Module 3: "Dynamic games of complete information"

Lecture 25: "Sequential Bargaining Problem"

The Lecture Contains:

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Bargaining Problem: The outline

- Players 1& 2 are bargaining over Re 1.
- They alternate in making offers- the game is dynamic
- Player 1 makes a proposal in the 1^{st} period- player 2 can either accept it or reject it.
- If the offer is rejected, then game goes to the second period
  - Player 2 now makes an offer
  - Player 1 can either accept it or reject it
- If the offer is rejected by player 1, then the game goes on to the next period and so on.
- Players discount payoffs received in the later periods by the discount factor δ per period where $0 < \delta < 1$

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Three period sequential bargaining model

1. In period 1, player 1 proposes to take a share s_1 of the rupee leaving ($1-s_1$) for player 2.

- Player 2 either accepts the offer (the game ends there and player 1 receives 's_1' and player 2 '$1-s_1$') or rejects the offer => game goes to period 2

2. Player 2 proposes that player 1 takes a share s_2 of the rupee leaving ($1-s_2$) for player 2
   Player 1 can either accept the offer (in which case payoffs to player 1& 2 are s_2 & ($1- s_2$) respectively) or reject the offer- game goes to period 3.
3. At the beginning at third period, player 1 receives a share 's' of rupee and player 2 receives '1-s' of a rupee where 's' is exogenously given and is known to both the players

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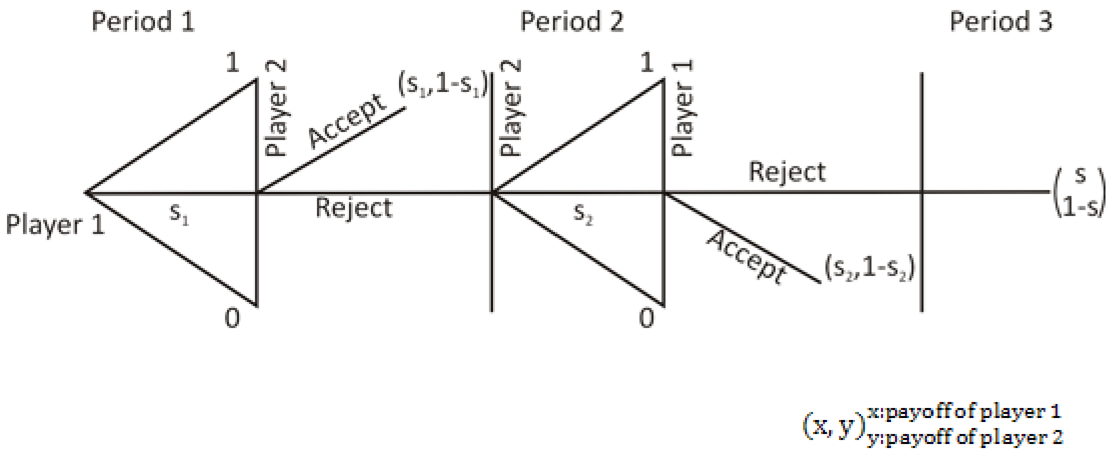
The game tree of the three period bargaining model is as follows



The game is solved by backward induction.

In third period, no player has to take any decisions as the third period settlement is given exogenously.

So one has to start from the 2nd period

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What is player 2's optimal offer in the 2^{nd} period?

Player 2 knows that player 1 can receive s in the 3^{rd} period by rejecting player 2's offer of s_2 in 2^{nd} period.

Present value in 2nd period of receiving s next period = δs

So player 1 will accept s_2 iff $s_2 \geq \delta s$

Player 2's second period decision problem : Offer $s_2 = \delta s$ & receive $1 - \delta s$ 2^{nd} period or receive (1-s) in 3^{rd} period which is $\delta\,(1-s)$ in 2nd period by offering $s_2 < \delta_s$

Now

$$1 - \delta_s > \delta(1-s)[\because\ \delta < 1]$$

$\Longrightarrow$ Optimal second period offer is $s_2^* = \delta s$. Hence if play reaches the 2nd period, player 2 will offer s_2^* and player 1 will accept.

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Module 3: "Dynamic games of complete information"

Lecture 25: "Sequential Bargaining Problem"

Period 1's problem

- Player 1 is rational & can solve player 2's decision problem in 2nd period.
- Player 1 knows that player 2 can receive $(1-s_2^*)$ in the 2nd period by rejecting player 1's offer of s_1 this period.

  PV to player 2 of receiving $(1-s_2^*)$ next period $=\delta(1-s_2^*)$
- Player 2 will accept $(1-\ s_1)$ iff $1-\ s_1 \geq\ \delta(1-s_2^*)$
  i.e. $s_1 \leq 1-\delta(1-s_2^*)$
- Player 1's decision problem :
  - Receive $1-\delta\,(1-s_2^*)$ in 1st period
    [by offering $1-s_1=\delta(1-s_2^*)$ to player 2]
  - or receive s_2^* next period i.e. δs_2^* this period
- Now
  $1-\delta(1-s_2^*)$
  $=1-\delta(1-\delta s)$
  $=1-\delta+\delta^2 s$
  $>\ \delta s_2^* =\ \delta^2 s$
  $(\because \delta\ < 1)$
- Player 1's optimal first period offer is $s_1^* = 1-\delta(1-s_2^*)$
  $=1-\delta(1-\delta s)$
- Outcome : Player 1 offers the settlement
  $(s_1^*, 1-s_1^*)$ to player 2. Player 2 accepts the offer.
  - Game ends in 1st period itself.

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