Module 2: "Static games of complete information"

Lecture 10: "Basic Structure of Cartel"

The Lecture Contains:

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Lecture 10: "Basic Structure of Cartel"

Basic Model of Cartel

- Two firms operate as a cartel
- Co-ordinate their production decisions
- Output quantities are chosen to maximize the joint or total Π,
  Max $[a - b(q_1 + q_2) - c][q_1 + q_2]$
- FOCs: $(a-c) - 2bq_1 - 2bq_2 = 0 \quad \left[\frac{\delta\Pi}{\delta q_1} = 0\right]$

  $(a-c) - 2bq_1 - 2bq_2 = 0 \quad \left[\frac{\delta\pi}{\delta q_2} = 0\right]$
- $q_1^* = q_2^* = \frac{a-c}{4b}$

  Total $\pi = \frac{(a-c)^2}{4b}$

  Each firm gets $\pi = \frac{(a-c)^2}{8b} > \pi$ in Cournot game

[**Note:** *it may be mentioned that the firms are assumed to be sharing the output and profit equally in the analysis. However, your basic argument for cartel instability qualitatively holds for other distributions as will.*]

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Instability of Cartel

- Each firm in the cartel would have an incentive to cheat the other firm
- Basically it wants to increase its own profit at the expense of other
- What it firm 2 produces Cartel qty $\left(\frac{a-c}{4b}\right)$?
  - what is the BRF of firm 1
  - $q_1 = \frac{a-c}{2b} - \frac{q_2}{2}$
  - If $q_2 = \frac{a-c}{4b}$, then
- $q_1 = \frac{3(a-c)}{8b} > \frac{a-c}{4b}$
- If q_1 increases, q_2 remaining same firm 1 gets better off
- Each firm will have such an incentive
- Cartel is unstable

  [**Note:** *it may be mentioned that the firms are assumed to be sharing the output and profit equally in the analysis. However, the basic argument for cartel instability qualitatively holds for other distributions as well.*]

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Bertrand model of Duopoly

- Cournot model, cartel
  - models for quantity competition
- Bertrand model
  - model for price competition
- Two firms are simultaneously choosing the prices
- $P = a - bQ$ i.e. $Q = \frac{a-P}{b}$; $MC_i = c$
- Homogenous product
- No difference in quality
- Consumers will purchase the product from sellers offering lower prices

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Representation of Bertrand model in NFG

- Players – firm 1 & firm 2
- Strategies

  $S_i = P_i \qquad S_i = [0, \infty]$
- Payoff Function:

  $\Pi^i = (P_i - c)q_i^d$

  where q_i^d is residual demand for firm i corresponding to the prices chosen by the two firms

$$q_i^d = \begin{cases} 0 & \text{if } P_1 > P_2 \\ \frac{1}{2}\frac{(a-P_1)}{b} & \text{if } P_1 = P_2 \\ \frac{a-P_1}{b} & \text{if } P_1 < P_2 \end{cases}$$

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Bertrand Model [Contd.]

- Payoff function
  - $\Pi_i(P_j, P_j) = \begin{cases} 0 & \text{if } P_i > P_j \\ (P_i - c)\frac{1}{2}\frac{a-P_i}{b}, P_i = P_j \\ (P_i - c)\frac{a-P_i}{b}, \; P_i < P_j \end{cases}$
- BRFi : $P_i^* = P_i(P_j)$

$$= \begin{cases} P_M \text{ if } P_j > P_M \\ P_j - \xi \text{ if } c < P_i \leq P_m \\ P_j \geq c \text{ if } P_j = c \\ > P_j \quad P_j < c \end{cases}$$

- How to find NE $(P_1 *, P_2 *)$
- Look at all possible combinations of (P_1, P_2)
  - check whether any firm has an incentive to deviate
    - if so, rule out that combination to be a NE

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Bertrand model [contd.]

Case I:

$P_1 > P_2 > c$

- Firm 1 has an incentive to undercut firm 2
- Not a NE

Case II:

$P_1 = P_2 > c$

- Each firm has an incentive to undercut the price of the rival

Case III:

$P_1 > P_2 = c$

- Firm 2 has an incentive to slightly raise its price but still make it lower to P_1

Case IV:

$P_1 = P_2 = P_M$

- Each firm would like to undercut the rival

- Rule out all possible cases in the same manner
- Left with $P_1 = P_2 = c$
  - Each firm earns zero Π,
  - Let $P_j = c$

    If $P_i < c$ i gets a negative Π

    $P_i = c \Rightarrow$ gets zero

    $P_i > c \Rightarrow$ gets zero

  - no unilateral incentive for i to deviate from $P_1 = P_2 = c$
    NE: $P_1^* = P_2^* = c$

[**Note:** *Here, for derivation of residual demand function at* $P_1^* = P_2^* = c$,*total demand is assumed to be shared equally.The same equilibrium prevails for unequal sharings as well*.]

- Bertrand paradox
  Everybody competing in price and earning zero profit
- It is a paradox as it is never seen in reality

- Do you see coke & Pepsi earning zero profits?

Bertrand Paradox

Can Bertrand paradox be eliminated ?

1. Product not homogenous

- product differentiation
- high expenditure on advertising
- brand loyalty among consumers
- even if price is raised, firm will not lose all customers
- $P_1^* = P_2^* = c$ no more a NE

2. Presence of infinitely repeated game

- Bertrand paradox breaks down
- Co-operation in the long run