Module 2: "Static games of complete information"

Lecture 11: "Application of NE : Tragedy of commons"

The Lecture Contains:

- Tragedy of commons

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Tragedy of commons

- Original paper by Hardin(1968)
- Basic idea :-
  - People respond only to private incentives
  - As a result , public goods will be under provided
  - Public resources over utilized
- Examples of the tragedy of commons
  - Historical example of herds of buffalo in the great plains of America
    - The once existing sprawling herds disappeared due to excessive hunting
  - Global warming
    - Burning coal oil or natural gas by driving cars using ACs and doing other things to heat homes.
    - Costs of each of this activity is global while benefits are borne by the users only.
    - Hence incentive to burn more coal, oil→ global warming

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Tragedy of commons : Theoretical Example

- N farmers in village
  - All graze their goats on village green
- Let the number of goats owned by i^{th} farmer - g_i
- Total number of goats in village denoted by $G = g_1 + \cdots + g_n$
- Cost of buying and caring for a goat - c
- Value to a farmer of grazing a goat on green when a total of G goats are grazing – V(G) per goat
- There is a maximum number of goats that can be grazed on the green - G_{max}
  - $V(G) > 0$ for $G < G_{max}$ but $V(G) = 0$ for $G > G_{max}$
  - For first few goats, where adding one more goat , there is a little harm done to those already grazing in relation to the case when there are many goats already grazing , where the harm is much more
    For $G < G_{max}$, $V'(G) < 0$ $V''(G) < 0$

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Tragedy of commons : Theoretical Example [Contd…]

- Simultaneous game
- Farmers simultaneously choose how many goats to graze
- A strategy for farmer $i - g_i$
- Payoff to farmer i form grazing g_i goats when the number of goats grazed by other farmers are $(g_i, \dots, g_{i-1}, g_i, g_{i+1}, \dots, g_n)$ is
  $[g_i V(g_i + \cdots + g_{i-1} + g_i + g_{i+1} + \cdots + g_n)] - cg_i$
- $(g_i*, \dots g_n*)$ is NE if for each i , g_i* solves the following problem
  $\underset{g_i}{Max}\, g_i V(g_i^* + \cdots + g_{i-1}^* + g_i + g_{i+1}^* + \cdots + g_n^*) - cg_i$

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Tragedy of commons : Theoretical Example [Contd…]

- First order condition for optimality
  - $V(g_i + g^*_{-i}) + g_i V'(g_i + g^*_{-i}) - c = 0$ ..... (A)
    Where $g^*_{-i} = g^*_i + \cdots + g^*_{i-1} + g^*_{i+1} + \cdots + g^*_n$

Substituting g_i* into (A) , summing over all n farmers first order conditions & dividing by n

$\Rightarrow V(G^*) + \frac{1}{n} G^* V'(G^*) - c = 0$ ..... (B)

Where $G^* = g^*_i + \cdots + g^*_n$

- The above therefore holds at NE
- For social optimum, one needs to solve the social welfare function i.e. choose G to solve the following
  $\max_G GV(G) - cG$

→ Yields the social optimum G^{**}

First-order Condition :

$V(G^{**}) + G^{**} V'(G^{**}) - c = 0$ ..... (C)

Comparison of (A) &(B) shows that $G^* > G^{**}$

- Suppose on the contrary that $G^* \leq G^{**}$
  $\Rightarrow V'(G^*) \geq V'(G^{**}) \because V' < 0$

  $V'(G^*) \geq V'(G^{**}) \because V'' < 0$

  Finally ,
  $G^*/n < G^{**}$

$\Rightarrow$ Left-hand side of (B) strictly exceeds the left-hand side of (C) which is impossible since both equal zero

$\Rightarrow G^* > G^{**}$

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Tragedy of commons : Theoretical Example [Contd…]

- The first order condition (A) considers the individual incentives faced by a farmer who is already grazing g_i goats but is considering adding one more.
- In Nash equilibrium, each farmer considers his/her actions and not the effect of his/her actions on other farmers
- Hence one observes $G^*V'(G^*)/n$ in condition (B) (NE) but $G^{**}V'(G^{**})$ in condition (C) (Social optimum)
- In NE, each thinks about his/her own benefits and hence ends up over utilizing the common resource
- Hence the resource gets depleted very fast
- This is called the “tragedy of commons”

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