Module 3: "Dynamic games of complete information"

Lecture 23: "Bertrand Paradox in an Infinitely repeated game"

The Lecture Contains:

Bertrand Paradox in an Infinitely repeated game

- Bertrand Paradox
  $P_1 = P_2 = C$
  - Everybody set price equal to MC
  - It's a paradox since each has market power but still price is set at the perfectly competitive level.
- Suppose both firms make an agreement
  - Both will choose the monopoly price to maximize the joint profit and the market share will be equally divided.
  - This is the co-operative agreement
  - Payoff will be $(\frac{\pi^m}{2}, \frac{\pi^m}{2})$ where π^m is the maximized joint profit/ monopoly profit
- Each firm has two strategies
  - to co-operate & stick to agreement (C)
  - not to co-operate (NC)

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NFG representation

- If both play strategy NC
  - then back to Bertrand Paradox
  - $P_1 = P_2 = c$
    $\Rightarrow \pi_1 = \pi_2 = 0$
- Suppose firm 2 cooperates but firm 1 non- cooperates.
  - If firm 2 plays P_m, best response to firm 1 is to choose P_m - ξ where ξ is a very small real number >0.
  - Firm 1 will get the entire market
  - $\pi_1^{Nc} = (P_m - \xi - C)D\,(P_m - \xi)$
    [D is the demand function]
  - As ξ is a very small number tending towards tero,
    Limiting value of $\pi_1^{Nc} = (P_m - C)D\,(P_m)$
    $= \pi^m$
- Hence the game is represented as

1 \ 2	C	NC
C	$(\frac{\pi^m}{2},\frac{\pi^m}{2})$	$(0,\pi^m)$
NC	$(\pi^m,0)$	$(0,0)$

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- Consider now that this game is repeated infinitely
- Each player has a discount factor δ.
- Each player's payoff in the repeated game is the present value of the players' payoffs from the different periods' games.
- Consider the trigger strategy (TS)
  - Start by playing C in 1^{st} period
  - it outcome in any of "t-1" periods $\neq$ (C,C), then play NC forever from period t
- If TS combination is a NE, then there will be co-operation always
  - The Bertrand paradox will be resolved in that case

Suppose player 1 plays TS

- Is there any incentive for player 2 to deviate from TS

Consider any arbitrary period t.

- Two situations

Situation 1: One of the previous period's outcome (C,C)

- In one period, outcome differs from (C,C). Since player 1 plays TS, after that period,
  player 1 would play NC for all future periods.
- Player 2 will then play NC for all future periods.
- Hence once deviation occurs and given that player 1 chooses TS, it is also optimal for player 2 to stick to TS.

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Situation 2: All of the previous periods' outcome **:** (C,C)

Present value of payoff of player 2 by playing TS

$= \frac{\pi^m}{2} + \delta\frac{\pi^m}{2} + \delta^2\frac{\pi^m}{2} + \ldots\ldots\ldots$ [outcome in tth $\text{period} = (C,C)$ - it will be(C,C)forever and in each periods , each gets $\frac{\pi^m}{2}$]

$$= \frac{\pi^m}{2}/1-\delta$$

Present value of payoff of player 2 by not playing TS

$$= \pi^m + 0 + 0 + \ldots\ldots\ldots$$

[$\because$ In tth period, outcome is (C, NC) Player 2 gets π^m in period t-from period 't+1' outcome will be (NC, NC) and payoff will be 0]

PV of payoff to player 2 by playing TS>PV of payoff to player 2 by deviating from TS

$$\Rightarrow If\ \frac{\pi^m/2}{1-\delta} > \pi^m$$

$$i{:}\ \delta > \frac{1}{2}$$

- If $\delta > \frac{1}{2}$, both firms will have no incentive to deviate from TS

$\Longrightarrow$ both firms will find it beneficial to play co-operative strategy and hence will charge the monopoly price in each period

$\Longrightarrow$ Bertrand paradox can be resolved in an infinitely repeated game.