Module 5:Understanding of applications of renewal theory, Stationary Process with discrete and continuous parameters

Lecture 21:Theoretical Backgground for the Simulation Study to study Sequential Sampling Procedures

The Lecture Contains:

- Simulation study using Gamma distribution
- Simulation study using Extreme Value Distribution
- Simulation study using Normal distribution
- Simulation study using Exponential distribution

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Simulation study using Normal distribution: Suppose $X \sim N(\mu, \sigma^2)$, and one is interested to estimate the location parameter, μ . Now in order to compare the efficiencies of the multistage sampling procedures we consider the bounded risk estimation problem of μ , for both (i) SEL and (ii) linear exponential (LINEX) loss functions. The estimate of μ is $\hat{\mu} = \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, and the bounded SEL and LINEX loss risk problems may be written as $\left\{R(\bar{X}_n, \mu) = E\left[(\bar{X}_n - \mu)^2\right] = \frac{\sigma^2}{n} \le w\right\}$ and $\left\{R(\bar{X}_n, \mu) = E\left[e^{-a(\bar{X}_n - \mu)} - a(\bar{X}_n - \mu) - 1\right] = \left(e^{\frac{a^2\sigma^2}{2n}} - 1\right) \le w\right\}$, respectively. The corresponding optimal sample sizes, with a known value of the scale parameter, σ, are then $D \ge \left(\frac{\sigma^2}{w}\right)$ and $D \ge \left\{\frac{a^2\sigma^2}{2\log_e(1+w)}\right\}$ respectively. In case s is unknown, then both the bounded risk problems may be solved using different adaptive sampling procedures.

Simulation study using Exponential distribution: Consider the exponential distribution, $X \sim E(\sigma, \lambda)$, with location parameter, λ and scale parameter as σ. We highlight the advantages of using sequential sampling procedures when one is interested to estimate, λ If $(X_1, X_2, \ldots, X_n)$ is the set of sample observations, then the best estimate of λ is $X_{(1)} = \min\{X_1, X_2, \ldots, X_n\}$, for both known and unknown σ values. Now the bounded SEL risk problem may be formulated as $\left\{R(X_{(1)}, \lambda) = E\left[(X_{(1)} - \lambda)^2\right] = \frac{\sigma^2}{n^2} \le w\right\}$, while the counterpart for LINEX loss bounded risk is $\left\{R(X_{(1)}, \lambda) = E\left[e^{-a(X_{(1)} - \lambda)} - a(X_{(1)} - \lambda) - 1\right] = \left[\left\{\frac{1}{1 - \left(\frac{a\sigma}{n}\right)}\right\} - \left(\frac{a\sigma}{n}\right) - 1\right] \le w\right\}$. The corresponding optimal sample sizes, are $D \ge \left(\sqrt{\frac{\sigma^2}{w}}\right)$ and $D \ge \frac{1}{2}\left\{a + |a|\left(1 + \frac{4}{w}\right)^{\frac{1}{2}}\right\}\sigma$ respectively. So with s being unknown, one may solve both these bounded risk problems using different multi-stage sampling methodologies.

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Simulation study using Gamma distribution: Next assume $X \sim G(\alpha, \beta. \gamma)$, i.e., the gamma distribution, where α , β and γ as the *shape*, *scale* and *location* parameters respectively. For simplicity assume $\beta = 1$, $\gamma = 0$, then one can easily see that α=E(X), for which the best estimate is $\hat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} X_i$. Considering the bounded SEL and LINEX risk problem for the gamma distribution the corresponding optimal sample sizes, are respectively given by $D \geq \left(\frac{\alpha}{w}\right)$ and $D \geq \left\{\frac{a^2\alpha}{2\log_e(1+w)}\right\}$ (with $-1 < a < n$). Thus with unknown σ one may solve both these bounded risk problems using any one of the sequential analysis methods discussed earlier.

Simulation study using Extreme Value Distribution: Finally consider $X \sim EVD(\mu, \sigma)$ as the Extreme Value Distribution (EVD) with m as its location parameter and s as the scale parameter. We know the estimate of σ is $\hat{\sigma} = \sqrt{\frac{6}{\pi^2}} \times \sqrt{\left(\frac{1}{n-1}\right) \times \sum_{i=1}^{n}\left(X_i - \bar{X}_n\right)^2}$, where $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, while that of μ is $\hat{\mu} = \bar{X}_n - \gamma_E \hat{\sigma}$. Hence the SEL bounded risk optimal sample size is $D \geq \left(\frac{\pi^2\sigma^2}{6w}\right)$. In case one considers the LINEX loss function, then the optimal sample size for the bounded LINEX loss risk is $D \geq \left[\frac{-a^2\sigma^2}{2\{\log_e(1+w+a\gamma\sigma)+a\sigma\}}\right]$, when $|a| \leq \frac{n}{\sigma}$ and $a \neq 0$. Hence to find the optimal estimate of μ , with unknown s, both these bounded risk problems may be solved employing any one of the sequential sampling methods discussed. Furthermore this estimate of μ may be used to calculate E(X). Hence one first finds the bounded risk estimate of μ and then calculates E(X) using $\hat{E}(X) = \hat{\mu}_N + \gamma_E \sigma$, where $\hat{\mu}_N$, is the estimate of μ found using any one of the sequential sampling plans.

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