Module 1: "Introduction to game theory"

Lecture 2: "Game representation"

**The Lecture Contains:**

- Representation of a Game
- Normal form representation(NFG)
- Examples of NFG
- Extensive Form Game (EFG)
- Example of EFG
- Example of a Static Game
- Prisoner's Dilemma – NFG representation

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**Representation of a Game**

**Games can be classified into two types**

- Static games
  - Players move simultaneously.
- Dynamic games
  - Players move sequentially.

[**Note:** *This is one way of classifying games. Another common way to classify games is according to the nature of information available – games of complete versus incomplete information (will cover this in future slides)*]

**Games can be represented in two main ways**

- Normal form game (NFG)
  - Usually represents static games.

*[**Note:** However NFG can also be used to represent dynamic games.]*

- Extensive form game (EFG)
  - Usually represents dynamic games.

    [Note: *EFG can also be used to represent static games*.]

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**Normal form representation**

**To describe a normal form game one needs the following**

1. A set of N players.

- I = {1,2,….,N}
- Eg: Battle of sexes: I = {husband, wife}; Matching pennies: I = {player1, player2}

2. Let $S_i$ denote a set of strategies available to player i.

- Eg: Battle of sexes: $S_H$ = {B, M}, $S_W$ = {B, M}

3. Let S = ($s_1$, $s_2$, …, $s_n$) denote the set of all possible combinations of strategies for all player, $s_i$ denotes an outcome.

- Eg: For Battle of sexes there can be 4 outcomes: $s_1$ = {B, M} where the first entry denotes the strategy of husband while the second entry denotes the strategy of wife.
- Or $s_2$ = {B, B} or $s_3$ = {M, B} or $s_4$ = {M, M} , S = {(B, M), (B, B), (M, B), (M, M)}

4. For each player i, a payoff function $\Pi^i$ which assigns a real number to every possible outcome of game.

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**Examples**

**Matching pennies**

I = {1, 2} $S_1$ = {H, T} $S_2$ = {H, T}

Set of outcomes = [ (H, H), (H, T), (T, H), (T, T) ]
Payoff determined by following rule:

- If both coins match, player 1 pays Re 1 to player 2 otherwise, player 2 pays Re1 to player 1.
- NFG represented by a payoff matrix.

| | | PLAYER 2 | |
|---|---|---|---|
| | | H | T |
| PLAYER 1 | H | (-1, 1) | (1, -1) |
| | T | (1, -1) | (-1, 1) |

**Note:**

- Player 1 usually represented left, Player 2 on top.
- (x, y); x refers to payoff of 1 and y represents payoff of 2.

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**Examples(continued)**

Battle of sexes

I = {H, W} $S_H$ = {B, M} $S_W$ = {B, M}

Set of outcomes = [(B, M), (M, B), (M, M), (B, B)]

Payoffs determined by following rules

- Payoffs greater if both stay together than staying separately.
- Payoff of Husband more than payoff of Wife if the outcome is to see boxing.
- Payoff of Wife more than payoff of Husband if the outcome is to see movie.
- NFG represented as

| | | HUSBAND | |
|---|---|---|---|
| | | B | M |
| WIFE | B | (3,1) | (0,0) |
| | M | (0,0) | (1,3) |

**Note:** There can be more than one payoff matrix to represent the game defined by the above rules.

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**Extensive Form Game (EFG)**

**To describe an EFG one needs the following**

- The list of players.
- When each player moves.
- What each player can do when he/she is entitled to move.
- What each player knows at each move.
- The payoffs each player will receive for all possible combinations of strategies chosen by the player.

EFG is represented by a Game Tree.

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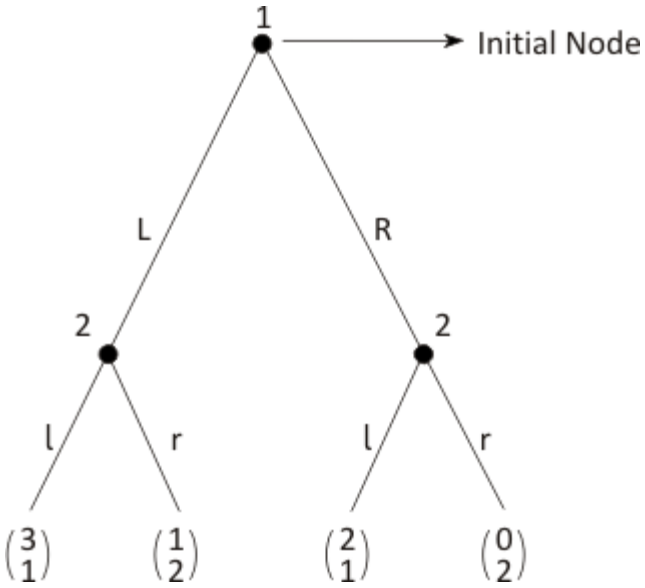
**Example of EFG**

**Players – Player 1, Player 2**
Ordering of moves:

- Player 1 is the first mover.
- Player 2 is the second mover.

Actions available to player1 – Left (L) & Right (R)
Actions available to player2 – Left (l) & Right (r)




$\binom{x}{y}$: x is payoff of player 1

y is payoff of player 2

**Note**

- EFG usually represents dynamic games.
- Can also be used to represent static games.

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**Example of a Static Game**

**Prisoner's Dilemma**

- Players: Two suspects/ prisoners
- Strategies: Confess (C), Not Confess (NC)
- Prisoners are questioned in different rooms at the same time – simultaneous game

**Rules for determining payoffs:**

- If neither confess both convicted of minor offense and sentenced to 1 month in jail.
- If both confess, then both confined to jail for 6 months.
- Finally, if one confesses but the other does not, then the confessor will be released immediately but the other prisoner will be confined to jail for 9 months.

**Strategic Independence:**
Whether are prisoner ends up in jail or gets freed depends not only on his/her own strategy but also on the strategy of the other prisoner.

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**Prisoner's Dilemma – NFG representation**

| | | PRISONER 2 | |
|---|---|---|---|
| | | C | NC |
| PRISONER 1 | C | (-6,-6) | (0,-9) |
| | NC | (-9,0) | (-1,-1) |

**Note:** Justification for the name of Prisoner's Dilemma will be given soon.

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