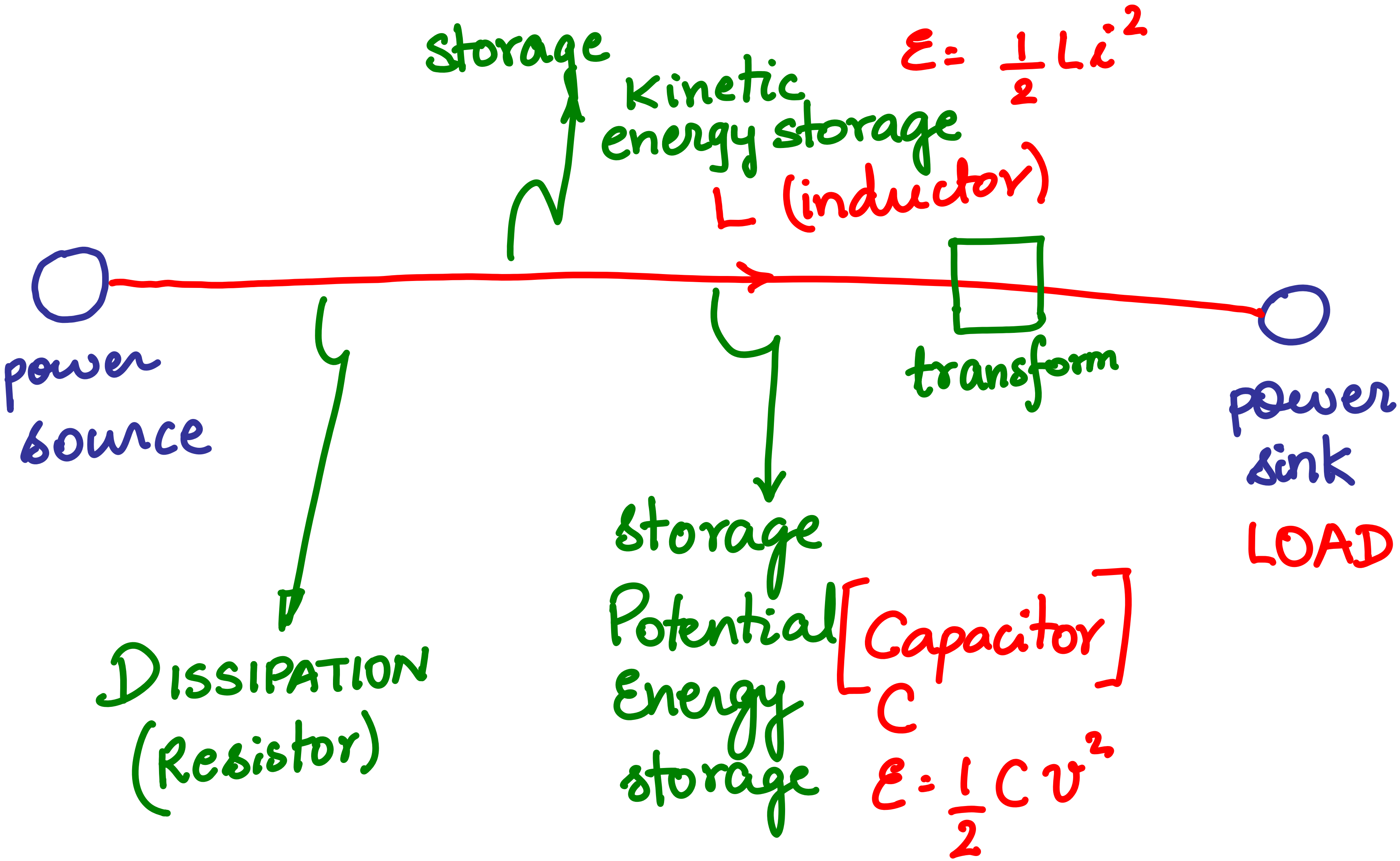
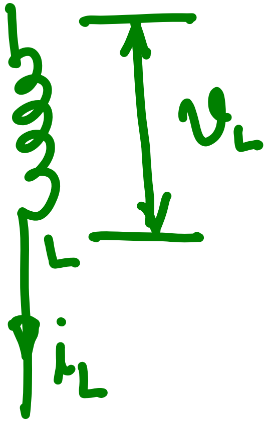
# State Space Representation

## State



Storage
Kinetic energy storage
$\mathcal{E} = \frac{1}{2} L i^2$
L (inductor)
power source
transform
power sink
LOAD
DISSIPATION (Resistor)
storage
Potential Energy storage
[Capacitor] C
$\mathcal{E} = \frac{1}{2} C v^2$

Inductor, L $\Longrightarrow$ $v_L = L\dfrac{di_L}{dt}$ — ①




or $\boxed{i_L = \dfrac{1}{L}\displaystyle\int v_L\,dt}$ — ②

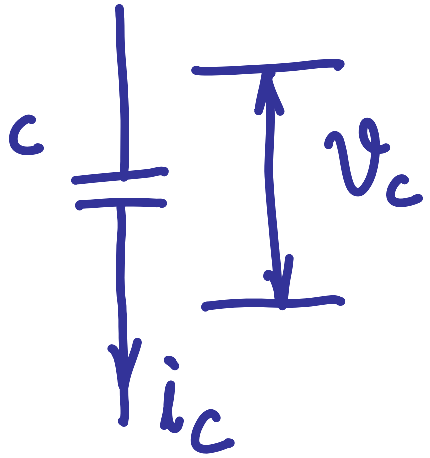
CAUSALITY

Integral Causal

Capacitor, C $=$ $$\boxed{v_c = \frac{1}{C}\int i_c \, dt} \quad -①$$

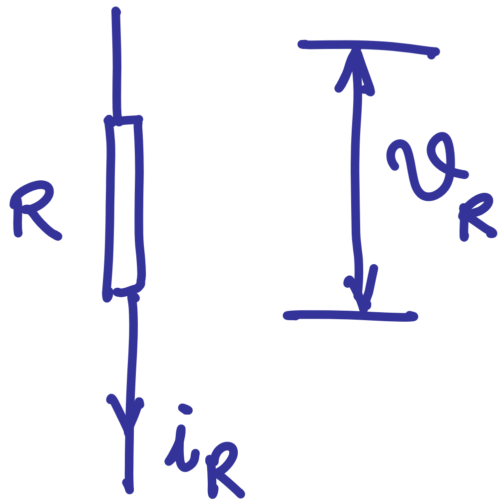
or

$$i_c = C\frac{dv_c}{dt} \quad -②$$




models

# Dissipator, Resistor, R




$$v_R = i_R \cdot R \quad — \quad 1$$

or

$$i_R = \frac{v_R}{R} \quad — \quad 2$$

$$\left\{\begin{aligned} \dot{x} &= \mathbb{A}x + \mathbb{B}u \\ y &= \mathbb{C}x + \mathbb{D}u \end{aligned}\right. \qquad \Big\| \quad \text{state space representation}$$

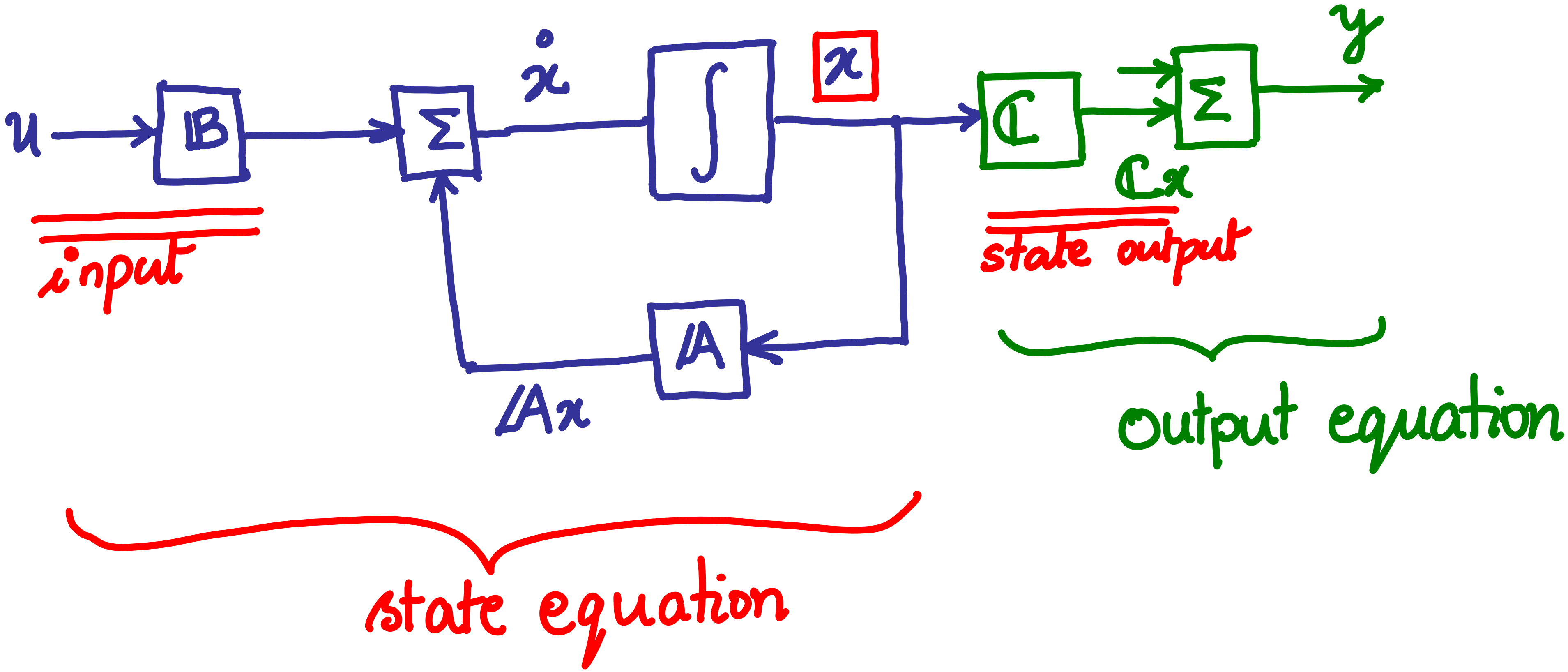
State Equation

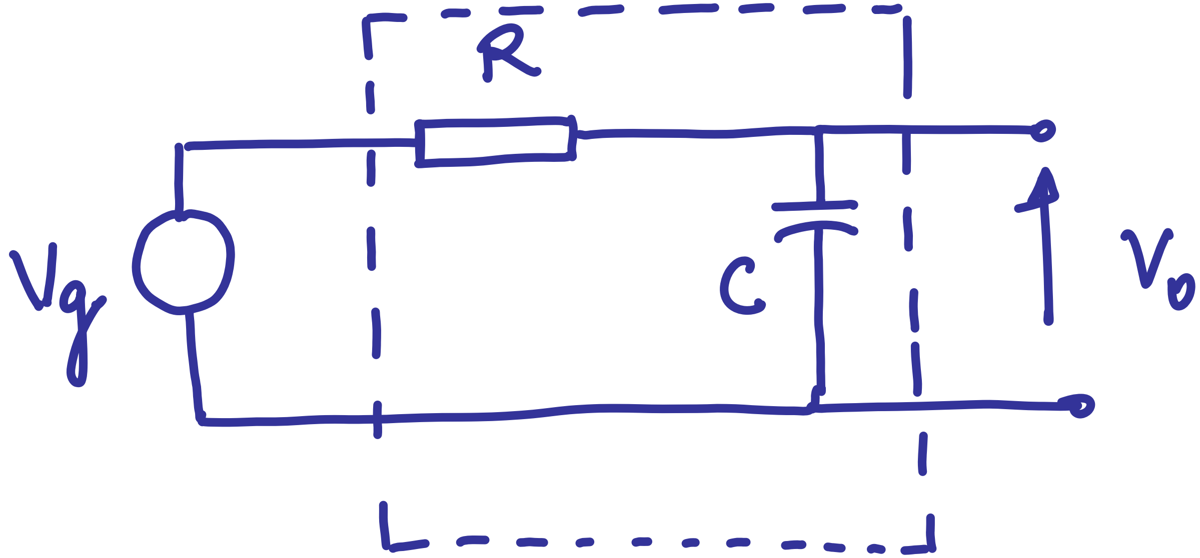
Output Equation

STANDARD

$$\dot{x} = \boxed{Ax} + \overline{Bu}$$

$$y = \underline{Cx}$$






Step 1: Identify energy storing elements

ORDER of system

= 1 first order system

Step 2 : Identify the variables that will be used for modelling

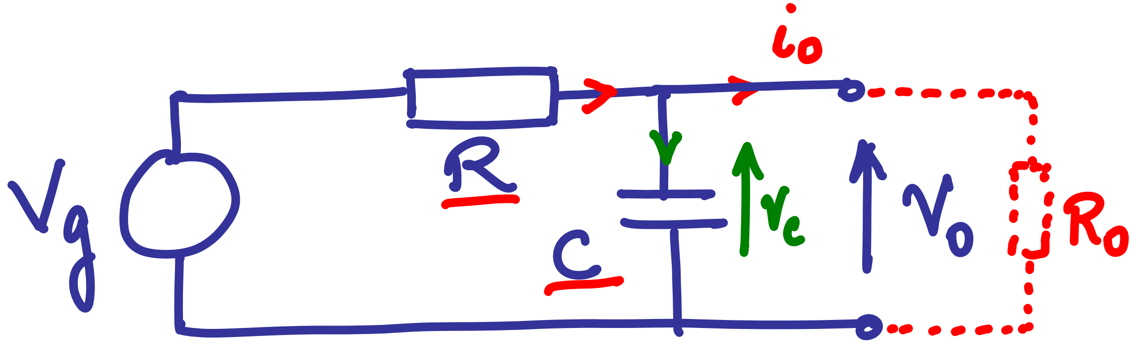
System's response $\Longrightarrow$ ① Input - $u$

② Integrator output values.

state variables

$x$

$u_s$ $x_s$




Step 3: Starting with dynamic elements obtain differential equations

$$\left[\ \overset{(u)}{V_g},\ \overset{(x)}{V_c},\quad \overbrace{R \ \& \ C}^{A}\ \right]$$

$R_o$

$(V_c = V_o)$

$$v_c = \frac{1}{C}\int \underline{i_c}\, dt$$

$$i_c \rightarrow i_R - i_o = \frac{\left(\frac{V_g - V_c}{R}\right) - \frac{V_c}{R_o}}{}$$

$$\frac{dv_c}{dt} = \frac{1}{CR}\left[V_g - V_c\right] - \frac{1}{CR_o}\cdot V_c$$

Step 4: Segregate into form

$$\dot{x} = \mathbb{A}\underline{\underline{x}} + \mathbb{B}\underline{\underline{u}}$$

state input

state space representation

$$\dot{x} = \mathbb{A}\,x + \mathbb{B}u$$

$$\boxed{\dot{v}_c = \left(-\frac{1}{RC} - \frac{1}{R_0 C}\right) v_c + \left(\frac{1}{RC}\right) v_g}$$ — state equation

$$y = \mathbb{C}\,x + \mathbb{D}u$$

$$\boxed{V_0 = [1][v_c] + [0]\,V_g}$$ — output equation

Steps

1. Identify the energy storage elements

2. Identify variables – $u$, $x$, $[\mathbb{A}]$

3. Starting from Dynamic components

4. Put all equations in the form

$$\dot{x} = \mathbb{A}x + \mathbb{B}u$$
$$y = \mathbb{C}x + \mathbb{D}u$$

# Practice