The Lecture Contains:

- Diffusion process

Example 6.1

Pick a non-trivial 2×2 and 3×3 matrices and calculate $P^{(2)}, P^{(4)}, P^{(8)}, P^{(16)}$, etc. On can also attempt to find, $P^{(3)}, P^{(5)}, P^{(7)}, P^{(9)}$, etc.

Consider an arbitrary but fixed state i. For that define $f_{i,j}^{n} = P\{X_n = j, X_v \neq j, v = 1,2,\ldots,n-1 | X_o = i\}; n \geq 1$.
Then $f_{i,j}^{n}$ is the probability (given we are in state i at time $t = 0$), of the first attainment of state j occuring at the n^{th} transition, which may be called the first passage time.

By definition we have $f_{i,j}^{0} = 0, \forall i, j$ and $f_{i,j}^{1} = P_{i,j}$, then one can claim that:
$P_{i,j}^{n} = \sum_{k=0}^{n} f_{i,j}^{k} P_{j,j}^{n-k}$, where $i \ngtr j$, $n \geq 0, i = j$, $n \geq 1$.

Diagrammatically it can be shown in Figure 6.3.

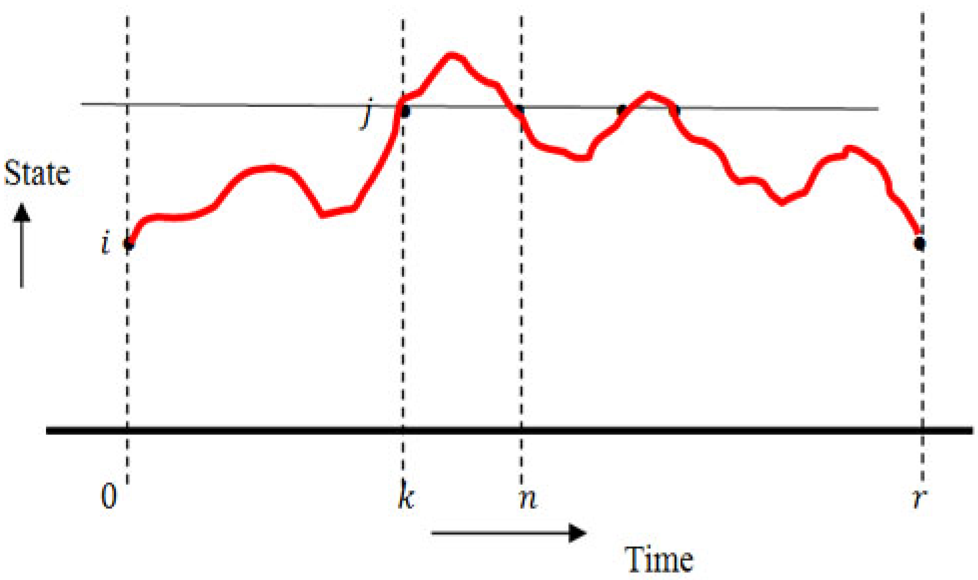

Figure 6.3: A simple stochastic process

Contrast this with the first order ***Markov chain***.

Previous Next

Define

$P_{i,j}(s) = \sum_{n=0}^{\infty} P_{i,j}^{n} s^{n} \quad |s| < 1$

$F_{i,j}(s) = \sum_{n=0}^{\infty} f_{i,j}^{n} s^{n} \quad |s| < 1$

Claim $F_{i,i}(s)P_{i,i}(s) = P_{i,i}(s) - 1$

Now

$\sum_{n=0}^{\infty}\left(P_{ii}^{0} f_{ii}^{n} + P_{ii}^{1} f_{ii}^{n-1} + \cdots . + P_{ii}^{n} f_{ii}^{0}\right)s^{n} = f_{ii}^{0} P_{ii}^{0} + \sum_{n=1}^{\infty}\left(\sum_{k=0}^{n} f_{ii}^{k} P_{ii}^{n-k}\right)s^{n}$, where $\left(\sum_{k=0}^{n} f_{ii}^{k} P_{ii}^{n-k}\right) = P_{ii}^{n}$ and $f_{ii}^{0} P_{ii}^{0} = 0$.

Moreover $P_{i,i}(s) = 1 + \sum_{n=1}^{\infty} P_{ii}^{n} s^{n}$, i.e., $\sum_{n=1}^{\infty} P_{ii}^{n} s^{n} = P_{i,i}(s) - 1$

Hence we have: $F_{i,i}(s)P_{i,i}(s) = P_{i,i}(s) - 1$, i.e., $P_{i,i}(s) = \left\{\frac{1}{1 - F_{i,i}(s)}\right\}$

Diffusion process

The use of diffusion process is used quite extensively in many areas, one being the modelling of stock price movement. This concept helps us to to cover topics as important as Black-Schole's model of option pricing which is derived using the basic concepts of stochastic differential equation. A good example of diffusion process is the ***Brownian motion***, and the basic concept of Brownian motion is derived from Wiener's process. For the interest of the reader we should mention that even though *Robert Brown* the Scottish botanist is credited with the discovery of this phenomenon, which is named after him, yet it was the Dutch physiologist, chemist and botanist, *Jan Ingenhousz* who discovered the Brownian motion phenomenon before *Robert Brown*.

Previous Next