Lecture - XXVII (Test IV)

1. Use the Seifert Van Kampen theorem to compute the fundamental group of the double torus.
2. Let K be a compact subset of $\mathbb{R}^3$ and regard S^3 as the one point compactification of $\mathbb{R}^3$. Show that $\pi_1(\mathbb{R}^3 - K) = \pi_1(S^3 - K)$.
3. If C is the circle in $\mathbb{R}^3$ given by the pair of equations
$$x^2 + z^2 = 1, \quad z = 0,$$
show that $\pi_1(\mathbb{R}^3 - C) = \mathbb{Z} \oplus \mathbb{Z}$. Let C' be the circle given by
$$(y-1)^2 + z^2 + 1, \quad x = 0.$$
Show that $\pi_1(\mathbb{R}^3 - C \cup C') = \mathbb{Z} \oplus \mathbb{Z}$. Hint: Use stereographic projection.
4. Show that the complement of a line in $\mathbb{R}^4$ is simply connected.
5. Calculate the fundamental group of $\mathbb{C}^2 - \{(z_1, z_2)/z_1 z_2 = 0\}$.