Module 5

Lecture 34

Topics

5.2 Growth Theory II

5.2.1 Solow Model

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5.2.1 Solow Model

- Robert Solow was quick to recognize that the instability inherent in the Harrod-Domar model is coming from the lack of flexibility in the production function -- characterized by fixed coefficient production function.

- Solow instead assumed a standard neo-classical production function with diminishing marginal product for both labor and capital.

$$Y(t) = F\big(K(t), L(t), A(t)\big) \tag{1}$$

$F_i > 0, F_{ii} > 0$ Where $i = K, L, A$ represents

The equation of motion (assuming continuous time) is given by

$$K^{\cdot}(t) = I(t) - \delta K(t) = sF\big(K(t), L(t), A(t)\big) - \delta K(t) \tag{2}$$

Where $K^{\cdot}(t) = \frac{\delta K}{\delta t}$ and $0 \leq s \leq 1$

- The labor force evolves following the formula

$$L = e^{nt} \tag{3}$$

- This means that the rate of growth of population is given by

$$\frac{L^{\cdot}(t)}{L(t)} = n \tag{4}$$

- The production function is assumed to be homogeneous of degree 1 in labor and capital. This means that $\lambda Y = F(\lambda K,\ \lambda L)$. For $\lambda = \frac{1}{L}$ we get,

$$\frac{Y}{L} = F\left(\frac{K}{L}, 1\right) \tag{5}$$

- We write the per capita variable by smaller case variables. So$y = \frac{Y}{L}, k = \frac{K}{L}$ and so on,

- Hence we get

$$y = f(k) \tag{6}$$

- In the steady state all variables should grow at a constant rate so capital-labor ratio must grow at a zero rate. At the steady state,

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = 0 \tag{7}$$

- We already know that, $\dot{K} = sF(K, L) - \delta K$. Hence, dividing both sides by K we get

$$\frac{\dot{K}}{K} = s\frac{F(K, L)}{K} - \delta \tag{8}$$

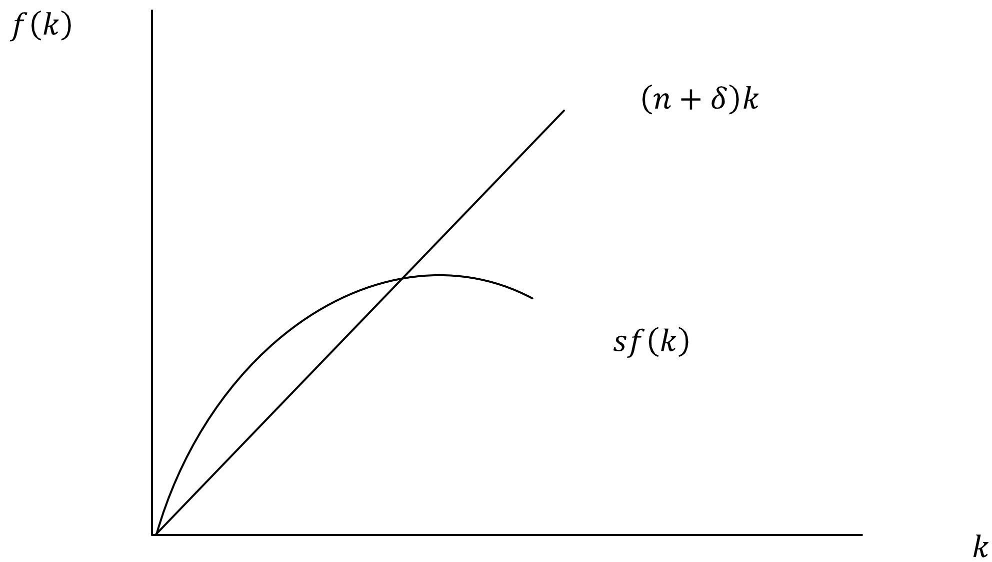
- Again, dividing both the numerator and denominator of the first term we get

$$\frac{\dot{K}}{K} = s\frac{f(k)}{k} - \delta \tag{9}$$

- Finally,

$$\frac{\dot{k}}{k} = s\frac{f(k)}{k} - \delta - n = 0 \tag{10}$$

- The steady state in Solow model is characterized by the condition $sf(k) = (n+\delta)k$



- Solow model was the first of its kind neo-classical growth model. In the steady state the per capita income does not grow. This is because of the assumption of the diminishing return to capital.

- Capital accumulation is the main driving force in these models and because of the diminishing return, the growth of per capita income comes to an end in the long run.

- However, one way to increase growth is to continuously increase savings rate. But savings rate has a maximum limit of 1. Hence, there is limit to achieve growth that way.

- One very interesting implication of the Solow model is the convergence debate.

- The equation of motion in Solow suggests that smaller values of k are associated with higher values of $\frac{\dot{k}}{k}$ – the growth rate of k. This means that countries with higher ki.e. richer countries should have lower growth rates of k and eventually of per capita income $f(k)$.

- But this suggests that poor countries grow faster and that there is a possibility of convergence of poor and rich countries over time.

- The hypothesis that a poorer country always grows faster than a rich country without any other qualification is known as the hypothesis of *absolute convergence.*

- However, data did not support absolute convergence. In fact there is a positive relation between per capita GDP and GDP growth rate. Rich countries tend to grow faster than the poor countries.

- This is not surprising as there are wide variations in factors determining growth across countries such as technology and institutions.

- Thus it makes sense to test for convergence hypothesis controlling for such factors. In other words, it is good to test the result among homogeneous countries. This notion leads to the hypothesis of *conditional convergence*.

- This hypothesis says that *controlling for other things* countries with low per capita income grows faster than the countries with high per capita income. This also implies that an economy grows faster when it is further from its own steady state value.

- The only problem with the neo-classical growth model is that a country eventually land up in a situation where per capita income does not grow. The only option for avoiding that is to have a situation where technology develops continuously.

- The technological change can be labor saving or capital saving. Hicks, Harrod and Solow defined *neutral technological progress* in different ways.

- **Hicks neutral technological progress:** A technological innovation is Hicks neutral if the ratio of marginal products remains the same for the same capital labor ratio. A Hicks neutral production function can be written as

$$Y = T(t)F(K,L) \tag{11}$$

- Harrod neutral technological progress: A technological innovation is Harrod neutral if the relative input shares $\frac{K.F_K}{L.F_L}$ remains constant for given **capital-labor** ratio. It turns out that such a production function must be of the form

$$Y = F\big(K, L.T(t)\big) \tag{12}$$

- This is also called *labor augmenting* technological progress.

- Solow neutral technological progress: An innovation is Solow neutral if the relative input shares, $\frac{L.F_L}{K.F_K}$ is constant for a given labor/output ratio.

- This implies that the production function is of the form

$$Y = F(K.T(t), L) \tag{13}$$

- This is capital augmenting technological progress.

- In neo-classical growth models with constant rate of technological progress, only labor augmenting technological progress is consistent with steady state.

- The intuition is easy to understand. An economy converges to a steady state only if capital shows diminishing return. Capital augmenting technological progress will counter act the diminishing return and will not allow the steady state take place

- The Solow model provides a counter point to the HarrodDomar model. The basic teaching of the HD models is that market is inherently unstable which requires the need for planning. This proposition goes against the basic tenet of neo classical economics. Solow's model can be seen as the defense of neo-classical economics extended to the dynamic setting.

- But why does the difference arise between Solow and Harrod's main result? The difference can be traced back to the different assumptions used to set up the models. The lack of substitution possibility between labor and capital assumed in Harrod's model leads to the instability. On the other hand, technology in Solow model assumes substitution possibility between labor and capital and diminishing return to capital. These two properties drive Solow's economy home – stability under market. Which set of assumptions are close to the reality can only be confirmed by empirical investigation and cannot be justified by the models themselves. This is an important thing to remember -- the results are sensitive to the assumptions its author makes. Ideally assumptions must be supported by empirical observations. But often they are chosen by the political view of the economist --- i.e. final result that he/she believes in.

- The second important observation is the convergence doctrine. The convergence doctrine predicts that poor countries will catch up with the rich ones which is consistent with the development agenda of World Bank-IMF. Such agenda has the West European experience in its core and expects that the rest of the world will catch up with the trajectory West Europe has taken. The theory of conditional convergence bears the same philosophy in its core in a revised form. It tells us that with some corrective measures in terms of technology and institutions mostly consistent with free market doctrine, less developed countries will catch up with the developed ones.

- In the next chapter we discuss endogenous growth which explains the process of technology development which is taken as exogenous in Solow model. Again, these models are not known for radically alternative view of growth. Yet, deeper understanding of technology development gives us some deeper understanding of the development process.