Module 2: "Static games of complete information"

Lecture 9: "Cournot Model of Duopoly"

The Lecture Contains:

Cournot Model of Duopoly

- Two sellers – Firm I & Firm II
- Produce a homogenous product
- Aggregate market demand for the product is

  $$P = a - bQ \qquad Q = q_1 + q_2$$

- Total cost

  $$C_i(q_i) = cq_i, c < a$$

- Firms choose their quantities simultaneously
- **oligopoly model**

  - Scope for strategic interaction
- Few sellers in the market
- Price determined jointly by their output decisions
- One ferm's payoff therefore dependent on other firms strategies (output choices)via the price

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Representation of Cournot case in NFG

- Players: Two firms
- Strategies :

  $s_i = q_i \qquad s_i \in S_i$

  $S_i = [0, \infty]$

- Payoff Function:
  - it should specify the payoff to firm i as a function of strategies chosen by other firms
  - $\Pi_i\left(q_i, q_j\right) = q_i\left[a - b\left(q_i + q_j\right) - c\right]$
- Solution of the game - NE
  - $\left(q_i^*, q_j^*\right)$ is a NE if for each player i,

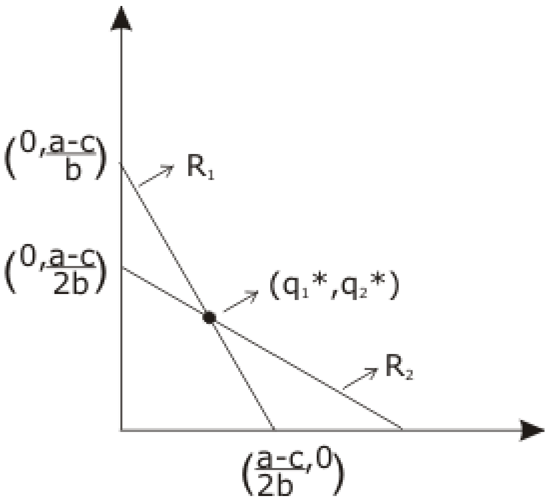
    $\Pi_i\left(q_i^*, q_j^*\right) \geq \Pi_i\left(q_i, q_j^*\right)$ for all $q_i \in S_i$

  i.e. q_i^* solves $\underset{q_i \in S_i}{\text{Max}}\, \Pi_i\left(q_i, q_j^*\right)$

Cournot model – [Contd.]

- FOC for i^th^ firm
- Best response function of firm i

$$q_i = \frac{a-c}{2b} - \frac{q_j^*}{2}$$



- (q_1^*, q_2^*) is a NE if,

$$q_1^* = \frac{a-c}{2b} - \frac{q_2^*}{2}$$

$$q_2^* = \frac{a-c}{2b} - \frac{q_1^*}{2}$$

- $q_1^* = q_2^* = \frac{a-c}{3b}$
- $NE\ is\ \left[\frac{a-c}{3b}, \frac{a-c}{3b}\right]$
- $\Pi_1^* = \frac{(a-c)^2}{9b} = \Pi_2^*$

Cournot model with different costs

- Firm are not identical
  - MC of firm $1 = c_1$
  - MC of firm $2 = c_2$
- NE (following some procedure)
- $q_1^* = \frac{a-2c_1+c_2}{3\,b}$; $\quad q_2^* = \frac{a-2c_2+c_1}{3b}$
- $\pi_1^* = \frac{(a-2c_1+c_2)^2}{9b}$; $\quad \pi_2^* = \frac{(a-2c_2+c_1)^2}{9b}$
- Suppose firm 1 is involved in R and D expenditure
  $\Rightarrow c_1$ falls $\Rightarrow q_1^*$ increases, q_2^* falls, π_1^* increases, π_2^* decreases

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Cournot model Different MCs [Contd.]

- Suppose firm 1 is a low cost firm
  - $c_1 < c_2 \Rightarrow q_1^* > q_2^*$
- c_1 falls further given c_2
  $\Rightarrow q_1^*$ rises, q_2^* falls
- $c_1, c_2 < a$
  Suppose $2c_2 > a$
- $q_2^* = \frac{a - 2c_2 + c_1}{3b}$
- c_2 staying constant, if c_1 falls, q_2^* may become zero or negative
- If cost difference is sufficiently high, for a particular combination of c_1 & c_2, q_2^* becomes negative
- **Main Point** :
  By spending money in R & D & succeeding in bringing about continuous innovations, a firm can push the rival outside the market.

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Numerical example of Cournot Game

- Market demand function is P = 6 –Q
- Each firm can choose only one of qty levels [0, 1, 2, 3, 4, 5, 6]
- $c_i = 0$
- Find the best response of firm 1 if firm 2 produces 4 units of output
- $\Pi_1 = [6 - q_1 - q_2]q_1$
- $\frac{\delta \pi_1}{\delta q_1} = 0 \Rightarrow q_1 = \frac{6-q_2}{2}$

  $\Downarrow$

  BRF of firm 1
- If $q_2 = 4,$ firm 1's best response is
  $q_1 = 1$