Business Analytics for Management Decision

Prescriptive Analytics 3 [51]

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Integer Programming Problems

- Bound
  - An upper or lower limit on the value of the objective function at a given stage of the analysis of an integer programming problem.

- Branch
  - Selection of an integer value of a decision variable to examine for a possible integer solution to a problem.

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Solving Integer Programming

- Branch and Bound method
  - Creates and solves a sequence of subproblems to the original problem that are increasingly more restrictive until an optimal solution is found.

- Enumeration Method
  - A method used with 0–1 problems that involves listing every possible outcome in order to identify the optimal solution.

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Integer Programming Problems

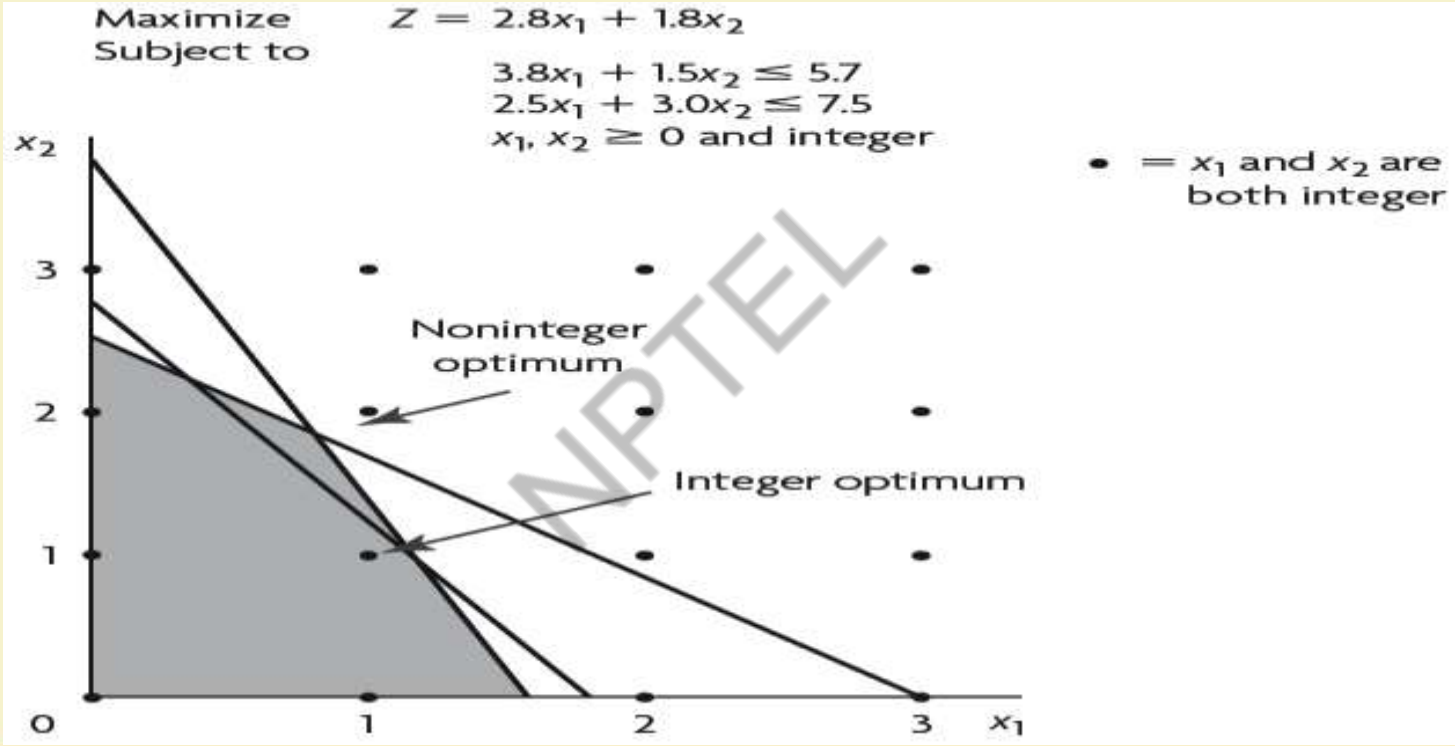
- Mixed-Integer Problem
  - Requires that some, but not all, decision variables have integer values.

- Pure-Integer Problem
  - requires that all decision variables have integer values.

- **Mixed-Integer Problem**
  - requires that some, but not all, decision variables have integer values.

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Graph of an Integer Programming Problem



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Integer Programming Problems and Sensitivity Analysis

- Integer programming problems do not readily lend themselves to sensitivity analysis as only a relatively few of the infinite solution possibilities in a feasible solution space will meet integer requirements.

- Trial-and-error examination of a range of reasonable alternatives involving completely solving each revised problem is required.

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Solving Integer Programming

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Formulating Integer Programming Problems with 0–1 Constraints

- Either-Or Alternatives
- *k*-Out-of-*n* Alternatives
- If-Then Alternatives
- Either-Or Constraints
- Variables that Have Minimum Level Requirements

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Specialized Integer Programming Problems

- Integer programming problems with 0–1 decision variables
  - Fixed-charge problem: minimize total costs
  - Set covering problem: minimize coverage costs
  - Knapsack problem: capacity-profit maximization
  - Facility location problem: multiple facility locations with capacity considerations
  - Traveling salesperson problem: minimize total costs of departing and returning the same location.

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Example 3: All Integer Problem

Consider this all-integer problem:

maximize $Z = 6x_1 + 8x_2$ (millions)

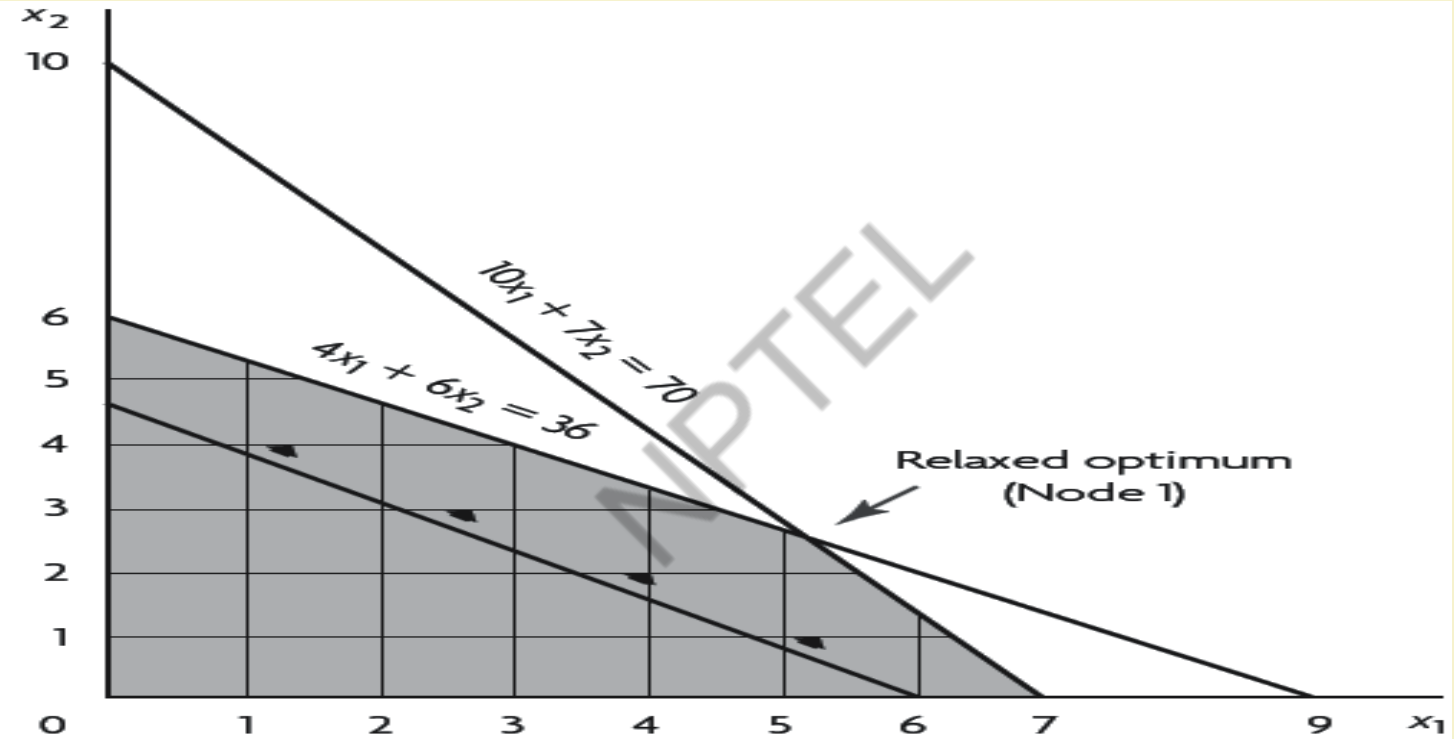
subject to

$$4x_1 + 6x_2 \leqslant 36$$
$$10x_1 + 7x_2 \leqslant 70$$
$$x_1 \text{ and } x_2 \geqslant 0 \text{ and integer}$$

Solve for the optimal integer solution.

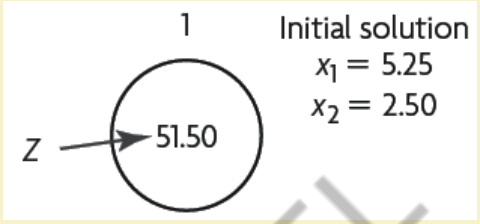
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The LP Relaxation Solution

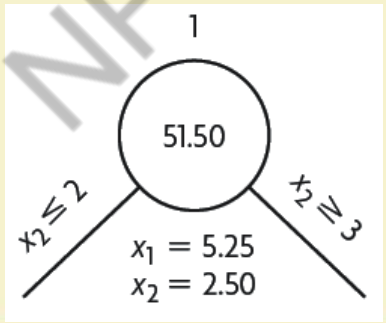


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Node 1 Summarizes the LP Relaxation Solution

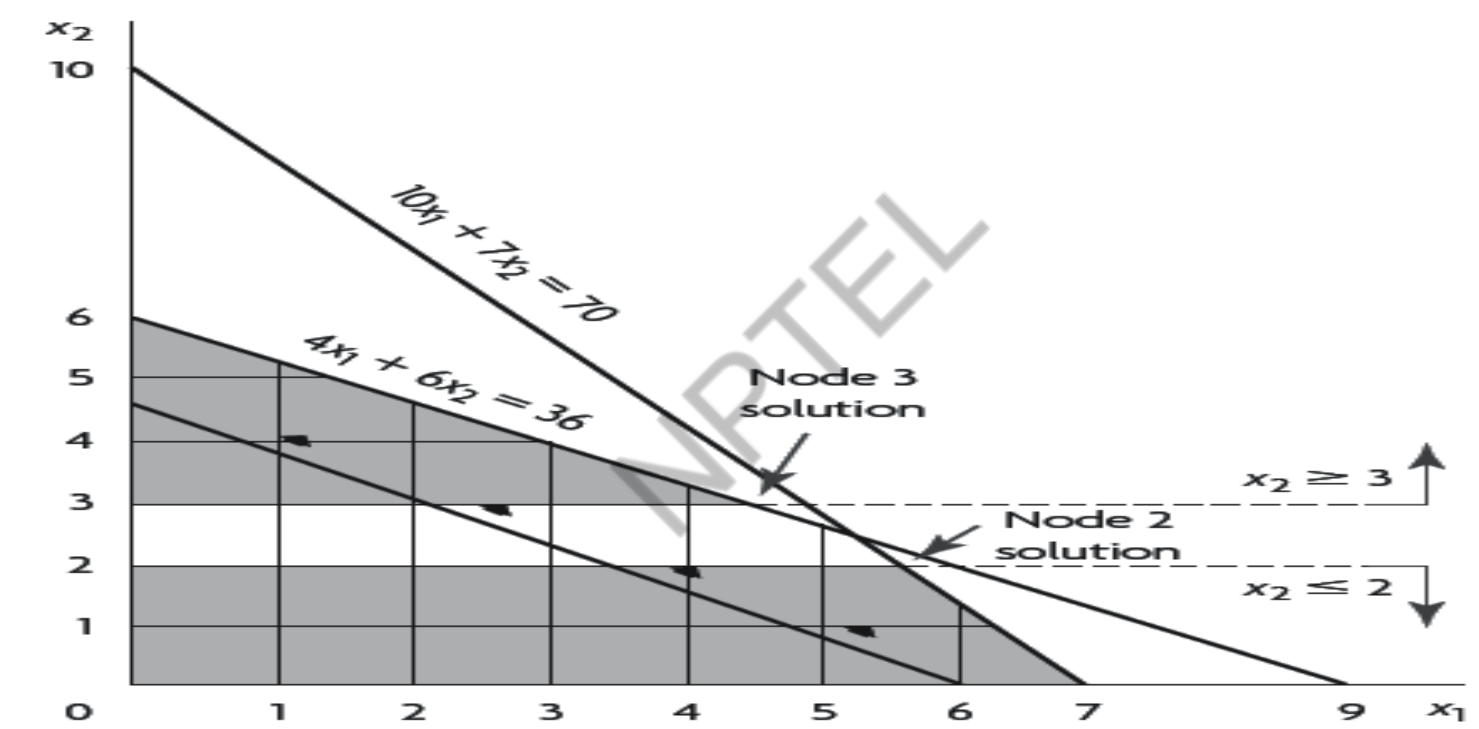


Branching from the First Node



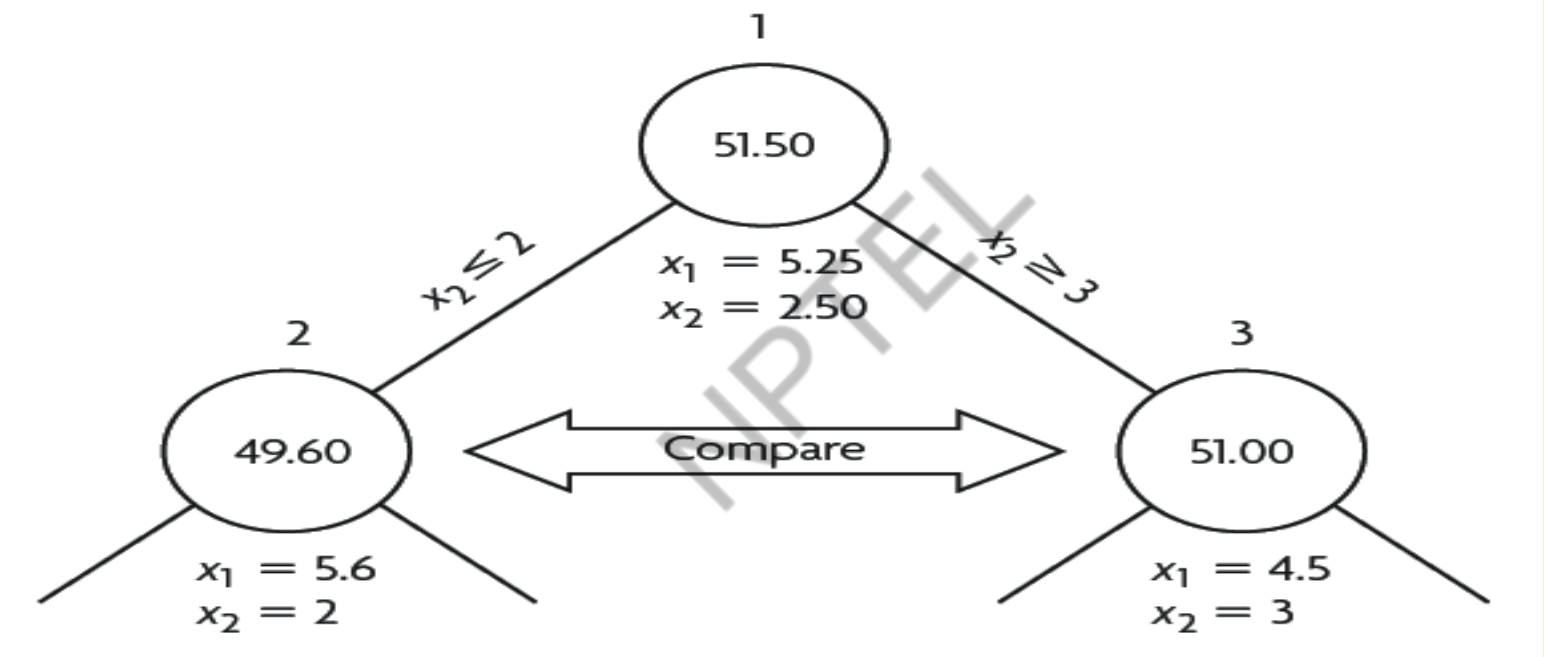
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Subproblems and Solution Points for Nodes 2 and 3



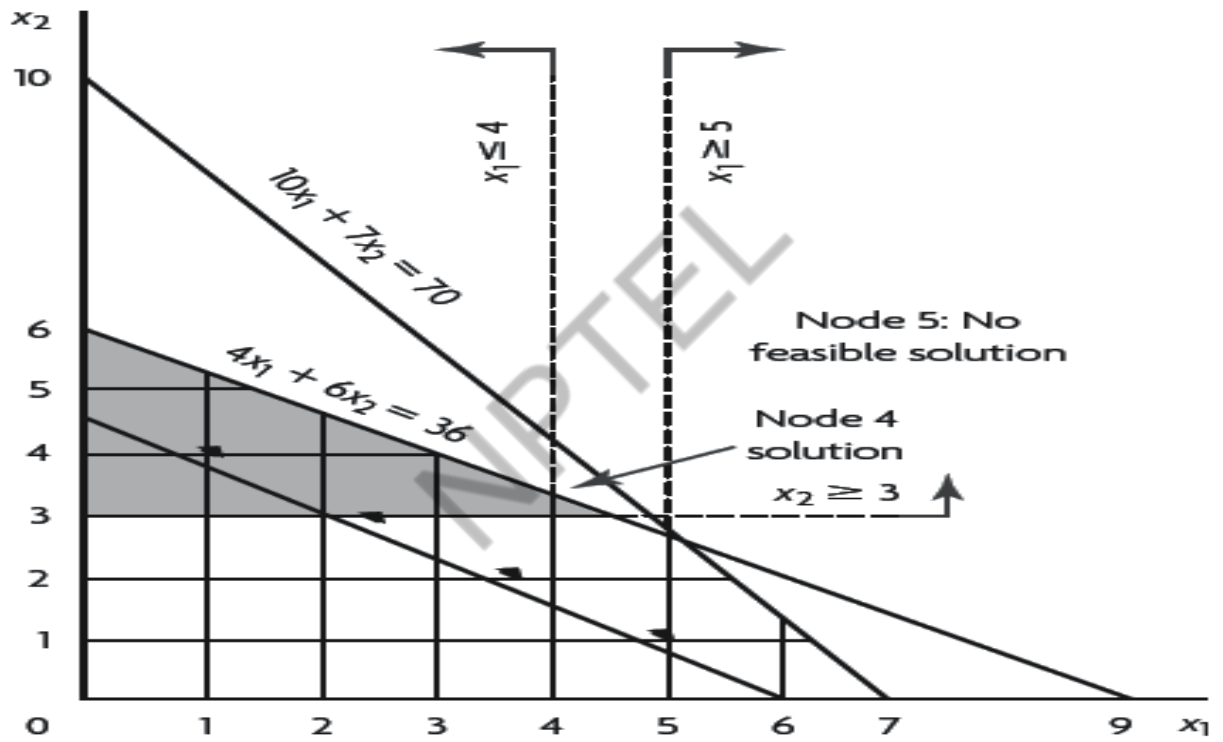
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Solutions for Nodes 2 and 3



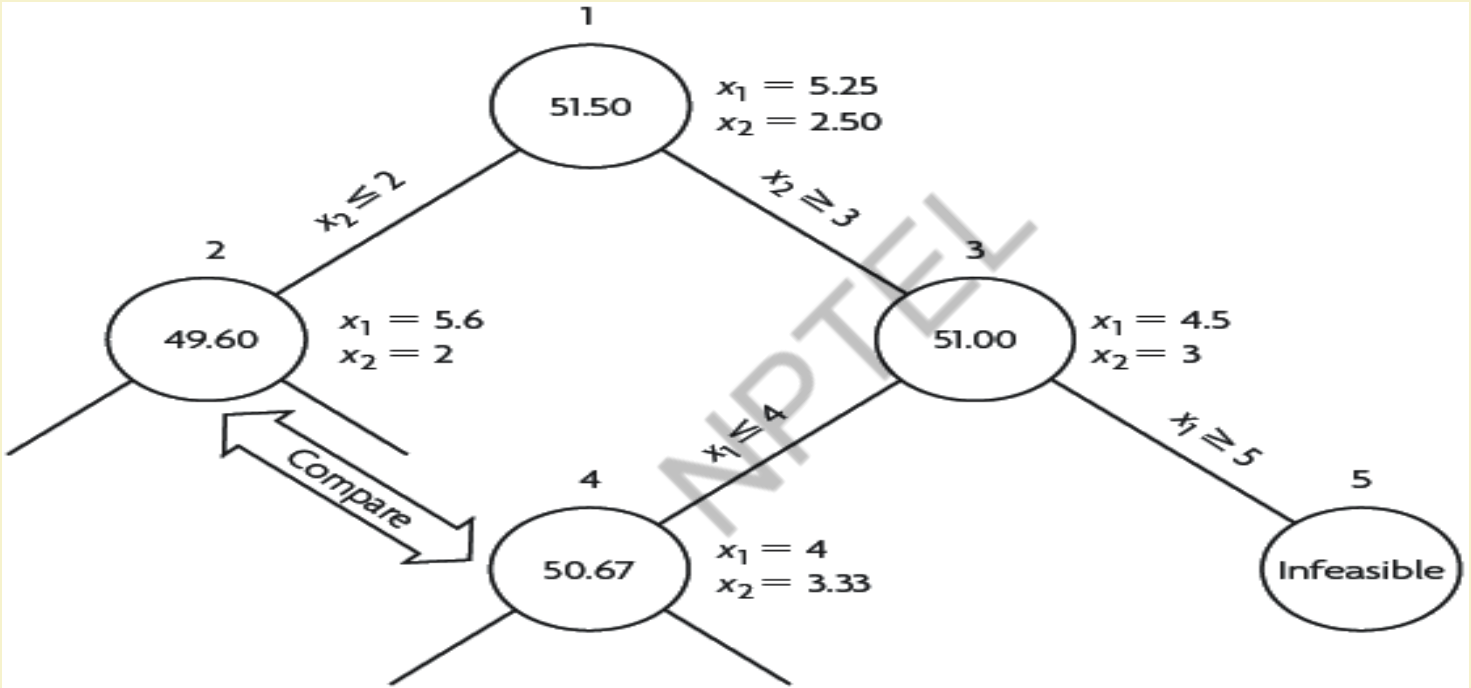
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Subproblem and Solution Point for Node 4



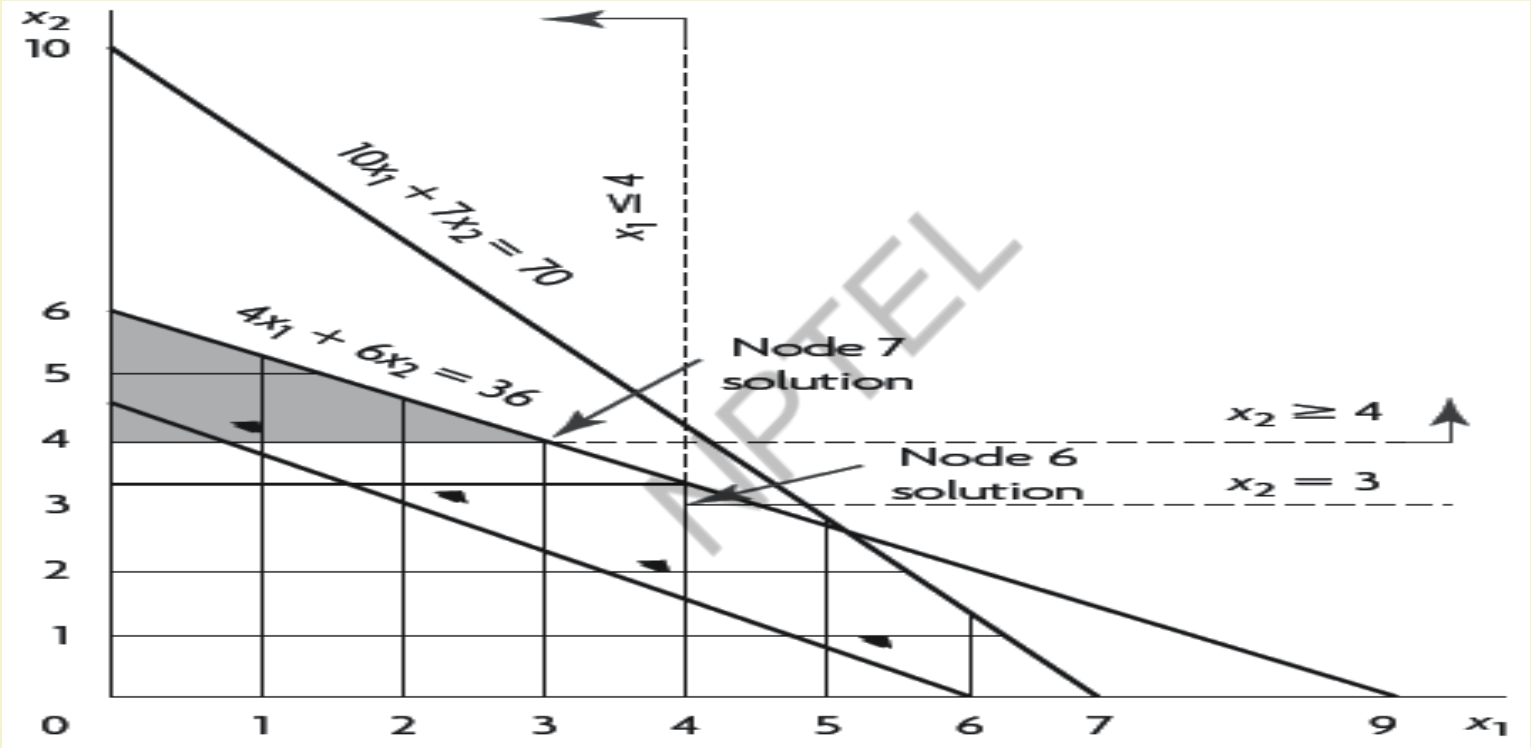
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Solutions for Nodes 4 and 5



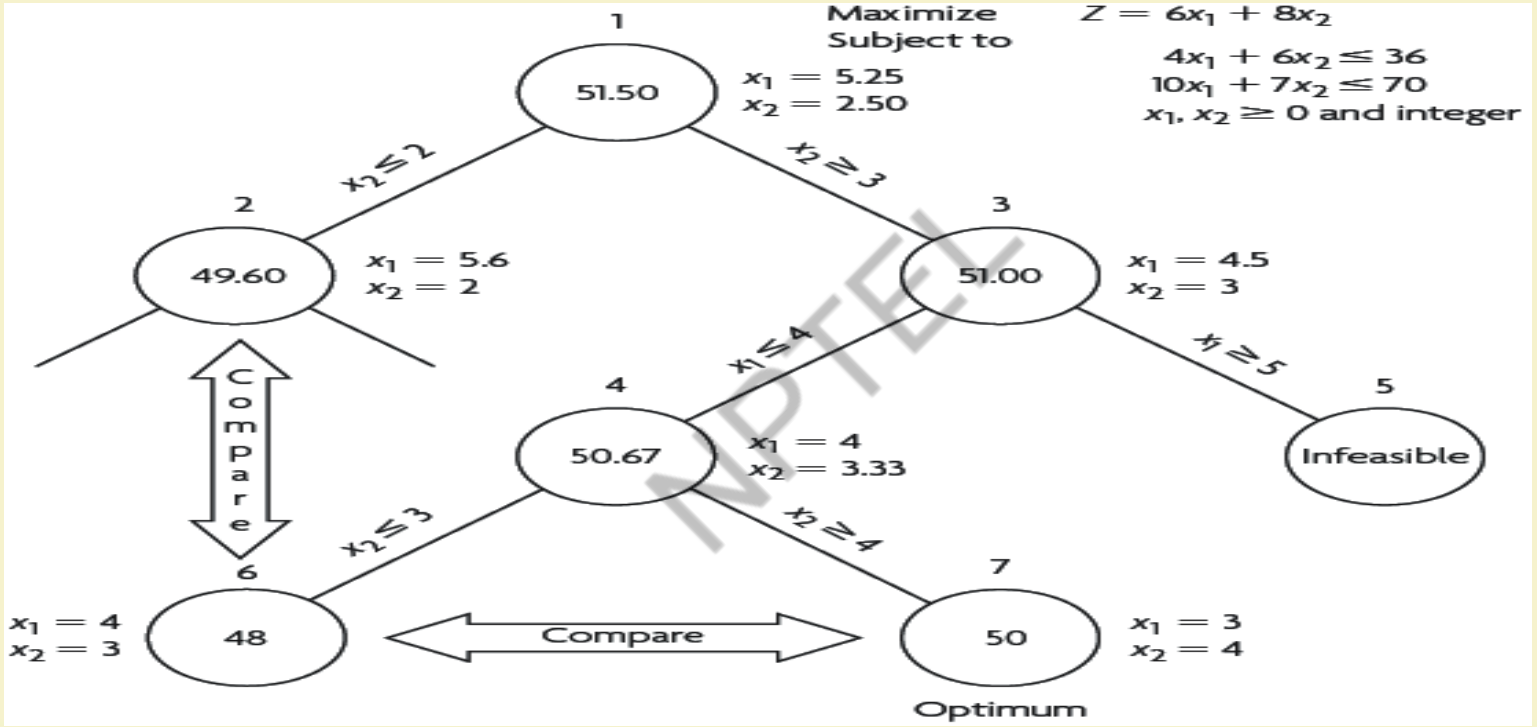
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Subproblems and Solutions for the Next Two Nodes



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Solutions for Nodes 6 and 7



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Summary of the All-Integer Maximization Problem

1. Solve the problem as standard LP problem: Ignore any integer requirements.
2. If all variables have integer values, this is the optimal integer solution. If any variables with integer requirements are non-integer, this is not optimal. Using the values of the variables for this solution, compute the value of the objective function. This is the upper bound: No feasible solution can yield a better value.
3. Examine the non-integer valued variables with integer requirements. Identify the one with the largest fractional part. In case of a tie, choose one arbitrarily. *Branch* on that variable: Round it both up and down to integer values. For the lower integer value, formulate a new constraint requiring that this variable cannot exceed this lower value. For the upper integer value, formulate a new constraint requiring that this variable must equal or exceed this upper value.

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Summary of the All-Integer Maximization Problem

4. Solve two new problems, one consisting of the original problem and the lower integer constraint and the other consisting of the original problem and the upper integer constraint. (For subsequent nodes, include the constraints for all branches leading to the node being considered.) For each node (problem), one of three possibilities will materialize:
   a. All required variables will be integer. This *may* be the optimal solution. Compute the upper bound for this solution. Stop branching on this portion of the tree. This upper bound is now a standard of comparison for other nodes: Stop branching from any node if its upper bound does not exceed this node's value. For nodes with higher upper bounds, return to step 3.
   b. The solution will be infeasible. Again, stop branching on this portion of the tree, but consider other portions: Return to step 3.

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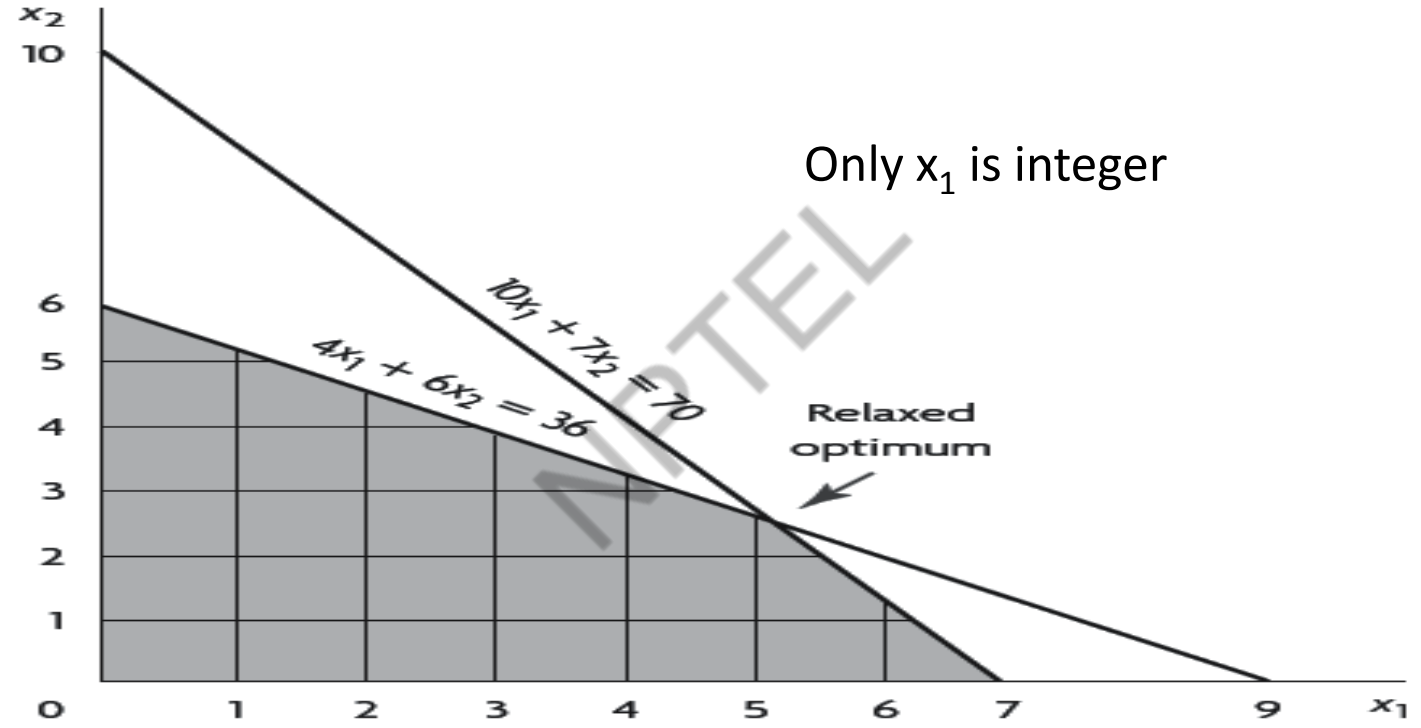
Summary of the All-Integer Maximization Problem

c. If some integer-required variables are still non-integer, compute the upper bound for this node. If any all-integer nodes exist, compare the upper bound of this node to the highest upper bound of the all-integer nodes. If it exceeds that value, return to step 3. Otherwise, stop branching from this node and consider other nodes using step 3.

5. The optimal integer solution is found when the value of the objective function at a node with all integer values is not exceeded by the upper bound of any other node, whether they represent all integers or not.

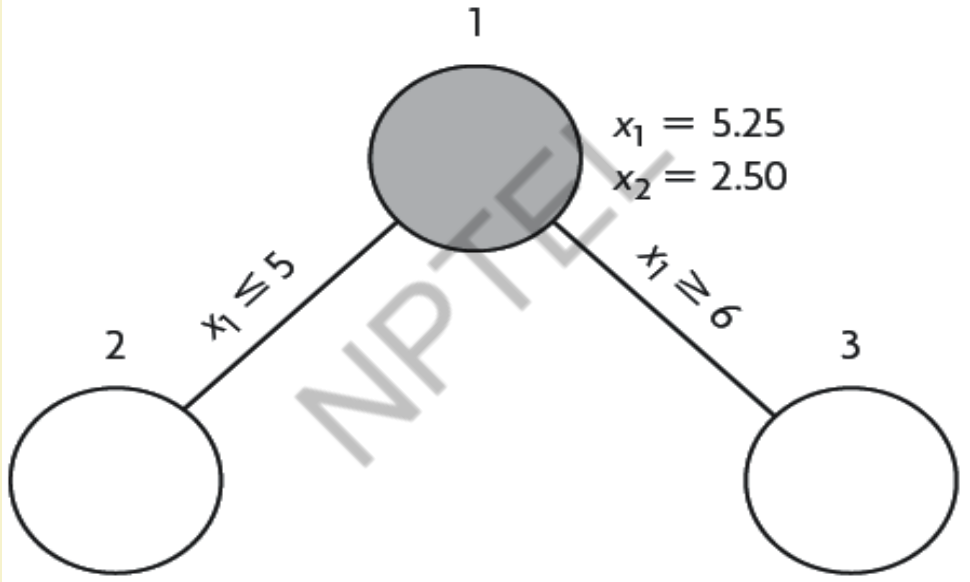
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LP Relaxation Solution Point (Mixed-Integer Problem)



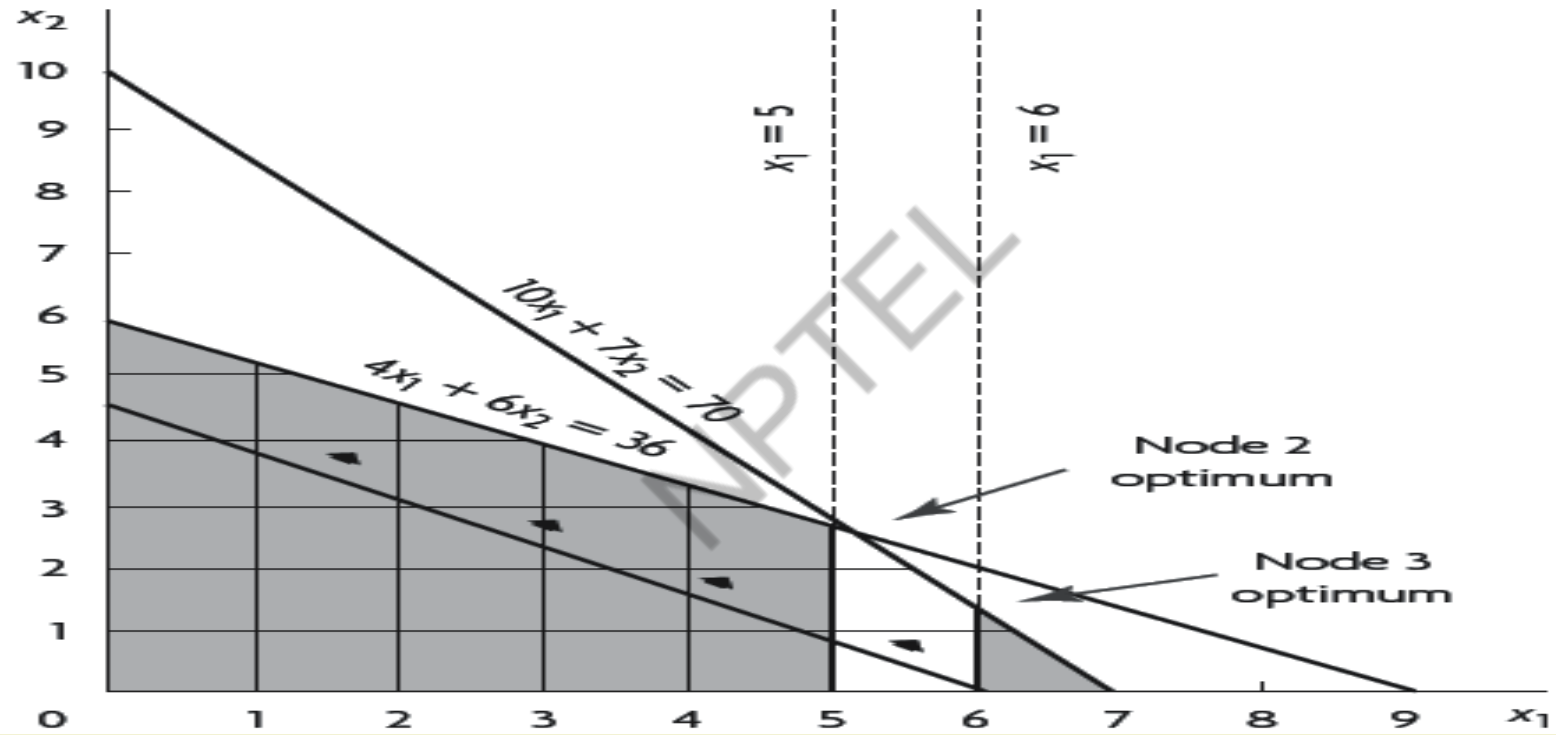
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Solution at Node 1 for Example 3 and Branches to Nodes 2 and 3



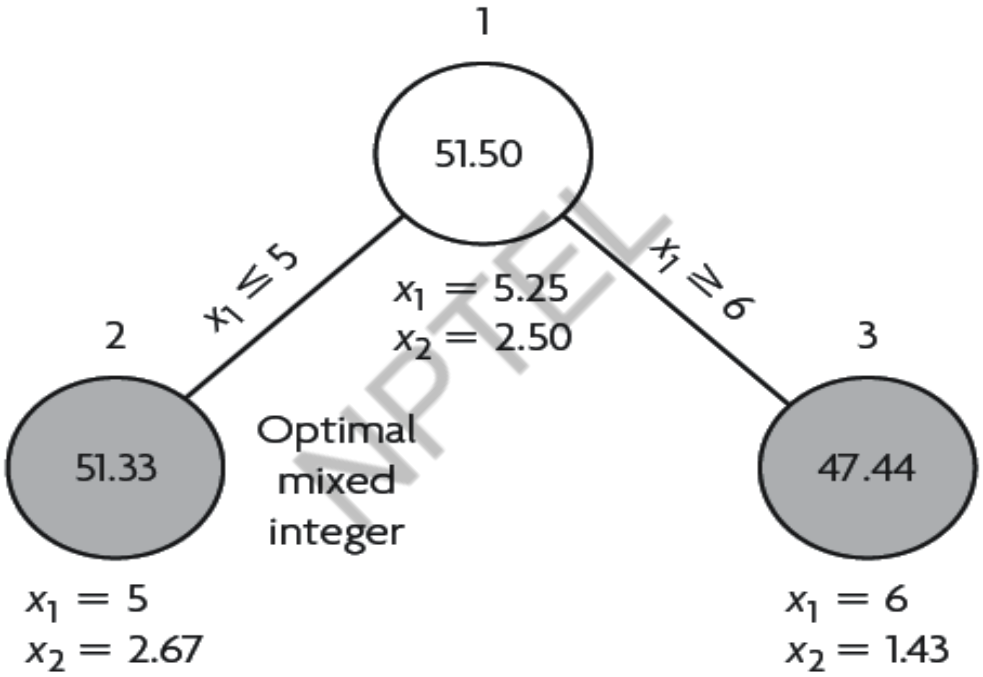
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Subproblems and Solution Points for Nodes 2 and 3



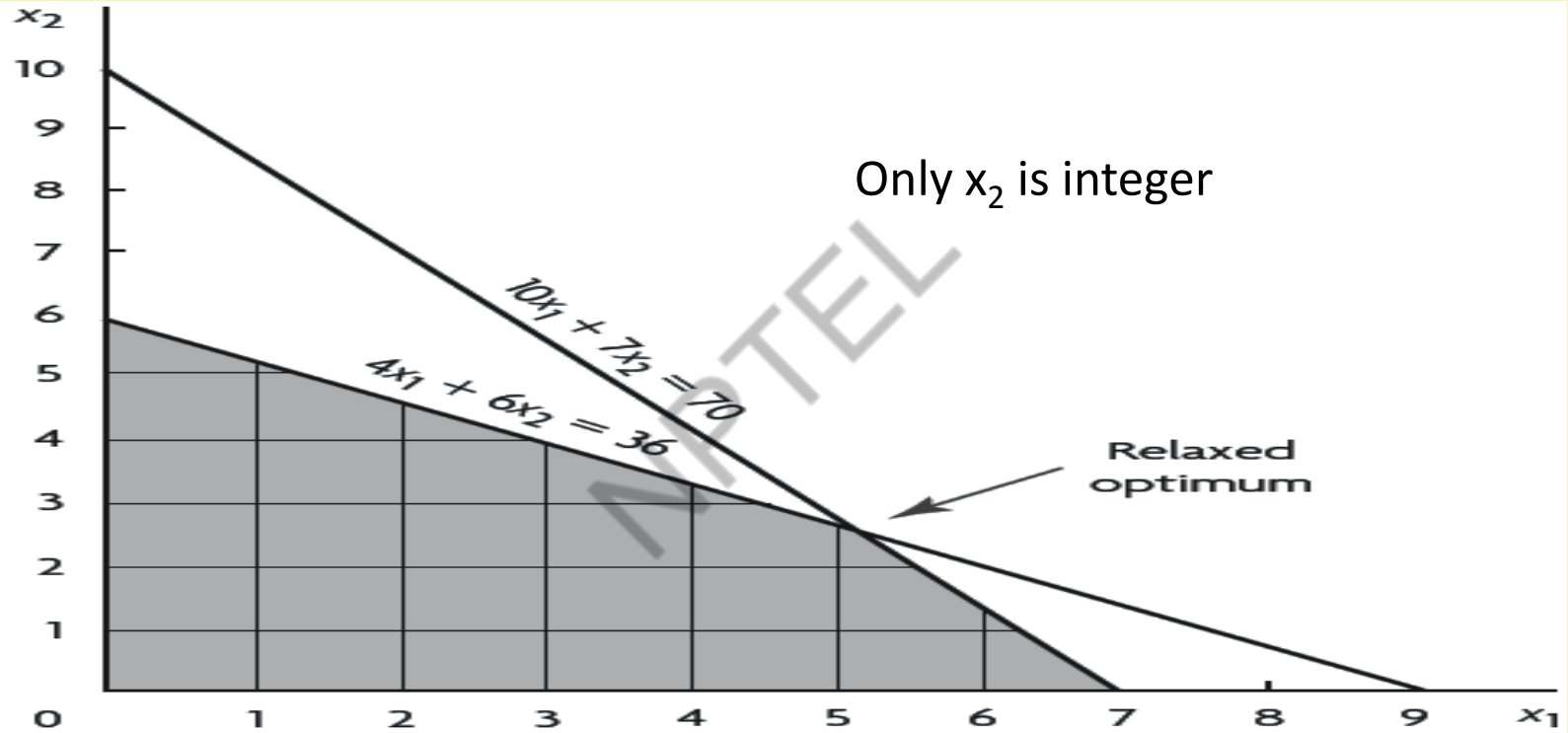
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Solutions for Nodes 2 and 3



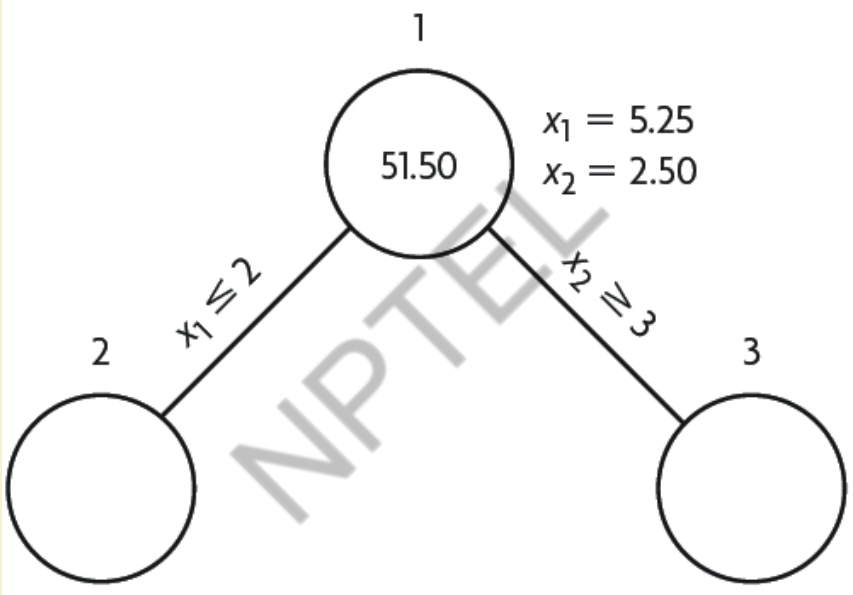
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The LP Relaxation Solution Point (Mixed-Integer Problem)



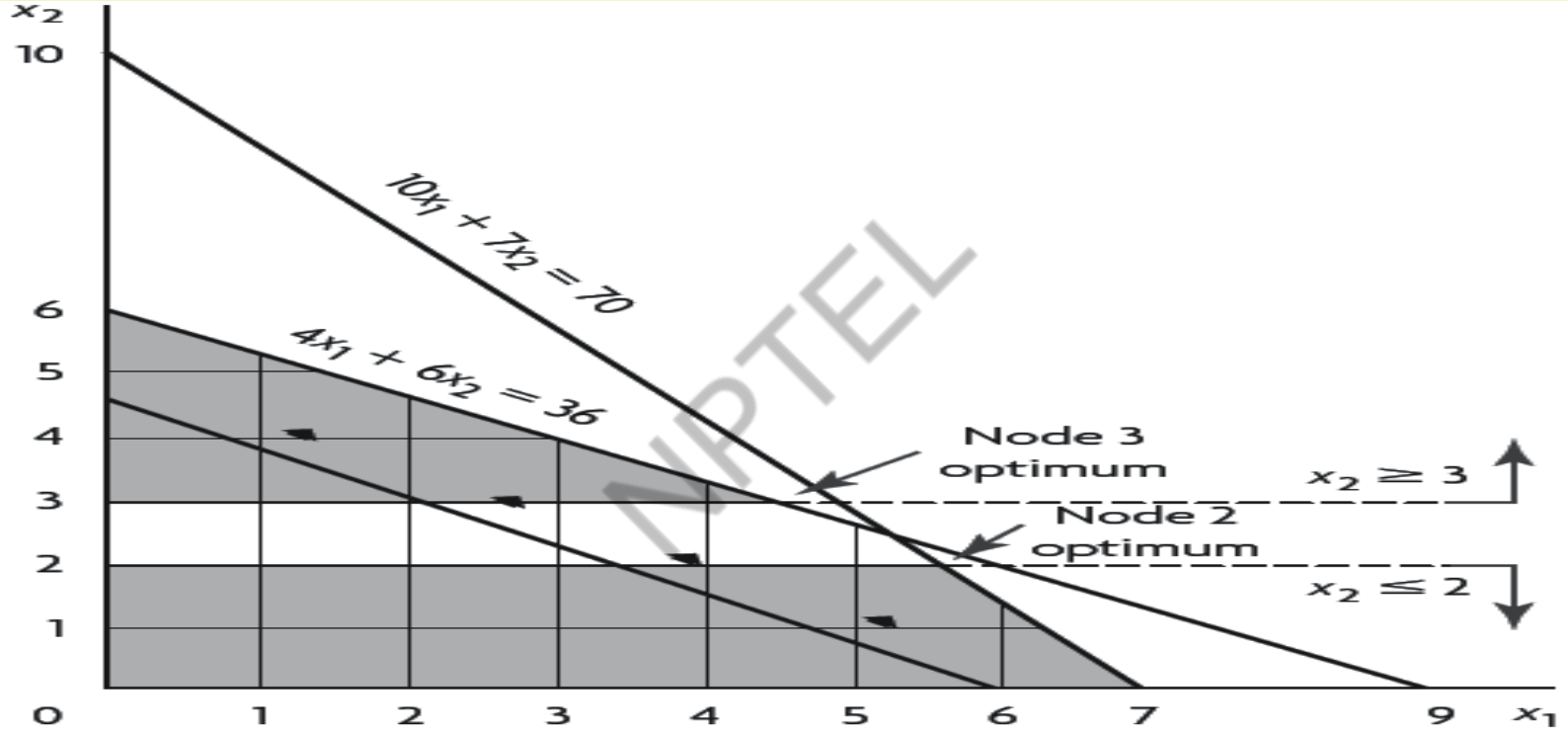
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Node 1 Solution and Branches



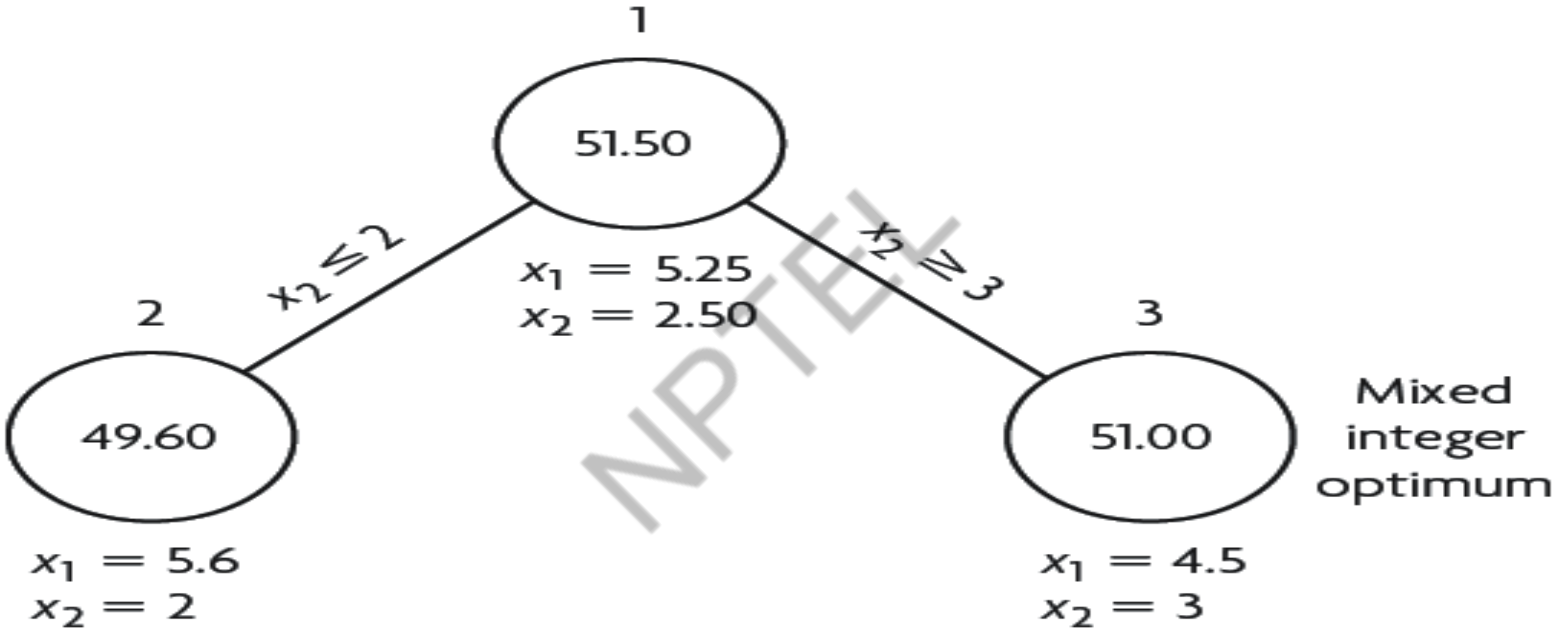
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Subproblems and Solution Points for Nodes 2 and 3



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Solutions for Nodes 2 and 3 (Node 3 Contains the Optimal)



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0-1 Problems

- Some integer programming problems relate to binary decisions involving choice between two possible values (0-NO or 1-YES) for each decision variable.
- Generally these problems involve multiple variables, such as the need to choose three locations for new convenience stores from a list of 20 potential sites. Even so, a yes-no decision must be made for each potential site (i.e., there are 20 decision variables).
- These can be solved using the **branch and bound** method or the **enumeration** method. The enumeration method is the simpler one for such problems.

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Enumeration Method

- Suitable when the number of alternatives is fairly small (say, not more than 16) and there are no more than 3 or 4 constraints, else, the process of evaluating alternatives becomes tedious.
- Generally, the number of alternatives is equal to 2^n, where n is the number of decision variables.
- Steps of the method:
  1. List all possible alternatives.
  2. Determine which alternatives are feasible.
  3. Find the value of the objective function for each feasible alternative.
  4. Select the feasible alternative with the best value of the objective function.

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Example 4

Solve the following problem using the enumeration method:

maximize $Z = 65x_1 + 70x_2 + 40x_3 + 50x_4$

subject to

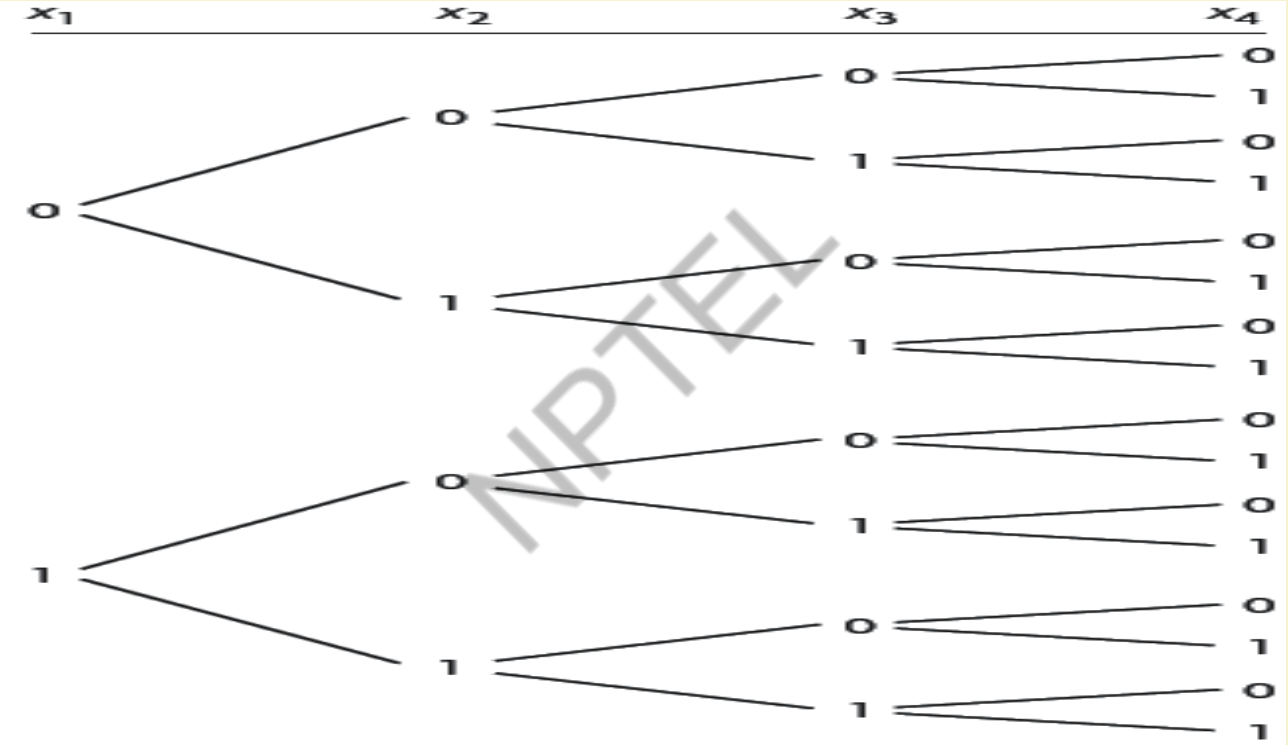
$$10x_1 + 12x_2 + 6x_3 + 8x_4 \leq 30 \quad (1)$$

$$3x_1 + x_2 + 2x_3 \leq 5 \quad (2)$$

All variables 0 or 1

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Tree Diagram for Example 4



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Enumeration and Evaluation of Alternatives for Example 4

	Alternative				Constraint 1		Constraint 2		Both Feasible?	Value of Objective Function
	x_1	x_2	x_3	x_4	Value	Feasible?	Value	Feasible?		
1	0	0	0	0	0	✓	0	✓	✓	0
2	0	0	0	1	8	✓	0	✓	✓	50
3	0	0	1	0	6	✓	2	✓	✓	40
4	0	0	1	1	14	✓	2	✓	✓	90
5	0	1	0	0	12	✓	1	✓	✓	70
6	0	1	0	1	20	✓	1	✓	✓	120
7	0	1	1	0	18	✓	3	✓	✓	110
8	0	1	1	1	26	✓	3	✓	✓	160
9	1	0	0	0	10	✓	3	✓	✓	65
10	1	0	0	1	18	✓	3	✓	✓	115
11	1	0	1	0	16	✓	5	✓	✓	105
12	1	0	1	1	24	✓	5	✓	✓	155
13	1	1	0	0	22	✓	4	✓	✓	135
14	1	1	0	1	30	✓	5	✓	✓	185*
15	1	1	1	0	28	✓	6	No	No	—
16	1	1	1	1	36	No	6	No	No	—

*Optimum

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Goal Programming

- Goal Programming (GP)
  - A variation of linear programming that allows multiple objectives (goals)—soft (goal) constraints or a combination of soft and hard (nongoal) constraints—that can deviate, allowing for tradeoffs in achieving satisficing rather than only optimal solutions.
  - GP models are similar to LP models in that both are formulated under the same requirements and assumptions (e.g., linearity, nonnegativity, certainty).
  - GP uses, like LP, graphical methods to illustrate linear programming concepts.

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Goal Programming Models

- Multiple objectives
- Satisfy rather than optimize
- Minimize underachievement of goals
- Two approaches
  - Weighted models
  - Ranked models

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Examples: Wilson Doors

	EXTERIOR	INTERIOR	COMMERCIAL	AVAILABILITY
Steel (lb/door)	4	3	7	9,000 pounds
Forming (hr/door)	2	4	3	6,000 hours
Assembly (hr/door)	2	3	4	5,200 hours
Selling price/door	$70	$110	$110	

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Examples: Wilson Doors

Maximize total sales = \$70E + \$110I + \$110C

subject to

$4E + 3I + 7C$	≤	9,000	(steel usage)
$2E + 4I + 3C$	≤	6,000	(forming time)
$2E + 3I + 4C$	≤	5,200	(assembly time)
E, I, C	≥	0	

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Examples: Wilson Doors, Multiple goals

Goal 1: Achieve total sales of at least $180,000

Goal 2: Achieve exterior doors sales of at least $70,000

Goal 3: Achieve interior doors sales of at least $60,000

Goal 4: Achieve commercial doors sales of at least $35,000

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Examples: Wilson Doors, Multiple goals

- Deviation variables

d_T^- = amount by which the total sales goal is underachieved

d_T^+ = amount by which the total sales goal is overachieved

d_E^- = amount by which the exterior doors sales goal is underachieved

d_E^+ = amount by which the exterior doors sales goal is overachieved

d_I^- = amount by which the interior doors sales goal is underachieved

d_I^+ = amount by which the interior doors sales goal is overachieved

$d_{\bar{C}}^-$ = amount by which the commercial doors sales goal is underachieved

$d_{\bar{C}}^+$ = amount by which the commercial doors sales goal is overachieved

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Examples: Wilson Doors, Multiple goals

- Multiple goals

$$70E + 110I + 110C + d_T^- - d_T^+ = 180{,}000 \quad \text{(total sales goal)}$$
$$70E + d_E^- - d_E^+ = 70{,}000 \quad \text{(exterior doors sales goal)}$$
$$110I + d_I^- - d_I^+ = 60{,}000 \quad \text{(interior doors sales goal)}$$
$$110C + d_C^- - d_C^+ = 35{,}000 \quad \text{(commercial doors sales goal)}$$

- Objective function

Minimize total underachievement of goals = $d_T^- + d_E^- + d_I^- + d_C^-$

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Examples: Wilson Doors, Multiple goals

subject to

$70E + 110I + 110C + d_T^- + d_T^+$	=	180,000	(total sales goal)
$70E + d_E^- + d_E^+$	=	70,000	(exterior doors sales goal)
$110I + d_I^- + d_I^+$	=	60,000	(interior doors sales goal)
$110C + d_C^- + d_C^+$	=	35,000	(commercial doors sales goal)
$4E + 3I + 7C$	≤	9,000	(steel usage)
$2E + 4I + 3C$	≤	6,000	(forming time)
$2E + 3I + 4C$	≤	5,200	(assembly time)
$E, I, C, d_T^-, d_T^+, d_E^-, d_E^+, d_I^-, d_I^+, d_C^-, d_C^+$	≥	0	

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Examples: Wilson Doors, Multiple goals

Formulating a weighted model

Goal	Weight
1 – Achieve total sales of at least $180,000	5
2 – Achieve exterior doors sales of at least $70,000	1
3 – Achieve interior doors sales of at least $60,000	1
4 – Achieve commercial doors sales of at least $35,000	1

- Objective function

Minimize total *weighted*
underachievement of goals = $5d_T^- + d_E^- + d_I^- + d_C^-$

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Examples: Wilson Doors, Multiple goals

Total sales goal is weighted 5 times as much as other goals.

Interior doors sales goal is overachieved by $28,000.

Commercial doors sales goal is underachieved by $13,000.

Wilson Doors (Weighted GP #1)

	E	I	C	d_T^-	d_T^+	d_E^-	d_E^+	d_I^-	d_I^+	d_C^-	d_C^+				
	Exterior doors	Interior doors	Comm doors	Under ach total sales	Over ach total sales	Under ach exter doors	Over ach exter doors	Under ach inter doors	Over ach inter doors	Under ach comm doors	Over ach comm doors				
Solution value	1000.00	800.00	200.00	0.00	0.00	0.00	0.00	0.00	28000.00	13000.00	0.00				
Goal weights				5		1		1		1		13000.00			
Constraints:															**Achieved**
Total sales goal	70	110	110	1	-1							180000.00	=	180000	180000.00
Exterior doors goal	70					1	-1					70000.00	=	70000	70000.00
Interior doors goal		110						1	-1			60000.00	=	60000	88000.00
Comm doors goal			110							1	-1	35000.00	=	35000	22000.00
Steel usage	4	3	7									7800.00	<=	9000	
Forming time	2	4	3									5800.00	<=	6000	
Assembly time	2	3	4									5200.00	<=	5200	
												LHS	Sign	RHS	



Entries show how much of each goal has been achieved.

Model includes three resource constraints and four goal constraints.

All goals are expressed as = constraints, using the deviation variables.

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Examples: Wilson Doors, Multiple goals

Total sales goal is weighted only 2.5 times as much as other goals.

Total sales goal is now underachieved.

Commercial doors sales goal is now fully achieved.

Wilson Doors (Weighted GP #2)

		E	I	C	d_T^-	d_T^+	d_E^-	d_E^+	d_I^-	d_I^+	d_C^-	d_C^+				
		Exterior doors	Interior doors	Comm doors	Under ach total sales	Over ach total sales	Under ach exter doors	Over ach exter doors	Under ach inter doors	Over ach inter doors	Under ach comm doors	Over ach comm doors				
5	Solution value	1000.00	642.42	318.18	4333.33	0.00	0.00	0.00	0.00	10666.67	0.00	0.0				
6	Goal weights				2.5		1		1		1		10833.33			
7	**Constraints:**															**Achieved**
8	Total sales goal	70	110	110	1	-1							180000.00	=	180000	175666.67
9	Exterior doors goal	70					1	-1					70000.00	=	70000	70000.00
10	Interior doors goal		110						1	-1			60000.00	=	60000	70666.67
11	Comm doors goal			110							1	-1	35000.00	=	35000	35000.00
12	Steel usage	4	3	7									8154.55	<=	9000	
13	Forming time	2	4	3									5524.24	<=	6000	
14	Assembly time	2	3	4									5200.00	<=	5200	
15													LHS	Sign	RHS	



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Screenshot 6-6B

Examples: Wilson Doors, Multiple goals

- Formulating a ranked goal model

Goal 1: Achieve total sales of at least $180,000

Goal 2: Achieve exterior doors sales of at least $70,000

Goal 3: Achieve interior doors sales of at least $60,000

Goal 4: Achieve commercial doors sales of at least $35,000

Goal 5: Achieve steel usage of as close to 9,000 pounds as possible

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Examples: Wilson Doors, Multiple goals

- Formulating a ranked goal model
  - Lower ranked goals considered only after higher ranked goals are met

Rank R_1: Goal 1

Rank R_2: Goal 5

Rank R_3: Goals 2, 3, and 4

- New deviation variable

d_S^- = amount by which the steel usage goal is underachieved

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Examples: Wilson Doors, Multiple goals

Objective function

Minimize ranked deviations = $R_1(d_T^-) + R_2(d_S^-) + R_3(d_E^- + d_I^- + d_C^-)$

subject to

$70E + 110I + 110C + d_T^- + d_T^+$	=	180,000	(total sales goal)
$4E + 3I + 7C + d_S^-$	=	9,000	(steel usage goal)
$70E + d_E^- + d_E^+$	=	70,000	(exterior doors sales goal)
$110I + d_I^- + d_I^+$	=	60,000	(interior doors sales goal)
$110C + d_C^- + d_C^+$	=	35,000	(commercial doors sales goal)
$2E + 4I + 3C$	≤	6,000	(forming time)
$2E + 3I + 4C$	≤	5,200	(assembly time)
$E, I, C, d_T^-, d_T^+, d_S^-, d_E^-, d_E^+, d_I^-, d_I^+, d_C^-, d_C^+$	≥	0	

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Examples: Wilson Doors, Multiple goals

Objective function – first LP model

Minimize rank R_1 deviations = d_T^-

Objective function – second LP model

Minimize rank R_2 deviations = d_S^-

Additional constraint

d_T^- = 0 (optimal value of rank R_1 goal)

Objective function – third LP model

Minimize rank R_3 deviations = $d_E^- + d_I^- + d_C^-$

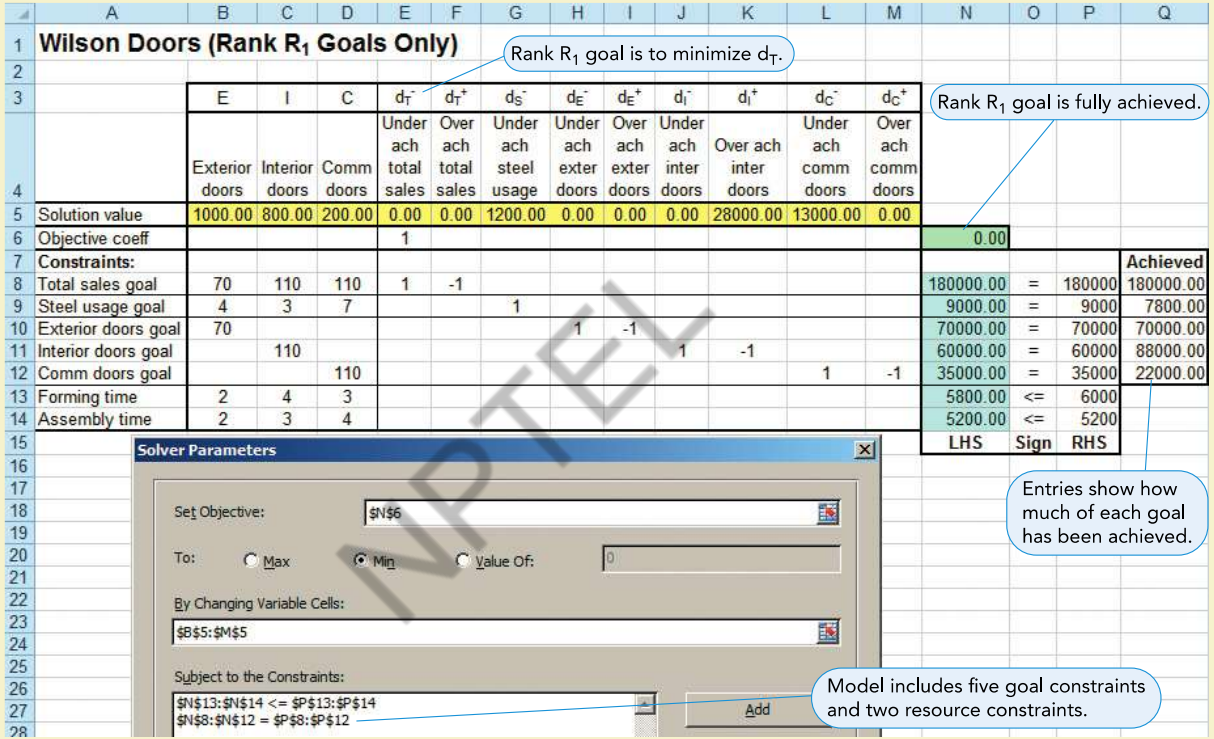
Additional constraints

d_T^- = 0 (optimal value of rank R_1 goal)

d_S^- = 0 (optimal value of rank R_2 goal)

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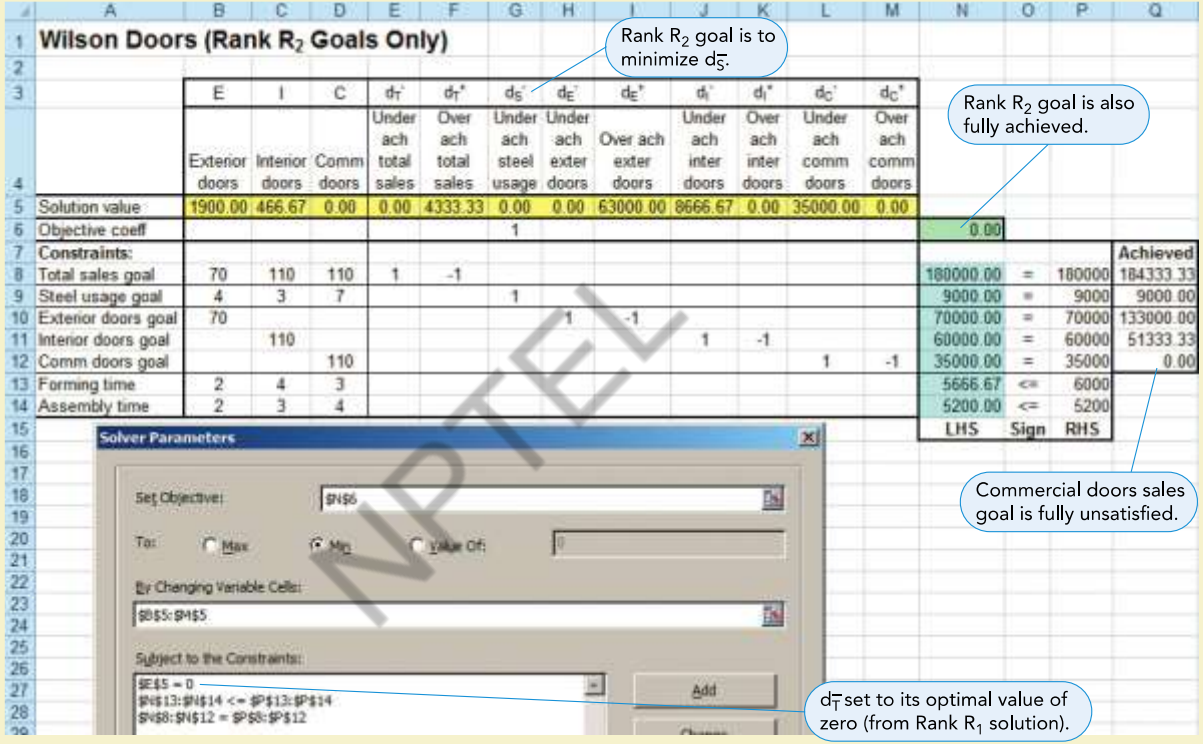
Examples: Wilson Doors, Multiple goals



	A	E	I	C	d_T^-	d_T^+	d_S^-	d_E^-	d_E^+	d_I^-	d_I^+	d_C^-	d_C^+				
4		Exterior doors	Interior doors	Comm doors	Under ach total sales	Over ach total sales	Under ach steel usage	Under ach exter doors	Over ach exter doors	Under ach inter doors	Over ach inter doors	Under ach comm doors	Over ach comm doors				
5	Solution value	1000.00	800.00	200.00	0.00	0.00	1200.00	0.00	0.00	0.00	28000.00	13000.00	0.00				
6	Objective coeff				1									0.00			
7	**Constraints:**																**Achieved**
8	Total sales goal	70	110	110	1	-1								180000.00	=	180000	180000.00
9	Steel usage goal	4	3	7			1							9000.00	=	9000	7800.00
10	Exterior doors goal	70						1	-1					70000.00	=	70000	70000.00
11	Interior doors goal		110							1	-1			60000.00	=	60000	88000.00
12	Comm doors goal			110								1	-1	35000.00	=	35000	22000.00
13	Forming time	2	4	3										5800.00	<=	6000	
14	Assembly time	2	3	4										5200.00	<=	5200	
15														LHS	Sign	RHS	

NPTEL

Examples: Wilson Doors, Multiple goals



Wilson Doors (Rank R_2 Goals Only)

	E	I	C	d_T^-	d_T^+	d_S^-	d_E^-	d_E^+	d_I^-	d_I^+	d_C^-	d_C^+				
	Exterior doors	Interior doors	Comm doors	Under ach total sales	Over ach total sales	Under ach steel usage	Under ach exter doors	Over ach exter doors	Under ach inter doors	Over ach inter doors	Under ach comm doors	Over ach comm doors				
Solution value	1900.00	466.67	0.00	0.00	4333.33	0.00	0.00	63000.00	8666.67	0.00	35000.00	0.00				
Objective coeff						1							0.00			
Constraints:																**Achieved**
Total sales goal	70	110	110	1	-1								180000.00	=	180000	184333.33
Steel usage goal	4	3	7			1							9000.00	=	9000	9000.00
Exterior doors goal	70						1	-1					70000.00	=	70000	133000.00
Interior doors goal		110							1	-1			60000.00	=	60000	51333.33
Comm doors goal			110								1	-1	35000.00	=	35000	0.00
Forming time	2	4	3										5666.67	<=	6000	
Assembly time	2	3	4										5200.00	<=	5200	
													LHS	Sign	RHS	

Rank R_2 goal is to minimize d_S^-.

Rank R_2 goal is also fully achieved.

Commercial doors sales goal is fully unsatisfied.

d_T^- set to its optimal value of zero (from Rank R_1 solution).

Solver Parameters

Set Objective: $N$6

To: Max Min Value Of: 0

By Changing Variable Cells: $B$5:$N$5

Subject to the Constraints:

$E$5 = 0
$N$13:$N$14 <= $P$13:$P$14
$N$8:$N$12 = $P$8:$P$12

Add

NPTEL

Screenshot 6-7B

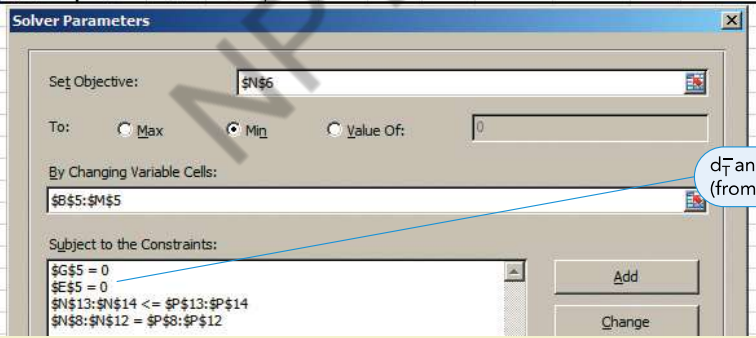
Examples: Wilson Doors, Multiple goals

Rank R_3 goal is to minimize ($d_E^- + d_I^- + d_C^-$).

Wilson Doors (Rank R_3 Goals Only)

	E	I	C	d_T^-	d_T^+	d_S^-	d_E^-	d_E^+	d_I^-	d_I^+	d_C^-	d_C^+				
	Exterior doors	Interior doors	Comm doors	Under ach total sales	Over ach total sales	Under ach steel usage	Under ach exter doors	Over ach exter doors	Under ach inter doors	Over ach inter doors	Under ach comm doors	Over ach comm doors				
Solution value	1694.74	421.05	136.84	0.00	0.00	0.00	0.00	48631.58	13684.21	0.00	19947.37	0.00				
Objective coeff							1		1		1		33631.58			
Constraints:																**Achieved**
Total sales goal	70	110	110	1	-1								180000.00	=	180000	180000.00
Steel usage goal	4	3	7			1							9000.00	=	9000	9000.00
Exterior doors goal	70						1	-1					70000.00	=	70000	118631.58
Interior doors goal		110							1	-1			60000.00	=	60000	46315.79
Comm doors goal			110								1	-1	35000.00	=	35000	15052.63
Forming time	2	4	3										5484.21	<=	6000	
Assembly time	2	3	4										5200.00	<=	5200	
													LHS	Sign	RHS	

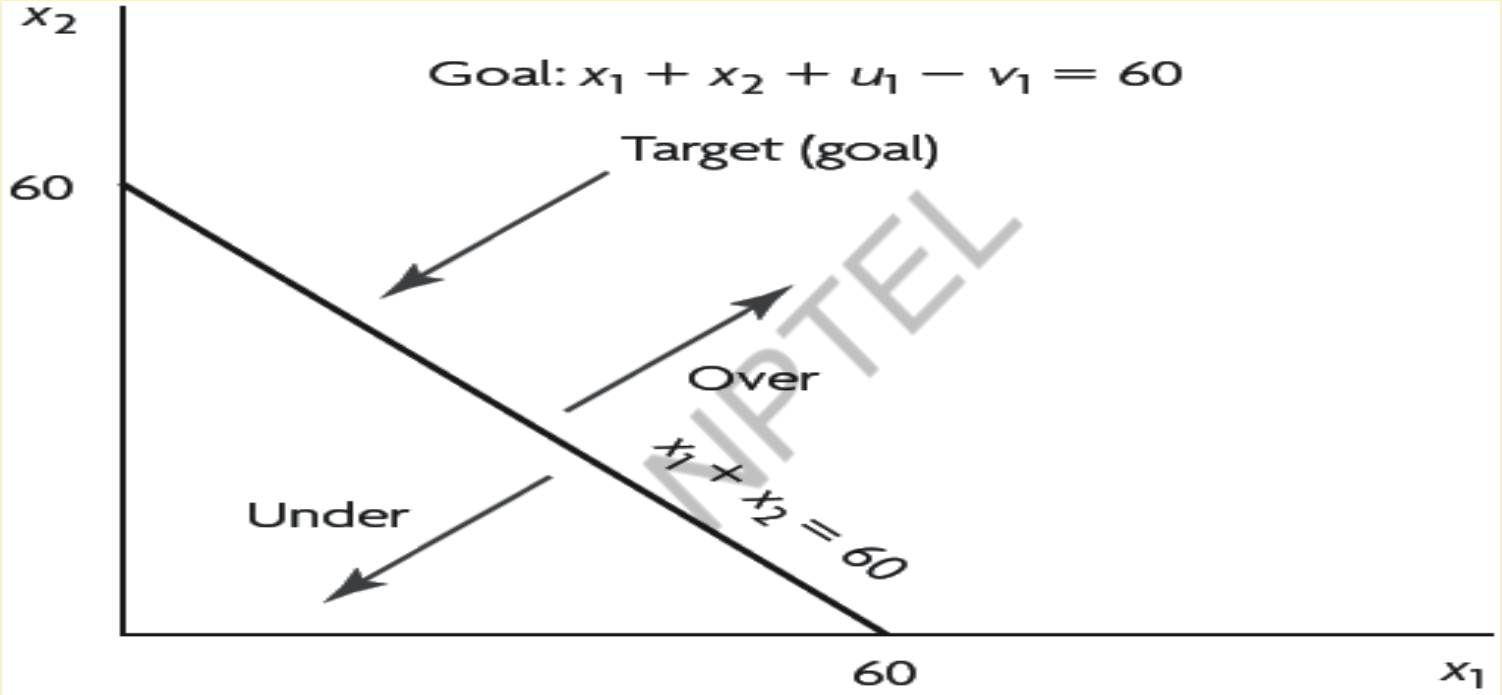
Rank R_3 goal cannot be fully achieved.



d_T^- and d_S^- both set to zero (from Rank R_1 and R_2 solutions).

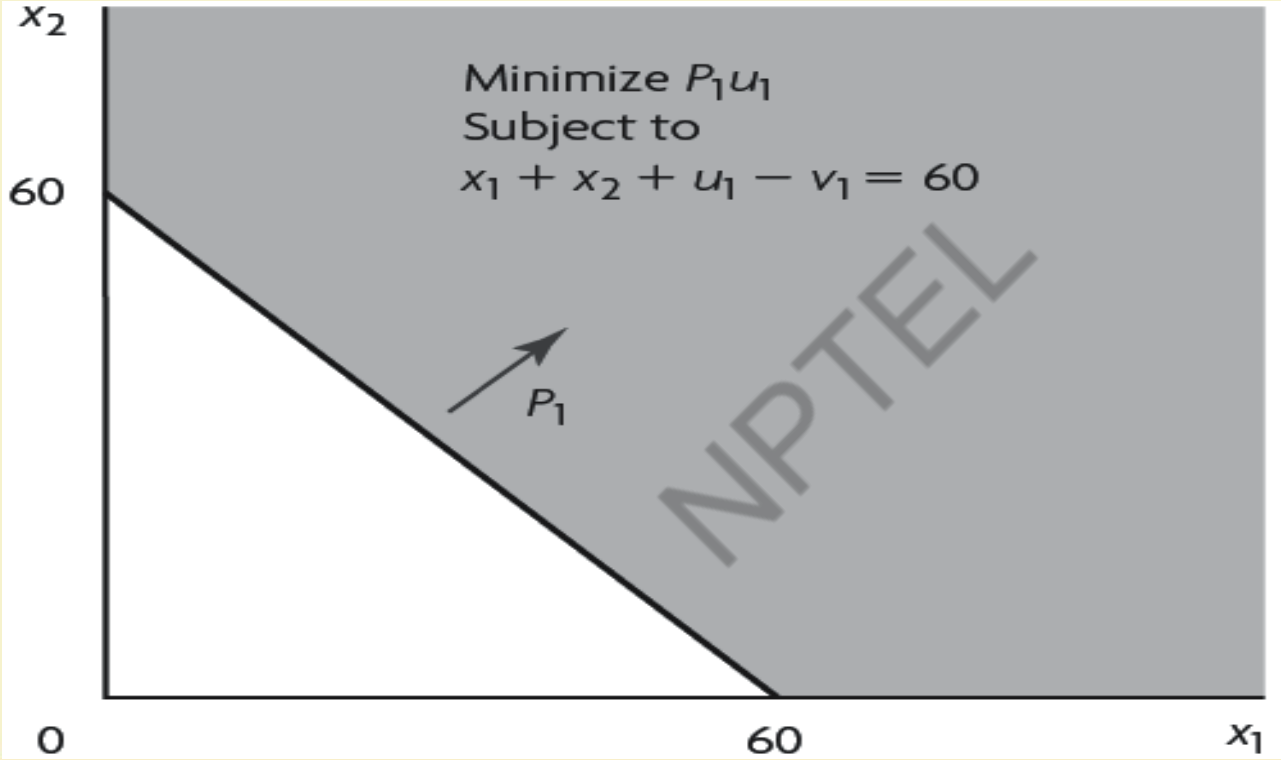
NPTEL

Plot of a Goal Constraint



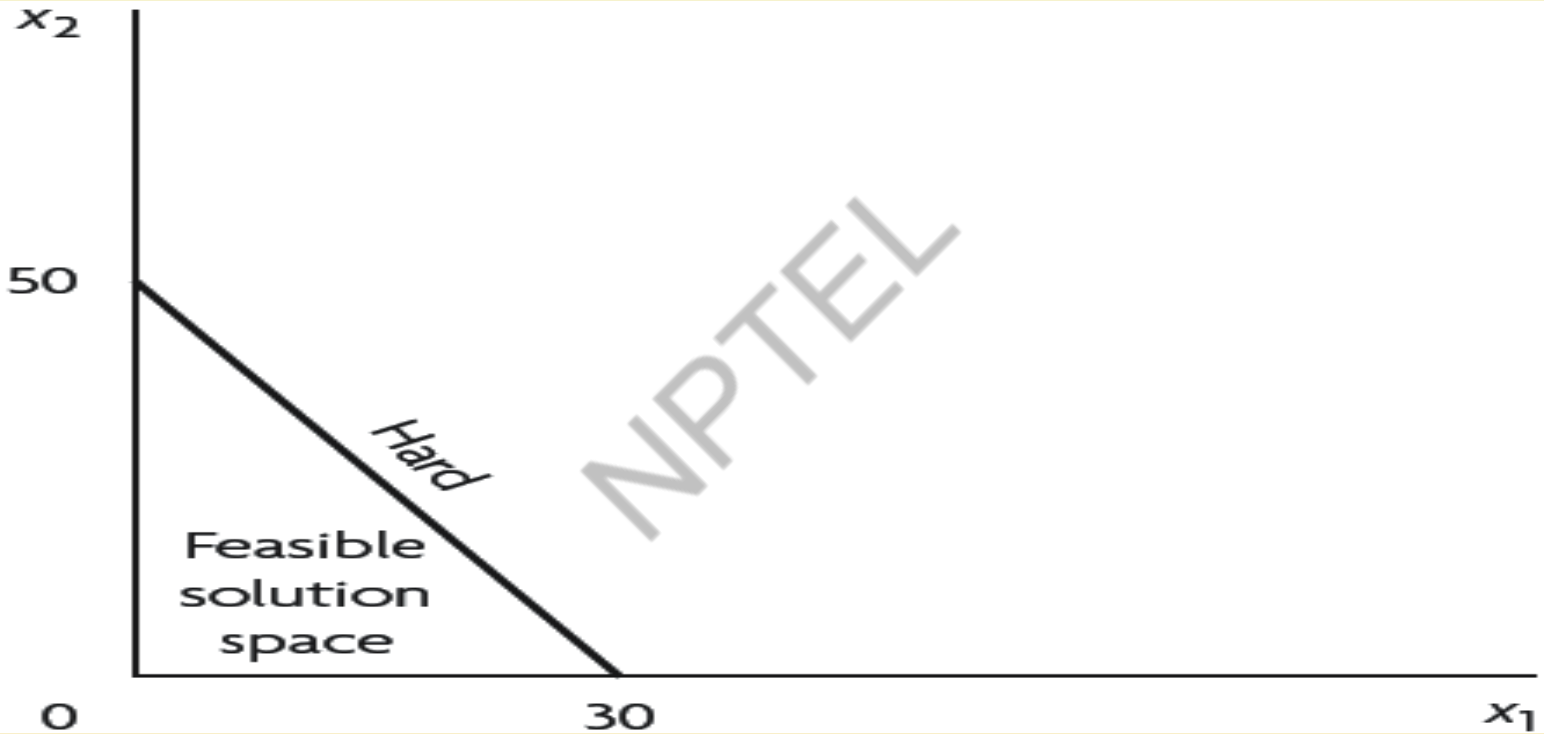
NPTEL

Designating Priority and Direction



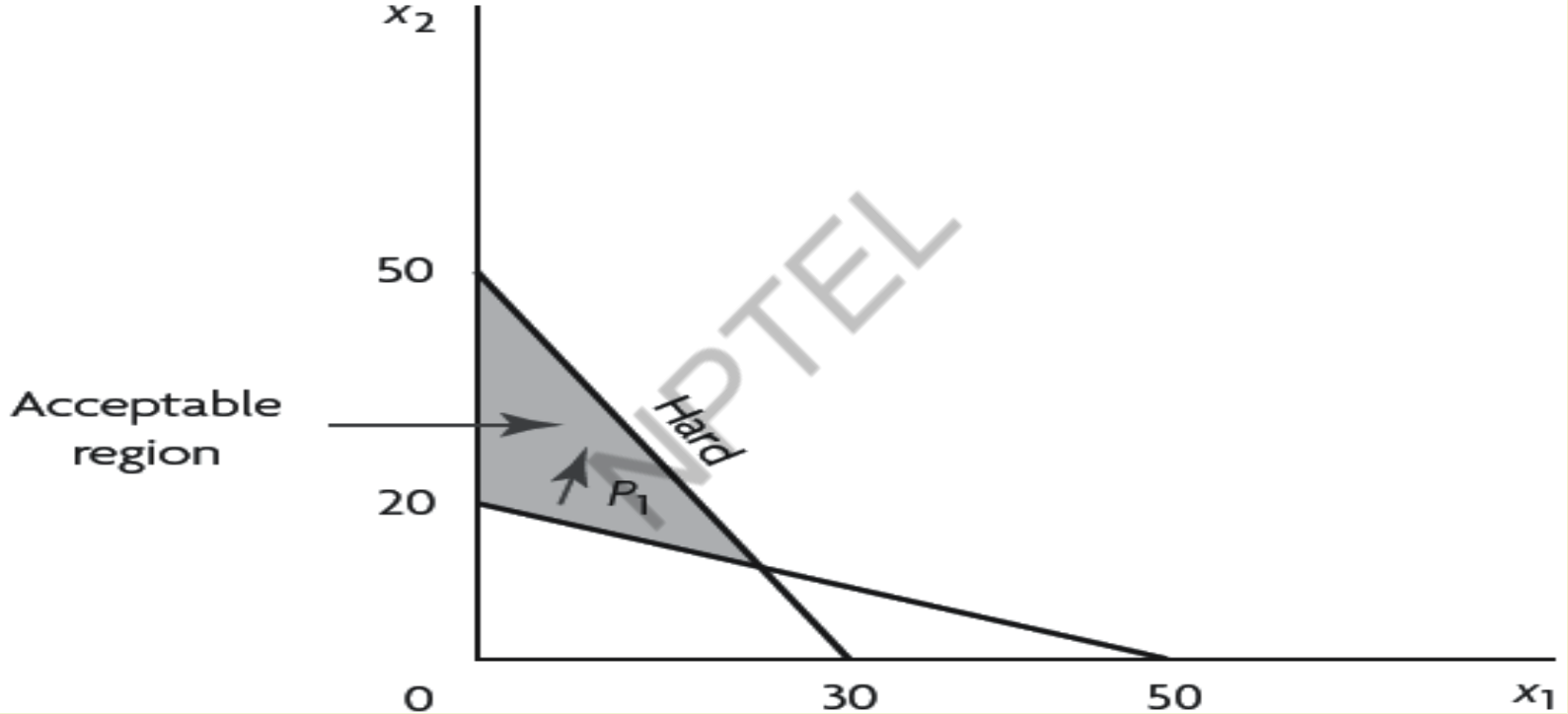
NPTEL

Plot of the Hard Constraint and the Feasible Solution Space



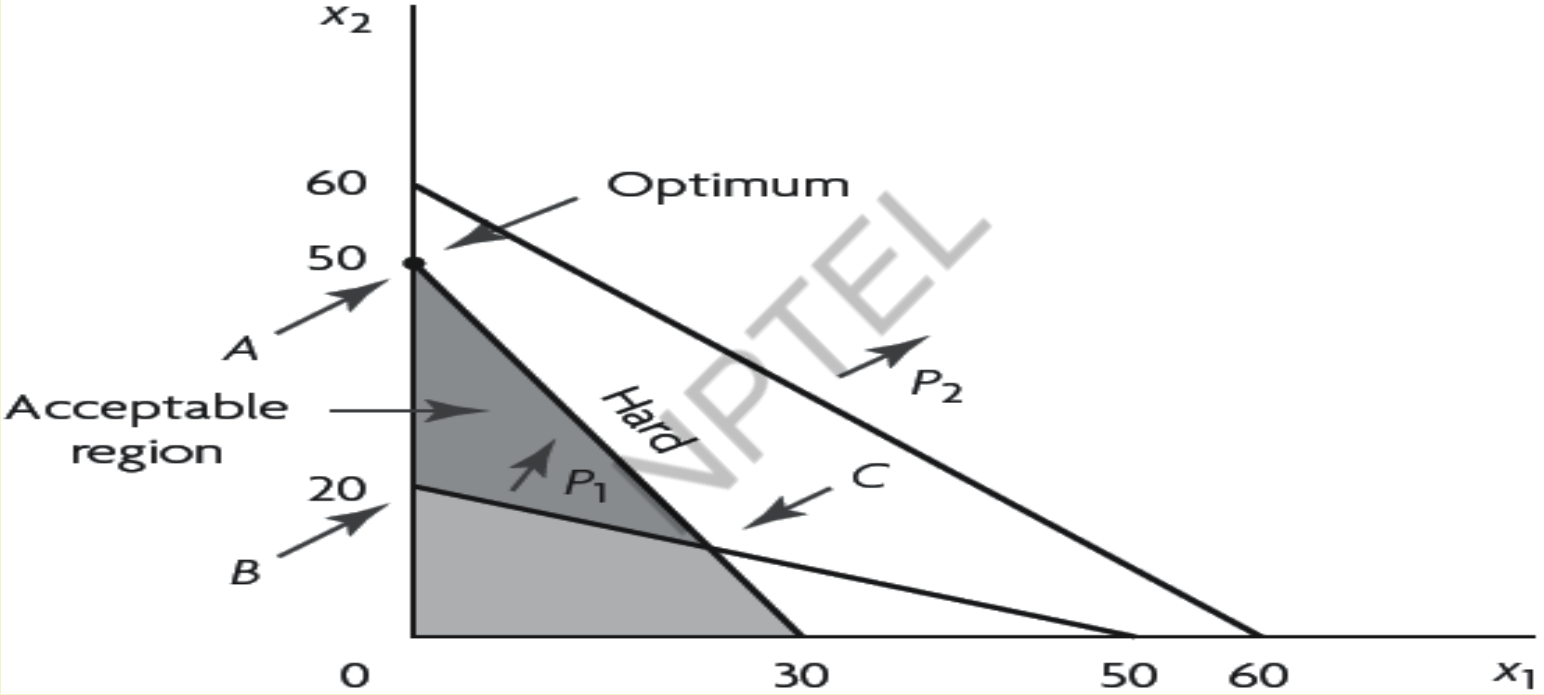
NPTEL

The Acceptable Region after Adding the First Goal Constraint



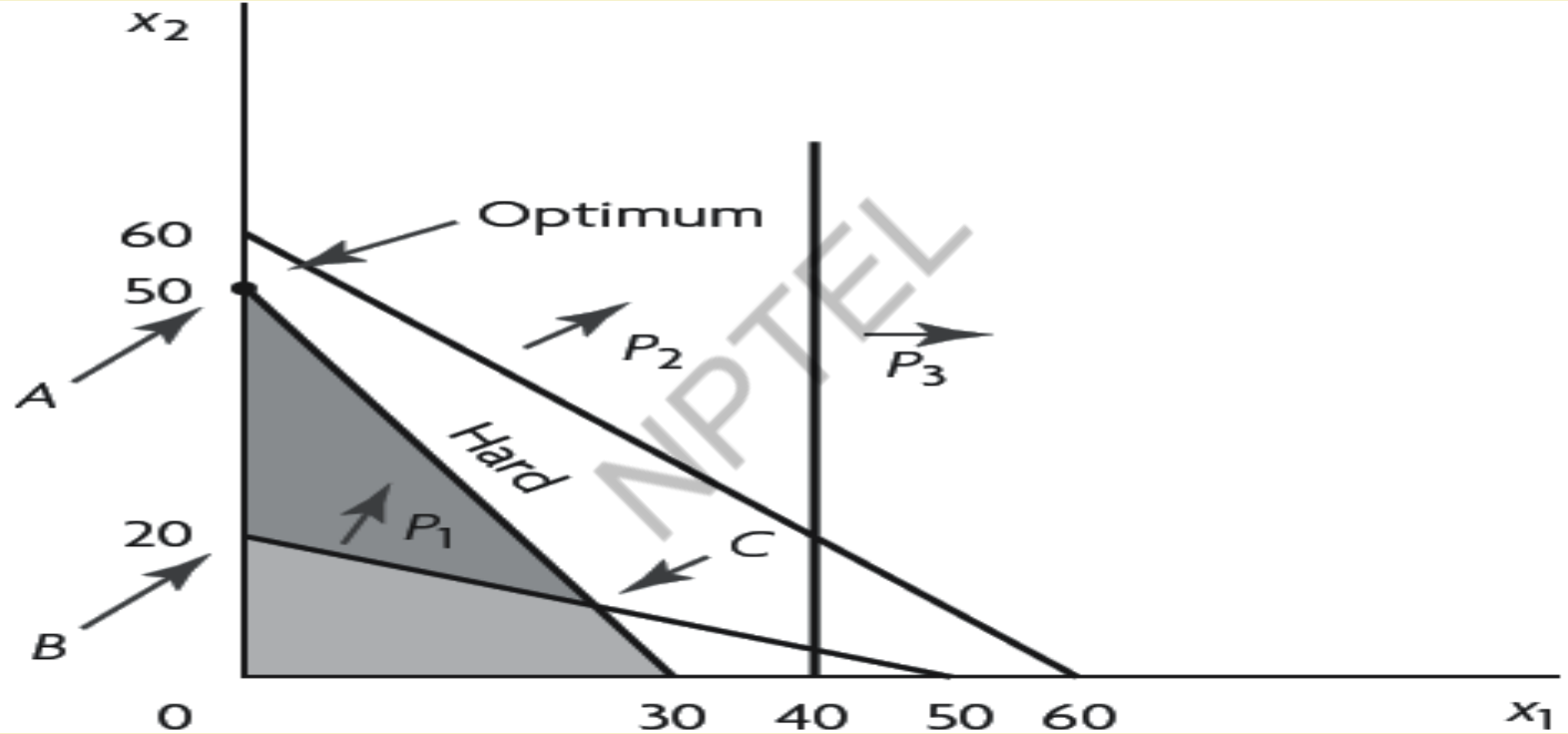
NPTEL

The Second Goal Is Added to the Graph



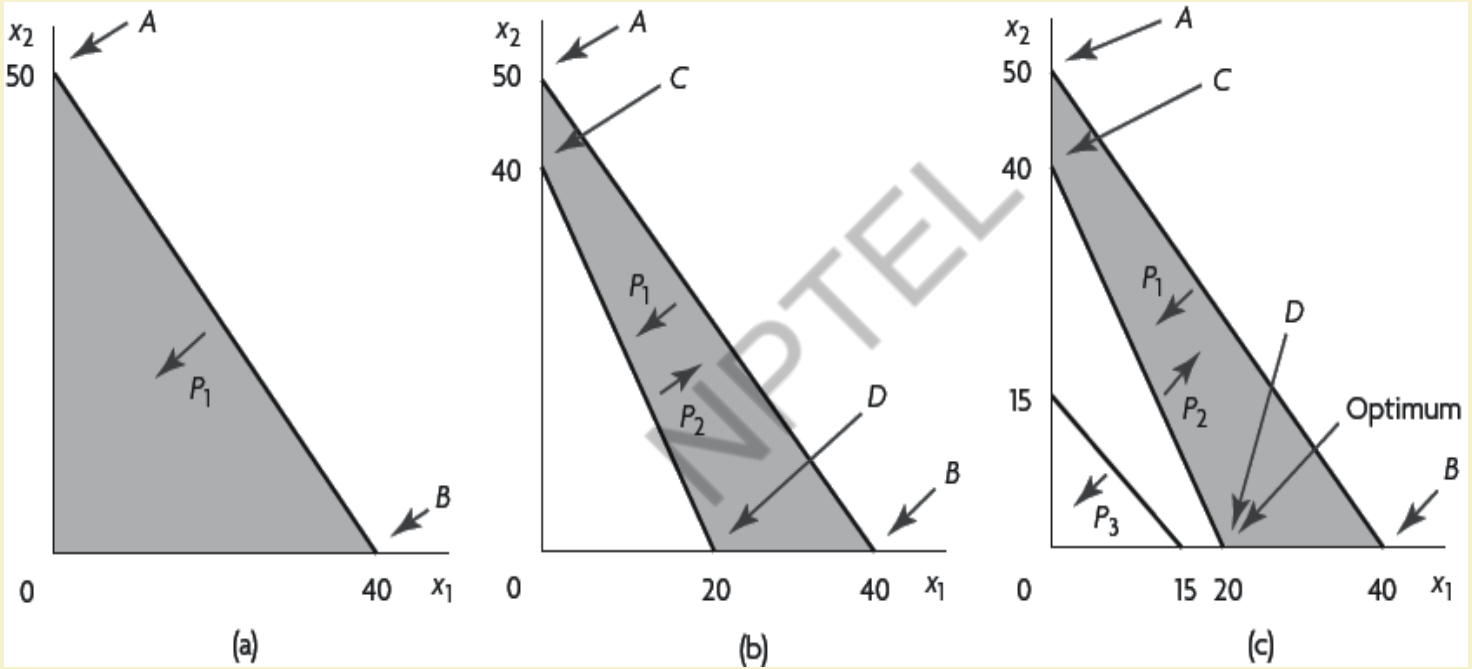
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The Third Goal Is Added, but It Doesn't Change the Solution



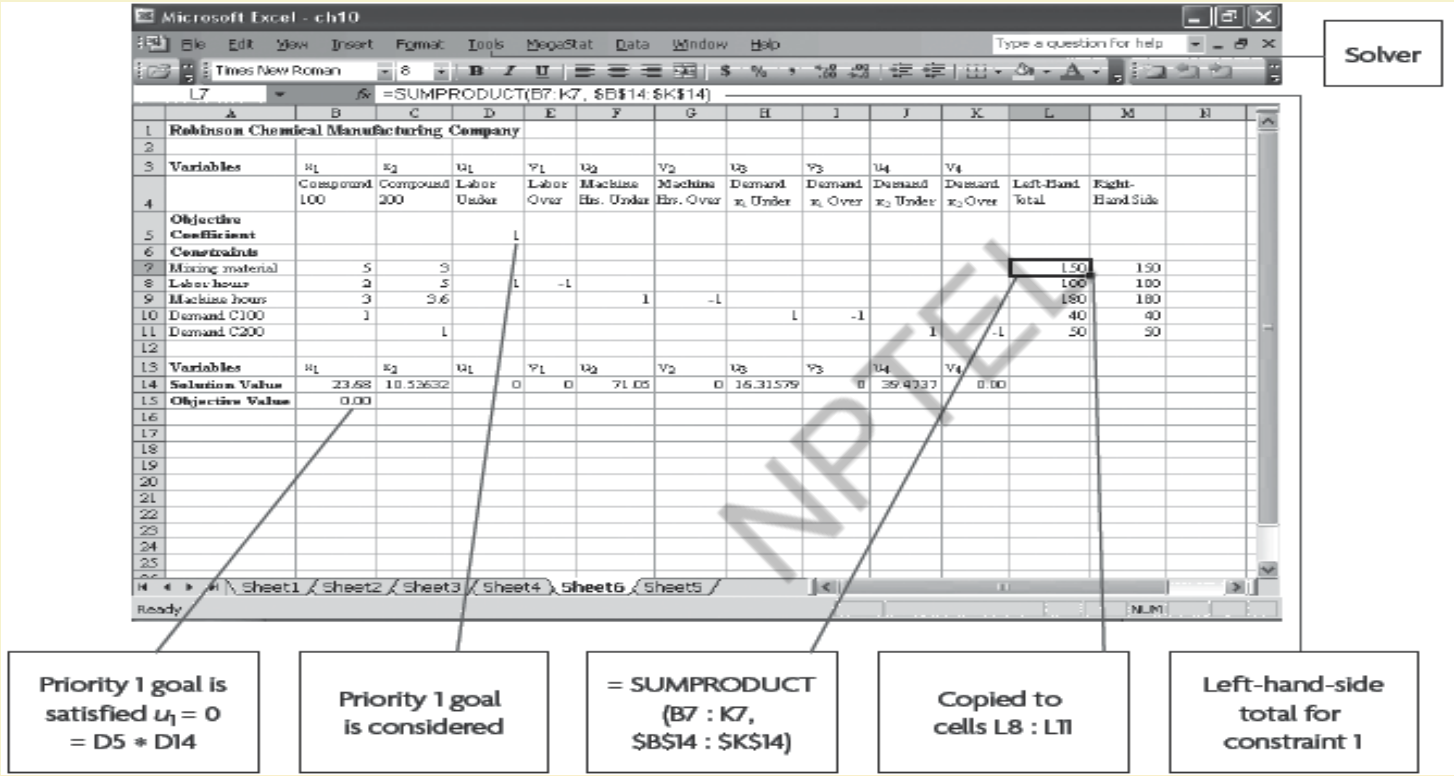
NPTEL

Plotting the Acceptable Region



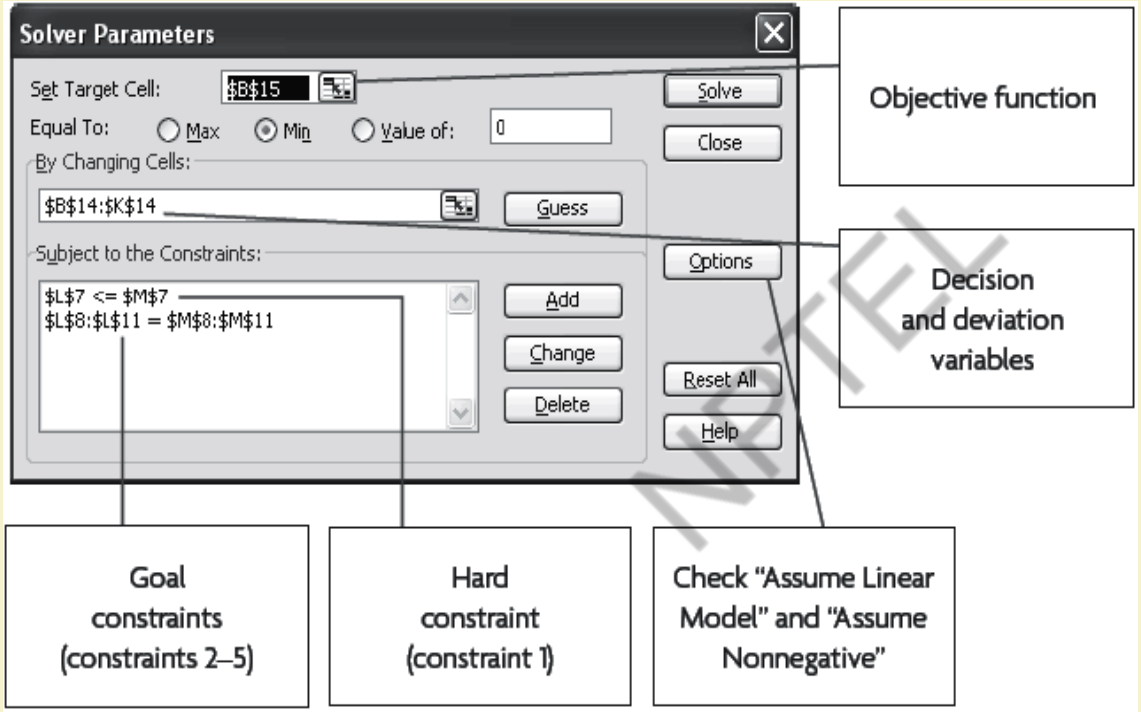
NPTEL

Excel Worksheet for Robinson Chemical Company Problem



Considering Only Priority 1 (Minimizing Labor Underutilization) Goal

Parameters Screen for Robinson Chemical Company Problem



Considering Only Priority 1 (Minimizing Labor Underutilization) Goal

NPTEL

Excel Worksheet for Robinson Chemical Company Problem

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**Robinson Chemical Manufacturing Company**													
2														
3	**Variables**	x_1	x_2	u_1	v_1	u_2	v_2	u_3	v_3	u_4	v_4			
4		Compound 100	Compound 200	Labor Under	Labor Over	Machine Hrs. Under	Machine Hrs. Over	Demand x_1 Under	Demand x_1 Over	Demand x_2 Under	Demand x_2 Over	Left-Hand Total	Right-Hand Side	
5	**Objective Coefficient**			0		1								
6	**Constraints**													
7	Mixing material	5	3									150	150	
8	Labor hours	2	5	1	-1							100	100	
9	Machine hours	3	3.6			1	-1					180	180	
10	Demand C100	1						1	-1			40	40	
11	Demand C200		1							1	-1	50	50	
12														
13	**Variables**	X_1	X_2	u_1	v_1	u_2	v_2	u_3	v_3	u_4	v_4			
14	**Solution Value**	0.00	50	0	150	0.00	0	40	0	3E-10	0.00			
15	**Objective Value**	0.00												
16														
17														
18														
19														
20														
21														
22														
23														
24														
25														

Sheet1 / Sheet2 / Sheet3 / Sheet4 / **Sheet6** / Sheet5

Ready NUM

Priority 2 goal is satisfied $u_2 = 0$ = F5 * F14

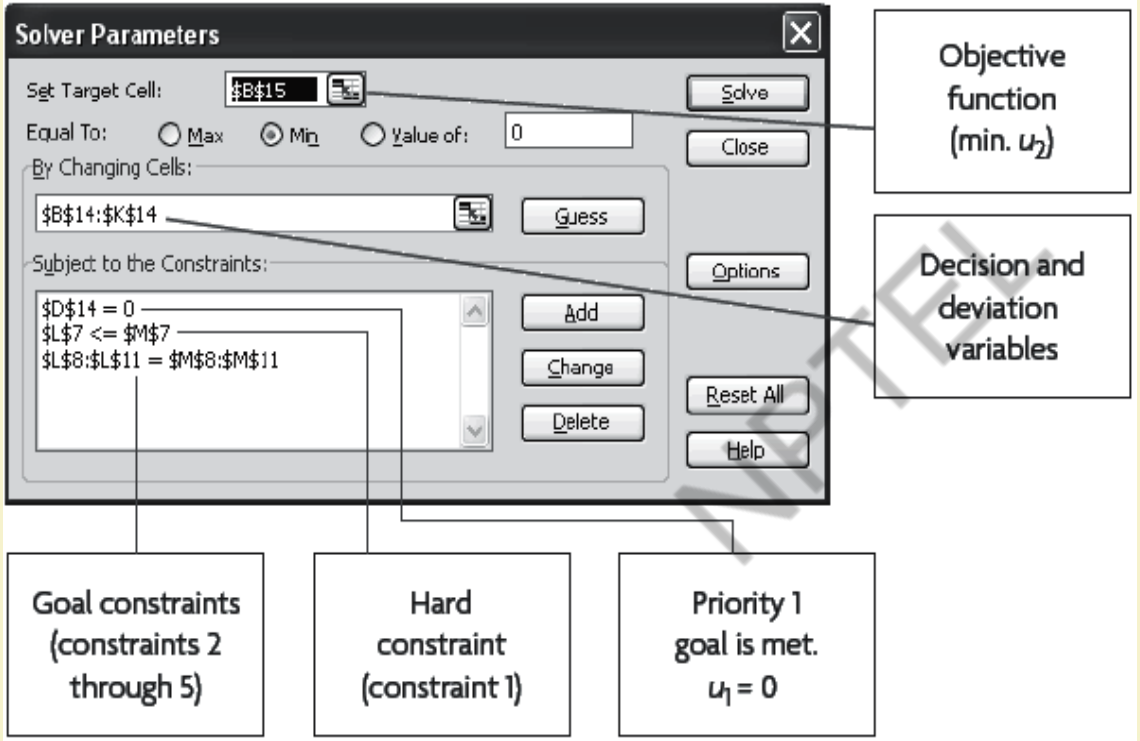
Priority 2 goal is considered, and priority 1 goal is satisfied $u_1 = 0$

= SUMPRODUCT (B7 : K7, $B$14 : $K$14) copied to cells L8 : L11

Considering Priority 1 (Minimizing Labor Underutilization) and Priority 2 (Minimizing Machine Hour Underutilization) Goals

NPTEL

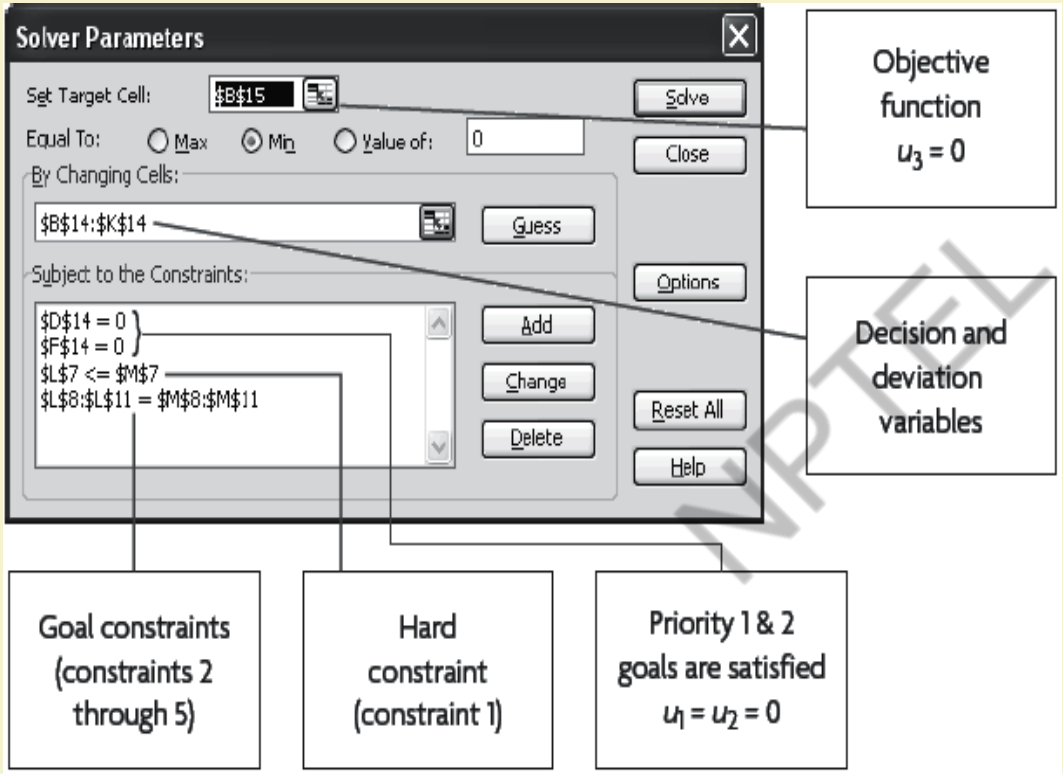
Excel Solver Parameters for Robinson Chemical Company Problem



Considering Priority 2 (Minimizing Machine Hours Underutilization) Goal While Ensuring That Priority 1 Goal Is Satisfied

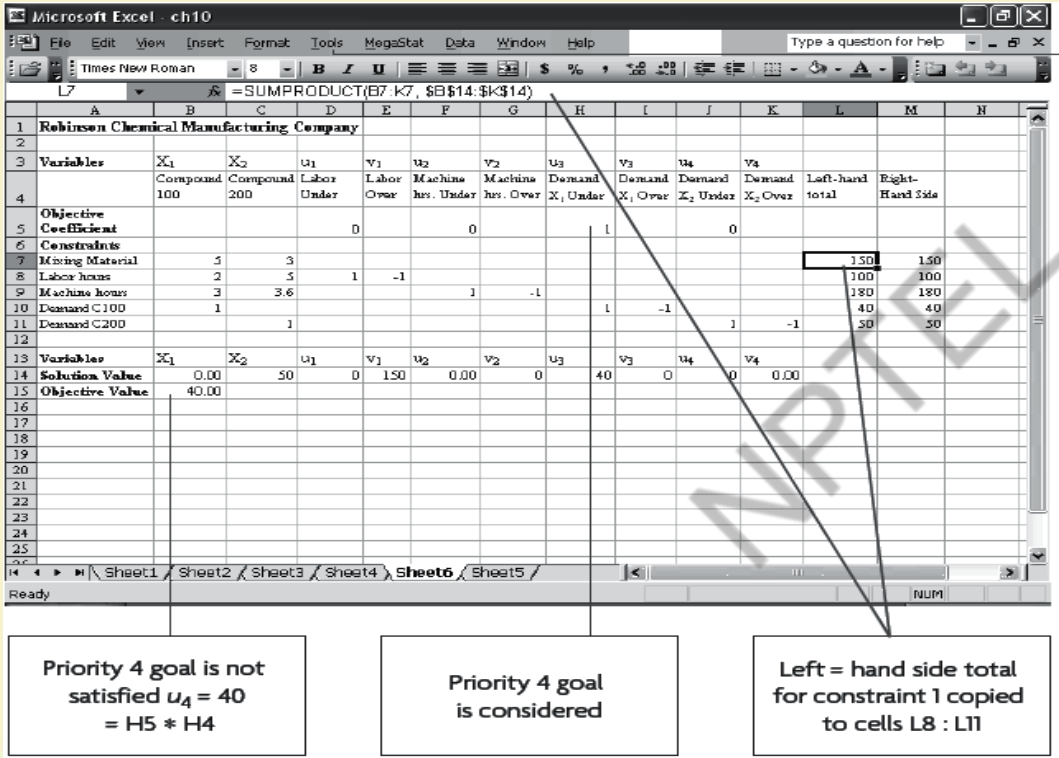
NPTEL

Excel Solver Parameters for Robinson Chemical Company Problem



Considering Priority 3 (Minimizing the Unsatisfied Demand for Compound 200) Goal While Ensuring That the Priority 1 and Priority 2 Goals Are Still Satisfied

Excel Worksheet for Robinson Chemical Company Problem

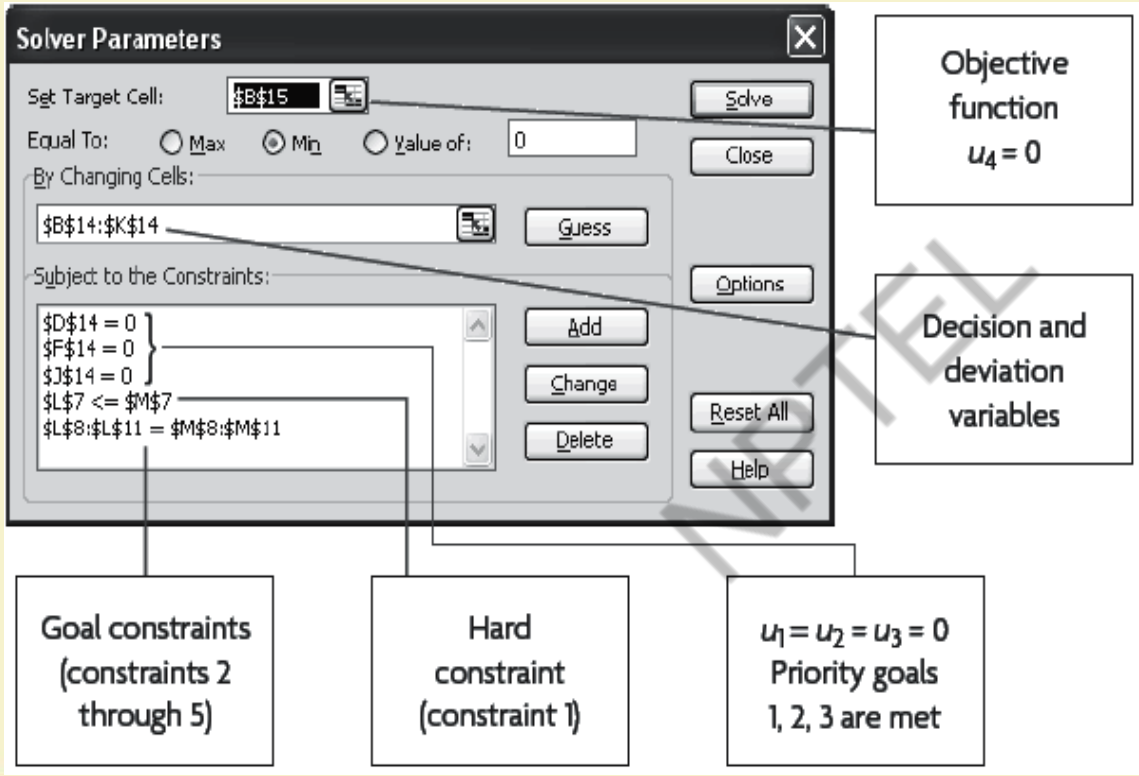


Solver

Considering Priority 4 Goal (Minimizing Unsatisfied Demand for Compound 200) While Ensuring That Priority 1 (Minimizing Labor Underutilization), Priority 2 (Minimizing Machine Hour Underutilization), and Priority 3 (Unsatisfied Demand for Compound 200) Goals Are Satisfied

NPTEL

Excel Solver Parameters for Robinson Chemical Company Problem



Considering Priority 4 (Minimizing the Unsatisfied Demand for Compound 100) Goal While Ensuring that Priority 1, priority 2, and Priority 3 Goals Are Still Satisfied

NPTEL

Excel Worksheet for Robinson Chemical Company Problem

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1	**Robinson Chemical Manufacturing Company**													
2														
3	**Variables**	x_1	x_2	u_1	v_1	u_2	v_2	u_3	v_3	u_4	v_4			
4		Compound 100	Compound 200	Labor Under	Labor Over	Machine Hrs. Under	Machine Hrs. Over	Demand x_1 Under	Demand x_1 Over	Demand x_2 Under	Demand x_2 Over	Left-Hand Total	Right-Hand Side	
5	**Objective Coefficient**			35	5	25		15		20				
6	**Constraints**													
7	Mixing material	5	3									150	150	
8	Labor hours	2	5	1	-1							100	100	
9	Machine hours	3	3.6			1	-1					180	180	
10	Demand C100	1						1	-1			40	40	
11	Demand C200		1							1	-1	50	50	
12														
13	**Variables**	x_1	x_2	u_1	v_1	u_2	v_2	u_3	v_3	u_4	v_4			
14	**Solution Value**	0.00	50	0	150	0.00	0	40	0	0	0.00			
15	**Objective Value**	1350.00												
16														
17														
18														
19														
20														
21														
22														
23														
24														
25														

Sheet1 / Sheet2 / Sheet3 / Sheet4 / **Sheet6** / Sheet5

Ready NUM

NPTEL

Cell	Copied to	Formula
L7	L8 : L11	= SUMPRODUCT (B7 : K7, $B$14 : $K$14)
B15		= SUMPRODUCT (B14 : K14, B5 : K5)

Business Analytics for Management Decision

Nonlinear Programming

Rudra P. Pradhan
Vinod Gupta School of Management
IIT KHARAGPUR

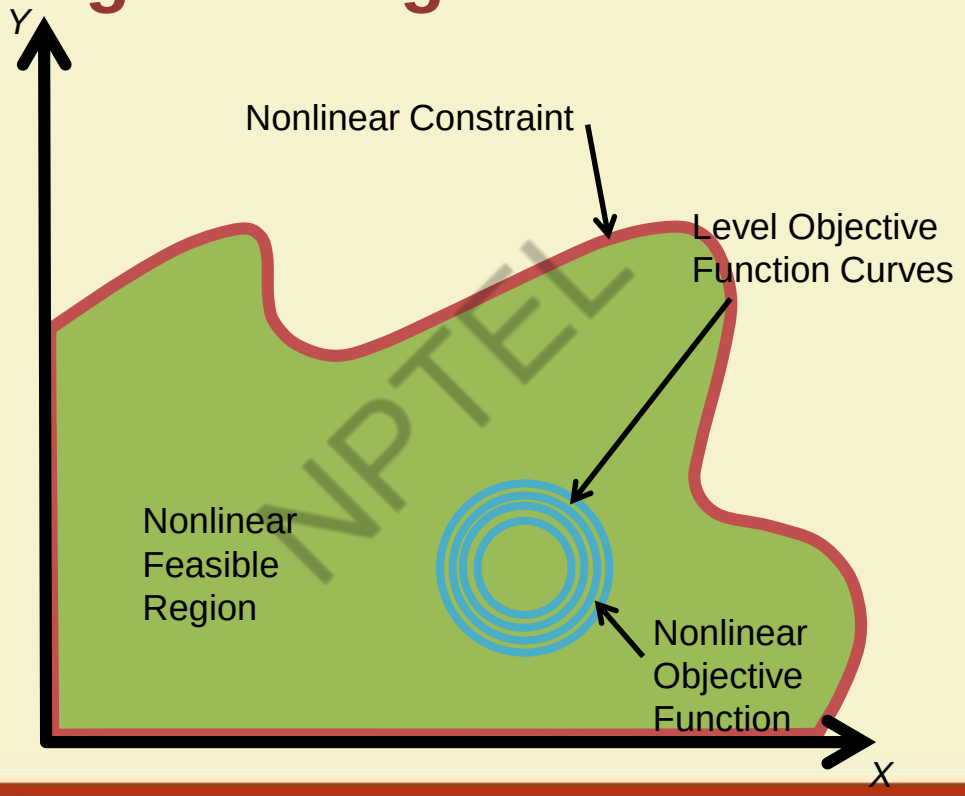
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Nonlinear Programming

- Objective function and one or more constraints may be nonlinear
- Difficult to solve
- Optimal solution not necessarily a corner point
- Local verses global optimization
- Starting point influences solution

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Nonlinear Programming



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Nonlinear Programming

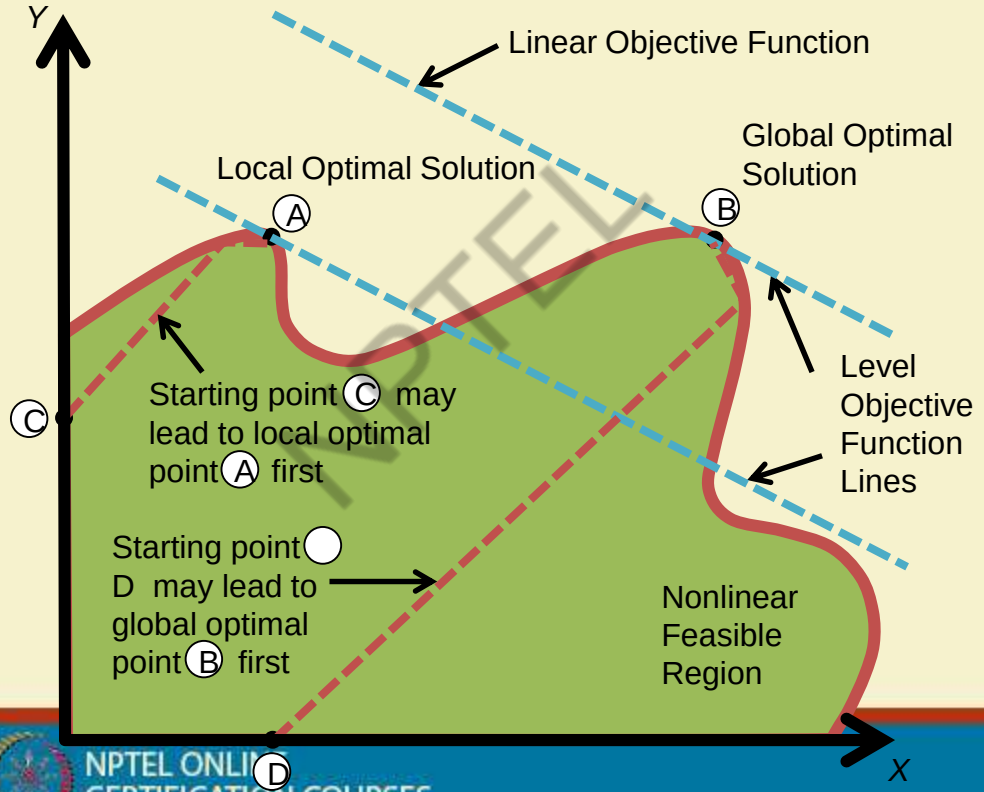


Figure 6.3

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Nonlinear Programming: Pickens Memorial Hospital

- Profit contribution and decision variables

Profit contribution per medical patient = \$45 + \$2M
Profit contribution per surgical patient = \$70 + \$3S + \$2M
Profit contribution per pediatric patient = \$60 + \$3P

where

M = number of medical patients admitted
S = number of surgical patients admitted
P = number of pediatric patients admitted

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Nonlinear Programming: Patient mix problem

	MEDICAL	SURGICAL	PEDIATRIC	AVAILABILITY
Number of x-rays/patient	1	3	1	560 x-rays
Marketing budget/patient	$3	$5	$3.5	$1,000
Number of lab tests/patient	3	3	3	140 hrs

Time required per lab test (in hours) = $0.2 + 0.001M$

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Nonlinear Programming: Pickens Memorial Hospital

Objective function

Maximize profit = ($45 + +$$M$) x M + ($70 + 3S$ + 2M$) x S
+ ($60 + 3P$) x P

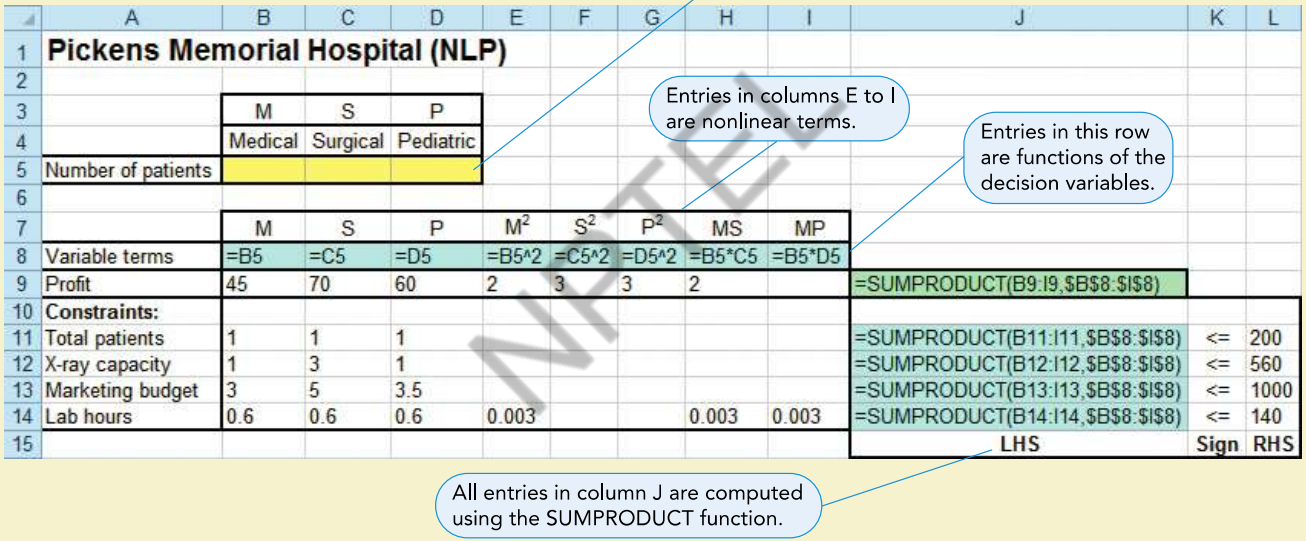
= 45M$ + 2M^2$ + 70S$ + 3S^2$ + 2MS$
+ 60P$ + 3P^2$

subject to

$M + S + P$	≤	200	(total patient capacity)
$M + 3S + P$	≤	560	(x-ray capacity)
$3M + 5S + 3.5P$	≤	1,000	(marketing budget, $)
$(0.2 + 0.001M) \times (3M + 3S + 3P)$	≤	140	(lab capacity, hrs)
M, S, P	≥	0	

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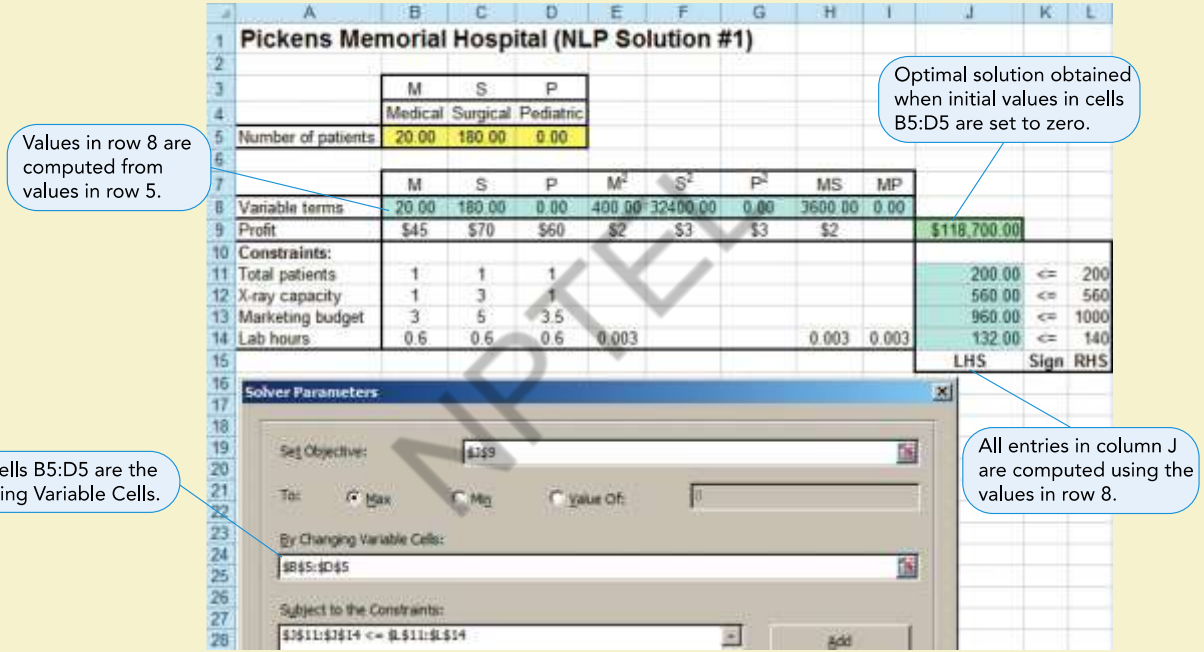
Solution: Pickens Memorial Hospital



	A	B	C	D	E	F	G	H	I	J	K	L
1	**Pickens Memorial Hospital (NLP)**											
2												
3		M	S	P								
4		Medical	Surgical	Pediatric								
5	Number of patients											
6												
7		M	S	P	M^2	S^2	P^2	MS	MP			
8	Variable terms	=B5	=C5	=D5	=B5^2	=C5^2	=D5^2	=B5*C5	=B5*D5			
9	Profit	45	70	60	2	3	3	2		=SUMPRODUCT(B9:I9,$B$8:$I$8)		
10	**Constraints:**											
11	Total patients	1	1	1						=SUMPRODUCT(B11:I11,$B$8:$I$8)	<=	200
12	X-ray capacity	1	3	1						=SUMPRODUCT(B12:I12,$B$8:$I$8)	<=	560
13	Marketing budget	3	5	3.5						=SUMPRODUCT(B13:I13,$B$8:$I$8)	<=	1000
14	Lab hours	0.6	0.6	0.6	0.003			0.003	0.003	=SUMPRODUCT(B14:I14,$B$8:$I$8)	<=	140
15										LHS	Sign	RHS

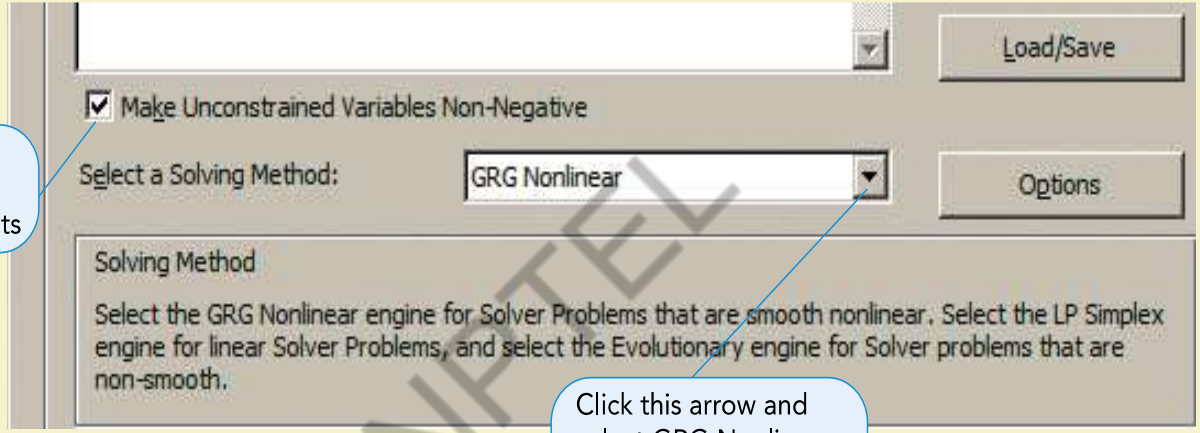
NPTEL

Solution: Pickens Memorial Hospital



NPTEL

Solution: Pickens Memorial Hospital

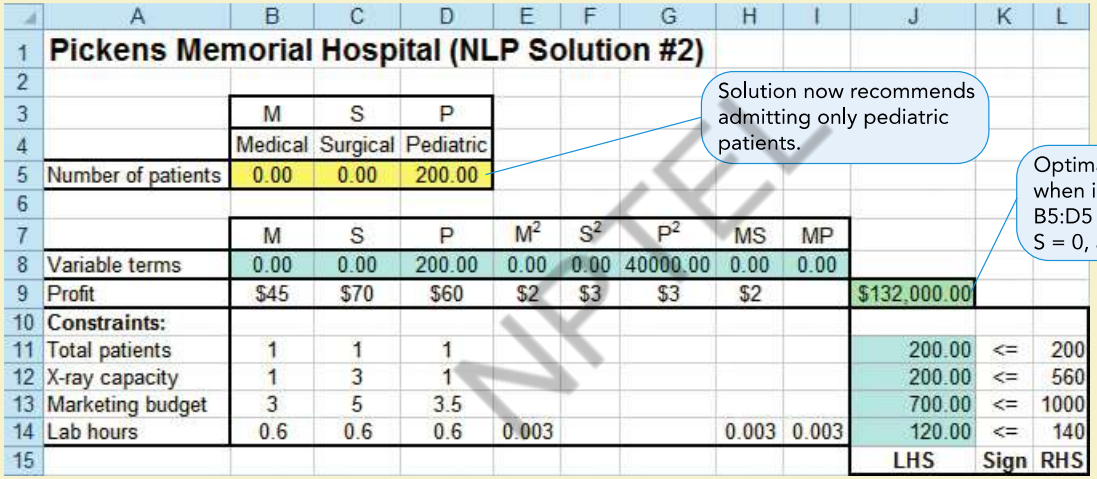


Checking this box enforces the non-negativity constraints

Click this arrow and select GRG Nonlinear as the solving method.

NPTEL

Solution: Pickens Memorial Hospital



	A	B	C	D	E	F	G	H	I	J	K	L
1	**Pickens Memorial Hospital (NLP Solution #2)**											
2												
3		M	S	P								
4		Medical	Surgical	Pediatric								
5	Number of patients	0.00	0.00	200.00								
6												
7		M	S	P	M^2	S^2	P^2	MS	MP			
8	Variable terms	0.00	0.00	200.00	0.00	0.00	40000.00	0.00	0.00			
9	Profit	$45	$70	$60	$2	$3	$3	$2		$132,000.00		
10	**Constraints:**											
11	Total patients	1	1	1						200.00	<=	200
12	X-ray capacity	1	3	1						200.00	<=	560
13	Marketing budget	3	5	3.5						700.00	<=	1000
14	Lab hours	0.6	0.6	0.6	0.003			0.003	0.003	120.00	<=	140
15										LHS	Sign	RHS

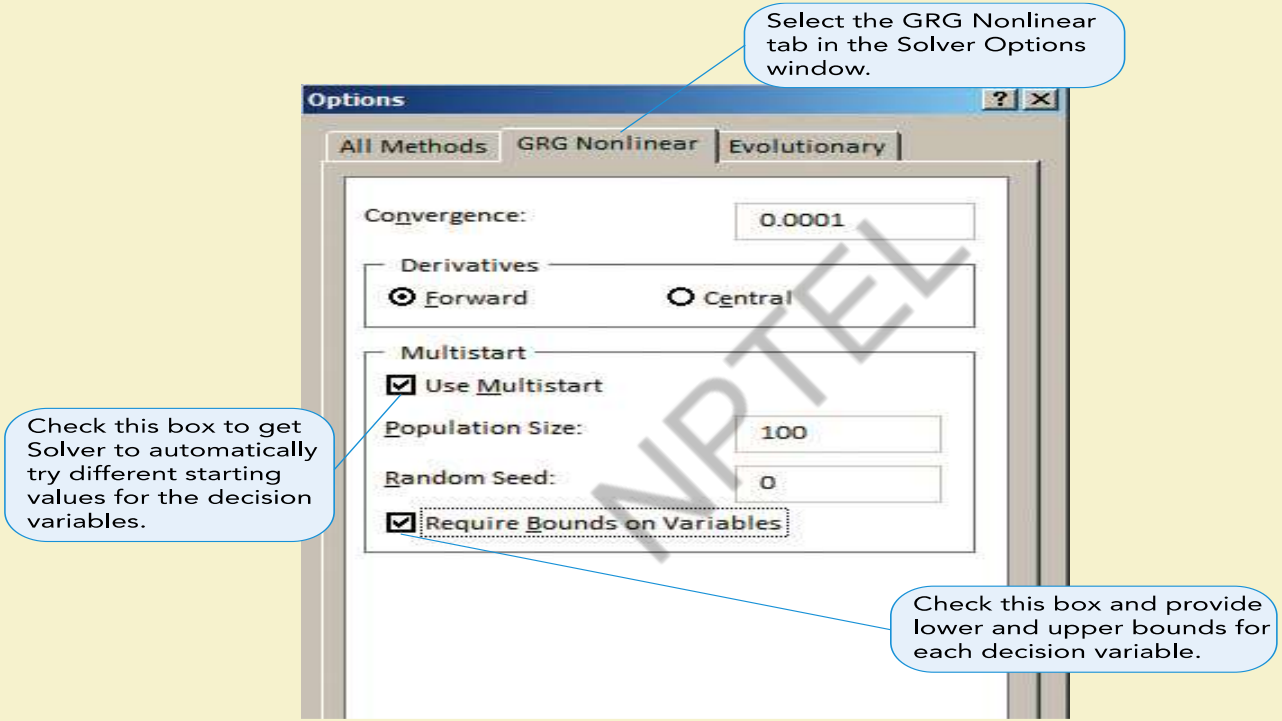
Solution now recommends admitting only pediatric patients.

Optimal solution obtained when initial values in cells B5:D5 are set to M = 100, S = 0, and P = 100.

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Screenshot 6-8C

Solution: Pickens Memorial Hospital



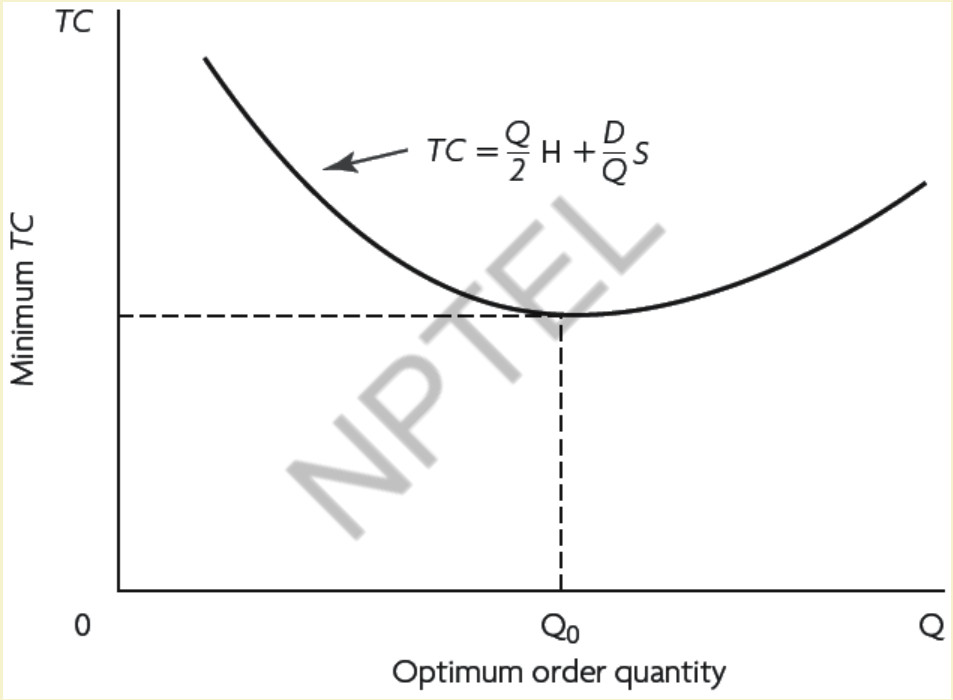
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Nonlinear Programming

- Characteristics of Nonlinear Models
  - Have one or more nonlinear components which cannot be handled by linear programming techniques.
  - Require nonlinear programming procedures which involves obtaining the first derivative of the objective function, finding all solutions for which the first derivative is equal to zero, and then checking second derivative conditions to ascertain the nature of the zero points (e.g., a local maximum or a local minimum).

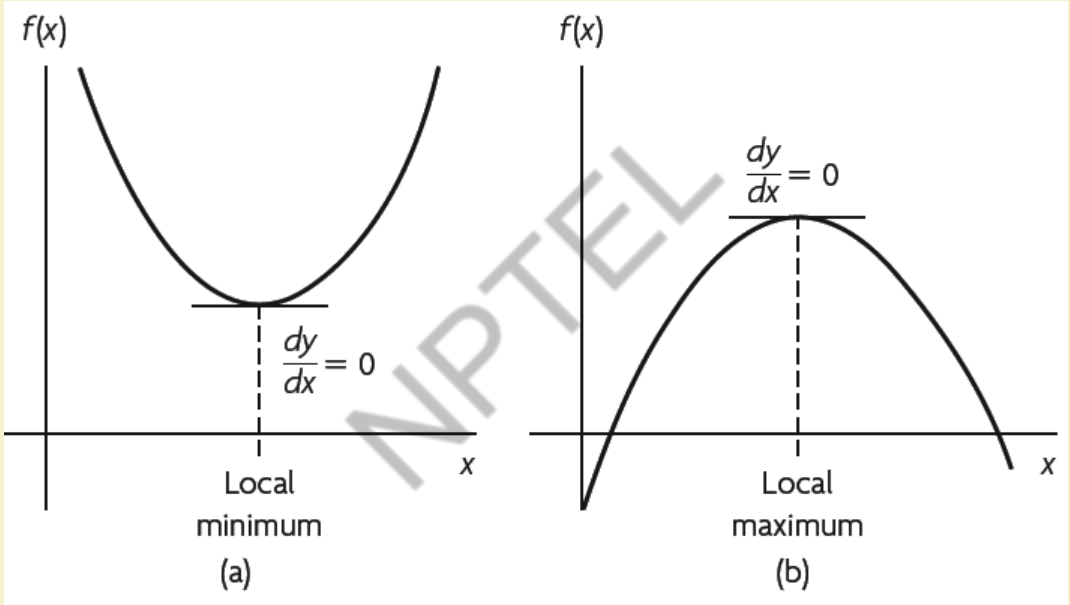
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U-Shaped Cost Function



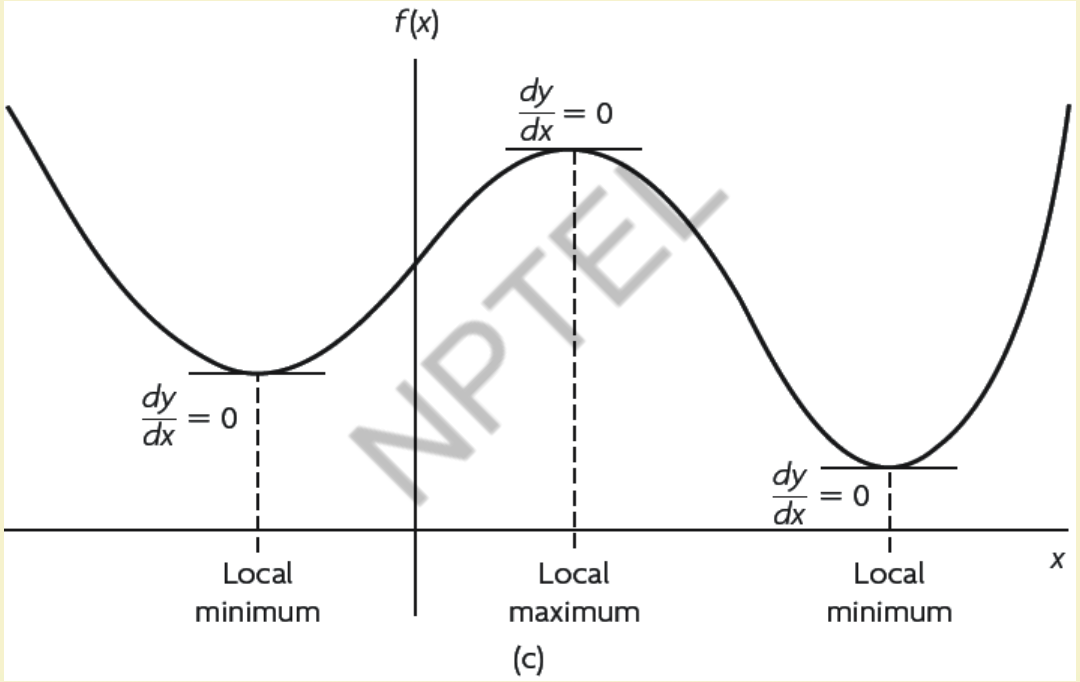
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Illustrations of Local Maximum and Local Minimum Points



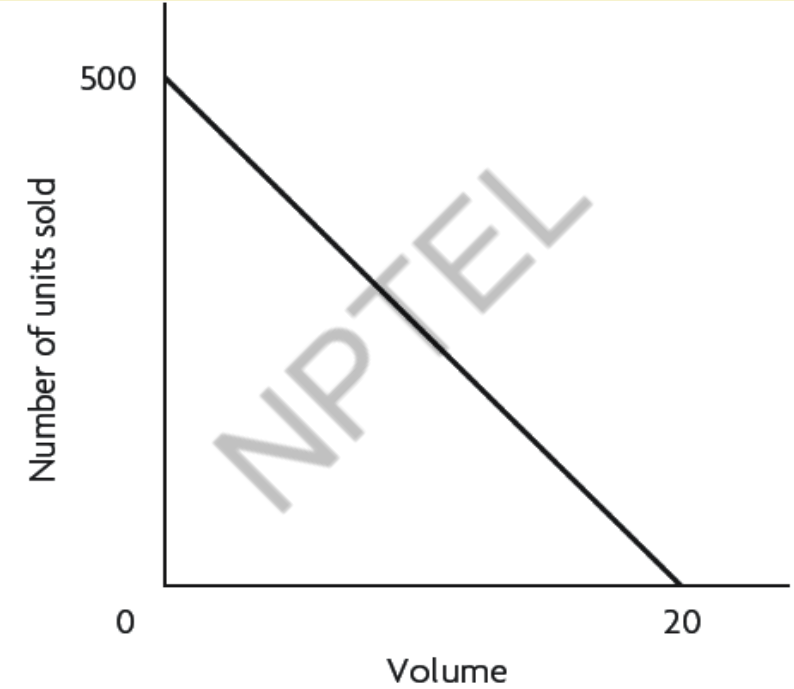
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Illustrations of Local Maximum and Local Minimum Points (Contd.)



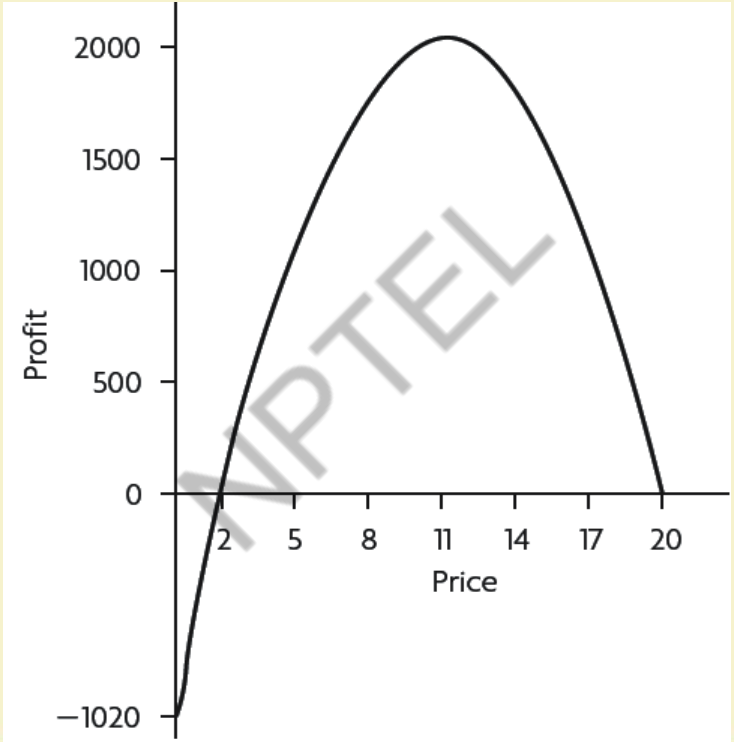
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Volume–Price Relationship for the X-Tech Inc. Problem



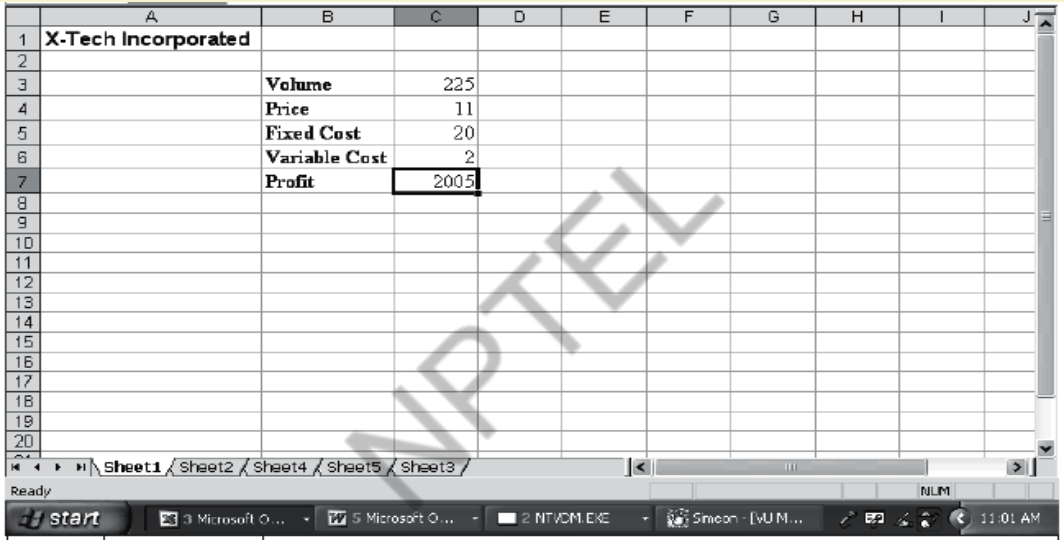
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Graph of the Profit Function for the X-Tech Inc. Problem



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Worksheet for the One-Decision-Variable, Unconstrained Problem (X-Tech Inc.)

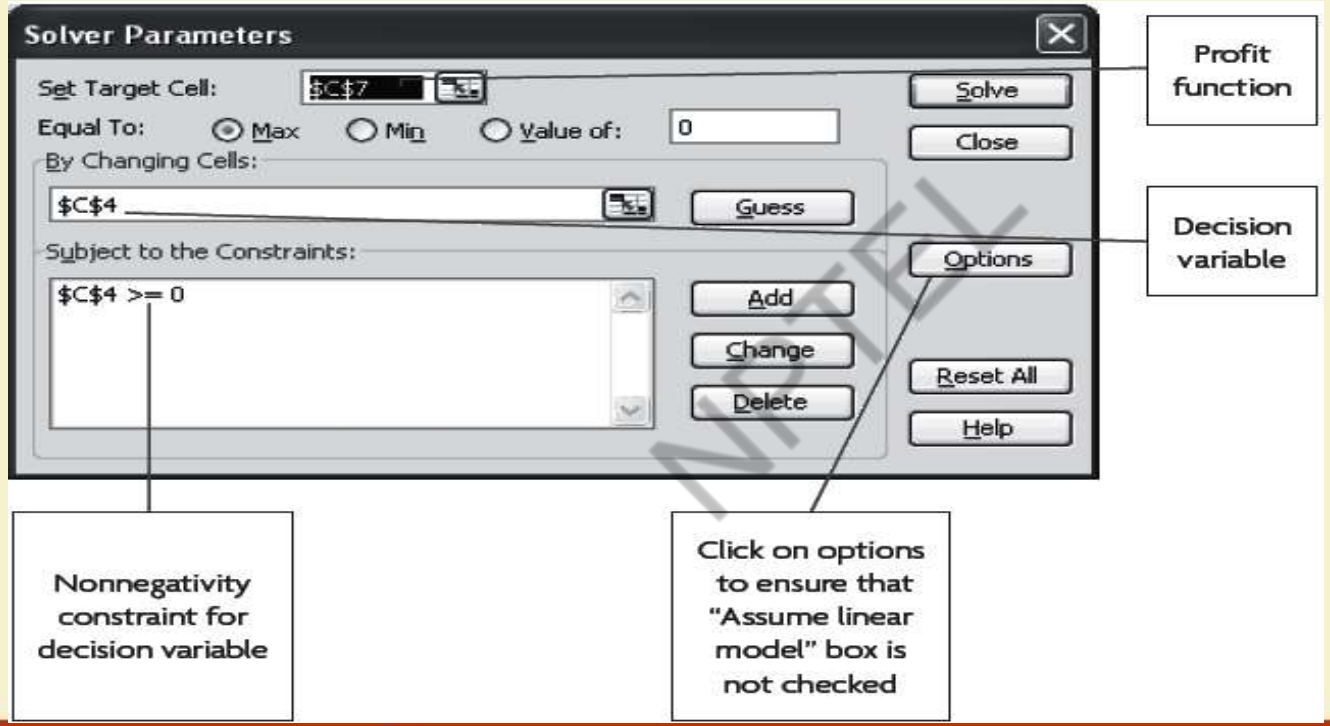


	A	B	C	D	E	F	G	H	I	J
1	**X-Tech Incorporated**									
2										
3		**Volume**	225							
4		**Price**	11							
5		**Fixed Cost**	20							
6		**Variable Cost**	2							
7		**Profit**	2005							
8										
9										
10										
11										
12										
13										
14										
15										
16										
17										
18										
19										
20										

Cell	Copied to	Formula
C7		= C4 * C3 − C3 * C6 − C5
C3		= 500 − 25 * C4

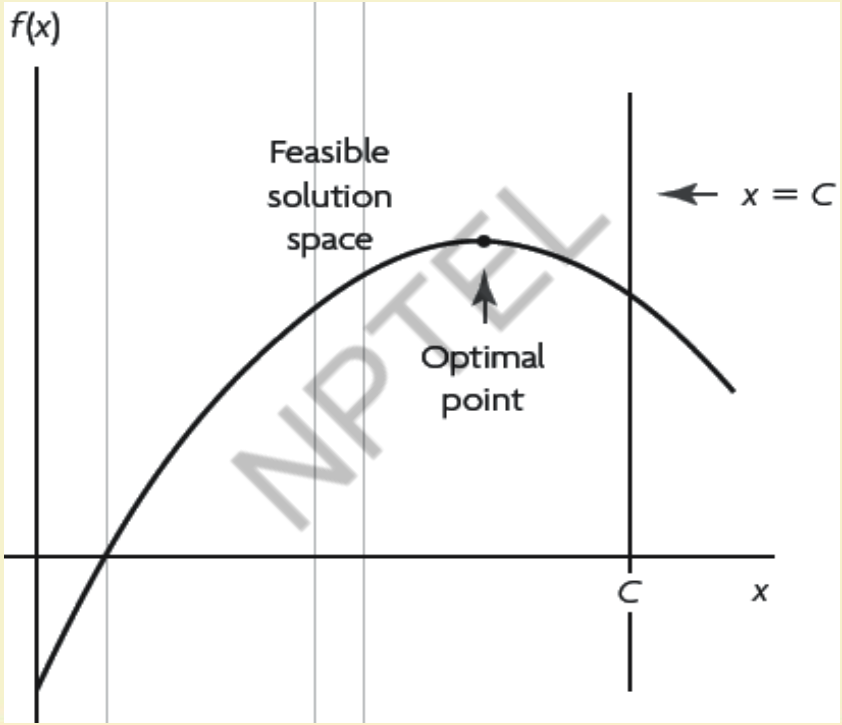
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Parameter Specification for the One-Decision-Variable, Unconstrained Problem (X-Tech Inc.)



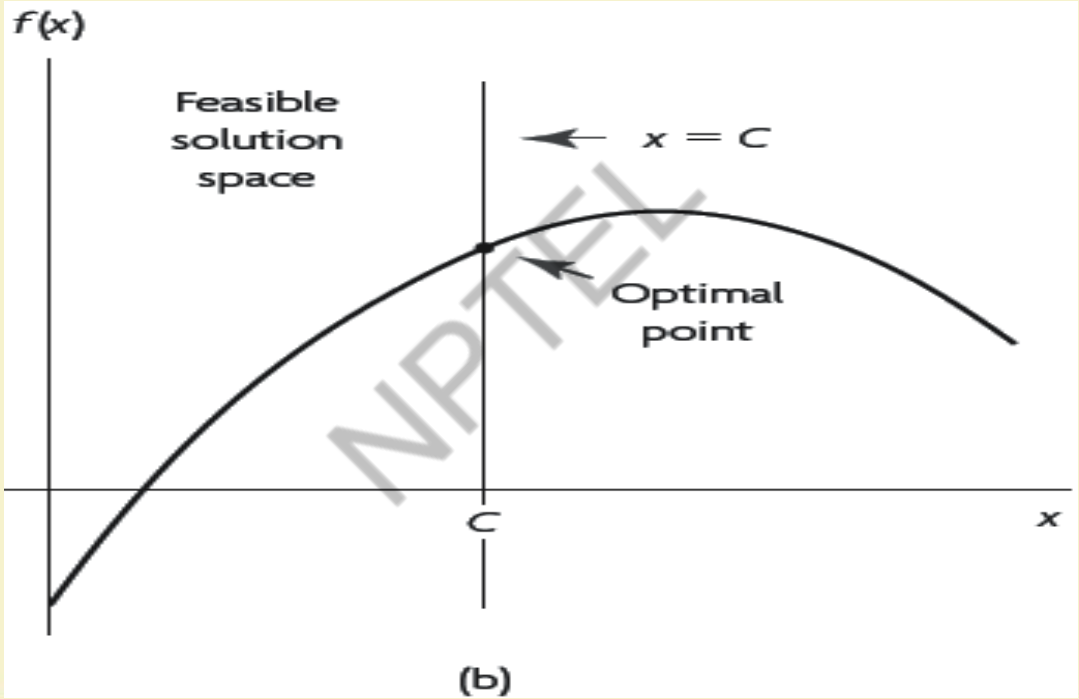
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The Optimal Point Lies within the Feasible Solution Space



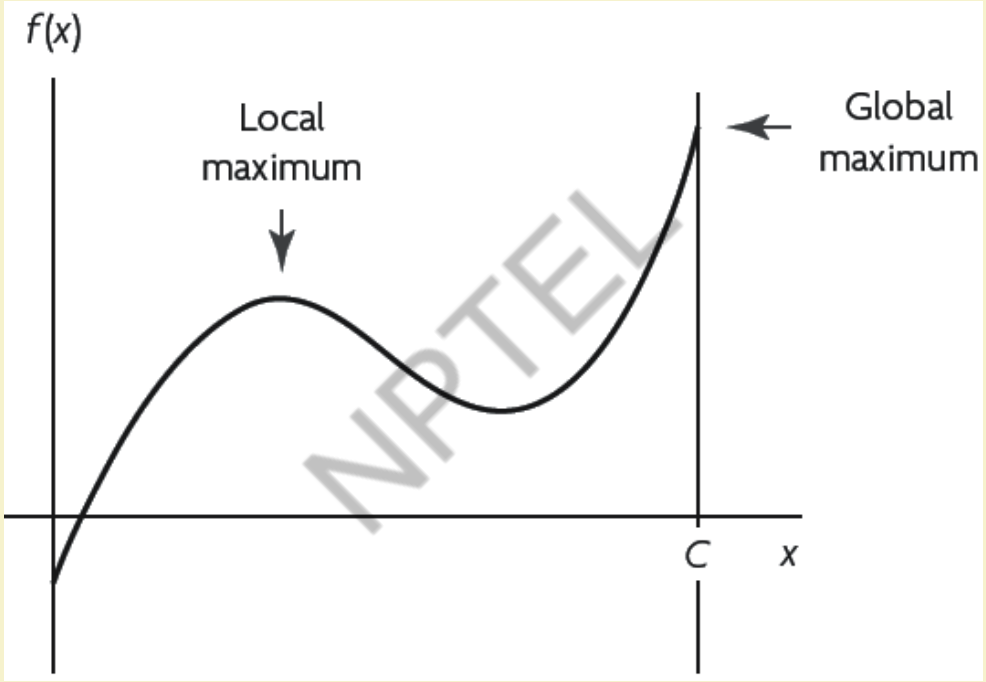
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The Optimal Point is on the Boundary of the Feasible Solution Space



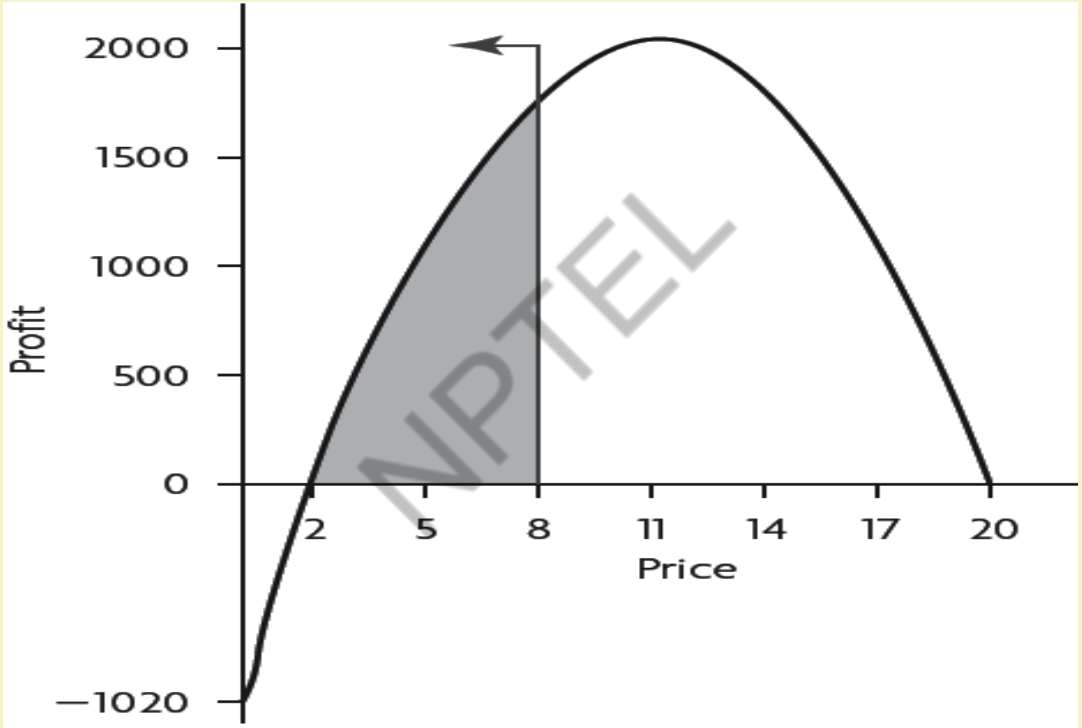
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The Global Maximum Can Be at the Point Where x = C



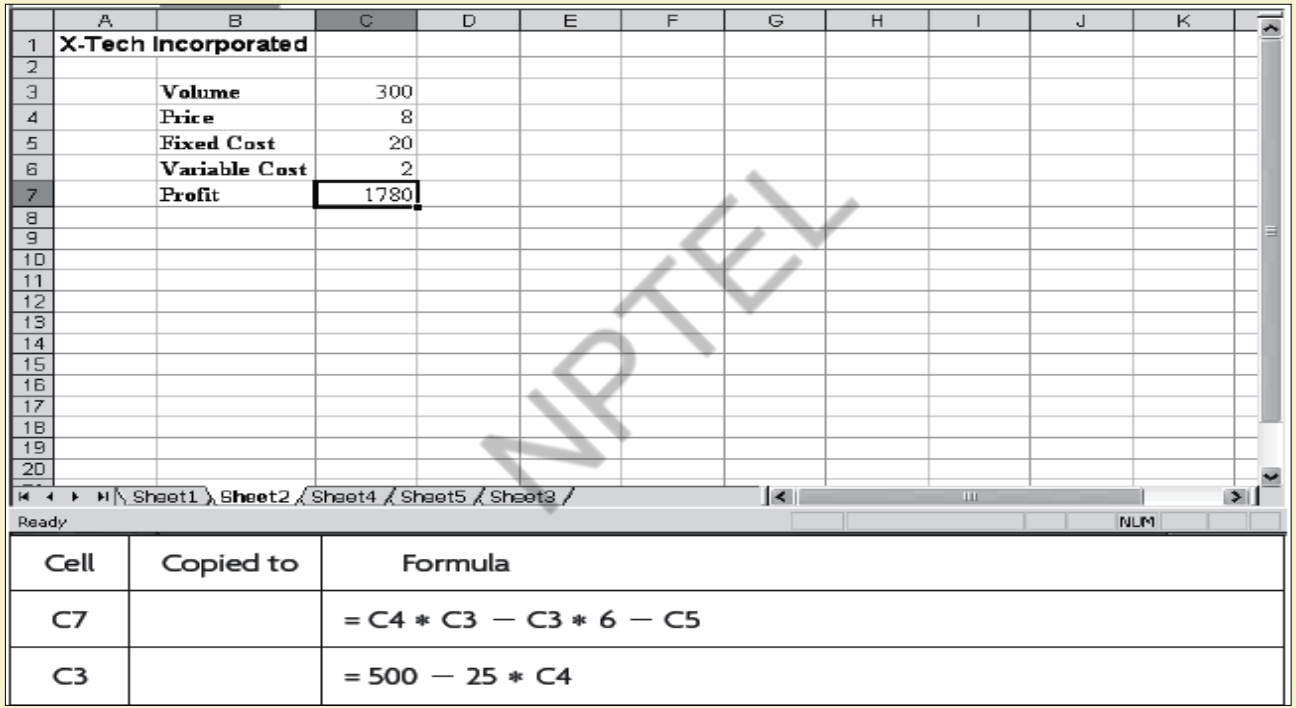
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Graph of the Profit Function and the Feasible Space for the X-Tech Inc. Problem



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Worksheet for the One-Decision-Variable, Constrained Problem (X-Tech Inc.)



	A	B	C	D	E	F	G	H	I	J	K
1	X-Tech Incorporated										
2											
3		Volume	300								
4		Price	8								
5		Fixed Cost	20								
6		Variable Cost	2								
7		Profit	1780								
8											
9											
10											
11											
12											
13											
14											
15											
16											
17											
18											
19											
20											

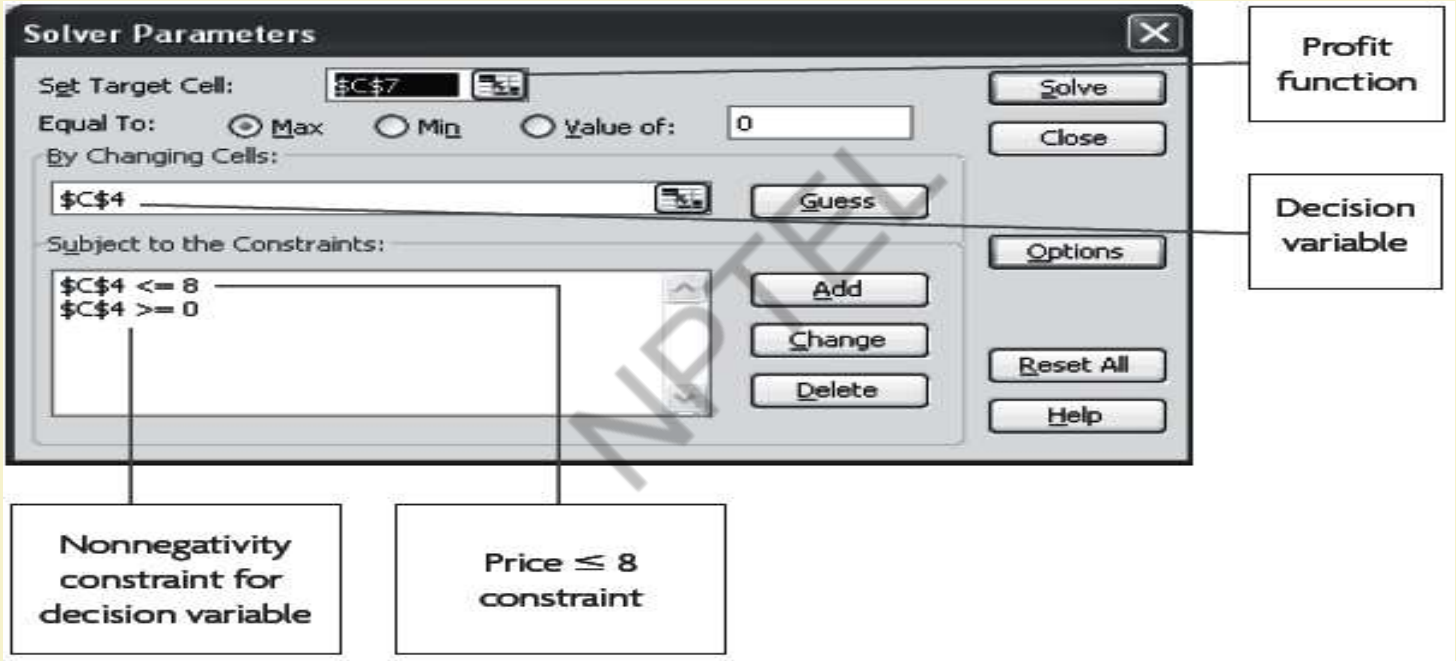
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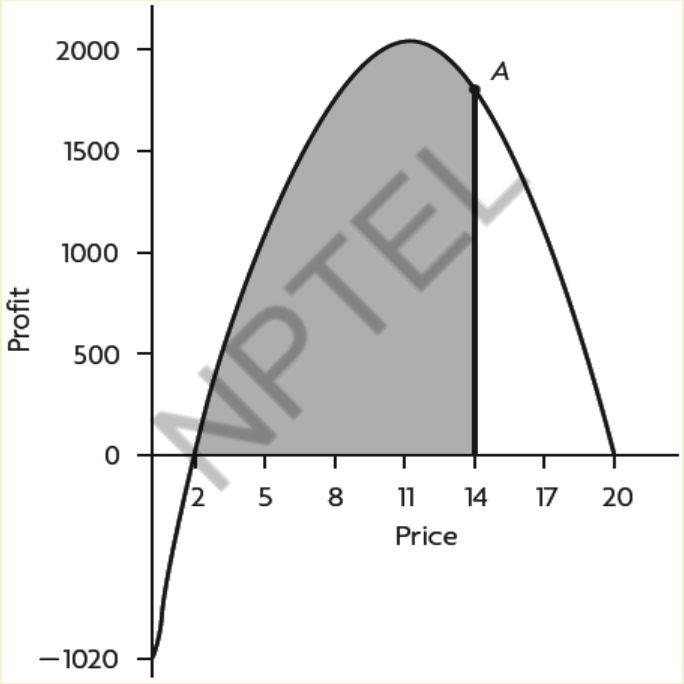
Cell	Copied to	Formula
C7		= C4 * C3 — C3 * 6 — C5
C3		= 500 — 25 * C4

Parameter Specification for the One-Decision-Variable, Constrained Problem (X-Tech Inc)



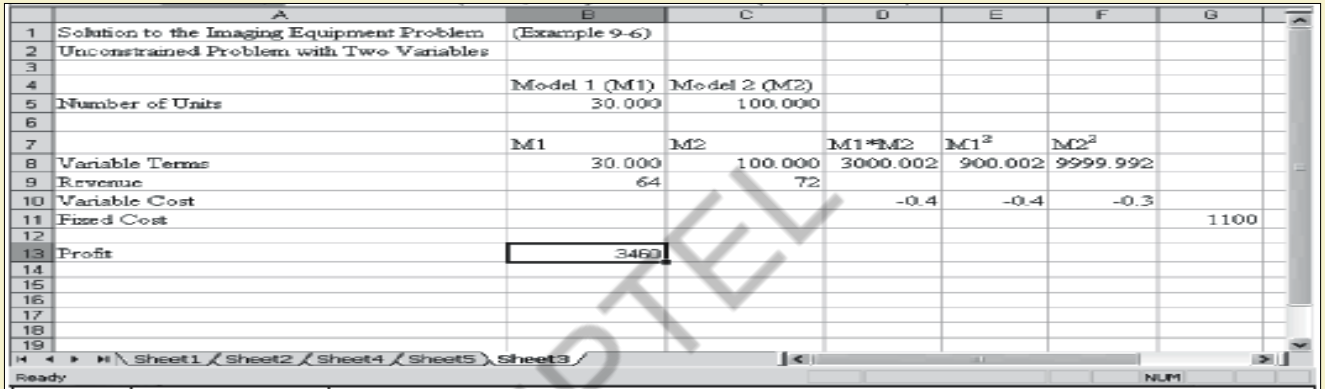
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Graph of the Profit Function and the Feasible Space for the X-Tech Inc. Problem



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Worksheet for the Two-Decision-Variable, Unconstrained Problem (Imaging Equipment)



	A	B	C	D	E	F	G
1	Solution to the Imaging Equipment Problem	(Example 9-6)					
2	Unconstrained Problem with Two Variables						
3							
4		Model 1 (M1)	Model 2 (M2)				
5	Number of Units	30.000	100.000				
6							
7		M1	M2	M1*M2	$M1^2$	$M2^2$	
8	Variable Terms	30.000	100.000	3000.002	900.002	9999.992	
9	Revenue	64	72				
10	Variable Cost			-0.4	-0.4	-0.3	
11	Fixed Cost						1100
12							
13	Profit	3460					
14							
15							
16							
17							
18							
19							

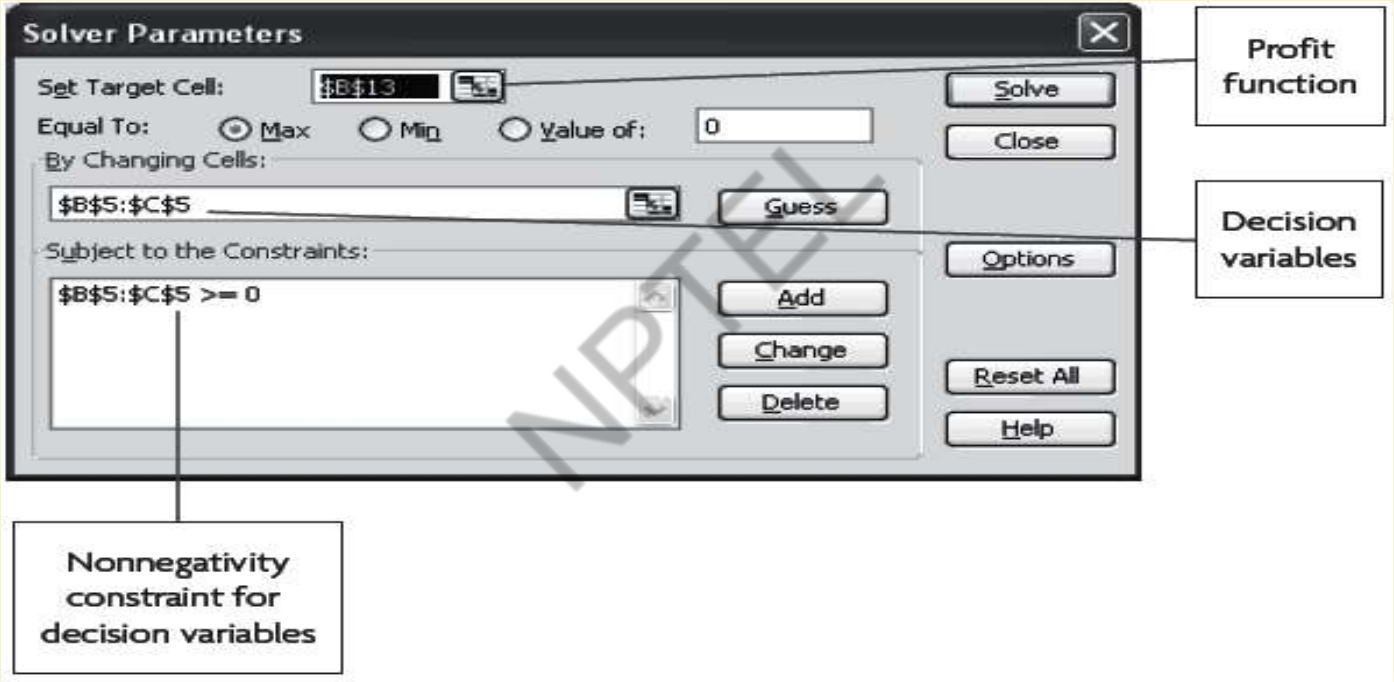
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Cell	Copied to	Formula
B8		= B5
C8		= C5
D8		= B5 * C5
E8		= B5 * B5
F8		= C5 * C5
B13		= SUM PRODUCT (B9 : C9, B8 : C8) + SUM PRODUCT (D8 : F8, D10 : F10) — G11

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Parameter Specification for the Two-Decision-Variable, Unconstrained Problem (Imaging Equipment)



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Worksheet for the Problem with Two Decision Variables and a Single Equality Constraint (Daisy-Fresh Company Deodorant

	A	B	C	D	E	F	G	H	I
1	**Solution to the Daisy Fresh Deodorant Problem (Example 9-8)**								
2	**Unconstrained Problem with Two Variables**								
3									
4		**Roll-On -X_1**	**Stick -X_2**						
5	Number of Units	80.000	66.667				Time Required	Time Available	
6									
7		X_1	X_2	X_1^2	X_2^2				
8	Variable Terms	80.000	66.667	6400.000	4444.445				
9	Revenue	3.2	4						
10	Variable Cost			-0.02	-0.03				
11	Fixed Cost					150			
12	Machine Time	20	30				3600	3600	
13	Profit	111.333							
14									
15									
16									
17									
18									

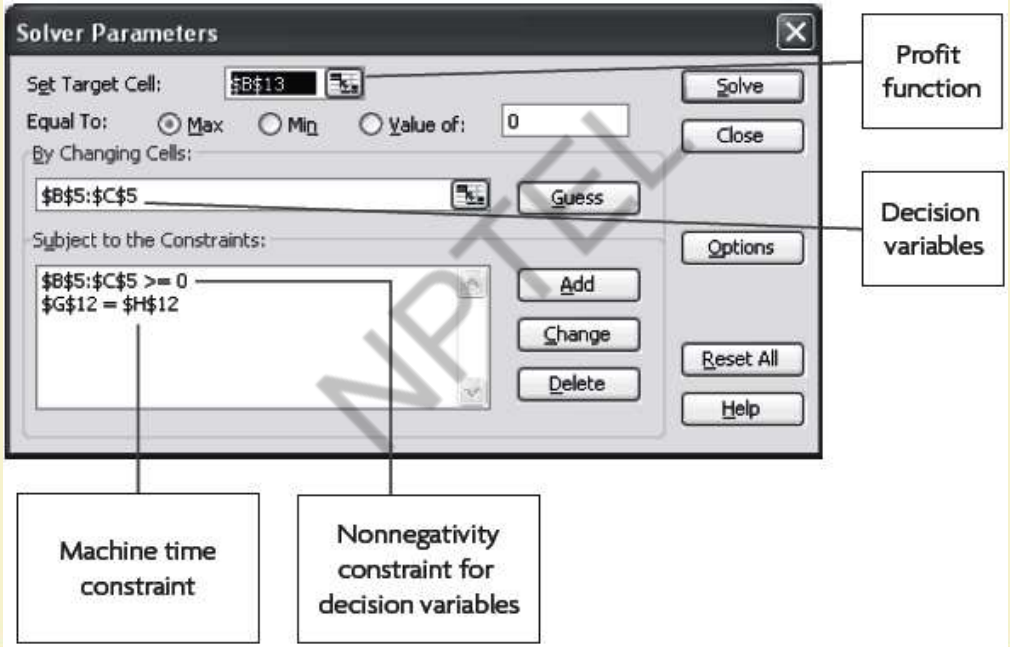
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Cell	Formula
G12	= SUMPRODUCT (B12 : C12, B5 : C5)
B8	= B5
C8	= C5
D8	= B5 * B5
E8	= C5 * C5
B13	= SUMPRODUCT (B9 : C9, B8 : C8) + SUMPRODUCT (D8 : F8, D10 : F10) — G11 * This formula is also in the formula bar

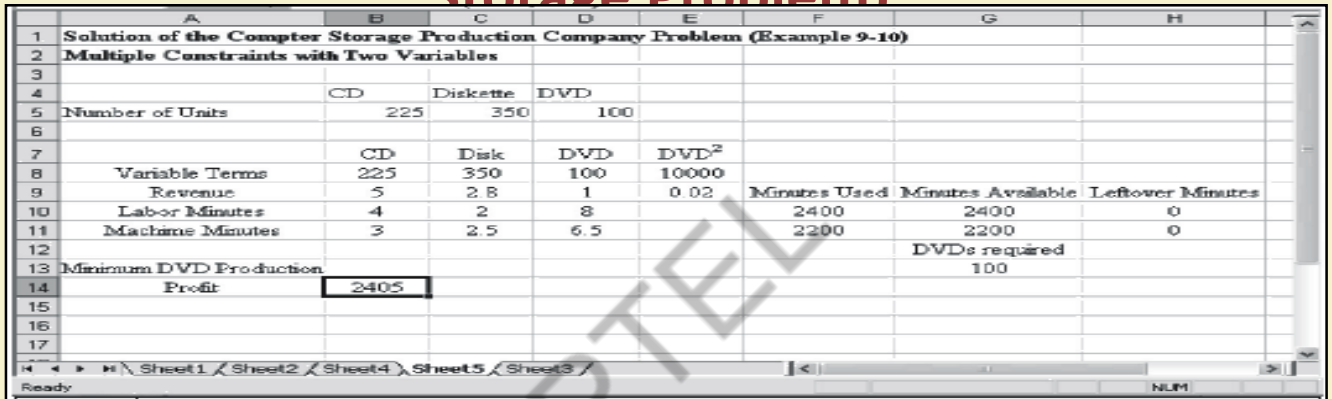
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Parameter Specification for Two Decision Variables and a Single Equality Constraint Problem (Daisy-Fresh Co. Deodorant Problem)



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Worksheet for a Two-Decision-Variable, Multiple-Constraint Problem (East-West Company Computer Storage Problem)



	A	B	C	D	E	F	G	H
1	**Solution of the Compter Storage Production Company Problem (Example 9-10)**							
2	**Multiple Constraints with Two Variables**							
3								
4		CD	Diskette	DVD				
5	Number of Units	225	350	100				
6								
7		CD	Disk	DVD	DVD^2			
8	Variable Terms	225	350	100	10000			
9	Revenue	5	2.8	1	0.02	Minutes Used	Minutes Available	Leftover Minutes
10	Labor Minutes	4	2	8		2400	2400	0
11	Machine Minutes	3	2.5	6.5		2200	2200	0
12							DVDs required	
13	Minimum DVD Production						100	
14	Profit	2405						
15								
16								
17								

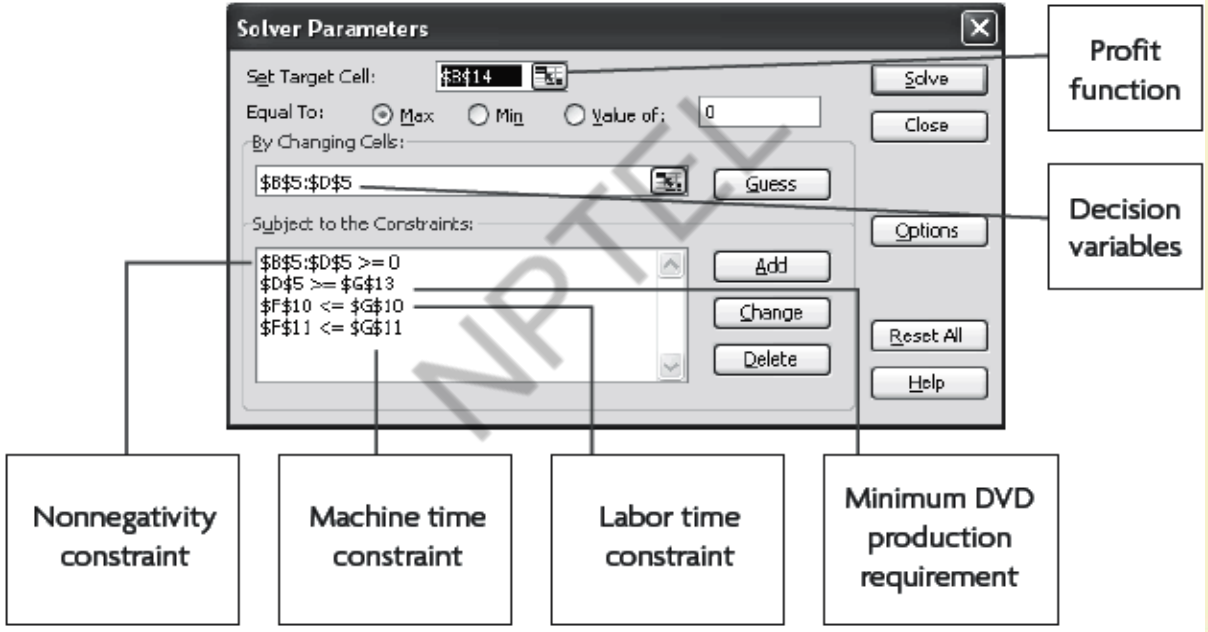
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Cell	Formula
B8	= B5
C8	= C5
D8	= D5
E8	= D5 * D5
B14	= SUMPRODUCT (B9 : E9, B8 : E8)
F10	= SUMPRODUCT (B10 : D10, B5 : D5)
F11	= SUMPRODUCT (B11 : D11, B5 : D5)

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Parameter Specification for the Two-Decision-Variable, Multiple-Constraints Problem (East-West Company Computer Storage Problem)



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Summary

- Graph of an Integer Programming Problem (IPP)
- Solving an IPP using Branch & Bound Method
- Solving an IPP using Enumeration Method
- Formulation of Goal Programming Problem
- Plot of Constraints & Feasible Solution Space
- Similarities & Differences between Linear & Non-Linear Programming
- Optimal Solutions of Non-Linear Programming Problems

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Key Terms

- Pure-Integer Problem
- Mixed-Integer Problem
- 0-1 Integer Problem
- Bound
- Branch
- Relaxed Optimum
- Tree Diagram
- Soft Constraints
- Hard Constraints
- Acceptable Region
- Feasible Solution Space
- Local Maximum
- Local Minimum
- Global Maximum
- Lagrange Multiplier
- Unconstrained Problem
- Constrained Problem
- Multiple Constrained Problem

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Thank You!!

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