Module 3:Branching process, Application of Markov chains, Markov Processes with discrete and continuous state space

Lecture 11:Application of Markov Process

The Lecture Contains:

Previous Next

Example 3.1

Let $p_0 = \frac{1}{4}$, $p_1 = \frac{1}{3}$, $p_2 = \frac{5}{12}$, i.e., $\Pr\{X_1 = 0|X_0 = 1\} = \frac{1}{4}$, $\Pr\{X_1 = 1|X_0 = 1\} = \frac{1}{3}$ and

$\Pr\{X_1 = 2|X_0 = 1\} = \frac{5}{12}$, such that

$\Pr\{X_1 = 0|X_0 = 1\} + \Pr\{X_1 = 1|X_0 = 1\} + \Pr\{X_1 = 2|X_0 = 1\} = \frac{1}{4} + \frac{1}{3} + \frac{5}{12} = 1$. Hence we can find

$$\mu = 0 \times \frac{1}{4} + 1 \times \frac{1}{3} + 2 \times \frac{5}{12} = \frac{14}{12} > 1.$$

Given this we are interested to find the probability of extinction and that is definitely not 1. Furthermore we have:

$$\pi_0 = \sum_{\forall i} \pi_0^i \times p_i$$

$$\pi_0 = \pi_0^0 \times \frac{1}{4} + \pi_0^1 \times \frac{1}{3} + \pi_0^2 \times \frac{5}{12}$$

$5\pi_0^2 - 3\pi_0 + 3 = 0$, i.e., $(5\pi_0 - 3)(\pi_0 - 1) = 0$, i.e., $\pi_0 = \frac{3}{5}$ and $\pi_0 = 1$.

Hence the probability of ultimate extinction is 0.6

Previous Next

Example 3.2

In a branching process the number of off springs per individual has a binomial distribution with parameters 2 and p. Starting with a single individual calculate (i) the extinction probability, (ii) the probability that the population becomes extinct for the first time in the third generation. Suppose that instead of starting with a single individual the initial population size, Z_0 is a random variable that is Poisson distributed with mean, λ. One can easily show that in this case the extinction probability is given for $p > \frac{1}{2}$ by $e^{\frac{\lambda(1-2p)}{p^2}}$.

Example 3.3

Consider a branching process in which the number of offspring per individual has a Poisson distribution with meanλ, $\lambda > 1$. Let π_0 denote the probability that, starting with a single individual the population eventually becomes extinct. Also let , $a < 1$ be such that $ae^{-a} = \lambda e^{-\lambda}$. We can prove that (i) $a = \lambda\pi_0$, (ii) conditional on eventual extinction, the branching process follows the same probability law as the branching process in which the number of offspring per individual is Poisson with mean a.

Previous Next

Markov Process with discrete state space in continuous time

Till now in all our analysis we considered time component to be discrete and as such transitions occurred at $n, n+1, \ldots.$ intervals. In case the time component is continuous we need to consider transitions which occur at small time intervals e.g., $t, t+\Delta t, ..$ and in the limiting case $\Delta t \to 0$.

As per our earlier convention let us assume $X(t)$ to be a continuous parameter ***Markov process*** with state space $N = \{0,1,2,,,\}$. In case $X(t)$ is ***time homogeneous*** then we know the transition probability from state i to j ***depends only on*** the state and time interval and ***not*** on the initial time when it was at state i.

Thus we have the following set of equations which are true for this case.

$$P\{X(T+t) = j | X(T) = i\} = p_{i,j}^{(T,T+t)} = p_{i,j}^{(t)} \qquad (3.2)$$

As the property of ***time homogeneous*** holds hence we also have

$$P\{X(t) = j | X(0) = i\} = p_{i,j}^{(0,t)} = p_{i,j}^{(t)} \qquad (3.3)$$

Furthermore the following facts are also true:

$$0 \le p_{i,j}^{(t)} \le 1 \text{ for each } i, j, t \qquad (3.4)$$

$$\sum_{\forall j} p_{i,j}^{(t)} = 1 \qquad (3.5)$$

Previous Next

Waiting time for a change of state

Suppose $X(t)$ is a homogeneous ***Markov process*** and that at time, $t_0 = 0$ the state of the process $X(t_0) = X(0) = i$ is know to us. Consider the change of the state from i to j in some arbitrary time τ, thus we have $P\{\tau > s + t | X(0) = i\}$.

Markov Process with continuous state space in continuous time

In this part we will consider that both state and space changes are ***continuous*** in nature. A well known example of this type of stochastic process is the ***Browian motion***. The process derives it name from that of the British botanist by the name of Robert Brown who noticed the random and erratic behavious of pollen grains. Other pioneering work related to this area was done by Einstein and Weiner. Due to its similarities with the concept of diffusion, ***Brownian motion*** is also called a ***diffusion process***.

At this stage of our discussion it is imperative to mention that we consider the one dimension Brownian motion only.

Previous Next