IIT ROORKEE

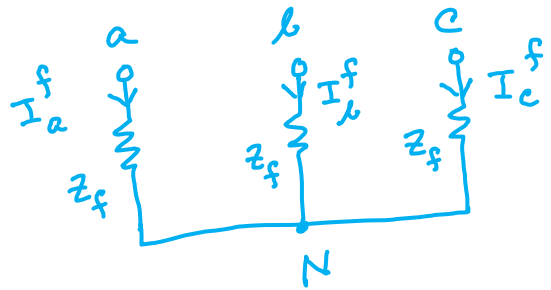
NPTEL ONLINE CERTIFICATION COURSE

Computer aided power system analysis

Fault Analysis (Contd.)

Prof. Biswarup Das
Department of Electrical Engineering

L-L-L fault.



Applying KCL at point 'N'

$$I_a^f + I_b^f + I_c^f = 0$$

$$\frac{V_a - V_N}{Z_f} + \frac{V_b - V_N}{Z_f} + \frac{V_c - V_N}{Z_f} = 0$$

$$\Rightarrow V_N = \frac{1}{3}(V_a + V_b + V_c)$$

$$\Rightarrow \frac{1}{Z_f}(V_a + V_b + V_c) = \frac{3}{Z_f} V_N$$

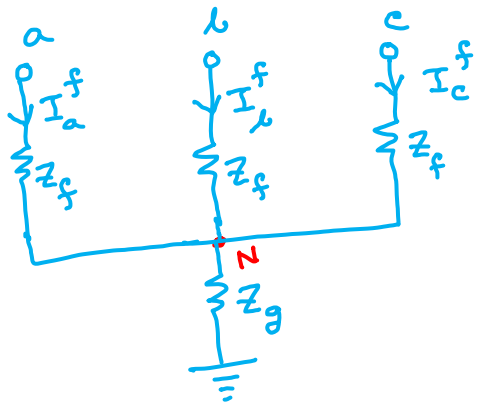
$$I_a^f = \frac{1}{Z_f}\left[V_a - \frac{1}{3}(V_a + V_b + V_c)\right] = \frac{2V_a - V_b - V_c}{3Z_f}$$

$$I_b^f = \frac{1}{Z_f}\left[V_b - \frac{1}{3}(V_a + V_b + V_c)\right] = \frac{-V_a + 2V_b - V_c}{3Z_f}$$

$$I_c^f = \frac{1}{Z_f}\left[V_c - \frac{1}{3}(V_a + V_b + V_c)\right] = \frac{-V_a - V_b + 2V_c}{3Z_f}.$$

$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \frac{y_f}{3}\underbrace{\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}}_{Y_{fault}}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \therefore \quad y_f = \frac{1}{Z_f}.$$

L-L-L-G fault.



Applying KCL at point N

$$\frac{V_a - V_N}{Z_f} + \frac{V_b - V_N}{Z_f} + \frac{V_c - V_N}{Z_f} = \frac{V_N}{Z_g}$$

$$\Rightarrow \frac{1}{Z_f}(V_a + V_b + V_c) = \left(\frac{3}{Z_f} + \frac{1}{Z_g}\right) V_N = \frac{3Z_g + Z_f}{Z_g Z_f} V_N$$

$$\Rightarrow V_N = \frac{Z_g}{3Z_g + Z_f}(V_a + V_b + V_c)$$

$$I_a^f = \frac{1}{Z_f}\left[V_a - \frac{Z_g}{3Z_g + Z_f}(V_a + V_b + V_c)\right]$$

$$\Rightarrow I_a^f = \frac{1}{Z_f}\left[\frac{(3Z_g + Z_f)V_a - Z_g V_a - Z_g V_b - Z_g V_c}{(3Z_g + Z_f)}\right]$$

$$\Rightarrow I_a^f = \frac{1}{Z_f}\left[\frac{(2Z_g + Z_f)V_a - Z_g V_b - Z_g V_c}{3Z_g + Z_f}\right] \quad \text{----(1)}$$

Now, Let $y_0 = \frac{1}{3Z_g + Z_f}$; $y_f = \frac{1}{Z_f}$

Then, $y_0 + 2y_f = \frac{1}{3Z_g + Z_f} + \frac{2}{Z_f} = \frac{Z_f + 6Z_g + 2Z_f}{Z_f(3Z_g + Z_f)} = \frac{3(2Z_g + Z_f)}{Z_f(3Z_g + Z_f)}$ --(2)

and $y_0 - y_f = \frac{1}{3Z_g + Z_f} - \frac{1}{Z_f} = \frac{Z_f - 3Z_g - Z_f}{Z_f(3Z_g + Z_f)} = \frac{-3Z_g}{Z_f(3Z_g + Z_f)}$

--(3)

From eqn (1)

$$I_a^f = \frac{2z_g + z_f}{z_f(3z_g + z_f)} V_a - \frac{z_g V_b}{z_f(3z_g + z_f)} - \frac{z_g V_c}{z_f(3z_g + z_f)}$$

$$= \frac{y_0 + 2y_f}{3} V_a + \frac{y_0 - y_f}{3} V_b + \frac{y_0 - y_f}{3} V_c \quad \text{--- (4)}$$

[utilizing eqn: (2) and (3)].

$$I_b^f = \frac{1}{z_f}\left[V_b - \frac{z_g}{3z_g + z_f}(V_a + V_b + V_c) \right]$$

$$= \frac{1}{z_f}\left[\frac{-z_g V_a + (2z_g + z_f) V_b - z_g V_c}{3z_g + z_f} \right] = \frac{y_0 - y_f}{3} V_a + \frac{y_0 + 2y_f}{3} V_b + \frac{y_0 - y_f}{3} V_c \quad \text{--- (5)}$$

$$I_c^f = \frac{1}{z_f}\left[V_c - \frac{z_g}{3z_g + z_f}(V_a + V_b + V_c) \right]$$

$$\Rightarrow I_c^f = \frac{1}{Z_f}\left[\frac{-Z_g V_a - Z_g V_b + (2Z_g + Z_f)V_c}{3Z_g + Z_f}\right]$$

$$\Rightarrow I_c^f = \frac{y_0 - y_f}{3} V_a + \frac{y_0 - y_f}{3} V_b + \frac{y_0 + 2y_f}{3} V_c \quad \text{----} (6)$$

From equations (4), (5) and (6),

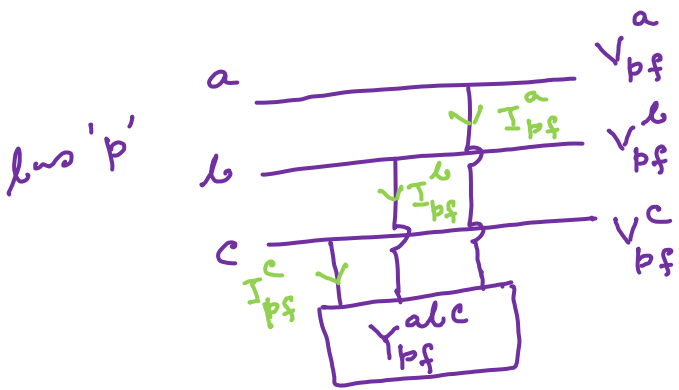
$$\begin{bmatrix} I_a^f \\ I_b^f \\ I_c^f \end{bmatrix} = \frac{1}{3}\begin{bmatrix} y_0 + 2y_f & y_0 - y_f & y_0 - y_f \\ y_0 - y_f & y_0 + 2y_f & y_0 - y_f \\ y_0 - y_f & y_0 - y_f & y_0 + 2y_f \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Y_{fault}

Any fault (L-G, L-L, L-L-G, L-L-L, L-L-L-G) can be represented by a (3×3) fault admittance matrix (Y_{fault})

Let, Y_{fault} be denoted as Y^{abc}_{pf}

- abc → denotes that it is a (3×3) matrix involving all phases 'a', 'b' and 'c'.
- p → stands for the bus at which the fault takes place
- f → stands for 'fault'



V^l_{pf} → fault voltage at bus 'p' corresponding to phase 'l' where 'l' = a, b, c

I^l_{pf} → fault current at bus 'p' corresponding to phase 'l' $l = a, b, c$