Lecture 33: "Signaling Game: Numerical Example"

The Lecture Contains:

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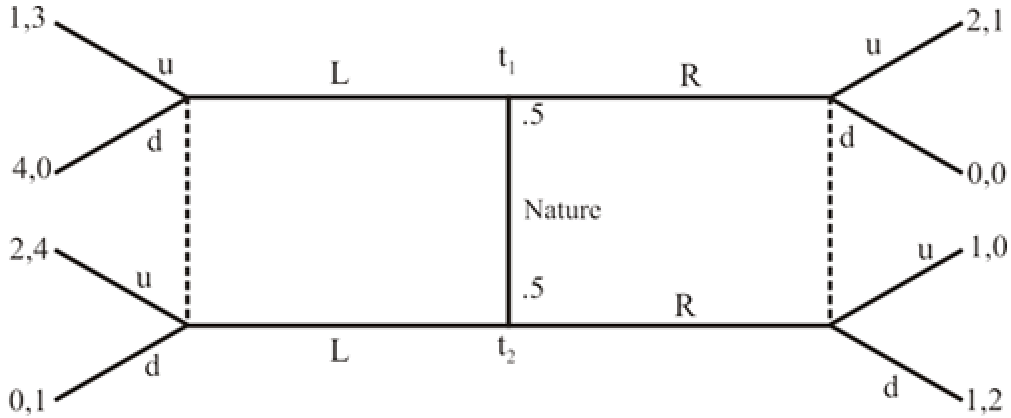
Signaling Game: Numerical Example



Find all the pure strategy PBNE for the above game. There are four possible pure strategy PBNE:

Pooling Equilibria:

1. Two types pooling on L
2. Two types pooling on R

Separating Equilibria:

Type 1 playing L and type 2 playing R
Type 1 playing R and type 2 playing L

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Numerical Example: [Contd.]

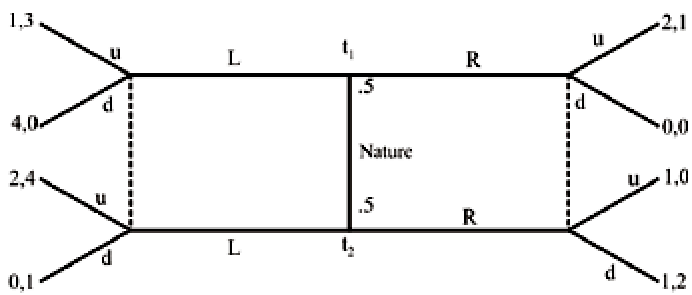
Let us analyze that whether there is pooling PBNE involving (L, L) strategy by the sender.

Let us denote $\mu\,(t_1/L) = p$ and $\mu\,(t_1/R) = q$

If there is a PBNE involving (L, L) strategy by sender

$\Longrightarrow$ p is fixed by requirement (3).

$$p = \frac{p(t_1)}{p(t_1)+p(t_2)} = \frac{.5}{.5+.5} = .5$$



$p = .5 \Longrightarrow$ Payoff to receiver from playing u

$= 0.5(3) + 0.5(4) = 3.5$

Payoff to receiver from playing d

$= 0.5\,(0) + 0.5(1) = 0.5$

$\Longrightarrow$ Hence seeing L, receiver plays u

$\Longrightarrow a^*(L) = u$

$\therefore$ at the strategy pair (L, L)

Sender of type t_1 earns a payoff =1 and sender of type t2 earns a payoff =2

To determine whether sender of any type has any incentive to deviate from sending L, it needs to be seen how the receiver responds if sender sends R.

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If receiver's response to R is U i.e. if a (R)=U, then t_1's payoff from playing R=2 > t_1's payoff from playing L=1.

If a(R)=d, then t_1 & t_2 earn payoffs of 0&1 from playing R and hence none of the types will have an incentive to deviate.

Hence for sender's strategy (L, L) to constitute part of equilibrium, receiver's response to R must be d. So receiver's strategy must be

$$\left(\underset{a(L)}{\underset{\uparrow}{u}},\ \underset{a(R)}{\underset{\uparrow}{d}}\right)$$

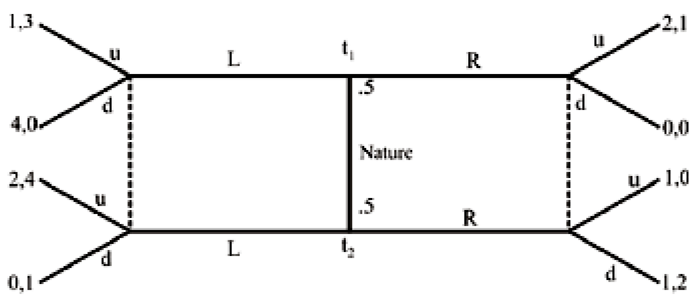
Seeing R is playing d optimal for receiver ? – that depends on belief $q\left(\mu(t_1/R)\right)$

q is off the eqm path-hence it is free to form.

Payoff to receiver from playing u

= q.1+(1-q).0 =q

Payoff to receiver from playing d



= q.0 + (1-q).2= 2-2q

Hence for q such that $q < 2 - 2q$ is $q \leq {}^2/_3$, receiver would play d facing R.

Hence $[(L, L), (U, d), p = .5, q < 2/3]$ a bunch of pooling PBNE.

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Numerical example [contd.]

Let's look for a pooling PBE involving the strategy (R,R) by the sender.

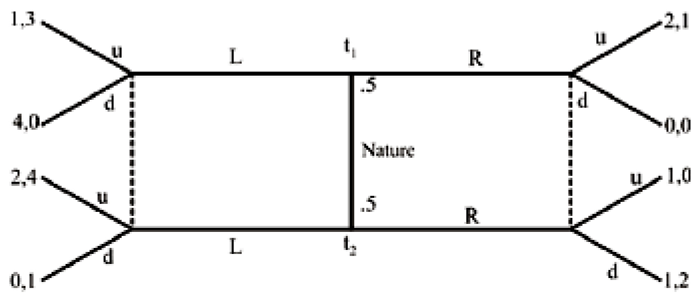
Any PBNE involving (R, R)

$\Longrightarrow$ q = .5

$\Longrightarrow$ a(R) = d ($\because$ payoff to receiver by playing u=0.5<1 = payoff to receiver by playing d)

$\Longrightarrow$ Payoff to sender at (R,R)

- Payoff of type t_1 = 0
- Payoff of type t_2 = 1



To check for incentives for deviation, let us find a(L)-

Now if receiver plays u in seeing L,

$\Longrightarrow$ Payoff = 3.p + (1-p).4

Plays d in seeing L

$\Longrightarrow$ Payoff = 0.p + 1. (1-p)

$\Longrightarrow$ So no matter what beliefs are a(L)=u

- t_1 earns 1 if he plays L
- t_2 earns 0 if he plays R

$\Longrightarrow$ t_1 always has an incentive to deviate.

$\Longrightarrow$ Hence there will be no pure strategy PBE involving (R,R)

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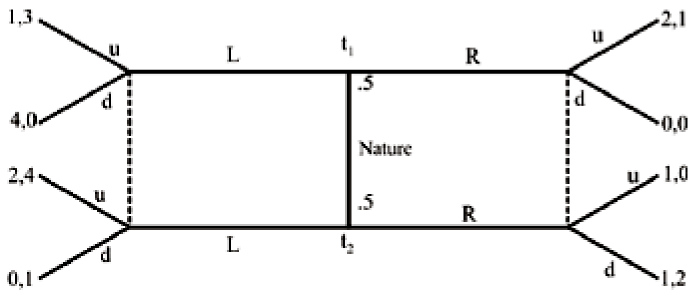
Numerical example [contd.]

Let us look for a separating PBE involving (L, R)

In a PBNE involving (L, R) both the beliefs p and q are determined by Bayers' rule.

$$p = \mu(t_1/L) = \frac{p(t_1)}{\begin{matrix} sum\ of\ probability\ of \\ different\ types\ that \\ send\ the\ message\ L \\ given\ the\ equilibrium \end{matrix}}$$



$$= \frac{p(t_1)}{p(t_1)} = \frac{0.5}{0.5} = 1$$

$$q = \mu(t_1/L) = \frac{0}{0.5} = 0$$

$\Longrightarrow$ a(L)=U;   a(R)=d

i.e. the receiver's strategy is (u,d)

$\Longrightarrow$ payoff to $t_1 = 1$

$\Longrightarrow$ payoff to $t_2 = 1$ Given the strategy (U,d) by the receiver. Is there any incentive for sender of any type to deviate?

No incentive for t_1

If t_2 deviates from R to L then payoff to $t_2 = 2$ [given the receiver's strategy a(L)=u]

>payoff to t_2 by playing R = 1

Given (u,d) response by receiver, sender of type 2 has an incentive to deviate & play L. So there is no separating PBNE involving (L,R)

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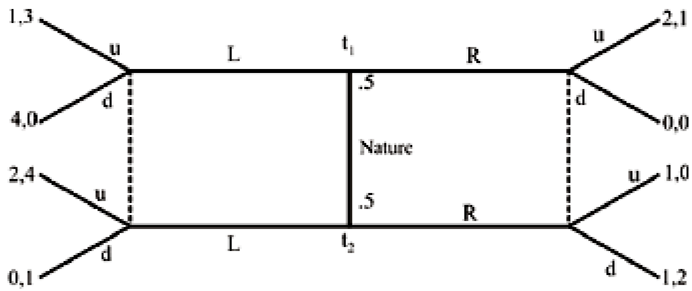
Module 5: "Dynamic games of incomplete information"

Lecture 33: "Signaling Game: Numerical Example"

Let us look for a PBNE involving the strategy (R, L) by the sender.

In a PBNE involving (R, L)



$\Longrightarrow$ p=0, q=1

$\Longrightarrow$ a(L)=U; a(R)=U

$\Longrightarrow$ i.e. receiver's strategy = (u, u)

$\Longrightarrow$ At the PBNE (R, L)

Payoff to type t_1 = 2

Payoff to type $t_2 = 2$

If t_1 deviates from R to L,

Payoff = 1 => so no incentive to deviate

Similarly if t_2 deviates from L to R,

Payoff = 1 => so no incentive to deviate.

Hence the following is a pure strategy PBNE

$[(R, L);\ (u, u), p = 0, q = 1]$ Separating PBNE