NPTEL Course on

Mathematics in India: From Vedic period to Modern times

Lecture 21

$\boldsymbol{L\bar{\imath}l\bar{a}vat\bar{\imath}}$ of Bhāskarācārya 2

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Outline

- Applications of right triangles, continued.
- $S\bar{u}c\bar{\imath}$ probems
- Construction of a quadrilateral: Discussion on earlier confusions
- To find the second diagonal, given the four sides and a diagonal of a quadrilateral
- Cyclic quadrilaterals
- Value of π, area of a circle, surface area of a sphere, volume of a sphere

A rational right triangle

Verse 140:

As in Brāhmasphuṭasiddhānta: Side $= a$, upright $= \frac{1}{2}\left(\frac{a^2}{n} - n\right)$, hypotenuse $= \frac{1}{2}\left(\frac{a^2}{n} + n\right)$. It is obviously true. But how to get this?

Gaṇeśa Daivajña explains in his *Buddhivilāsinī* commentary.

Let side $= a$. Let hypotenuse - upright $= n$. Let upright $= x$.

$$\therefore\ a^2 + x^2 = (x+n)^2 = x^2 + n^2 + 2xn$$

$$\therefore\ x = \frac{a^2 - n^2}{2n} = \frac{1}{2}\left(\frac{a^2}{n} - n\right).$$

$$\text{Hypotenuse } = x + n = \frac{1}{2}\left(\frac{a^2}{n} + n\right).$$

Or given the hypotenuse a, and an assumed number n. Then by a similar rule in Verse 142, the upright is $\frac{2an}{n^2+1}$, and the side is found to be

$$n \times \text{ upright - Hypotenuse} = \frac{2an^2}{n^2+1} - a = \frac{a(n^2-1)}{(n^2+1)}.$$

Bamboo problem

Suppose a bamboo of height a , standing vertically, is broken at height x, and the tip falls to the ground at a distance b from the root of the bamboo. A right triangle is now formed with the side b, upright x and hypotenuse $h = a - x$. Verse 147 states that the upright , $x = \frac{1}{2}(a - \frac{b^2}{a})$ and the hypotenuse , $h = \frac{1}{2}(a + \frac{b^2}{a})$:

वंशाग्रमूलान्तरभूमिवर्गो वंशोद्धृतस्तेन पृथग्युतोनौ।
वंशो तदर्धे भवतः क्रमेण वंशस्य खण्डे श्रुतिकोटिरूपे॥

Bamboo problem

> “The square of the ground intercepted between the root and the tip is divided by the (length of the) bamboo, and the quotient severally added to , and subtracted from , the bamboo: the halves (of the sum and difference) will be the two portions of it representing hypotenuse and upright.”

Here

$$h + x = a.$$

Also

$$h^2 = x^2 + b^2.$$

$$\therefore\ h^2 - x^2 = b^2.$$

$$\therefore\ h - x = \frac{h^2 - x^2}{h + x} = \frac{b^2}{a}.$$

So, from *Saṅkramaṇa*, upright $x = \frac{1}{2}\left(a - \frac{b^2}{a}\right)$, Hypotenuse, h $= \frac{1}{2}\left(a + \frac{b^2}{a}\right)$. This is essentially Gaṇeṣa’s explanation of the result.

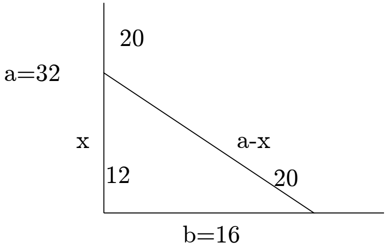
Example

Example in Verse 148.

यदि समभुवि वेणुः दन्तपाणिप्रमाणो
गणक! पवनवेगादेकदेशे स भग्नः।
भुवि नृपमितहस्तेष्वेव लग्नस्तदग्रं
कथय कतिषु मूलादेष भग्नः करेषु॥ १५६ ॥

“If a bamboo, measuring 32 cubits and standing upon level ground, be broken in one place by the force of the wind and the tip of it meets the ground at 16 cubits; say mathematician at how many cubits from the root it is broken.”

Bamboo problem: Solution



$$
\begin{aligned}
x &= \frac{1}{2}\left(a - \frac{b^2}{a}\right) \\
\therefore x &= \frac{1}{2}\left(32 - \frac{256}{32}\right) = \frac{1}{2}(32-8) = 12 \\
h &= a - x = 32 - 12 = 20
\end{aligned}
$$

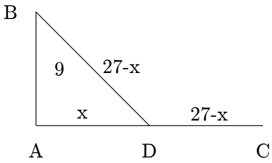
Snake-Peacock problem

Snake -Peacock Problem in Verse 150 :

अस्ति स्तम्भतले बिलं तदुपरि क्रीडाशिखण्डी स्थितः
स्तम्भो नन्दकरोच्छ्रितस्त्रिगुणितस्तम्भप्रमाणान्तरे।
दृष्ट्वाहिं बिलमाव्रजन्तमपतत् तिर्यक् स तस्योपरि
क्षिप्रं ब्रूहि तयोर्बिलात् कतिमिते साम्येन गत्योर्युतिः॥

“A snake’s hole is at the foot of a pillar, 9 cubits high, and a peacock is perched on its summit. Seeing a snake at the distance of thrice the pillar gliding towards his hole, he pounces obliquely upon him. Say quickly at how many cubits from the snakes’s hole they meet, both proceeding an equal distance.”

Snake-Peacock problem: Solution



Snake - Peacock problem

$AB = 9$ cubits, $AC = 3 \times 9 = 27$ cubits. Let $AD = x, CD = 27 - x = BD$. Now

$$\begin{aligned}
BD^2 - AD^2 &= AB^2 \\
\therefore\ (27-x)^2 - x^2 &= 9^2 \\
\therefore\ (27-x+x)(27-x-x) &= 81 \\
\therefore\ 27 \cdot (27-2x) &= 81 \\
\therefore 27 - 2x = \frac{81}{27} &= 3 \\
\therefore\ 2x = 27 - 3 &= 24 \\
\therefore\ x &= 12
\end{aligned}$$

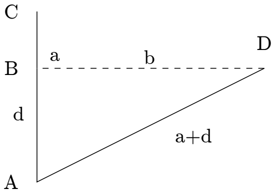
Snake, Peacock meet at distance $x = 12$, from the hole.

Lotus problem

Lotus probem in Verse 154 :

सखे पद्मतन्मज्जनस्थानमध्यं भुजः कोटिकर्णान्तरं पद्मदृश्यम्।
नलः कोटिरेतन्मितं स्याद्यतोऽम्भो वदैवं समानीय पानीयमानम्॥
१५४ ॥

“Friend, the space between the lotus (as it stood) and the spot where it submerged, is the side. The lotus as seen (above water) is the difference between the hypotenuse and upright. The stalk is the upright, for the depth of water is measued by it. say, what the depth of the water is.”



Lotus Problem

Lotus problem: Solution

AC : original position of the lotus $= a + d$. $BC = a$: portion above water. $AB = d$: portion inside water $=$ Depth of water. Due to wind the lotus is swept and assumes the position $AD = a + d$. (So it is just submerged at D). Suppose $BD = b$ is given. Find d (Depth of water).

Solution :

$$(a+d)^2 = d^2 + b^2, \text{ or } d = \frac{b^2 - a^2}{2a}.$$

Example in Verse 153.

$$a = 1 \text{ span} = \tfrac{1}{2} \text{ cubit. } b = 2 \text{ cubits. } d = \frac{2^2 - 1/4}{2 \cdot \frac{1}{2}} = \frac{15}{4}.$$

Apes problem

वृक्षाद्धस्तशतोच्छ्रयाच्छतयुगे वाप्यां कपिः कोऽप्यगात्
उड्डीयाथ परो द्रुतं श्रुतिपथात् प्रोड्डीय किञ्चिद् द्रुयात् ।
जातैवं समता तयोर्युतिरपि प्रोड्डीयमानं कियद्
विद्वन् वेत्सि परिश्रमोऽस्ति गणिते क्षिप्रं तदाचक्ष्व मे ॥

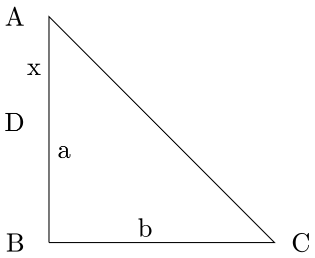
“From a tree, a hundred cubits high, an ape descended and went to a pond, two hundred cubits distant; while another ape, vaulting to some height off the tree, proceeded with a velocity diagonally to the same spot. If the space traveled by them be equal, tell me quickly, learned man, the height of the leap, if thou have diligently studied calculation.”

Solution given in (earlier) Verse 154.

Solution of Apes problem

द्विनिघ्नतालोच्छ्रितिसंयुतं यत् सरोऽन्तरं तेन विभाजितायाः।
तालोच्छ्रितेस्तालसरोन्तरघ्नया उड्डीयमानं खलु लभ्यते तत्॥

"The height of the tree multiplied by its distance from the pond, is divided by twice the height of the tree, added to the space between the tree and the pond: the quotient will the measure of the leap."



Ape Problem

Solution of Apes problem

This is essentially the same as the ”Two ascetics problem” in *Brāhmasphuṭasiddhānta*.

$BD = a =$ Height of the tree; $BC = b =$ Distance between tree and pond. Let the leap be $x = AD$

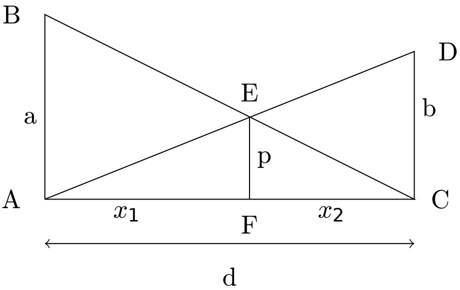
Given that

$$BD + BC = a + b = AD + AC = x + AC = x + \sqrt{(a+x)^2 + b^2}.$$

$$\begin{aligned}
\therefore x + \sqrt{(a+x)^2 + b^2} &= a + b \\
\therefore (a+x)^2 + b^2 &= (a+b-x)^2 = x^2 + (a+b)^2 - 2x(a+b) \\
\therefore a^2 + b^2 + 2ax + x^2 &= x^2 + (a+b)^2 - 2x(a+b) = x^2 + a^2 \\
&\quad + b^2 + 2ab - 2x(a+b). \\
\therefore x &= \frac{ab}{2a+b}
\end{aligned}$$

Two bamboo pillars: Segments and Perpendicular

Verse 159.



Two bamboos and perpendicular at the junction

Two bamboos of heights $AB = a, CD = b$. Distance between them $= AC = d$. Top of each bamboo joined by string to the bottom of the other. $p = EF$ is the perpendicular from the intersection of the strings at E to the base AC at F. Find p and the segments $AF = x_1, FC = x_2$.

Segments and Perpendicular

From similar triangles, $$\frac{EF}{AB} = \frac{p}{a} = \frac{FC}{AC} = \frac{x_2}{x_1 + x_2} = \frac{x_2}{d}$$

Similarly, $$\frac{EF}{CD} = \frac{p}{b} = \frac{AF}{AC} = \frac{x_1}{x_1 + x_2} = \frac{x_1}{d}$$

$$\therefore p\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{x_1 x_2}{d} = \frac{d}{d} = 1$$

.

$$\therefore p = \frac{ab}{a+b}$$

Also $x_1 = \dfrac{pd}{b} = \dfrac{ad}{(a+b)}$. Similarly, $x_2 = \dfrac{bd}{(a+b)}$

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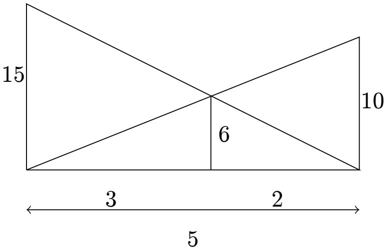
Example

Example Verse 160.



An example

Given Bamboos; 15, 10. Distance: 5. Then $p = \dfrac{15 \times 10}{15 + 10} = 6$.
$x_1 = 3, x_2 = 2$.

Triangles and quadrilaterals

In Verse 161. It is stated: In any rectilinear figure, one side cannot be greater than the sum of the other sides.

Verse 163,164: Given the segments and the perpendicular (altitude) in terms of the two sides and the third side (base). Also it is stated that:

$$\text{Area} = \frac{1}{2} \times \text{ Base } \times \text{ Altitude}$$

Verse 166. Case of a triangle with an obtuse angle.

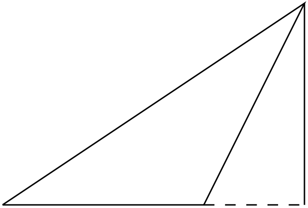
दशसप्तदशप्रमौ भुजौ त्रिभुजे यत्र नवप्रमा मही।
अबधे वद लम्बकं तथा गणितं गणितकाशु तत्र मे॥ १६६ ॥

“In a triangle, wherein the sides measure ten and seventeen , and the base nine, tell me promptly, expert mathematician, the segments, perpendicular, and area.”

Negative seqment

Here, the quotient $\frac{b^2-c^2}{2a}$ is 21. This cannot be subtracted from the base; wherefore the base is subtracted from it : (अनेन भूरूना न स्यात् अस्मादेव भूरपनीता) . (One of) the segment is negative , that is to say, in the contrary direction. (ऋणगताऽबाधा दिग्वैपरीत्येनेत्यर्थः). The two segments are found 15 and 6(negative) . Perpendicular is 8 and the area is 36.

The negative segment is shown by the dotted line.



Obtuse triangle

Construction of a quadrilateral and its area

Verse 167.

सर्वदोर्युतिदलं चतुःस्थितं बाहुभिर्विरहितं च तद्धतेः।
मूलमस्फुटफलं प्रजायते, स्पष्टमेवमुदितं त्रिबाहुके॥ १६७ ॥

"Half the sum of all the sides is set down in four places and the sides are severally subtracted. The remainders multiplied together, the square root of the product is the area, inexact for quadrilateral, but pronounced exact for triangle."

Bhāskara, Area of quadrilateral,

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}, \; s = \frac{(a+b+c+d)}{2}$$

is "Inexact" in a quadrilateral, but exact for a triangle (by putting one of the sides =0).

Construction of a quadrilateral

Verse 169-172:

चतुर्भुजस्याऽनियतौ हि कर्णौ कथं
ततोऽस्मिन् नियतं फलं स्यात्।
प्रसाधितौ तच्छ्रवणौ यदा द्वौ स्वकल्पितत्वादितरत्र न स्तः॥ १६९ ॥
तेष्वेव बाहुष्वपरौ च कर्णावनेकधा क्षेत्रफलं ततश्च॥ १७० ॥
लम्बयोः कर्णयोर्नैकं समुद्दिश्यापरान् कथम्।
पृच्छत्यनियतत्वेऽपि नियतं चापि तत्फलम्॥ १७१ ॥
स पृच्छकः पिशाचो वा गणको नितरां ततः।
यो न वेत्ति चतुर्बाहौ क्षेत्रस्याऽनियतां स्थितिम्॥ १७२ ॥

"Since the diagonals of a quadrilateral are indeterminate how should the area be in this case, determinate? The diagonals found as assumed by the ancients do not answer in another case. with the same sides, there are other diagonals; and the area of the quadrilateral is accordingly manifold.

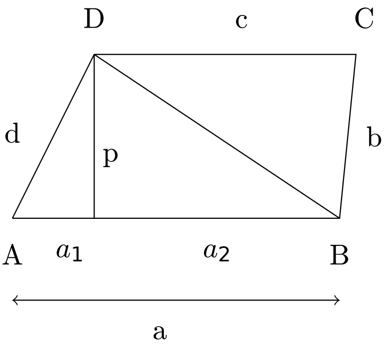
Construction of a quadrilateral

For in a quadrilateral, opposite angles being made to approach, contract their diagonal as they advance inwards. While the other angles receding outwards lengthen the diagonal. Therefore it is said that with the same sides there are other diagonals.

How can person, neither specifying one of the perpendicular nor the either of the diagonals, ask the rest? Or how can he demand a determinate area, while they are indefinite?

Such a questioner is a blundering devil($piśaca$). Still more so is he, who answers the question. For he considers not the indefinite nature of the lines in a quadrilateral figure.”

Construction of a quadrilateral



A quadrilateral

AB, AD, CD, CB are the sides. To specify D, we should be given p (or equivalently $B\hat{A}D$) or $DB = D_1$: one of the diagonals. Later if d, D_1, a are known, p can be found. Similarly, if p, d are known, a_1, a_2 are known, and $D_1 = \sqrt{p^2 + a_2^2}$.

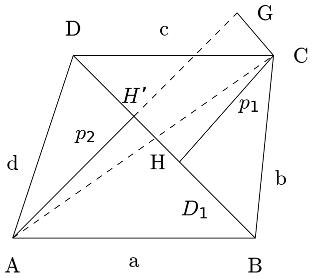
Finding the second diagonal

Determination of the second diagonal given the four sides and one diagonal in Verses 181-182 :

इष्टोऽत्र कर्णः प्रथमं प्रकल्प्यः त्र्यस्रे तु कर्णोभयतः स्थिते ये।
कर्णं तयोः क्ष्मामितरौ च बाहू प्रकल्प्य लम्बावबधे प्रसाध्ये॥१८१॥
आबाधयोरेककुप्स्थयोः यत् स्यादन्तरं तत्कृतिसंयुतस्य।
लम्बैक्यवर्गस्य पदं द्वितीयः कर्णो भवेत् सर्वचतुर्भुजेषु॥ १८२ ॥

"In the figure, first a diagonal is assumed. In the two triangles situated on each side of the diagonal, tis diagonal is made the base of each; and the other sides are given; the perpendiculars and segments must be found. Then the square of the difference of two segments on the same side being added to the square of the sum of the perpendiculars, the square root of the sum of those squares will be the second diagonal in all quadrilaterals."

Determination of the second diagonal



Determination of the second diagonal

In the figure, BD is a diagonal. To find the other diagonal AC. p_1, p_2 are perpendicular to the first diagonal $BD = D_1$ can be determined. $DH = D_{1c}$ and $HB = D_{1b}$: segments associated with p_1. $DH' = D_{1d}$ and $H'B = D_{1a}$: segments associated with p_2. Extend AH' to G such that $H'G = CH = p_1$. Clearly, CG is perpendicular to $H'G$.

$$CG = H'H = DH - DH' = D_{1c} - D_{1d}$$

. Then the second diagonal $AC = D_2$ is given by

$$AC^2 = AG^2 + CG^2 = (p_1 + p_2)^2 + (D_{1c} - D_{1d})^2$$

Example

Example: $d = 68, a = 75, c = 51, b = 40, D_1 = 77..$

$$DH = D_{1c} = \frac{D_1 + \dfrac{(c^2 - b^2)}{D_1}}{2} = \frac{77 + \dfrac{(51^2 - 40^2)}{77}}{2} = \frac{77 + \dfrac{91 \times 11}{77}}{2} = \frac{77 + 13}{2} = 45$$

$$DH' = \frac{D_1 - \dfrac{(a^2 - d^2)}{D_1}}{2} = \frac{77 - \dfrac{(75^2 - 68^2)}{77}}{2} = \frac{77 - \dfrac{143 \times 7}{77}}{2} = \frac{77 - 13}{2} = 32.$$

$$DH' = D_{1d} = 32, D_{1c} - D_{1d} = 13.$$

.

$$p_1 = \sqrt{b^2 - D_{1b}^2} = \sqrt{40^2 - 32^2} = 8\sqrt{5^2 - 4^2} = 24$$

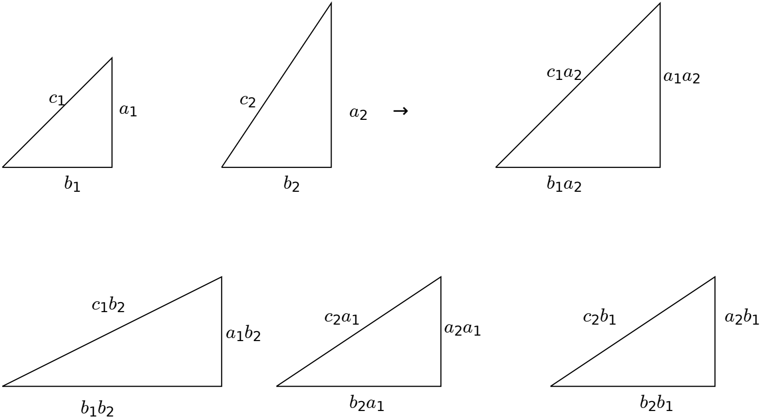
Similarly, $p_2 = 60$.

$$\therefore AC^2 = (p_1 + p_2)^2 + (D_{1c} - D_{1d})^2 = 84^2 + 13^2 = 7225 = 85^2$$

$\therefore AC = 85$ $\qquad (85^2 = (84+1)^2 = 84^2 + 2 \cdot 84 + 1 = 84^2 + 169 = 84^2 + 13^2)$

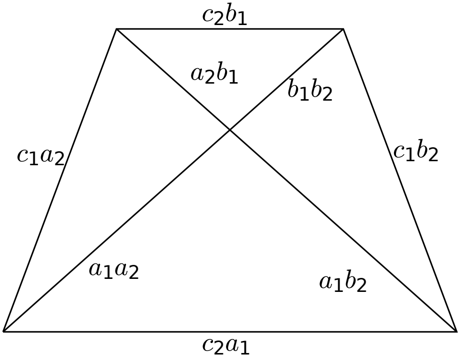
Construction of a cyclic quadrilateral

Bhāskara gives the method to construct a cyclic quadrilateral beginning with two right triangles in Verses 191-192 : Same as the construction in *Brāhmasphuṭasiddhānta*.



Triangles used to construct a cyclic quadrilateral

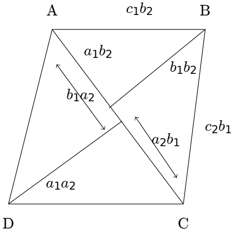
Construction of a cyclic quadrilateral



A cyclic quadrilateral constructed from 4 right triangles

Another cyclic quadrilateral with same sides

Another cyclic quadrilateral can be constructed from the four right triangles, by interchanging the sides c_2b_1 and c_1b_2:



Another cyclic quadrilateral from the same 4 right triangles

Here one diagonal $AC = a_1b_2 + a_2b_1$.

$$\begin{aligned}(\text{other diagonal})^2 &= (a_1a_2 + b_1b_2)^2 + (b_1a_2 - a_1b_2)^2 \\ &= (a_1^2 + b_1^2)(a_2^2 + b_2^2) \\ &= c_1^2c_2^2.\end{aligned}$$

$$\therefore \text{ Other diagonal } BD = c_1c_2.$$

We may also note that in this construction, the diagonal BD is the diammeter of the circle.

Circle

After triangles and quadrilaterals, Bhāskara discusses Circles . He has many new things to say regarding circles.

Verse 201.

व्यासे भनन्दाग्निहते विभक्ते
खबाणसूर्यैः परिधिः सुसूक्ष्मः।
द्वाविंशतिघ्ने विहतेऽथ शैलैः
स्थूलोऽथवा स्याद् व्यवहारयोग्यः॥ २०७ ॥

“When the diameter of a circle is multiplied by 3927 and divided by 1250, the quotient is nearly the circumference: or multiplied by 22 and divided by 7, it is the gross circumference adapted to the practice”.

Ratio of circumference and diameter

$$\pi = \frac{\text{Circumference}}{\text{Diameter}} \underbrace{\approx}_{\text{'Near'}} \frac{3927}{1250} = 3.1416$$

$$\text{Rough value: } \pi = \frac{22}{7}.$$

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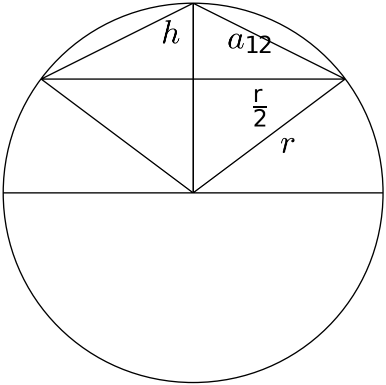
In his *Buddhivilāsini*, Gaṇeśa Daivajña explains the value $\frac{3927}{1250}$ for π.

Inscribed hexagon



Side of an inscribed hexagon $= r$. Circumference $\approx 6r$.

Inscribed dodecagon

Side of an inscribed dodecagon (12 sides), a_{12} is obtained as follows:

$$(r-h)^2+\left(\frac{r}{2}\right)^2=r^2.$$

$$\therefore\ r^2+h^2-2rh+\frac{r^2}{4}=r^2.$$

$$\therefore\ h^2-2rh+\frac{r^2}{4}=0$$

$$\therefore\ h=r-\frac{1}{2}\sqrt{4r^2-r^2}=r-\frac{\sqrt{3}}{2}r.$$

$$a_{12}^2=h^2+\frac{r^2}{4}.$$

$$\therefore\ a_{12}^2=r^2\left\{\left(1-\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}\right\}=r^2\{2-\sqrt{3}\}.$$

$$\therefore\ a_{12}=r\sqrt{2-\sqrt{3}}=d\sqrt{\frac{1}{2}-\frac{\sqrt{3}}{4}}\approx\frac{d\sqrt{669.87}}{100}.$$

Gaṇeśa gives $a_{12}\approx d\frac{\sqrt{673}}{100}$.

$$\therefore\ \text{Circumference}\approx 12a_{12}\approx\left(\frac{12\sqrt{673}}{100}\right)\times d.$$

Better value of π

In this manner, he says if we have a polygon of 384 sides. $(384 = 12 \times 32 = 12 \times 2^5)$.

$$\text{Circumference } \approx \left(\frac{\sqrt{98683}}{100} \right) \times d\,.$$

$$\therefore\ \pi \approx \frac{\sqrt{98683}}{100} \approx \frac{3927}{1250}.$$

(check: $\sqrt{98683} \times 1250 \approx 392673 \approx 392700$).

Area of a circle, Surface area and Volume of a Sphere

Area of a circle, Surface area and Volune of a Sphere in Verse 203 :

> वृत्तक्षेत्रे परिधिगुणितव्यासपादः फलं यत्
> क्षुण्णं वेदैरुपरि परितः कन्दुकस्येव जालम्।
> गोलस्यैवं तदपि च फलं पृष्ठजं व्यासनिघ्नं
> षड्भिर्भक्तं भवति नियतं गोलगर्भे घनाख्यम्॥ २०३ ॥

"In a circle, a quarter of the diameter multiplied by the circumference is the area. That multiplied by four is the net all around the ball.This content of the surface of the sphere, multiplied by the diameter and divided by six, is the precise solid, termed cubic, content within the sphere. "

Area etc.

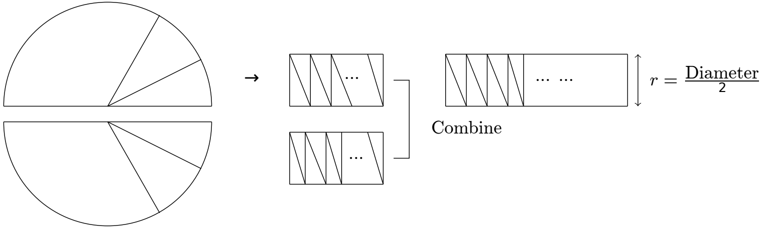
$$\text{Area of a circle} = \frac{\text{Circumference} \times \text{Diameter}}{4} = \frac{\pi d^2}{4}.$$

$$\text{Surface area of a sphere} = \text{Circumference} \times \text{Diameter}$$

$$= \pi d^2 = 4\pi r^2.$$

$$\text{Volume of a sphere} = \text{Circumference} \times \frac{(\text{Diameter})^2}{6} = \frac{4}{3}\pi r^3.$$

Gaṇeśa's derivation for the area of a circle

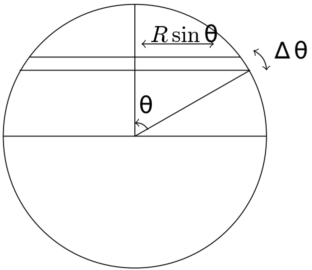


Rectangle with sides: $\dfrac{\text{Circumference}}{2}$ and $\dfrac{\text{diameter}}{2}$.

$$\therefore \text{Area} = \frac{1}{4} \times \text{Circumference} \times \text{Diameter}.$$

Surface area of a sphere

Surface area of a sphere. *Siddhāntaśiromaṇi - Golādhyāya - Vāsanā*.



Divide hemisphere into 24 strips.

$$\text{Area of strip} = 2\pi R\sin\theta R\Delta\theta.$$

$$R\Delta\theta = \frac{\pi \cdot R}{2 \cdot 24}$$

$$\theta_i = i \times \frac{\pi}{2 \times 24}$$

$$R\sin\theta_i \rightarrow \text{Given in table.}$$

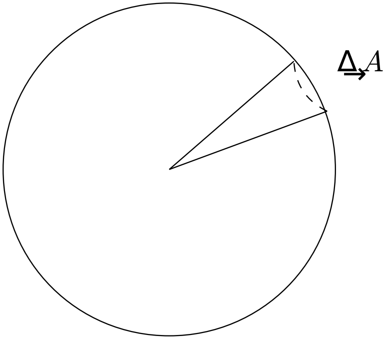
Area of hemisphere

$$\therefore \text{ Area of hemisphere } = 2\pi \left(\sum R \sin\theta_i\right)(R\Delta\theta).$$

Bhāskara carries out the sum explicitly using the 24 Rsine values from the table and reports the measure of the hemisphere to be $2\pi R^2$. (Actually, half the product of the circumference and the diameter). *Yuktibhāṣā* proves that the area of the hemisphere is $2\pi R^2$ by integration: essentially

$$\int_0^{\frac{\pi}{2}} \sin\theta d\theta = \cos\theta|_0^{\frac{\pi}{2}}$$

Volume of a sphere



Volume of the cone bit (Fig.68) $= \frac{1}{3}$ Area $\times$ height $= \frac{1}{3}\Delta A \times R$.
Then Volume of sphere $= \frac{1}{3} \times$ Area of sphere $\times R = \frac{1}{3} \times 4\pi R^3$.

Relation among chord, *śara* and diameter, as in *Brāhmasphuṭasiddhānta* and *Gaṇitasārasaṅgraha*.

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Thanks!

Thank You