Module 5: "Dynamic games of incomplete information"

Lecture 32: "Signaling Game"

The Lecture Contains:

Previous Next

Signaling Game: Main features

- A signaling Game is a dynamic game of incomplete information involving 2 players: a sender (S) and a Receiver (R)
- The timing of the game is as follows:-
  1. Nature draws a type t_i for the sender from the set of feasible types $T = \{t_1, \dots\dots, t_I\}$ according to a probability distribution $p(t_i)$, where $p(t_i) > 0$ for every i and $p(t_1) + \dots\dots + p(t_I) = 1$
  2. Sender observes ti and then chooses a message m_j from a set of feasible messages.
     $M = \{m_1, \dots\dots\dots, m_J\}$
  3. The receiver observes m_j (but not t_i and then chooses an action a_k from a set of feasible actions A= $\{a_1, \dots\dots, a_K\}$
  4. Payoffs are given by $U_s(t_i, m_j, a_k) \,\&\, U_R(t_i, m_j, a_k)$

Previous Next

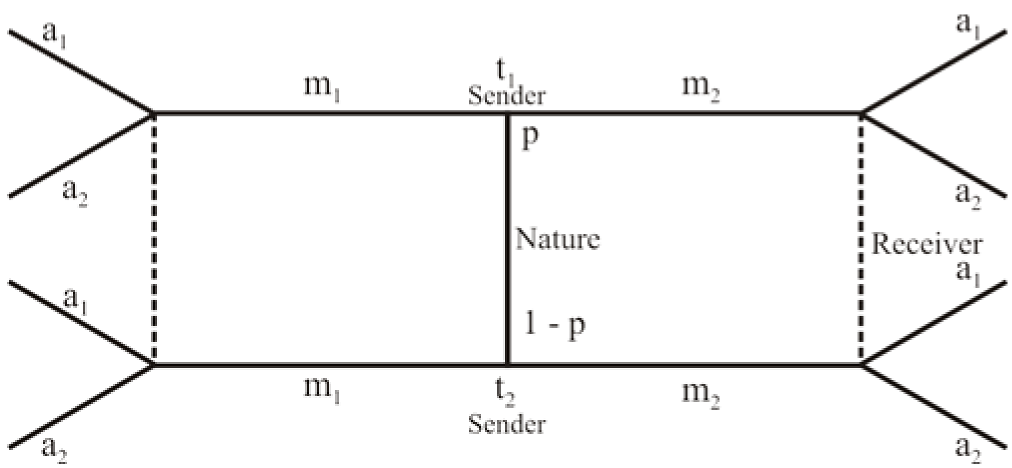
Pooling and Separating strategies

Consider a simple case:

$T = \{t_1, t_2\}$, $M = \{m_1, m_2\}$

$A = \{a_1, a_2\}$, Probability (type=t_1)= p (common knowledge)

The signaling game with the timing described above can be depicted as



Receiver knows whether the sender has given message m_1 or m_2 but does not know the exact type of the sender.

Previous Next

Strategy is a complete plan of action, i.e. it is a plan of all feasible actions for every possible contingent situation.

Strategy for sender: m(ti): message chosen for each possible type.

Strategy for receiver $a(m_j)$: action chosen for each possible message sent by sender.

=> In this game set-up, there are four strategies of sender

$$\text{Sender:}\quad \left.\begin{matrix} (m_1, m_1) \\ (m_2, m_2) \end{matrix}\right\} \text{Pooling strategy: here each type sends the same message}$$

$$\left.\begin{matrix} (m_1, m_2) \\ (m_2, m_1) \end{matrix}\right\} \text{Separating Strategy: each type sends a different message.}$$

Previous Next

PBNE for signaling games: Restatement

Signaling Requirement 1:

After observing any message mj from M, the receiver must have a belief about the types that have sent m_j.

Let the belief be represented by the probability distribution $\mu(t_i/m_j)$ where $\mu(t_i/m_j) \geq 0$ for each t_i and $\sum_{t_i \in T} \mu(t_i/m_j) = 1$

Signaling requirement 2R:

For each m_j, $a^*(m_j)$ solves

$$\max_{a_k \in A} \sum_{t_i \in T} \mu(t_i/m_j)\, U_R(t_i, m_j, a_k)$$

Signaling requirement 2S:

For each t_i in T, the sender's message $m^*(t_i)$ solves

$$\max_{m_j \in M} U_s\left(t_i, m_j, a*(m_j)\right)$$

PBNE: Restatement [Cont.]

Signaling Requirement 3:

For each $m \in M$, if there exits $t_i \in T$ such that $m^*(t_i) = m_j$, then the receiver's belief at the I-set corresponding to m_j must follow from Bayes' rule and senders' strategy.

$$\mu(t_i/m_j) = p(t_i) \Big/ \sum_{t_i \in T} p(t_i)$$

Definition: A pure strategy PBNE in a signaling game is a pair of strategies $m^*(t_i)$ and $a^*(m_j)$ and a belief $\mu(t_i/m_j)$ satisfying requirements (1), (2) (2S and 2R) and (3).

Previous Next