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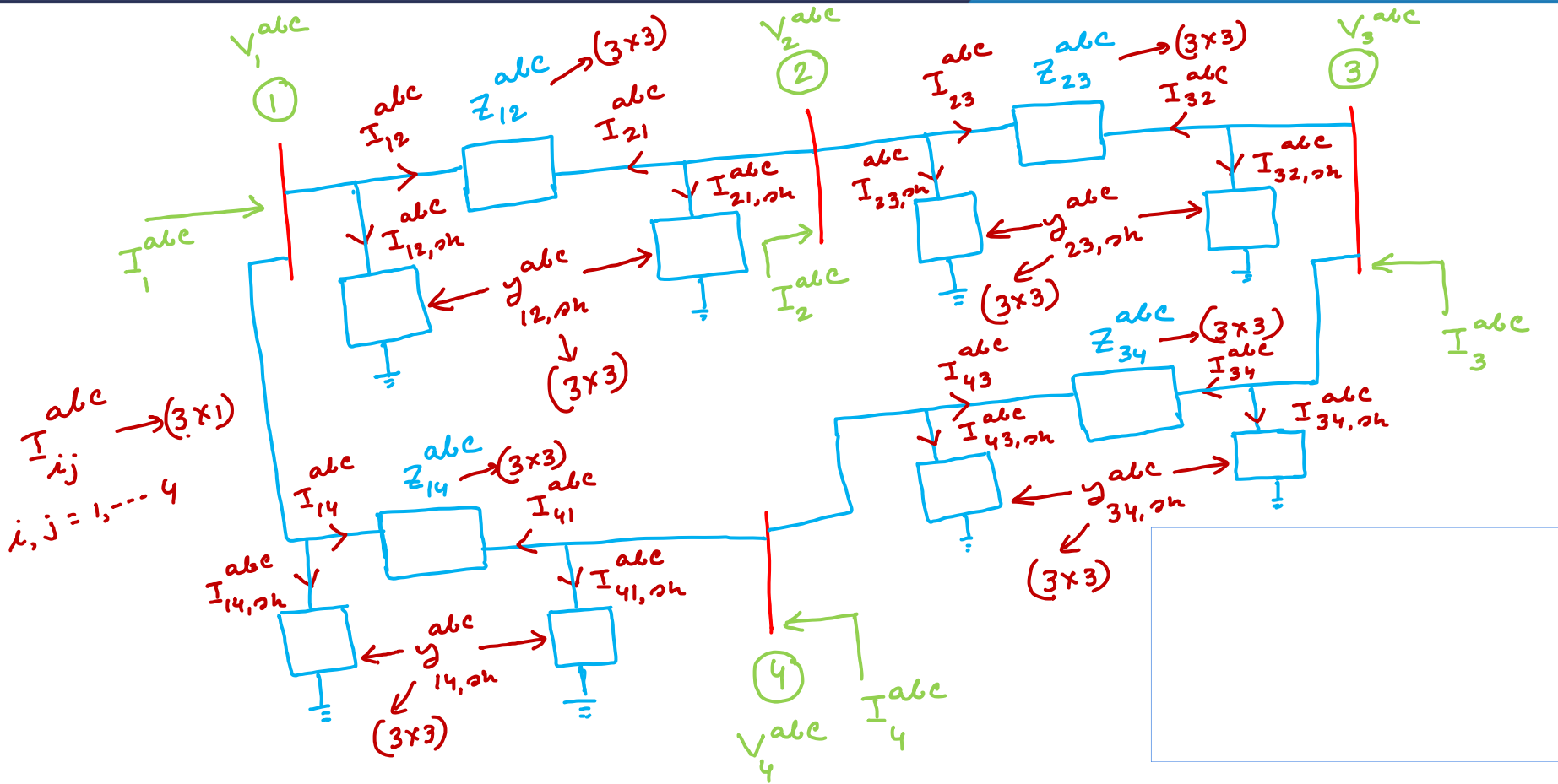
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# Computer aided power system analysis

## Fault Analysis (Contd.)

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## Equations for the complete system

Bus 1

$$\begin{aligned}
 I_1^{abc} &= I_{12,sh}^{abc} + I_{12}^{abc} + I_{14,sh}^{abc} + I_{14}^{abc} \\
 &= y_{12,sh}^{abc} V_1^{abc} + y_{12}^{abc} (V_1^{abc} - V_2^{abc}) + y_{14,sh}^{abc} V_1^{abc} + y_{14}^{abc} (V_1^{abc} - V_4^{abc}) \\
 &= \left( y_{12,sh}^{abc} + y_{12}^{abc} + y_{14,sh}^{abc} + y_{14}^{abc} \right) V_1^{abc} - y_{12}^{abc} V_2^{abc} - y_{14}^{abc} V_4^{abc} \quad \dots (1) \\
 y_{12}^{abc} &= (Z_{12}^{abc})^{-1}; \quad y_{14}^{abc} = (Z_{14}^{abc})^{-1}; \quad y_{23}^{abc} = (Z_{23}^{abc})^{-1}
 \end{aligned}$$

Bus 2

$$\begin{aligned}
 I_2^{abc} &= I_{21,sh}^{abc} + I_{21}^{abc} + I_{23,sh}^{abc} + I_{23}^{abc} \\
 &= y_{12,sh}^{abc} V_2^{abc} + y_{12}^{abc} (V_2^{abc} - V_1^{abc}) \\
 &\quad + y_{23,sh}^{abc} V_2^{abc} + y_{23}^{abc} (V_2^{abc} - V_3^{abc})
 \end{aligned}$$



$$\Rightarrow I_2^{abc} = -y_{12}^{abc} V_1^{abc} + (y_{12,2h}^{abc} + y_{12}^{abc} + y_{23,2h}^{abc} + y_{23}^{abc}) V_2^{abc} - y_{23}^{abc} V_3^{abc} \quad \dots (2)$$

Ans 3

$$I_3^{abc} = I_{32,2h}^{abc} + I_{32}^{abc} + I_{34,2h}^{abc} + I_{34}^{abc}$$

$$= y_{23,2h}^{abc} V_3^{abc} + y_{23}^{abc} (V_3^{abc} - V_2^{abc}) + y_{34,2h}^{abc} V_3^{abc} + y_{34}^{abc} (V_3^{abc} - V_4^{abc})$$

$$\Rightarrow I_3^{abc} = -y_{23}^{abc} V_2^{abc} + (y_{23,2h}^{abc} + y_{23}^{abc} + y_{34,2h}^{abc} + y_{34}^{abc}) V_3^{abc} - y_{34}^{abc} V_4^{abc} \quad \dots (3)$$

$$y_{34}^{abc} = (Z_{34}^{abc})^{-1}$$

Ans-4

$$I_4^{abc} = I_{41,2h}^{abc} + I_{41}^{abc} + I_{43,2h}^{abc} + I_{43}^{abc}$$



$$\begin{aligned}
 I_4^{abc} &= y_{14,2n}^{abc} V_4^{abc} + y_{14}^{abc} (V_4^{abc} - V_1^{abc}) + y_{34,2n}^{abc} V_4^{abc} + y_{34}^{abc} (V_4^{abc} - V_3^{abc}) \\
 &= -y_{14}^{abc} V_1^{abc} - y_{34}^{abc} V_3^{abc} + \left( y_{14,2n}^{abc} + y_{14}^{abc} + y_{34,2n}^{abc} + y_{34}^{abc} \right) V_4^{abc} \\
 &\quad \dots \textcircled{4}
 \end{aligned}$$



In matrix form

$$\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \\ I_3^{abc} \\ I_4^{abc} \end{bmatrix} = \begin{bmatrix} y_{11}^{abc} & -y_{12}^{abc} & 0 & -y_{14}^{abc} \\ -y_{12}^{abc} & y_{22}^{abc} & -y_{23}^{abc} & y_{24}^{abc} \\ 0 & -y_{23}^{abc} & y_{33}^{abc} & -y_{34}^{abc} \\ -y_{14}^{abc} & y_{24}^{abc} & -y_{34}^{abc} & y_{44}^{abc} \end{bmatrix} \begin{bmatrix} V_1^{abc} \\ V_2^{abc} \\ V_3^{abc} \\ V_4^{abc} \end{bmatrix}$$

Dimensions:  $(4 \times 1)$  for  $I$ ,  $(4 \times 4)$  for  $y$ ,  $(4 \times 1)$  for  $V$ .  
 Matrix  $y$  is  $(12 \times 12)$  in total when considering the three phases (a, b, c).  
 $N = 4$

$$\begin{aligned}
 y_{44}^{abc} &= y_{14,2h}^{abc} + y_{14}^{abc} + y_{34,2h}^{abc} + y_{34}^{abc} \\
 y_{33}^{abc} &= y_{23,2h}^{abc} + y_{23}^{abc} + y_{34,2h}^{abc} + y_{34}^{abc} \\
 y_{22}^{abc} &= y_{12,2h}^{abc} + y_{12}^{abc} + y_{23,2h}^{abc} + y_{23}^{abc}
 \end{aligned}$$

$$y_{11}^{abc} = y_{12,2h}^{abc} + y_{12}^{abc} + y_{14,2h}^{abc} + y_{14}^{abc}$$



For a  $N$ -Bus system,  $\bar{I}^{abc} \rightarrow (3N \times 1) \rightarrow$

Three phase  
injection  
current vector

$$\bar{V}^{abc} \rightarrow (3N \times 1)$$

$N \rightarrow$  no. of  
bus

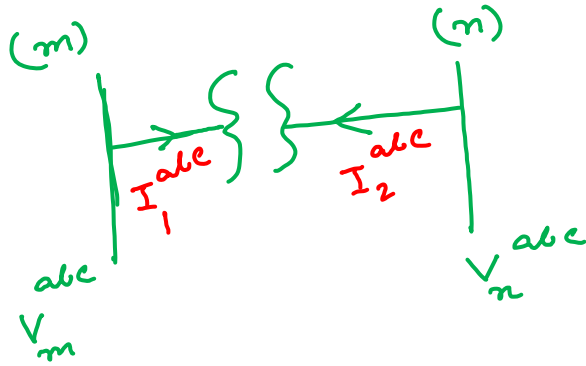
$$Y^{abc} \rightarrow (3N \times 3N)$$

Three-  
phase bus  
voltage vector

Three-phase bus impedance  
matrix



## Consideration of three-phase Transformers in $Y_{BUS}$ matrix



$$I_1^{abc} = \begin{bmatrix} I_1^a & I_1^b & I_1^c \end{bmatrix}^T$$
$$I_2^{abc} = \begin{bmatrix} I_2^a & I_2^b & I_2^c \end{bmatrix}^T$$

$$\begin{array}{c}
 \left. \begin{array}{c} abc \\ I_1 \end{array} \right\} \begin{bmatrix} I_1^a \\ I_1^b \\ I_1^c \end{bmatrix} \\
 \left. \begin{array}{c} abc \\ I_2 \end{array} \right\} \begin{bmatrix} I_2^a \\ I_2^b \\ I_2^c \end{bmatrix}
 \end{array}
 =
 \begin{bmatrix}
 x & x & x & x & x & x \\
 x & \underbrace{Y_I}_{(3 \times 3)} & x & x & x & x \\
 x & x & x & x & x & x \\
 x & x & x & x & x & x \\
 x & x & x & x & x & x \\
 x & x & x & x & x & x
 \end{bmatrix}
 \begin{bmatrix}
 x & x & x & x & x & x \\
 x & \underbrace{Y_{II}}_{(3 \times 3)} & x & x & x & x \\
 x & x & x & x & x & x \\
 x & x & x & x & x & x \\
 x & x & x & x & x & x \\
 x & x & x & x & x & x
 \end{bmatrix}
 \begin{bmatrix}
 V_m^a \\ V_m^b \\ V_m^c \\ V_n^a \\ V_n^b \\ V_n^c
 \end{bmatrix}
 \begin{array}{c}
 \left. \begin{array}{c} abc \\ V_m \end{array} \right\} \\
 \left. \begin{array}{c} abc \\ V_n \end{array} \right\} \rightarrow (6 \times 1)
 \end{array}$$

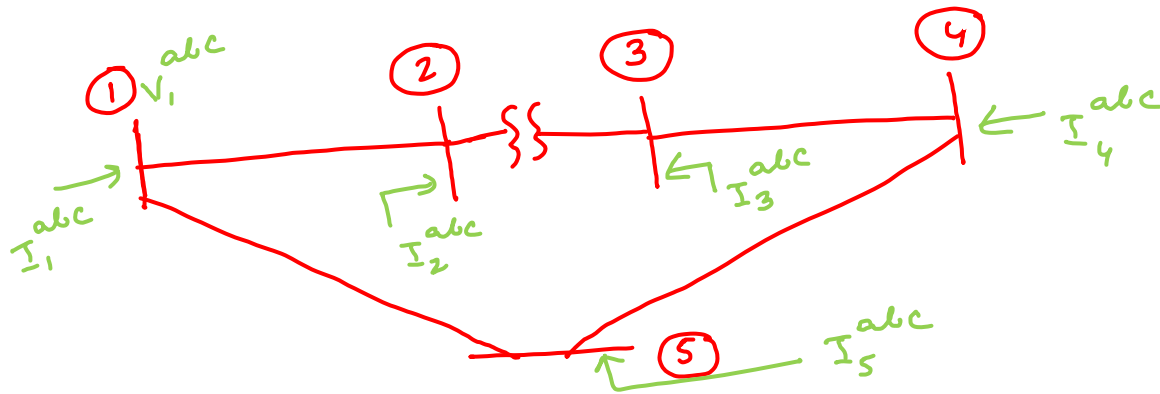
$(6 \times 1)$        $(6 \times 6)$



$$\begin{bmatrix} I_1^{abc} \\ I_2^{abc} \end{bmatrix} = \begin{bmatrix} Y_I & Y_{II} \\ Y_{III} & Y_{IV} \end{bmatrix} \begin{bmatrix} V_m^{abc} \\ V_n^{abc} \end{bmatrix}$$

$$\Rightarrow I_1^{abc} = Y_I V_m^{abc} + Y_{II} V_n^{abc}$$

$$I_2^{abc} = Y_{III} V_m^{abc} + Y_{IV} V_n^{abc}$$



Line-models

$\gamma_{mn}^{abc}$

$\gamma_{mn,sh}^{abc}$

| $m$ | $n$ |
|-----|-----|
| 1   | 2   |
| 3   | 4   |
| 1   | 5   |
| 4   | 5   |



Ques-1

$$I_1^{abc} = y_{11}^{abc} V_1^{abc} - y_{12}^{abc} V_2^{abc} - y_{15}^{abc} V_5^{abc} \dots \textcircled{1}$$

$$y_{11}^{abc} = y_{12}^{abc} + y_{12, oh}^{abc} + y_{15}^{abc} + y_{15, oh}^{abc}$$

