The Lecture Contains:

Note

Before we start this chapter few nomenclature conventions should be made clear for the reader in order to avoid any confusion later on

- $N(t)$:Denotes the number of events which occurs in the time interval $(0, t]$
- X_i : Denotes the time interval difference between the occurrence of the i^{th} and the event $(i-1)^{th}$
- S_n: Sum of all the times till the occurrence of all the n^{th} events, i.e.,$S_n = \{X_1 + \cdots + X_n\}$

Previous Next

Definition of a Counting process

A stochastic process is said to be a ***counting process*** if represents the total number of events that have occurred up to time . One should remember that for many practical examples like how many times will the number 1 occur when one rolls a dice 15 number of times, or the numbers of red coloured cars passing through a certain road crossing/junction when one is standing at the junction for the past 1 hours, etc., can be cited as simple examples to illustrate the concept of a ***counting process***.

In general a ***counting process*** should satisfy few properties which are as follows:

1. $N(t) \geq 0$: Which is logical and implies that the number of occurrences of the event which we are interested to count should definitely be positive.

2. $N(t)$ is an integer: This means that the number is always an integer and for the experiment when a person tosses a coin, then intuitively one can definitely comment that the number of heads one can get within a stipulated time can never be a fraction or decimal, but will always be an integer.

3. If $s < t$, then $N(s) \leq N(t)$: Implies that it may be possible that the number of occurrences of the stochastic process within the stipulated time interval of $(t - s)$ may be zero for which the number of arrivals in $(0, t]$, and $(0, s]$ will always be the same, else the number of occurrences in the time interval $(0, t]$ will be greater than that in $(0, s]$.

4. For $s < t$, $N(t) - N(s)$ equals the number of events that have occurred in the time interval between s and t, i.e.,$(s, t]$ and this is a follow up of # 3.

Previous Next

There are other two important properties for ***counting process*** worth mentioning and they are:

1. *Independent increments*: Which means that the numbers of events occurring between disjoint time intervals are independent. One can refer to Figure 2.1 which shows two disjoint time intervals $(0, t]$ and $(t, t+s]$, such that numbers of events occurring in these two disjoint intervals which are respectively $N(t)$ and $N(s)$ are assumed to be ***independent***.

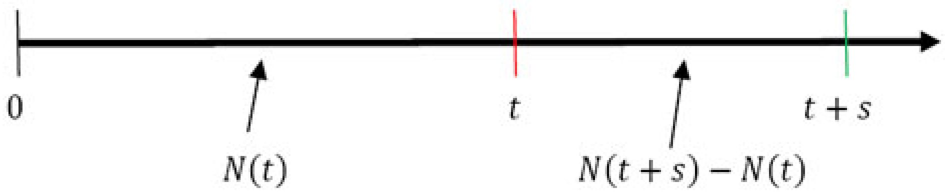

Figure 2.1: Illustration of counting process and the concept of independent increments

2. ***Stationary increments*** : When a counting process is said to have stationary increments, then it means that the number of events that occur only depends on the length of the respective time interval and not on the time interval's end points.It also means that the number of events in the interval $(t_1+s, t_2+s]$ has the same distribution as the number of events in the interval $(t_1, t_2]$. Thus if we refer to Figure 2.2 we see that the number of events $\{N(t_2+s)-N(t_1+s)\}$ shown in the green time interval and that of $\{N(t_2)-N(t_1)\}$ shown in the red time interval would have the same distribution. Furthermore one can comment that the number of events occurring between $(t, t+h]$, where h is positive depends on h only and not on t.

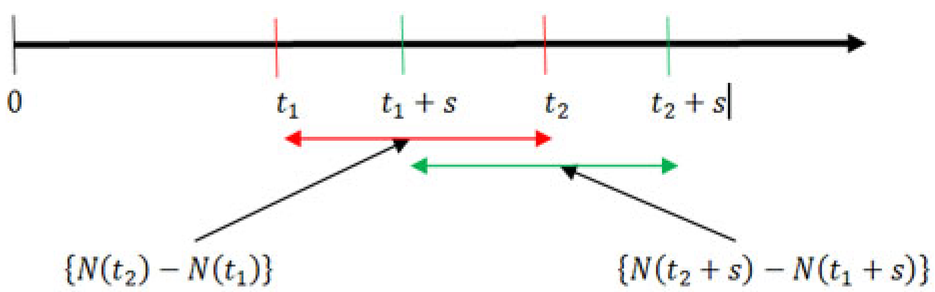

Figure 2.2: Illustration of counting process and the concept of stationary increments

Two important examples of counting process which we will consider in this course are
(i) Poisson Process and
(ii) Renewal process.

1. In this module/chapter we discuss in details the concepts of ***Poisson process*** and few

interesting examples of Poisson process and the relevance of Kolmorogov equation and its use.

Module 2:Poisson Process and Kolmorogov equations

Lecture 5:Posson Process

Poisson process

Definition 1: A counting process denoted by $\{N(t), t \geq 0\}$ is said to be a ***Poisson process*** if it has the following three properties which are:

1. $N(0) = 0$: Which means that the number of events when the process has just started is zero.

2. The process has ***independent increments***.

3. The number of events in the time interval $(0, t]$, i.e., of length t, where t can be of any length is Poisson distributed with mean λt. Thus for all values of s and $t \geq 0$ we have $P\{N(t+s) - N(s) = n\} = e^{-(\lambda t)} \times \frac{(\lambda t)^n}{n!}$, where λ is the ***rate of the process***.

Definition 2 : A counting process denoted by $\{N(t), t \geq 0\}$ is said to be a ***Poisson process*** with rate λ if it has the following four properties which are:

1. $N(0) = 0$: This means the number of occurrences at time $t = 0$, i.e., when the process has just started is zero.

2. The process has ***stationary (time homogeneity)*** and ***independent increments***.

3. $P\{N(h) = 1\} = \lambda h + o(h)$: Which means that the probability of the number of events in the time interval $(0, h]$ being exactly equal to 1 is equal to the product of the rate of the process and the time interval plus some incremental function of time interval, i.e.,h .

4. $P\{N(h) \geq 2\} = o(h)$: The fourth property denotes that in case we are interested to find the probability that the number of events in the time interval $(0, h]$ is equal to 2 or more, then that probability becomes zero as the time interval shrinks or is made smaller and smaller. For the benefit of the reader we like to mention that a function $g(x)$ is said to be $o(x)$ if we have $\lim_{x \to 0}\left\{\frac{g(x)}{x}\right\} = 0$

Previous Next

Note

Definition # 1 and definition # 2 hold true in the iff sense which implies that definition # 1 $\Leftrightarrow$ definition # 2.

Thus a Poisson process gives us the basic model for the arrival of some information based on which further studies may be done. Suppose we have a function that counts the number of times a specific event occurs, e.g., the number of gamma rays being emitted by a radioactive isotope in the time interval of 5 minutes, or the number of phone calls taking place within a time frame of say 1 hour, or the number of customers arriving at the bank teller counter within 5 hours, etc.

Using the same nomenclature used previously, $N(t)$, would mean the number of events which occur in the time interval $(0,t]$. One should understand and it is also quite obvious that this function, $N(t)$, is a non-decreasing step function, as shown in Figure 2.3.

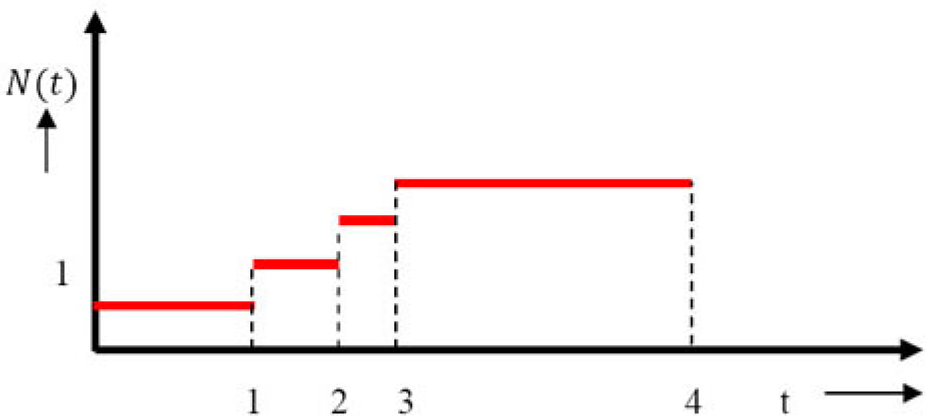

Figure 2.3: A hypothetical example of counting process which is also stationary process with respect to mean

If $N(t)$ has a probability mass function, then it is necessary continuous. However continuous r.v. need not posses density function, and for that recall examples of a ***continuous*** and a ***discrete*** distribution function which are respectively of the forms $f(x) = \lambda e^{-\lambda x}$, $x > 0$ and $f(x) = \frac{\lambda^n e^{-\lambda}}{n!}$, $n \geq 0$

Previous Next