Limitations of Classical Logic

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Classical logic is like a person who comes to a party dressed in a black suit, a white, starched shirt, a black tie, shiny shoes, and so forth.
And Fuzzy Logic is like a person dressed informally, in jeans, tee shirt, and sneakers. In the past, this informal dress won't have been acceptable. Today, it's the other way.
Lotfi A. Zadeh Communications of the Association for Computing Machinery(ACM), Volume 27, 1984

Philosophical Logic

Classical Logic

1. Logic: A systematic study of *argumentation*, principles of *valid reasoning*.
2. Classical Logic (First order Logic: CL): Propositional and Predicate Logic: Good starting point of study of reasoning.
3. Most appropriate for mathematical reasoning, it is bivalent (only two truth values), based on material implication.
4. CL is not appropriate for formalizing human reasoning.

Basic Principles of Logic

1. Law of Identity: P is P
2. Law of Excluded Middle, Principle of Bivalence: $P \vee \neg P$.
3. Law of non- contradiction. $\neg(P \wedge \neg P)$. P cannot be both P and $\neg P$ at the same time and the same sense.
   A contradiction occurs when one statement excludes the possibility of another and yet both are claimed to be true. Truth is not self-contradictory

Laws of Logic

Three laws: Foundation for mathematical, physical, and rational thinking

`https://school.carm.org/amember/files/demo3/2_logic/3logic.htm`

Are all these laws complete?

A variety of arguments can easily be produced to show that these laws are incomplete; i.e., they do not specify all reality, for parts of reality can be shown to contradict one or more of Aristotle's laws.

Hiraclitus(500BC): The problem of change

Heraclitus pointed out that, for a thing to change, it must turn into something else, and then asked how a thing could be something other than itself?

You cannot step in to the same river twice.

1. if Aristotle's laws are taken to be all the fundamental laws of logic, then logically there can be no change whatsoever, because change negates all three laws. I.e., either change does not exist or it is totally illogical.
2. Since all measurements, detections, thoughts, and perceptions are simply changes, then it follows that these operations logically cannot exist.

Online Reference

http://www.cheniere.org/books/aids/appendixIII.htm

Resolution of Motion paradox

1. Aristotle's three laws must specify or apply to only that which is not changing, since change violates or negates all three laws.
2. If change is to logically exist, there must exist at least a fourth law of logic, one which applies to change;
3. This fourth law must contain the negations of each of the first three laws, since change negates them;
4. To be consistent, in any particular logical case, either the three laws explicitly apply or the fourth law explicitly applies (i.e., either change explicitly exists in that particular case or it does not)
5. Since all four laws must apply at all times, when the fourth law applies explicitly, the three laws must be implicit.

Contd...

1. Classical logic fails to provide satisfactory account of the following: conditionals, arguments involving possibility, necessity, logic of knowledge and belief, vagueness etc.
2. Non-classical logics are developed to overcome several defects of clasical logic.
3. Classical logics obeys transitivity, property of bivalence, and monotonic. But common sense reasoning is non-monotonic.

Two reasons for doing non-classical Logic

1. Classical logic is of no help to represent intensional concepts like modality and time. But, these notions are pervasive in common sense reasoning.
2. Human knowledge may be incomplete, and inconsistent. Classical Logic cannot express incomplete and inconsistent information.
3. It Fails to explain vagueness, which is part and parceal of our life.

Non-classical Logic:

Complementary Logics

1. Modal Logics: Tesnse Logic, Epistemic Logic, doxastic logic, Deontic logic, Dynamic Logic, Conditional Logic, Intensional Logic
2. Modal Logic: Normal and non normal Modal Logic

Deviant Logic

Intutionistic logic, para-consistent Logic, Many-valued Logic (Fuzzy Logic)

Modal Sentences

Examples

1. If it is necessarily the case that 2 is the smallest prime number, then 2 is the smallest prime number.
2. It is known that Ravi is richer than Ramesh, then Ravi is richer than Ramesh.
3. If it is morally obligatory that you love your neighbour, then you love love your neighbour. False.

Paradox of Material Implication

The Paradoxes of Material Implication concern some logical consequences(entailments) which are valid according to the principles of propositional logic but which contradict our universal linguistic intuitions.

Instances of Paradox of Material Implication:

1. $p \vdash q \rightarrow p$.
2. $\neg q \models p \rightarrow q$
3. $p \rightarrow s \models (p \wedge q) \rightarrow s$
4. $(p \wedge \neg p) \rightarrow q$
5. $\models p \rightarrow (q \vee q)$
6. $\models p \rightarrow (q \rightarrow p)$
7. $p \wedge q \models p \rightarrow q$
8. $\models (p \rightarrow q) \vee (q \rightarrow p)$

Examples:

Example

$\neg p$: There is no oil in my coffee.
q: I like it.

Example

1. p: I will play football tomorrow.
2. q: I break my leg today.

Paradox of Material Implication

$P \rightarrow Q =_{\text{Def}} \neg P \vee Q$

1. $P \models Q \rightarrow P$: I am alive to if I am dead, I am alive
2. $P \models \neg P \rightarrow Q$. I'm alive to if I'm not alive, Im famous.
3. $(P \wedge Q) \rightarrow R \models (P \rightarrow R) \wedge (Q \rightarrow R)$.

Irrelevance: Bad Taste in Reasoning

1. if the earth is flat, then today is Thursday.
2. If today is Thursday, then 2+2=4.
3. Therefore If the earth is flat, then 2+2 = 4.

1. $(p \wedge \neg p) \rightarrow q$.
2. $(p \wedge \neg p) \rightarrow (q \vee \neg q)$.
3. All are classically true, but

Aristotle's Sea battle argument

A general is contemplating whether or not to give an order to attack. The general reasons as follows:

1. If I give the order to attack, then, necessarily, there will be a sea battle tomorrow
2. If not, then, necessarily, there will not be one.
3. Now, I give the order or I do not.
4. Hence, either it is necessary that there is a sea battle tomorrow or it is necessary that none occurs.

Conclusion

The conclusion is that either it is inevitable that there is a sea battle tomorrow or it is inevitable that there is no battle. So, why should the general bother giving the order?

Future Contingent sentences

1. There will be a sea-battle tomorrow
2. There will not be a sea-battle tomorrow

According to the Law of the Excluded Middle, it seems that exactly one of these must be true and the other false.
But if (1) is now true, then there must be a sea-battle tomorrow, and there cannot fail to be a sea-battle tomorrow. The result, according to this puzzle, is that nothing is possible except what actually happens: there are no unactualized possibilities.
If it was true 10,000 years ago that there would be a sea-battle tomorrow, then its truth was a fact about the past; if the past is now unchangeable, then so is the truth value of that past utterance.
Thus it is necessarily true that there will be a sea-battle tomorrow.

Sorites Paradox

Consider a heap of grains of sand. Take away one solitary grain from the heap–you still have a heap, for one grain of sand is not enough to make the transition from heap to non-heap. So, if a pile of 10,000 grains of sand makes a heap (call this statement h10,000) then a pile of 9,999 grains also makes a heap.

1. A heap of sand is comprised of a large collection of grains. (Premise 1)
2. A heap of sand minus one grain is still a heap. (Premise 2)

The premises seem true.
The argument seems valid.
The conclusion seems false

Sorites Paradox

The paradox is tricky for philosophers because they must explain why one of the two premises, or the conclusion, is wrong even though they appear to be self evident.

Possible Solutions

1. Deny that the problem is legitimately set up. That is, hold that logic does not apply to vague expressions.
2. Accept that logic does legitimately apply here but hold that this particular argument is invalid.
3. Accept both that logic applies in such cases and that the argument is valid, but deny one of the premises.
4. Accept the argument and the premises, and hence embrace the conclusion also.

Liars Paradox

The liar paradox is an ancient conundrum of logic. It was originally cast in the form of a fable.

In ancient times, all the inhabitants of Crete were incapable of making a true statement. Epimenides, who lived in Crete, made the following statement: All Cretans are liars. Is Epimenides lying?

Liar's Paradox

Essence of Liar's Paradox

This sentence is False

Any attempt to assign a truth value to the statement in (1) leads to a vicious cycle of contradiction (sometimes called a vicious circle. If the statement is true, then it is false. But if it is false, then it must be true. Thus we have a vicious cycle of contradictory statements

Self Referential statements

1. Epimenides: All Cretans are liars.
2. **This sentence is false**.
   Is the sentence true or false?
3. Card Paradox:
   1. The sentence on the other side of the card is true
   2. the sentence of the other side of card is false.

The Liars Paradox

Let S be the statement S is false.Other way of saying is: This statement is false.

1. S is the statement s is false (from Definition).
2. Exactly one of the following is true:
   1. S is true
   2. S is false.
3. If S is true, then S is false (result of 1).
4. S is not true (2,3)
5. If S is false then S is true
6. S is not false. (2,5 MP)

Together with 4,6 we can say that S is neither true nor false.

beginframe Consider set of all those sets which are not members of itself.
$r = \{x : x \notin x\}$

1. **Case1:** If r is a member of itself, then it is one of the sets that is not not a member of itself, so r is not a member of itself.

2. **Case 2:** If r is not a member of itself, then it is one of the sets in r, and hence it is a member of itself.

3. $r \in r$ **iff** $r \notin r$

Frege's letter to Russell:

A scientist can hardly meet with anything more undesirable than to have the foundation give way just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell when the work was nearly through the press.

Barbers Paradox:

Suppose there is a town with just one male barber; and that every man in the town keeps himself clean-shaven: some by shaving themselves, some by attending the barber. It seems reasonable to imagine that the barber obeys the following rule: He shaves all and only those men in town who do not shave themselves.
Does the barber shave himself?

1. If the barber does not shave himself, he must abide by the rule and shave himself.
2. If he does shave himself, according to the rule he will not shave himself.

Omnipotence Paradox

1. An omnipotent being can do absolutley anything.
2. So can an omnipotent being create a stone that is too heavy for him to lift?
3. If he can create it, then there is one thing he can't do: lift the stone. If he can't create it, then there is one thing he can't do: create such a stone.
4. Either way, there is something he can't do, which contradicts the assumption that he is **omnipotent.** This is a paradox.
5. It has been taken by some to show that there cannot be an omnipotent being, and by others to show merely that the concept of omnipotence is misunderstood (It does not involve the power to bring about logical impossibilities).

Omnipotence Paradox

1. If God can make a rock He cannot lift, then God is not omnipotent.
2. If God cannot make a rock He cannot lift, then God is not omnipotent.
3. Either God can make a rock He cannot lift, or God cannot make a rock He cannot lift.
4. Therefore, 4, God is not omnipotent. (From 1,2,3. Constructive Dilemma).

The End