Module 5: "Dynamic games of incomplete information"

Lecture 31: "PBNE: Numerical Example"

The Lecture Contains:

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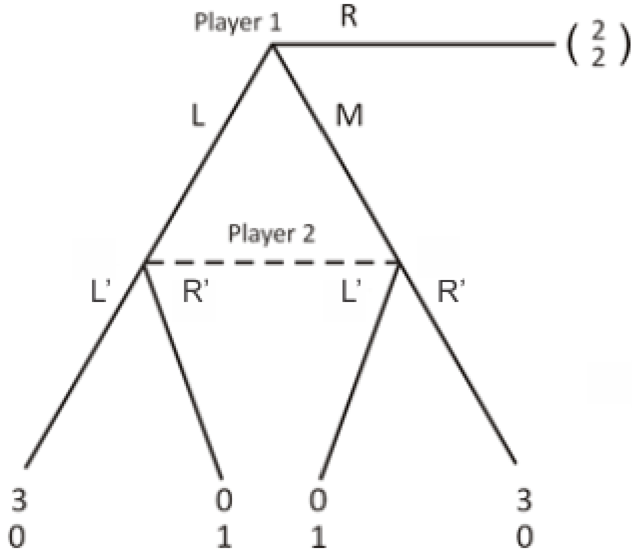
PBNE: Numerical Example

- Show that there does not exist a pure-strategy PBNE in the following game, Give a mixed strategy PBNE.



Let's look for a PBNE with

$\sigma_1(L) = 1$

$\sigma_1(L) = 1 \ \Rightarrow p = 1$

p=1 => player 2 will play R'.

However in response to $\sigma_2(R') = 1$ the best response from player 1 is to play $\sigma_1(M) = 1$.

Hence there is no PBNE with $\sigma_1(L) = 1$

Let's look for PBNE with $\sigma_1(M) = 1$

$\sigma_1(M) = 1 \Rightarrow p = 0$

p=0 => player 2 will play L' for sure i.e. $\sigma_2(L') = 1$

Given that $\sigma_2(L') = 1$, the best response of player 1 is to play L for sure i.e. $\sigma_1(L) = 1$

Hence there is no PBNE with $\sigma_1(M) = 1$

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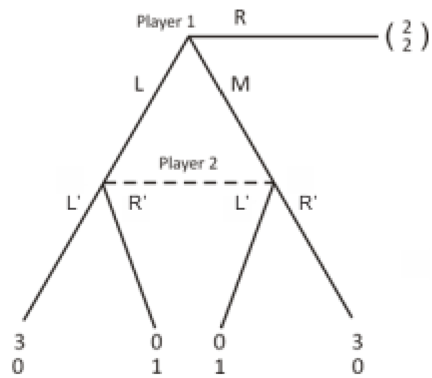

Let's look for PBNE with $\sigma_1(R) = 1$

=> p is not pinned down by Bayes rule

Expected utility to player 2 from playing $L' = p.0 + (1-p).1 = 1-p$

Expected utility to player 2 from playing $R' = p.1 + (1-p).0 = p$

Player 2 will play L' for sure if $p < \frac{1}{2}$

Player 2 will play R' for sure if $p > \frac{1}{2}$

Suppose $p < \frac{1}{2}$ => players 2 plays L' for sure

=> player 1 will play L for sure, so $\sigma_1(R) = 1$ is not a best response to $\sigma_2(L') = 1, p < \frac{1}{2}$

$\sigma_1(R) = 1$ is not a best response to $\sigma_2(L') = 0, p > \frac{1}{2}$

Also, [$\because p > \frac{1}{2} \Rightarrow \sigma_2(L') = 0$ i. e $\sigma_2(R') = 1$ => player 1's best response is $\sigma_1(M) = 1$]

Hence there is no pure strategy PBNE with $\sigma_1(R) = 1$. If $p = \frac{1}{2}$ => player 2 is indifferent between L' and R' => can mix in any manner between L' and R'

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If $p = \frac{1}{2} \Rightarrow$ player 2 is indifferent between L' & R' $\Rightarrow$ can mix in any manner between L' & R'

$p = \frac{1}{2} \Rightarrow \ \sigma_2(L') = \ \sigma_2(R') = \frac{1}{2}$

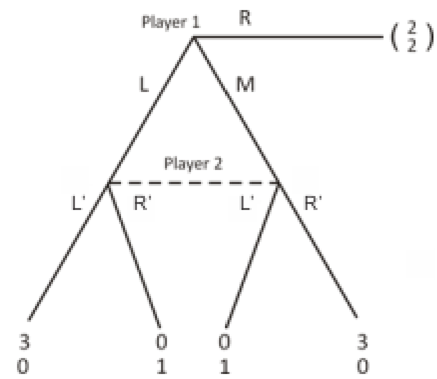
Payoff to 1 from playing L=3/2

< 2= payoff to 1 from playing R

Payoff to 1 from playing M= 3/2

< 2 = payoff to 1 from playing R

$\Rightarrow \sigma_1(R) = 1$



So $\sigma_1(R) = 1$ is a best response to $p = \frac{1}{2}, \sigma_2(L') = \ \sigma_2(R') = \frac{1}{2}$

Suppose $\sigma_1(R) = 1$

=> p is free to form let $p = \frac{1}{2}$

=> payoff from playing L'= payoff from playing R'

So mixing in any manner between L' and R' is the best response

$\Rightarrow \sigma_1(R) = 1 \Rightarrow p = \frac{1}{2}, \sigma_2(L') = \sigma_2(R') = \frac{1}{2}$ is a best response.

One mixed strategy PBNE for this game is

$\sigma_1(R) = 1, \sigma_2(L') = \sigma_2(R') = \frac{1}{2}, p = \frac{1}{2}$

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