

Module 3: "Dynamic games of complete information"

Lecture 21: "Repeated Games"

The Lecture Contains:

- ☰ Repeated Games
- ☰ Infinitely repeated games

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Repeated Games

A repeated game is a one shot simultaneous game that is repeated more than once.

		2	
		L ₂	R ₂
1	L ₁	1, 1	5, 0
	R ₁	0, 5	4, 4

- Suppose this game is repeated twice
 - once in period 1 & once in period 2
- Players are concerned with the aggregate payoff

This therefore is a special kind of dynamic game.

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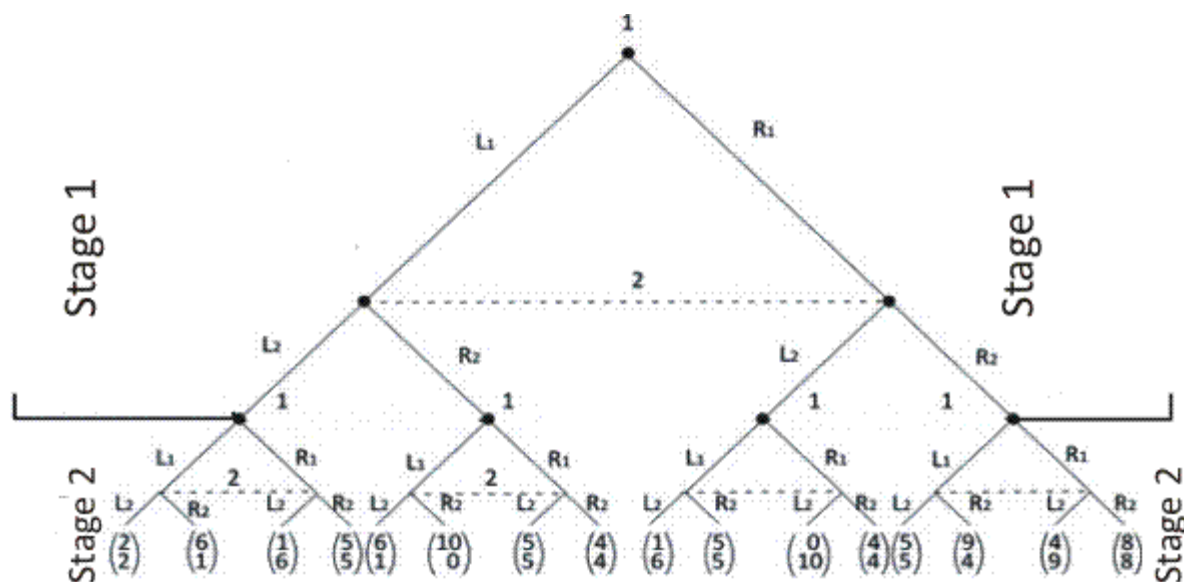
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Repeated Game :

Example continued

The repeated game is a dynamic game with the following features :

- (1) Players move simultaneously at each stage
 - (2) Players action sets do not vary over time i.e. if L_1 & R_1 are available to player 1 at period 1 & the game is a repeated one, then in period 2 also , the actions available are L_1 & R_1
 - (3) Each period's outcome is observed before play begins at the next period.
- This game can be represented in the following extensive form.



To find SPNE, one has to use backward induction. Here there are four subgames. If game is repeated twice, there will be 16 subgames. If game is repeated for more number of times the number of subgames rises even further.

Hence to solve finitely repeated game we use the method of backwards induction in a different manner.

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NE in this static game (not repeated)

		2	
		L ₂	R ₂
L ₁		1, 1	5, 0
R ₁		0, 5	4, 4

(L_1, L_2) is NE

Co-operative outcome would have been (R_1, R_2) - it fetches higher payoffs to both

- however everybody has an Incentive to cheat and unilaterally deviate.
 - Is there a scope for punishing the deviating person if the game is repeated.
 - In other words, can co-operation be sustained in a finitely repeated game?
- Suppose the game is repeated twice.
- what will be the NE?

Using the method of backwards induction, one starts from stage 2.

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Stage 2

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The game ends after stage 2

Now consider stage 1

- Stage 2 equilibrium is (L_1, L_2)
- (R_1, R_2) cannot be an equilibrium as each has an incentive to deviate.
 - (R_1, R_2) is sustainable as an equilibrium if cost of unilaterally deviating from R_1 or $R_2 > \text{Benefit of unilaterally deviating from } R_1 \text{ or } R_2$
 - Benefit for each player to unilaterally deviate (to L_i) from $R_i=1$
- Whatever be the outcome in 1st period, outcome in 2nd period is always (L_1, L_2) .
 - Hence cost of unilateral deviation=0
[∴ there is no punishment.]
- Hence everybody has an incentive to deviate.
[don't cross]

This is true for any finitely repeated game.

For any finitely repeated game, the non co-operative equilibrium repeated at each stage is the equilibrium outcome.

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Infinitely repeated games

Problems:

- (1) Cannot identify the last period - difficult to use the method of backwards induction.
- (2) In an infinitely repeated game, payoffs become infinite.

To counter this, one uses the method of discounting.

Suppose player 1 gets Re 1 as a payoff in period 1. She can invest this Re 1 on an income earnings asset which yields a rate of interest of $r\%$ per year.

1 rupee now $\simeq (1+r)$ rupee in next period.

\Rightarrow 1 rupee next period $\simeq \frac{1}{1+r}$ rupee now.

Hence future payoffs are discounted by the discount factor $\delta = \frac{1}{1+r}$, $\delta < 1$

In an infinitely repeated game, the player is concerned with the present value (PV) of sum of payoffs.

Suppose payoff to a player is π^1 in each period by playing a particular strategy. Hence

PV of payoff by playing this strategy

$$= \pi^1 + \delta\pi^1 + \delta^2\pi^1 + \dots = \pi^1[1 + \delta + \delta^2 + \dots]$$