Module 10:Application of stochastic processes in areas like finance

# Lecture 35:Option Pricing

## The Lecture Contains:

- Option pricing
- Call (it is the option which gives the owner of the option the right to buy)
- How do we model for stock, bond prices and then value an option considering

## Option pricing

Let us consider an example here, such that:

$S_0$: Denotes the stock price at time $t = 0$
$B_0$: Denotes the bond price at time $t = 0$

Diagrammatically Figures 13 (a) & (b) denote the situations which show the price movement of a typical stock and a bond. Thus we have the following situations

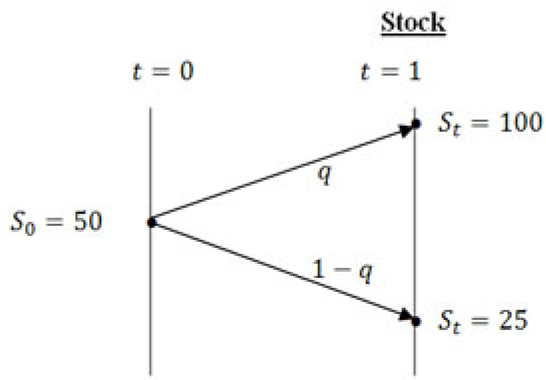


Figure 10.1: A typical stock price movement in upward or downward direction

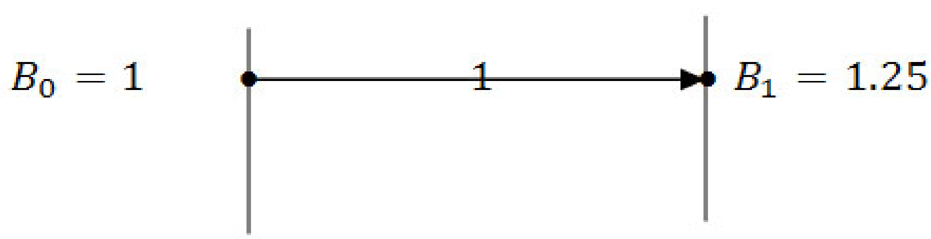


Figure 10.2: A typical bond price movement in upward direction

Furthermore consider the call.

## Call (it is the option which gives the owner of the option the right to buy)

Price as of today ($t = 0$) is *c*

In case the call option has the strike price of $K = 50$, then the strategy at time $t = 0$ is as follows

| Strategy at t=0 | Inflow at t=0 | Outflow at t=1 if S=25 | Outflow at t=1 if S=100 |
|---|---|---|---|
| Write 3 calls | +3c | 0 | -150 |
| Buy 2 share of stock | -100 | +50 | +200 |
| Borrow 40 | +40 | -50 | -50 |
| Net flow | (+3c-100+40) | 0 | 0 |

If we look carefully then we note that to avoid arbitrage we should have $+3c - 100 + 40 = 0$, such that $c = 20$.

The main problem is how do we incorporate the model so that we have one stock (Figure 10.1), plus one bond (Figure 10.2). This is the question we try to answer through the simple illustrations of Figures 10.3 &10.4

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How do we model for stock, bond prices and then value an option considering $\delta t = (t_j - t_{j-1})$

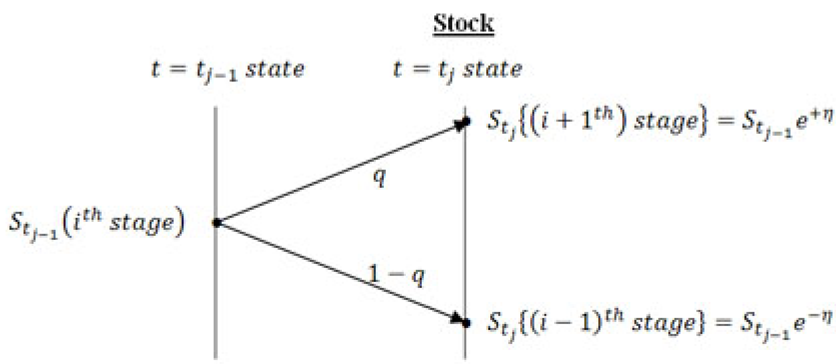


Figure 10.3: Upward and downward price fluctuation of typical stock

Here $\eta$ is the rate of interest for the stock market price increase, which more generally can be described as the average price increase on a continuous compounding case.

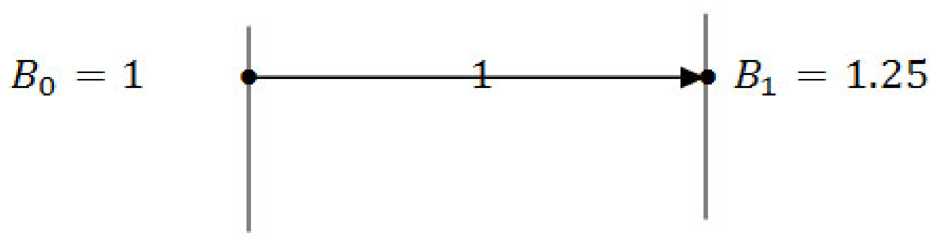


Figure 10.4: Price movement of a typical bond

Here $r$ is the rate of interest for the bond market price increase, which more generally can be described as the average price increase on a continuous compounding case.

In case we have such *n* time periods, i.e., $t(=T) - t(=0) = n \times \delta t$ then the corresponding calculations will be as shown as below with the help of Figure 15.

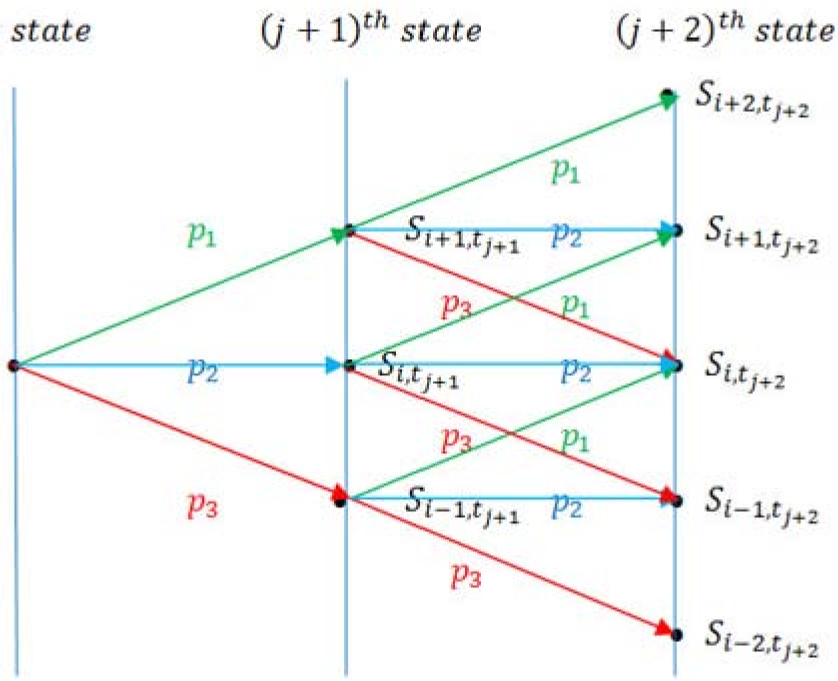


Figure 10.5: Price fluctuation of a typical stock for three time periods, and three states of price movement

At time $n$ let the state of the system be $j$, then we have $S_j = S_0 e^{j\eta}$

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In case someone is interested to understand this concept using the basic idea of transition, then we have the following equivalent diagram (Figure 16) which would make things clear for the reader.

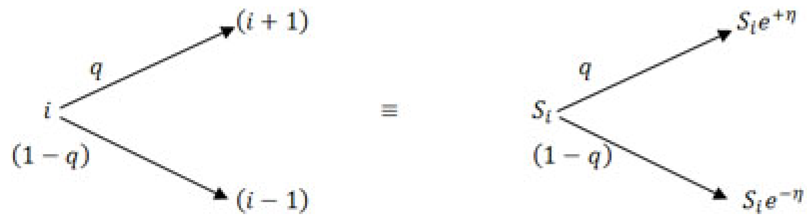


Figure: 10.6: Equivalent concept of price movement for a stock considering binomial tree

Let $C_i^n$ =Value of a claim at time $n$ in state $i$, where as per the usual notation we have the call as the option to buy, while put is that which denotes the option to sell.

Consider the contract maturity case for the example (Figure 10.7) when we have the call option and for which $K$ as per notation is the strike price.

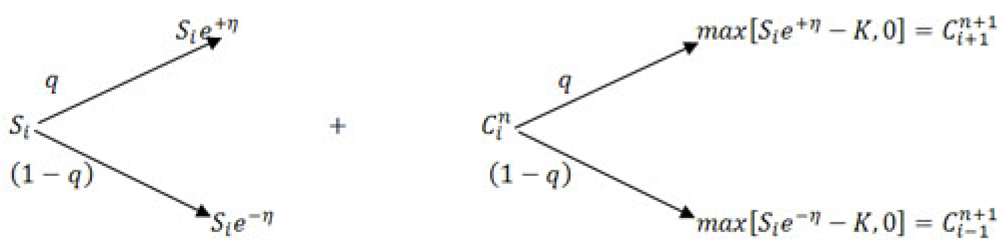


Figure: 10.7: Concept of price movement for a stock as well as the option, where the portfolio consists of the stock and option

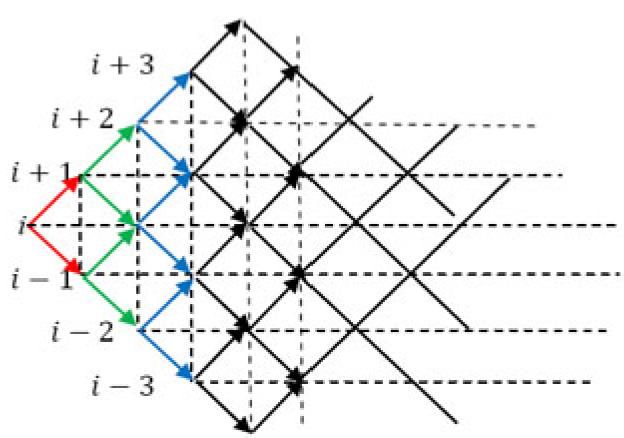


Figure: 10.8: Binomial tree concept extended for more than one stage and more than one state

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The concept illustrated with this simple two tree diagram is denoted by the binomial tree concept (Figure 18), and in case there are more than two such outcomes we have the multi-nomial tree concept as shown below through Figure 19.

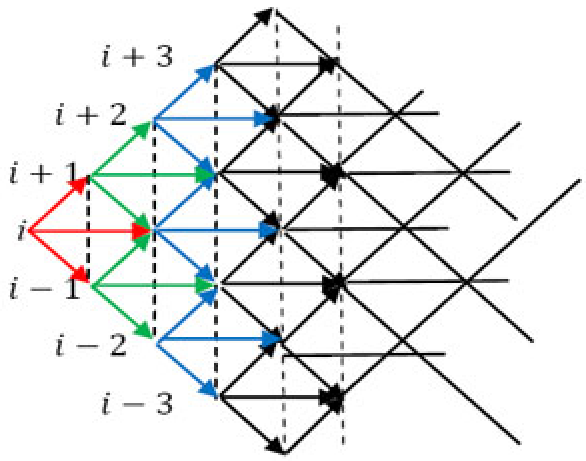


Figure: 10.9: Multinomial (for three) tree concept extended for more than one stage and more than one state

Now consider that we are interested to form a hedged portfolio where the portfolio is a stock and a claim, hence the portfolio value can either increase or decrease by a certain value as depicted in the diagram below (Figure 20)

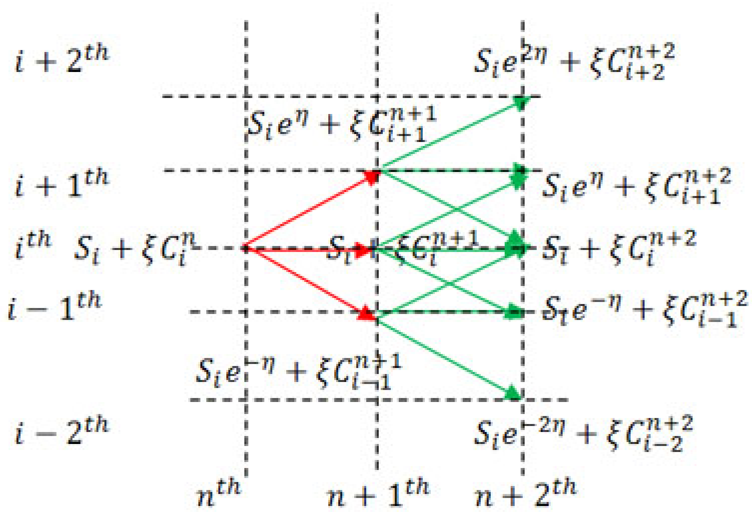


Figure 10.10: A combination of a stock and a claim

Here $\xi$ is a fixed number and the portfolio as we can see consists of (i) ***one stock*** and (ii) ***one claim***.

Now our task it to choose the value of $\eta$ (say $\xi^*$) such that the pay offs are same in each state, which means that

$S_i e^{\eta}+\xi C_{i+1}^{n+1}=S_i e^{-\eta}+\xi C_{i-1}^{n+1}$, from which we have: $\xi^*=-\left\{\frac{S_i e^{\eta}-S_i e^{-\eta}}{C_{i+1}^{n+1}-C_{i-1}^{n+1}}\right\}$

Thus we have created a ***risk free*** portfolio or investment to avoid any arbitrage opportunity. Now one should remember that this investment or portfolio should grow at the ***risk free interest rate***, $r$, else the opportunity of making extra-ordinary profit is evident.

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