Lecture 14:Renewal Theory

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Introduction

First let us consider few examples to initiate the reader for a better understanding of the concept of ***renewal process***, ***renewal theory*** and ensuing theorems based on the concept of ***renewal process*** and ***renewal theory***.

Example 4.1

Assume you are playing a game in which you roll a dice and you are interested to find the pattern of occurrence of the number 2. Let us define the event which corresponds to the pattern that the number 2 occurs if the rolling of the dice is repeated continuously. Then this event can be defined as a simple example of ***renewal process***.

Example 4.2

In the second example consider you as the marketing manager of a refrigerator manufacturing firm and you are conducting a survey to find the need of giving a free service contract to all the customers who have purchased your company's refrigerator in the past. In case the working life of the refrigerator which is being used is more than 4 years then you would give the customer the ***new free service scheme***, otherwise no such scheme or benefit is given to that customer. Suppose now you are interested to find the event which corresponds to the pattern of finding refrigerators which are more than 4 years old, then, you can term this study as an illustration of ***renewal process***.

Example 4.3

Let us continue our discussion about renewal process with another example. You are the maintenance-in-charge of a sophisticated CNC machine. The machine has n number of similar components which are critical for the working of the CNC machine. Each of the n components has an operational life which is distributed exponentially with a mean of years. You as the maintenance-in-charge may either replace the component or let it fail till you change it. Thus if you define the event as the number of component failures in some specific time, say T, then one can study this phenomena using the concepts of ***renewal theory***.

Example 4.4

Consider you have a light socket in your room and you put a CFL light in it. The moment the first light bulb fails you replace that with a similar type of second bulb. In case the second bulb fails you replace that with the third CFL bulb and so on. Then the event which denotes the successive lifetimes of bulbs which are being replaced can be explained using the concept of ***renewal process***.

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Definition

A ***renewal*** or a ***counting process*** nomenclature by $\{N(t), t \geq 0\}$ is a non-negative stochastic process which denotes the occurrences of an ***event*** (this is what we were discussing in the above four examples) during the time interval $(0, t]$ where the time duration between consecutive events are (i) ***positive***, (ii) ***independent*** and (iii) ***identically distributed*** random variables, i.e., *i.i.d*. r.v.

Let $\{X_k\}_{k=1}^{\infty}$ denote the successive occurence times of components which are counted (number 2 in Example # 4.1 or the refrigerators for which you are willing to give free service facilities in Example # 4.2 or the components which you may replace in case of failure or in case of general replacement schedule which you have planned for the machine in Example # 4.3 or in Example # 4.4 where you replace the electric bulb the moment the earlier one stops working), and successively placed into service, such that X_i is the elapsed time from the occurrence of the $(i-1)^{st}$ event until the occurrence of the i^{th} event.

Then we can write $F(x) = P\{X_k \leq x\}$ for $k = 1,2 \ldots ..$ which will denote the common probability distribution of $\{X_k\}$. A quick look at the probability distribution will make it obvious that the following is true, which is $F(0) = 0$, and it signifies that X_k's are positive random variable.

Now let us define $S_n = X_1 + X_2 + \cdots + X_n$ for $n \geq 1$ and it is also obvious from simple logic and convention that $S_0 = 0$. Moreover S_n is the ***waiting time*** (***total waiting time***) until the occurrence of the n^{th} event. Note $N(t)$ is equal to the number of indices n for which $0 < S_n \leq t$, hence we depict the situation using Figure 4.1.

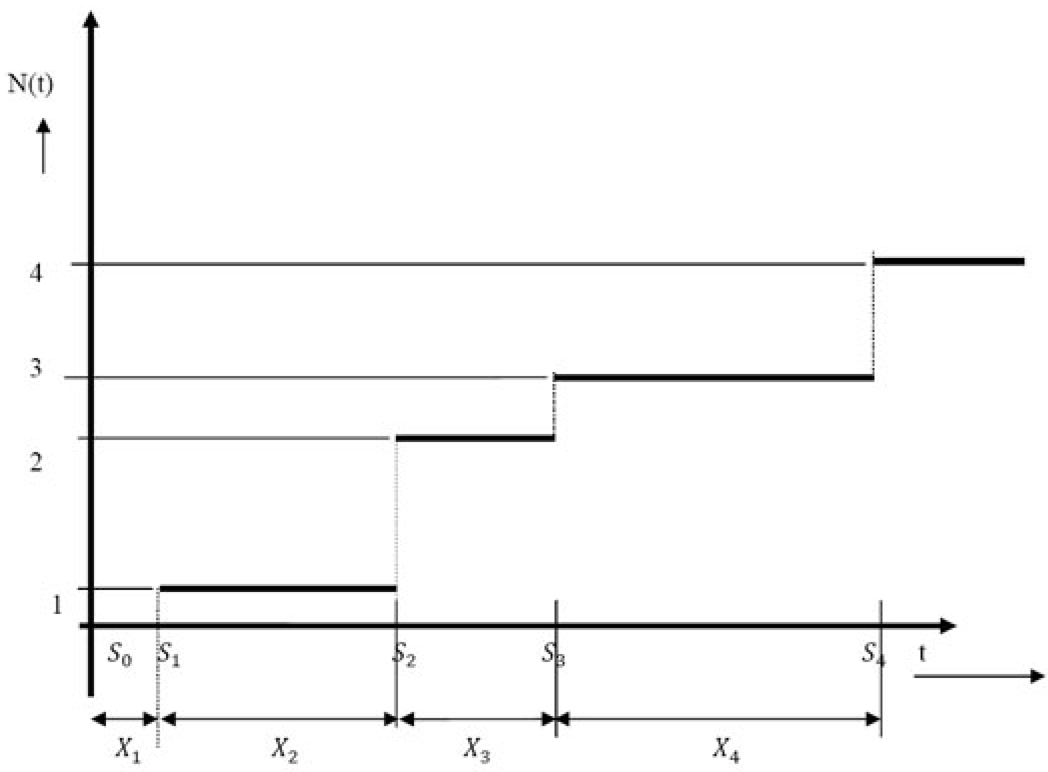

Figure 4.1: Simple concept of renewal process using diagrammatic illustration

Note

Thus if one considers Figure 4.1 then we have (i) $\{N(t), t \geq 0\}$ as the ***counting process***, while (ii) $\{S_n, n \geq 0\}$ as the ***partial sum process***. Thus (i) or (ii) interchangeable is called the ***renewal process***.

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