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CERTIFICATION COURSES

Business Analytics for Management Decision

Prescriptive analytics [46]

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Course Contents

Weeks		Lecture Names
Week 1	:	Introduction to Business Analytics
Week 2	:	Exploring Data and Analytics on Spreadsheets
Week 3	:	Descriptive Analytics
Week 4	:	Inferential Analytics 1
Week 5	:	Inferential Analytics 2
Week 6	:	Predictive Analytics 1
Week 7	:	Predictive Analytics 2
Week 8	:	Predictive Analytics 3
Week 9	:	Prescriptive Analytics 1
Week 10	:	Prescriptive Analytics 2
Week 11	:	Prescriptive Analytics 3
Week 12	:	Decision Analytics

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Sample Example

The Server Problem

General description A firm that assembles computers and computer equipment is about to start production of two new Web server models. Each type of model will require assembly time, inspection time, and storage space. The amounts of each of these resources that can be devoted to the production of the servers is limited. The manager of the firm would like to determine the quantity of each model to produce in order to maximize the profit generated by sales of these servers.

Additional information In order to develop a suitable model of the problem, the manager has met with design and manufacturing personnel. As a result of those meetings, the manager has obtained the following information:

	Type 1	Type 2
Profit per unit	$60	$50
Assembly time per unit	4 hours	10 hours
Inspection time per unit	2 hours	1 hour
Storage space per unit	3 cubic feet	3 cubic feet

The manager also has acquired information on the availability of company resources. These (daily) amounts are:

Resource	Amount Available
Assembly time	100 hours
Inspection time	22 hours
Storage space	39 cubic feet

The manager also met with the firm's marketing manager and learned that demand for the servers was such that whatever combination of these two models of servers is produced, all of the output can be sold.

x_1 = quantity of server model 1 to produce

x_2 = quantity of server model 2 to produce

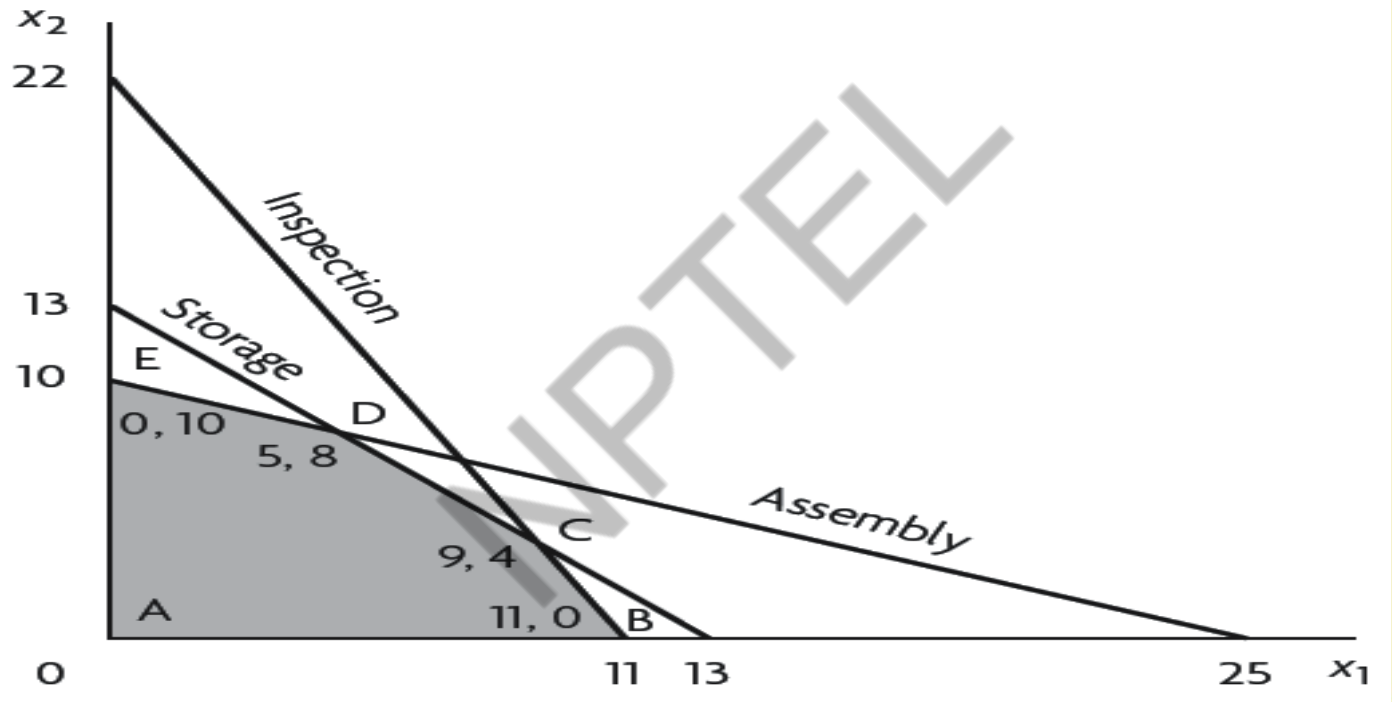
maximize $Z = 60x_1+50x_2$

Subject to:

$$\begin{aligned} \text{Assembly} \quad & 4x_1 + 10x_2 \le 100 \text{ hours} \\ \text{Inspection} \quad & 2x_1 + 1x_2 \le 22 \text{ hours} \\ \text{Storage} \quad & 3x_1 + 3x_2 \le 39 \text{ cubic feet} \\ & x_1, x_2 \ge 0 \end{aligned}$$

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Solution Space: Extreme Points Case



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Summary of Solutions

Point	Coordinates x_1	Coordinates x_2	How Determined	Value of the Objective Function
A	0	0	Inspection	\$60(0) + \$50(0) = \$0
B	11	0	Inspection	\$60(11) + \$50(0) = \$660
C	9	4	Simultaneous equations	\$60(9) + \$50(4) = \$740 (largest)
D	5	8	Simultaneous equations	\$60(5) + \$50(8) = \$700
E	0	10	Inspection	\$60(0) + \$50(10) = \$500

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Unit Objectives

- Explain how sensitivity analysis can be useful to a decision maker.
- Explain why it can be useful for a decision maker to extend the analysis of a linear programming problem beyond determination of the optimal solution.
- Explain how to analyze and interpret the impact of a change in the value of the objective function coefficient.
- Explain how to analyze and interpret the impact of a change in the right-hand-side value of a constraint.
- Explain what a dual is and formulate the dual of a problem.
- Interpret dual problem and relate dual solution to primal solution.
- Explain economic interpretation of dual variables and the dual solution.
- Determine if adding another variable to a problem will change the optimal solution mix of the original problem.

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Sensitivity analysis

- Benefits of sensitivity analysis
  - Enables the decision maker to determine how a change in one of the values of a model will impact the optimal solution and the optimal value of the objective function while holding all other parameters constant.
  - Provides the decision maker with greater insight about the sensitivity of the optimal solution to changes in various parameters of a problem.
  - Permits quick examination of changes due to improved information relating to a problem or because of the desire to know the potential impact of changes that are contemplated.

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Changes in Parameter Values

- Categories of model parameters subject to potential changes
  - The value of an objective function coefficient
  - The right-hand side (RHS) value of a constraint
  - A coefficient of a constraint
- Concerns about ranges of changes
  - Which range pertains to a given situation?
  - How can the range be determined?
  - What impact on the optimal solution does a change that is within the range have?

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Optimality and the Objective Function Coefficient

- Range of optimality
  - Finding the range of objective function values for which the optimal values of the decision variables would not change.
  - A value of the objective function that falls within the range of optimality will not change the optimal solution, although the optimal value of the objective function will change.

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Feasibility

- Range of feasibility
  - The range of values over which the right-hand-side (RHS) value can change without causing the shadow price to change.
  - Within this range of feasibility, the same decision variables will remain optimal, although their values and the optimal value of the objective function will change.
  - Analysis of RHS changes begins with determination of a constraint's shadow price in the optimal solution.

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Example

The Server Problem

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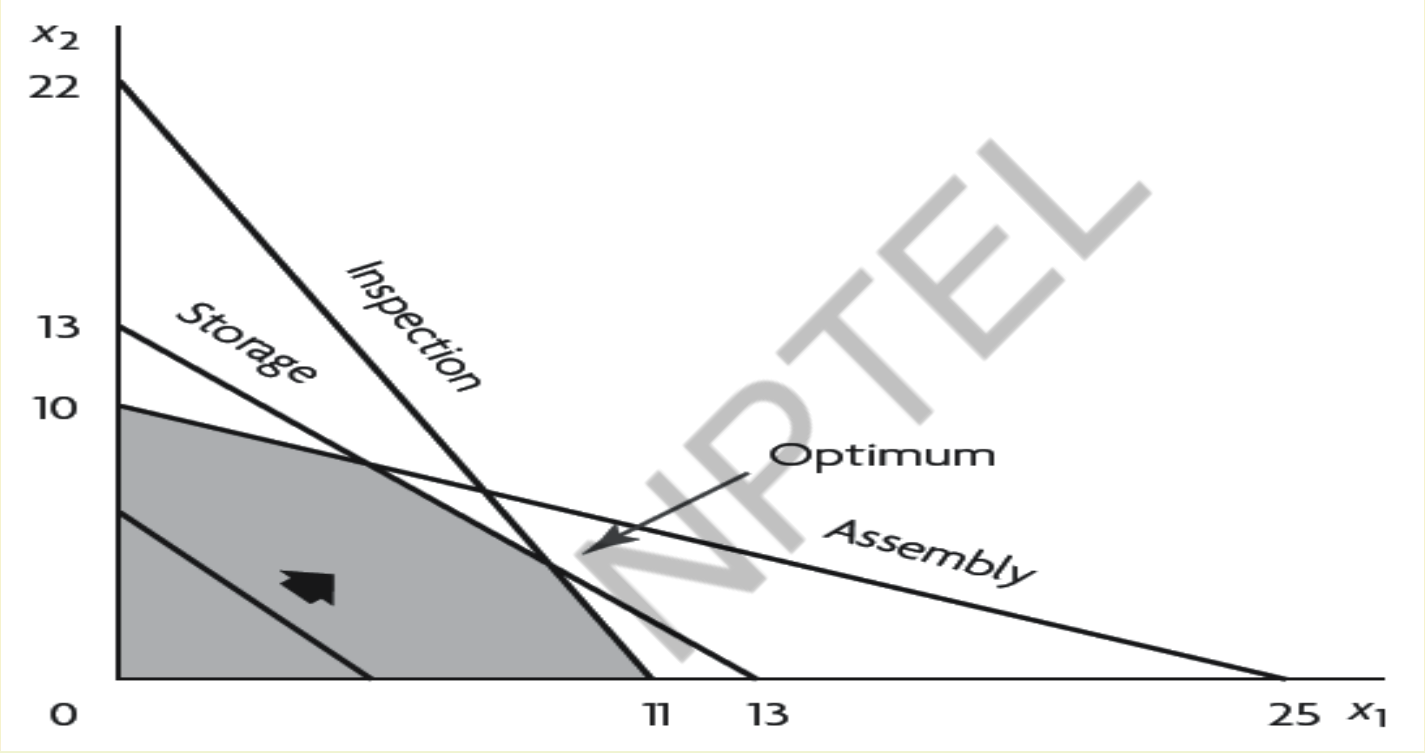
maximize $Z = 60x_1+50x_2$

Subject to:

$$\begin{aligned} \text{Assembly} \quad & 4x_1 + 10x_2 \leq 100 \text{ hours} \\ \text{Inspection} \quad & 2x_1 + 1x_2 \leq 22 \text{ hours} \\ \text{Storage} \quad & 3x_1 + 3x_2 \leq 39 \text{ cubic feet} \\ & x_1, x_2 \geq 0 \end{aligned}$$

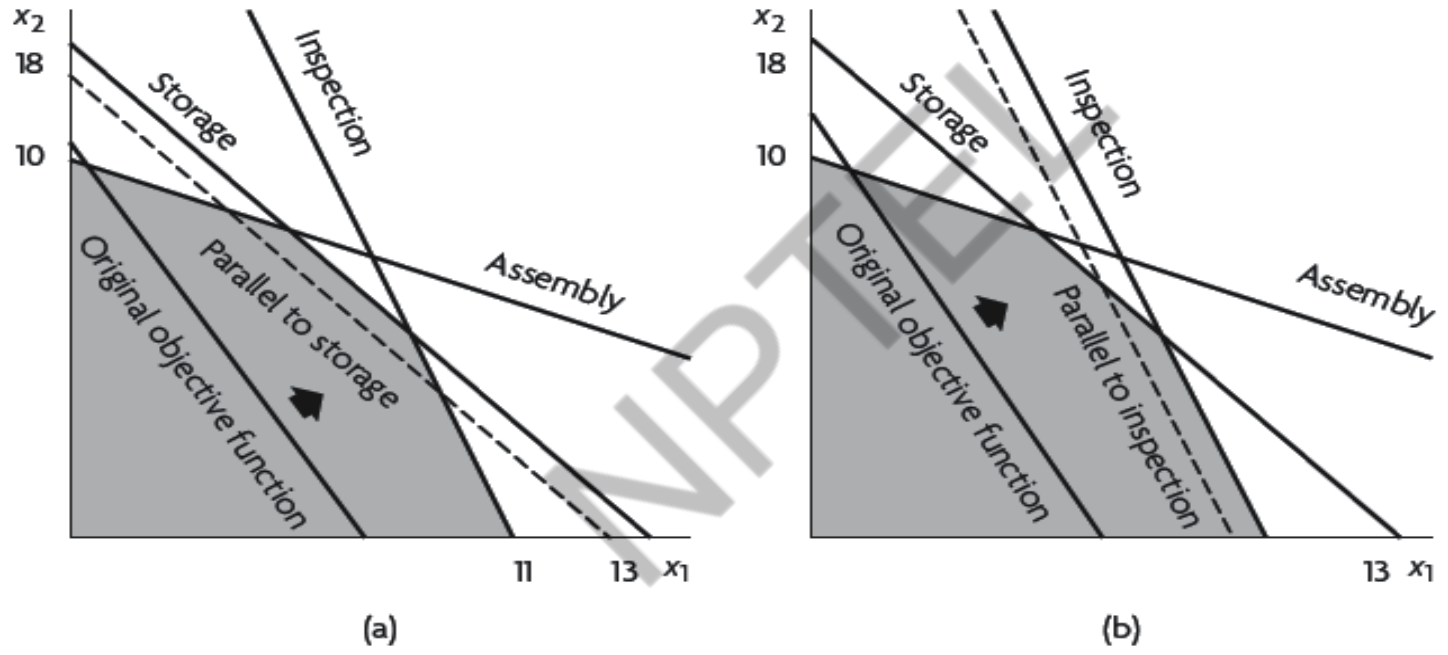
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Feasible Solution Space



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Structure of a Change in the Objective Function Coefficients



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The optimum values of the server problem are

$x_1 = 9$
$x_2 = 4$
$Z = 740$

In this chapter, we just found the range of optimality for each coefficient of the objective function for the server problem. They were

For the x_1 coefficient, 50 to 100
For the x_2 coefficient, 30 to 60

Given this information, analyze the impact of a change in the x_1 coefficient of the objective function from 60 to 82.50. Specifically, what impact will there be on the optimal values of the decision variables and the objective function?

Solution

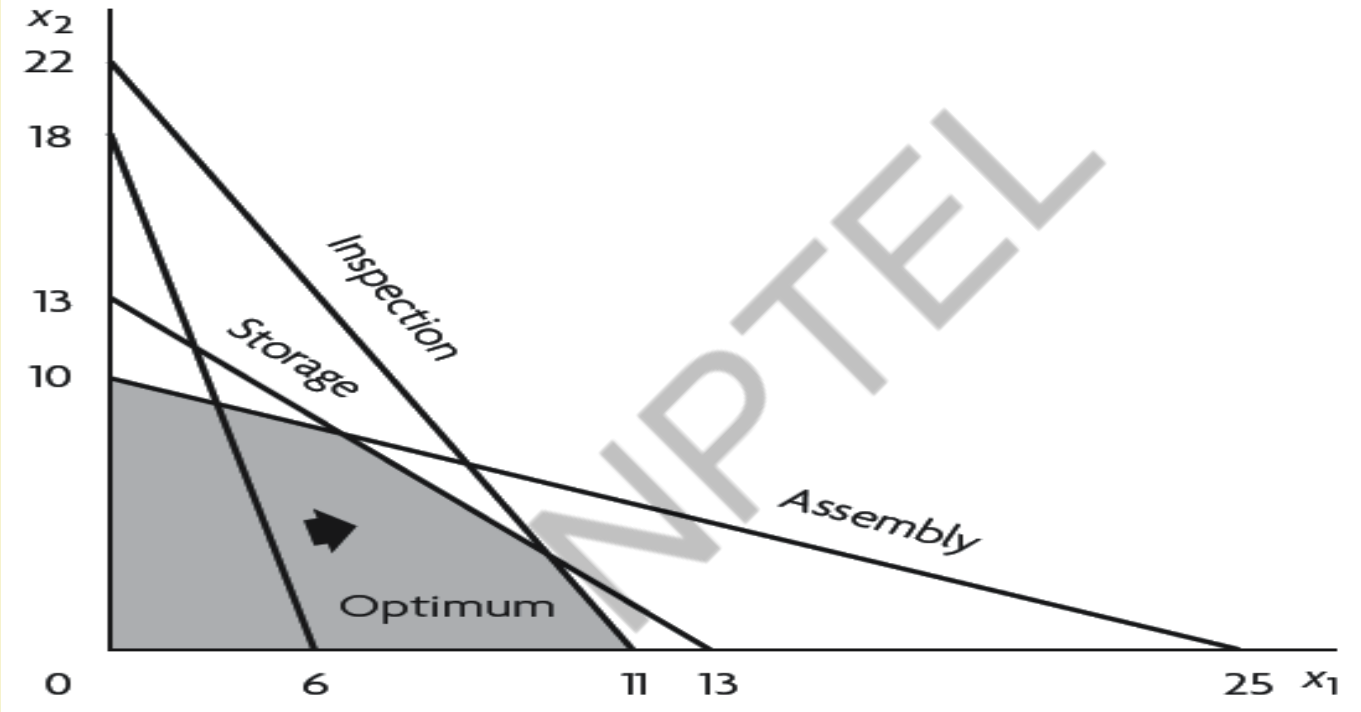
First, check to see if the new value of the x_1 coefficient is within the range of optimality. We can see that 82.50 is within the range of 50 to 100. Consequently, there will be no change in the optimal values of x_1 and x_2.

However, the optimal value of Z will change. To find its new value, we must substitute the new coefficient into the objective function with the optimal values of the decision variables and solve. Thus:

$Z = 82.50(9) + 50(4) = 942.50$

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Solution for Revised Server Problem



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Example with the same problem extended

Determine the range of feasibility for each of these binding constraints in the server problem:

1. The inspection constraint.
2. The storage constraint.

Solution

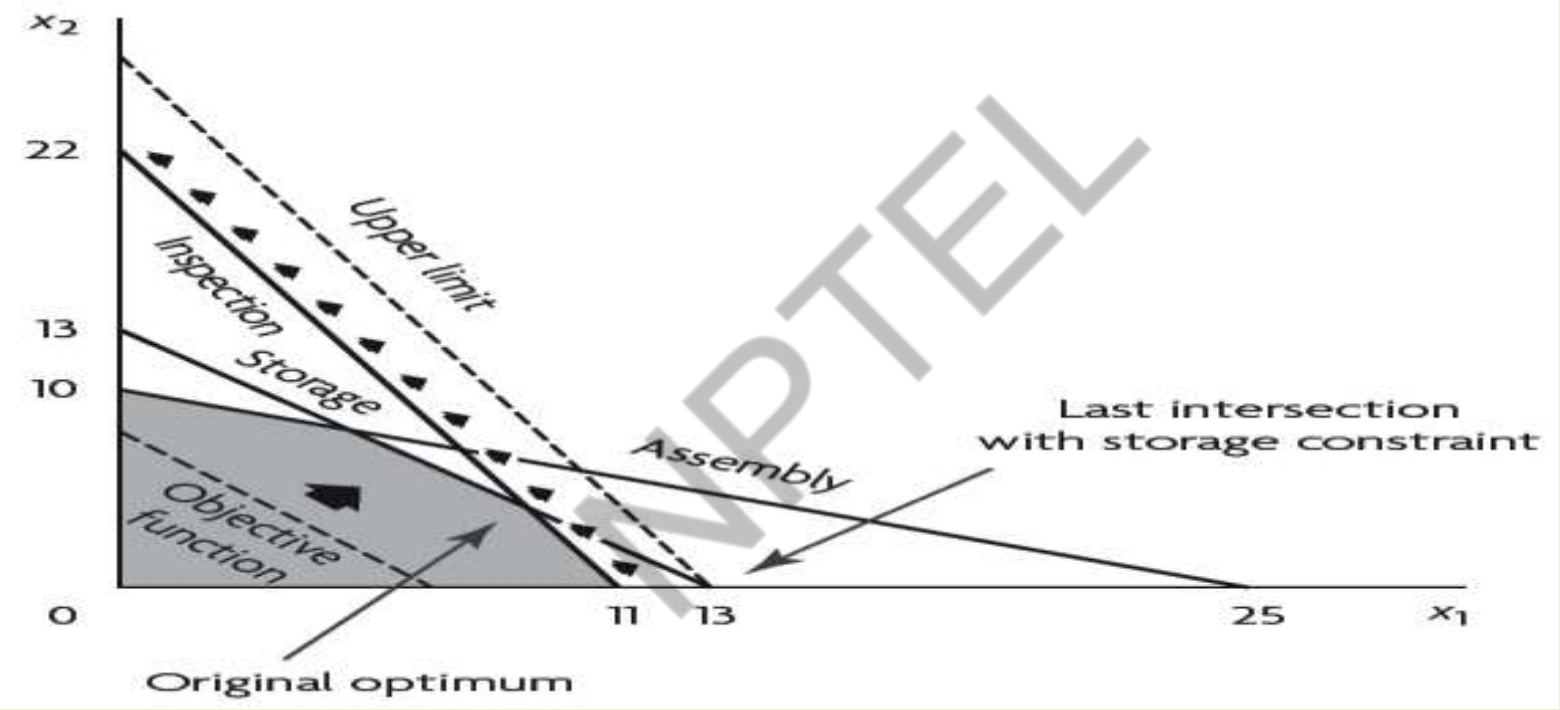
1. Following the above procedure, we determine the last point where the storage constraint intersects with the line that is parallel to the inspection constraint line, moving away from the origin. See Figure 4.

   Next, we repeat the procedure moving toward the origin. See Figure 5.

   The coordinates of the upper limit, the point where the storage constraint intersects with the line parallel to the inspection constraint line, are easy to obtain directly from the graph: $x_1 = 13, x_2 = 0$. For the lower limit, note that at that last point, the storage and assembly lines intersect. Hence, we can determine the coordinates of that point by solving the equations of the storage and assembly lines simultaneously. This will yield $x_1 = 5$ and $x_2 = 8$.

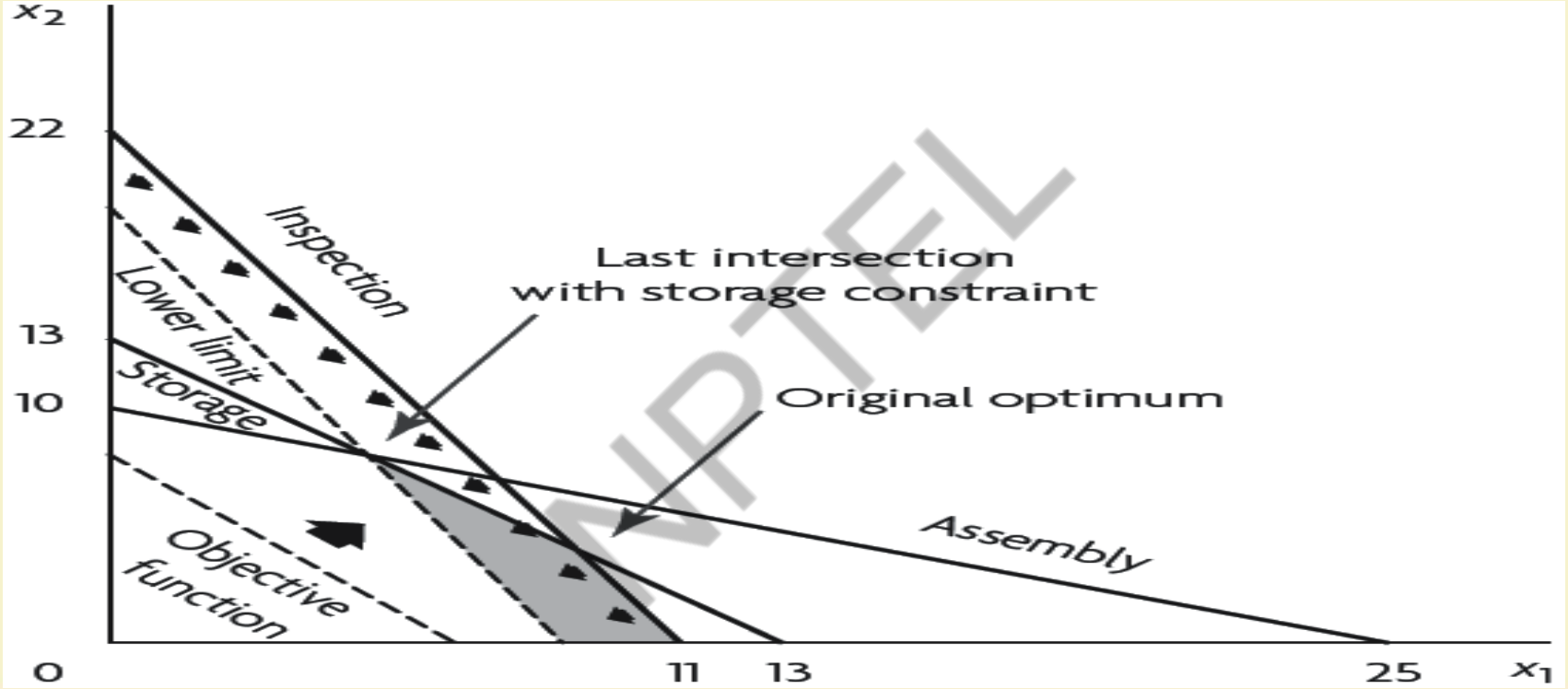
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The Upper Limit on Using Additional Inspection Time



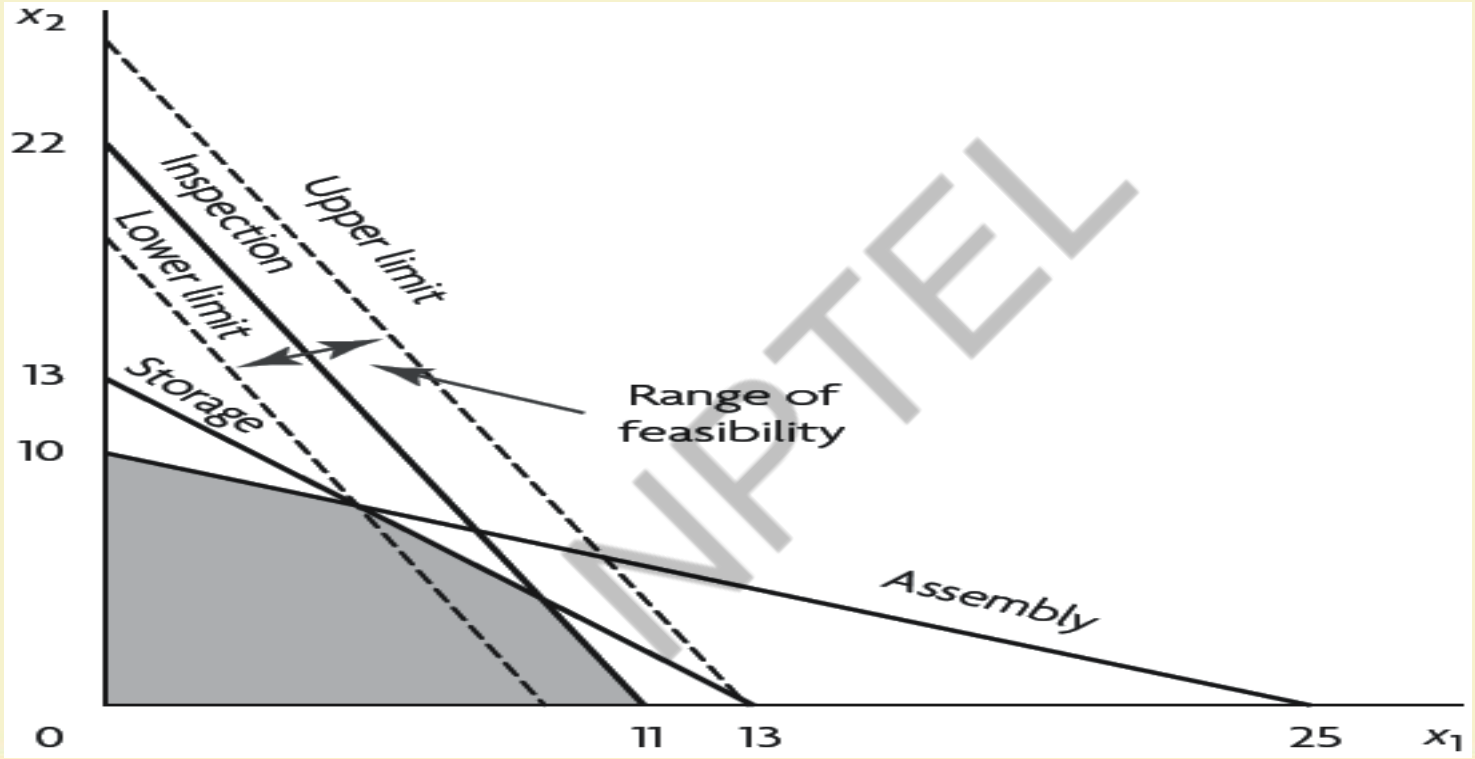
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Lower Limit on Inspection Time



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Range of Feasibility for Changes in Inspection Time



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Example: Continued

Hence, the range of feasibility for the RHS of the inspection constraint is 18 to 26.

The range of feasibility is bordered by these two limits, as illustrated in Figure 6.

2. Repeating the process for the storage constraint, we determine the last points (upper limit and lower limit) for which the line parallel to *storage* constraint intersects the *inspection* constraint (see Figure 7)

We can see on the graph in Figure 7 that the lower limit intersects at $x_1 = 11, x_2 = 0$, and at the upper limit the inspection and assembly lines cross. Solving those two equations simultaneously yields $x_1 = 7.5$ and $x_2 = 7$.

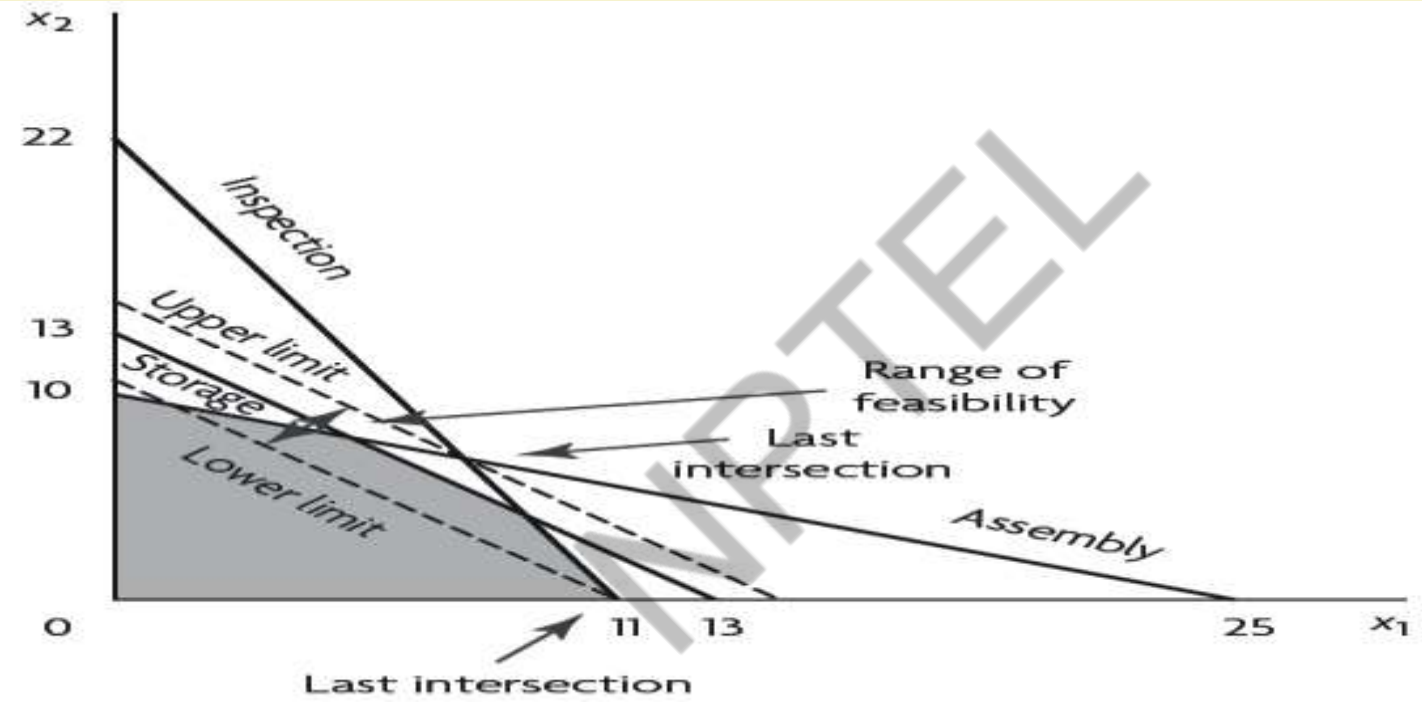
Substituting these two sets of values into the storage constraint gives us

$3(7.5) + 3(7) = 43.5$ (upper limit)
$3(11) + 3(0) = 33$ (lower limit)

Hence, the range of feasibility for the RHS of the storage constraint is 33 cubic feet to 43.5 cubic feet.

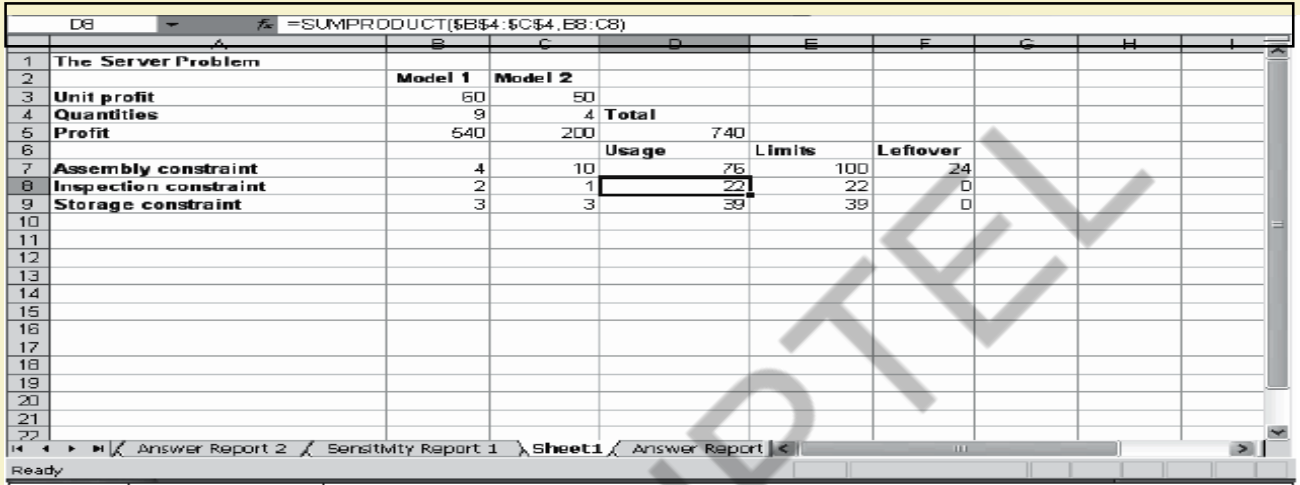
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Range of Feasibility for the Storage Constraint



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Excel Input Screen for the Server Problem



Cell	Copied to	Formula
D7	D8, D9	= SUMPRODUCT ($B$4 : $C$4, B7 : C7)
F7	F8, F9	= E7 — D7
D5		= SUM (B5 : C5)
B5	C5	= B3 * B4

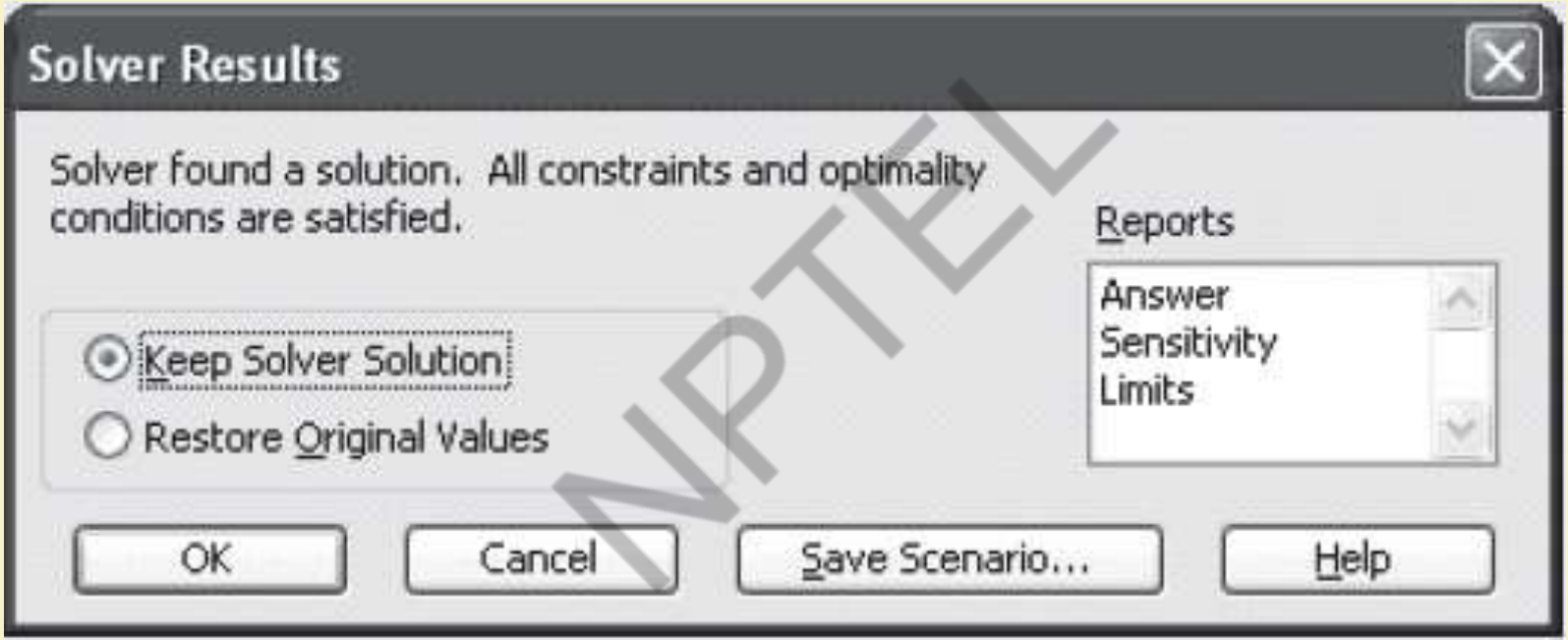
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Solver Input Specification (parameter) Screen



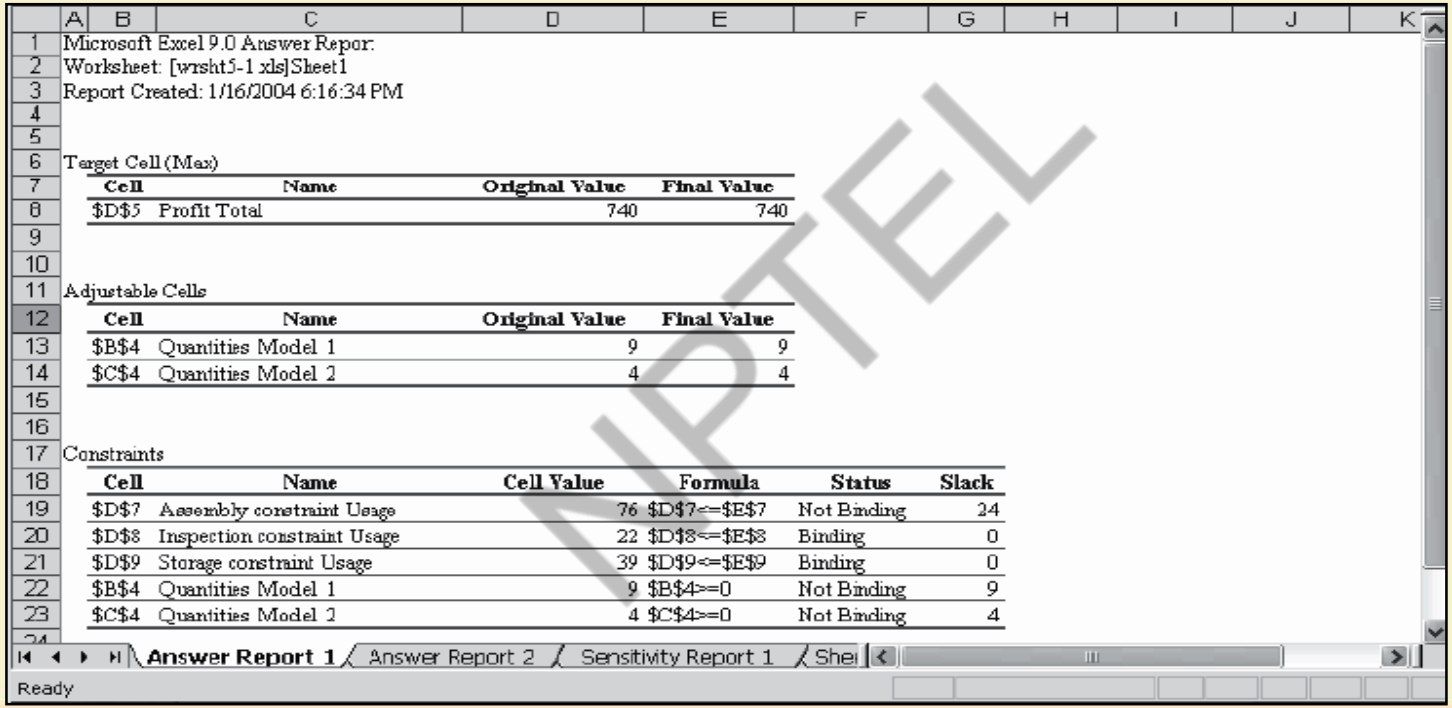
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Optimization Output Screen



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Excel Basic Output (Answer) Report for the Server Problem



Microsoft Excel 9.0 Answer Repor:
Worksheet: [wrsht5-1 xls]Sheet1
Report Created: 1/16/2004 6:16:34 PM

Target Cell (Max)

Cell	Name	Original Value	Final Value
$D$5	Profit Total	740	740

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$4	Quantities Model 1	9	9
$C$4	Quantities Model 2	4	4

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$D$7	Assembly constraint Usage	76	$D$7<=$E$7	Not Binding	24
$D$8	Inspection constraint Usage	22	$D$8<=$E$8	Binding	0
$D$9	Storage constraint Usage	39	$D$9<=$E$9	Binding	0
$B$4	Quantities Model 1	9	$B$4>=0	Not Binding	9
$C$4	Quantities Model 2	4	$C$4>=0	Not Binding	4

Answer Report 1 | Answer Report 2 | Sensitivity Report 1 | Shei

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Excel Sensitivity Analysis Report for the Server Problem

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [wrsht5-1.xls]Sheet1
Report Created: 1/16/2004 6:17:00 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Quantities Model 1	9	0	60	40	10
$C$4	Quantities Model 2	4	0	50	10	20

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$D$7	Assembly constraint Usage	76	0	100	1E+30	24
$D$8	Inspection constrain: Usage	22	10	22	4	4
$D$9	Storage constraint Usage	39	13.33333333	39	4.5	6

Answer Report 1 | Answer Report 2 | Sensitivity Report 1 | Shee

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Another example

Let us consider how the solution to the server problem would be affected by multiple changes to *either* its objective function coefficients *or* the RHS values of its constraints. Let's consider these changes:

a. A change in the objective function from maximize $60x_1 + 50x_2$ to maximize $70x_1 + 56x_2$.
b. A change in the RHS of the inspection constraint from 22 to 23 hours and a change in the storage constraint from 39 to 36 cubic feet.

Solution

(Values for allowable increases and decreases are taken from Exhibit 5.)

a. The change in the x_1 coefficient is +10 (i.e., 60 to 70), and the change in the x_2 coefficient is +6 (i.e., 50 to 56). From Exhibit 5, the allowable increase for the x_1 coefficient is 40, and the allowable increase for the x_2 coefficient is 10. Thus, we have

Variable	x_1	x_2
Change	+10	+6
Allow. inc. or dec.	+40	+10
percent change	25% + 60% = 85%	

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Example (Continued)

b. For the RHS changes, we have

Constraint	Inspection	Storage
Change	1	−3
Allow. inc. or dec.	4	−6
percent change	25%	+ 50% = 75%

The total change would be 75 percent. Because this does not exceed 100 percent, it is within the range of sensitivity. Consequently, the shadow prices (10.00 and 13.33) do not change. The optimal value of Z will change. The amount of change will be

$$+1(10.00) - 3(13.33) = -30$$

Hence, the optimal value of Z will be $740 - 30 = 710$.

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Summary of Results of Changes That Are within Ranges of Optimality and Feasibility

Objective Function Coefficient Change within the Range of Optimality	
Factor	**Result**
Variables in solution	Same variables still in solution
Optimal values of solution variables	Don't change
Optimal value of *Z*	Will change
RHS Change within the Range of Feasibility	
Factor	**Result**
Shadow price	Remains the same
List of variables with positive values	Remains the same
Values of variables with positive values	Change
Optimal value of *Z*	Changes

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Business Analytics for Management Decision

Prescriptive analytics [47]

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Duality Problem

- The Dual
  - An alternate formulation of a linear programming problem as either the original problem or its mirror image, *the dual*, which can be solved to obtain the optimal solution.
  - Its variables have a different economic interpretation than the original formulation of the linear programming problem (the primal).
  - It can be easily used to determine if the addition of another variable to a problem will change the optimal.

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Formulation of a Dual

- Dual
  - The number of decision variables in the primal is equal to the number of constraints in the dual.
  - The number of decision variables in the dual is equal to the number of constraints in the primal.
  - *Since it is computationally easier to solve problems with less constraints in comparison to solving problems with less variables, the dual gives us the flexibility to choose which problem to solve.*

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Example

Write the dual of the following:

minimize $40x_1 + 44x_2 + 48x_3$

subject to

$$1x_1 + 2x_2 + 3x_3 \geq 20$$
$$4x_1 + 4x_2 + 4x_3 \geq 30$$
$$x_1, x_2, x_3 \geq 0$$

Solution

The dual of this problem is

maximize $20y_1 + 30y_2$

subject to

$$1y + 4y_2 \leq 40$$
$$2y_1 + 4y_2 \leq 44$$
$$3y_1 + 4y_2 \leq 48$$
$$y_1, y_2 \geq 0$$

A comparison of these two versions of the problem will reveal why the dual might be termed the “mirror image” of the primal. Table 2 shows how the primal problem is transformed into its dual.

We can see in Table 2 that the original objective was to minimize, whereas the objective of the dual is to maximize.

In addition, the coefficients of the primal’s objective function become the right-hand-side values for the dual’s constraints, whereas the primal’s right-hand side values become the coefficients of the dual’s objective function.

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Table: Transforming the Primal into Its Dual

		Primal		Dual	
Objective function and right-hand-side values	minimize	$40x_1 - 44x_2 - 48x_3$		maximize	$20y_1 + 30y_2$
	subject to			subject to	
	1		$\geqslant 20$	1	$\leqslant 40$
	2		$\geqslant 30$	2	$\leqslant 44$
				3	$\leqslant 48$

		Primal	Dual	
Constraint Coefficients	1	$1x_1 + 2x_2 + 3x_3 \geqslant$	1	$1y_1 + 4y_2 \leqslant$
	2	$4x + 4x_2 + 4x_3 \geqslant$	2	$2y_1 + 4y_2 \leqslant$
			3	$3y_1 + 4y_2 \leqslant$

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Server Problem

Formulate the dual of the server problem.

Solution

The original (primal) problem was

maximize $Z = 60x_1 + 50x_2$

subject to

$$4x_1 + 10x_2 \leq 100$$
$$2x_1 + 1x_2 \leq 22$$
$$3x_1 + 3x_2 \leq 39$$

To formulate the dual, we can follow this sequence:

1 For the objective function:
   a. Because the primal is a maximization, the dual will be a minimization.
   b. The right-hand-side values for the constraints of the primal become the co-efficients of the objective function of the dual.

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Economic Interpretation of The Dual

- Economic interpretation of dual solution results
  - Analysis enables a manager to evaluate the potential impact of a new product.
  - Analysis can determine the marginal values of resources (i.e., constraints) to determine how much profit one unit of each resource is equivalent to.
  - Analysis helps the manager to decide which of several alternative uses of resources is the most profitable.

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Example

Susan Frank is the operations manager of Arc Manufacturing Inc. She has developed a linear programming model to help her determine the product mix on one of the three production lines for Arc Manufacturing Inc.:

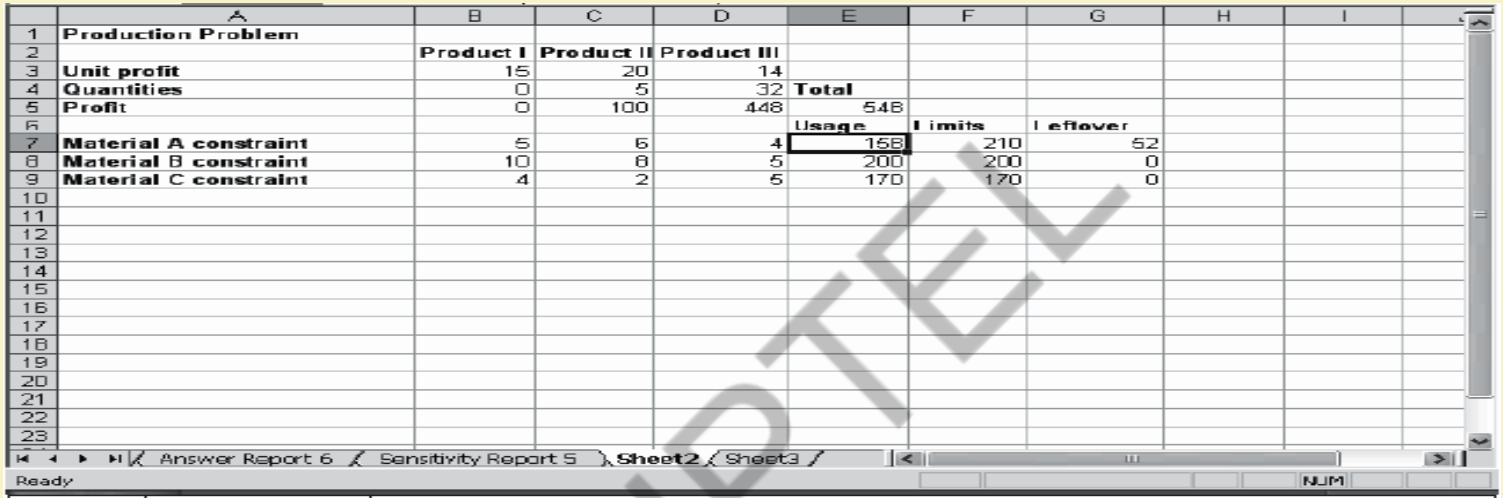
maximize $Z = 15x_1 + 20x_2 + 14x_3$

subject to

$5x_1 + 6x_2 + 4x_3 \leq 210$ pounds per day (Material A constraint)
$10x_1 + 8x_2 + 5x_3 \leq 200$ pounds per day (Material B constraint)
$4x_1 + 2x_2 + 5x_3 \leq 170$ pounds per day (Material C constraint)
$x_1, x_2, x_3 \geq 0$ (nonnegativity constraint)

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Excel Worksheet for Arc Manufacturing Inc.



	A	B	C	D	E	F	G	H	I
1	**Production Problem**								
2		**Product I**	**Product II**	**Product III**					
3	**Unit profit**	15	20	14					
4	**Quantities**	0	5	32	**Total**				
5	**Profit**	0	100	448	548				
6					**Usage**	**Limits**	**Leftover**		
7	**Material A constraint**	5	6	4	158	210	52		
8	**Material B constraint**	10	8	5	200	200	0		
9	**Material C constraint**	4	2	5	170	170	0		
10									
11									
12									
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20									
21									
22									
23									

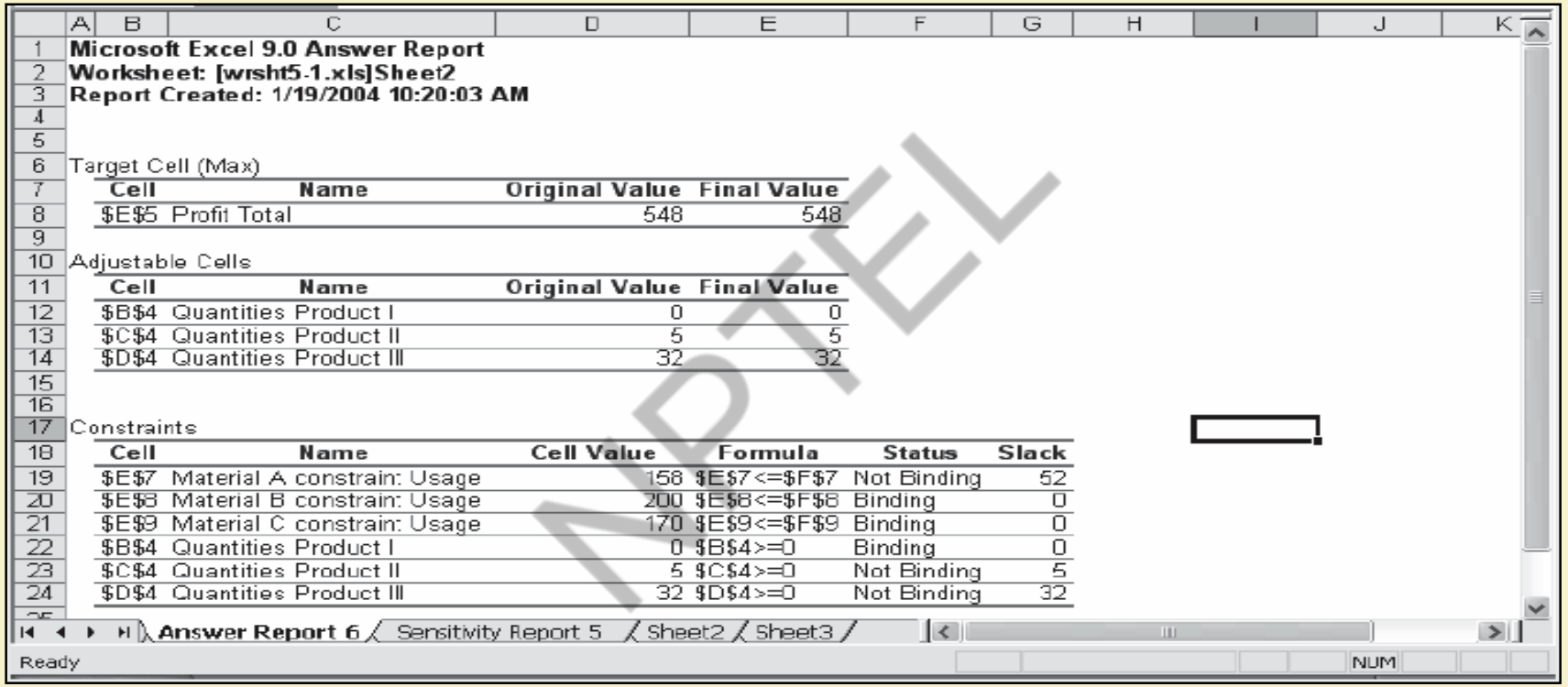
Answer Report 6 | Sensitivity Report 5 | Sheet2 | Sheet3

Ready NUM

Cell	Copied to	Formula
E7	E8, E9	= SUMPRODUCT ($B$4 : $D$4, B7 : D7)
G7	G8, G9	= F7 — E7
E5		= SUM (B5 : D5)
B5	C5, D5	= B3 * B4

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Excel Basic Output Report for Arc Manufacturing Inc.



Microsoft Excel 9.0 Answer Report
Worksheet: [wrsht5-1.xls]Sheet2
Report Created: 1/19/2004 10:20:03 AM

Target Cell (Max)

Cell	Name	Original Value	Final Value
$E$5	Profit Total	548	548

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$4	Quantities Product I	0	0
$C$4	Quantities Product II	5	5
$D$4	Quantities Product III	32	32

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$7	Material A constrain: Usage	158	$E$7<=$F$7	Not Binding	52
$E$8	Material B constrain: Usage	200	$E$8<=$F$8	Binding	0
$E$9	Material C constrain: Usage	170	$E$9<=$F$9	Binding	0
$B$4	Quantities Product I	0	$B$4>=0	Binding	0
$C$4	Quantities Product II	5	$C$4>=0	Not Binding	5
$D$4	Quantities Product III	32	$D$4>=0	Not Binding	32

Answer Report 6 | Sensitivity Report 5 | Sheet2 | Sheet3

Ready NUM

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Excel Sensitivity Analysis for Arc Manufacturing Inc.

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [wrsht5-1.xls]Sheet2
Report Created: 1/19/2004 10:20:03 AM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Quantities Product I	0	-10.6	15	10.6	1E+30
$C$4	Quantities Product II	5	0	20	2.4	10.6
$D$4	Quantities Product III	32	0	14	36	1.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$7	Material A constrain: Usage	158	0	210	1E+30	52
$E$8	Material B constrain: Usage	200	2.4	200	70.90909091	30
$E$9	Material C constrain: Usage	170	0.4	170	30	120

Sensitivity Report 4 | Answer Report 6 | Sensitivity Report 5

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Excel Sensitivity Report for Solved Problem

Microsoft Excel 11.0 Sensitivity Report
Worksheet: [wrsht5-1.xls]Sheet3
Report Created: 3/18/2005 5:13:23 PM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Quantities Product I	2	0	10	2	5.5
$C$4	Quantities Product II	6.666666667	0	6	24	1
$D$4	Quantities Product III	0	-2.2	5	2.2	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$7	Material A constrain: Usage	24	0	25	1E+30	1
$E$8	Material B constrain: Usage	22	0.4	22	1.25	16.66666667
$E$9	Material C constrain: Usage	32	1.6	32	4.999999999	10

Sensitivity Report 5 | Sheet2 | Sensitivity Report 6 | Sheet3

Ready

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Table: Excel Reports for Problem

A. Answer Report

Target Cell (Max)

Cell	Name	Original Value	Final Value
$E$5	Profit Total	588	588

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$4	Quantities Product I	48	48
$C$4	Quantities Product II	0	0
$D$4	Quantities Product III	28	28

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$7	Labor Usage	360	$E$7<=$F$7	Binding	0
$E$8	Machine Usage	264	$E$8<=$F$8	Not Binding	36
$E$9	Material Usage	600	$E$9<=$F$9	Binding	0

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Table: Excel Reports for Problem (continued)

B. Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$4	Quantities Product I	48	0	7	6.5	1
$C$4	Quantities Product II	0	−4.500000001	2.999999999	4.500000001	1E+30
$D$4	Quantities Product III	28	0	9	1.5	4.333333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R. H. Side	Allowable Increase	Allowable Decrease
$E$7	Labor Usage	360	1.3	360	257.1428571	93.33333333
$E$8	Machine Usage	264	0	300	1E+30	36
$E$9	Material Usage	600	0.2	600	210	90

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Finding Dual of a LP problem: Another example

Primal	Dual
Maximize $Z = 4x_1 + 3x_2$	Minimize $Z' = 6000y_1 - 2000y_2 + 4000y_3$
Subject to $x_1 + \frac{2}{3}x_2 \le 6000$ $x_1 - x_2 \ge 2000$ $x_1 \le 4000$ x_1 unrestricted $x_2 \ge 0$	Subject to $y_1 - y_2 + y_3 = 4$ $\frac{2}{3}y_1 + y_2 \le 3$ $y_1 \ge 0$ $y_2 \ge 0$ $y_3 \ge 0$

Note: Second constraint in the primal is transformed to $-x_1 + x_2 \le -2000$ before constructing the dual.

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Primal-Dual relationships

- If one problem (either primal or dual) has an optimal feasible solution, other problem also has an optimal feasible solution. The optimal objective function value is same for both primal and dual.
- If one problem has no solution (infeasible), the other problem is either infeasible or unbounded.
- If one problem is unbounded the other problem is infeasible.

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Business Analytics for Management Decision

Prescriptive analytics [48]

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Dual Simplex Method

Simplex Method verses Dual Simplex Method

1. Simplex method starts with a non-optimal but feasible solution where as dual simplex method starts with an optimal but infeasible solution.

2. Simplex method maintains the feasibility during successive iterations where as dual simplex method maintains the optimality.

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Dual Simplex Method: Iterative steps

Steps involved in the dual simplex method are:

1. All the constraints (except those with equality (=) sign) are modified to 'less-than-equal-to' sign. Constraints with greater-than-equal-to' sign are multiplied by -1 through out so that inequality sign gets reversed. Finally, all these constraints are transformed to equality sign by introducing required slack variables.
2. Modified problem, as in step one, is expressed in the form of a simplex tableau. If all the cost coefficients are positive (i.e., optimality condition is satisfied) and one or more basic variables have negative values (i.e., non-feasible solution), then dual simplex method is applicable.

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Dual Simplex Method: Iterative steps…contd.

3. **Selection of exiting variable:** The basic variable with the highest negative value is the exiting variable. If there are two candidates for exiting variable, any one is selected. The row of the selected exiting variable is marked as pivotal row.
4. **Selection of entering variable:** Cost coefficients, corresponding to all the negative elements of the pivotal row, are identified. Their ratios are calculated after changing the sign of the elements of pivotal row, i.e.,

The column corresponding to minimum ratio is identified as the pivotal column and associated decision variable is the entering variable.

$$ratio = \left(\frac{Cost\ Coefficients}{-1 \times Elements\ of\ \ pivotal\ row} \right)$$

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Dual Simplex Method: Iterative steps…contd.

5. **Pivotal operation:** Pivotal operation is exactly same as in the case of simplex method, considering the pivotal element as the element at the intersection of pivotal row and pivotal column.

6. **Check for optimality:** If all the basic variables have nonnegative values then the optimum solution is reached. Otherwise, Steps 3 to 5 are repeated until the optimum is reached.

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Dual Simplex Method: An Example

Consider the following problem:

$$
\begin{aligned}
\text{Minimize} \quad & Z = 2x_1 + x_2 \\
\text{subject to} \quad & x_1 \geq 2 \\
& 3x_1 + 4x_2 \leq 24 \\
& 4x_1 + 3x_2 \geq 12 \\
& -x_1 + 2x_2 \geq 1
\end{aligned}
$$

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Dual Simplex Method: An Example…contd.

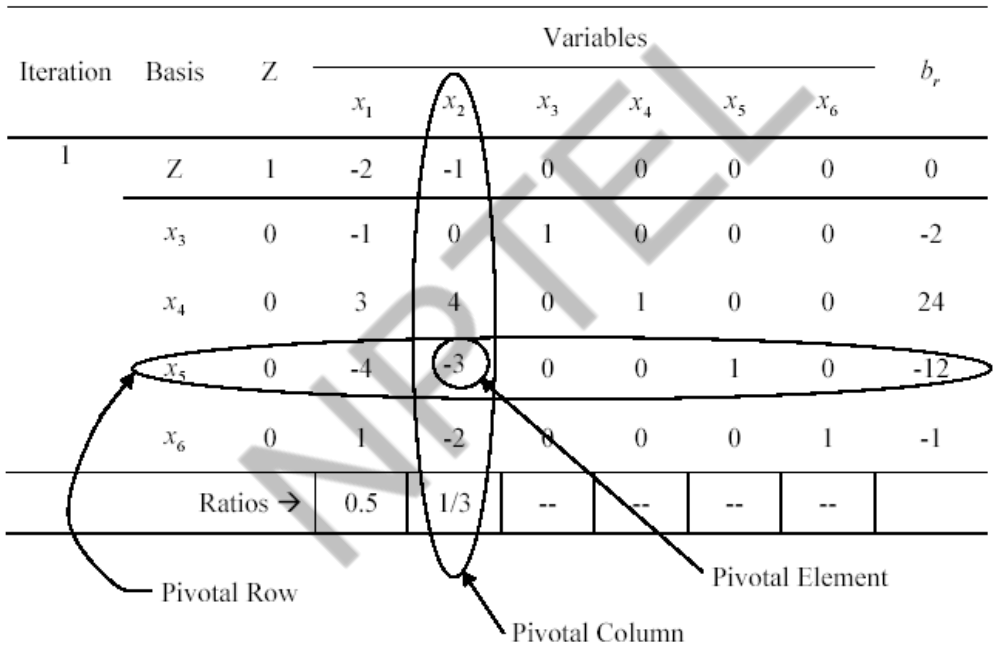
After introducing the surplus variables the problem is reformulated with equality constraints as follows:

$$
\begin{aligned}
\text{Minimize} \quad & Z = 2x_1 + x_2 \\
\text{subject to} \quad & -x_1 + x_3 = -2 \\
& 3x_1 + 4x_2 + x_4 = 24 \\
& -4x_1 - 3x_2 + x_5 = -12 \\
& x_1 - 2x_2 + x_6 = -1
\end{aligned}
$$

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Dual Simplex Method: An Example…contd.

Expressing the problem in the tableau form:



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Dual Simplex Method: An Example…contd.

Successive iterations:

Iteration	Basis	Z	Variables						b_r
			x_1	x_2	x_3	x_4	x_5	x_6	
	Z	1	-2/3	0	0	0	-1/3	0	4
	x_3	0	-1	0	1	0	0	0	-2
2	x_4	0	-7/3	0	0	1	4/3	0	8
	x_2	0	4/3	1	0	0	-1/3	0	4
	x_6	0	11/3	0	0	0	-2/3	1	7
	Ratios →		2/3	--	--	--	--	--	

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Dual Simplex Method: An Example…contd.

Successive iterations:

Iteration	Basis	Z	Variables						b_r
			x_1	x_2	x_3	x_4	x_5	x_6	
	Z	1	0	0	-2/3	0	-1/3	0	16/3
	x_1	0	1	0	-1	0	0	0	2
3	x_4	0	0	0	-7/3	1	4/3	0	38/3
	x_2	0	0	1	4/3	0	-1/3	0	4/3
	x_6	0	0	0	11/3	0	-2/3	1	-1/3
	Ratios →		--	--	--	--	0.5	--	

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Dual Simplex Method: An Example…contd.

Successive iterations:

Iteration	Basis	Z	Variables						b_r
			x_1	x_2	x_3	x_4	x_5	x_6	
	Z	1	0	0	2.5	0	0	-0.5	5.5
	x_1	0	1	0	-1	0	0	0	2
4	x_4	0	0	0	5	1	0	2	12
	x_2	0	0	1	-0.5	0	0	-0.5	1.5
	x_5	0	0	0	-5.5	0	1	-1.5	0.5
	Ratios →								

As all the b_r are positive, optimum solution is reached.

Thus, the optimal solution is $Z = 5.5$ with $x_1 = 2$ and $x_2 = 1.5$

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Solution of Dual from Primal Simplex

Primal

Maximize $Z = 4x_1 - x_2 + 2x_3$

subject to $2x_1 + x_2 + 2x_3 \le 6$

$x_1 - 4x_2 + 2x_3 \le 0$

$5x_1 - 2x_2 - 2x_3 \le 4$

$x_1, x_2, x_3 \ge 0$

Dual

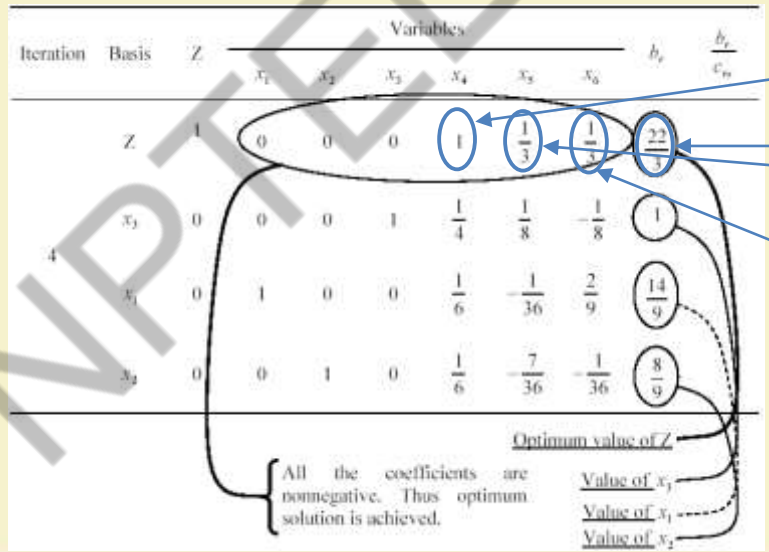
Minimize $Z' = 6y_1 + 0y_2 + 4y_3$

subject to $2y_1 + y_2 + 5y_3 \ge 4$

$y_1 - 4y_2 - 2y_3 \ge -1$

$2y_1 + 2y_2 - 2y_3 \ge 2$

$y_1, y_2, y_3 \ge 0$



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Summary

Sensitivity analysis

Post-optimal analysis

Primal

Dual

Dual simplex

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Key Terms

Primal simplex

Objective function

Decision variables

Sensitivity Analysis

Post-optimal Analysis

Duality

Dual simplex

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Books to read

- Business Analytics by J R Evans
- Business Analytics for Managers by H N Laursen

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Thank You!!

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