Module 4: "Static games of incomplete information"

Lecture 27: "Cournot Game with Incomplete Information: A Numerical Example"

The Lecture Contains:

- Cournot game with incomplete information
  - A numerical example

Cournot game with incomplete information: An example

- Two players- firm 1 & firm 2
- Firms 1 & 2 simultaneously choose non- negative levels of output- q_1 and q_2
- A firm has a marginal cost of c^H or $c^L, c^H > c^L$
- A firm knows its own cost but does not know the exact cost of the other firm.
- The following probability distribution over MCs is common knowledge

		Firm 2 c^H	*Firm 2* c^L
Firm 1	c^H	1/3	1/3
Firm 1	c^L	1/3	0

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Prob $(c_2 = c^H / c_1 = c^L)$

$= \frac{P(c_2=c^H, c_1=c^L)}{Prob(c_1=c^L)}$

$= \frac{1/3}{1/3} = 1$

$Prob(c_2 = c^H / c_1 = c^H)$

$\frac{P(C_2 = C^H, C_1 = C^H)}{Prob(C_1 = C^H)}$

$= \frac{1/3}{2/3} = 1/2$

There probabilities are derived using Bayesian formula

These probabilities are used in finding the best response of each firm.

Now depending on the cost the firm chooses a quantity i.e. $q_i = q_i(c^i)$

Let firm 1 has a high cost i.e. $c_1 = c^H$

It will choose a q_1 that will maximize its payoff.

So its optimization problem will be,

$$\underset{q_1{}^H}{Max} \left[\left\{ \left\{ a - \left(q_1{}^H + q_2^{*L} \right) \right\} q_1{}^H \right\} Prob(c_2 = c^L / c^1 = c^H) \right.$$
$$\left. + \left\{ \left\{ a - \left(q_1{}^H + q_2^{*H} \right) \right\} q_1{}^H \right\} Prob(c_2 = c^H / c^1 = c^H) \right]$$

$$\underset{q_1{}^H}{Max} \left[\frac{1}{2} \left\{ a - \left(q_1{}^H + q_2^{*L} \right) \right\} q_1{}^H + \frac{1}{2} \left\{ a - \left(q_1{}^H + q_2^{*H} \right) \right\} q_1{}^H \right]$$

$\Longrightarrow$ Best response function relating q_1^H to q_2^{*L} and q_2^{*H}

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Example [contd.]

Best response function of firm 1 when it is a low cost firm i.e. $C_1 = C^L$

Optimization problem

$$\underset{q_1{}^L}{Max}[\{a-(q_1{}^L+q_2^{*L})\}\,q_1{}^L \times Prob(c_2=c^L/c_1=c^L)$$
$$+\{a-(q_1{}^L+q_2^{*H})\}q_1{}^L \times Prob(c_2=c^H/c_1=c^L)]$$

i.e. $\underset{q_1}{Max}[0.\{a-(q_1{}^L+q_2{}^L)\}q_1+1.\{a-(q_1{}^L+q_2{}^H)\}q_1{}^L]$

Best response function relating $q_1{}^L\ to\ q_2{}^H\ \&\ q_2{}^L$

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Example [contd.]

- Symmetric Bayesian Nash equilibrium

$$\Rightarrow q_1^{*H} = q_2^{*H}$$
$$q_1^{*L} = q_2^{*L}$$

- **Four equations**

$q_1^{*L} = \frac{a - c_L}{2} - \frac{q_2^{*H}}{2}$ [BRF of high cost firm 1]

$q_1^{*H} = \frac{a - c_H}{2} - \frac{q_2^{*L} + q_2^{*H}}{4}$ [BRF of low cost firm 1]

$q_1^{*H} = q_2^{*H}$

$q_1^{*L} = q_2^{*L}$

Four unknowns
$q_1^{*H}, q_1^{*L}, q_1^{*H}, q_2^{*L}$
Hence the system can be solved.

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Example [contd.]

Solution of the game:

$$q_1^{*L} = q_2^{*L} = \frac{3a - 5c_L + 2c_H}{9}$$

This is the symmetric Bayesian Nash equilibrium of Cournot game with incomplete information.

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