Module 5:Understanding of applications of renewal theory, Stationary Process with discrete and continuous parameters

## Lecture 23:More Practical Application of Sequential Sampling Procedure

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## Example 5.2

Let us consider another example of renewal theory, where we discuss the problem relate to arrival of customers in a grocer shop or departmental store. Let us consider the grocer store has $J$ number of different items on display and amongst these $J$ number of items the customer chooses his/her product(s). Let the inter-arrival times of customers have a distribution, $F$. Furthermore one can safely assume that the amounts desired of any product amongst $J$ is independent, and the distribution is $G_j$, $j = 1,2.,,,J$. The store uses the $(s_j,S_j)$ ordering policy, which means if the level of stock for the $j^{th}$ item falls below $s_j$, then the inventory of the same is brought back to $S_j$ and that is done instantaneously (which is practically not possible, yet we assume this for simplicity).

Thus the inventory level after serving the customer if be denoted by $x_j$ then we have:

$S_j - x_j$ if $x_j < s_j$

0 if $x_j \geq s_j$

In case if we denote $X_j(t)$ as the inventory level for the $j^{th}$ item at time $t$, then our main concern is to find $\lim_{t\to\infty} P\{X_j(t) \geq x_j\}$ which is given by $\left\{\frac{E\left(\sum_{i=1}^{N_{x_j}} X_{j,i}\right)}{E\left(\sum_{i=1}^{N_{s_j}} X_{j,i}\right)}\right\}$, where

$\sum_{i=1}^{N_{x_j}} X_{j,i}$ = Amount of time in the cycle when the inventory is not ordered.

$\left(\sum_{i=1}^{N_{s_j}} X_{j,i}\right)$ = Amount of total time of the cycle.

## Example 5.3

As a third example suppose a sophisticated machine consists of $n$ crucial parts/components which function independently (even though this is not valid, yet we take this assumption to be true for our example). Each part of the machine has an exponential alternating renewal process in which it fluctuates by going up and down (like a switch with ON and OFF positions) for which the averages times are $\lambda_i$ and $\mu_i$. The sophisticated machine is said to function if at least one of the $n$ component is in ON position. Let us also denote by $N(t)$ the number of time the sophisticated machine breaks down in the time interval of $[0,t]$.

Then the aim of the shop floor manager is to calculate the mean time between breakdowns of this sophisticated machine. In order to calculate this, he/she will try to find out the probability of breakdown in the time interval of $(t,t+h)$, where $h$ is very small. A simple concept using which we can find this value is to first consider the case when only one part/component is functioning and the rest $(n-1)$ are down and then after that have all the parts/components fail. Using calculations we can show that the value is given by $\lim_{t\to\infty} P\{system\ is\ down\ at\ t\} = \prod_{j=1}^{n}\left(\frac{\mu_j}{\lambda_j+\mu_j}\right)$.

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## Example 5.4

Consider passengers arrive at Jhakarkati bus depot in Kanpur, India, in accordance with renewal process with a mean arrival time of $\mu$ minutes. Now the scheduling manager of the bus depot decides that when ever there are $N$ number of passengers the bus (which are also in a queue) leaves. Also assume that when the number of passengers are $n$ then the bus depot incurs a costs like running the air conditioning system etc., and that this cost is $nc$ per unit time. Apart from this cost, an additional cost of $K$ is incurred by the bus company when ever a bus leaves with passengers. Our main concer is to calculate the average cost per unit time incurred by the bus depot. Assume the cycle is such that it is completed when a bus leaves and this in very simple terms is a renewal process.

Thus $E[length\, of\, cycle] = N\mu$. Now if we denote the time between $n^{th}$ and $(n+1)^{th}$ arrival in a cycle, then the expected cost of a cyle can be expressed as:

$$E[cost\ of\ a\ cycle] = E[cX_1 + 2cX_2 + \cdots. + (N-1)cX_{N-1}] + K$$

$$= \frac{c\mu N(N-1)}{2} + K$$

Thus average cost is $\left\{\frac{c(N-1)}{2} + \frac{K}{\mu N}\right\}$

## Example 5.4

Consider $U$ and $V$ are independent normally distributed random variables with zero means and unit variances and along with that let us refer to the diagram given below which is a circle with radius $1$.

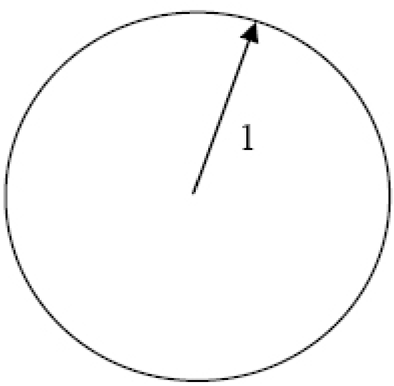


Figure 5.1: Circle with circumference and radius

Let us also assume $T$ is the circumference such that $T = [0, 2\pi]$. Also assume: $Y(t) = U\sin(t) + V\cos(t)$ and $Z(t) = -U\cos(t) + V\sin(t)$. Let us also denote $X(t) = (Y(t), Z(t))$ as a bivariate process. It is very clear from the example given above that (i) $E[Y(t)] = E[Z(t)] = 0$ and (ii) $E[Y(t)^2] = E[Z(t)^2] = 1$ since $\sin^2(t) + \cos^2(t) = 1$.
Since $Y(t)$ and $Z(t)$ have a joint normal distribution, hence we need to find their covariance. Moreover we can easily see that $E[Y(t)Z(t)] = 0$. Thus the distribution of $X(t)$ is the same as that of $X(t+\theta)$ for any value of $\theta$ which proves that fact that $\{X(t_1), X(t_2), \ldots, X(t_k)\}$ is a stationary process.

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## Example 5.5

Consider the financial time series which for case we take the example of BASF (a script in German stock exchange index DAX) prices from February 7, 2006 through February 6, 2007 and as required we first illustrate in Figure 5.2 and Figure 5.3 the prices and returns for this particular stock.

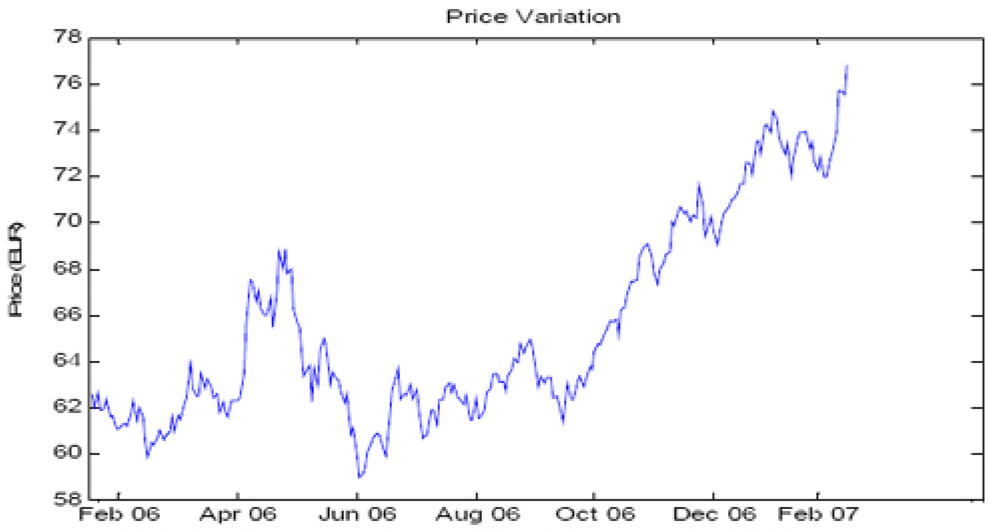


Figure 5.2: BASF prices from 7-Feb-2006 to 6-Feb-2007

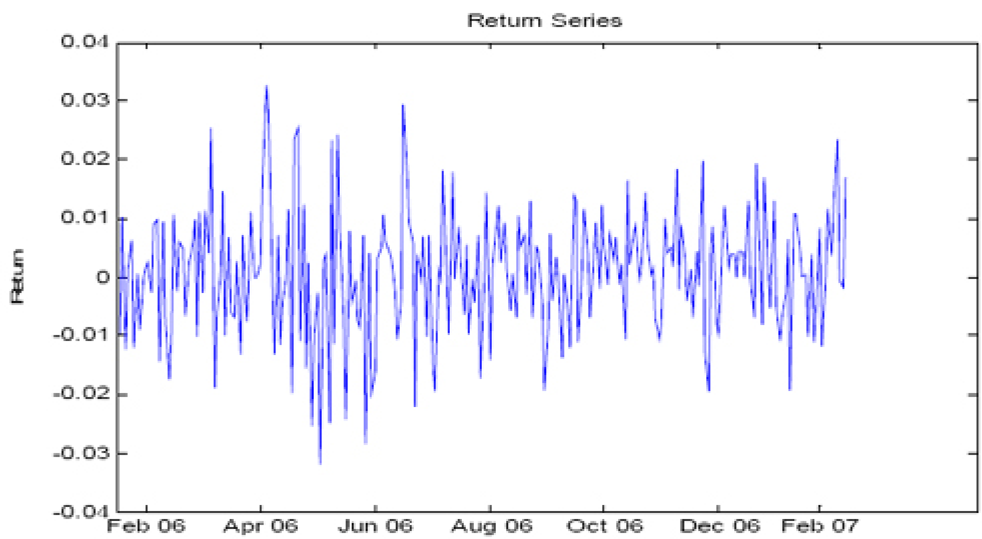


Figure 5.3: BASF return series from 7-Feb-2006 to 6-Feb-2007

One already knows that a very simple concept to check the stationarity of a stochastic process is by considering either the first or second difference of the stochastic cprocess.

Now again going back to our example. We estimate the model parameters and then examine the estimated GARCH model (time series model), and the parameters are estimated and their standard errors obtained for *BASF* are shown in Figure.5.4:

```
Mean: ARMAX(0,0,0); Variance: GARCH(1,1)

Conditional Probability Distribution: Gaussian
Number of Model Parameters Estimated: 4

                                 Standard          T
Parameter        Value            Error        Statistic
-----------   -----------    ------------   -----------
          C    0.0010764      0.0006622         1.6254
          K    2.8386e-006    3.1143e-006       0.9115
   GARCH(1)    0.92801        0.047386         19.5842
    ARCH(1)    0.047358       0.029413          1.6101
```

Figure.5.4. Parameter estimates and their Standard Errors of the GARCH (1,1) model for BASF

Now, substituting these estimates in the definition of the GARCH (1,1) the constant conditional mean GARCH(1,1) conditional variance model that best fits the observed data is:

$$y_t = .0010764 + \varepsilon_t$$
$$\sigma_t^2 = 2.8386\,e-006 + .047358\ \varepsilon_{t-1}^2 + .92801\ \ \sigma_{t-1}^2$$

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Now I the next step we analyze the residuals, conditional standard deviations and returns and as required Figure.5.5 shows the relationship between the innovations (i.e., residuals) derived from the fitted model (i.e., GARCH (1,1)), the corresponding conditional standard deviations, and the observed returns for *BASF*

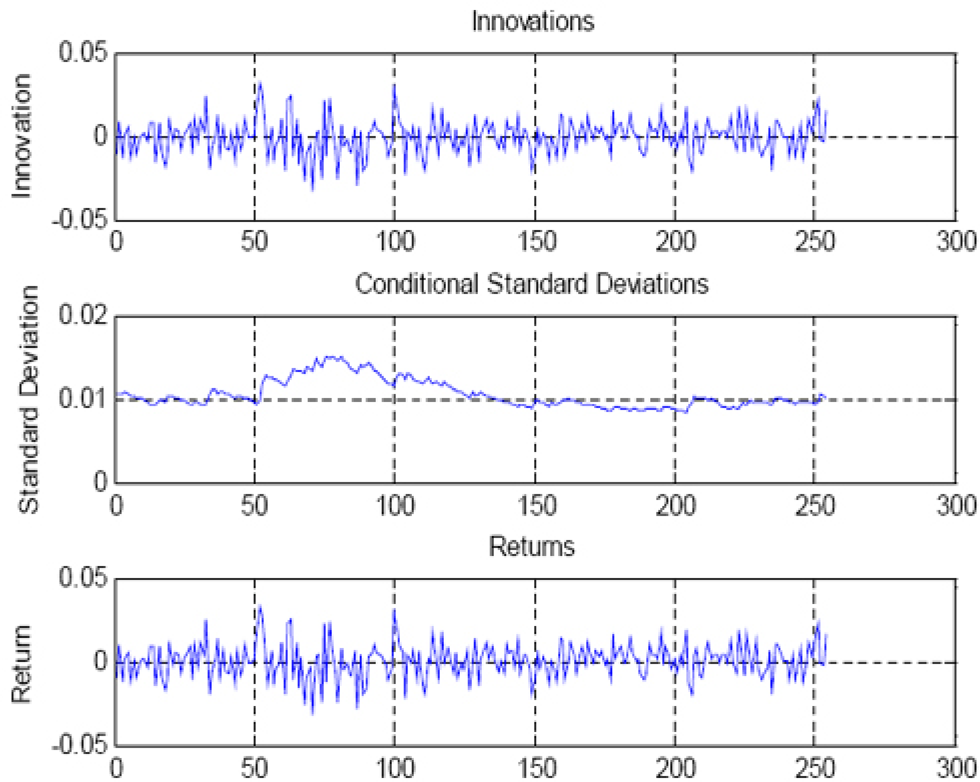


Figure.5.5. Innovations, Conditional Standard Deviations, and Returns of ***BASF***.

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