

## Lecture XIV (Test - II)

1. Suppose  $X$  is a metric space and  $A$  is a retract of  $X$ . Show that  $A$  is closed in  $X$ . Is the space homeomorphic to the letter  $Y$  a deformation retract of a space homeomorphic to  $E^2$ ?
2. Show that if  $X$  has the fixed point property and  $A$  is a retract of  $X$  then  $A$  also has the fixed point property.
3. Find the degree of the following maps  $f : S^1 \longrightarrow S^1$  given by:  
(i)  $f(z) = \exp(z - \operatorname{Re} z)$ . (ii)  $f(z) = \bar{z}^2 z^3$ .
4. Show that  $S^1$  is not homeomorphic to any subset of  $\mathbb{R}$ . Can  $S^2$  be homeomorphic to a subset of  $\mathbb{R}^2$ ?
5. Determine  $\pi_1(\mathbb{R}P^2 - \{p\})$  where  $p$  is any point of  $\mathbb{R}P^2$ .
6. For the map  $f : S^1 \longrightarrow S^1 \times S^1$  given by  $f(z) = (z^p, z^q)$ , where  $p$  and  $q$  are positive integers, find the induced group homomorphism  $f_* : \mathbb{Z} \longrightarrow \mathbb{Z} \times \mathbb{Z}$ .