Switched Mode Power Conversion

Inductors

Devices for Efficient Power Conversion

Switches
Inductors
Transformers
Capacitors

Switched Mode Power Conversion

Inductors

Inductors Store Energy

Inductors Store Energy in a Magnetic Field

In Power Converters Energy Storage Elements Smoothen Power Flow

Switched Mode Power Conversion

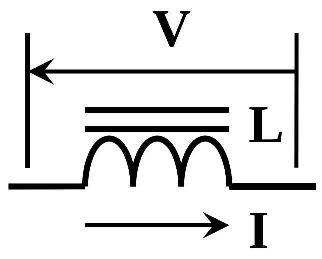
Inductors

Inductors are also used as Switch Protection Elements

In Switching Aid Networks Inductors Provide Turn-On di/dt Protection

Switched Mode Power Conversion

Inductors



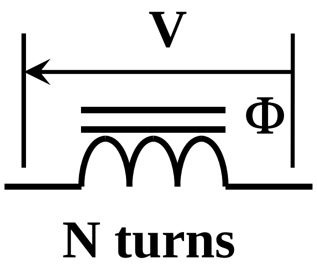
$$V = L\frac{dI}{dt}$$

Electrical Circuit Element Eqaution

V, L, I are Electrical Circuit Quantities

Switched Mode Power Conversion

Inductors



$$V = N\frac{d\Phi}{dt}$$

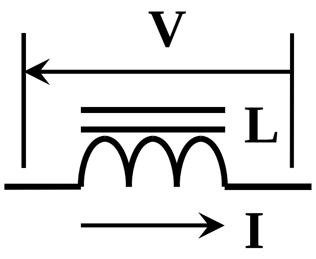
Electromagnetic Equation – Faraday's Law

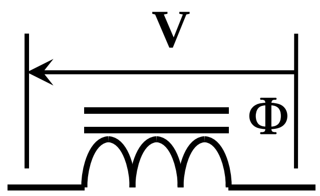
V is Electrical Circuit Quantity

N, Φ are Magnetic Circuit Quantities

Switched Mode Power Conversion

Inductors





$$V = L\frac{dI}{dt}$$

$$V = N\frac{d\Phi}{dt}$$

$$LI = N\Phi$$

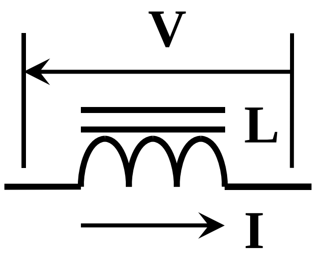
Relationship Between

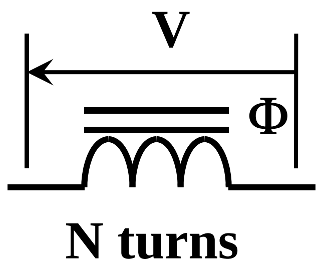
L, I & N, Φ

NΦ is also defined as Flux Linkage in the Inductor

Switched Mode Power Conversion

Inductors





$$V = L\frac{dI}{dt} \qquad L\,I = N\,\Phi \qquad V = N\frac{d\Phi}{dt}$$

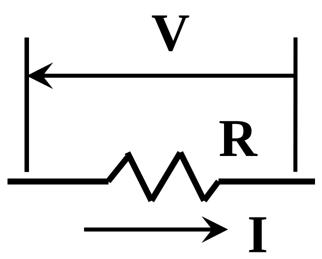
How to Relate the Electrical Circuit V, L, I

and

The Magnetic Circuit of the Inductor N, Φ, I & R

Switched Mode Power Conversion

Ohm's Law

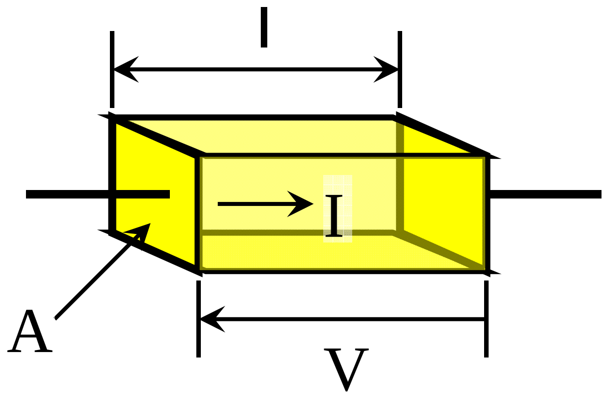


$$V = IR$$

Bulk Ohm's Law

Switched Mode Power Conversion

Ohm's Law at a Point



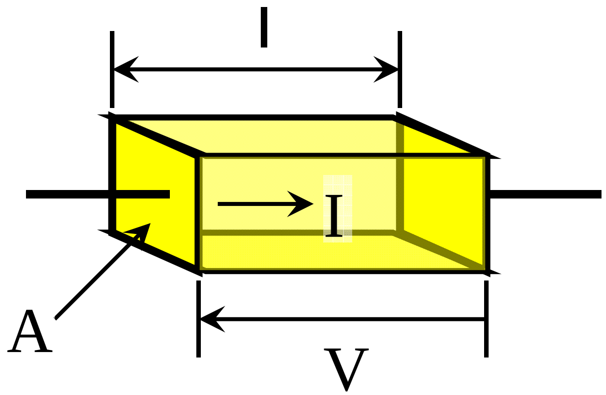
$$V = IR = JA\frac{\rho\ell}{A} = J\rho\ell$$

$$\frac{V}{\ell} = J\rho$$

$$J = \sigma\varepsilon$$

Switched Mode Power Conversion

Ohm's Law at a Point

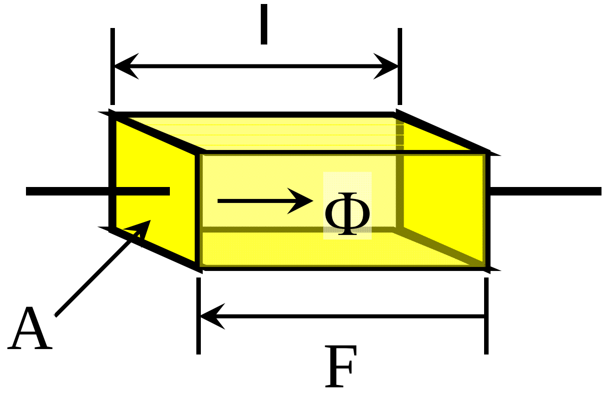


$J = \sigma E$

In a Conducting Material
Electrical Current Density (J)
is Proportional to (σ), and
Electrical Field Intensity (E)
σ is the conductivity of the material

Switched Mode Power Conversion

Magnetic Ohm's Law at a Point



$$B = \mu H$$

In a Magnetic Material

Magnetic Flux Density (B)

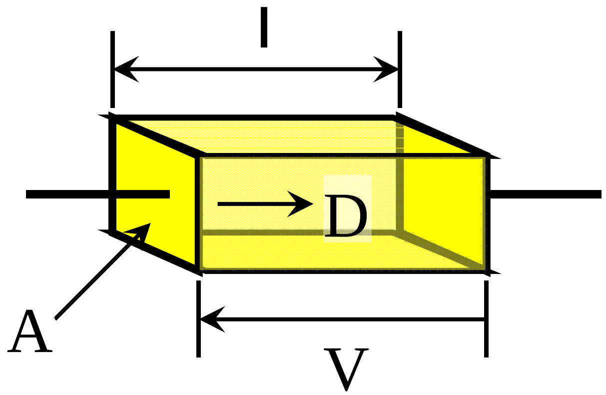
is Proportional to (μ), and

Magnetic Field Intensity (H)

μ is the permeability of the material

Switched Mode Power Conversion

Dielectric Ohm's Law at a Point



$D = \varepsilon E$

In a Dielectric Material

Dielectric Flux Density (D)

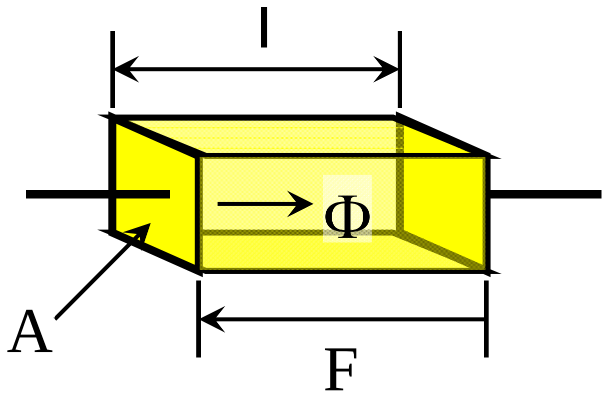
is Proportional to (ε), and

Electrical Field Intensity (E)

ε is the permittivity of the material

Switched Mode Power Conversion

Magnetic Ohm’s Law



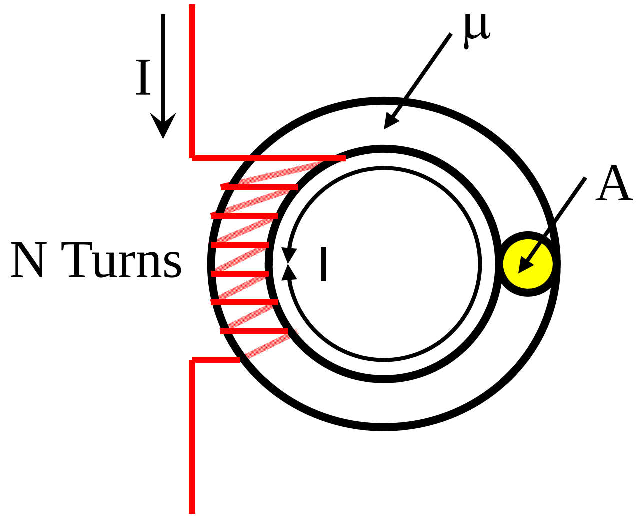
$$BA = \mu A \frac{NI}{\ell} = \Phi = \frac{NI}{\ell / A\mu} = \frac{F}{\mathcal{R}}$$

$$\Phi = \frac{F}{\mathcal{R}}$$

Bulk Magnetic Ohm’s Law

Switched Mode Power Conversion

Magnetic Circuit Relationships

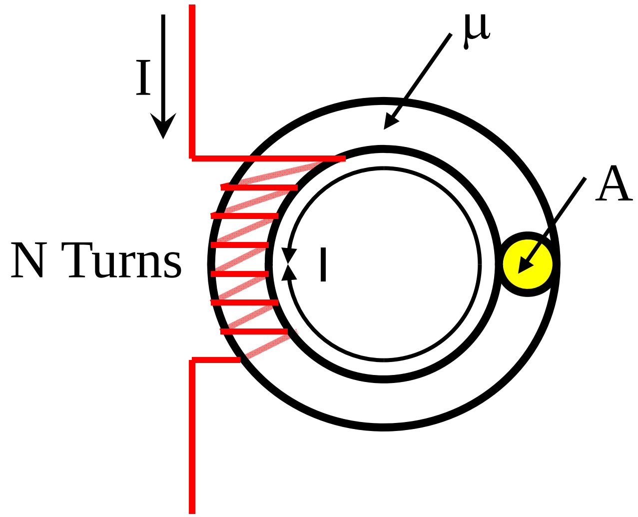


$$\Phi = \frac{F}{\mathcal{R}} = \frac{(NI)}{\left(\ell / A\mu\right)}$$

Flux in Magnetic Circuit

Switched Mode Power Conversion

Inductance of an Electromagnetic Circuit

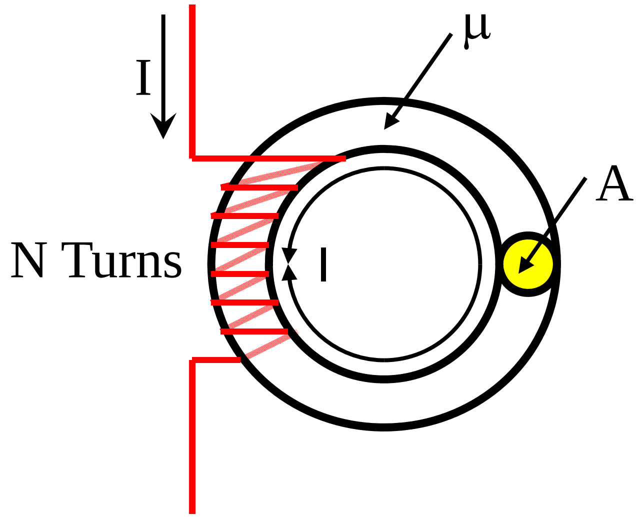


$$L = \frac{N\Phi}{I} = \frac{N^2}{\left(\ell / A\mu\right)} = \frac{N^2 A\mu}{\ell}$$

Inductance of a Magnetic Circuit

Switched Mode Power Conversion

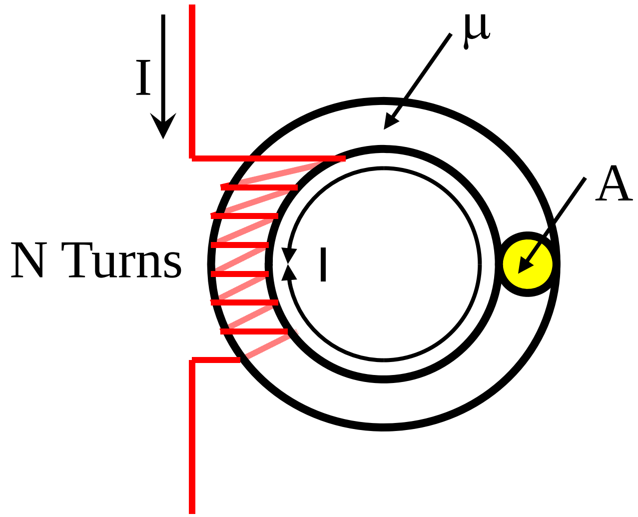
Conceptual Design of an Inductor



For the desired L and I,
To select a magnetic core (A, μ, l),
number of turns N
and conductor size a

Switched Mode Power Conversion

Design of an Inductor

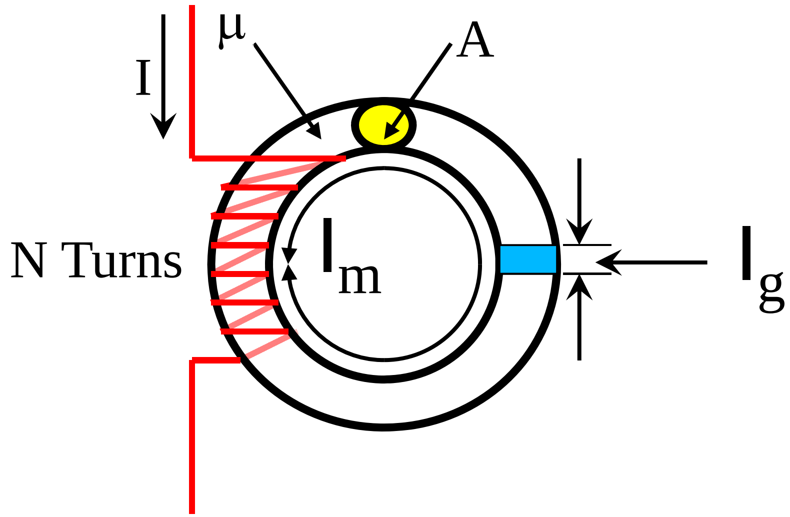


μ of a magnetic material varies widely.

l is not finely controllable.

Switched Mode Power Conversion

Practical Design of an Inductor



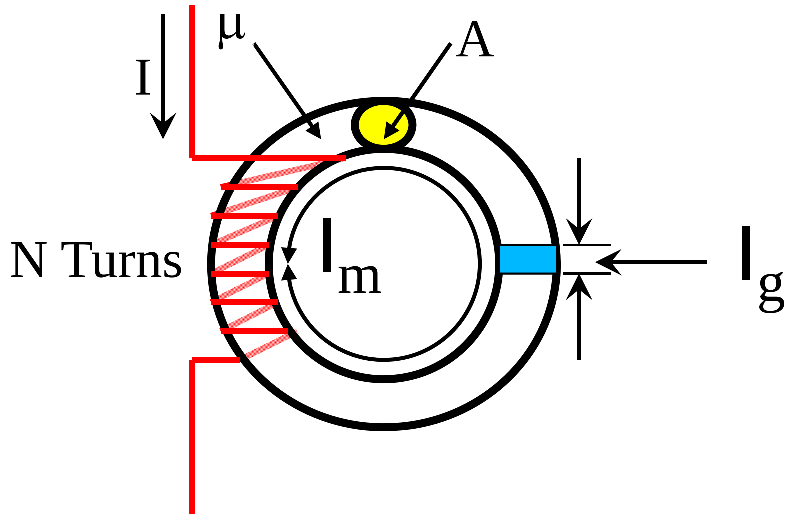
Magnetic path is made of:

Large Path of High Permeability and a

Small Controlled Path of Low Permeability (Air)

Switched Mode Power Conversion

Practical Design of an Inductor



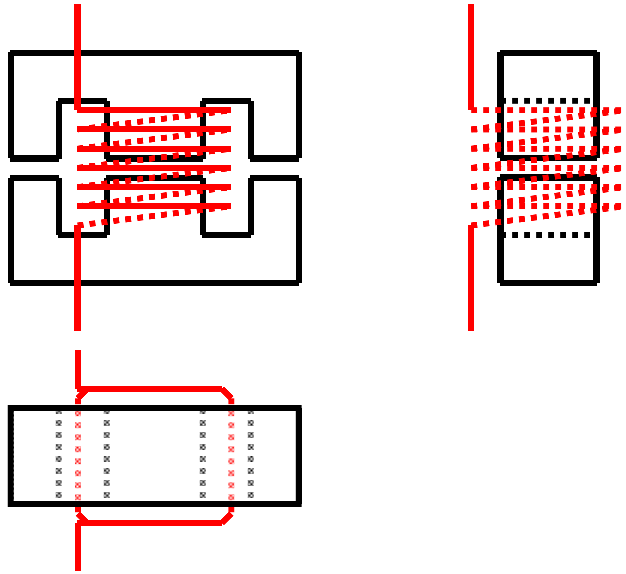
$$L = \frac{N\Phi}{I} = \frac{N^2}{\left(\ell_m / A\mu_o\mu_m + \ell_g / A\mu_o \right)} \cong \frac{N^2 A\mu_o}{\ell_g}$$

Inductance is Independent of Core Material (μ)

Inductance is Independent of Core Shape (l_m)

Switched Mode Power Conversion

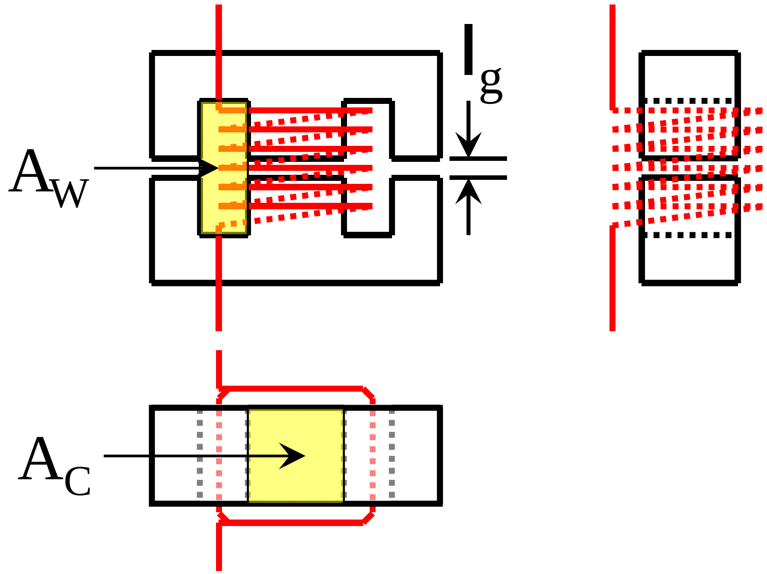
Popular Geometry of Inductor (EE)



EE Cores

Switched Mode Power Conversion

Popular Geometry of Inductor (EE)

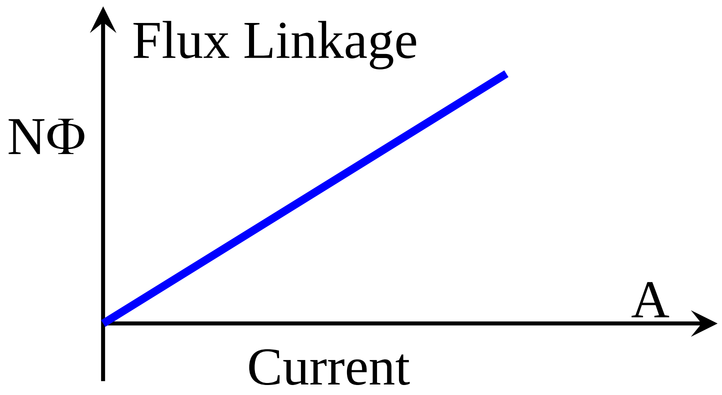


A_C is the area of Magnetic Path supporting the Flux

A_W is the area of Window Supporting the Electric Current

Switched Mode Power Conversion

Definition of Inductance

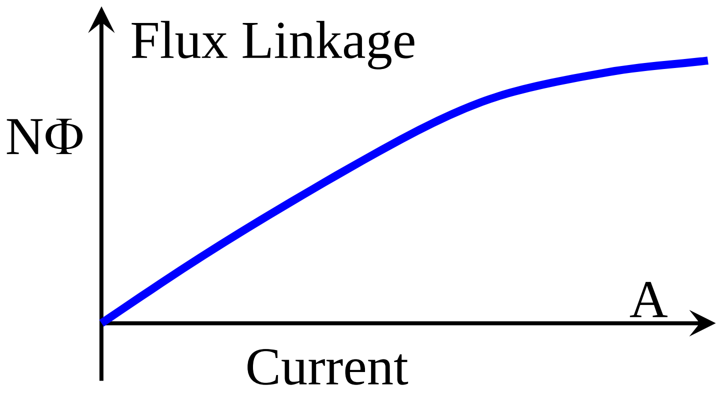


Inductance is Defined as Flux Linkage per Ampere

Slope of NΦ vs I Curve is Inductance

Switched Mode Power Conversion

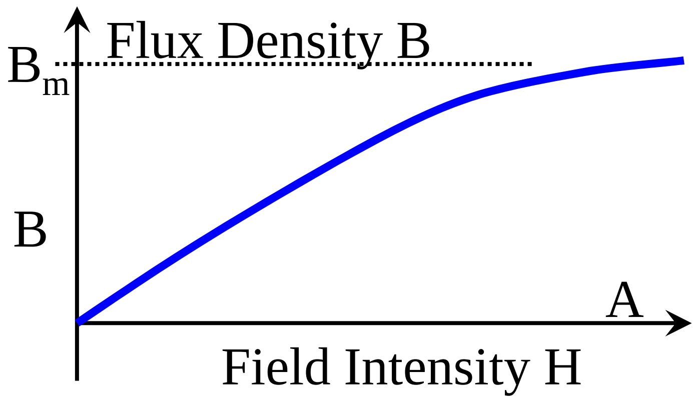
Saturation Effect



Flux Linkage Saturates in Most Magnetic Materials Beyond a Certain Level

Switched Mode Power Conversion

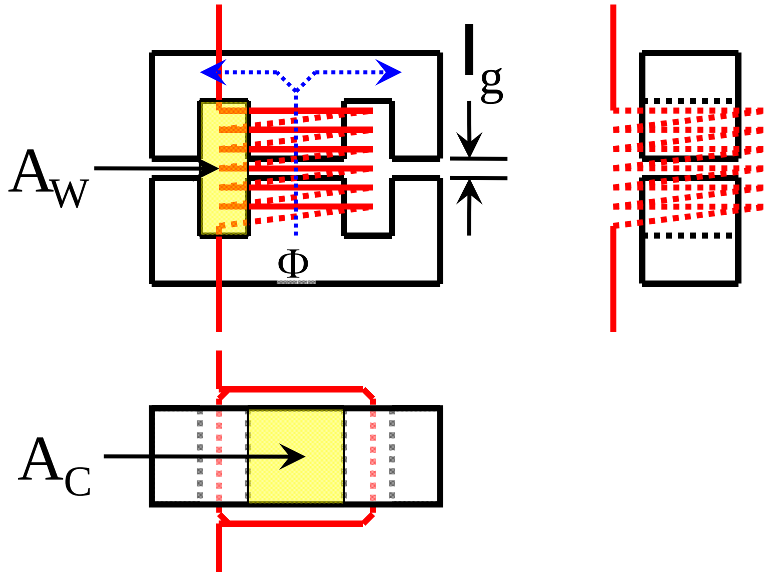
Saturation Flux Density



Peak Flux Density in Magnetic Materials is Limited by B_m

Switched Mode Power Conversion

Saturation Limit

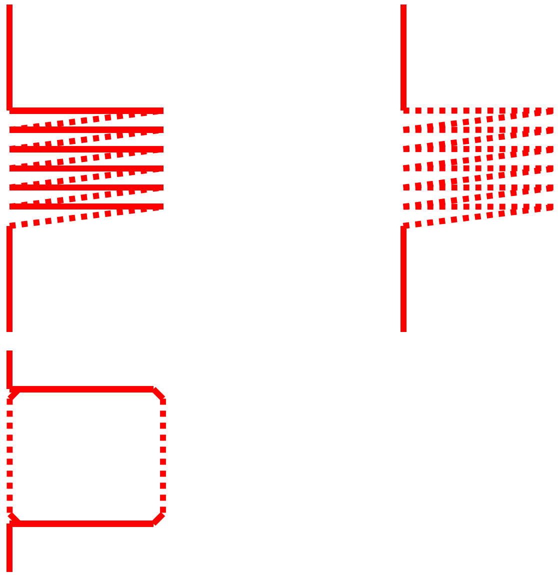


$$L\,I_m = N\,\Phi_m = N\,B_m A_C$$

Magnetic Path Constraint

Switched Mode Power Conversion

Heat Produced in the Conductors

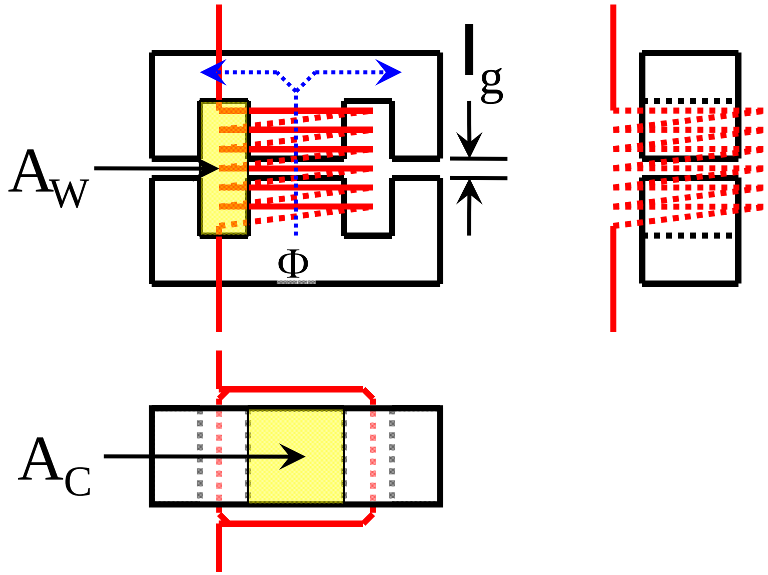


$$Q = I^2Rt$$

I^2R Heating in the Conductors

Switched Mode Power Conversion

Thermal Limit



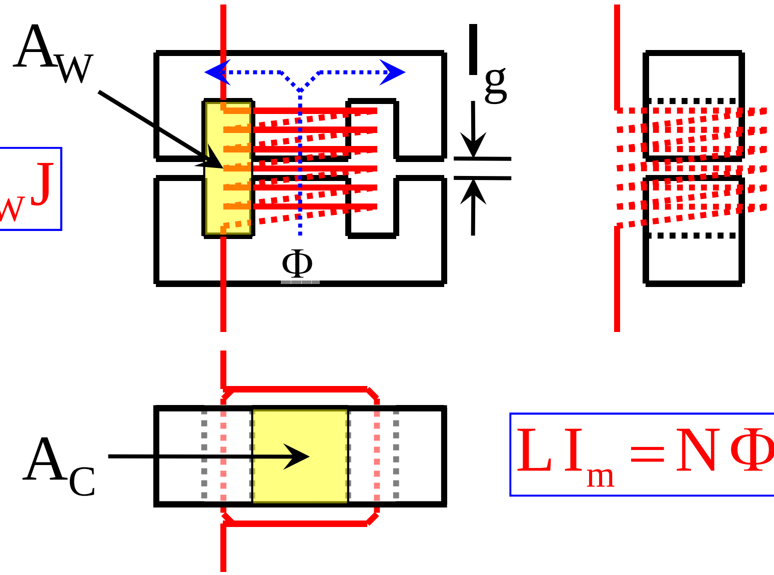
$$N\,I_{rms} = k_W A_W J$$

Current Carrying Capacity Constraint

Switched Mode Power Conversion

Energy Equation

$$\boxed{N\,I_{rms} = k_W A_W J}$$



$$\boxed{L\,I_m = N\,\Phi_m = N\,B_m A_C}$$

$$\frac{L\,I_m\,I_{rms}}{k_w B_m J} = A_C A_W$$

Energy (Area Product) Equation for Inductor Design

Switched Mode Power Conversion

Size of Magnetic Core

$$\frac{L\, I_m\, I_{rms}}{k_w B_m J} = A_C A_W$$

The Magnetic Core has to be Big Enough to Handle the Stored Energy in the Core

The Core Size is Inversely Proportional to

Saturation Flux Density B_m

Peak Current Carrying Capacity J

Window Space factor k_W

Switched Mode Power Conversion

Typical Design Constraints

The Core Size is Inversely Proportional to Saturation Flux Density B_m

B_m = 0.2 T for Ferrite Cores

Peak Current Carrying Capacity J

$J = 3\ A/mm^2$ for Copper, Natural Cooling

Window Space factor k_W

k_W = 0.35 for round Conductors

Switched Mode Power Conversion

Sample Core Selection

L = 20 μH; DC Current of 5 A;
High Frequency (20 kHz) Application

I_m = 5 A; I_{rms} = 5 A; L = 20 μH

$$A_C A_W = \frac{20\mu * 5 * 5}{0.35 * 0.2 * 3 * 10^6} = 2381 \text{mm}^4$$

Switched Mode Power Conversion

Sample Core Selection

Type Number	A_C	A_W	$A_C A_W$	
	mm^2	mm^2	mm^4	
E16/8/5	20.1	37.6	755.8	
E21/9/5	21.6	66.0	1425.6	
E20/10/6	32.1	57.4	1842.5	
E25.4/10/7	38.2	80.0	3056.0	
E25/13/7	52.5	87.0	4567.5	

Core Data Sheet

Select Core E25.4/10/7

Switched Mode Power Conversion

No. of Turns

Type Number	A_C mm^2	A_W mm^2	$A_C A_W$ mm^4
E25.4/10/7	38.2	80.0	3056.0
L	2.00E-05		
I	5		
N	13		

$$N = \frac{L\,I}{B_m A_C} = \frac{20*10^{-6}*5}{0.2*38.2*10^{-6}}$$

Select 13 Turns

Switched Mode Power Conversion

Wire Size

Type Number	A_C mm^2	A_W mm^2	$A_C A_W$ mm^4
E25.4/10/7	38.2	80.0	3056.0
L	2.00E-05		
I	5		
N	13		
a_W mm^2	1.67		

$$a_W = \frac{I}{J} = \frac{5}{3} = 1.67\,mm^2$$

Select a_W 1.67 mm^2 which is 16 SWG

Switched Mode Power Conversion

Air Gap

Type Number	A_C mm^2	A_W mm^2	$A_C A_W$ mm^4
E25.4/10/7	38.2	80.0	3056.0
L	2.00E-05		
I	5		
N	13		
a_W mm^2	1.67		
l_g mm	0.41		

$$\ell_g = \frac{N^2 A \mu}{L}$$

Select Airgap of 0.205 mm on each limb

Switched Mode Power Conversion

A Few Other Geometries (EE)

Low Profile Cores

ELP

Low Profile Cores

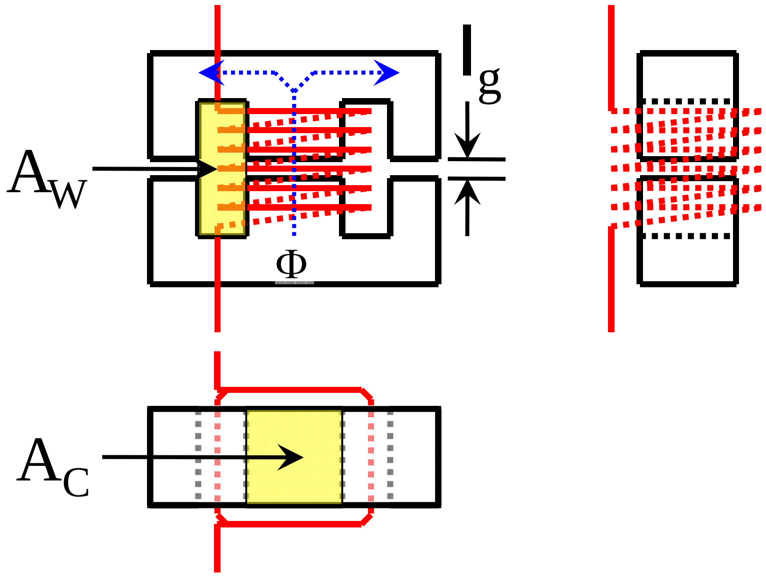
EFD

Pot Cores

P

Switched Mode Power Conversion

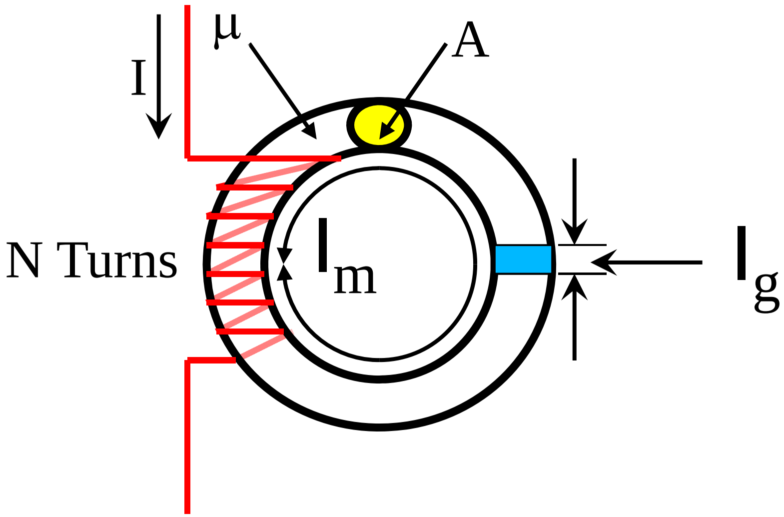
Back to the Design



L = 20 μH; DC Current of 5 A;
13 Turns of 16 SWG
Core Type E25.4/10/7
Airgap 0.205 mm x 2

Switched Mode Power Conversion

Practical Design of an Inductor



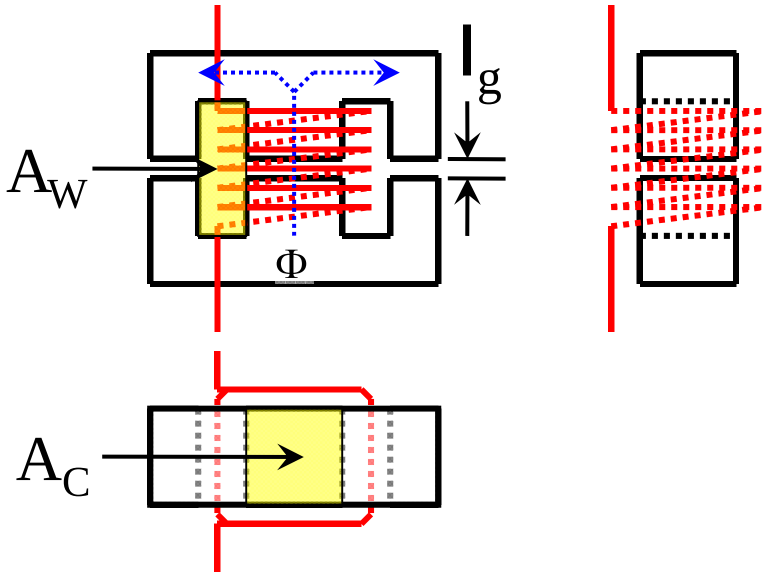
$$L = \frac{N\Phi}{I} = \frac{N^2}{\left(\ell_m / A\mu_o\mu_m + \ell_g / A\mu_o \right)} \cong \frac{N^2 A\mu_o}{\ell_g}$$

Inductance is Independent of Core Material (μ)

Inductance is Independent of Core Shape (l_m)

Switched Mode Power Conversion

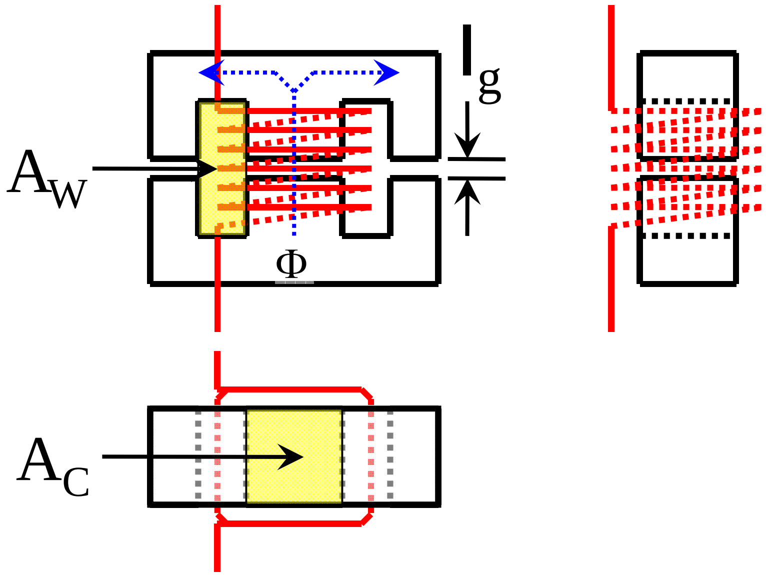
Back to the Design



Assumptions:
Core Reluctance << Gap Reluctance
Fringing Field at Gap is Negligible

Switched Mode Power Conversion

Core Reluctance : Gap Reluctance

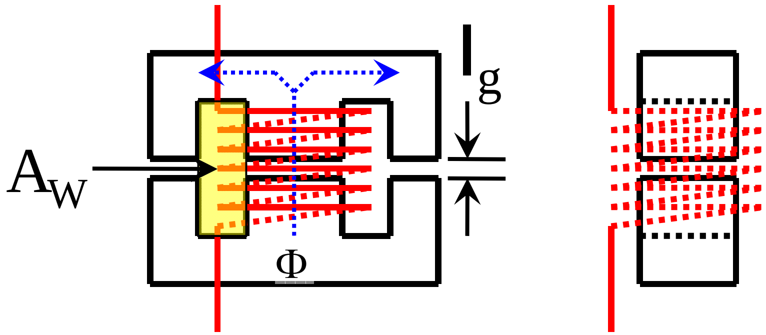


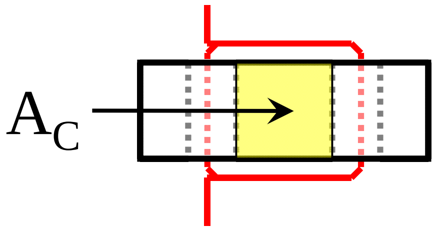
$$\left(\ell_m / \mu_m \ll \ell_g \right)$$

$$\frac{47.2}{1510} = 0.031 \ll 0.205$$

Switched Mode Power Conversion

Fringing Effect



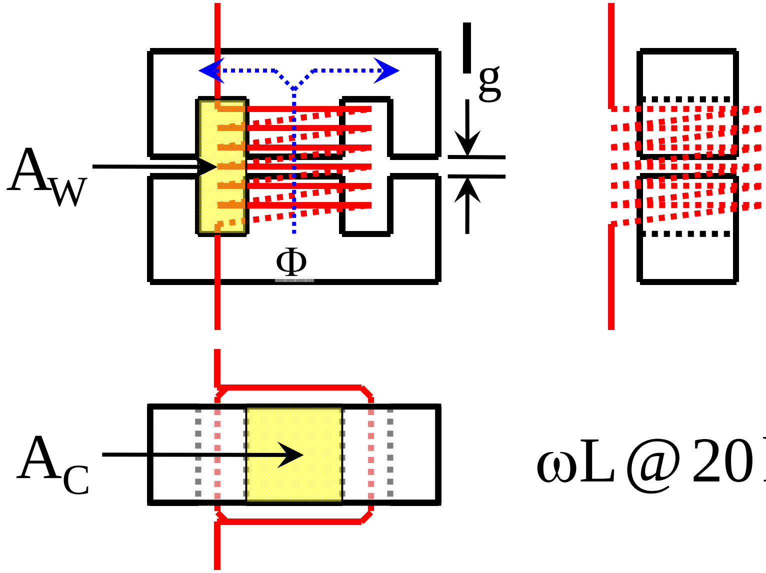


$$\left(\ell_g << \sqrt{A_C} \right)$$

$$0.205 << 6.2$$

Switched Mode Power Conversion

Parasitic Resistance of Inductor



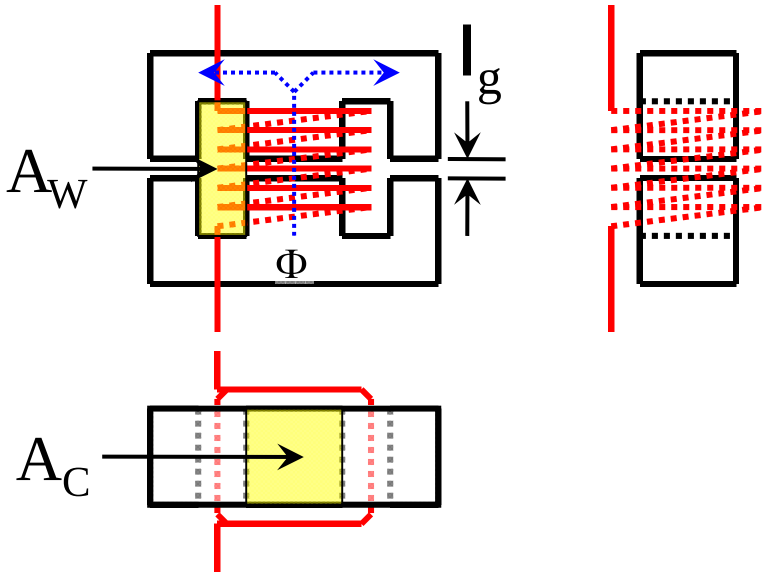
$\omega L @ 20\,kHz$ is 2.51Ω

$$\left(R_\ell = \frac{\rho L_W}{a_W} \right)$$

$$R_\ell = \frac{1.76*10^{-6}*13*40*10^{-1}}{1.67*10^{-2}}\Omega = 5.5\,m\Omega$$

Switched Mode Power Conversion

Losses in the Inductor



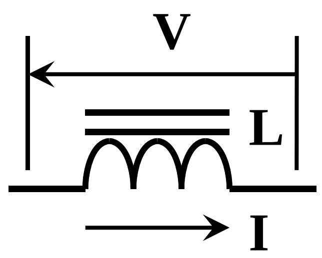
0.36 W per set

If the DC Output Voltage is 7.5 V,

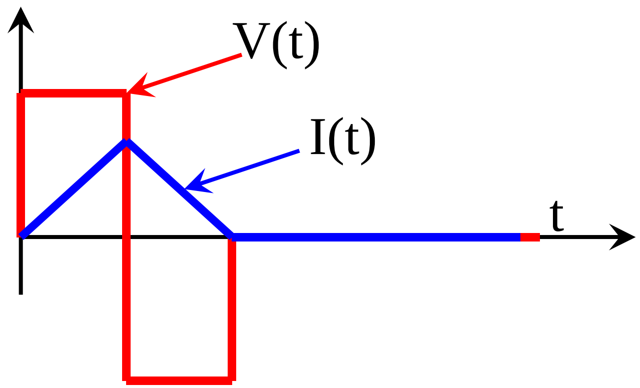
This loss is 0.1%.

Switched Mode Power Conversion

Measurement of L



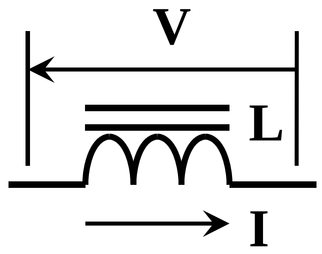
$$V = L\frac{dI}{dt}$$



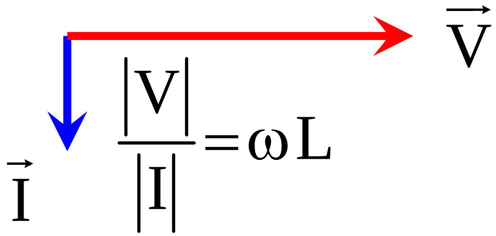
Pulsed Voltage and Current Rise

Switched Mode Power Conversion

Measurement of L with LCR Meter



$$\vec{I} = \frac{\vec{V}}{j\omega L}$$

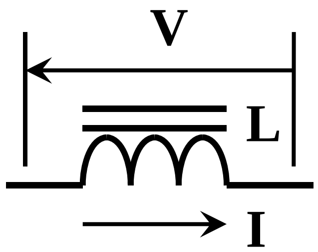


$$\frac{|V|}{|I|} = \omega L$$

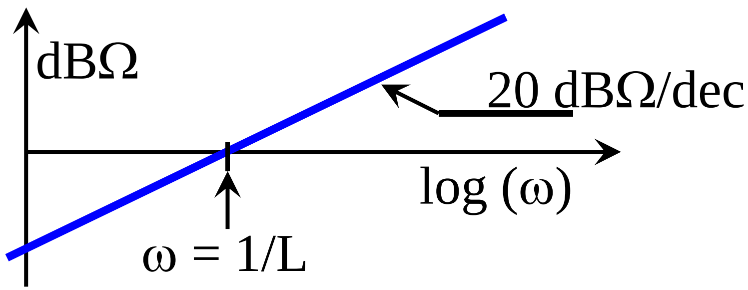
Sinusoidal Voltage and Current

Switched Mode Power Conversion

Impedance as a Function of ω



$$\vec{Z} = \frac{\vec{V}}{\vec{I}} = j\omega L$$



Impedance Plot [dBΩ vs log (ω)]

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