Lecture 35: "Signaling Game: Practical Application"

The Lecture Contains:

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Signaling Game : Practical Application

The Problem

- There are 2 types of Assistant Professor(AP)-high ability(t=H) & low ability(t-L)
- AP knows her/his own ability but the department which is recruiting does not know the ability.
- A randomly selected AP is of high ability with probability λ where $0.5 > \lambda > 0$ - this is in common knowledge
- Timing of the Game
  - AP moves first , choosing a number of papers, $n \geq 0$ to write.
  - The department observes and then decides whether to grant tenure $(\tau = 1)$ or deny tenure $(\tau = 0)$.

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The Problem[Contd…]

- The payoff function of AP is given by $\prod_{AP}(t, n, \tau) = \begin{matrix} \tau - \alpha n & \text{if } t = H \\ \tau - \beta n & \text{if } t = L \end{matrix}$

  $\beta > \alpha > 0$

- Payoff function of the department is, $\prod_{D}(t, n, \tau) = 1 \text{ if } \tau = 1 \,\&\, t = H$

  $= -1 \text{ if } \tau = 1 \,\&\, t = L$

  $= 0 \text{ if } \tau = 0$

Let $\mu(t/n)$ be the department's belief that the AP is of type t, given that he has written n papers.

Consider pure strategy equilibrium

a. Define PBNE for this game .
b. Characterize all pooling equilibria.
c. Give a separating PBE.

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PBNE of the Game

- A PBNE for the game is

$n^*()$: mapping from type to a number of papers

$\{H, L\} \to z_t$ $[z_t \sim \text{real number}]$

$\tau^*()$: $Z_t \to [0,1]$

[mapping from the number of papers to the decision as to whether or not to grant the tenure.]

$\mu^*(H/n)$: mapping from z_t $\to$ prob distribution over types

Such that

i. $n^*(t)$ solves $\underset{n}{\text{Max}} \prod_{AP}(t, n, \tau_n^*)$

ii. $\tau^*(n)$ solves $\underset{\tau\epsilon[0,1]}{Max} \left[\mu(H/n)\prod_D(H, n, \tau) + \mu(L/n)\prod_D(L, n, \tau)\right]$

Where the beliefs are
$$\mu_{(H/n)} = \begin{cases} 1 & if\; n^*(H) = n, n^*(L) \neq n \\ \lambda & if\; n^*(H) = n^*(L) = n \\ 0 & if\; n^*(H) \neq n, n^*(L) = n \end{cases}$$

$\mu(H/n)$ is arbitrary otherwise

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Answer to (b)

Let $n^*(H) = n^*(L) = \hat{n} \geq 0$

$\Longrightarrow \mu(H/\hat{n}) = \lambda \ \forall \hat{n} \geq 0$

What's the department's best response to $n = \hat{n}$?

If $\tau = 1$,

$\Pi_{D=\lambda_0 1+(1-\lambda)(-1)<0} \qquad \therefore \lambda < 5$

If $\tau = 0$

$\prod_D = 0$

Hence given the AP's s strategy,
$\tau^*(\hat{n}) = 0 \ \forall \hat{n} \geq 0$ is the best response from the department.

Now given $\tau = 0$, what is AP's best response,
If AP is of low type,

Payoff = - $\beta\hat{n} < 0$ for $\hat{n} > 0$

If AP is of high type , Payoff = - $\alpha\hat{n} < 0 \ for \ \hat{n} > 0$

Hence whatever be the type,

$$\left.\begin{matrix}\text{Payoff} < 0 \ \text{ for n} > 0 \\ \text{Payoff} = 0 \ \ if \ \text{n} = 0\end{matrix}\right\} \Longrightarrow \begin{matrix} n^*(H) \\ = n^*(L) = 0\end{matrix}$$

Given that the department never tenures , all APs should write zero papers.

$\Longrightarrow$ There is only one pooling equilibrium for this game :

$n^*(H) = n^*(L) = 0$

$\tau^* = 0$

$\mu(H/n^*) = \lambda$

Answer to (C)

Let us look for a separating equilibrium with,

$n^*(H) = n\,, n^*(L) = 0$

let's take the AP's strategy ($n^*(H) = n\,, n^*(L) = 0$) as given & find the department's best response.

$n^*(H) = n\,, n^*(L) = 0$

$\Longrightarrow \mu(H/n) = 1\,, \mu(L/n) = 0$

If the department sees n papers,

Then Π_D if $\tau = 1 = 1.1 + 0.(-1) = 1$.

Π_D if $\tau = 0 = 0$

$\Longrightarrow\ \tau^*(n) = 1$

If the department sees 0 paper then Π_D if $\tau = 1$

$= 1.\mu(H/n) + \mu(L/n)(-1)$

$= 1.0 + 1(-1) = -1$

Π_D it $\tau = 0$

= 0

$\Longrightarrow\ \tau^*(0) = 0$

Let us consider the strategy of the department to be given : $\tau^*(n) = 1, \tau^*(0) = 0$

What is the AP's best response to that strategy. Payoff

Number of Papers	High Type	Low Type
n	$1 - \alpha n$	$1 - \beta n$
0	0	0

For $n^*(H) = n \Longrightarrow 1 - \alpha n \geq 0 \Longrightarrow n \leq \dfrac{1}{\alpha}$

$n^*(L) = 0 \Longrightarrow 1 - \beta n \leq 0 \Longrightarrow \dfrac{1}{\beta} \leq n$

$$\frac{1}{\beta} \leq n \leq \frac{1}{\alpha}$$

Condition on n such that AP follows the strategy $n^*(H) = n, n^*(L) = 0$ given the department's strategy.

Hence one separating PBE is

$$n^*(H) = n, n^*(L) = 0, \frac{1}{\beta} \leq n \leq \frac{1}{\alpha}$$

$$\tau^*(n) = 1, \tau^*(0) = 0$$

$$\mu(H/n) = 1$$