Lecture VI (Test - I)

1. Prove that the intervals (a, b) and $[a, b)$ are non-homeomorphic subsets of $\mathbb{R}$. Prove that if A and B are homeomorphic subsets of $\mathbb{R}$, then A is open in $\mathbb{R}$ if and only if B is open in $\mathbb{R}$. Is an injective continuous map $f : \mathbb{R} \longrightarrow \mathbb{R}$ a homeomorphism onto its image?

2. Using Tietze's extension theorem or otherwise construct a continuous map from $\mathbb{R}$ into $\mathbb{R}$ such that the image of $\mathbb{Z}$ is not closed in $\mathbb{R}$.

3. If K is a compact subset of a topological group G and C is a closed subset of G, is it true that KC is closed in G? What if K and C are merely closed subsets of G?

4. Removing three points from $\mathbb{R}P^2$ we get an open set G and a continuous map $f : G \longrightarrow \mathbb{R}P^2$ given by $f([x_1, x_2, x_3]) = [x_2x_3, x_3x_1, x_1x_2]$. Which three points need to be removed? Prove the continuity of f.

5. Let $C = \{(\mathbf{v}_1, \mathbf{v}_2) \in S^2 \times S^2 \ / \ \langle \mathbf{v}_1, \mathbf{v}_2 \rangle = 0\}$. Is C connected? Is C homeomorphic to $SO(3, \mathbb{R})$?

6. Prove that $\mathbb{R}P^1$ is homeomorphic to S^1.