne et Orientalle* (1787) by Jean Sylvain Bailly (1736-1793). This was reviewed in detail by John Playfair (1748-1819) in the Transactions of Royal Society in 1790.

The Asiatic Society was founded in 1784 by William Jones (1746-1794). The Journal *Asiatic Researches,* started in 1788, carried articles by William Jones, Samuel Davis and John Bentley on Indian Astronomy.

Translations and Editions of Indian Texts on Astronomy and Mathematics in 19th Century

The *Bījagaṇita* of Bhāskara, was translated into English from the Persian translation of Ata Allah Rushdi (1634) by Edward Strachey (1812-1901) with notes by Samuel Davis (London, 1813). This was closely followed by the translation of *Līlāvatī* by John Taylor (Bombay, 1816).

Henry Thomas Colebrooke (1756-1837) published several articles on Indian Astronomy and also on Law, Linguistics Philosophy etc. His most important work is *Algebra with Arithmetic and Mensuration from the Sanskrit of Brahmagupta and Bhascara* (London, 1817), which included a tranlsation of *Gaṇitādhyāya* and *Kuṭṭakādhyāya* of *Brāhmasphuṭasiddhānta* as well as the *Līlāvatī* and *Bījagaṇita* of Bhāskara II, along with notes drawn from some of the ancient commentaries.

Translations and Editions of Indian Texts

John Warren (1769-1830) wrote on Indian calendrical computations based on both the *siddhānta* and *vākya* methods in *Kālasaṅkalita* (1825). The book also includes notes of exchanges between Warren, B. Heyne and C. M. Whish on the various infinite series with which the contemporary south Indian astronomers seem to have been acquainted with.

Charles Matthew Whish (1794-1833) collected several important manuscripts of the Kerala School and made extensive notes on them. His seminal article on the Kerala School was published in 1835.

Lancelot Wilkinson, political agent at Bhopal, edited the *Siddhāntaśiromaṇi* of Bhāskarācārya with *Vāsanā* (Calcutta 1842) and *Grahalāghava* of Gaṇeśa with commentary of Mallāri (Calcutta 1843). His translation of *Golādhyāya* of *Siddhāntaśiromaṇi* was edited by Bapudeva Sastri (Calcutta 1861).

Fitz-Edward Hall (1825-1901), in collaboration with Bapudeva Sastri, edited *Sūryasiddhānta* with the commentary of Raṅganātha (Calcutta 1854).

Translations and Editions of Indian Texts

Rev. Ebenzer Burgess published an English translation of *Sūrya-siddhānta* with detailed notes with the help of William Dwight Whitney (New Haven 1860).

Albrecht Weber edited the *Vedāṅga Jyotiṣa* with the commentary Somākara (Berlin 1862)

Johann Hendrick Caspar Kern (1833-1917) edited the *Bṛhatsaṃhitā* of Varāhamihira (Calcutta 1865) and partially translated it (JRAS, 1873). He also edited the *Āryabhaṭīya* with the commentary of Parameśvara (Leiden 1875)

George Frederick William Thibaut (1848-1914) edited the *Baudhāyana-Śulvasūtra* with the commentary of Dvārakānātha (1874). He also edited *Vedāṅga-Jyotiṣa* (1877) and *Kātyāyana-Śulvasūtra* (in part) with commentary (1882). In collaboration with Sudhakara Dvivedi, he edited and translated the *Pañcasiddhāntikā* of Varāha-mihira (1884). Thibaut's essay, *The Śulvasūtras*, was reprinted as a book (Calcutta 1875). He also wrote an overview *Astronomie Astrologie und Mathematik* (Strassburg 1899).

Rediscovering the Tradition (1850-1900)

Some editions and translations into Bengali, Telugu, Marathi etc, of Indian source-works such as *Līlāvatī, Bījagaṇita, Grahalāghava,* were published in the first half of 19th century. Around the same time, several Indian scholars, who were often from traditional learned families but educated in the English education system, embarked on a process of rediscovery of Indian tradition. We mention some of the seminal figures in this movement.

Bapudeva Sastri (1821-1900) studied with Pandit Dhundiraja Misra and later Pandit Sevarama and Wilkinson at Sehore Sanskrit College. He became a Professor at Benares Sanskrit College where he is said to have taught Euclidean Geometry. He published editions of *Siddhāntaśiromaṇi* with *Vāsanā* of Bhāskarācārya (1866) and *Līlāvatī* with his own commentary (1883). He collaborated with Lancelot Wilkinson and edited his translation of *Golādhyāya* of *Siddhānta-śiromaṇi* (1861).

Bhau Daji Laud (1821-1874), trained in medicine at Grant's College, Mumbai, he was also a Sanskritist and an expert in numismatics. He was the first to locate a manuscript of *Āryabhaṭīya* in 1864.

Rediscovering the Tradition (1850-1900)

Shankar Balakrishna Dikshit (1853-1898), a mathematics teacher and Principal of Teacher's Training College, Pune, wrote a voluminous history of Indian Astronomy, *Bhāratīya Jyotiṣa Śāstrāchā Prācīna āṇi Arvācīn Itihās* in Marathi (Pune, 1896). Along with Robert Sewell, he also wrote on the *Indian Calendar* (London, 1896)

Sudhakara Dvivedi (1855-1910) studied with Pandits Devakrishna and Bapudeva Sastri at Benares Sanskrit College and became a Professor there. He edited a very large number of ancient texts which became the main source for all later studies.

Some of the important texts edited by Dvivedi are: *Līlāvatī* (1878), *Karaṇakutūhala* of Bhāskarācārya II (1881), *Yantrarāja* with Malayendu's commentary (1882), *Siddhānta-tattvaviveka* with *Śeṣavāsanā* of Kamalākara (1885), *Śiṣyadhīvṛddhida* of Lalla (1886), *Bījagaṇita* with commentary (1888), *Bṛhatsaṃhitā* with Utpala's commentary (1895-7), *Triśatikā* of Śrīdhara (1899), *Karaṇaprakāṣa* of Brahmadeva (1899), *Brāhmasphuṭasiddhānta* with his own Sanskrit commentary (1902) and *Grahalāghava* with commentaries of Mallāri and Viśvanātha (1904).

Rediscovering the Tradition (1850-1900)

Sudhakara Dvivedi also edited *Yājuṣa-Jyautiṣam* with Somākara commentary (1908), *Mahāsiddhānta* of Āryabhaṭa II with his own commentary (1910), *Āryabhaṭīya* with his own commentary (1911), *Sūryasiddhānta* with his own commentary (1911). With Thibaut, he edited *Pañcasiddhāntikā* with his own Sanskrit commentary (1889).

Dvivedi wrote many original works such as *Dīrghavṛtta-lakṣaṇa* (1878), *Vāstavacandra-śṛṅgonnati-sādhanā* (1880), *Bhābhramarekhā-nirūpaṇa* (1882), *Calanakalana* on differential calculus in Hindi (1886) and *Gaṇakataraṅgiṇī* (1890) on the lives of Indian mathematicians and astronomers. In 1910 he wrote *A History of Mathematics, Part I (Arithmetic)* in Hindi.

Development of Higher Education in India (1850-1900)

The Universities of Calcutta, Bombay and Madras were set up in 1857.

It has often been remarked that these (and the later Universities in India) were established as examining bodies with affiliated colleges on the model of the then London University and not on the model of the renowned Oxford and Cambridge Universities with extensive research and teaching activities.

The Indian Association of Cultivation of Science was established by Mahendra Lal Sircar (1833-1904) in 1876 with the object of enabling Indians “to cultivate science in all its departments with a view to its advancement by original research”. However, during the first thirty years, the main efforts of the institution were directed towards the development of science teaching at the collegiate level.

In 1855, there were 15 Arts Colleges with 3246 students, and 13 Professional Colleges with 912 students. By 1901, there were 5 Universities, 145 Arts Colleges with 17,651 students and 46 Professional Colleges with 5,358 students in the whole of India (including Burma).

Development of Higher Education in India (1850-1900)

	1855	1882	1901-2
Universities	-	4	5
Number of Students	-	NA	NA
Arts Colleges	15	38	145
Number of Students	3,246	4,252	17,651
Professional Coleges	13	96	46
Number of Students	912	3,670	5,358
Secondary Schools	169	1,363	5,493
Number of Students	18,335	44,605	6,22,768
Primary Schools	1,202	13,882	97,854
Number of Students	40,041	6,81,835	32,04,336
Special Schools	7	83	1,084
Number of Students	197	2,814	36,380
Total Recognised Institutions	1,406	15,462	1,04,627
Number of Students	62,731	7,37,176	38,86,493

Development of Higher Education in India (1850-1910)

Yesudas Ramchundra (1821-1880), a teacher of science in Delhi College, wrote a *Treatise on Problems of Maxima and Minima* (1850), which approached these problems purely algebraically. Augustus de Morgan got it republished, with his own introduction, from London in 1859.

The Indian Mathematical Society began as the “Analytical Club” in 1907 at the initiative of V. Ramaswamy Aiyar, a civil servant (then a Deputy Collector at Gutti). It was renamed Indian Mathematical Society in 1910.

It started a journal in 1909, edited by M. T. Narayaniyengar and S. Narayana Aiyar. This was soon renamed the *Journal of Indian Mathematical Society.* The Society also started the journal *Mathematics Student* in 1932.

The Calcutta Mathematical Society was formed in 1908 at the initiative of Prof. Ashutosh Mukherjee (1864-1924) the then Vice Chancellor of Calcutta University. It also began publishing the *Bulletin* of the Society in 1909.

Srinivasa Ramanujan (1887-1920)

Srinivasa Ramanujan, the greatest mathematician India has produced in recent times, was born on December 22, 1887 at Erode.

In 1892, he enrolled in primary school in Kumbakonam.

In 1898, having passed his primary examinations with distinction, he joined the Town High School of Kumbakonam. He passed out of the school as an outstanding student in 1904 and received a scholarship to study at the Government College, Kumbakonam.

While at school, he got a copy of Loney's *Plane Trigonometry* which he soon mastered, but also was surprised to see there some of the results that he had obtained himself.

Around 1903, Ramanujan went through G. S. Carr's *Synopsis of Pure and Applied Mathematics* (1880), a compendium of about 5000 results, which is said to have influenced him considerably.

Early Work of Ramanujan

Ramanujan seems to have started discovering new results and recording them his Notebook by 1904.

However, at the college, “owing to his weakness in English” as Hardy notes, Ramanujan failed and lost his scholarship. Ramanujan joined the Pachaiyappa’s College at Madras, but soon he had to discontinue due to bad health. He appeared for F.A. privately in 1907, but failed once again.

Ramanujan got married in 1909 and with great effort managed to get a job in Madras Port Trust as a clerk in 1912, by the good will and support of various personalities who were impressed by his mathematical abilities.

Early Work of Ramanujan

During 1911-1913, Ramanujan published 5 papers in the Journal of Indian Mathematical Society.

His first paper on “Some Properties of Bernoulli Numbers” (1911) is said to contain “eight theorems, ...embodying arithmetical properties of the B’s. Of these proofs are indicated for three.. but the theorems on which these proofs would depend...and the corresponding propositions about the series....are never proved. Two other theorems...are stated to be corollaries...and (three)... are stated merely as conjectures.”[7]

[7] G. H. Hardy et al Eds, *Collected Papers of Srinivasa Ramanujan,* Cambridge 1929, p.335.

Early Work of Ramanujan

During 1911-13, Ramanujan also posed about 30 problems in the Journal of Indian Mathematical Society for nearly twenty of which he also provided the solution (as they were not solved by others in six months).

Here is a sample question published in 1911:

Question 289 (III 90):

Find the Value of

(i) $\sqrt{(1+2\sqrt{\{1+3\sqrt{(1+\ldots)}\}})}$
(ii) $\sqrt{(6+2\sqrt{\{7+3\sqrt{(8+\ldots)}\}})}$

Early Work of Ramanujan

Solution by Srinivasa Ramanujan, IV 226:

(i) $n(n+2) = n\sqrt{\{}1+(n+1)(n+3)\}$.
Let $\qquad n(n+2) = f(n);$

Then we see that

$$\begin{aligned} f(n) &= n\sqrt{\{}1+f(n+1)\} \\ &= n\sqrt{\{}1+(n+1)\sqrt{(}1+f(n+2))\} \\ &= n\sqrt{(}1+(n+1)\sqrt{\{}1+(n+2)\sqrt{(}1+f(n+2))\}) \\ &= \ldots\ldots \end{aligned}$$

That is,

$$n(n+2) = n\sqrt{(}1+(n+1)\sqrt{\{}1+(n+2)\sqrt{(}1+(n+3)\sqrt{1}+\ldots)\})$$

Putting $n=1$, we have

$$\sqrt{(}1+2\sqrt{\{}1+3\sqrt{(}1+\ldots)\}) = 3.$$

Early Work of Ramanujan

One of Ramanujan's early papers is on the "Modular equations and approximations to π". Though published later from London in 1914 (QJM 1914, 350-372), it is said to embody "much of Ramanujan's early Indian work." Here is a sample of his results:

$$\frac{1}{3\pi\sqrt{3}} = \frac{3}{49} + \frac{43}{49^3}\,\frac{1}{2}\,\frac{1.3}{4^2} + \frac{83}{49^5}\,\frac{1.3}{2.4}\,\frac{1.3.5.7}{4^2.8^2} + \ldots, \ldots\ldots$$

$$\frac{2}{\pi\sqrt{11}} = \frac{19}{99} + \frac{299}{99^3}\,\frac{1}{2}\,\frac{1.3}{4^2} + \frac{579}{99^5}\,\frac{1.3}{2.4}\,\frac{1.3.5.7}{4^2.8^2} + \ldots, \ldots\ldots$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1103}{99^2} + \frac{27493}{99^6}\,\frac{1}{2}\,\frac{1.3}{4^2} + \frac{53883}{99^{10}}\,\frac{1.3}{2.4}\,\frac{1.3.5.7}{4^2.8^2} + \ldots$$

In November 1985, R.W.Gospar used the last series above to compute π to 17,526,100 digits which was a record at that time. In 1989, Jonathan and Peter Borwein proved all the 17 series for $\frac{1}{\pi}$ given in Ramanujan's paper. Discovering similar such series continues to be an active area of research.

Approaching British Mathematicians (1912-13)

In 1912, Ramanujan sent a sample of his results to Prof. M. J. M. Hill of University College, London, through Prof. C. L. T. Griffith of the Madras College of Engineering.

Prof. Hill wrote back that Ramanujan had “fallen into the pitfalls of ...divergent series” and advised that he consult Bromwich’s book on infinite series.

Ramanujan is said to have also contacted Profs H. F. Baker and E. W. Hobson at Cambridge and received no response.

Approaching British Mathematicians (1912-13)

On January 16, 1913 Ramanujan wrote to Prof Godfrey Harold Hardy (1877-1947) at Cambridge, enclosing an eleven page list of over one hundred results such as the following:

$$
\begin{aligned}
\text{If} \qquad u &= \frac{x}{1+}\,\frac{x^5}{1+}\,\frac{x^{10}}{1+}\,\frac{x^{15}}{1+}\,\frac{x^{20}}{1+}\cdots \\
\text{and} \qquad v &= \frac{\sqrt[5]{x}}{1+}\,\frac{x}{1+}\,\frac{x^2}{1+}\,\frac{x^3}{1+}\cdots, \\
\text{then} \qquad v^5 &= u\frac{1-2u+4u^2-3u^3+u^4}{1+3u+4u^2+2u^3+u^4}.
\end{aligned}
$$

$$
\frac{1}{1+}\,\frac{e^{-2\pi}}{1+}\,\frac{e^{-4\pi}}{1+}\,\frac{e^{-8\pi}}{1+}\cdots = \left(\sqrt{\frac{5+\sqrt{5}}{2}} - \frac{\sqrt{5}+1}{2}\right)\sqrt[5]{e^{2\pi}}.
$$

Approaching British Mathematicians (1912-13)

The impact of this letter can be gauged from the fact that on February 2, 1913, Bertrand Russell wrote to Lady Morell that he “found Hardy and Littlewood in a state of wild excitement, because they have discovered a second Newton, a Hindu clerk on 20 Pounds a year.”

There is also a note of Littlewood to Hardy in March 1913 with the comment “I can believe that he is at least a Jacobi”

On February 8, Hardy wrote expressing his interest in the work of Ramanujan, while adding,

> “But I want particularly to see your proofs of your assertions here. You will understand that in this theory everything depends on rigorous exactitude of proof.”

The same point is repeated twice again in the same letter.

Approaching British Mathematicians (1912-13)

In his reply of February 27, 1913 (which also included a 10 page supplement with many results), Ramanujan recounted his experience with a Professor in London (Prof. Hill) and said:

> "I find in many a place in your letter rigorous proofs are required and so on and you ask me to communicate the methods of proof. If I had given you my method of proof I am sure you will follow the London Professor. ...
>
> I dilate on this simply to convince you that **you will not be able to follow my methods of proof if I indicate the lines on which I proceed in a single letter.** You may ask how you can accept results based upon wrong premises. What I tell you is this. Verify the results I give and if they agree with your results, got by treading on the groves in which the present day mathematicians move, you should at least grant that there may be some truths in my fundamental basis....
>
> You may judge me hard that I am silent on the methods of proof. I have to re-iterate that I may be misunderstood if I give in a short compass the lines on which I proceed."

Ramanujan’s Work in England

Ramanujan arrived in London on April 14, 1914 and left for India on February 27, 1919. Of the nearly five years he spent there, he was very ill for more than two years. From around the spring of 1917, he was in hospitals most of the time.

On his work during 1914, Ramanujan wrote to his friend B. Krishna Rao on November 14, 1914:

> “I changed my plan of publishing my results. I am not going to publish any of the old results in my notebook till the war is over. After coming here I have learned some of their methods. **I am trying to get new results by their methods so that I can easily publish these results without delay.** In a week or so I am going to send a long paper to the London Mathematical Society. The results in this paper [on highly composite numbers] have nothing to do with those of my old results. I have published only three short papers....”

Ramanujan’s Work in England

Ramanujan reiterated this in a letter to S. M. Subramanian in January 1915:

> “I am doing my work very slowly. My notebook is sleeping in a corner for these four or five months. I am publishing only my present researches as I have not yet proved the results in my notebooks rigorously. I am at present working in arithmetical functions...”

Ramanujan's Work in England

During 1914-1919, Ramanujan wrote about 30 papers, 7 of them in collaboration with Hardy, which were mostly concerning properties of various arithmetical functions.

His work was highly acclaimed. On March 16, 1916 he was awarded Bachelor of Science degree by Research from the Cambridge University. He was elected a Fellow of Royal Society on February 28, 1918, the second Indian to be so honoured. On October 23, 1918 he was elected a Fellow of the Trinity College (It appears that the College failed to elect Ramanujan as a fellow in 1917 for various non-academic reasons).

In late 1918, the Madras University also offered a matching grant of 250 Pounds a year. On receipt of this communication, Ramanujan wrote to the Registrar of the University on January 11, 1919 that, after meeting his basic expenses, the surplus "should be used for some educational purpose, such in particular as the reduction of school-fees for poor boys and orphans and provision of books in schools."

Ramanujan’s Last Letter and “Lost Notebook”

On March 27, 1919 Ramanujan returned to India. He was in very poor health. He stayed for a while in Madras and then moved to Kodumudi, then to Kumbakonam, and finally returned to Madras by January 1920.

Though seriously ill, he was continuing his work all the while. On January 12, 1920, Ramanujan wrote to Hardy (for the first time after returning to India):

> “I discovered very interesting functions recently which I call “Mock” θ-functions. Unlike the “False” θ-functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics as beautifully as the ordinary θ-functions. I am sending you with this letter some examples...”.

This was followed by a few pages containing definitions, some examples and properties of the mock θ-functions.

Ramanujan's Last Letter and "Lost Notebook"

The so called "Lost Notebook" of Ramanujan is a sheaf of over hundred sheets containing about 600 results that Ramanujan had found during the last year of his life.

This seems to have been sent to Hardy along with all other papers of Ramanujan in 1923.

It was finally discovered by George Andrews in 1976 (amongst Watson papers) in Trinity College.

Ramanujan passed away in Madras on April 26, 1920.

Hardy's Assessment of Ramanujan (1921)

Soon after Ramanujan's death, Hardy wrote an Obituary Notice in Proc. Lond. Math. Soc. (19, 1921, pp. 40-58), which was later reproduced in the *Collected Papers of Ramanujan* (Cambridge 1927). There, Hardy first presents his assessment of Ramanujan when he arrived in England (1914):

"The limitations of his knowledge were as startling as its profundity. Here was a man who could work out modular equations, and theorems of complex multiplication, to orders unheard of, whose mastery of continued fractions was, on the formal side at any rate, beyond that of any mathematician in the world, who had found for himself the functional equation of the Zeta-function, and the dominant terms of many of the most famous problems in the analytic theory of numbers; and he had never heard of a doubly periodic function or of Cauchy's theorem, and had indeed but the vaguest idea of what a function of a complex variable was. **His ideas as to what constituted mathematical proof were of the most shadowy description. All his results, new or old, right or wrong had been arrived at by a process of mingled argument, intuition and induction, of which he was entirely unable to give any coherent account.**"

Hardy's Assessment of Ramanujan (1921)

Hardy then notes that, after their interaction, "In a few years time, he [Ramanujan] had a very tolerable knowledge of the theory of functions, and the analytic theory of numbers. He was never a mathematician of the modern school...."

Hardy also states that Ramanujan **"adhered with a severity most unusual in an Indian resident in England to the religious observance of his caste; but his religion was a matter of observance and not of intellectual conviction, and I remember well his telling me (much to my surprise) that all religions seem to him more or less to be equally true."**

Hardy then raises the issue: "I have often been asked whether Ramanujan had any special secret; whether his method differed in any kind from those of other mathematicians; whether there was anything really abnormal in his mode of thought. I cannot answer these questions with any confidence or conviction; but I do not believe it. My belief is that all mathematicians think, at bottom, in the same kind of way, and that Ramanujan was no exception...."

Hardy's Assessment of Ramanujan (1921)

Hardy then goes onto declare:

> "It was his insight into algebraic formulae, transformation of infinite series, and so forth, that was most amazing. On this side most certainly I have never met his equal, and **I can compare him only with Euler or Jacobi.** He worked, far more than the majority of modern mathematicians, by induction from numerical examples; all of his congruence properties of partitions, for example, were discovered this way. But with his memory, his patience, and his power of calculation, he combined a power of generalisation, a feeling for form, and a capacity for rapid modification of his hypothesis that were often really startling..."

Hardy's Assessment of Ramanujan (1921)

Hardy concludes with the observation:

> "Opinions may differ as to the importance of Ramanujan's work, the kind of standard by which it should be judged, and the influence which it is likely to have on the mathematics of the future**. It has not the simplicity and the inevitability of the very greatest work; it would be greater if it were less strange.** One gift it has which no one can deny, profound and invincible originality**. He would probably have been a greater mathematician if he had been caught and tamed a little in his youth; he would have discovered more that was new, and that, no doubt of greater importance.**"

Hardy's Assessment of Ramanujan's Early Work (1940)

In his lectures on *Ramanujan,* published in 1940, Hardy again gave an assessment of Ramanujan's early work before he went to England:

> "He [Ramanujan] published abundantly... but he also left a mass of unpublished work which has never been assessed properly until the last few years. **This work includes a great deal that is new, but much more that is rediscovery, and often imperfect rediscovery; and it is sometimes still impossible to distinguish between what he must have rediscovered and what he may somehow have learnt.**"

Hardy's Assessment of Ramanujan's Early Work (1940)

"It was inevitable that a very large part of Ramanujan's work should prove on examination to have been anticipated. He had been carrying an impossible handicap, **a poor and solitary Hindu pitting his brains against the accumulated wisdom of Europe.** He had had no real teaching at all; there was no one in India from whom he had anything to learn... **I should estimate that about two-thirds of Ramanujan s best Indian work was rediscovery,** and comparatively little of it was published in his life time... The great deal of Ramanujan s published work was done in England....In particular he learnt what was meant by proof..."[8]

[8] G. H. Hardy, *Ramanujan,* Cambridge 1940, pp. 1, 10

Ramanujan's Work on Partitions

The number of partitions $p(n)$ is the number of distinct ways of representing n as a sum of positive integers, without taking the order into account. $p(0)$ is taken to be1.

Partitions of 4 are: $1+1+1+1, 1+1+2, 1+3, 2+2, 4$. Hence, $p(4)=5$

In a couple of papers published in 1919 and 1920, and a paper published posthumously in 1921, Ramanujan discovered and proved the congruences:

$$\begin{aligned} p(5m+4) &\equiv 0 \pmod{5} \\ p(7m+5) &\equiv 0 \pmod{7} \\ p(11m+6) &\equiv 0 \pmod{11} \end{aligned}$$

Ramanujan also conjectured that 5, 7 and 11 are the only primes for which such congruences hold; Ahlgren and Boylan proved this in 2003.

Ramanujan's Work on Partitions

In his 1919 paper, Ramanujan also conjectured:

If $\quad d = 5^a 7^b 11^c,$ and $24\lambda \equiv 1 \pmod{d}$, then, $p(\lambda) \equiv 0 \pmod{d}$.

In a later unpublished manuscript, Ramanujan proved the above conjecture for arbitrary a, with $b = c = 0$.

In 1934, S. Chawla disproved the general conjecture, by noting that

$$p(243) = 13,397,825,934,888 \text{ is not divisible by } 7^3$$

$$\text{even though } 24.243 \equiv 1 \ (\text{mod } 7^3)$$

However, it has later been established that

$$\text{If } 24\lambda \equiv 1 \ (\text{mod } 5^a 7^b 11^c), \text{ then}$$

$$p(\lambda) \equiv 0 \ (\text{mod } 5^a 7^{b'} 11^c)$$

$$\text{where } b' = b \text{ if } b = 0, 1, 2 \text{ and } b' = \left[\frac{(b+2)}{2}\right] \text{ if } b > 2$$

Ramanujan's Work on Partitions

In 1918, Hardy and Ramanujan obtained an infinite asymptotic series for $p(n)$, of which the first term is of the form

$$p(n) \sim \frac{1}{4n\sqrt{3}} \exp\left(\pi\sqrt{\frac{2n}{3}}\right)$$

It has been noted by Berndt that:

> "In their classic paper [of 1918] Hardy and Ramanujan introduced their famous 'circle method', which remains today as the primary tool of number theorists using analytical techniques in studying problems of additive number theory. The principal idea behind the 'circle method' can be found in Ramanujan's notebooks..., although he did not rigorously develop his ideas....Despite its genesis in Ramanujan's work, today it is often called Hardy-Littlewood circle method, because Hardy and J. E. Littlewood extensively developed the method in a series of papers."[9]

---

[9] B. Berndt, *Number Theory in the Spirit of Ramanujan*, AMS, New York 2006, pp.22-23.

Ramanujan's Work on Partitions

One of the results communicated by Ramanujan to Hardy, in his first letter of January 16, 1913, was

$$\text{The coefficient of } x^n \text{ in } \frac{1}{1-2x+2x^4-2x^8+2x^{16}-\dots}$$
$$= \text{the nearest integer to } \frac{1}{4n}\left\{\cosh(\pi\sqrt{n}) - \frac{\sinh(\pi\sqrt{n})}{\pi\sqrt{n}}\right\}$$

Commenting on this, Hardy wrote in 1940 that:

> "The function in ...[the right hand side] is a genuine approximation to the coefficient, though not at all so close as Ramanujan imagined, and Ramanujan's false statement was one of the most fruitful he ever made, since it ended by leading us to all our joint work on partitions"

It is altogether another story that, in the above conjecture, Ramanujan seems to have clearly anticipated the exact form of $p(n)$, which was found later by Radmacher in 1937.

Ramanujan's Tau Function

In his paper "On certain arithmetic functions" (1916) Ramanujan defined the Tau function via the identity

$$\sum_{n\geq 1} \tau(n)q^n = q\prod_{n\geq 1}(1-q^n)^{24}$$

Two of the properties of the Tau function stated by Ramanujan were proved by Mordell in 1917.

$$\tau(mn) = \tau(m)\tau(n) \text{ if gcd } (m,n) = 1.$$
$$\tau(p^\lambda) = \tau(p)\tau(p^{\lambda-1}) - p^{11}\tau(p^{\lambda-2}) \text{ for all prime } p.$$

Ramanujan also conjectured that if p is any prime

$$|\tau(p)| \leq 2p^{\frac{11}{2}}$$

This was proved by Pierre Deligne in 1974 as a consequence of his proof of Weil's conjecture. Deligne was awarded the Fields Medal in 1978 and Abel's prize in 2013.

Selberg’s Assessment of Ramanujan’s Work (1988)

In 1988, during the centenary of Ramanujan, Atle Selberg (1917-2007) one of the leading number theorists of 20th century and winner of the Fields Medal (1950) (and later the first recipient (honorary) of the Abel Prize in 2002), presented his reflections on Ramanujan’s work:

> “Srinivasa Ramanujan’s work played a very important role in my own development as a mathematician...
>
> Ramanujan’s particular talent will seem to be primarily of an algebraic and combinatorial nature. He developed it, for a long time in complete isolation really without any contact with other mathematicians. **He had on his own acquired an extraordinary skill of manipulation of algorithms, series, continued fractions and so forth, which certainly is completely unequalled in modern times....**
>
> **...in what has been left of his work, there seems quite clear evidence that he had developed, on his own, a theory of modular forms and equations,** for instance, but the precise form of this theory has to be guessed from the isolated results that he wrote in the *Notebooks.*”

Selberg's Assessment of Ramanujan's Work (1988)

Selberg, who himself independently discovered Radmacher's exact form for the partition function $p(n)$, is very positive that Ramanujan had the same result, but somehow Hardy was not convinced about it, and so they ended up proving only the asymptotic form.

> "If one looks at Ramanujan's first letter to Hardy, there is a statement there which has some relation to his later work on the partition function, namely about the coefficient of the reciprocal of a certain theta series... It gives the leading term in what he claims as an approximate expression for the coefficient. If one looks at that expression, one sees that it is the exact analogue of the leading term in Radmacher's formula for $p(n)$, which shows that Ramanujan, in whatever way he had obtained this, had been led to the correct form of that expression."

Selberg's Assessment of Ramanujan's Work (1988)

"In the work on the partition function, studying the paper it seems clear to me that it must have been, in a way, Hardy who did not fully trust Ramanujan's insight and intuition, when he chose the other form of the terms in their expression, for a purely technical reason, which one analyses as not very relevant.

I think that if Hardy had trusted Ramanujan more, they should have inevitably ended with the Radmacher series. There is little doubt about that.

Littlewood and Hardy were primarily working with hard analysis and they did not have a strong feeling for modular forms and such things; the generating function for the partition function is essentially a modular form, particularly if one puts an extra factor of $x^{\frac{-1}{24}}$ to the power series. This must have been something that came quite naturally to Ramanujan from the beginning..."

Selberg's Assessment of Ramanujan's Work (1988)

Selberg also refers to the assessment of Ramanujan's work by Louis J. Mordell (1888-1972)

> "Louis J. Mordell questioned Hardy's assessment that Ramanujan was a man whose native talent was equal to that of Euler and Jacobi. Mordell...claims that one should judge a mathematician by what he has actually done, by which Mordell seems to mean the theorems he has proved. By the way I should say Mordell clearly at no stage seems to have had access to or seen Ramanujan's Notebooks. Mordell's assessment seems quite wrong to me.
>
> **I think that a felicitous but unproved conjecture may be of much more consequence for mathematics than the proof of many a respectable theorem.**"

Selberg's Assessment of Ramanujan's Work (1988)

Selberg also emphasises that the Ramanujan's stature in Mathematics has indeed grown over the decades since his death:

> "Ramanujan's recognition of the multiplicative properties of the coefficients of modular forms that we now refer to as cusp forms and his conjectures formulated in this connection, and their later generalisation have come to play a more central role in the mathematics of today, serving as a kind of focus for the attention of quite a large group of the best mathematicians of our time. Other discoveries like the mock theta functions are only in the very early stages of being understood and no one can yet assess their real importance. **So the final verdict is not really in, and it may not be in for a long time, but the estimates of Ramanujan's stature in mathematics certainly have been growing over the years. There is no doubt about it.**"

Selberg's Assessment of Ramanujan's Work (1988)

Finally Selberg talks about Hardy's assessment of Ramanujan:

> "One might speculate, although it may be somewhat futile, about what would have happened if Ramanujan had come in contact not with Hardy but with a great mathematician of more similar talents, someone who was more inclined in the algebraic directions, for instance, E. Hecke in Germany. This might have perhaps proved much more beneficial and brought out new things in Ramanujan that did not come to fruition by his contact with Hardy...
>
> **I do not think that Hardy fully understood how the interest for Ramanujan's work would be growing when he speaks of the influence which it is likely to have on the mathematics of the future. It seems rather clear that he underestimated that. Later developments have certainly shown him wrong on that point."**[10]

[10] Atle Selberg, Reflections around Ramanujan Centenary, Rep. in *Resonance*, 1996.

How is Ramanujan's Work Assessed Today

Till the latter half of 20th century, the corpus of work of Ramanujan that was generally available comprised of the 37 papers that he had published in various Journals during 1911-1920. These, together with, the 57 questions and solutions published by him in the Journal of Indian Mathematical Society, and extracts from his two letters of 1913 to Hardy which contained statements of around 120 results, were edited by G. H. Hardy, P. V. Seshu Aiyar and B. M. Wilson and published by the Cambridge University in 1927 as the *Collected Papers of Srinivasa Ramanujan.*

This of course excluded most of Ramanujan's work done both before he left for England and after his return to India. The corpus of work done before leaving to England is available in the form of three notebooks which are said to contain around 3250 results. The corpus of work done after the return from England is contained mainly in the "Lost Notebook" which is said to have about 600 results.

Detailed analysis of this large corpus began only in the last quarter of the 20th century. Though much of the work is still in progress, it has already revolutionised our understanding and appreciation of Ramanujan's work.

The Saga of Ramanujan's Notebooks

It seems Ramanujan started recording his results in a Notebook around the time he entered the Government College of Kumbakonam in 1904. Sometime during 1911-13, Ramanujan copied these results in to a second notebook. As Ramanujan noted in his letters to Krishna Rao and Subramaniam, he perhaps did not add any further results to these notebooks, nor did he try to publish the results contained in them, during his stay in England.

The following is a brief description of the notebooks due to Bruce Berndt:

"Ramanujan left three notebooks. The first notebook totalling 351 pages contains 16 chapters of loosely organised material with the remainder unorganised. ...The second notebook is a revised enlargement of the first. This notebook contains 21 chapters comprising 256 pages followed by 100 pages of miscellaneous material. The third short notebook contains 33 pages of unorganised entries. ...in preparing *Ramanujan's Notebooks Parts I-V,* we counted 3254 results,..."[11]

---

[11] B. Berndt, An Overview of Ramanujan's Notebooks, 1998

The Saga of Ramanujan's Notebooks

In January 1921, Prof. K. Ananda Rau of Presidency College wrote to Hardy:

> "Mr. R. Ramachandra Rao told me that you had written to him some months ago that Ramanujan was working on a certain topic in his last days and possibly there may be some record of this work left. If you will please tell us the nature of this investigation, we may find it easier to sift the papers. The whole of the manuscripts will of course be sent to you in accordance with the resolution of the syndicate. You will have noticed also in the Minutes that the syndicate has asked Mr. Seshu Iyer and me to arrange for the preparation of a transcript of Ramanujan's note book, with a view to having it incorporated as an Appendix to the Memorial volume. I do not know if this will serve any useful purpose. **I fear it may look a little incongruous by the side of his mature work. But there are some here, who think that the Note Book may contain valuable algorithms providing starting points for future investigations.**"

On August 30, 1923, transcripts of Ramanujan's notebooks and a packet of miscellaneous papers were despatched by the Registrar of University of Madras, to Hardy, with a note that "You may decide whether any or all of them should find a place in the proposed Memorial Volume".

But the final volume of *Collected Papers* edited by Hardy et al in 1927, did not include any of this material.

In a letter to B. M. Wilson in June 1925, Hardy had expressed his opinion that the notebooks may be published later.

The Saga of Ramanujan's Notebooks

In a recent article on the occasion of Ramanujan's 125th birthday, Bruce Berndt has recounted the saga of publication of Ramanujan's notebooks:

> "As it transpired, ...published with the *Collected Papers* [of Ramanujan], were the first two letters that Ramanujan had written to Hardy, which contained approximately one hundred twenty mathematical claims. Upon their publication, these letters generated considerable interest, with the further publication of several papers establishing proofs of these claims. Consequently, either in 1928 or 1929, at the strong suggestion of Hardy, Watson and B. M. Wilson, ... agreed to edit the notebooks..."

The Saga of Ramanujan's Notebooks

"In an address to the London Mathematical Society on February 5, 1931, Watson cautioned (in retrospect, far too optimistically), 'We anticipate that it, together with the kindred task of investigating the work of other writers to ascertain which of his results had been discovered previously, may take us five years.' Wilson died prematurely in 1935, and although Watson wrote approximately thirty papers on Ramanujan's work, his interest evidently flagged in the late 1930s, and so the editing was not completed....

Finally, in 1957, the notebooks were made available to the public when the Tata Institute of Fundamental Research in Bombay published a photocopy edition, but no editing was undertaken...."

The Saga of Ramanujan's Notebooks

"In February 1974, while reading two papers by Emil Grosswald, in which some formulas from the notebooks were proved, we [Berndt and coworkers] observed that we could prove these formulas by using a transformation formula for a general class of Eisenstein series that we had proved two years earlier. We found a few more formulas in the notebooks that could be proved using our methods, but a few thousand further assertions that we could not prove. In May 1977, the author [Berndt] began to devote all of his attention to proving all of Ramanujan's claims in the notebooks. With the help of a copy of the notes from Watson and Wilson's earlier attempt at editing the notebooks and with the help of several other mathematicians, the task was completed in five volumes in slightly over twenty years."[12]

[12] B. Berndt, Notices of AMS 2012, pp.1532-33

In the same article, Bruce Berndt has also presented the following overall assessment of Ramanujan s notebooks:

> **"Altogether, the notebooks contain over three thousand claims, almost all without proof. Hardy surmised that over two-thirds of these results were rediscoveries. This estimate is much too high; on the contrary, at least two-thirds of Ramanujan's claims were new at the time that he wrote them, and two-thirds more likely should be replaced by a larger fraction. Almost all the results are correct; perhaps no more than five to ten are incorrect."**

"The topics examined by Ramanujan in his notebooks fall primarily under the purview of analysis, number theory and elliptic functions, with much of his work in analysis being associated with number theory and with some of his discoveries also having connections with enumerative combinatorics and modular forms. Chapter 16 in the second notebook represents a turning point, since in this chapter he begins to examine the q-series for the first time in these notebooks and also to begin an enormous devotion to theta functions."[13]

[13] B. Berndt, Notices of AMS 2012, p.1533

Ongoing Work on Ramanujan's "Lost Notebook"

The manuscript of Ramanujan discovered in the Trinity College Library (amongst Watson papers) by G. E. Andrews in 1976, is generally referred as Ramanujan's "Lost Notebook". This seems to pertain to work done by Ramanujan during 1919-20 in India.

This manuscript of about 100 pages with 138 sides of writing has around 600 results.

This seems to have been sent to Hardy along with all other papers of Ramanujan in 1923. It might have been passed on by Hardy to Watson sometime in 1930s.

This notebook along with some other unpublished manuscripts of Ramanujan were published during Ramanujan Centenary in 1987.

G. E. Andrews and B. Berndt have embarked on an edition of all this material in five volumes; of which the first three have appeared in 2005, 2009 and 2013.

Ongoing Work on Ramanujan's "Lost Notebook"

Andrews and Berndt note in the first volume of their edition of the Lost Notebook that:

> "...only a fraction (perhaps 5%) of the notebook is devoted to the mock theta functions themselves. ... A majority of the results fall under the purview of q-series. These include mock theta functions, theta functions, partial theta function expansions, false theta functions, identities connected with the Rogers-Fine identity, several results in the theory of partitions, Eisenstein series, modular equations, the Rogers-Ramanujan continued fraction, other q-continued fractions, asymptotic expansions of q-series and q-continued fractions, integrals of theta functions, integrals of q-products, and incomplete elliptic integrals. Other continued fractions, other integrals, infinite series identities, Dirichlet series, approximations, arithmetic functions, numerical calculations, Diophantine equations, and elementary mathematics are some of the further topics examined by Ramanujan in his lost notebook."

Ramanujan's Mock Theta Functions

The last letter of Ramanujan to Hardy contained 17 examples of mock theta functions such as the following:

$$\sum_{n=0}^{\infty} \alpha(n) q^n := 1 + \sum_{n=1}^{\infty} \frac{q^{n^2}}{(1+q)^2(1+q^2)^2 \dots (1+q^n)^2}.$$

Ramanujan also gave the asymptotic formula

$$\alpha(n) \sim \frac{(-1)^{n-1}}{2\sqrt{n-\frac{1}{24}}} e^{\pi\sqrt{\frac{n}{6}-\frac{1}{144}}}$$

This has later been proved by Andrews and Dragnette (1966)

Ramanujan's Mock Theta Functions

In the 1935 Presidential address to the London Mathematical Society, G. N. Watson had declared:

> "Ramanujan's discovery of the mock theta function makes it obvious that his skill and ingenuity did not desert him at the oncoming of his untimely end. As much as any of his earlier work, the mock theta functions are an achievement sufficient to cause his name to be held in lasting remembrance."

Around the time of Ramanujan's centenary (1987), the famous theoretical physicist Freeman J. Dyson had remarked:

> "The Mock-theta functions give us tantalizing hints of a grand synthesis to be discovered. Somehow it should be possible to build them into a coherent group theoretical structure, analogous to the structure of the modular forms which Hecke built around the old theta-functions of Jacobi. This remains a challenge for the future."

Ramanujan's Mock Theta Functions

The following is the abstract of a recent article, M. Griffin, K. Ono and L. Rollen, Ramanujan's Mock Theta Functions, Proc. Nat. Acad Sc. 2013:

> "In his famous deathbed letter, Ramanujan introduced the notion of a mock theta function, and he offered some alleged examples. Recent work by Zwegers [2001 and 2002]... has elucidated the theory encompassing these examples. They are holomorphic parts of special harmonic weak Maaas forms. Despite this understanding, little attention has been given to Ramanujan's original definition. Here we prove that Ramanujan's examples do indeed satisfy his original definition."

The Enigma of Ramanujan's Mathematics

For the past hundred years, the problem in comprehending and assessing Ramanujan's mathematics and his genius has centred around the issue of "proof".

In 1913, Hardy wrote to Ramanujan asking for proofs of his results. Ramanujan responded by asserting that he had a systematic method for deriving all his results, but that could not be communicated in letters.

Ramanujan's published work in India, and a few of the results contained in the notebooks have proofs, but they were often found to be sketchy, not rigorous, incomplete and sometimes even faulty.

Ramanujan, however, had no doubts whatsoever about the validity of his results, but still he was often willing to wait and supply proofs in the necessary format so that his results could be published.

But, all the time, he was furiously discovering more and more interesting results.

Edward Shils has recorded the following very remarkable account of Ramanujan s ceaseless creativity as recounted by Littlewood:

> "Professor Littlewood once told me that he had been assigned by Hardy to the task of bringing Ramanujan up to date in the more rigorous methods of European mathematics which had emerged subsequently to the state reached by Ramanujan's studies in India; he said that it was extremely difficult because every time some matter, which it was thought Ramanujan needed to know, was mentioned, Ramanujan's response was an avalanche of original ideas which made it almost impossible for Littlewood to persist in his original intention."[14]

[14] E. Shils: Reflections on Tradition, Centre Periphery and Universality of Science: The Importance of the Life of S. Ramanujan, Minerva, 29, 1991, p.416.

The Enigma of Ramanujan's Mathematics

The Greco-western tradition of mathematics almost equates mathematics with proof, so that the process of discovery of mathematical results can only be characterised vaguely as "intuition", "natural genius" etc. Since mathematical truths are believed to be non-empirical, there are no systematic ways of arriving at them except by pure logical reason. There are some philosophers who have argued that this "philosophy of mathematics" is indeed barren: that it has little validity when viewed in terms of mathematical practice, either in history or in our times.

Incidentally, Hardy was amongst those who swore by the non-emipirical nature of mathematics. In his *A Mathematicians Apology* written in 1940, he speaks of the "immortality" of mathematics, of the Greek genre.

> "The Greeks were the first mathematicians who are still 'real' to us today. Oriental mathematics may be an interesting curiosity, but Greek mathematics is the real thing....So Greek mathematics is 'permanent'. ... 'Immortality' may be a silly word, but probably a mathematician has the best chance of whatever it may mean."[15]

[15] G. H. Hardy, *A Mathematician's Apology*, 2nd ed., Cambridge 1967, pp.80-81.

In the Indian mathematical tradition, as is known from the texts of the last two to three millennia, mathematics was not equated with proof. Mathematical results were not perceived as being non-empirical and they could be validated in diverse ways. In this way, the process of discovery and the process of validation were not completely divorced from each other. Proof or logical argumentation to demonstrate the results was important. But proofs were mainly for the purpose of obtaining assent for one's results in the community of mathematicians.

Ramanujan: Not A Newton But A Mādhava

In 1913, Bertrand Russell had jocularly remarked about Hardy and Littlewood having discovered a “second Newton” in a “Hindu clerk”. If parallels are to be drawn, Ramanujan may indeed be compared to the legendary Mādhava.

It is not merely in terms of his methodology and philosophy that Ramanujan is clearly in continuity with the earlier Indian tradition of mathematics. Even in his extraordinary felicity in handling iterations, infinites series, continued fractions and transformations of them, Ramanujan is indeed a successor, a very worthy one at that, of Mādhava, the founder of the Kerala School.

References

1. S. Ramanujan, *Collected Papers,* Cambridge University Press, Cambridge 1927.
2. G. H. Hardy, *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work,* Cambridge 1940.
3. S. Nurullah and J. P. Naik, *A History of Education in India,* Macmillan, New Delhi 1951.
4. Dharampal, *The Beautiful Tree: Indigenous Indian Education in the Eighteenth Century,* Impex, New Delhi 1983.
5. S. N. Sen, Survey of Studies in European Languages, in S. N. Sen and K. S. Shukla, Eds., *History of Astronomy,* INSA, New Delhi 1985, pp.49-121.
6. B. C. Berndt, *Ramanujan's Notebooks,* Parts I-V, Springer New York 1985-1998.
7. B. C. Berndt and R. A. Rankin, *Ramanujan: Letters and Commentary,* AMS, Providence 1995.

References

8. R. Kanigal, *The Man Who Knew Infinity: The Life of the Genius Ramanujan,* Scribner, New York 1995.
9. B. C. Berndt and R. A. Rankin, *Ramanujan: Essays and Surveys,* AMS, Providence 2001.
10. G. E. Andrews and B. C. Berndt, *Ramanujan's Lost Notebook,* Volumes 1-3, Springer, New York 2005-2013.
11. Krishnaswami Alladi (ed), Srinivasa Ramanujan Going Strong at 125, Notices of the AMS, 59, December 2012, pp.1522-37 and ibid. 60, January 1913, pp.10-22 [Articles by K. Alladi, G. E. Andrews, B. C. Berndt, J. M. Borwein, K. Ono, K. Soundararajan, R. C. Vaughan and S. O. Warnaar]

Thanks!

Thank You