Module 4

Demographic Models

Lecture 11: Modelling Population Phenomena

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MATHEMATICAL AND STATISTICAL MODELING

As discussed in Module 1, population research deals with quantitative and qualitative studies of five demographic processes: fertility, mortality, migration, mobility and marriage. It uses both primary and secondary data. The data collected by the researcher is called primary data and the data that already exists in the form of census reports, publications of national sample survey (NSS), administrative statistics collected by different government departments, hospital records, etc. is called secondary data. For those who use quantitative methodology (also considered to be scientific methodology by them) the methods of data collection include sample survey and experiments. For those who use qualitative methodology the methods include fieldwork, observation, ethnographic methods, unstructured and in-depth interviews, and participatory rapid appraisal methods. Quantitative methods are more suited to hypothesis testing while the qualitative methods generate hypothesis and provide subjective meanings.

Among those who prefer quantitative methodology, mathematical models are used quite frequently for estimation, policy experimentation and theory building. They are also used for description of demographic phenomena, interpretation of data, and often to smooth questionable data (Halli and Rao, 1992). They provide answers to many questions which cannot be answered otherwise due to lack of data or observations. For example they can answer:

- How many people have ever lived on earth?
- In a country having a life expectancy of 70 years what proportion of people are likely to survive till the age of 65?
- If the present schedule of fertility (i.e., the set of age-specific-fertility rates) remains unchanged how many children a woman will produce in her life time?

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- If the prevalence rate of HIV is reduced to half what will be its impact on life expectancy?
- If all couples follow a stopping rule of α sons and β daughters what will be its effect on the sex ratio of the population (Keyfitz, 1968)?
- Are there errors in census age data? What corrections in age data need to be made before using them for prediction of India's population?
- If the data is available on proportion married by age, how can one calculate average age of marriage?

MATHEMATICAL AND STATISTICAL MODELS

Models are mathematical and/or statistical expressions of relationships between variables describing some major, chosen aspects of a phenomenon. In population studies they are often used to explore relationship between different components of demographic systems, and to explore relationship between demographic variables and socio-economic and cultural variables. The relationships expressed in mathematical form of any kind are called mathematical model. Exponential growth model of population (such as $P_t = P_0$ Exp (r.t)) is an example of a simple mathematical model which states that population growth follows the exponential model. When the relationships are developed using statistical methods such as regression analysis they are called statistical models. They contain an element of uncertainty. Statistical models have been developed for studying growth of population as a stochastic process, i.e., a process which can be described only in terms of probability distributions (Kendall, 1949). Thus statistical models are closer to reality than mathematical models. They are particularly suited for the studies of random errors. Models using probability distributions, Monte Carlo computer simulation methods (because of their reliance on repeated computation of random or pseudo-random numbers), regression analysis, discriminant analysis, factor analysis are all statistical models.

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HISTORY OF MATHEMATICAL MODELS

For long, scholars from all fields have looked for ways of representing social and scientific phenomena in the form of abstract laws, principles and mathematical forms. Population studies is no exception to this. One of the earliest and most forceful arguments in favour of modelling was, however, given by Keyfitz. In the first volume of *Population and Development Review* Keyfitz (1975) published an article entitled "How do we know the facts of demography?" He argued that the empirical relationships based on regression analysis can be misleading as they depend heavily on the cases for which data are available. Also without modelling questions regarding cause and effect, multiple causation, and nature of relationship cannot be answered. In this article he showed that the empirical relationship between percent aged 65 and over and growth rate of population was dependent on: (a) number of countries for which data are available; and (b) homogeneity among the countries. In such situations analytical and mathematical models can provide a better understanding of the relationship.

The most important of all the demographic models is the stable population model (Smith and Keyfitz, 1977; Keyfitz, 2005). It says that if individuals are born at a constant rate of 1 person per unit of time, and the survival probability of a person aged x is p(x), then at any time the expected size of the population is given by

$$E[X] = \int_0^{\alpha} p(t)dt$$

where E[X] refers to the expected size of population at age x, p(t) refers to chance of survival from birth to age t, and α refers to the upper limit of the age distribution.

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It may be noted that the stable population model is a deterministic model and results in a stationary population in which the number of persons at any age (x) does not change with time. Further, suppose the individuals are born at rate $Be^{\rho t}$ where ρ is a constant and t is time. Then the size of population at time t, or

$$V(t) = Be^{\alpha t} \int_0^{\infty} e^{-\rho x} p(x) dx$$

This population grows or declines at rate ρ , and its age distribution, i.e., proportion surviving to age x is proportional to $e^{-\rho x} p(x)$. Using the property of stable populations that age distributions of two stable populations never cross each other census growth rate and proportion of population up to a certain age (normally 35 years) were used to estimate birth and death rates for those populations which lacked reliable and complete data on them. Development of model populations, showing probability of surviving from birth to different ages at different levels of life expectancy, for different regions of the world helped the demographers working on the populations of developing countries immensely (UN, 1983). Subsequently, the stable population theory was modified to accommodate for changes in mortality or fertility or both. Nath and Kalita (1989) developed a two-sex quasi-stable population in the presence of immigration.