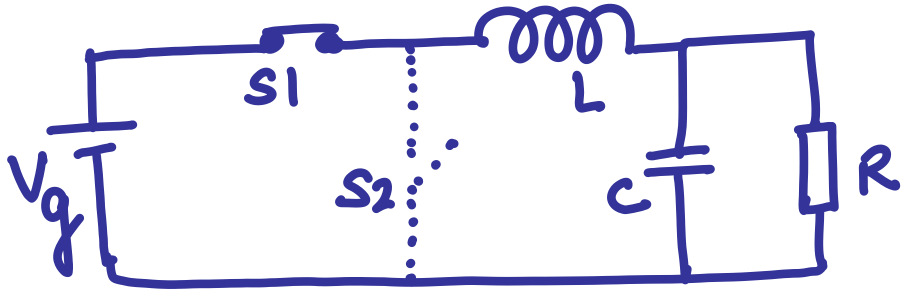
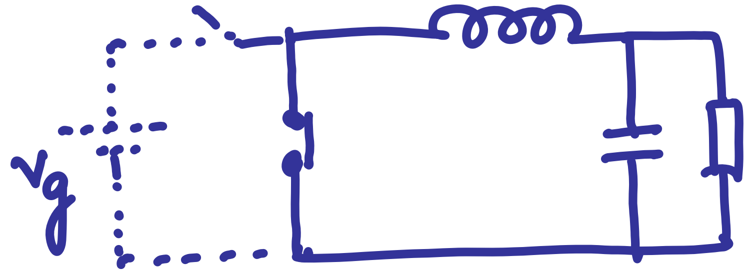
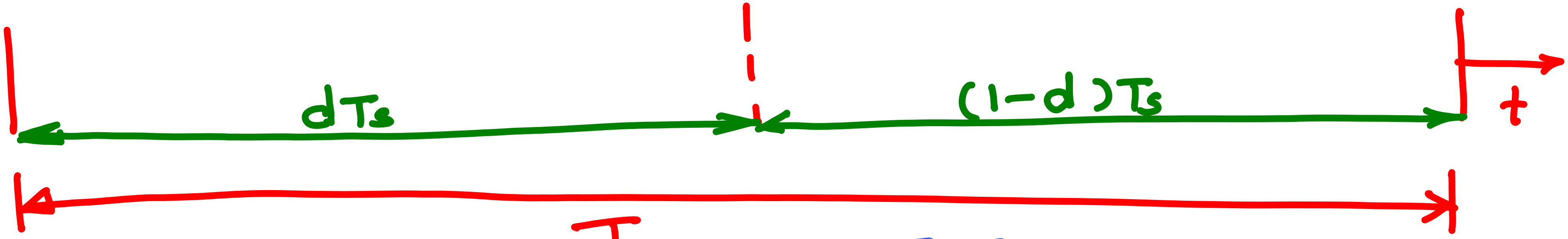
Circuit Averaging Method





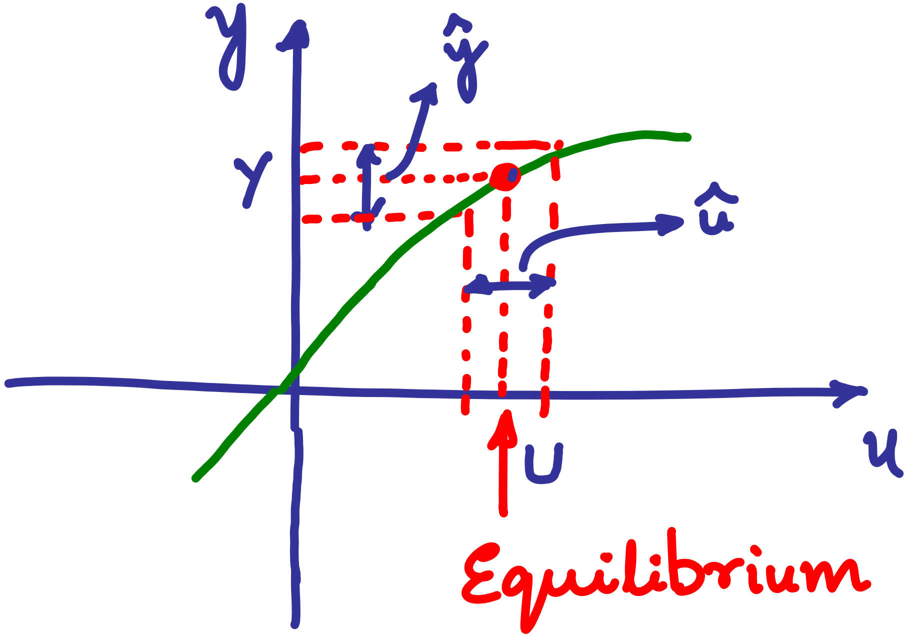


$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} [v_g]$$

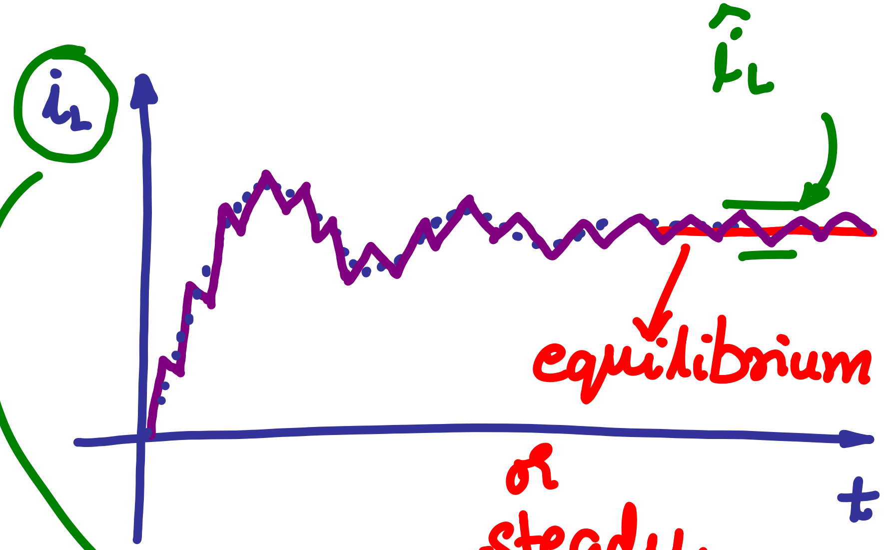
$$\begin{bmatrix} \dot{i}_L \\ \dot{v}_c \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} [v_g]$$

$$\underbrace{v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + [0][v_g]}$$

$$\underbrace{v_o = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_L \\ v_c \end{bmatrix} + [0]\, v_g}$$



Equilibrium or Steady state



equilibrium or steady state part, I_L

$$\underbrace{i_L}_{\text{large signal}} = \underbrace{I_L}_{\text{steady state}} + \underbrace{\hat{i}_L}_{\text{small signal}}$$

$$u = \underbrace{U}_{\sim} + \underbrace{\hat{u}}_{\sim} \quad \text{wherein} \quad \frac{\hat{u}}{U} \ll 1$$

$$d = D + \hat{d} \qquad \frac{\hat{d}}{D} \ll 1$$

$$\left.\begin{matrix} i_L \\ v_C \end{matrix}\right\rangle \; x = X + \hat{x} \qquad \frac{\hat{x}}{X} \ll 1$$

$$y = \underbrace{Y}_{\sim} + \underbrace{\hat{y}}_{\sim} \qquad \frac{\hat{y}}{Y} \ll 1$$

$$\dot{x} = \mathbb{A}_1 x + \mathbb{B}_1 u$$
$$y = \mathbb{C}_1 x + \mathbb{D}_1 u$$

$\underbrace{\hspace{6em}}_{dT_s}$

large signal

$$\dot{x} = \mathbb{A}_2 x + \mathbb{B}_2 u$$
$$y = \mathbb{C}_2 x + \mathbb{D}_2 u$$

$\underbrace{\hspace{6em}}_{(1-d)T_s}$

large signal model

d $\quad$ $(1-d)$

$$\left.\begin{aligned} \dot{x} &= \mathbb{A} x + \mathbb{B} u \\ y &= \mathbb{C} x + \mathbb{D} u \end{aligned}\right\} \text{Averaged large signal model}$$

$$\underline{\dot{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}$$

$$\underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$$

AVERAGED LARGE SIGNAL MODEL

$$\rightarrow \quad \underline{\underline{A}} = \underline{\underline{A}}_1 \cdot \underline{\underline{d}} + \underline{\underline{A}}_2 \cdot (1 - \underline{\underline{d}})$$

$$\rightarrow \quad \underline{\underline{B}} = \underline{\underline{B}}_1 \cdot \underline{\underline{d}} + \underline{\underline{B}}_2 \cdot (1 - \underline{\underline{d}})$$

$$\underline{\underline{C}} = \underline{\underline{C}}_1 \cdot \underline{\underline{d}} + \underline{\underline{C}}_2 \cdot (1 - \underline{\underline{d}})$$

$$\underline{\underline{D}} = \underline{\underline{D}}_1 \cdot \underline{\underline{d}} + \underline{\underline{D}}_2 \cdot (1 - \underline{\underline{d}})$$

$$\dot{\underline{x}} = \mathbb{A}\underline{x} + \mathbb{B}\underline{u}$$

$$x = X + \hat{x}$$

$$u = U + \hat{u}$$

$$(\dot{X + \hat{x}}) = \mathbb{A}\cdot(X+\hat{x}) + \mathbb{B}\cdot(U+\hat{u})$$

$$\boxed{\underbrace{(\dot{X} + \dot{\hat{x}})}_{0} = [\mathbb{A}_1 \underset{(D+\hat{d})}{d} + \mathbb{A}_2 \underset{(1-D-\hat{d})}{(1-d)}]\cdot(X+\hat{x}) + [\mathbb{B}_1\cdot \underset{(D+\hat{d})}{d} + \mathbb{B}_2 \underset{(1-D-\hat{d})}{(1-d)}](U+\hat{u})}$$

$$\mathbb{A}X + \mathbb{B}U = 0$$

$$X + \hat{x} \qquad \frac{\hat{x}}{X} \ll 1$$

1

$$(\hat{x} \cdot \hat{d}) \Rightarrow \text{neglect}$$

2

$$X \qquad \frac{dX}{dt} = 0$$

$$\dot{x} = AX + BU = 0$$

$$B_1 \cdot D + B_2(1-D)$$

$$\dot{\hat{x}} = A\hat{x} + B\hat{u} + \left[(A_1 - A_2)X + (B_1 - B_2)U\right]\hat{d}$$

$$A_1 \cdot D + A_2(1-D)$$

Control input

$$y = Cx + Du$$

$$Y + \hat{y} = \left[C_1 d + C_2(1-d)\right](X + \hat{x}) + \left(D_1 d + D_2(1-d)\right) \cdot (U + \hat{u})$$

$$\hat{y} = C\hat{x} + D\hat{u} + \left[(C_1 - C_2)X + (D_1 - D_2)U\right]\hat{d}$$

Averaged Model of Buck converter

$$A = A_1 D + A_2 (1-D)$$

$$= \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \cdot D + \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} (1-D)$$

$$= \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix}$$

$$\mathbb{B} = \mathbb{B}_1 \cdot D + \mathbb{B}_2 (1 - D)$$

$$= \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} \cdot D + \begin{bmatrix} 0 \\ 0 \end{bmatrix} (1 - D)$$

$$= \begin{bmatrix} \frac{d}{L} \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix}$$

$$\mathbb{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$\mathbb{D} = \begin{bmatrix} 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\hat{i}}_L \\ \dot{\hat{v}}_c \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_c \end{bmatrix} + \begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} \underline{\underline{\hat{v}_g}} + \begin{bmatrix} \frac{D}{L} \cdot V_g \\ 0 \end{bmatrix} \underline{\underline{\hat{d}}}$$

$$\boxed{\begin{bmatrix} \dot{\hat{i}}_L \\ \dot{\hat{v}}_c \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_c \end{bmatrix} + \begin{bmatrix} \frac{D}{L} & \frac{DV_g}{L} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{v}_g \\ \hat{d} \end{bmatrix}}$$

SMALL SIGNAL MODEL

$$\hat{y} = \mathbb{C}\hat{x} + \underset{\substack{\downarrow \\ 0}}{\mathbb{D}}\hat{u} +$$

$$= \mathbb{C}\hat{x}$$

$$= \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{i}_L \\ \hat{v}_c \end{bmatrix}$$

Practice

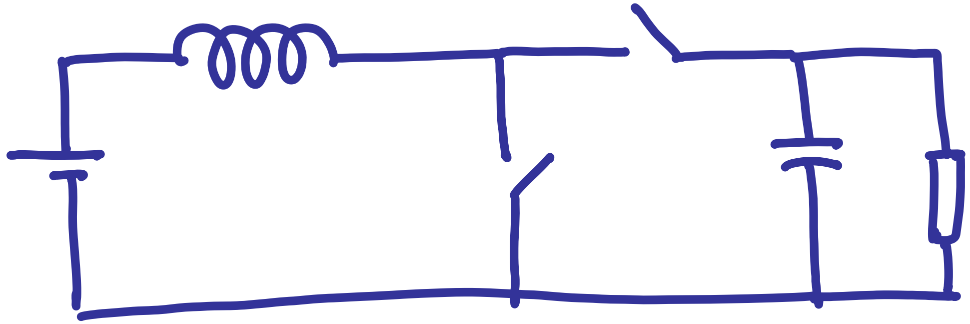
1. Large signal model for each mode of switch operation.

2. Averaged large signal model.

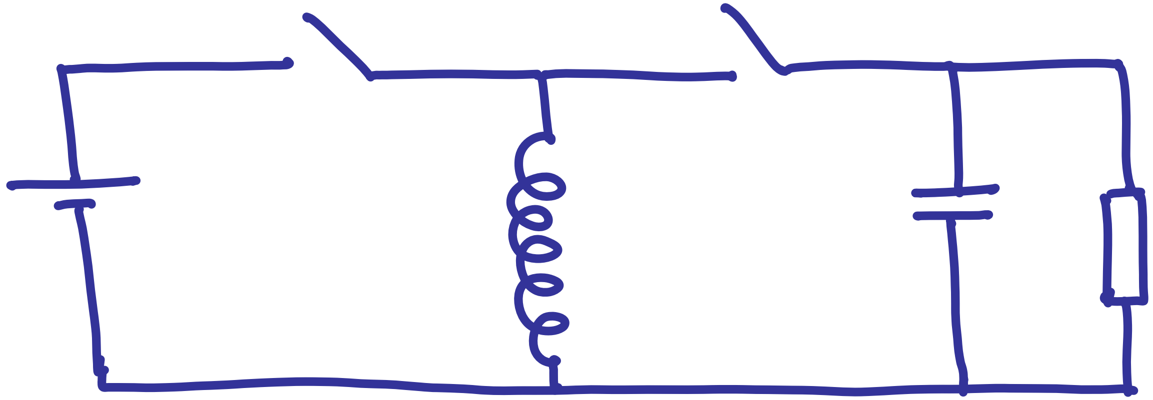
3. $\mathbb{A}X + \mathbb{B}u = 0$

Subtract the steady the portion & simplify

Small Signal Model



Boost



Buck-Boost converter