Module 2: "Static games of complete information"

Lecture 12: "Mixed strategies"

The Lecture Contains:

- Definition
- Usefulness of mixed strategies
- How to find out mixed strategy NE?

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Definition

- A mixed strategy for player i is a probability distribution over strategies in his strategy set S_i

example:

Game of matching pennies

- Pure strategy : Actions that players play for sure
  - Head & tail
- Mixed strategy is a probability distribution (q,1-q)
  - q - probability of playing head
  - 1-q - probability of playing tail

Note: Mixed strategy $(0,1) \simeq$ pure strategy tail

Mixed Strategy $(1,0) \simeq$ pure strategy head

Generalized Definition

Suppose player i has K pure strategies $S_i = \{s_{i1}, \dots, s_{iK}\}$

Mixed strategy for player i (σ) is a probability distribution $(p_{i1}, \dots, p_{iK})$

Where p_{ik} is the probability that player i will play strategy s_{ik}

$$0 \leq p_{ik} \leq 1 \quad \forall\, k = 1, 2, \dots K$$

$$p_{i1} + \dots \;..+ p_{iK} = 1$$

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Usefulness of mixed strategies

IEDS Method

	L	R
U	2,0	-1,0
M	0,0	0,0
D	-1,0	2,0

- Player 1's strategy M is not dominated by U or D.
- Player 2 also does not have a dominated strategy
  - Cannot find an outcome in pure strategies by IEDS method
- If player 1 plays U with probability $\frac{1}{2}$ and D with probability $\frac{1}{2}$,
  - guaranteed an excepted payoff of 1/2 irrespective of what player 2 plays
  - 50% mix of U and D strictly dominates M
- A pure strategy
  - May not be strictly dominated by another pure strategy
  - But may be strictly dominated by a mixed strategy

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Usefulness of mixed strategies: NE

	L	R
T	3,-	0,-
M	0,-	3,-
B	2,-	2,-

- B is not a best response for player 1 to either L or R by player 2
  - However if player 2 plays mixed strategy $\left(\frac{1}{2},\frac{1}{2}\right)$, then B is the best response from player 1[Player 1 gets expected pay off=2 by playing B>expected pay off =1.5 by playing T or M]
  - A pure strategy can be a best response to mixed strategy even if it is not a best response to any pure strategy

Theorem:

In the n-player game $\{S_1, \ldots, S_i, \pi_1, \ldots \pi_n\}$ if n is finite and S_i is finite for every i, there exists at least one NE, possibly involving mixed strategies

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How to find out mixed strategy NE?

Game of matching pennies

		Player 2	
Player 1	H	-1,-1	1,-1
	T	1,-1	-1,1

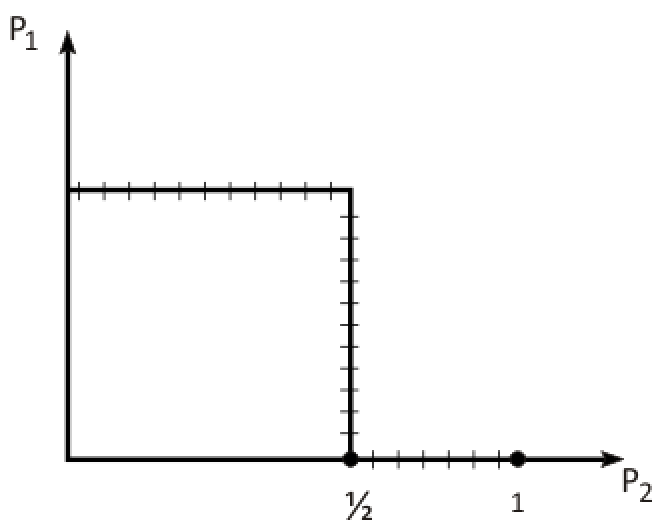
Use of best response functions:

- $P_1(P_2(H))$: best response function of player 1 that shows the probability that player 1 assigns to head as a function of probability that player 2 assigns to head
- $P_2(P_1(H))$ player 2's best response function defined similarly
- Derivation of $P_1(P_2(H))$
  - Player 1's expected payoff by playing H
    $= P_2(-1) + (1 - P_2).1$
    $= 1 - 2p_2$
  - player 1's expected payoff by playing T
    $= P_2.1 + (1 - P_2)(-1) = 2P_2 - 1$

    $\Rightarrow \therefore \quad P_1(H) = 1 \quad \text{if } P_2 < \frac{1}{2}$

    $= 0 \quad \text{if } P_2 > \frac{1}{2}$

    $= [0,1] \quad \text{if } P_2 = \frac{1}{2}$[indifferent between H & T]



How to find mixed strategy NE? [contd.]

Derivation of $P_2(P_1(H))$

- player 2's expected payoff by playing H if player 1 plays H with probability P_1

  $= P_1.1 + (1 - P_1)(-1) = 2P_1 - 1$

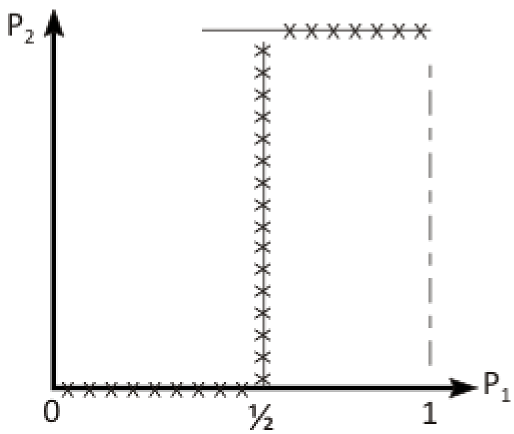
  Payoff to 1 from playing tail

  $= P_1(-1) + (1 - P_1)(-1) = 1 - 2P_1$

  $P_2(P_1(H)) = 1 \text{ if } P_1 > \frac{1}{2}$

  $= 0 \text{ if } P_2 > \frac{1}{2}$

  $= [0,1] \text{ if } P_2 = \frac{1}{2}$ [indifferent between H & T]



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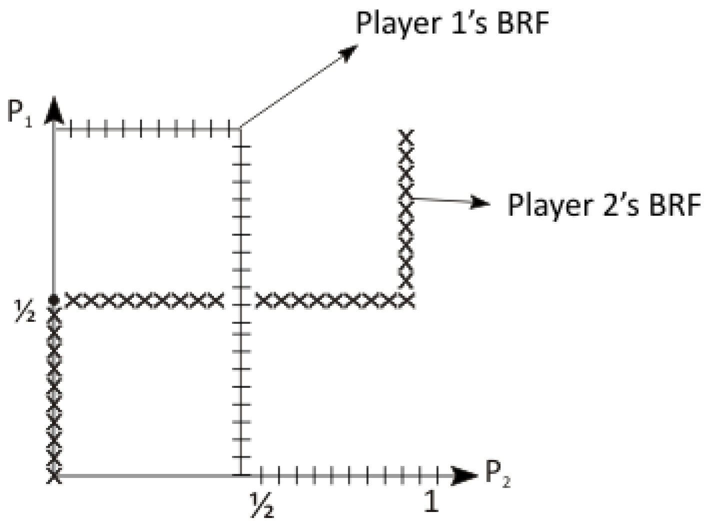
How to find out mixed strategy NE? [Contd.]



Two BRFs intersect at $\left(P_1 = \frac{1}{2}, P_2 = \frac{1}{2}\right)$

- $\left(\frac{1}{2}, \frac{1}{2}\right)$is the mixed strategy NE
- if player i plays $\left(\frac{1}{2}, \frac{1}{2}\right)$ then best response of player j is to play $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Battle of sexes

		W: O	W: F
H	O	2,1	0,0
H	F	0,0	1,2

- sketch the BRFs and find out the mixed strategy NE,