Module 2: "Static games of complete information"

Lecture 8: "NE: Case study & infinite Action set"

The Lecture Contains:

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NE: Case Study from animal kingdom

NE: An application to biology

- Animal Kingdom
  - Let us take spiders
- Spiders compete for scarce resources
  - fight over on existing web
- Two stylized facts
  1. Fighting between spiders is likely when stakes are higher
  2. Most of conflicts settled without fighting; winner of the conflict is endowed differently than the losers
- Can NE help to explain any of these stylized facts?

- consider the following case of conflict between 2 spiders
  - value of having web =10
  - if one spider fights and the other concedes, then one who fights gets the web.
  - if neither fight, both have a 50:50 chance to get the web

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NE: An application [contd.]

- If both fight, then again there is a 50-50 chance that either gets the web
  - Likelihood however that spider will be harmed by fighting

$$\text{Payoff} = x \underset{(>)}{<} 0 \text{ if Physical costs } \underset{(>)}{<} \begin{matrix}\text{expected}\\ \text{value of}\\ \text{web}\end{matrix}$$

	Concede	Fight
Concede	(5,5)	(0,10)
Fight	(10,0)	(x,x)

$x > 0$ $\Rightarrow$ stakes to fighting are higher

$\Rightarrow$ NE $\rightarrow (F, F)$

- Stylized Fact 1
  Fighting likely as stakes are higher

$x < 0$

$\Rightarrow$ 2 NE $\Rightarrow$ (Fight, concede), (concede, fight)

- Conflict settled without fighting

Suppose now, payoff to (F,F) (x, y)

Suppose $x < 0 < y$

spider 2 is heavier/more powerful than spider 1

NE (C, F).

Winner endowed with higher attribute gets the web without fighting.[Stylized Fact 2]

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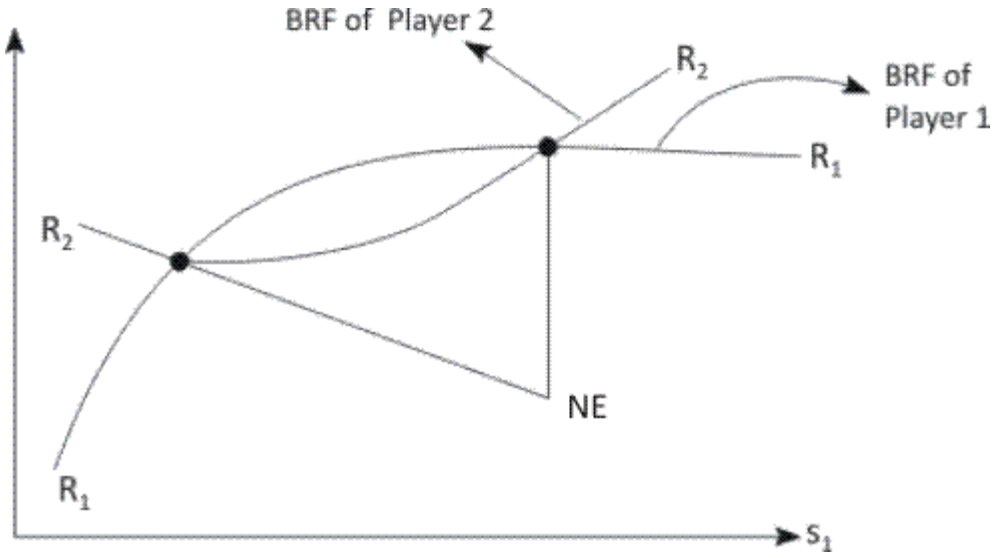
NE for infinite Strategy Set

- Use of best response functions
- Formal definition of best response function:
  - In a N- player game for any player i, the BRF $R^i(s^{-i})$ is such that for any given strategy profile s^{-i} chosen by all the players other then i,

$s_i^* = R^i(s^{-i})$ maximizes player i's payoff $\pi^i(s^i, s^{-i})$

If an outcome $\hat{s} = (\hat{s_1}, \hat{s_2}, \ldots, \hat{s_n})$ is a NE outcome, then for each player i, $\hat{s^i} = R(\hat{s}^{-i})$ should hold

One can plot the BRFs and intersection of BRFs gives the NE.



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Infinite Action Set : Use of BRF: An example

- Consider a line segment of unit length
- Population of voters uniformly distributed across this line set
- 2 candidates seeking an election to same office
- Candidate with majority of votes wins
- Voter prefers to vote for the candidate whose campaign platform is closer to voter's residence
- Candidates simultaneously choose their campaign platforms
- What campaign platforms should candidates choose ?

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Infinite Action Set: example [contd.]

BRF of player 1: $R^1(s^2)$

If player 2 chooses any platform to left of mid-point i.e. , if $s_2<0.5$

then player 1's best response is to choose slightly to the right of player 2's platform

[Get's majority of votes & wins]

$s_2 < 0.5$

Similarly

$R^1(s^2) = s_2 - \varepsilon \qquad s_2 > 0.5$

$R^1(s^2) = s_2 \qquad s^2 = 0.5$

cannot improve payoff by choosing anywhere other than $s^2 = 0.5$

Likewise

$$R^2(s^1) = \begin{cases} s_1 + \varepsilon & \text{if } s_1 < 0.5 \\ 0.5 & s_1 = 0.5 \\ s_1 - \varepsilon & s_1 > 0.5 \end{cases}$$

BRFs meet at (0.5, 0.5)

NE $\Rightarrow (0.5,0.5)$

None of the candidates have any incentive to deviate from this outcome

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Infinite Action Set: Another Example

- Players 1 & 2 are bargaining over Re 1
- Each player i simultaneously proposes a share (s_i) to an arbitrator
- Rules
  - If $s_1 + s_2 \leq 1$, then each player gets his/ her share

i.e. $\pi_1 = s_1, \pi_2 = s_2$

  - if $s_1 + s_2 > 1$, then arbitrator takes away the money i.e. $\Pi_1 = 0,\ \Pi_2 = 0$
  - Sketch the BRFs & find the NE.

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