The Lecture Contains:

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Module 6:Random walks and related areas

Lecture 27:Martingles

Martingales

1. Suppose $X(t)$ is a real values state space in either discrete or continuous time. Then $\{X(t): t \in T\}$ is a ***martingale*** iff

a. $E\{X(t)\} < \infty, \forall\, t \in T$

b. $\forall\, t_1 < t_2 < \cdots . < t_{n+1}$

$E\{X(t_{n+1})|X(t_1) = a_1, X(t_2) = a_2, \ldots, X(t_n) = a_n\} = a_n\ \forall$ a, t

Martingle talks of expected value and states that the expected value in finite.

2. $\{X(t): t\epsilon T\}$ is a Markov Process iff $\forall\, t_1 < t_2 < \cdots . < t_n < t$ for which

$P\{a < X(t) \leq b|X(t_1) = x_1, X(t_2) = x_2, \ldots.., X(t_n) = x_n\} = P\{a < X(t) \leq b|X(t_n) = x_n\}\ \forall$ x, t.
One must remember that *a* and *b* are arbitrary.

Markov Process talks about distribution function : Hence the concept of a Martingle states that the expected value of future depends only on present. While the Markov process states that the distribution function of future depends only on the present.

Let A be an interval of the real line, then $P(x, s: t, \mathcal{A}) = P(X(t) \in \mathcal{A}|X(s) = x)$, $t > s$ is called the transition probability function. Diagrammatically we can denote this in Figure 6.7

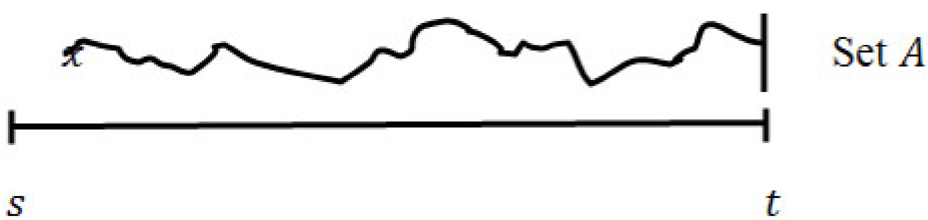

Figure 6.7: Diagrammatic representation of transition probability function

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Classification of states of a Markov Chain

State j is said to be ***accessible*** from state i if from some integer n, $P_{ij}^{n} > 0\ (n \geq 0)$, where n denotes the n stage transition.
Thus diagrammatically we may denote it simply as $i \rightarrow j$, which means that j is ***accessible*** from state i. On the other hand two states i and j which are ***accessible*** to each other are said to ***communicate*** and the way to mark it schematically is $i \leftrightarrow j$

The three characteristics which define equivalence relationship when two states communicate are

1. Reflexcive : Which is implied by $i \leftrightarrow i$
2. Symmetric : Which is denoted by $i \leftrightarrow j \Rightarrow\ j \leftrightarrow i$
3. Transitive : Which means that $i \leftrightarrow j, j \leftrightarrow k\ \Rightarrow i \leftrightarrow k$

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Module 6:Random walks and related areas

Lecture 27:Martingles

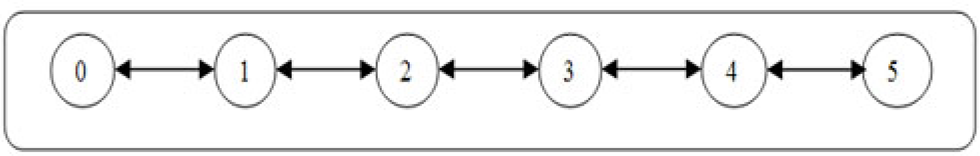
Consider the probability matrix given below which denotes the communication network between 6 nodes marked 0, 1, 2, 3, 4 and 5.

	0	**1**	**2**	**3**	**4**	**5**
0	1	0	0	0	0	0
1	½	0	½	0	0	0
2	0	½	0	½	0	0
3	0	0	½	0	½	0
4	0	0	0	½	0	½
5	0	0	0	0	0	1

A careful look at the matrix would make it apparent that following are the equivalence classes

Now if we have the matrix as

	0	1	2	3	4	5
0	0	2	0	0	0	0
1	½	0	½	0	0	0
2	0	½	0	½	0	0
3	0	0	½	0	½	0
4	0	0	0	½	0	½
5	0	0	0	0	1	0



Then we have only one equivalent class and it is easily discernible that one can reach any state from any other state.

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