The Lecture Contains:

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Branching process

Let us consider a population such that it consists of individuals which produce off springs of the same kind in the next generation and this continues from generation to generation. If an individual produces i number of off springs, then p_i is the corresponding probability, and this probability is ***independent*** of how the other reproduces or propagate. For illustration let us consider the growth of bacteria or amoeba, where for simplicity we assume the following:

Step 1: X_0: The zeroth generation, i.e., from where we begin the whole process and let us term this as the ancestor.

Step 2: X_1: Off spring of X_0 and is called the first generation (wrt to X_0).

Step 3: X_2: Off spring of X_1 and is called the second generation (wrt to X_0).

Step 4:………

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Step n: X_n: Offspring of X_{n-1} and called the n^{th} generation (wrt to X_0).

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Module 3:Branching process, Application of Markov chains, Markov Processes with discrete and continuous state space

Lecture 9:Branching process

Now remember that individuals of any generation may reproduce without depending on any other individual of any previous generations or any one of the same generation.

Thus we have X_i as the random variable (r.v) which denotes the offspring of an individual and $P(X_i = i) = p_i$, $i = 0,1,2,\ldots$, such that $\sum_{\forall i} p_i = 1$ and $p_i \geq 0$ hold true. Here p_i is the offspring distribution of an individual and it is generally denoted by $\{p_i : i = 0,1,2,3 \ldots\}$. This process can be considered a simple example of ***Markov Chain*** and we are interested in the distribution of the *nth* generation size.

Also $P(X_n = 0) = 1$, would simply imply the extinction of the species at any *nth* generation, and also remember $X_n = 0 \Rightarrow X_m = 0$, $\forall m > n$.

To start the process we assume $X_0 = 1$ and μ and σ^2 as the mean and the variance of the off spring distribution, i.e., of X_1 assuming $X_0 = 1$. Then: $\mu = E(X) = \sum_{\forall i}(i \times p_i)$ and $\sigma^2 = V(X) = \sum_{\forall i}(i - \mu)^2 \times p_i$ holds true.

With $X_0 = 1$, the following diagram (Figure 3.1) will illustrate the ***branching process*** more clearly

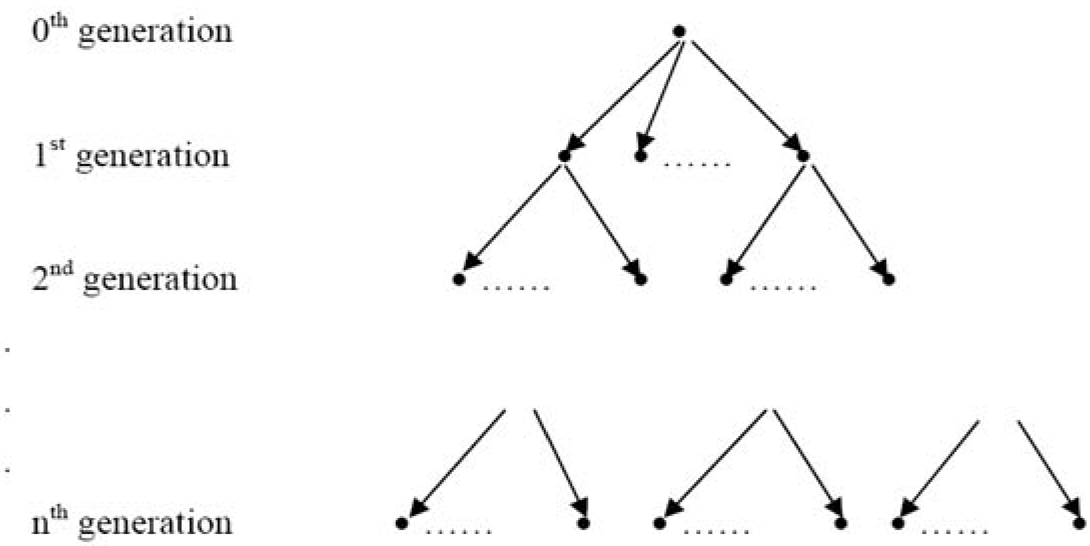

Figure 3.1: A typical example of branching process

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WALD'S EQUATION

Let Y_j denotes the number of the off springs of the j^{th} individual in the $(n-1)^{th}$ generation, such that $X_n = \sum_{j=1}^{n-1} Y_j$. One should remember that both number of off springs as well as generation number are non-deterministic.

Expectation

$E(X_n) = E\left(\sum_{j=1}^{n-1} Y_j\right)$, where $Y_j^{'}s$ are *i.i.d* random variables (r.v's). Thus

$$E(X_n) = E\left\{E\left(\sum_{j=1}^{n-1} Y_j \,|X_{n-1}\right)\right\} = E(X_{n-1} \times \mu) = \mu \times E(X_{n-1})$$

$$E(X_n) = \mu \times E(X_{n-1}) = \mu \times E\left\{E\left(\sum_{j=1}^{n-2} Y_j \,|X_{n-2}\right)\right\} = \mu \times E(X_{n-2} \times \mu) = \mu^2 \times E(X_{n-2})$$

$$E(X_n) = \mu^2 \times E(X_{n-2}) = \mu^2 \times E\left\{E\left(\sum_{j=1}^{n-3} Y_j \,|X_{n-3}\right)\right\} = \mu^2 \times E(X_{n-3} \times \mu) = \mu^3 \times E(X_{n-3})$$

Generalizing we have : $E(X_n) = \mu^n E(X_0) = \mu^n$.

The equation $E(Z) = E\left(\sum_{i=1}^{Y} X_i\right) = E(Y) \times E(X_i)$, where $Z = \sum_{i=1}^{Y} X_i$ is the sum of random number of random variables is frequently in sequential analysis. Now as $E(X_0 = 1) = 1$, hence one can easily prove that $E(X_n) = \mu^n$

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