The Lecture Contains:

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Consider the following game:

	L_2	R_2
L_1	1, 1	5, 0
R_1	0, 5	4, 4

Suppose this game is infinitely repeated. Further suppose that each person's discount factor is δ and each player's payoff in the repeated game is the present value of the players' payoffs from the stage games.

- it can be shown that co-operation i.e. (R_1, R_2) can occur in every stage even though the only NE in each individual stage of game is the non-cooperative outcome (L_1, L_2).

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Trigger Strategy

- Suppose player i begins the infinitely repeated game by cooperating in the first period (by playing Ri) and then cooperates in each subsequent stage only if both players have cooperated in every previous stage**.**
- Formally player i's strategy is**:**
    - Play Ri in the first stage
    - In the t^{th} period, if the outcome of all "t-1" preceeding stages has been (R_1, R_2), then play R_i, otherwise play L_i.

      This is called the Trigger strategy

    - Player i goes on cooperating until someone fails to cooperate. If cooperation is broken by one player, then that deviation from the cooperative action triggers off a switch to non-cooperation (L_1, L_2) for all the future periods
- If both players adopt this trigger strategy, then the outcome of this infinitely repeated game is co-operation always[i.e. (R_1, R_2) in each stage]

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Proposition : If δ is close enough to 1, it is a NE of the infinitely repeated game for both the players to adopt the trigger strategy**.** (TS)

Proof : Consider any arbitrary period t there are 2 possible situations:

Situation 1:
In one previous period, outcome differs from (R_1, R_2)

Situation 2:
For all periods upto "t-1" (R_1, R_2) is the outcome.

In order to show that (TS, TS) is a NE, one has to show that if player j has adopted TS (j= 1, 2), then player i $(i \neq j)$ has no unilateral incentive to deviate from TS.

Situation 1 :

- In one period, outcome differs from (R_1, R_2). Since player j has adopted TS, after that period, player j will play Lj for all future periods.
- The best response for player i then is to also choose L_i for all future periods.
- Hence once the deviation occurs and given that player 2 has chosen TS, it is also optimal for player 1 to choose TS.

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Proposition [Contd...]

Situation 2:

All the proceeding outcomes in period upto 't-1' (R_1, R_2)

If Player j is playing TS, then since there is no deviation from (R_1, R_2) in previous periods**;** player j will be playing R_j in the present period

Present Value of payoff of player i by playing TS

$$= 4 + 4\delta + 4\delta^2 + \ldots\ldots$$

$$= {}^{4}/_{1-\delta}$$

Suppose player i deviates from TS & plays Li. Playing Li will yield payoff of 5 in period t.

- triggers non co-operations by player j (since j plays TS) => player j hence forth will always play Lj
- Best response of i is then to play Li also
- In all the periods after deviation, outcome is (L_1, L_2) and payoff is therefore 1 in all future periods.

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PV of payoff of player i by deviation from TS

$$= 5 + \delta + \delta^2 + \dots\dots.. = \frac{5-4\delta}{1-\delta}$$

PV of pay off of player i by playing $\mathrm{TS} > \mathrm{PV}$ of payoff of play i by deviating from TS

$\Rightarrow 4/1-\delta > \dfrac{5-4\delta}{1-\delta} \Rightarrow \delta > \dfrac{1}{4}$

Given $\delta > \frac{1}{4}$, $(\mathrm{TS}, \mathrm{TS})$ will be a NE $\Rightarrow$ there will always be co-operation.