Module 9:Application of stochastic processes in areas like manufacturing

Lecture 34:Profit Maximization in Manufacturing Process

The Lecture Contains:

- Profit maximization considering partial backlogging
- Example for backorder cases
- Backorders case with Poisson demands and constant procurement lead time
- The lost sales case for constant lead times A study of effect of information sharing and lead-times on bullwhip effect in a serial supply chain
  - Demand Process
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Profit maximization considering partial backlogging

Let us assume the case when each unit of demand results in a revenue of p, while each lost sales is backlogged with probability $(1-\beta_{LS})$, where β_{LS} is the probability of lost sales. One should remember that each lost sales results in a loss of unit revenue of value p and an additional unit cost of c_{LS} which may be due to loss of good will. Finally c_{BL} is the unit cost due to special ordering which is due to loss of good will and as such the main focus then is to expedite the order in order to regain back the good will lost.

Now considering both expected revenue and expected cost, one can find out the expected profit/loss which is given by the formulae given below in (9.1)

$$(p-c)\mu-(c+c_H)(y-\mu)-\{c_{BL}+c_H+\beta_{LS}(p+c_{LS}-c_{BL})\}\int_y^{\infty}(\xi-y)d\Phi(\xi) \qquad (9.1)$$

Example for backorder cases

The system under consideration is such that one is interested to find the optimal order quantiy, Q, at some reorder point r. Few relevant assumptions for the model are:

1. Unit cost of the item is C, which is constant and is independent of Q.
2. The per unit back ordered cost is π.
3. Maximum number of order outstanding is 1.
4. Cost for operating the information system in order to study the production process is independent of either Q or r.

For a better understanding of this order model one can refer to Figure 9.2, where orders are made at T_1, T_2, etc. time periods and the order level is r. Now considering the safety stock is s, then the average on hand inventory is given by $\frac{1}{2}(Q+s)+\frac{1}{2}s$.

If procurement lead time, L, is constant then the safety stock level is given by $\int_0^{\infty}(r-x)f(x;r)dx$. But in case lead time is stochastic with a probability distribution of $g(\tau)$, then the safety stock value is calculated as $\int_0^{\infty}f(x;r)g(\tau)d\tau$.

In case $h(x)$ represents the marginal distribution of lead time demand, and in case we consider that to be normally distributed then the annual variable cost $\mathcal{K}$ is given by the following expression (9.2).

$$\frac{\lambda}{Q}A+IC\left(\frac{Q}{2}+r-\mu\right)+\frac{\pi\lambda}{Q}\left\{(\mu-r)\Phi\left(\frac{r-\mu}{\sigma}\right)+\sigma\phi\left(\frac{r-\mu}{\sigma}\right)\right\} \qquad (9.2)$$

where:

1. A = Cost of placing an order
2. λ = Average annual demand
3. I = Inventory carrying cost
4. C = Unit cost of the item
5. π = Per unit back order cost
6. Q = Optimal order quantity
7. μ = Expected value of demand distribution, which is denoted by D

8. σ = Standard deviation of demand distribution, which is denoted by D

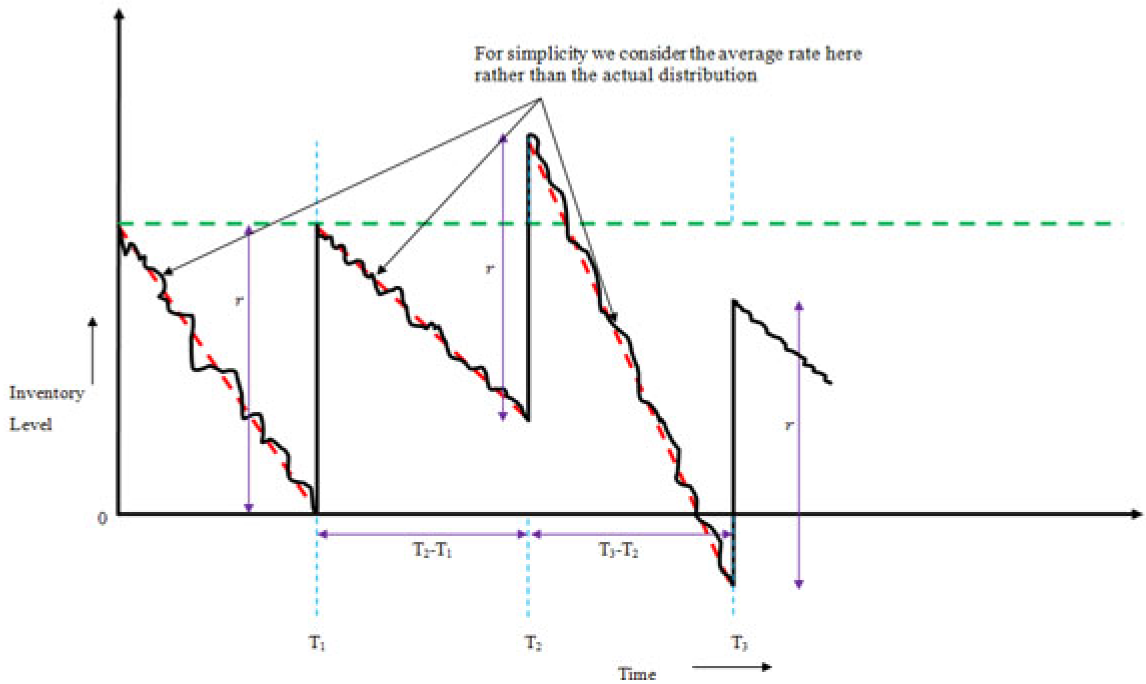

Figure 9.2: The Q, r system

Example for lost sales cases

For the lost sales case minimization of annual cost is equivalent to the maximization of the annual profit. The only difference between lost sales and back order models is in evaluating the safety stock expression.

In case $h(x)$ represents the marginal distribution of lead time demand, and in case we consider that to be normally distributed then the annual variable cost $\mathcal{K}$ is given by the following expression (9.3).

$$\frac{\lambda}{Q}A + IC\left(\frac{Q}{2} + r - \lambda\right) + \left(IC + \frac{\pi\lambda}{Q}\right) \times \left\{(\mu - r)\Phi\left(\frac{r-\mu}{\sigma}\right) + \sigma\phi\left(\frac{r-\mu}{\sigma}\right)\right\} \qquad (9.3)$$

where:

1. A = Cost of placing an order
2. λ = Average annual demand
3. I = Inventory carrying cost
4. C = Unit cost of the item
5. π = Per unit back order cost
6. Q = Optimal order quantity
7. μ = Expected value of demand distribution, which is denoted by D
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Backorders case with Poisson demands and constant procurement lead time

Here we would like to state the assumptions and the variables used thereof where the Poisson process generates the times between demands and they are:

1. λ = Mean rate of demand in units per year
2. τ = Procurement lead time which is constant
3. Q = Order quantity which is discrete
4. r = Reorder point, which is also considered as discrete

Few simple calculations will result in the following which are :

1. Expected number of backorders is $E(Q,r)$ and it is given by $E(Q,r) = \frac{\lambda}{Q}\{\alpha(r) - \alpha(r+Q)\}$, where $\alpha(v) = \lambda\tau P(v;\lambda\tau) - vP(v+1;\lambda\tau)$.
2. Expected number of backlogs is $B(Q,r)$ and it is given by $B(Q,r) = \frac{1}{Q}\{\beta(r) - \beta(r+Q)\}$, where $\beta(v) = \lambda\tau P(v;\lambda\tau) - vP(v+1;\lambda\tau)$.
3. Expected on hand inventory is $D(Q,r)$ and it is given by $D(Q,r) = \frac{Q+1}{2} + r - \mu + B(Q,r)$.
4. Average annual variable cost is $\mathcal{K}$ and it is given by $\mathcal{K} = \frac{\lambda}{Q}A + IC\left(\frac{Q}{2} + \frac{1}{2} + r - \mu\right) + \pi E(Q,r) + (\pi + IC)B(Q,r)$.

Few other examples where one can have different concepts of Q, r, S, s, etc are shown in Figures 9.3, 9.4 and 9.5

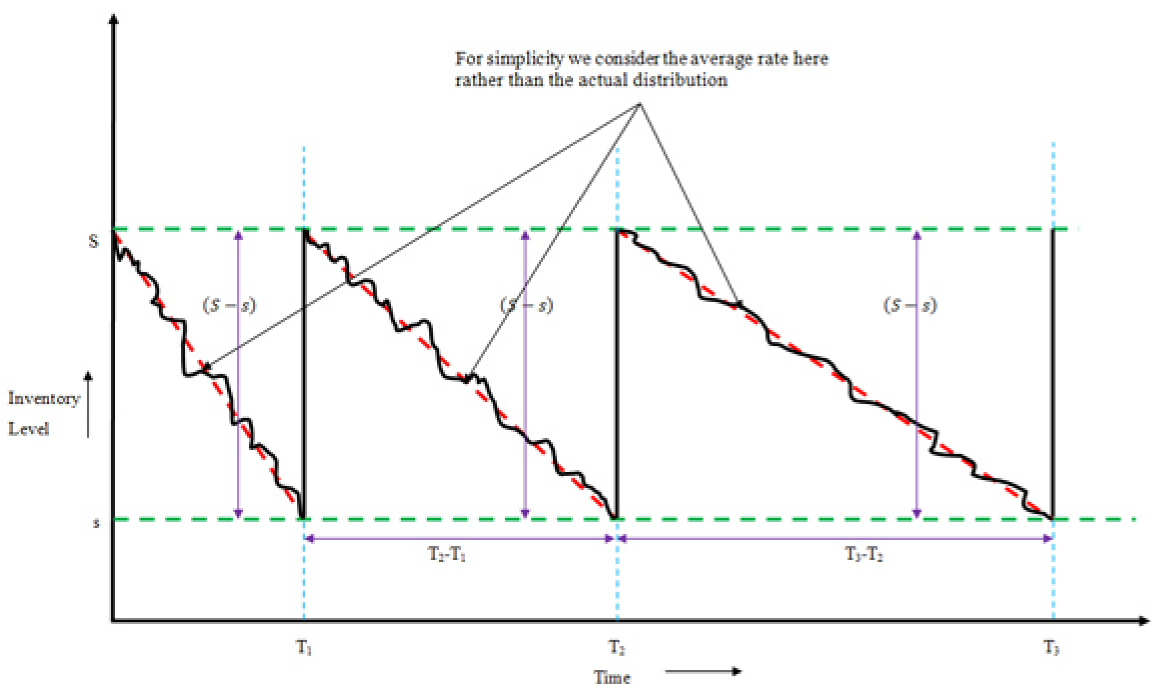

Figure 9.3: Ordering time periods are different, quantum of ordering is same, lead time is deterministic, base stock level is defined

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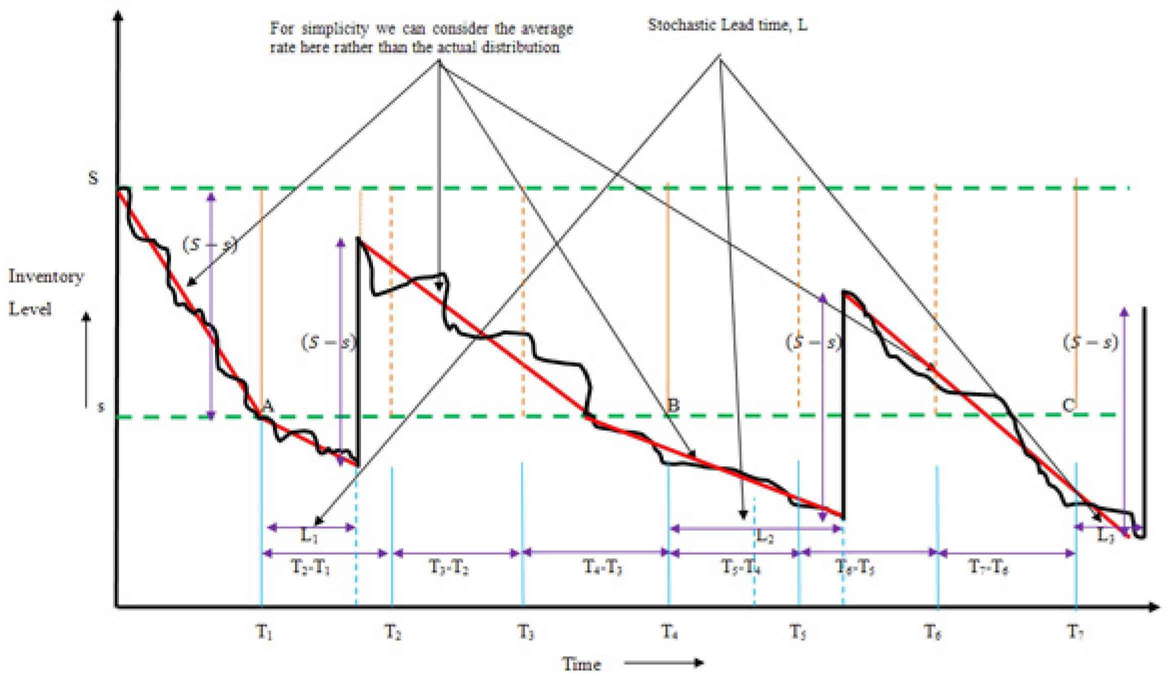

Figure 9.4: Ordering time periods are different, quantum of ordering is the same, lead time is stochastic, base stock level is defined

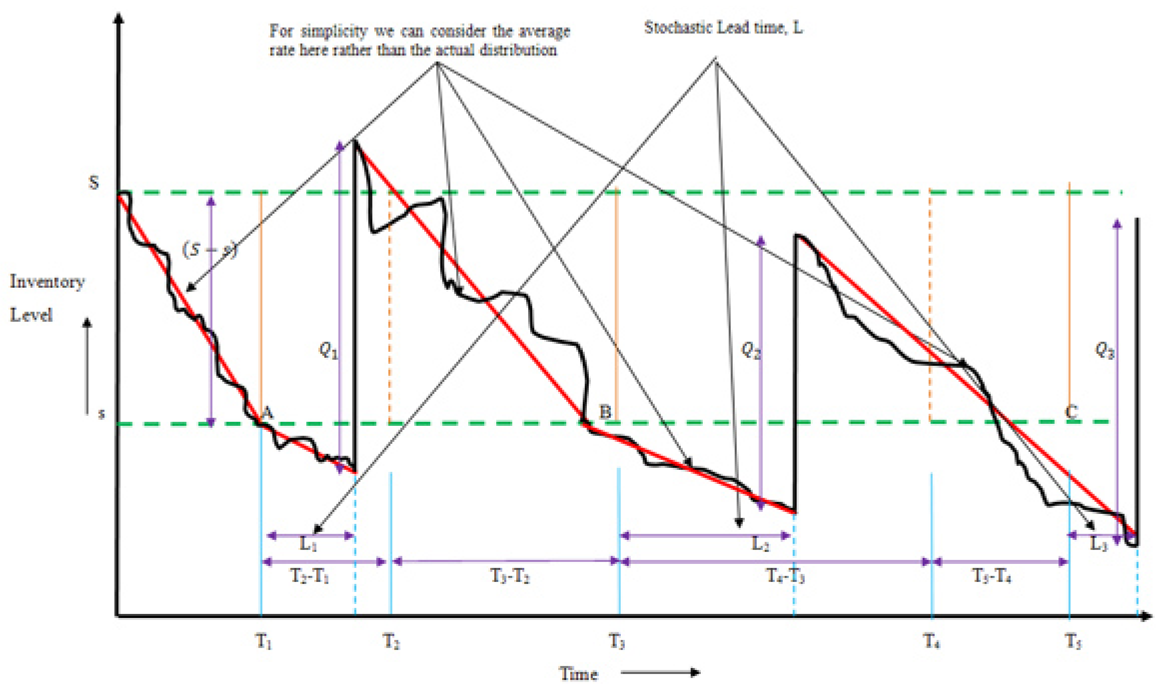

Figure 9.5: Ordering time periods are different, quantum of ordering is different, lead time is stochastic, base stock level is defined

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The lost sales case for constant lead times A study of effect of information sharing and lead-times on bullwhip effect in a serial supply chain

Increasing competition in the market generally leads to a high fluctuation in the demand of products. Such fluctuations pose a very severe problem at each stage of the supply chain, i.e., customer, retailer, warehouse, supplier and manufacturing, in deciding about the suitable inventory levels to maintain a good service level with minimum amount of holding cost. The problems compound further when the lead-times of replenishment are long and uncertain. Because of the longer lead-time, the uncertainty in the forecasting of the future demand increases, and consequently the variability of the order quantity increases. Any stage of the chain, apart from using the forecast values from its lower echelon, also has to do its own forecasting analysis while ordering to its next higher echelon. This naturally increases the ***variability*** of the order quantity. The phenomenon where there is an increase in the variance of the order quantity, as we move away from the end customer to the supplier in a supply chain, is defined as the ***bullwhip effect***. The demand process, lead-times, inventory policies and the forecasting models employed have significant bearing on the bullwhip effect. Among these, forecasting models, inventory policies and to some extent lead-times are controllable and hence can be suitably decided upon to reduce the bullwhip effect. Further more, the importance of sharing of relevant information across various stages of the supply chain is being increasingly realized and has been found to reduce the over all bullwhip effect.

Few of the relevant assumptions when considering models (Figure 9.1) like these are:

- Demand Process : Many models assume a probability distribution with known parameters to represent the demand process. The stationary Poisson distribution is widely used to model the demands in inventory models; however, seasonal type items, short product life cycles, and volatility in the market place suggest that the probability distribution of demand tends to change over time. One flexible correlative demand process that has been studied in the supply chain literature is AR(1) model which is an autoregressive process of the first order. For our simple discussion we consider the end customer demand to be AR(1).
- Forecasting Model : An mean squared error (MSE) optimal forecasting scheme is generally employed for the assumed AR(1) demand process.
- Replenishment Policy : One can safely assume that all stages operate with a periodic-review policy with a common review period. At the end of a period, the on-hand inventory (or backorders) is calculated using the inventory balance equation. In each period each stage observes the demand either from an external customer or from its downstream stages, and places orders to its suppliers so as to replenish the demand of the product based on the observed demand. For low demand situation, generally one-for-one or base-stock policies are used for inventory replenishment. We simpy assume that each stage operates with an adaptive base-stock replenishment policy.

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Hence we can easily calculate the bullwhip at retailer and it is given by

$$B_{(z,q)} = \frac{Var[q_t]}{Var[z_t]} = \left[\frac{\left(1-\phi_1^{L+1}\right)^2 - \phi_1^2\left(1-\phi_1^{L}\right)\left(1+\phi_1^{L}\left(1-2\phi_1\right)\right)}{\left(1-\phi_1\right)^2}\right] \qquad (9.4)$$

while that at retailer it is

$$B_{(z,r)} = \frac{Var[r_t]}{Var[z_t]} = \left[\frac{\left(1-\phi_1^{L+K+1}\right)^2 - \phi_1^2\left(1-\phi_1^{L+K}\right)\left(1+\phi_1^{L+K}\left(1-2\phi_1\right)\right)}{\left(1-\phi_1\right)^2}\right] \qquad (9.5)$$

here:

q = Retailer order quantity
z = Customer demand quantity
r = Warehouse order quantity
ϕ_1 = Correlation coefficient for the demand process

For a better illustration we show the variation of the bullwhip effect with lead time and correlation coefficient and Figure 9.6 shows the variations at the warehouse level.

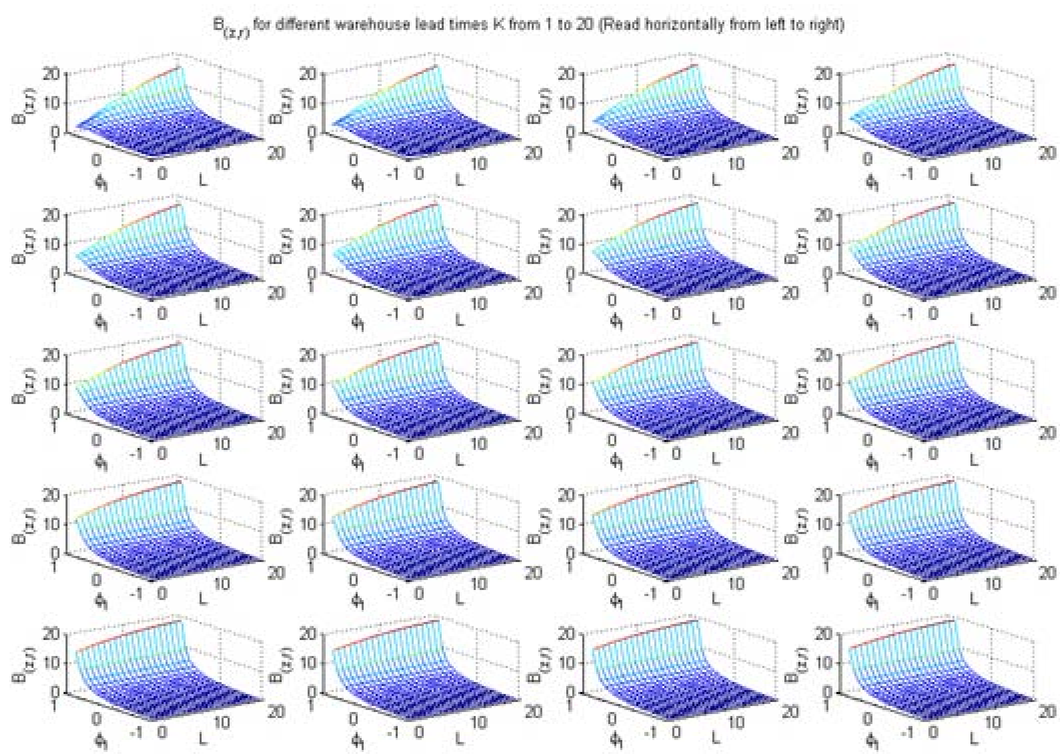

Fig. 9.6: Bullwhip effect at the warehouse level, $B_{(z,r)}$, for different values of K, i.e., lead time

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