Module 5: "Dynamic games of incomplete information"

Lecture 37: "Signaling Game : Limit pricing model [Cont.]"

The Lecture Contains:

Backward Induction Second stage Game : When Maria enters

- Market becomes a duopoly
- Both play a Cournot duopoly game.
- $P(Q_J, Q_M) = 16 - \frac{1}{100}(Q_J + Q_M)$
- Joe's second – Period profits are
  $\Pi^H_{J^2} = (P(Q_J, Q_M) - 5).Q_J \; if \; C_J = 5$
  $\Pi^L_{J^2} = (P(Q_J, Q_M) - 7).Q_J \; if \; C_J = 7$
- Maria's profit function is,
  $\Pi_M = (P(Q_J, Q_M) - 7).Q_M = 600$
- Cournot output & prices are given in the following table

	Joe's costs	
	High	*Low*
Joe's pizza output	300	433.33
Joe's pizza price	10.00	9.33
Joe's profit	900.00	1,877.78
Maria's pizza output	300	233.33
Maria's pizza price	10.00	9.33
Maria's profit	300.00	-55.56

Table 2 : Cournot profit in 2^{nd} stage

Previous Next

Separating & pooling Equilibrium in this context:

- From Table 2 on slide 1 it is clear that if Joe's costs are high , Maria earns a profit of 300 units upon entry but suffers a loss of 55.56 units if Joe has low cost.
- Maria will enter if she knows that Joe has high costs and will stay out if Joe has low costs.
- However Maria does not know Joe's cost.
- If Joe can signal credibly to Maria that he has low costs , he will deter entry of Maria.
- We will look for PBNE
  - Separating and pooling
- Separating PBNE- Joe will signal differently depending or whether his costs are high or low
- Pooling PBNE – Joe will choose the same signal irrespective of the costs.

Previous Next

Finding a separating equilibrium

- Let P^H & P^L be the prices that Joe will charge in the 1st period depending on whether he has high or low cost respectively.
- Suppose that Maria believes that a price of P^H means that Joe has high costs & a price of P^L means that Joe has low costs.
- If Joe has high costs, it must be the case that his payoff is higher by charging P^H in first time period then by charging P^L

i.e.

$$\Pi_{J_1}^{H}(P^H) + 900 > \Pi_{J_1}^{H}(P^L) + 2025$$

900 = Second period profit of Joe[duopoly profit - since Maria enter]

2025 = Second period profit of Joe[monopoly profit - Maria does not enters]

Or $\Pi_{J_1}^{H}(P^H) - \Pi_{J_1}^{H}(P^L) > 1125$ .... (A)

Previous Next

Separating Equilibrium [Contd…]

- If Joe has high costs and if the equilibrium is separating , then Joe must expect a price of P^H to signal to Maria that his costs are high → Maria will enter industry
  - Joe therefore expects deterence to fail in this case
  - Set P^H at short run profit maximizing level of $11.50 \Rightarrow \Pi_{J_1}^{H}\left(P^H\right) = 2025$
- Putting value of $\Pi_{J_1}^{H}\left(P^H\right)$ in (A)

$\Rightarrow \Pi_{J_1}^{H}(P_L) \leqslant 900$

i.e. $\left(P^L - 7\right)\left(1600 - 100P^L\right) \leqslant 900$

  - $P^L \leqslant 8.146$

Hence if Joe has low costs, the highest low price that will retain the separating equilibrium is 8.14 units.

Previous Next

Finding Separating Eqn [Contd….]

- Let's now suggest a hypothetical separating PBNE

  $P^H = 11.50 \; P^L = 8.14$

  $\mu(H/P_J > 8.14) = 1$

  $\mu(H/P_J \leq 8.14) = 0$

  Maria enters if Price = 11.50
  Maria stays out if price = 8.14

- It has already been shown that given Maria's beliefs and strategies , if Joe has high costs, his best response is to choose a high price $P^H = 11.50$
- If Joe has low costs,
  - If he chooses $P^L < 8.14$
    - Then gets lower profit in first period & same profit in second period
    - Hence so incentive to choose $P^L < 8.14$
  - If Joe chooses $P^L > 8.14$
    - Given Maria's beliefs, Maria will enter
    - He will choose SR profit maximizing price → earns 3025 units in first time period
    - Maria enters & Joe earns second-period profits of 1877.78units
    - Total Profit = 4902.78
  - If Joe chooses $P^L = 8.14$
    - Joe earns profit = 2468.04 is first period
    - Maria will not enter → profit = 3025 in second period
    - Total profit = 5493.04
- Best response for Joe if costs are low is to charge $P^L = 8.14$
- Given Joe's strategy & Maria's beliefs , Maria's strategy is also optimal
  - When $P \leq 8.14$
    Maria expected profit = -55.56 if she enters
    = 0 otherwise $\Longrightarrow$ Maria stays out
  - When P > 8.14
    Maria's expected profit = 300 if she enters
    = otherwise $\Longrightarrow$ Maria enters
  - Hypothetical BNE is verified to be the BNE.

Previous Next

Finding a pooling Equilibrium

- Pooling Equilibrium means Joe chooses the same first period price when his costs are high as well as low.
- If a pooling equilibrium exists , entry must be deterred
  - Suppose entry is not deterred
  - If Maria enters in 2nd period Joe will know that beforehand as she knows Maria's payoff.
  - Joe will be better off by charging the first period profit maximizing price
    - That price depends on cost
      = 11.50 if costs are high
      = 10.50 if costs are high
    - Different prices mean no pooling equilibrium exists.
- Hence in order for an equilibrium pooling price to exist , Maria must not enter.
  - Maria's expected profit from entering
    < 0 = Payoff from not entering
    Here $\text{expected profit} = 0.9(-55.56) + 0.1(300) = -20$
- Hence Maria will not enter

Previous Next

Finding a pooling Equilibrium[Contd…]

- Pooling Equilibrium
  - No updating of prior beliefs
- To check whether Joe has any incentives to deviate from pooling equilibrium , one has to compare Joe's expected profits it he chooses pooling price with that if he does not choose the pooling price.
- We need to specify Maria's off equilibrium beliefs
- Suppose Maria believes that for any price above the pooling price (say x) Joe has high cost with probability 1 & price < x →Joe has low cost with probability 1
- Hypothetical pooling equilibrium:
  $P(C_H) = P(C_L) = 11.50$
  - For high cost, $P(C_H) = 11.50$ is definitely optimal.
  - For low cost, if $P(C_H) = 10.50$ alternatively,
    - then Maria believes Joe has low cost & will not enter
    - allows Joe to choose 10.50 in second period also.
    - Joe's profits will be higher if $P(C_L) = 10.50$ and not 11.50
- Hence no such pooling equilibrium exists.

Previous Next

Module 5: "Dynamic games of incomplete information"

Lecture 37: "Signaling Game : Limit pricing model [Cont.]"

- Another hypothetical pooling equilibrium
  $P(C_H) = P(C_L) = 10.50$
- For low cost , Joe's best strategy is to choose $P(C_L) = 10.50$
- What if Joe has high costs?
  - If Joe charges a higher price of 11.50 instead → given Maria's beliefs, this will induce entry.
    Joe's total profit will then be = 2025+900 = 2925
  - If Joe chooses the pooling price of 10.50, entry is deterred:
    Joe's total profit in this case
    $= 1925(\text{1st period}) + 2025(\text{2nd period}) > 2925$
- Joe has no incentive to deviate from $P(C_H) = P(C_L) = 10.50$
  Hence pooling equilibrium exists where $P(C_H) = P(C_L) = 10.50$

Previous Next