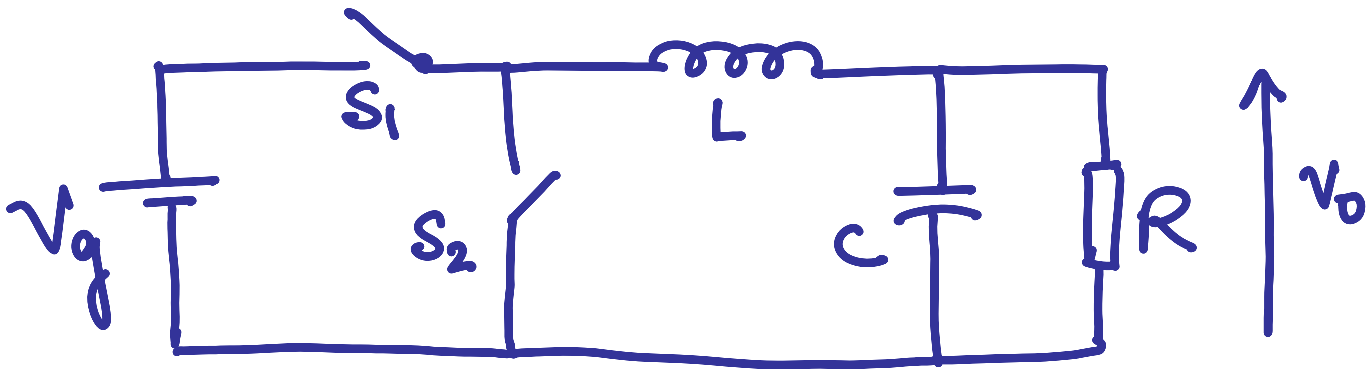
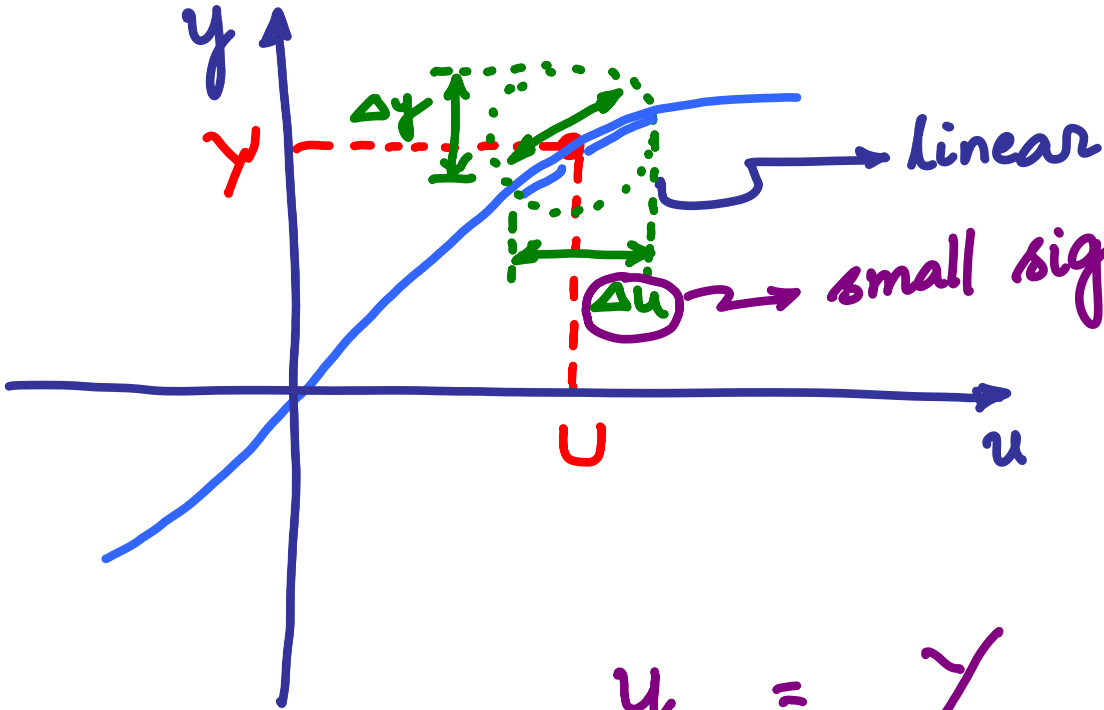
Circuit Averaging Method ...



Large Signal System (MODEL)

Small Signal (MODEL) (Linear) $\Rightarrow$ CONTROL



small signal deviations about the operating point

$$y = Y + \Delta y \qquad \hat{y}$$

$$u = U + \Delta u \qquad \hat{u}$$

$$y = Y + \hat{y}$$

$$u = U + \hat{u}$$

$$x = X + \hat{x}$$

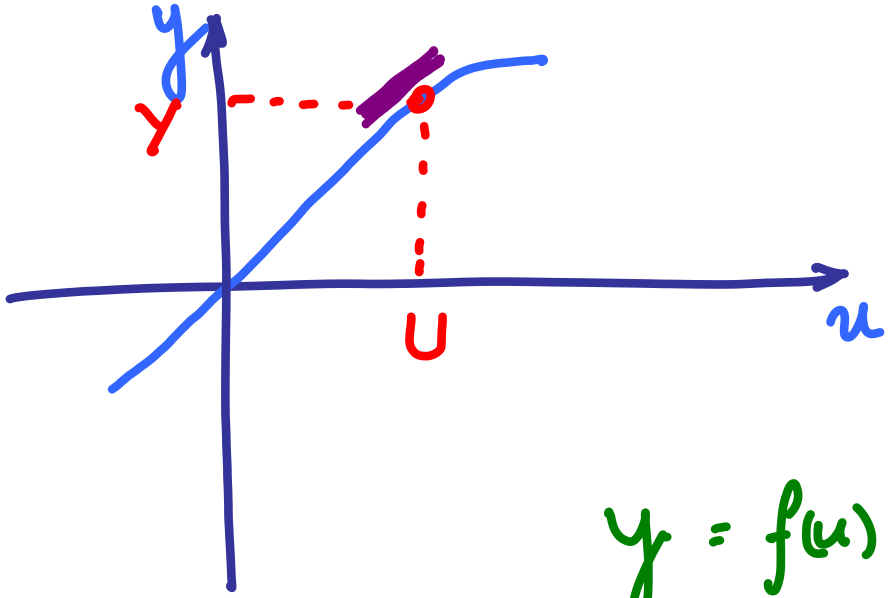
$$d = D + \hat{d}$$

Large signal : $\dot{x} = \mathbb{A}x + \mathbb{B}u$ | actual system

Steady State : $\dot{X} = 0 = \mathbb{A}X + \mathbb{B}U$ Equilibrium DESIGN of converter

Small signal : $\underbrace{\dot{\hat{x}} = \mathbb{A}\hat{x} + \mathbb{B}\hat{u}}_{\text{dynamics variations about OPERATING POINT}}$ || CONTROLLER DESIGN

linear model



deviations

$$y = f(u) = f(U) + \underbrace{(\qquad\qquad)}_{\left.\frac{df}{du}\right|_{u=U}(u-U) \;+\; \cancel{\frac{1}{2!}\left.\frac{d^2f}{du^2}\right|_{u=U}(u-U)^2 + \ldots}}$$

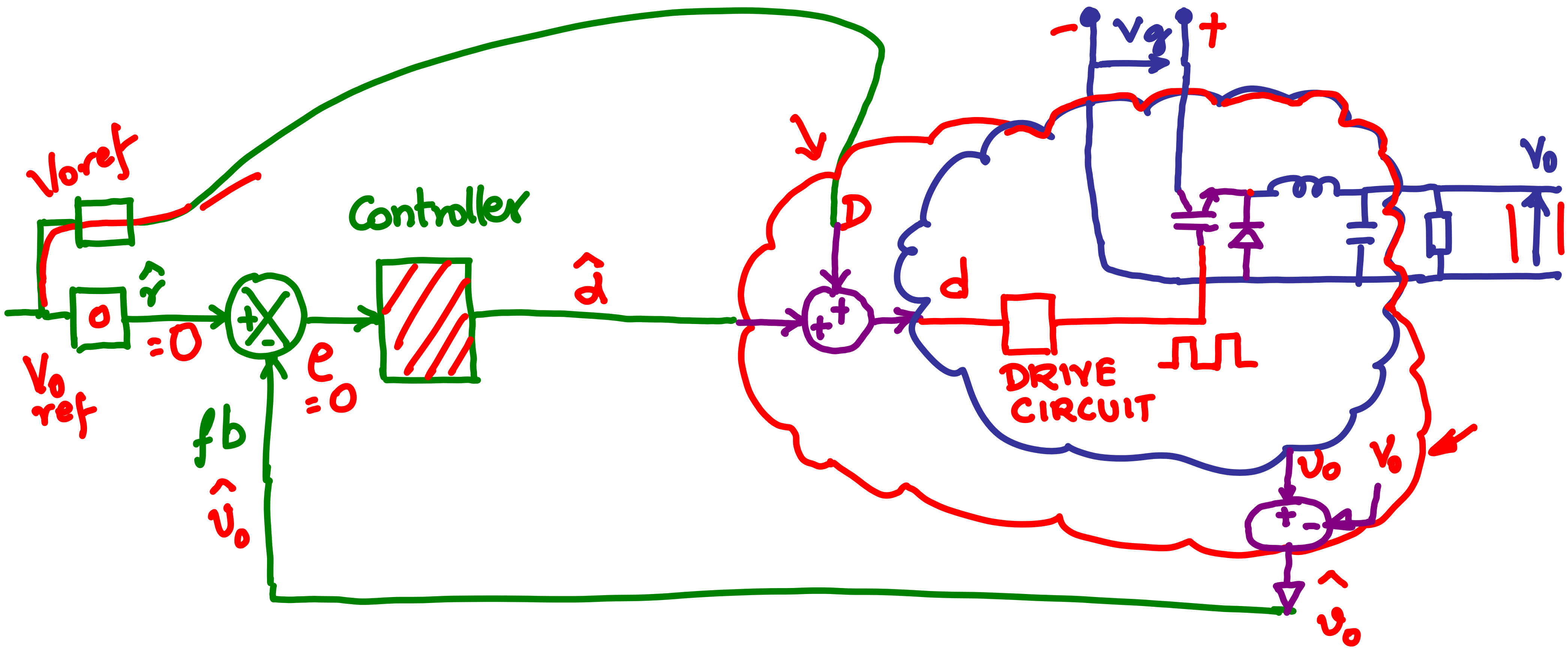
TAYLOR SERIES

$$y = f(U) + \underbrace{\frac{df}{du}}_{}(u-U)$$

$$(y - Y) = K\,(u-U)$$

$$\boxed{\hat{y} = K\,\hat{u}}$$

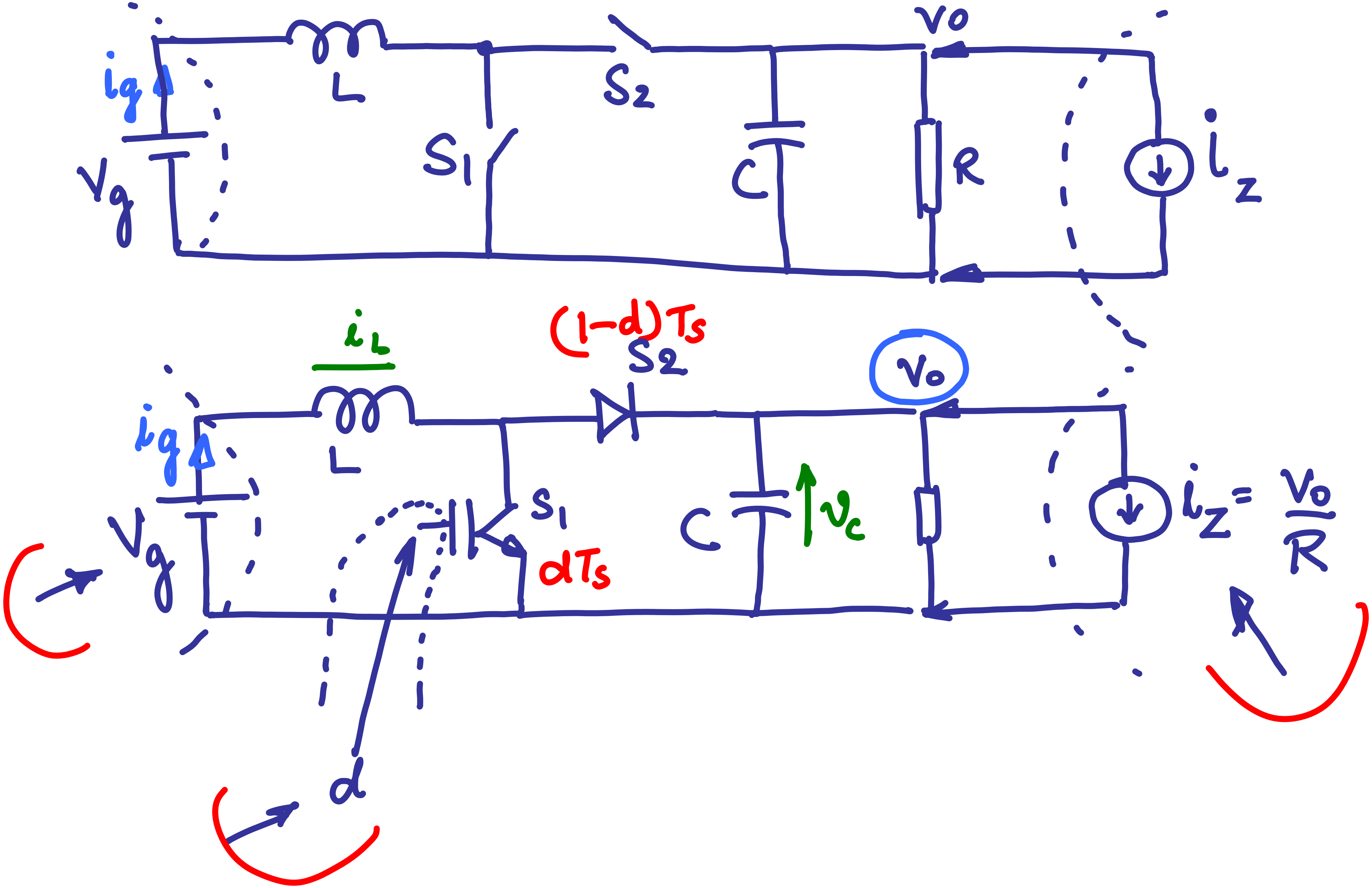
linear



Voref
$\hat{r}$
0
=0
V_o ref
e
=0
fb
$\hat{v}_o$
Controller
$\hat{d}$
D
d
DRIVE CIRCUIT
V_g
V_o
v_o
V_o
$\hat{v}_o$

BOOST CONVERTER

State space representation by Circuit Averaging Method



i_g
V_g
L
S_1
S_2
C
R
v_o
i_z
i_L
$(1-d)T_s$
S_2
v_o
i_g
V_g
L
S_1
dT_s
C
v_c
$i_z = \frac{v_o}{R}$
d

dT_s :

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \overset{\mathbb{A}_1}{\begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{RC} \end{bmatrix}} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \overset{\mathbb{B}_1}{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}} \begin{bmatrix} v_g \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{C} \end{bmatrix} \begin{bmatrix} i_Z \end{bmatrix}$$

STATE EQUATIONS

$(1-d)T_s$:

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \overset{\mathbb{A}_2}{\begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \overset{\mathbb{B}_2}{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}} \begin{bmatrix} v_g \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{C} \end{bmatrix} \begin{bmatrix} i_Z \end{bmatrix}$$

dT_s :

$$v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

$$i_g = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} v_o \\ i_g \end{bmatrix} = \overset{\mathbb{C}_1}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

$(1-d)\,T_s$:

$$\begin{bmatrix} v_o \\ i_g \end{bmatrix} = \overset{\mathbb{C}_2}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

OUTPUT EQUATIONS

$$\mathbb{D}_1 = \mathbb{D}_2 = [0]$$

AVERAGED LARGE SIGNAL MODEL

$$\mathbb{A} = \mathbb{A}_1 d + \mathbb{A}_2 (1-d)$$

$$\mathbb{B} = \mathbb{B}_1 d + \mathbb{B}_2 (1-d)$$

$$\mathbb{C} = \mathbb{C}_1 d + \mathbb{C}_2 (1-d)$$

$$\mathbb{D} = \mathbb{D}_1 d + \mathbb{D}_2 (1-d)$$

$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \overset{A}{\begin{bmatrix} 0 & -\frac{(1-d)}{L} \\ \left(\frac{1-d}{C}\right) & -\frac{1}{RC} \end{bmatrix}} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \overset{B}{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}} [v_g] + \begin{bmatrix} 0 \\ -\frac{1}{C} \end{bmatrix} [i_z]$$

$$\begin{bmatrix} v_o \\ i_g \end{bmatrix} = \overset{C}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \begin{bmatrix} i_L \\ v_c \end{bmatrix}$$

Averaged Large Signal Model

Steady state Model

$$\dot{x} = 0$$

$$d = D$$

$$v_g = V_g$$

$$i_g = I_g$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \overset{\mathbb{A}}{\begin{bmatrix} 0 & -\dfrac{(1-D)}{L} \\ \left(\dfrac{1-D}{C}\right) & -\dfrac{1}{RC} \end{bmatrix}} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \overset{\mathbb{B}}{\begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix}} \begin{bmatrix} V_g \end{bmatrix} + \begin{bmatrix} 0 \\ -\dfrac{1}{C} \end{bmatrix} \begin{bmatrix} I_Z \end{bmatrix}$$

$$\begin{bmatrix} V_o \\ I_g \end{bmatrix} = \overset{\mathbb{C}}{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}} \begin{bmatrix} I_L \\ V_c \end{bmatrix}$$

Steady State Model

$$0 = \mathbb{A}X + \mathbb{B}U$$

$$\boxed{X = -\mathbb{A}^{-1}\mathbb{B}U}$$

$$Y = \mathbb{C}X = -\underline{\underline{\mathbb{C}\mathbb{A}^{-1}\mathbb{B}}}U$$

$$\frac{Y}{U}$$

$$Y = \mathbb{C} X$$

$$V_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} I_L \\ V_o \end{bmatrix}$$

$$\frac{V_o}{V_g} \quad V_o = -\mathbb{C} A^{-1} \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_g$$ assume $I_z = 0$

$$\frac{V_o}{I_z} \quad V_o = -\mathbb{C} A^{-1} \begin{bmatrix} 0 \\ -\frac{1}{C} \end{bmatrix} I_z$$ assume $V_g = 0$

$$V_o = -\underbrace{\begin{bmatrix} 0 & 1 \end{bmatrix}}_{\mathbb{C}} \underbrace{\begin{bmatrix} 0 & x \\ -\left(\frac{L}{1-D}\right) & x \end{bmatrix}}_{= \mathbb{A}^{-1}} \underbrace{\begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}}_{\mathbb{B}} V_g$$

$$= -1 \cdot \left(\frac{L}{1-D}\right) \cdot \frac{1}{L} = \frac{1}{1-D}$$

$$\boxed{\frac{V_o}{V_g} = \frac{1}{1-D}}$$

$$\begin{bmatrix} \dot{\hat{i}}_L \\ \dot{\hat{v}}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D-\hat{d})}{L} \\ \frac{(1-D-\hat{d})}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L+\hat{i}_L \\ V_C+\hat{v}_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} [V_g+\hat{v}_g] + \begin{bmatrix} 0 \\ -\frac{1}{C} \end{bmatrix} [I_Z+\hat{i}_Z]$$

$$\begin{bmatrix} V_o+\hat{v}_o \\ I_g+\hat{i}_g \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} I_L+\hat{i}_L \\ V_C+\hat{v}_C \end{bmatrix}$$

Small Signal Model

+ Steady state

$$d = D + \hat{d}$$

$$v_o = V_o + \hat{v}_o$$

$$v_g = V_g + \hat{v}_g$$

$$i_Z = I_Z + \hat{i}_Z$$

$$i_L = I_L + \hat{i}_L$$

$$\begin{bmatrix} \dot{\hat{i}}_L \\ \dot{\hat{v}}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D-\hat{d})}{L} \\ \frac{(1-D-\hat{d})}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L+\hat{i}_L \\ V_C+\hat{v}_o \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cancel{\begin{bmatrix} V_g+\hat{v}_g \end{bmatrix}} + \begin{bmatrix} \\ -\frac{1}{C} \end{bmatrix} \cancel{\begin{bmatrix} I_Z+\hat{i}_Z \end{bmatrix}}$$

$$\underbrace{\cancel{\begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{1-D}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix}}}_{AX} + \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{1-D}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_C \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & \frac{\hat{d}}{L} \\ -\frac{\hat{d}}{C} & 0 \end{bmatrix} \begin{bmatrix} I \\ V_o \end{bmatrix}}_{\begin{bmatrix} \frac{V_o}{L} \\ -\frac{I}{C} \end{bmatrix} \hat{d}}$$

$$AX + BU = 0$$

$$\begin{bmatrix} \dot{\hat{i}}_L \\ \dot{\hat{v}}_c \end{bmatrix} = \begin{bmatrix} 0 & -\frac{(1-D)}{L} \\ \frac{(1-D)}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_c \end{bmatrix} + \begin{array}{c} \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \begin{bmatrix} \frac{1}{L} & 0 & \frac{V_o}{L} \\ 0 & -\frac{1}{C} & -\frac{I_L}{C} \end{bmatrix} \end{array} \cdot \begin{bmatrix} \hat{v}_g \\ \hat{i}_2 \\ \hat{d} \end{bmatrix}$$

$$\dot{\hat{x}} = \mathbb{A} \qquad \hat{x} + \qquad \mathbb{B} \qquad u$$

$$\begin{bmatrix} \hat{v}_o \\ \hat{i}_g \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_c \end{bmatrix}$$

$$y = \mathbb{C} \; x$$

Small Signal Model

1. Temporal operative modes

dT_s, $(1-d)T_s$

$\dot{x} = A_1 x + B_1 u$ | $A_1 d + A_2(1-d)$

$\dot{x} = A_2 x + B_2 u$ | $B_1 d + B_2(1-d)$

2. Averaged large signal model.

3. derivatives of equilibrium variables $\Rightarrow 0$

all variables represent steady state quantities

$d \Rightarrow D$ $x \Rightarrow X$, $y \Rightarrow Y$

$0 = \mathbb{A}X + \mathbb{B}U$

4. $d = D + \hat{d}$

$x = X + \hat{x}$

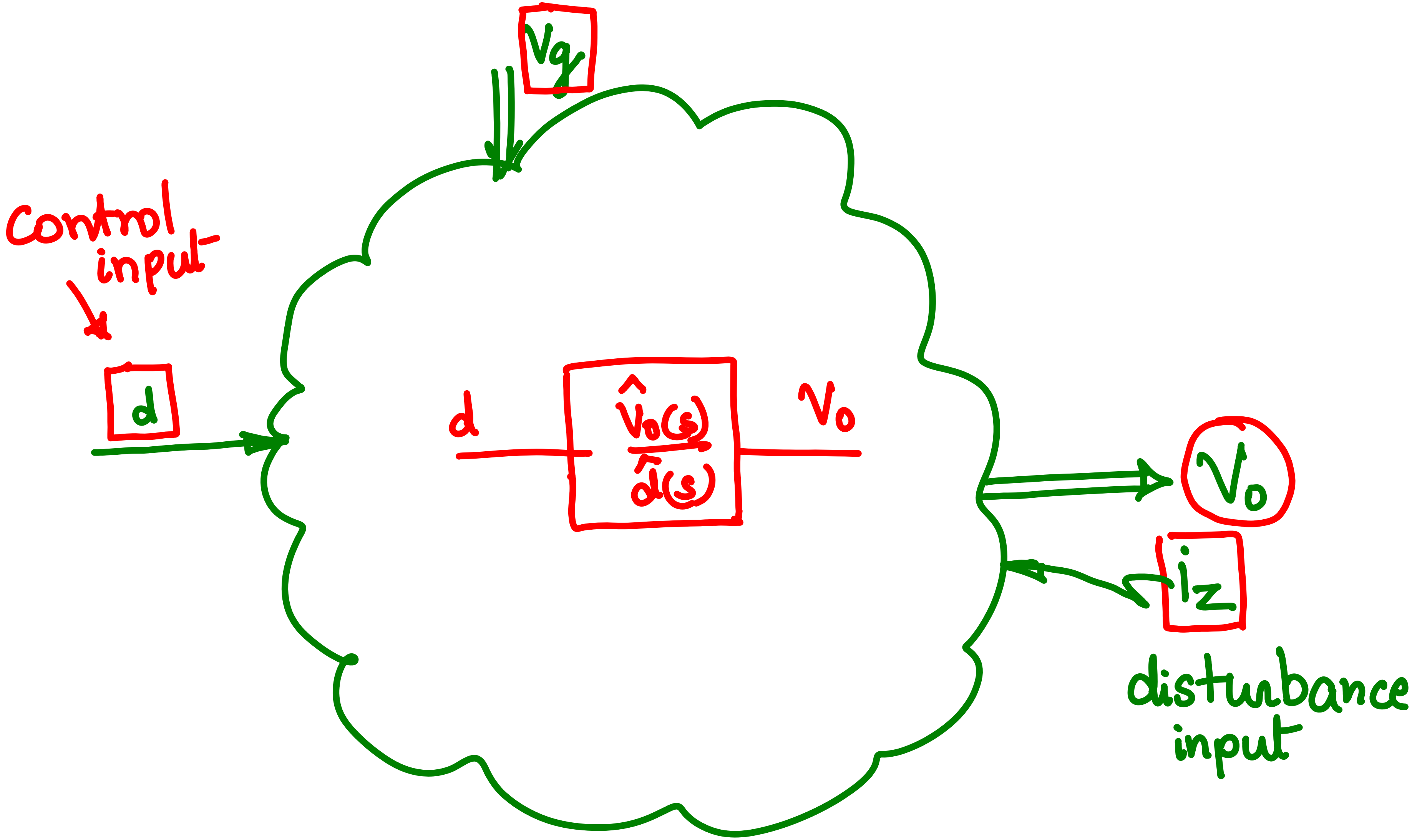
$y = Y + \hat{y}$

$\hat{d}\hat{x} \Rightarrow$ neglected

5. Small Signal model.

$\Downarrow$

Control controller design.



Control input
d
v_g
d
$\frac{\hat{v}_o(s)}{\hat{d}(s)}$
v_o
v_o
i_z
disturbance input

$$\dot{x} = \mathbb{A}x + \mathbb{B}u \quad : \quad \boxed{sIX(s) = \mathbb{A}X(s) + \mathbb{B}U(s)}$$

$$y = \mathbb{C}x \qquad\qquad Y(s) = \mathbb{C}X(s)$$

$$X(s) = (sI - \mathbb{A})^{-1}\mathbb{B}U(s)$$

$$\boxed{Y(s) = \mathbb{C}(sI - \mathbb{A})^{-1}\mathbb{B}U(s)}$$

$$\underbrace{Y(s)}_{\hat{v}_o} = C\,(sI - A)^{-1}\,\underbrace{B}_{\begin{bmatrix} \frac{V_c}{L} \\ -\frac{I_L}{C} \end{bmatrix}}\,\underbrace{U(s)}_{\hat{d}}$$

$$\frac{\hat{v}_o}{\hat{d}} = C\,(sI - A)^{-1}\,B_3$$

$$\frac{\hat{v}_o}{\hat{v}_g} = C\,(sI - A)^{-1}\,B_1$$

$$\frac{\hat{v}_o}{\hat{i}_z} = \mathbb{C}\left(s\mathbb{I} - \mathbb{A}\right)^{-1}\mathbb{B}_2$$

$$\frac{\hat{v}_g}{\hat{i}_g}(s) \Rightarrow \text{Input Impedance @ } \hat{d}=0,\ \hat{i}_z=0$$

$$\hat{i}_g = [1 \ \ 0]\, x$$

$$= \underline{\underline{[1 \ \ 0]\,(sI-\mathbb{A})^{-1}\mathbb{B}_1}}\ \hat{v}_g$$

$$\frac{\hat{v}_o}{\hat{i}_z}(s) = \text{Output Impedance @ } \hat{d}=0,\ \hat{v}_g=0$$

$$\mathbb{C}\,(sI-\mathbb{A})^{-1}\mathbb{B}_2$$

$$\frac{\hat{v}_o}{\hat{v}_g}(s) = \text{Audio Susceptibility @ } \hat{d}=0;\ \hat{i}_z=0$$

$$\underline{\mathbb{C}(sI-\mathbb{A})^{-1}\mathbb{B}_1}$$

$$\frac{\hat{v}_o}{\hat{d}}(s) = \text{Control voltage gain @ } \hat{v}_g=0,\ \hat{i}_z=0$$

$$\mathbb{C}(sI-\mathbb{A})^{-1}\mathbb{B}_3$$

$$\frac{\hat{i}_g}{\hat{d}}(s) = \text{Control current gain @ } \hat{v}_g=0,\ \hat{i}_z=0$$

$$[1\ \ 0](sI-\mathbb{A})^{-1}\mathbb{B}_3$$