

1. Calculate the homology groups of the double torus.
2. Show that any homeomorphism of  $E^n$  onto itself must preserve the boundary.
3. Show that  $\mathbb{R}P^n$  is not a retract of  $\mathbb{R}P^{n+1}$ . Use the lifting criterion.
4. Regard  $S^2$  as the Riemann sphere and calculate the degree of the map  $f : S^2 \longrightarrow S^2$  given by  $f(z) = z^n$ .
5. Use the previous exercise to prove the fundamental theorem of algebra.
6. Show that  $\mathbb{R}P^{2n}$  has the fixed point property. Does  $\mathbb{R}P^3$  have the fixed point property?