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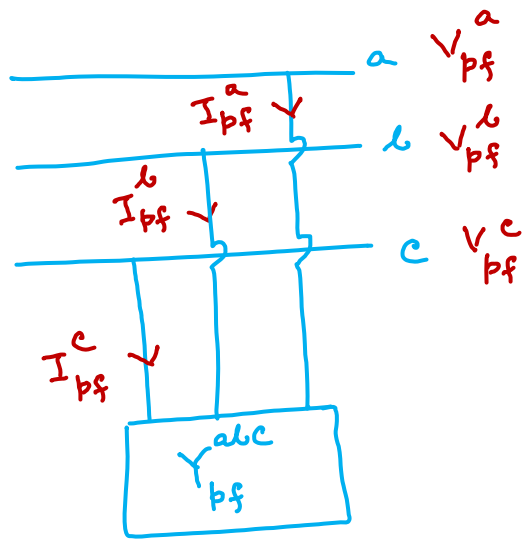
Computer aided power system analysis

Fault Analysis (Contd.)

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bus 'p'

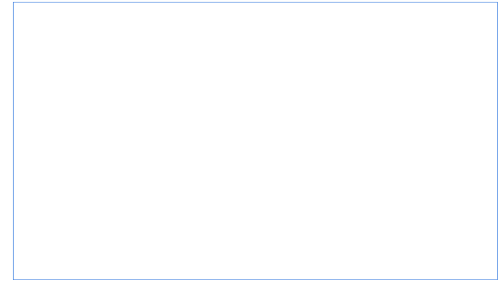


$$\text{Let } V_{pf}^{abc} = \begin{bmatrix} V_{pf}^a & V_{pf}^b & V_{pf}^c \end{bmatrix}^T \rightarrow (3 \times 1)$$

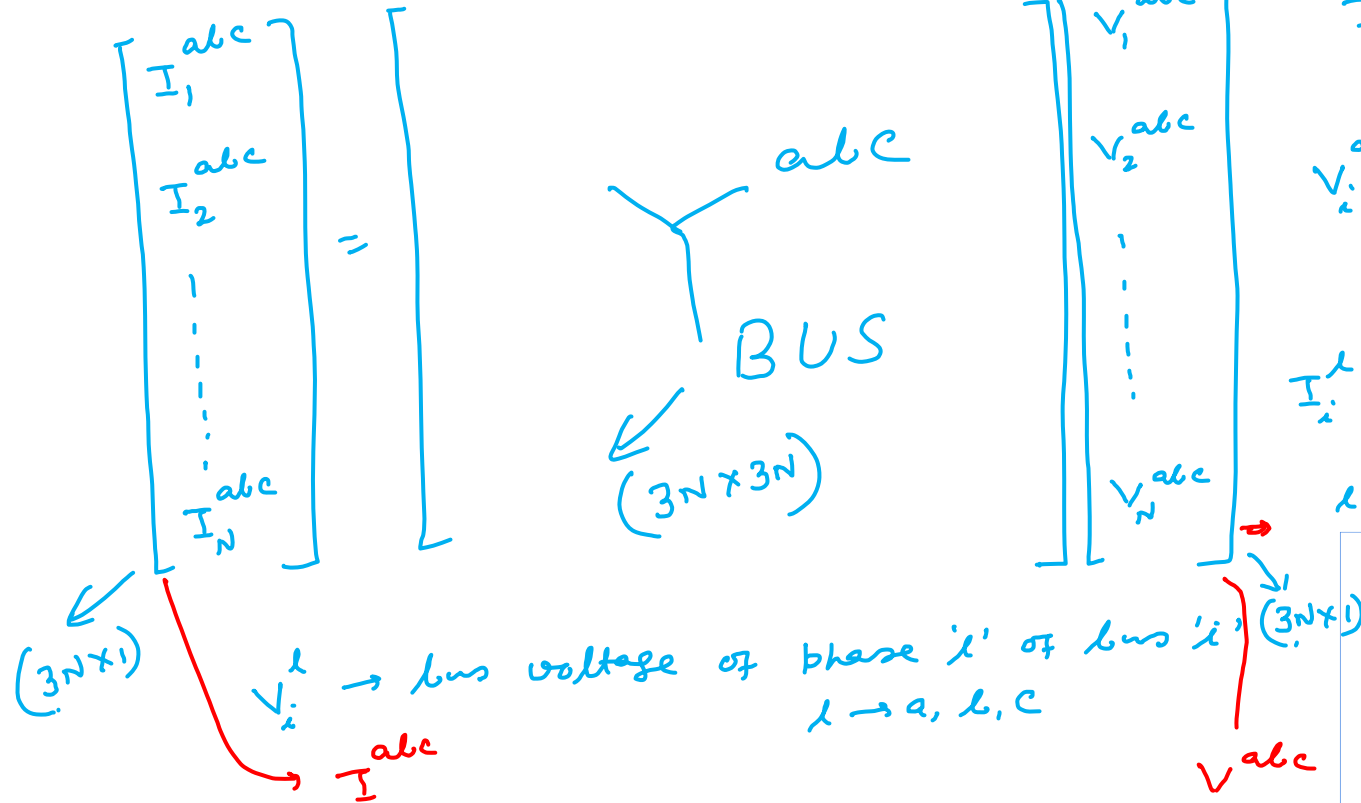
$$I_{pf}^{abc} = \begin{bmatrix} I_{pf}^a & I_{pf}^b & I_{pf}^c \end{bmatrix}^T \rightarrow (3 \times 1)$$

$$I_{pf}^{abc} = Y_{pf}^{abc} V_{pf}^{abc}$$

$\begin{matrix} \swarrow & \swarrow & \swarrow \\ (3 \times 1) & (3 \times 3) & (3 \times 1) \end{matrix}$



Now, for a N-Bus three-phase system,



$$I_i^{abc} = \begin{bmatrix} I_i^a & I_i^b & I_i^c \end{bmatrix}^T \rightarrow (3 \times 1)$$

$$V_i^{abc} = \begin{bmatrix} V_i^a & V_i^b & V_i^c \end{bmatrix}^T \rightarrow (3 \times 1)$$

$i = 1, 2, \dots, N$

$I_i^l \rightarrow$ injected current at phase i' of bus i'
 $l \rightarrow a, b, c$

$$\begin{matrix} \swarrow \\ (3N+1) \end{matrix} I^{abc} = Y_{BUS}^{abc} \begin{matrix} \searrow \\ (3N \times 3N) \end{matrix} V^{abc} \rightarrow (3N \times 1)$$

$$\Rightarrow V^{abc} = Z_{BUS}^{abc} I^{abc}$$

$$\begin{matrix} \swarrow \\ (3N \times 3N) \end{matrix} Z_{BUS}^{abc} = [Y_{BUS}^{abc}]^{-1}$$

Bus impedance matrix
(Three-phase)

$$\begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ \vdots \\ V_N^{abc} \end{bmatrix} = \begin{bmatrix} Z_{11}^{abc} & Z_{12}^{abc} & \dots & Z_{1N}^{abc} \\ Z_{21}^{abc} & Z_{22}^{abc} & \dots & Z_{2N}^{abc} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1}^{abc} & Z_{N2}^{abc} & \dots & Z_{NN}^{abc} \end{bmatrix} \begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ \vdots \\ I_N^{abc} \end{bmatrix}$$

$\begin{matrix} \swarrow \\ (3 \times 3) \end{matrix} Z_{pi}^{abc} \rightarrow (3 \times 3) \quad \forall p, i$



$$\Rightarrow V_i^{abc} = Z_{i1}^{abc} I_1^{abc} + Z_{i2}^{abc} I_2^{abc} + \dots + Z_{iN}^{abc} I_N^{abc} \quad \forall i=1, \dots, N$$

Suppose a fault occurs at bus $p \rightarrow I_p^{abc}$ will flow (away from the bus)

$$\begin{bmatrix} V_{1f}^{abc} \\ \vdots \\ V_{pf}^{abc} \\ \vdots \\ V_{Nf}^{abc} \end{bmatrix} = \begin{bmatrix} Z_{11}^{abc} \\ \vdots \\ Z_{p1}^{abc} \\ \vdots \\ Z_{N1}^{abc} \end{bmatrix} \begin{bmatrix} Z_{12}^{abc} & \dots & Z_{1N}^{abc} \\ \vdots & \ddots & \vdots \\ Z_{p2}^{abc} & \dots & Z_{pN}^{abc} \\ \vdots & \ddots & \vdots \\ Z_{N2}^{abc} & \dots & Z_{NN}^{abc} \end{bmatrix} \begin{bmatrix} I_1^{abc} \\ \vdots \\ I_p^{abc} - I_{pf}^{abc} \\ \vdots \\ I_N^{abc} \end{bmatrix}$$

$$\Rightarrow V_{if}^{abc} = Z_{i1}^{abc} I_1^{abc} + Z_{i2}^{abc} I_2^{abc} + \dots + Z_{ip}^{abc} (I_p^{abc} - I_{pf}^{abc}) + \dots + Z_{iN}^{abc} I_N^{abc}$$



$$\Rightarrow V_{if}^{abc} = V_i^{abc} - Z_{ip}^{abc} I_{pf}^{abc}$$

$$\forall i = 1, 2, \dots, p, \dots, N$$

Now, at bus p :

$$V_{pf}^{abc} = V_p^{abc} - Z_{pp}^{abc} I_{pf}^{abc}$$

$$= V_p^{abc} - Z_{pp}^{abc} Y_{pf}^{abc} V_{pf}^{abc}$$

(3x3) identity matrix

$$\Rightarrow V_{pf}^{abc} \left[I_3 + Z_{pp}^{abc} Y_{pf}^{abc} \right] = V_p^{abc}$$

$$\Rightarrow V_{pf}^{abc} = \left[I_3 + Z_{pp}^{abc} Y_{pf}^{abc} \right]^{-1} V_p^{abc}$$

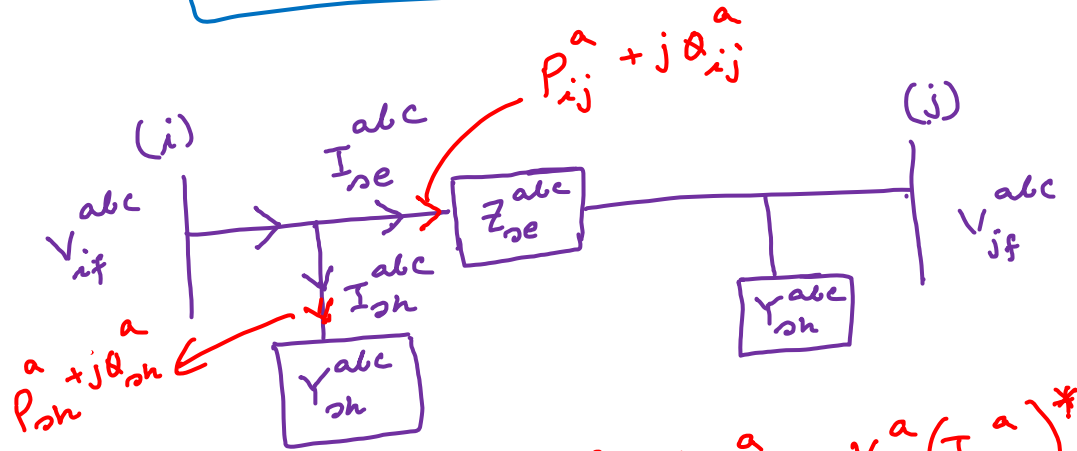
$$\Rightarrow I_{pf}^{abc} = Y_{pf}^{abc} \left[I_3 + Z_{pp}^{abc} Y_{pf}^{abc} \right]^{-1} V_p^{abc}$$



$\Rightarrow$

$$V_{if}^{abc} = V_i^{abc} - Z_{ip}^{abc} Y_{pf}^{abc} \left[I_3 + Z_{pp}^{abc} Y_{pf}^{abc} \right]^{-1} V_p^{abc}$$

$\forall i=1, \dots, N$



$\Rightarrow$ Calculate all line currents under the fault condition.

$$P_{ij}^a + jQ_{ij}^a = V_{if}^a (I_{se}^a)^*$$

$$P_{sh}^a + jQ_{sh}^a = V_{if}^a (I_{sh}^a)^*$$

