Switched Mode Power Conversion

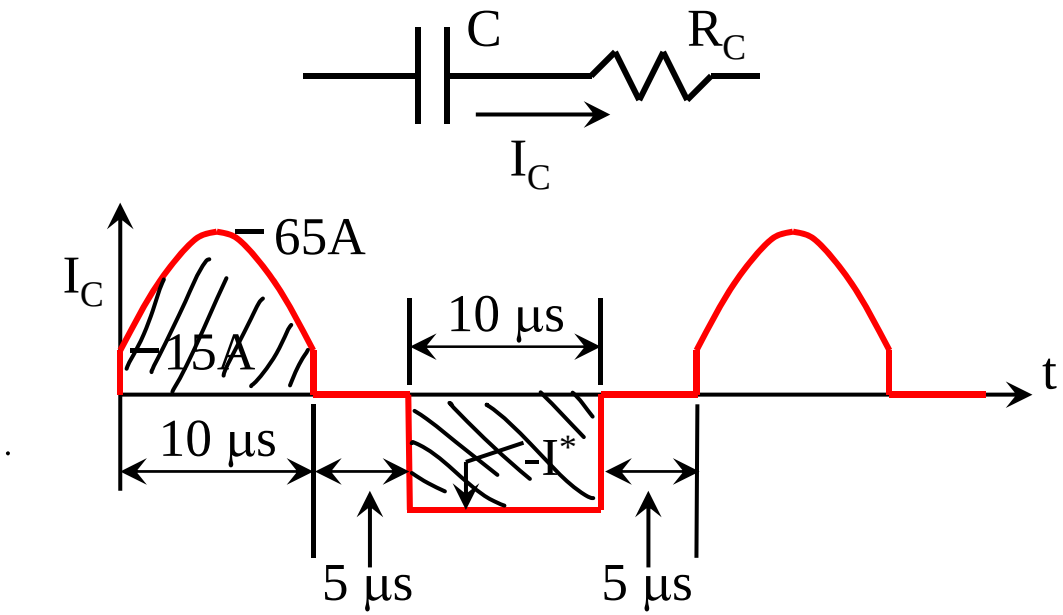
Problem Set 02

Selected Problems:
Devices for Efficient Power Conversion

Switched Mode Power Conversion

Problem Set 02

Problem No. 1

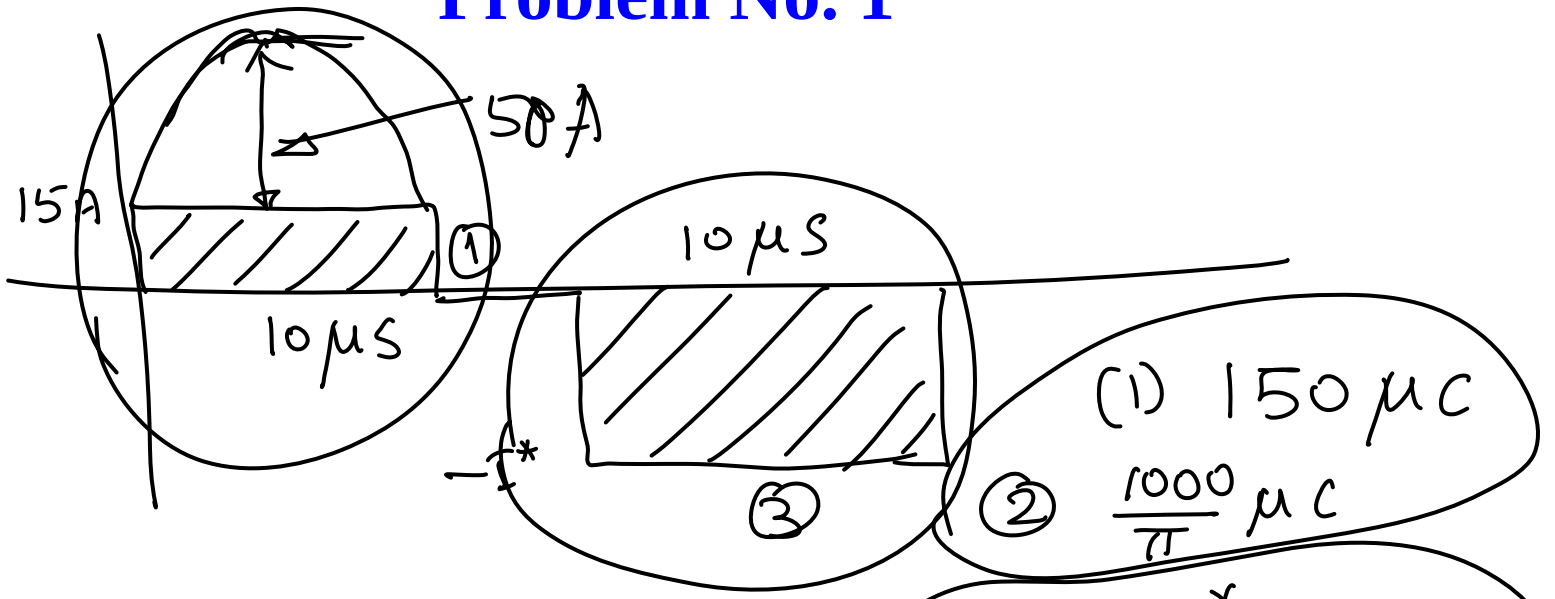


$C = 10\ \mu F$; $R_C = 0.02\ \Omega$; $V_C(0) = 0\ V$

Switched Mode Power Conversion

Problem Set 02

Problem No. 1



$A = 10\mu s \times \frac{50 \times 2}{\pi}$

$=$

$150\mu C + 318.3\mu C = 468.3\ \mu C$

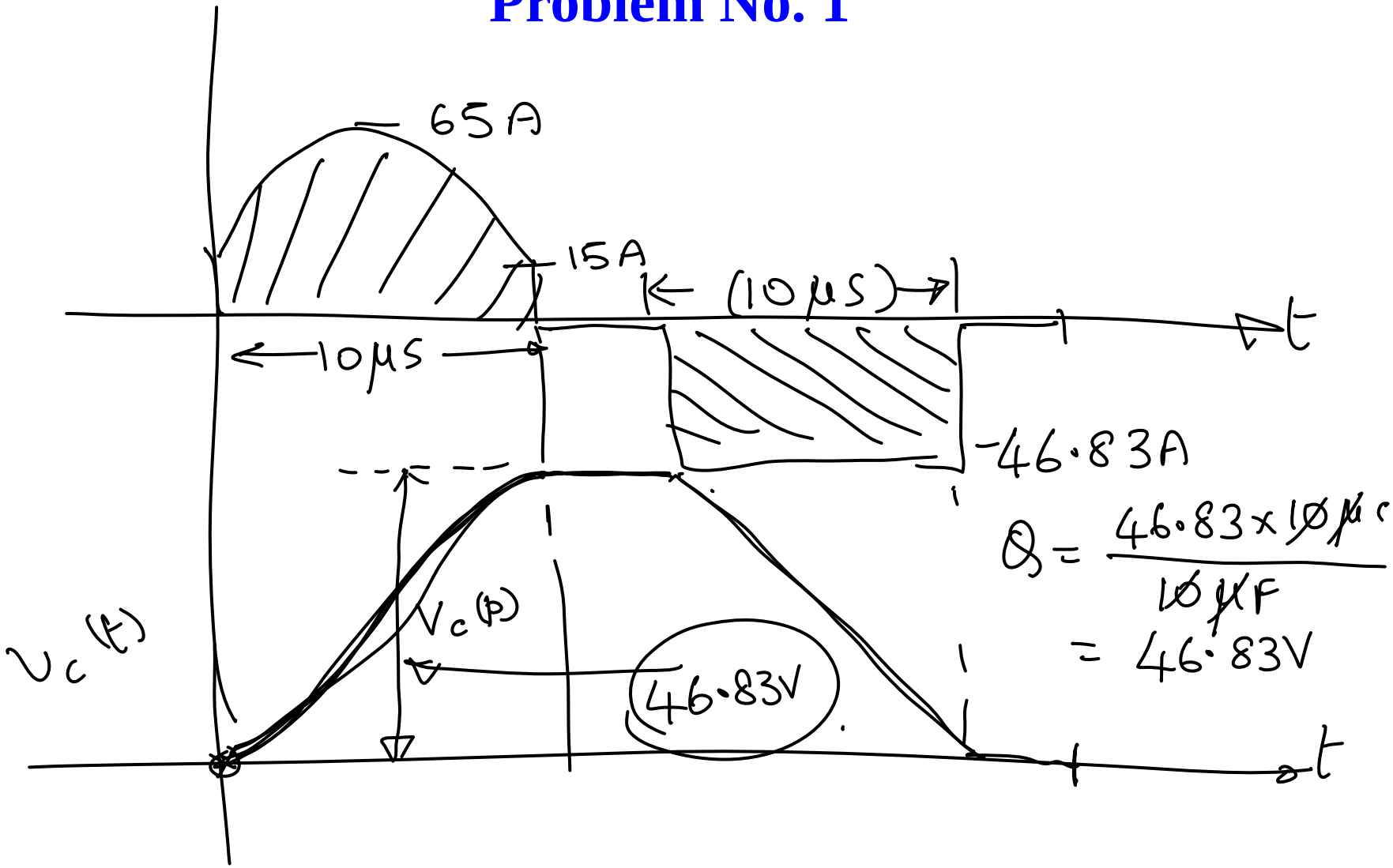
$I^* \times 10\mu C = 468.3\ \mu C$

$I^* = \underline{\underline{46.83\ A}}$

Switched Mode Power Conversion

Problem Set 02

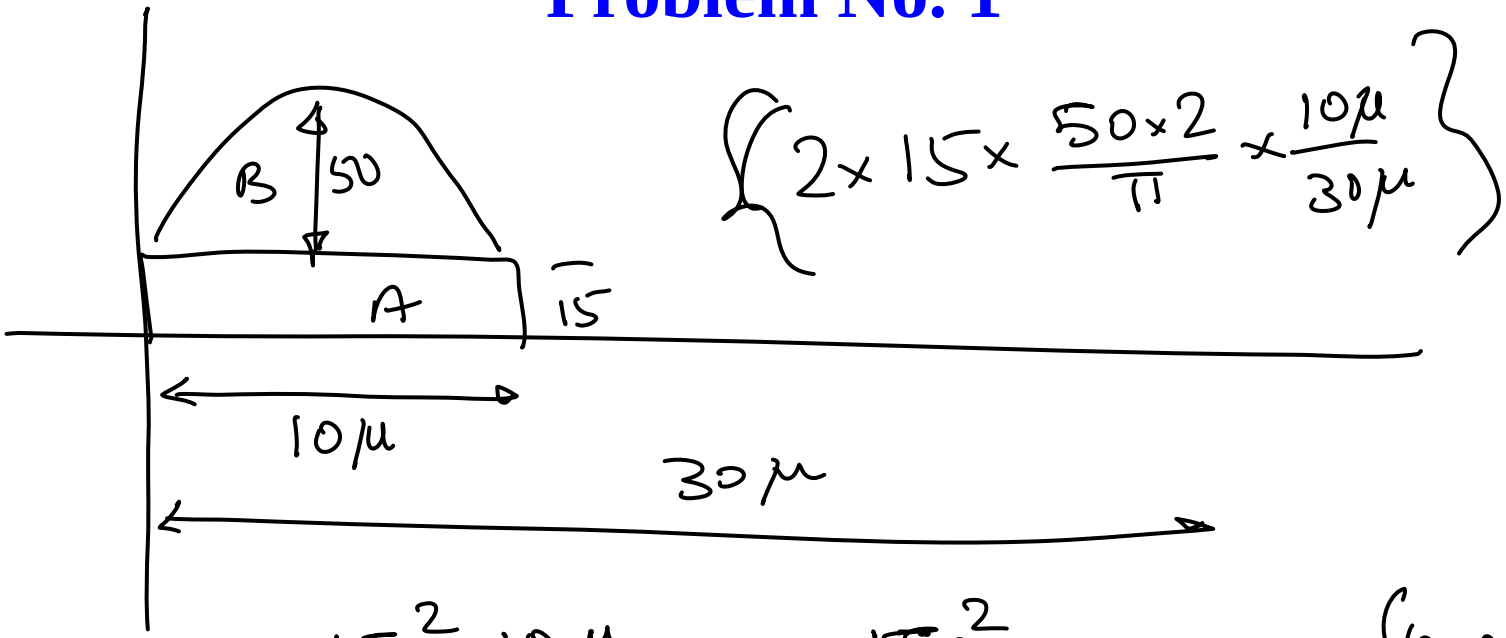
Problem No. 1



Switched Mode Power Conversion

Problem Set 02

Problem No. 1



$$\left\{ 2 \times 15 \times \frac{50 \times 2}{\pi} \times \frac{10\mu}{30\mu} \right\}$$

$$15^2 \frac{10\mu}{30\mu}$$

A

$$\frac{50^2}{2} \frac{10\mu}{30\mu}$$

B

$$2\int (A+B)\,dt$$

$$2 \times 15 \int 50 \sin \omega t \, d\omega t$$

Switched Mode Power Conversion

Problem Set 02

Problem No. 1

$$15^2 \frac{1}{3} + \frac{50^2}{2} \times \frac{1}{3} + 30 \times \frac{100}{\pi} \frac{1}{3} + 46.83^2 \frac{1}{3}$$

$$= I_{rms}^2 = 1533.7 \text{ A}^2$$

$$\text{Loss} = I_{rms}^2 R_e = 1533.7 \times 0.02$$

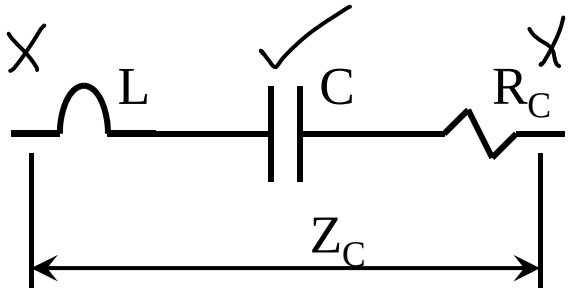
$$= 30.67 \text{ W}$$

$$\text{VA} = 65 \times 46 \simeq 3000 \text{ VA}$$

Switched Mode Power Conversion

Problem Set 02

Problem No. 2



$Z_C = \frac{1}{\omega C}$

$Z_L = \omega L$

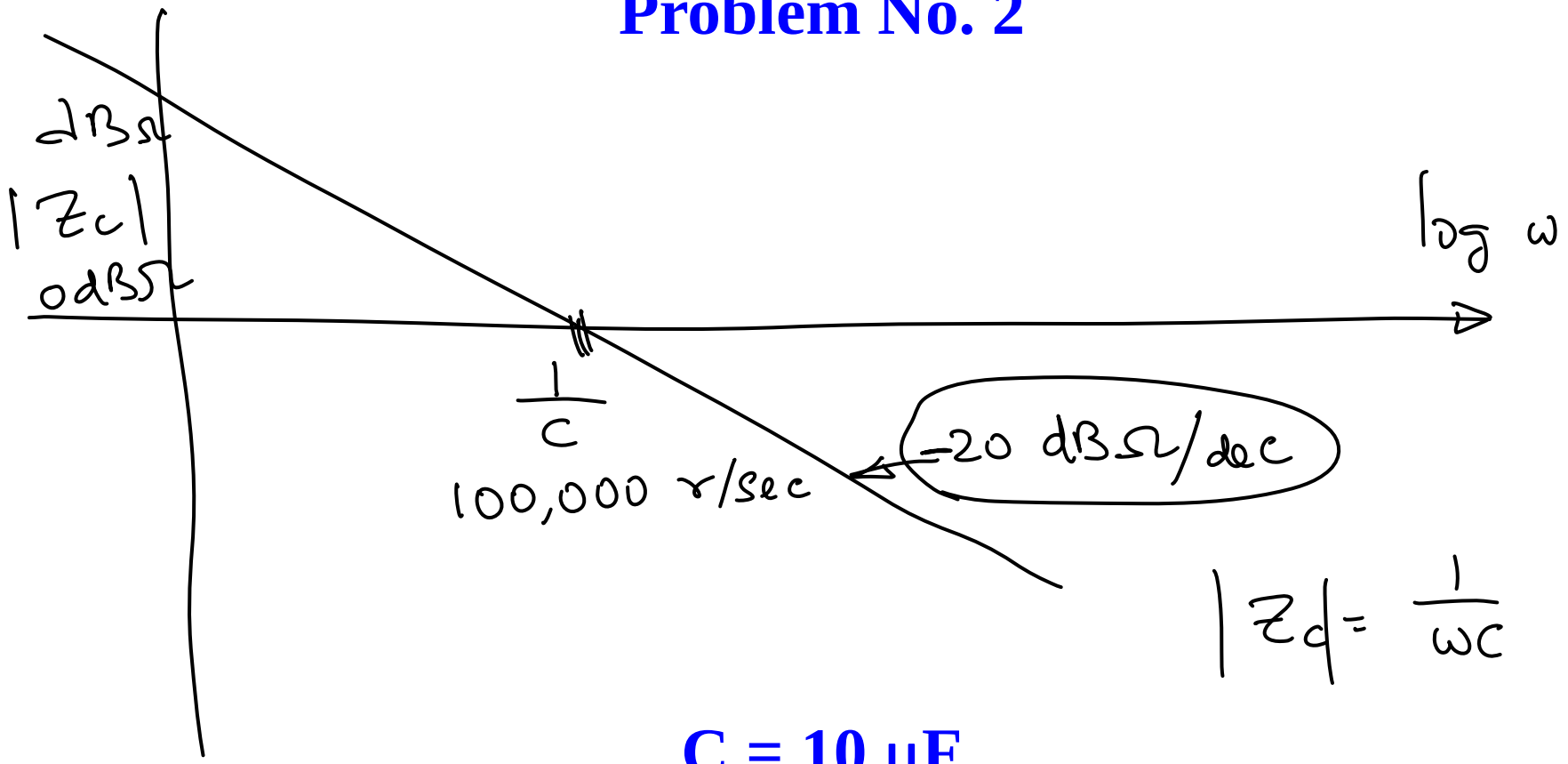
$Z_R = 30\,m\Omega$

C = 10 μF; R_C = 30 mΩ; L = 75 nH

Switched Mode Power Conversion

Problem Set 02

Problem No. 2

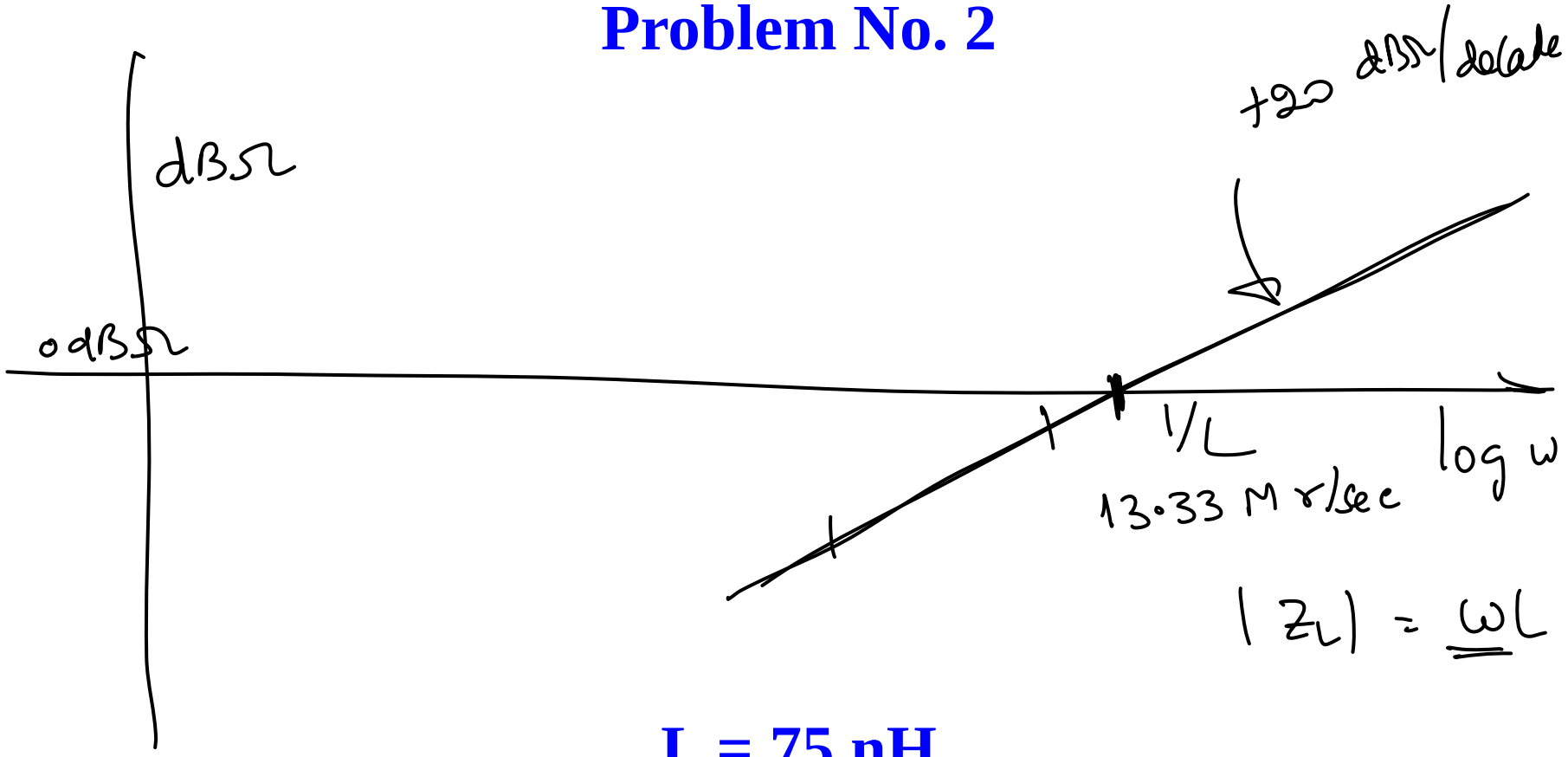


C = 10 μF

Switched Mode Power Conversion

Problem Set 02

Problem No. 2

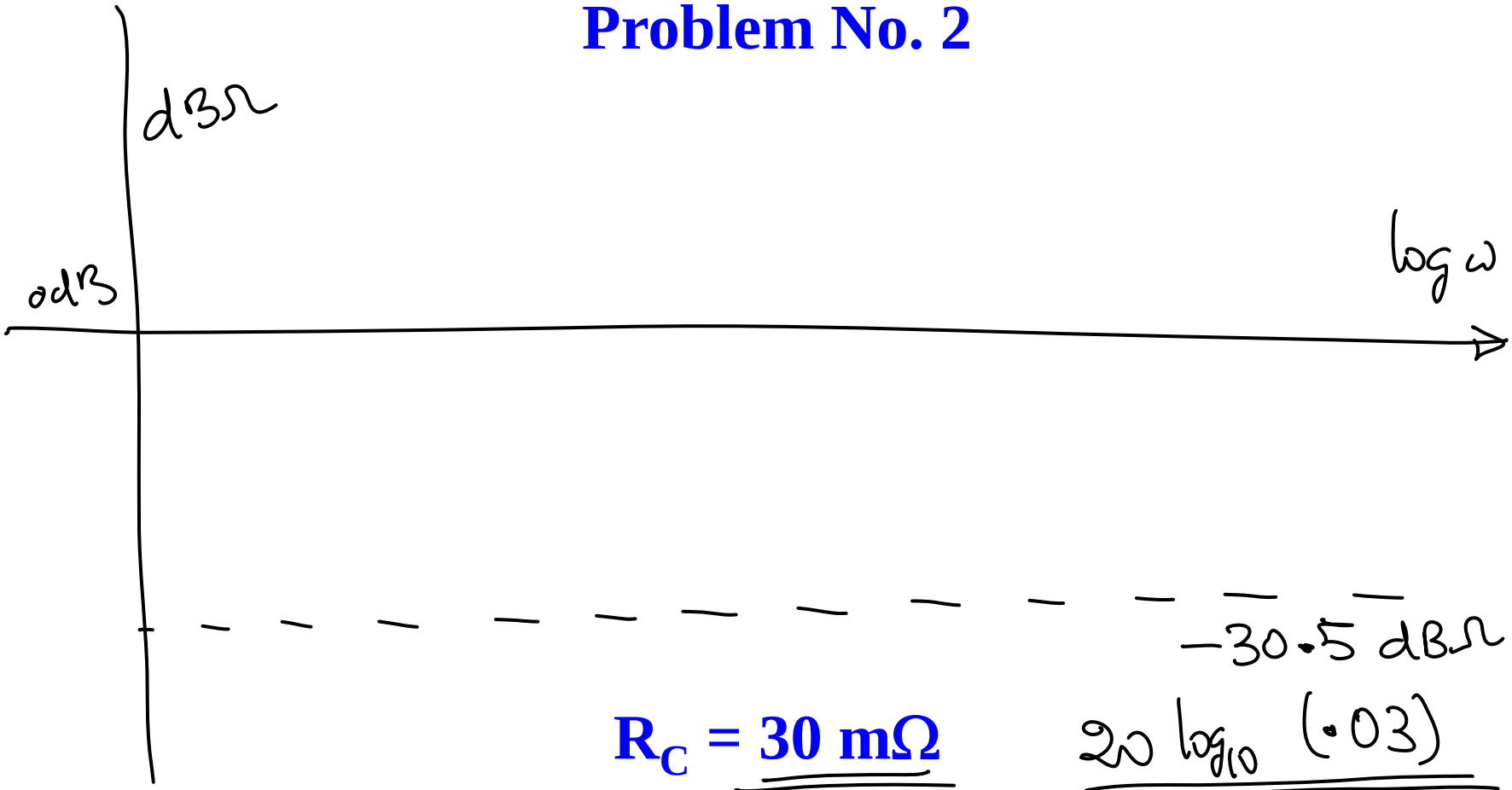


L = 75 nH

Switched Mode Power Conversion

Problem Set 02

Problem No. 2



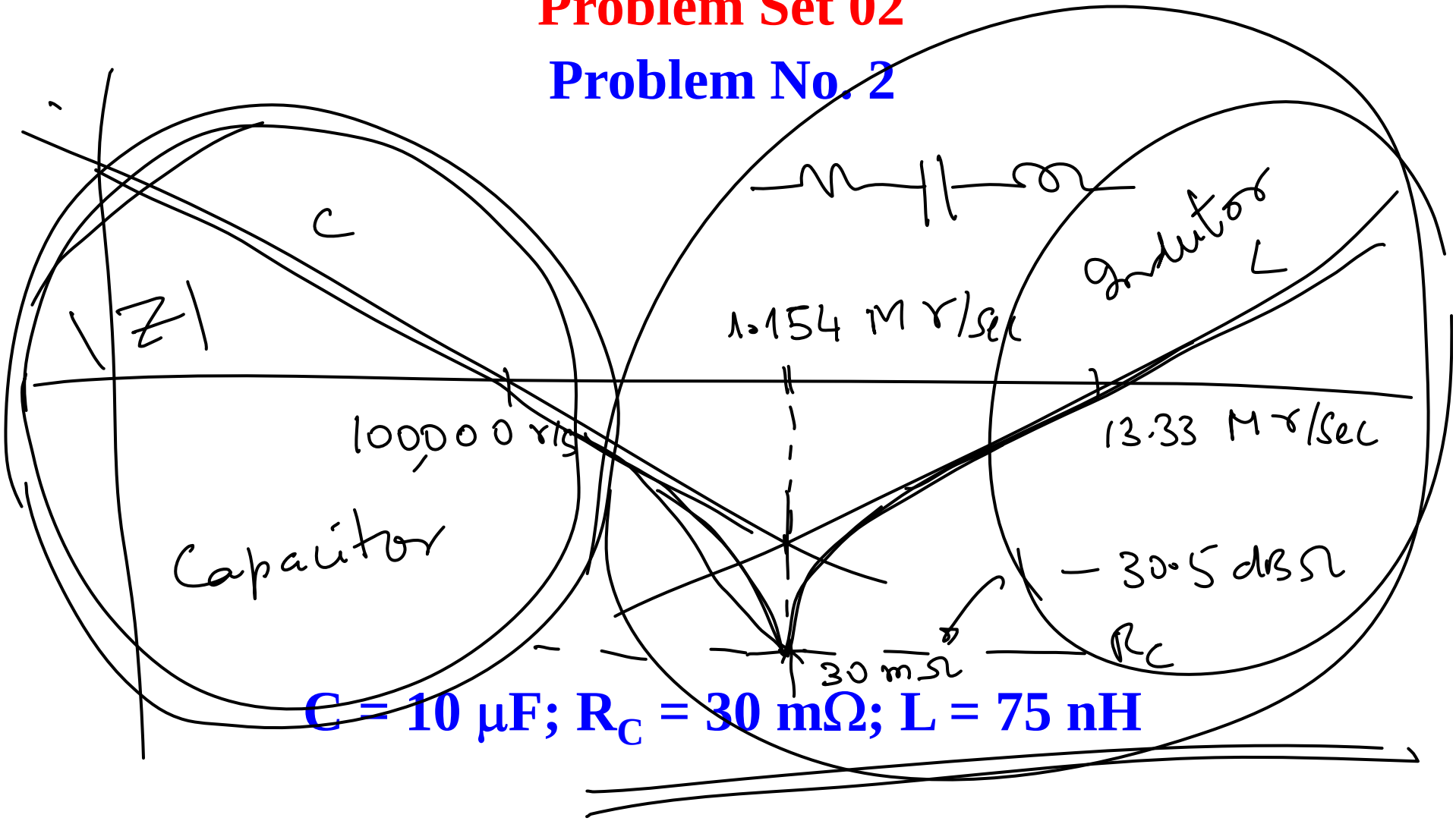
$R_C = 30\ m\Omega$

$20 \log_{10} (.03)$

Switched Mode Power Conversion

Problem Set 02

Problem No. 2

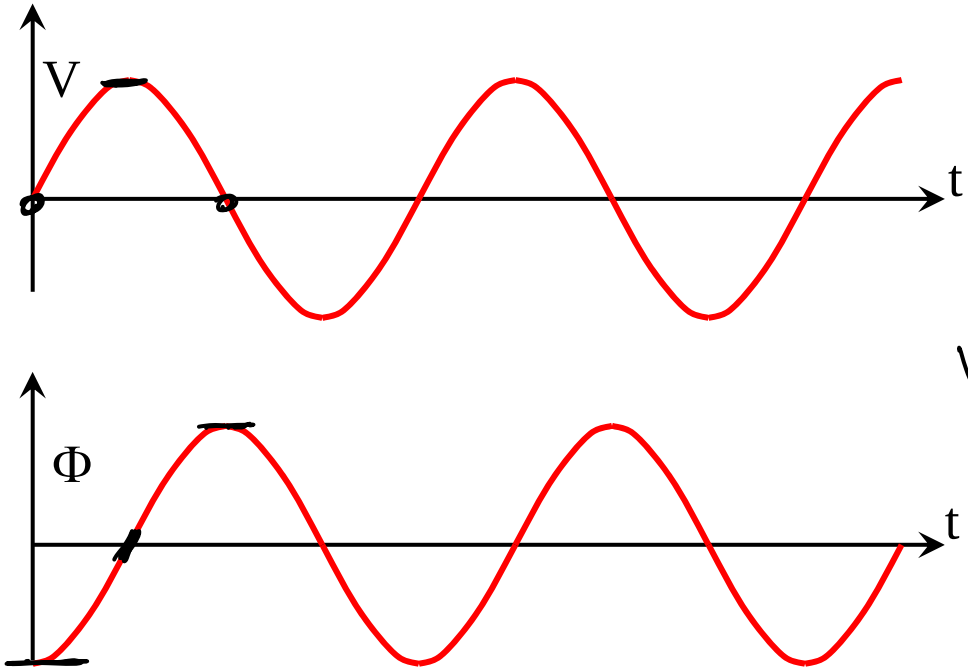


C = 10 μF; R_C = 30 mΩ; L = 75 nH

Switched Mode Power Conversion

Problem Set 02

Problem No. 3



VOLTAGE

$$V = N \frac{d\phi}{dt}$$

$$\phi = \int_0^t \frac{1}{N} V \, dt$$

Transformer with Sinusoidal Excitation

Switched Mode Power Conversion

Problem Set 02

Problem No. 3

$$\phi(t) = -\phi_M \cos \omega t$$

$$V(t) = N \frac{d\phi}{dt} = N \phi_M \omega \sin \omega t$$

$$V(t) = 2\pi f \phi_M N \sin \omega t$$

$$V_{rms} = \frac{2\pi}{\sqrt{2}} f B_m A_c N$$

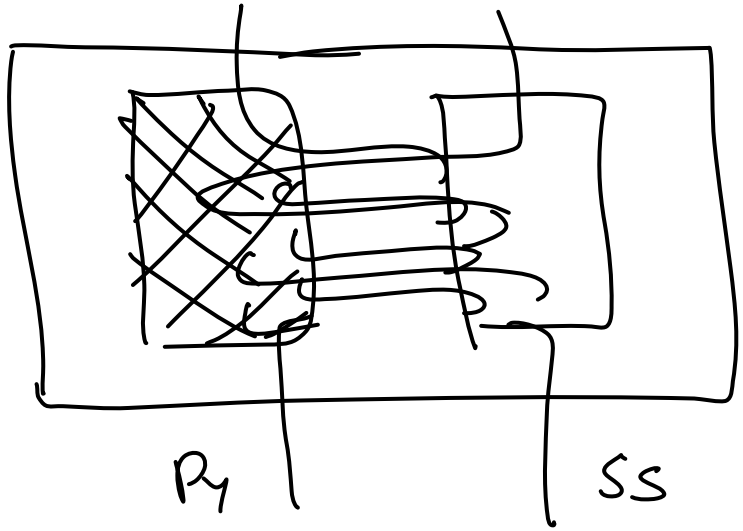
$$V_{rms} = 4.44 f B_m A_c N$$

Transformer with Sinusoidal Excitation

Switched Mode Power Conversion

Problem Set 02

Problem No. 3



$$K_w A_w = \{N_1 a_1 + N_2 a_2\}$$

$$= 2 N_1 a_1 = 2 N_1 \frac{I_{1rms}}{J}$$

Transformer with Sinusoidal Excitation

Switched Mode Power Conversion

Problem Set 02

Problem No. 3

$$V_{rms} = 4.44 f B_m A_c N_1$$

$$2 N_1 \frac{I_{rms}}{J} = K_w A_w$$

$$V_{rms} I_{rms} = 2.22 J B_m f K_w A_c A_w$$

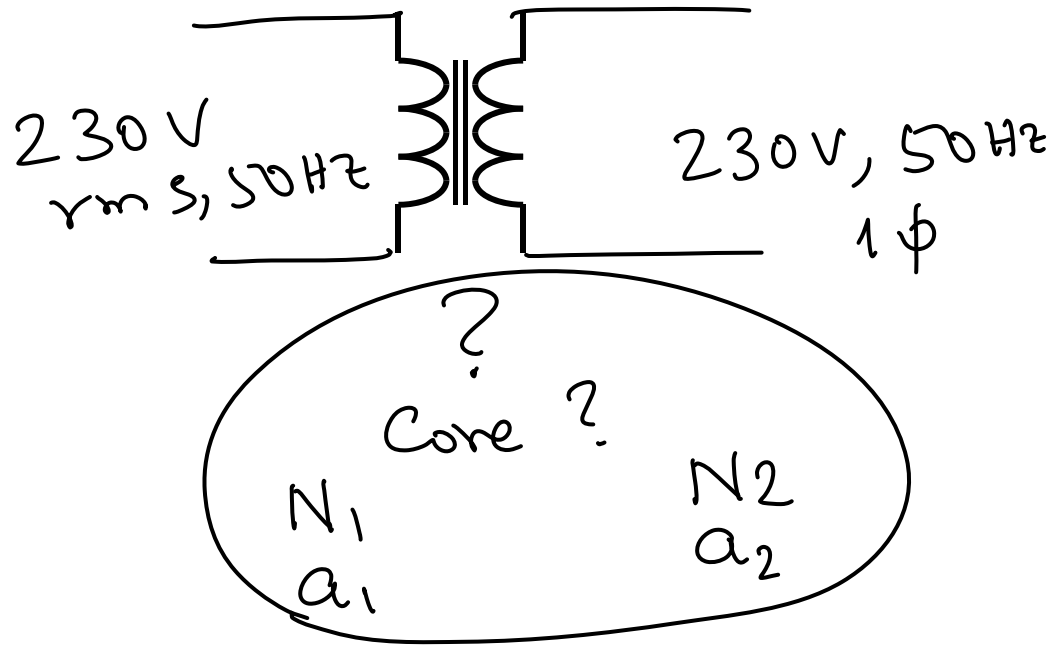
$$A_c A_w = \frac{V_{rms} I_{rms}}{2.22 J B_m f K_w}$$

Transformer with Sinusoidal Excitation

Switched Mode Power Conversion

Problem Set 02

Problem No. 4



150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer

Switched Mode Power Conversion

Problem Set 02

Problem No. 4

$$A_c A_w = \frac{V_{rms} I_{rms}}{2.22 f J B_m K_w} = \frac{150}{2.22}$$

$f = 50$

$J = 2.5 \times 10^6 \ A/m^2$

$B_m = 1.2 \ T$

$K_w = 0.35$

$$A_c A_w = 12,870 \ mm^4$$

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer

Switched Mode Power Conversion

Problem Set 02

Problem No. 4

$A_c A_w \Rightarrow$ T16

$A_c = 1452\ mm^2$

$A_w = 1093\ mm^2$

$$230 = 4.44\ 50\ 1.2\ 1452\ 10^{-6} \times N$$

$$N = \frac{230}{4.44\ 50\ 1.2\ 1452\ 10^{-6}}$$

$N_1 = N_2 = 595$ TURNS

Switched Mode Power Conversion

Problem Set 02

Problem No. 4

$$I_{rms} = \frac{150\,VA}{230\,V} = 0.65\,A$$

$$J = 2.5\ A/mm^2 \Rightarrow a_w = \frac{0.65}{2.5}$$

$$= 0.26\ mm^2$$

→ 23 SWG

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer

Switched Mode Power Conversion

Problem Set 02

Problem No. 4

T 16 Core EI

$A_c = 1452\ mm^2$

$N_1 = 595\ T$

$a_{w1} = 23\ SWG$

$A_w = 1092\ mm^2$

$N_2 = 595\ T$

$a_{w2} = 23\ SWG$

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer

Switched Mode Power Conversion

Problem Set 02

Problem No. 4

$A_c A_w$

Select a Core

A_c

N_1 N_2 - - -

I_1 I_2

a_{w1} a_{w2}

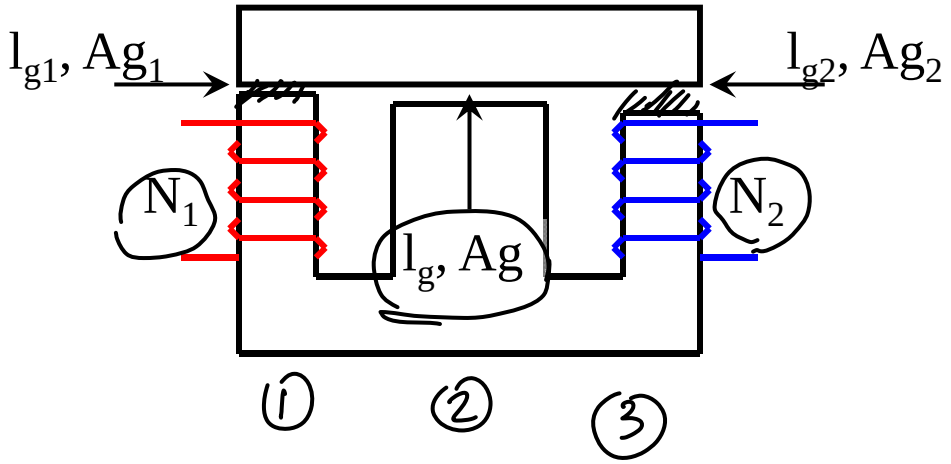
SWG SWG

150 VA, 230 V, 1:1, 50 Hz, Isolation Transformer

Switched Mode Power Conversion

Problem Set 02

Problem No. 5



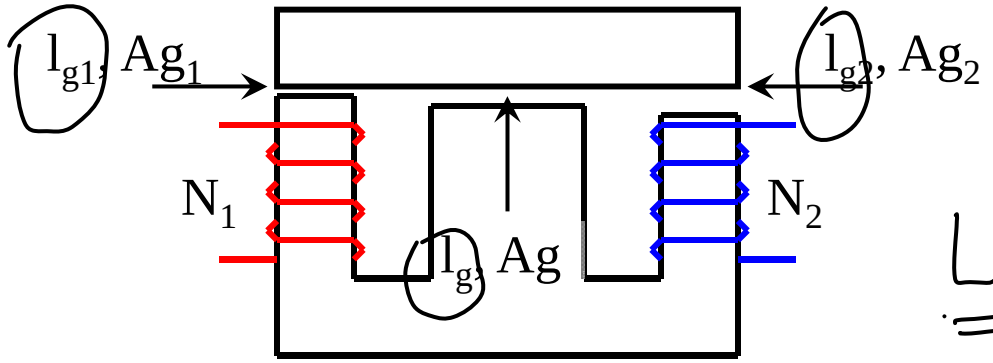
Self
Mutual

Coupled Inductor

Switched Mode Power Conversion

Problem Set 02

Problem No. 5



L_1 & L_2

L_{12} L_{21}

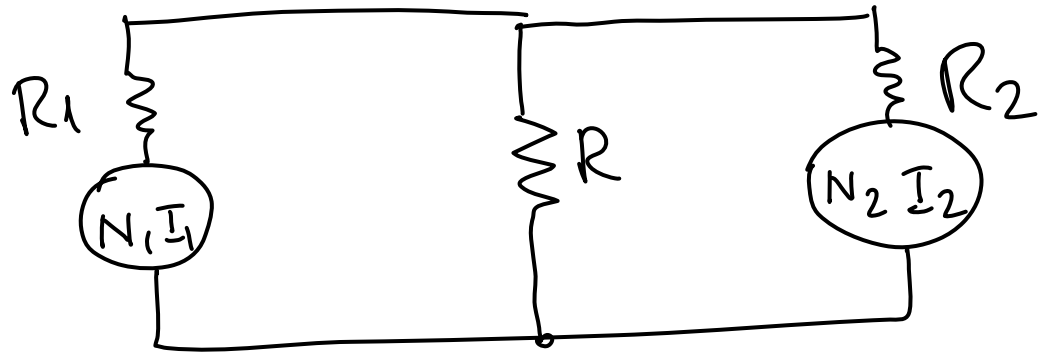
N_1 = 100 T; N_2 = 200 T; $Ag_1 = Ag_2$ = 40 mm^2;

Ag = 80 mm^2; l_{g1} = 1 mm; l_{g2} = 2 mm; l_g = 1.5 mm

Switched Mode Power Conversion

Problem Set 02

Problem No. 5



$$R_1 = \frac{l_{g1}}{A_{g1}\,\mu_0} \qquad R = \frac{l_g}{A_g\,\mu_0} \qquad R_2 = \frac{l_{g2}}{A_{g2}\,\mu_0}$$

Reluctance Model

Switched Mode Power Conversion

Problem Set 02

Problem No. 5

$$R_1 = \frac{l_{g1}}{A_{g1}\mu_0} = \frac{1 \times 10^{-3}}{40\ 10^{-6}\ 4\pi\ 10^{-7}}$$

$$= 19.9 \times 10^6 \quad 1/H$$

$$R_2 = \frac{l_{g2}}{A_{g2}\mu_0} = 39.8 \times 10^6 \quad 1/H$$

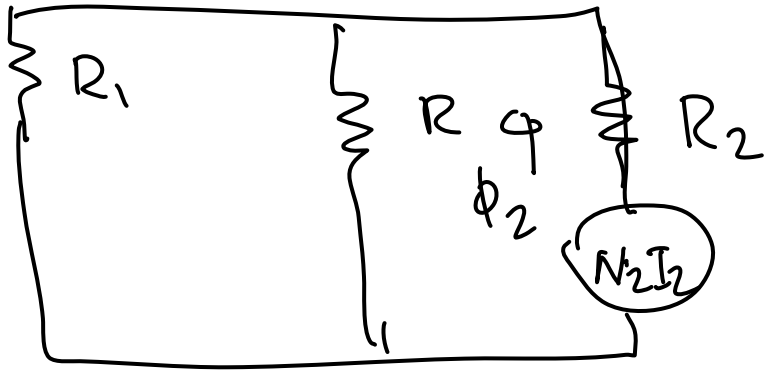
$$R = \frac{l_g}{A_g \mu_0} = \frac{1.5 \times 10^{-3}}{80 \times 10^{-6}\ 4\pi\ 10^{-7}} = 14.92\ 10^6 \frac{1}{H}$$

Reluctance Calculation

Switched Mode Power Conversion

Problem Set 02

Problem No. 5



$\Phi_1\big|_{i_1}$

$\Phi_2\big|_{i_2}$

$$R_{net} = R_1 + (R \parallel R_2)$$

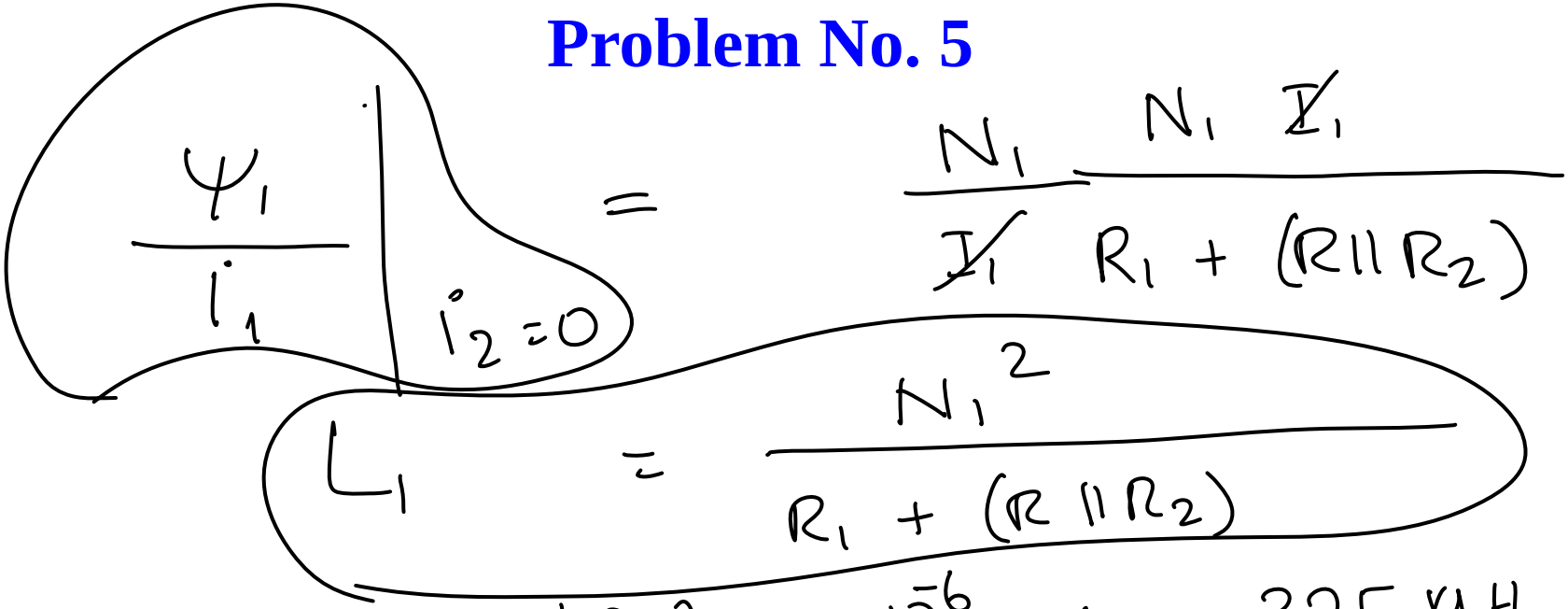
$$\Rightarrow R_2 + (R \parallel R_1)$$

Reluctance Calculation

Switched Mode Power Conversion

Problem Set 02

Problem No. 5



L_1

Switched Mode Power Conversion

Problem Set 02

Problem No. 5

$$L_2 = \frac{N_2^2}{R_2 + (R \,||\, R_1)}$$

$$L_2 = \left.\frac{\psi_2}{i_2}\right|_{i_1=0} \Rightarrow \frac{N_2 \frac{N_2 I_2}{\mathcal{R}}}{\frac{I_2}{\mathcal{R}} R_2 + (R \,||\, R_1)}$$

$$\frac{200 \times 200}{(39.8 + 8.52)} 10^{-6} \Rightarrow 828\ \mu H$$

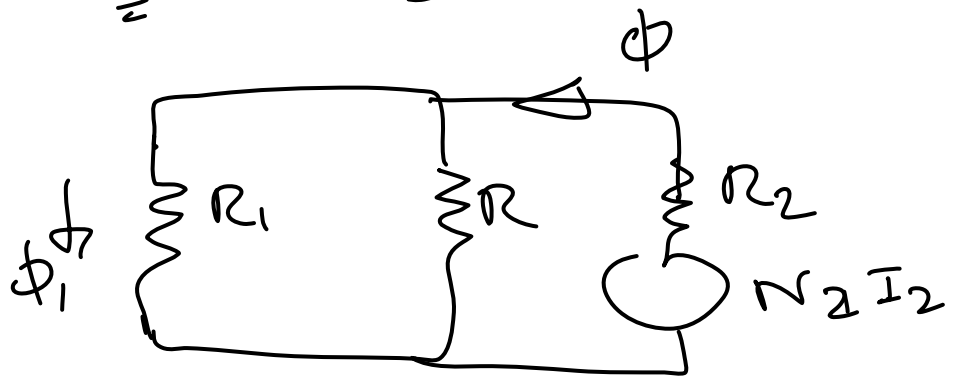
L_2

Switched Mode Power Conversion

Problem Set 02

Problem No. 5

$$\left.\frac{\psi_1}{i_2}\right|_{i_1=0} = L_{12}$$



$$\left.\frac{N_1\ \Phi_1}{i_2}\right|_{i_1=0} = \frac{N_1}{\cancel{i_2}}\ \frac{N_2\ \cancel{i_2}}{R_2+(R \,||\, R_1)}\ \frac{R}{R+R_1}$$

$$L_{12}$$

Switched Mode Power Conversion

Problem Set 02

Problem No. 5

$$L_{21} = \frac{N_1 N_2}{R_2 + (R \,||\, R_1)} \cdot \frac{R}{R + R_1}$$

$$= \frac{100 \quad 200}{19.9 + 10.85} \cdot \frac{14.92}{14.92 + 39.79} \quad 10^{-6}$$

$$= 177\ \mu H$$

$$L_{12} = 177\ \mu H$$

$$L_{21}$$

Switched Mode Power Conversion

Problem Set 02

Problem No. 5

$L_1 = 356\,\mu H$

$L_2 = 828\,\mu H$

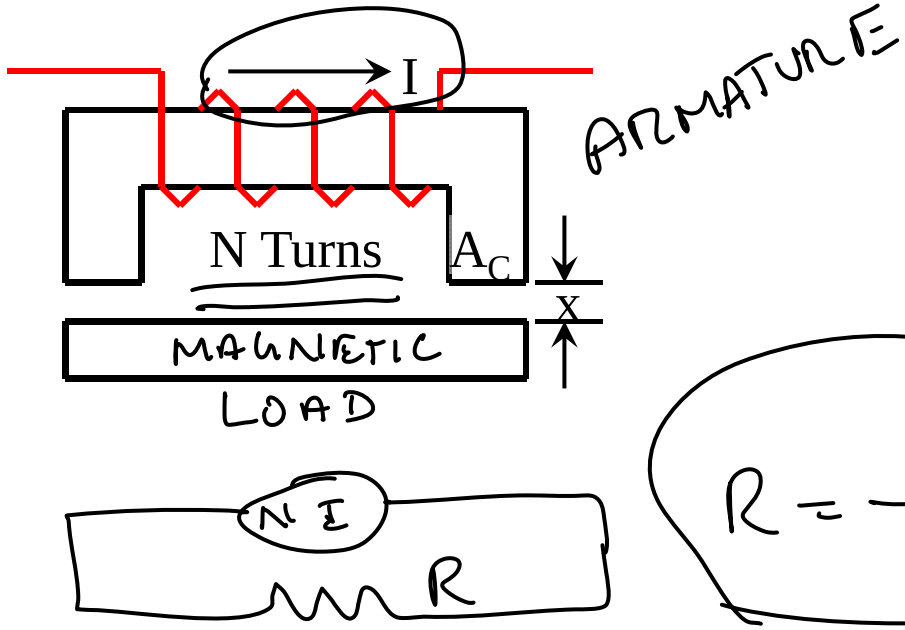
$L_{12} = L_{21} = 177\,\mu H$

Self and Mutual Inductances

Switched Mode Power Conversion

Problem Set 02

Problem No. 6



$$L = \frac{N^2}{R}$$

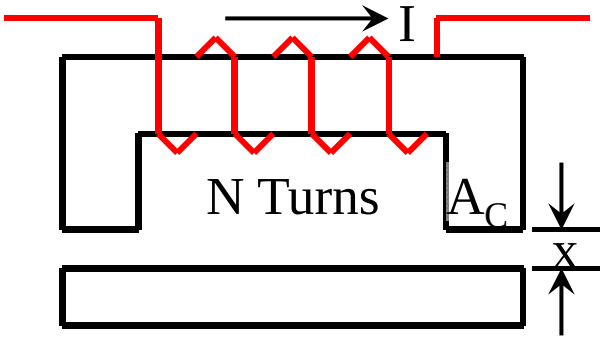
$$R = \frac{2x}{A_c \mu_0}$$

Lifting Magnet

Switched Mode Power Conversion

Problem Set 02

Problem No. 6



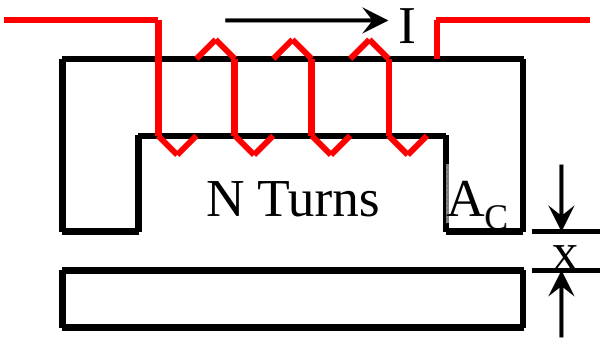
$$R = \frac{l_g}{A_C \mu_O \mu_r} = \frac{2x}{A_C \mu_O}$$

Reluctance

Switched Mode Power Conversion

Problem Set 02

Problem No. 6



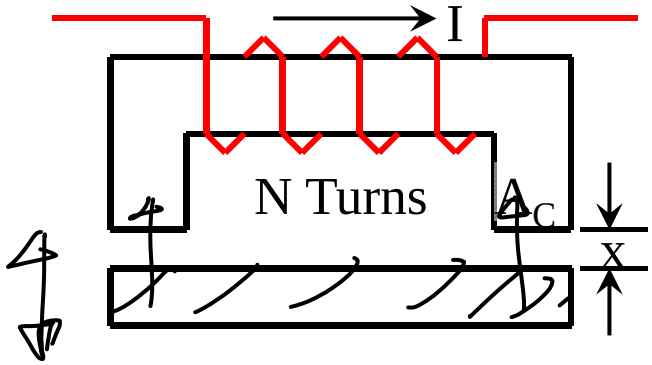
$$L = \frac{N^2}{R} = \frac{N^2 A_C \mu_O}{2x} =$$

Inductance

Switched Mode Power Conversion

Problem Set 02

Problem No. 6



$$E = \frac{1}{2} L I^2 = \frac{N^2 A_C \mu_O I^2}{4x}$$

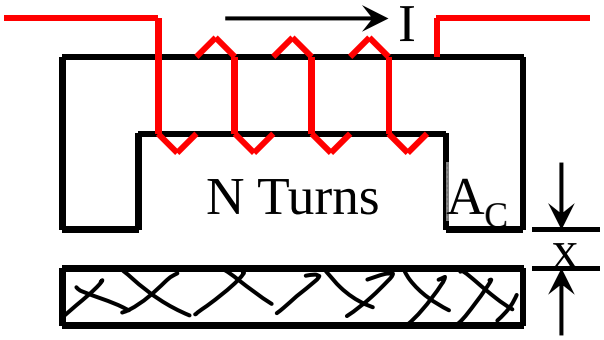
$E = f(x)$

Stored Energy

Switched Mode Power Conversion

Problem Set 02

Problem No. 6



$$F = -\frac{dE}{dx} = \frac{N^2 A_C \mu_O I^2}{4x^2}$$

Lifting Force

Switched Mode Power Conversion

Problem Set 02

Problem No. 6

$$F = \frac{N^2 A_C \mu_O I^2}{4x^2}$$

$$F = \frac{200 \;\; 200 \;\; .04 \;\; 4\pi \, 10^{-7} \;\; 500 \;\; 500}{4 \qquad .01}$$

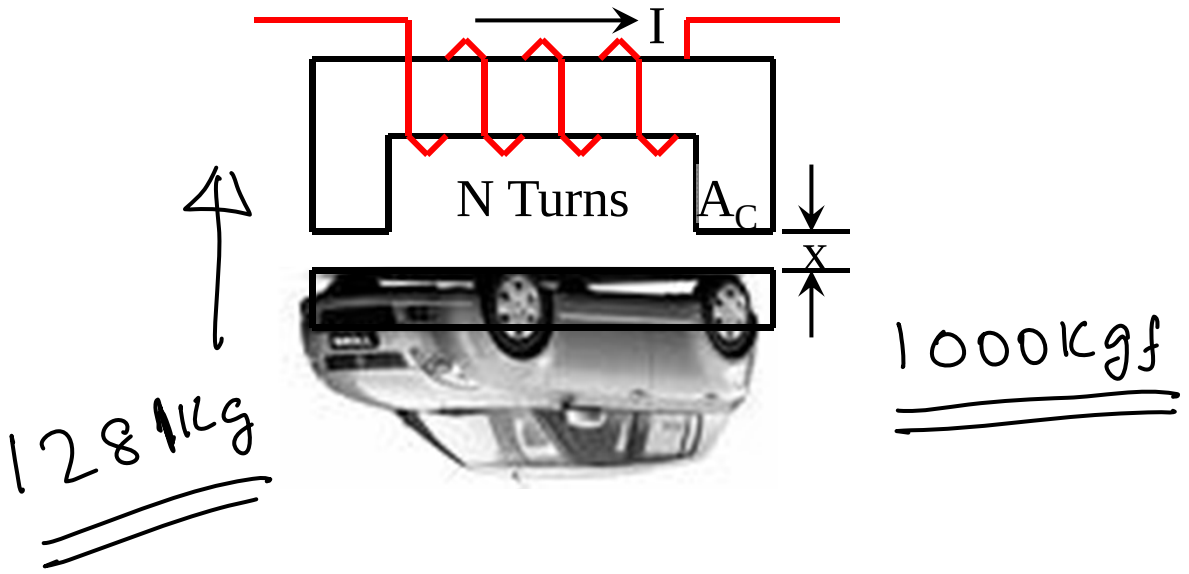
$$= 12{,}566 \text{ Nw} \Rightarrow 1281 \text{ Kgf}$$

Lifting Force: N = 200, A_C = 0.04 m^2, I = 500 A, x = 100 mm = 0.1 m

Switched Mode Power Conversion

Problem Set 02

Problem No. 6

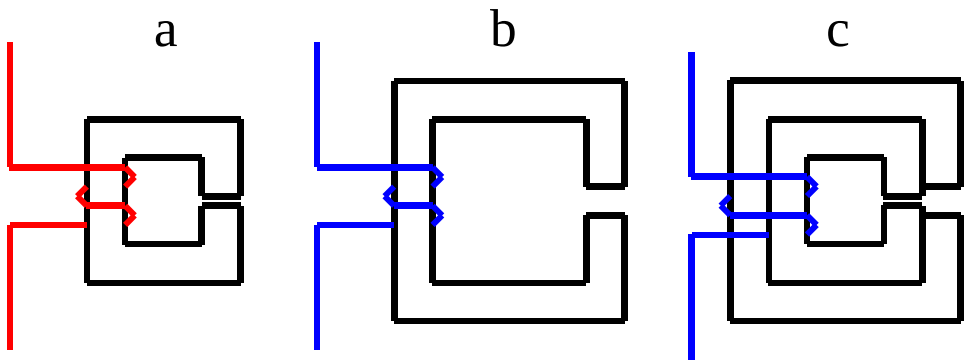


Lifting Force: N = 200, A_C = 0.04 m^2, I = 500 A, x = 100 mm = 0.1 m

Switched Mode Power Conversion

Problem Set 02

Problem No. 7



Gap = 1 mm
Area = 25 mm^2
N = 100 T
B_m = 0.2 T

Gap = 5 mm
Area = 25 mm^2
N = 100 T
B_m = 0.2 T

Composite Core

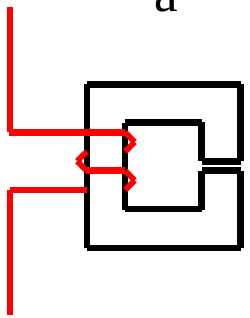
Non-linear Inductor

Switched Mode Power Conversion

Problem Set 02

Problem No. 7

a



Gap = 1 mm

Area = 25 mm^2

N = 100 T

B_m = 0.2 T

$$L = \frac{N^2}{R} = \frac{100^2 \; 25\times10^{-6} \; 4\pi 10^{-7}}{1\times 10^{-3}}$$

$$= 314 \; \mu H \quad \checkmark \qquad 0 < i < 1.59A$$

$I_m \Rightarrow B_m$

$$L \; I_m = N \; B_m \; A_c$$

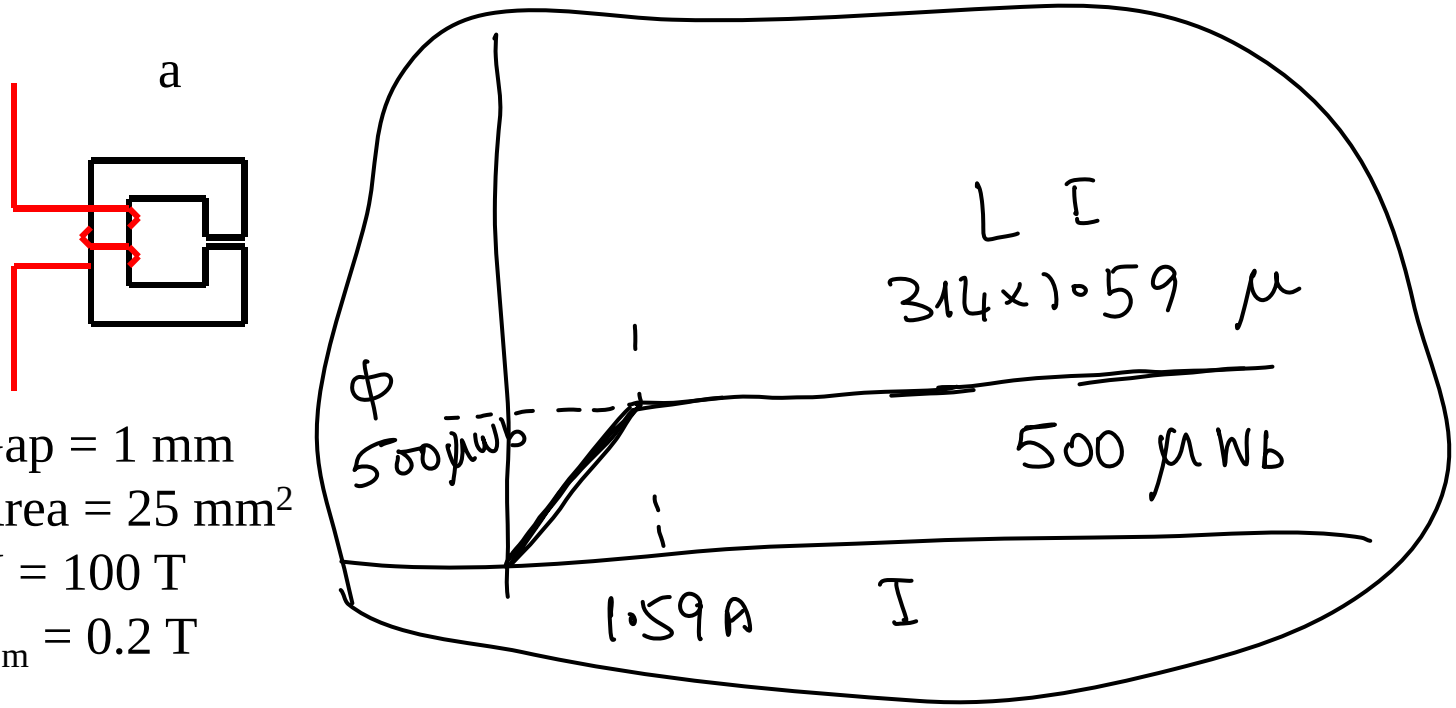
$$I_m = \frac{100 \cdot 2 \; 25\times10^{-6}}{314 \; 10^{-6}}$$

$$= 1.59A$$

Switched Mode Power Conversion

Problem Set 02

Problem No. 7



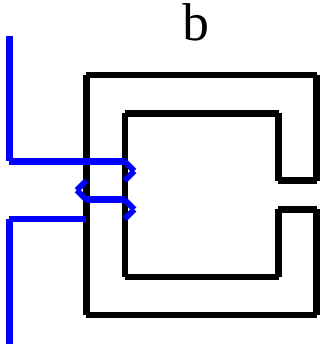
Gap = 1 mm
Area = 25 mm^2
N = 100 T
B_m = 0.2 T

Flux – Current Characteristics

Switched Mode Power Conversion

Problem Set 02

Problem No. 7



Gap = 5 mm

Area = 25 mm^2

N = 100 T

B_m = 0.2 T

$$L = \frac{100 \times 100 \quad 25 \times 10^{-6} \quad 4\pi 10^{-7}}{5 \times 10^{-3}}$$

$$= 62.8\ \mu H$$

$$L I_m = N B_m A_c$$

$$62.8\ I_m = 100 \cdot 2 \quad 25\ 10^{-6}$$

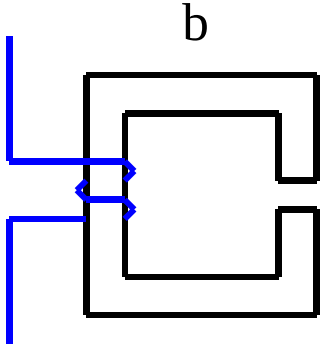
$$I_m = \frac{100 \cdot 2 \quad 25\ 10^{-6}}{62.8 \quad 10^{-6}}$$

$$= 7.96\ A$$

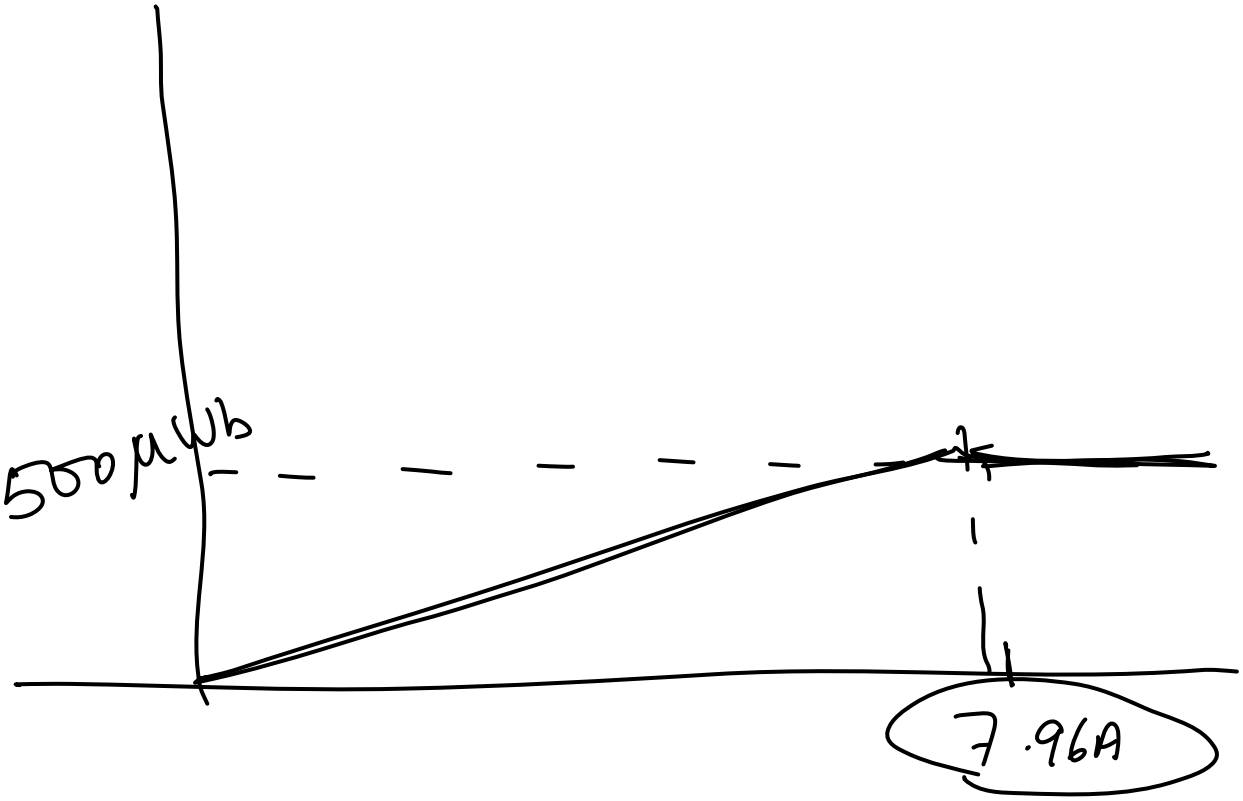
Switched Mode Power Conversion

Problem Set 02

Problem No. 7



Gap = 5 mm
Area = 25 mm^2
N = 100 T
B_m = 0.2 T

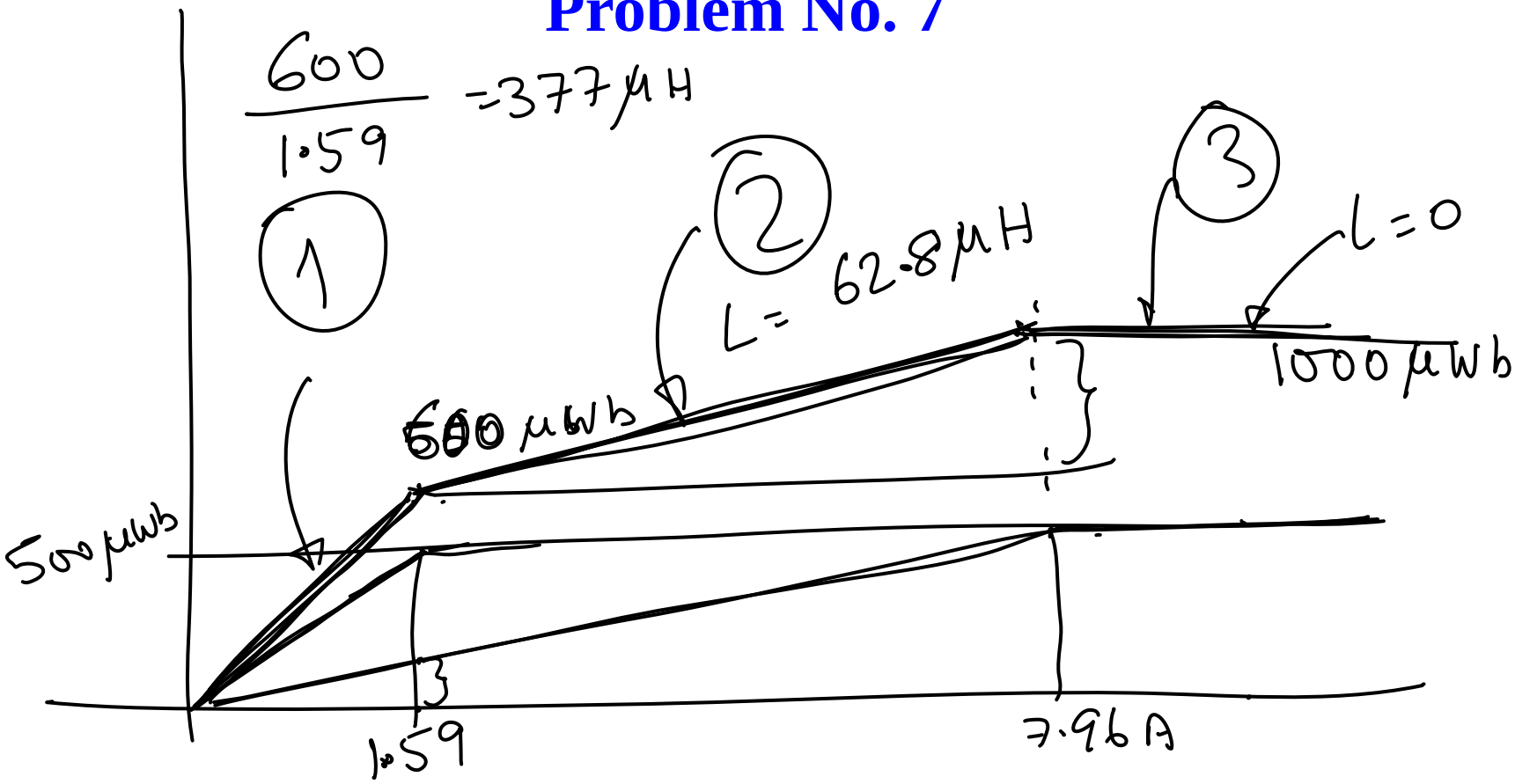


Flux – Current Characteristics

Switched Mode Power Conversion

Problem Set 02

Problem No. 7

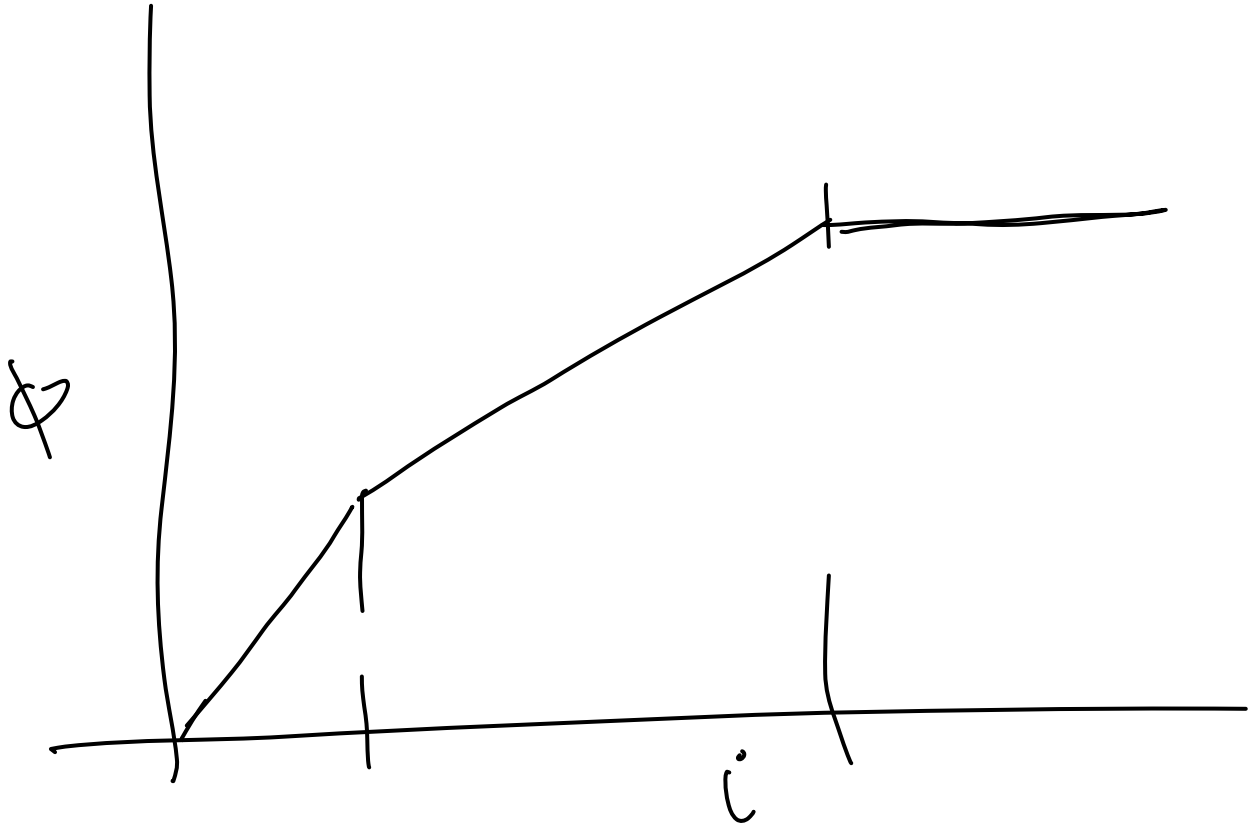


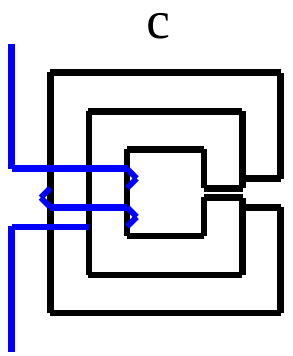
Flux – Current Characteristics (Total)

Switched Mode Power Conversion

Problem Set 02

Problem No. 7





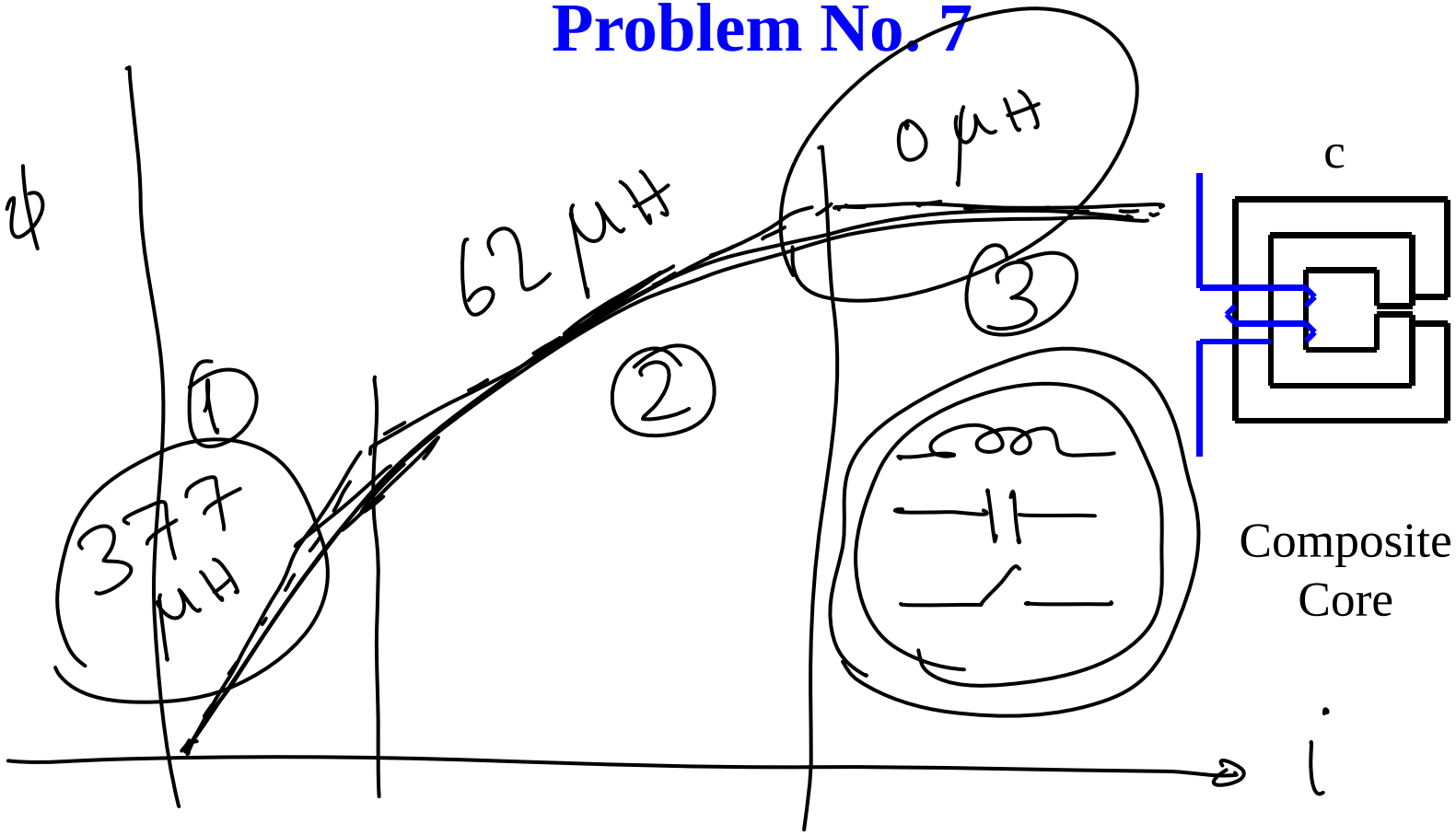
Composite Core

Idealised Flux – Current Characteristics

Switched Mode Power Conversion

Problem Set 02

Problem No. 7



Real Flux – Current Characteristics