Module 5: "Dynamic games of incomplete information"

Lecture 30: "PBNE: Requirements 3 & 4, Numerical Examples"

The Lecture Contains:

- Requirement 3 & Requirement 4 of PBNE
- Numerical Example

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PBNE : Definition [Contd.]

Requirement 3

- Beliefs should be reasonable
- To impose such requirements or players beliefs', distinction should be made between I-set on equilibrium path and I-set off equilibrium path.
- Definition: For a given equilibrium in a given extensive form game, an <u>I-Set is on equilibrium path</u> if it will be reached with positive probability if the game is played according to equilibrium strategies and is <u>off the equilibrium path</u> if it is certain not to be reached if the game is played according to equilibrium strategies.

Requirement 3 :-

At I-sets on equilibrium path, beliefs are determined by Bayes ' rule and the players' equilibrium strategies.

$$\text{Prob}\left(\underbrace{x}_{\text{Node}}/\overset{\text{I-set}}{h},\ \underbrace{\delta}_{\text{Equillibrium Strategy}}\right) = \frac{\text{Prob}\,(x/\delta)}{\sum_{x'\in h} \text{Prob}\,(x'/\delta)}$$

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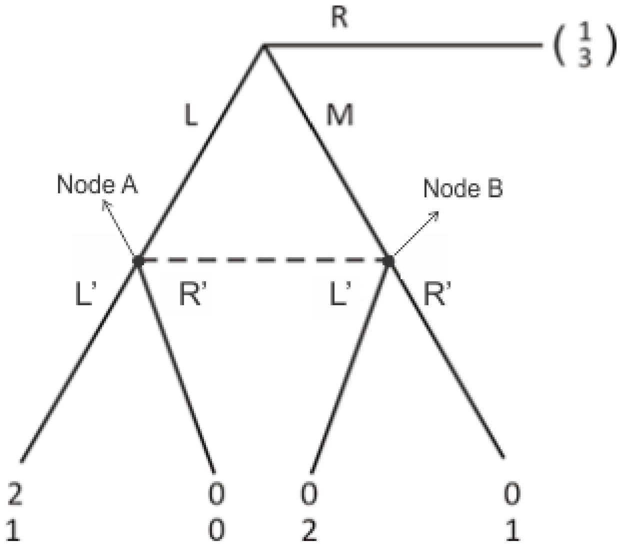
Requirement 3 : Example

Suppose for the previous example



there is a mixed strategy equilibrium in which player1 plays L with probability q_1, M with probability q_2 and R with probability 1-q_1- q_2

i.e. $\sigma_1(\mathrm{L}) = q_1$, $\sigma_1(\mathrm{M}) = q_2$

$\sigma_1(\mathrm{R}) = 1 - q_1 - q_2$

Then

p=Prob(Node A/h, σ)

$$= \frac{q_1}{q_1 + q_2}$$

In Subgame perfect NE (L, L')

$$P = \frac{q_1}{q_1 + 0} = 1$$

Definition:

A PBNE consists of strategies and beliefs satisfying Requirements 1 through 3.

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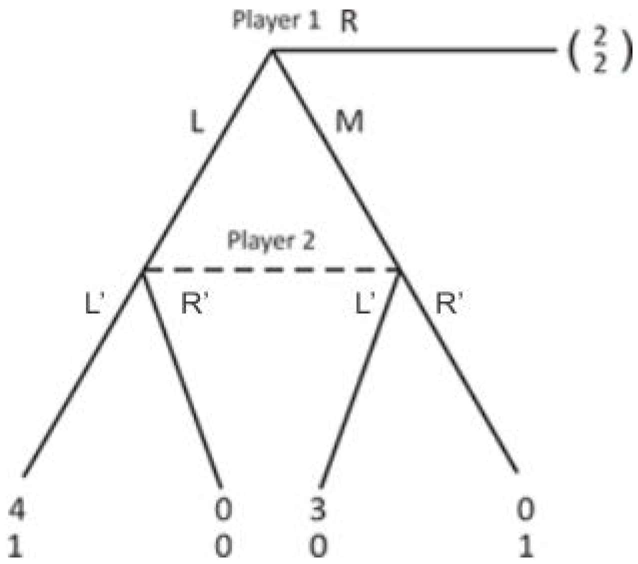
PBNE: Numerical Problem

Find PBNE of the above game.



We first look for PBNE with $\sigma_1(R) = 1$

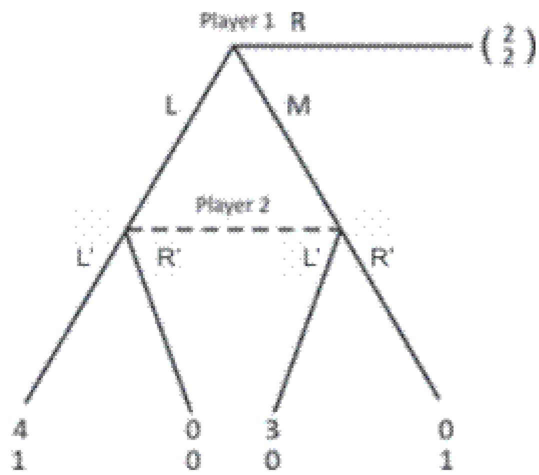
If there is such an equilibrium, the information set is off the equilibrium path.

- What Player 1's best response is, that depends on what player 2 is doing which depends on beliefs of player 2.
- Such beliefs for player 2 have to be found so that given the beliefs it is best for player 1 to play $\sigma_1(R) = 1$

Numerical Problem [Contd.]

- Payoff to player 1 by playing R
  $(\text{i. e. } \sigma_1(R) = 1) = 2$
  Let $\sigma_2(L')$ be the probability with which player 2 plays L'
  Payoff to player 1 by playing L
  $= \sigma_2(L').4 + (1 - \sigma_2(L')).0 = 4\sigma_2(L')$

Similarly, payoff to player 1by playing M = $3\sigma_2(L')$



- Player 1 will play R iff
  $4\sigma_2(L') \le 2 \text{ i. e. } \sigma_2(L') \le \frac{1}{2}$
- Beliefs for player 2 should be such that player 2 plays L with
  $\sigma_2(L') \le \frac{1}{2}$

Given the belief p, Payoff from playing $\mathrm{L}' = 1.\mathrm{p} + 0(1 - \mathrm{p}) = \mathrm{p}$

Payoff from playing $\mathrm{R}' = 0.\mathrm{p} + (1 - \mathrm{p}).1 = 1 - \mathrm{p}$

- Player 2 will play L' for sure if $\mathrm{p} > 1 - p \; i.e. p > \frac{1}{2}$

  $\therefore p < \frac{1}{2} = 1, \sigma_2(L') = 0 \Rightarrow \sigma_1(R) = 1$
  Hence there exists a bunch of PBE charecterised by

  $\sigma_1(R) = 1, \sigma_2(L') = 0, p < \frac{1}{2}$

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Numerical Problem [Contd.]

- Look for a PBE with $\sigma_1(R) < 1$
- I-set is on the equilibrium path
- Beliefs pinned down by requirement (3)
  $\sigma_1(R) < 1 \Rightarrow \sigma_2(L') > 0$
  [$\because$ this is the only way that player 1 can gain by playing L or M (i.e. by giving up R)]
  $\sigma_2(L') > 0 \Rightarrow \sigma_1(M) = 0$
  [$\because$ Payoff from playing $L = 4\sigma_2(L')$ > Payoff from playing $M = 3\sigma_2(L')$]
  $\sigma_1(R) < 1 \Rightarrow \sigma_1(M) = 0 \Rightarrow \sigma_1(L) > 0$

  By requirement (3)

  $p = 1\left[= \frac{q_1}{q_1 + q_2}\right]$
  Here $q_1 = \sigma_1(L) > 0$
  $q_2 = \sigma_2(M) = 0$
- $p = 1 \Rightarrow \sigma_2(L') = 1 \Rightarrow \sigma_1(L) = 1$
- One unique PBNE:
  $[\sigma_1(L) = 1, \sigma_2(L') = 1 \,\&\, p = 1]$;
  Bunch of PBNE
  $[\sigma_1(R) = 1, \sigma_2(L') = 0, p < \frac{1}{2}]$