Module 3: "Dynamic games of complete information"

Lecture 24: "Collusion between Cournot duopolists in an Infinitely Pepeated Game Structure"

The Lecture Contains:

Infinitely repeated game example

An example - Collusion in Cournot Competition

- Two players- Firm 1 & Firm 2
- Competing in quantities
- Market demand function
  $P = a - Q = a - (q_1 + q_2)$
- Cournot Equilibrium
  $q_1{}^* = q_2{}^* = \frac{a-c}{3}$
- Monopoly quantity, $q_m = \frac{a-c}{2}$

Note, aggregate Cournot quantity

$= \frac{2(a-c)}{3} <$ Monoply output $= \frac{a-c}{2}$

$\Longrightarrow$ Both firms could be better off if each produce $q_i = {}^{q_m}/_2$

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Collusion between Cournot duopolists: A game

If both non cooperate, then each chooses $q_1 = q_2 = \frac{a-c}{3}$, then

Profit $\Pi_c [= (1-c)^2/9]$

If both co-operate, then each chooses

$q_1 = q_2 = {}^{q_m}/_2$, then

Profit $= {}^{\Pi_m}/_2 \left(= \frac{(a-c)^2}{8} \right)$

If one co-operates, i.e. if one

produces $q_1 = \frac{a-c}{2}$, then the quantity that maximizes firm 2's profit in the current period solves,

$$\max_{q_2} \left(a - q_2 - \frac{q_m}{2} - c \right) q_2$$

$\Longrightarrow q_2{}^D = \frac{3(a-c)}{8}$ [Output by defecting]

$\Longrightarrow \Pi_2{}^D = \frac{9(a-c)^2}{64}$ [Profit by defecting]

Then $\Pi_1{}^{coop,D} = \left[a - \frac{3(a-c)}{8} - \frac{(a-c)}{3} \right] \frac{(a-c)}{3}$

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Collusion between Cournot duopolies: game [contd.]

The game in each period has the following payoff structure

Firm2

		Firm 2	
		Cooperate	Non Cooperate
Firm 1	Cooperate	$\left(\frac{\Pi_m}{2}, \frac{\Pi_m}{2}\right)$	$\left(\Pi^1_{coop,d}, \Pi^2_d\right)$
	Non Cooperate	$\left(\Pi^1_d, \Pi^2_{coop,d}\right)$	$\left(\Pi_c, \Pi_c\right)$

Consider the following trigger strategy:

Produce half the monopoly quantity, ${q_m}/{2}$ in first period.

In t^{th} period produce ${q_m}/{2}$ if both firms have produced ${q_m}/{2}$ in each of 't-1'previous periods, otherwise produce Cournot quantity, q_c

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Collusion: Game [contd.]

Consider the infinitely repeated game based on Cournot stage same, where both firms have a discount factor, δ

There will be a range of δ, for which it is a sub game perfect Nash equilibrium of this infinitely repeated game for both firms to play the above trigger strategy.

The analysis goes in similar manner to the one discussed in the previous lecture.

Payoff to firm i by co-operating

$$= \frac{\Pi_m}{2} + \frac{\delta\,\Pi_m}{2} + \frac{\Pi_m}{2}\,\delta^2 + \cdots$$

$$= \frac{\Pi_m/_2}{1-\delta}$$

Payoff to firm i by defecting

$$= \Pi^D + \delta\Pi_c + \delta^2\Pi_c + \cdots$$

Players will adopt the trigger strategy only if

Payoff to player i by co-operating

$\geq$ Payoff to players i by defecting from trigger strategy

i.e. $\frac{1}{1-\delta}\,\frac{1}{2}\Pi_m \geq \Pi_d + \frac{\delta}{1-\delta}\Pi_c$

Substituting the values if $\Pi_m, \Pi_d\ and\ \Pi_C$ in the above inequality, one gets

$\delta \geq {}^9/_{17}$

Hence for any $\delta \geq {}^9/_{17}$, the sub game-perfect Nash equilibrium of this infinitely repeated game for both firms is to play trigger strategy.

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