Lecture 19: "Dynamic Game: Application: Stackelberg Model of Duopoly"

The Lecture Contains:

Previous Next

Stackelberg Model: Basic Structure

- Two players - Two firms
- Sequential Move Game
  - a dominant firm (leader) moves first
  - the subordinate firm (follower) moves second
- Actions- Choice of quantities
- Timing of game
  1. Firm 1 chooses a quantity $q_1 \geq 0$
  2. Firm 2 observes and then chooses a quantity $q_2 \geq 0$
- Payoffs: Payoff to firm is given by the profit function $\pi_i(q_i, q_j) = q_i[P(q) - c]$ where $P(Q) = a - Q$, Q: aggregate quantity $i.e. Q = q_1 + q_2$, C: marginal cost of production

Previous Next

Solving the Stakelberg model

- Solve the model by backward induction
- In stage 2, Firm 2's best response to an arbitrary quantity by firm 1 is given by $R_2(q_1)$
  $R_2(q_1)$ solves

  $$\underset{q_{2\geq 0}}{Max} \quad \pi_2(q_1, q_2) = \underset{q_{2\geq 0}}{Max} \quad q_2[a - q_1 - q_2 - c]$$

  $\Rightarrow R_2(q_1) = \dfrac{a - q_1 - c}{2}$ provided $q_1 < a - c$
- Note in case of simultaneous move Cournot game, one finds same $R_2(q_1)$ but there is a difference in the concepts
  - $R_2(q_1)$ is really firm 2's reaction to firm 1's observed quantity in Stackelberg case
  - $R_2(q_1)$ is firm 2's best response to a hypothesized quantity which is simultaneously chosen by firm 1, in Cournot case

Previous Next

Solving the Stackelberg model [Contd...]

- Firm 1 knows firm 2's problem and hence can solve it as well
- Firm 1 therfore can anticipate that following his quantity choice q_1, the follower will choose q_2 according to $R_2(q_1)$
- Hence in first stage, firm 1 chooses q_1 to solve.

$$\underset{q_{1\geq 0}}{Max}\quad [\pi_1(q_1, R_2(q_1))]$$

$$= \underset{q_{1\geq 0}}{Max}\quad [q_1[a - q_1 - R_2(q_1) - c]]$$

$$= \underset{q_{1\geq 0}}{Max}\quad \left[q_1\left(\frac{a - q_1 - c}{2}\right)\right]$$

$$\Rightarrow q_1^* = \frac{a-c}{2}$$

SPNE: $q_1^* = \frac{a-c}{2}$

$$q_2 = R_2(q_1) = \frac{a - q_1 - c}{2}$$

- Note: In this dynamic game for firm 1 both action and strategy meant the same quantity choice. However for firm 2, the strategy is a function- quantity choice of firm 2 as a function of quantity chosen by firm 1
- Backward induction outcome in SPNE:

$$q_1^* = \frac{a-c}{2} \; \& \; R_2(q_1^*) = \frac{a-c}{4}$$

Comparison with cournot game

- Aggregate quantity in backward induction outcome $3(a-c)4$
- Note, aggregate quantity in NE of Cournot Game: $2\left(\frac{a-c}{3}\right)$
- Aggregate quantity is higher in the Stackelberg game
- Market-clearing price is lower in the Stackelberg game
- Aggregate profits are lower in Stackelberg game
- However leader's profits are higher in Stackelberg game
- This implies that firm 2 or the follower is worse off in Stackelberg than in the Cournot game.

Previous Next