Module 4: "Static games of incomplete information"

Lecture 26: "Games of Incomplete Information: Example-Cournot Game"

The Lecture Contains:

- Games of Incomplete information
- Cournot game with Asymmetric information

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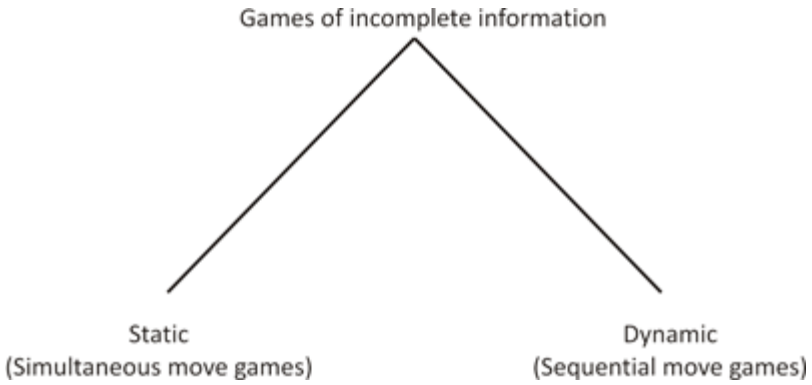
Lecture 26: "Games of Incomplete Information: Example-Cournot Game"

Games of Incomplete info

- In games of complete information, the payoff functions of all the players are common knowledge.
- Games of incomplete information
  - Payoff functions of at least some players are not in common knowledge
  - At least some players do not know the payoff functions of their rivals



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Example: Cournot game with Asymmetric information

- Two firms
- Simultaneous quantity choice
- Demand function
  $P(Q) = a - q; Q = q_1 + q_2$
  - Cost function
    $c_1(q_1) = cq_1$ [In Common knowledge]
    - Firm 2's cost function
      $c_2(q_2) = c_H\, q_2$ with probability θ
      $= c_L q_2$ with probability $(1-\theta)$
- Information is incomplete/ asymmetric
  - Firm 2 knows its own cost function as well as the cost function of firm 1.
  - Firm 1 knows its own cost but is unsure about the exact cost of firm 2.
  - Unsure about the exact payoff function of firm 2.
- All of the above points are in common knowledge
- Firm 1 knows that firm 2 has better information. Firm 2 knows that firm 1 knows this and so on.

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Solution of the game

- Firm 2 chooses a different quantity depending on the realization of the MC.
  $q_2^*(c_H), q_2^*(c_L)$: Firm 2's optimal choices as functions of its cost.
- q_1^* - firm 1's optimal choice
- $q_2^*(c_H)$ solves the following:
  $\max_{q_2}[(a - q_1^* - q_2) - c_H]q_2$
- $q_2^*(c_L)$ solves the following:
  $\max_{q_2}[(a - q_1^* - q_2) - c_L]q_2$

Note just like in a static Cournot game firm 2 is taking q_1^* as given when it determines it's optimal quantity choice

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Solution of the game [contd.]

Firm 1 knows that firm 2's cost is c_H with probability θ & c_L with probability $1-\theta$

Firm 1 anticipates that firm 2's quantity choice will be $q_2^*(c_H)$ with probability θ & $q_2^*(c_L)$ with probability $(1-\theta)$

Firm 1 chooses q_1^* to solve

$$\underset{q_1}{Max}[\{\{a-(q_1+q_2^{*L})\}_{q_1}\}Prob(c_2=c^L)]+[\{a-(q_1+q_2^{*L})\}q_1]Prob(c_2=c^H)$$

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Solution of the game [contd.]

Three first order conditions:

$$q_2^*(C_H) = \frac{a - q_1^* - C_H}{2}$$

$$q_2^*(c_L) = \frac{a - q_1^* - c_L}{2}$$

$$q_1^* = \frac{[a - q_2^*(c_H) - c]\theta + (1-\theta)[a - q_2^*(c_L) - c]}{2}$$

- Three equations, three unknowns
- **Solution:**

$$q_2^*(c_H) = \frac{a - 2c_H + c}{3} + \frac{(1-\theta)}{6}(c_H - c_L)$$

$$q_2^*(c_L) = \frac{a - 2c_L + c}{3} + \frac{\theta}{6}(c_H - c_L)$$

$$q_1^* = \frac{(a-2c) + \theta c_H + (1-\theta)c_L}{3}$$

- Bayesian Nash Equilibrium
  Relevant solution concept for static games with incomplete information

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