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NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

Fault Analysis (Contd.)

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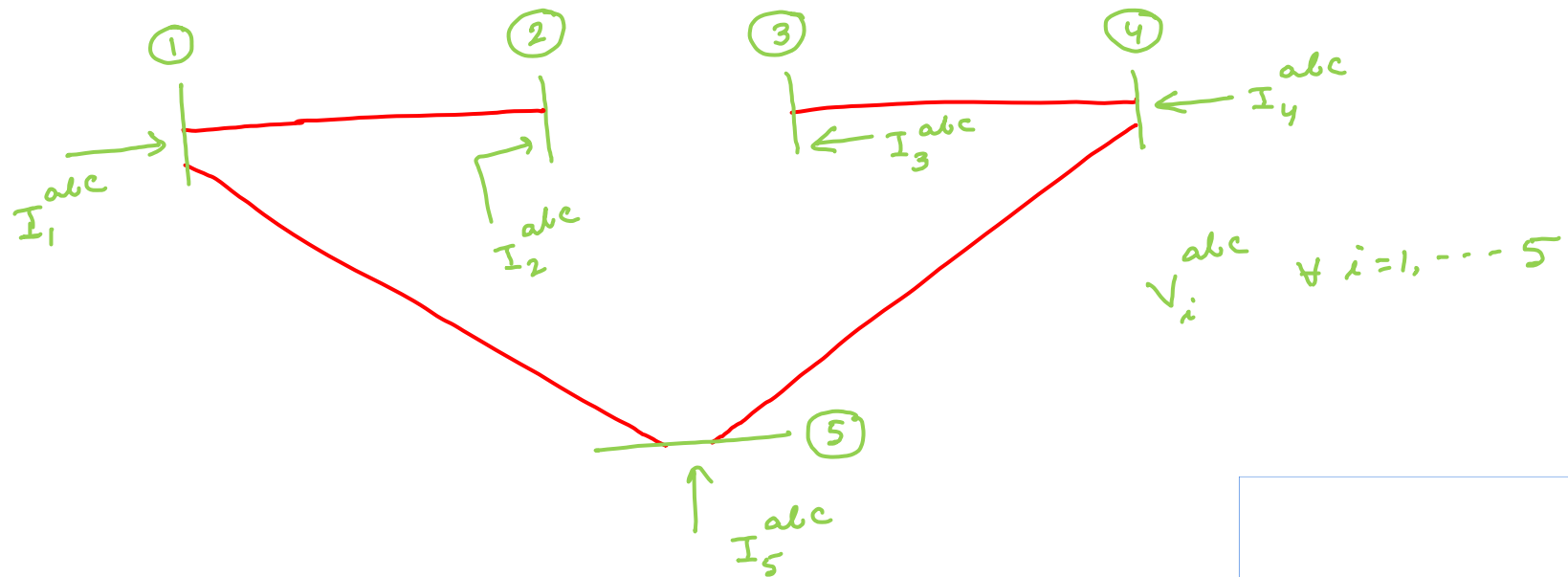
In matrix form

$$\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ I_3^{abc} \\ I_4^{abc} \\ I_5^{abc} \end{bmatrix} = \begin{bmatrix} y_{11}^{abc} & -y_{12}^{abc} & 0 & 0 & -y_{15}^{abc} \\ -y_{12}^{abc} & Y_I + y_{12}^{abc} + y_{12,sh}^{abc} & Y_{II} & 0 & 0 \\ 0 & Y_{III} & Y_{IV} + y_{34}^{abc} + y_{34,sh}^{abc} & -y_{34}^{abc} & 0 \\ 0 & 0 & -y_{34}^{abc} & y_{44}^{abc} & -y_{45}^{abc} \\ -y_{15}^{abc} & 0 & 0 & -y_{45}^{abc} & y_{55}^{abc} \end{bmatrix} \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ V_3^{abc} \\ V_4^{abc} \\ V_5^{abc} \end{bmatrix} \rightarrow (15 \times 1)$$

$\downarrow (15 \times 1)$ $\downarrow (15 \times 15)$ Y_{bus}



Reconsider the system by removing the transformer



Write down the Y_{BUS} matrix of this reduced system.

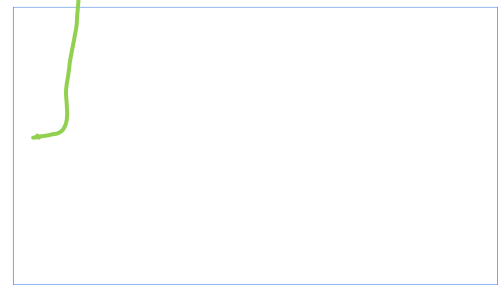


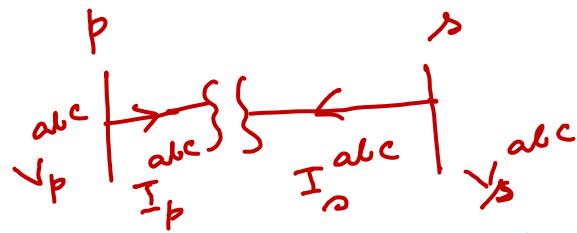
$$\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ I_3^{abc} \\ I_4^{abc} \\ I_5^{abc} \end{bmatrix} = \begin{bmatrix} y_{11}^{abc} & -y_{12}^{abc} & 0 & 0 & -y_{15}^{abc} \\ -y_{12}^{abc} & y_{12}^{abc} + y_{12,2h}^{abc} * Y_{II} & 0 & 0 & 0 \\ 0 & * Y_{III} & y_{34}^{abc} + y_{34,2h}^{abc} * Y_{IV} & -y_{34}^{abc} & 0 \\ 0 & 0 & -y_{34}^{abc} & y_{44}^{abc} & -y_{45}^{abc} \\ -y_{15}^{abc} & 0 & 0 & -y_{45}^{abc} & y_{55}^{abc} \end{bmatrix} \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ V_3^{abc} \\ V_4^{abc} \\ V_5^{abc} \end{bmatrix}$$

(15×1) (15×15) $22, 23, 32, 33$ Y_{red} BUS

$p = 2$
 $n = 3$
 $p p, p n, n p, n n$
 $\swarrow \quad \swarrow \quad \swarrow \quad \swarrow$
 $Y_I \quad Y_{II} \quad Y_{III} \quad Y_{IV}$

$\rightarrow (15 \times 1)$



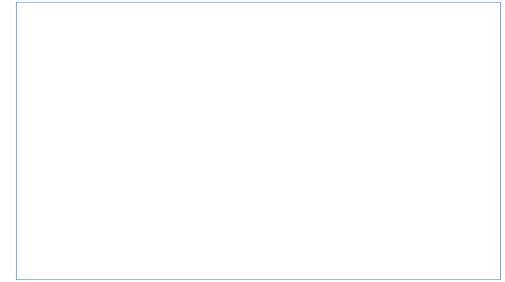


$p \rightarrow \text{primary}, s \rightarrow \text{secondary}.$

$$\begin{bmatrix} I_p^{abc} \\ I_s^{abc} \end{bmatrix} = \begin{bmatrix} Y_I \xrightarrow{p/s} \\ Y_{III} \xrightarrow{s/p} \end{bmatrix} \begin{bmatrix} Y_{II} \xrightarrow{p/s} \\ Y_{IV} \xrightarrow{s/p} \end{bmatrix} \begin{bmatrix} V_p^{abc} \\ V_s^{abc} \end{bmatrix}$$

$\left\{ \begin{array}{l} p/p, p/s, s/p, s/s \\ p'p', p's', s'p', s's' \end{array} \right\}$
 $N \rightarrow \text{Bus}$
 $T \rightarrow \text{Transformer}$

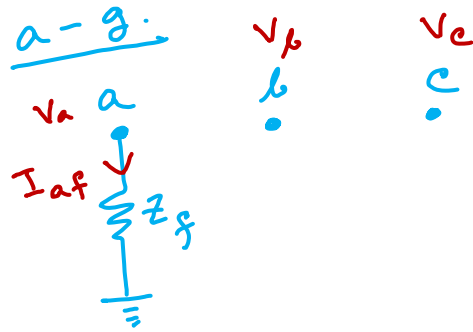
Let there are two Transformers
 p, s p', s'



Fault admittance matrices of different types of fault.

L-G	a-g, b-g, c-g	a-g
L-L	a-b, b-c, c-a	b-c
L-L-G	a-b-g, b-c-g, c-a-g	b-c-g
L-L-L	a-b-c ✓	a-b-c
L-L-L-G	a-b-c-g ✓	a-b-c-g





$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

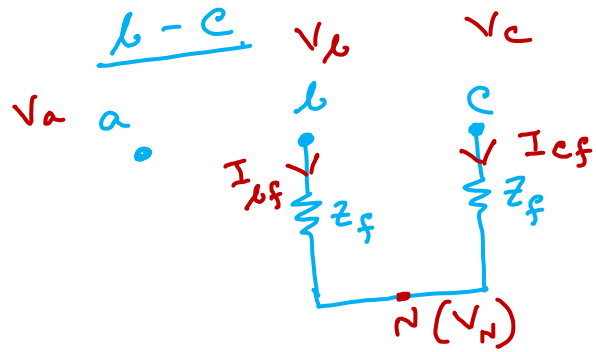
$$I_{bf} = 0 ; I_{cf} = 0$$

$$I_{af} = \frac{V_a}{Z_f} = Y_f V_a$$

$$Y_f = \frac{1}{Z_f}$$

$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \begin{bmatrix} Y_f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Fault admittance matrix



$I_{af} = 0$
Applying KCL at point N

$$I_{bf} + I_{cf} = 0$$

$$\frac{V_b - V_N}{Z_f} + \frac{V_c - V_N}{Z_f} = 0 \Rightarrow \cancel{2} \cancel{Z_f} (V_b - V_N) + \cancel{2} \cancel{Z_f} (V_c - V_N) = 0$$

$$\Rightarrow \cancel{2} \cancel{Z_f} (V_b + V_c) = 2 \cancel{Z_f} V_N \Rightarrow V_N = \frac{1}{2} (V_b + V_c)$$

$$I_{bf} = \cancel{Z_f} (V_b - V_N) = \cancel{Z_f} \left[V_b - \frac{1}{2} (V_b + V_c) \right]$$

$$= \frac{\cancel{Z_f}}{2} (V_b - V_c)$$

$$I_{cf} = \cancel{Z_f} (V_c - V_N) = \cancel{Z_f} \left[V_c - \frac{1}{2} (V_b + V_c) \right] = \frac{\cancel{Z_f}}{2} (V_c - V_b)$$

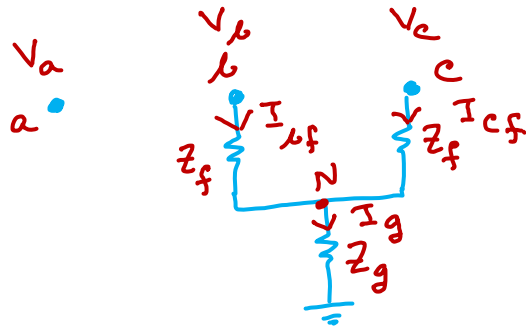


$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \frac{y_f}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Y_{fault}

↓ Fault admittance matrix = Y_{fault}

b-c-g



$$\begin{aligned} I_{af} &= 0 \\ \text{at point } N \\ I_{bf} + I_{cf} &= I_g \end{aligned}$$

$$\Rightarrow \partial_f (V_b - V_N) + \partial_f (V_c - V_N) = \partial_g V_N \quad \partial_g = \frac{1}{2g}$$

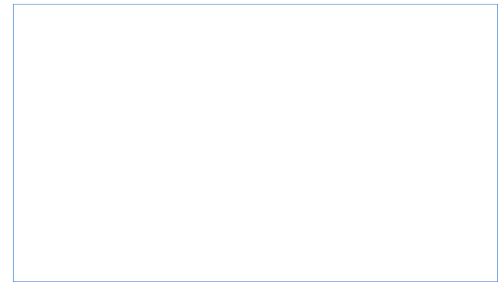
$$\Rightarrow \partial_f (V_b + V_c) = (\partial_g + 2\partial_f) V_N$$

$$\Rightarrow V_N = \frac{\partial_f}{\partial_g + 2\partial_f} (V_b + V_c)$$

$$I_{bf} = \partial_f (V_b - V_N) = \partial_f \left[V_b - \frac{\partial_f}{\partial_g + 2\partial_f} (V_b + V_c) \right] = \partial_f \left[\frac{\partial_g + \partial_f}{\partial_g + 2\partial_f} V_b - \frac{\partial_f}{\partial_g + 2\partial_f} V_c \right]$$

$$I_{cf} = \partial_f (V_c - V_N) = \partial_f \left[V_c - \frac{\partial_f}{\partial_g + 2\partial_f} (V_b + V_c) \right]$$

$$= \partial_f \left[-\frac{\partial_f}{\partial_g + 2\partial_f} V_b + \frac{\partial_g + \partial_f}{\partial_g + 2\partial_f} V_c \right]$$



$$\text{Now, } y_f \cdot \frac{y_g + y_f}{y_g + 2y_f} = \frac{1}{z_f} \left[\frac{\frac{1}{z_g} + \frac{1}{z_f}}{\frac{1}{z_g} + \frac{2}{z_f}} \right] = \frac{1}{z_f} \left[\frac{z_f + z_g}{z_f + 2z_g} \right] = \frac{z_f + z_g}{z_f^2 + 2z_g z_f} = a_1 \text{ (let)}$$

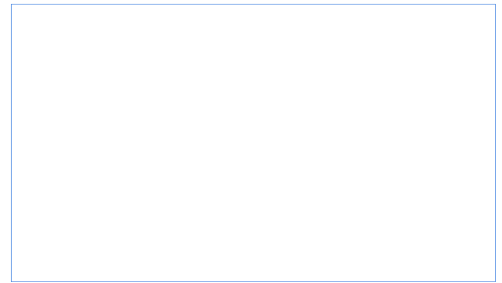
$$\text{And } \frac{y_f^2}{y_g + 2y_f} = \frac{1}{z_f^2} \cdot \frac{1}{\frac{1}{z_g} + \frac{2}{z_f}} = \frac{z_g z_f}{z_f + 2z_g} \cdot \frac{1}{z_f^2} = \frac{z_g}{z_f^2 + 2z_g z_f} = b_1 \text{ (say)}$$

Therefore,

$$I_{bf} = a_1 V_b - b_1 V_c$$

$$I_{cf} = -b_1 V_b + a_1 V_c$$

$$I_{af} = 0$$



$$\begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & a_1 & -b_1 \\ 0 & -b_1 & a_1 \end{bmatrix}}_{Y_{\text{fault}}} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$