Lecture XXI - Test - III

1. Show that a homeomorphism of E^2 onto itself must preserve the boundary. That is it must map a boundary point to a boundary point.

2. Is it true that $\mathbb{R}P^3$ minus a point deformation retracts to a space homeomorphic to $\mathbb{R}P^2$?

3. Let G be the infinite grid
$$G = \{(x, y) \in \mathbb{R}^2 \ / \ x \in \mathbb{Z} \text{ or } y \in \mathbb{Z}\}.$$
Consider the covering map from G onto the figure eight loop $(S^1 \times \{1\}) \cup (\{1\} \times S^1)$ given by
$$p(x, y) = (\exp(2\pi i x),\ \exp(2\pi i y)).$$
Determine the deck transformations of this covering. Is this a regular covering?

4. Given topological spaces X and Y, a map $p : X \longrightarrow Y$ is said to be a local homeomorphism if each $x_0 \in X$ has a neighborhood N_{x_0} such that the restriction map
$$f\Big|_{N_{x_0}} : N_{x_0} \longrightarrow f(N_{x_0})$$
is a homeomorphism. Show that a local homeomorphism which is a proper map is a covering projection.

5. Show that the map $f : \mathbb{C} - \{0, 1, -1\} \longrightarrow \mathbb{C} - \{\pm 2\}$ is a local homeomorphism. Is this map a covering projection? If so what is the group of deck transformations?