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CERTIFICATION COURSE

Computer aided power system analysis

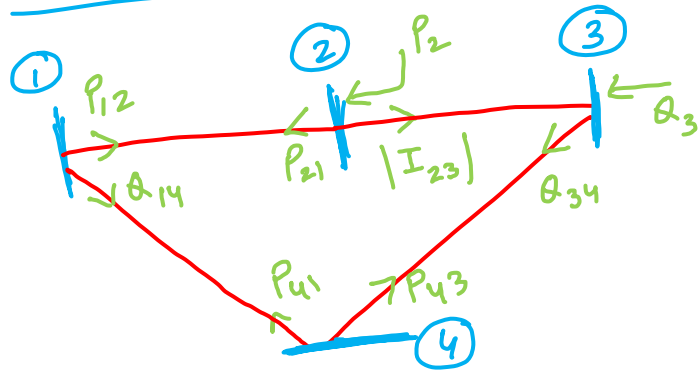
Power system state estimation (Contd.)

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A simple example



$$N=4 ; n=2 \times 4 - 1 = 7$$

$$\bar{x} = [v_1 \quad v_2 \quad v_3 \quad v_4 \quad \theta_2 \quad \theta_3 \quad \theta_4]^T$$

$$= [\theta_2 \quad \theta_3 \quad \theta_4 \quad v_1 \quad v_2 \quad v_3 \quad v_4]^T$$

$$\downarrow \quad \downarrow \quad \dots \quad \downarrow$$

$$x_1 \quad x_2 \quad \dots \quad x_9$$

$P_{12} \neq -P_{21}$ (because of loss)

$$\bar{z} = [P_{12} \quad P_{21} \quad Q_{14} \quad P_{41} \quad P_{43} \quad Q_{34} \quad |I_{23}| \quad P_2 \quad Q_3]^T$$

$$\downarrow \quad \downarrow$$

$$z_1 \quad z_2$$

$$\downarrow$$

$$z_9$$



Jacobian matrix

$H =$

$$\begin{bmatrix} \frac{\partial p_{12}}{\partial \theta_2} & \frac{\partial p_{12}}{\partial \theta_3} & \dots & \frac{\partial p_{12}}{\partial v_4} \\ \frac{\partial p_{21}}{\partial \theta_2} & \frac{\partial p_{21}}{\partial \theta_3} & \dots & \frac{\partial p_{21}}{\partial v_4} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial q_3}{\partial \theta_2} & \frac{\partial q_3}{\partial \theta_3} & \dots & \frac{\partial q_3}{\partial v_4} \end{bmatrix}$$

$\leftarrow p_{12}$ (first row)

$\leftarrow p_{21}$ (second row)

$$p_{12} = f_1(v_1, v_2, \theta_2)$$

$$p_{21} = f_2(v_1, v_2, \theta_2)$$

$$\frac{\partial p_{12}}{\partial \theta_3} = \frac{\partial p_{12}}{\partial \theta_4} = \frac{\partial p_{12}}{\partial v_3} = \frac{\partial p_{12}}{\partial v_4} = 0$$

$\leftarrow q_3$ (9th row)



$H =$

θ_2	θ_3	θ_4	v_1	v_2	v_3	v_4	
x	0	0	x	x	0	0	$\leftarrow P_{12}$
x	0	0	x	x	0	0	$\leftarrow P_{21}$
0	0	x	x	0	0	x	$\leftarrow Q_{14}$
0	0	x	x	0	0	x	$\leftarrow P_{41}$
0	x	x	0	0	x	x	$\leftarrow P_{43}$
0	x	x	0	0	x	x	$\leftarrow Q_{34}$
x	x	0	0	x	x	0	$\leftarrow I_{23} $
x	x	0	x	x	x	0	$\leftarrow P_2$
x	x	x	0	x	x	x	$\leftarrow Q_3$

$$P_2 = \sum_{j=1}^4 v_2 v_j \gamma_{2j} \cos(\theta_2 - \theta_j - d_{2j})$$

$$Q_{14} = f_3(v_1, v_4, \theta_4)$$

$$P_{43} = g_1(\theta_3, \theta_4, v_3, v_4)$$

$$|I_{23}| = g_2(\theta_2, \theta_3, v_2, v_3)$$

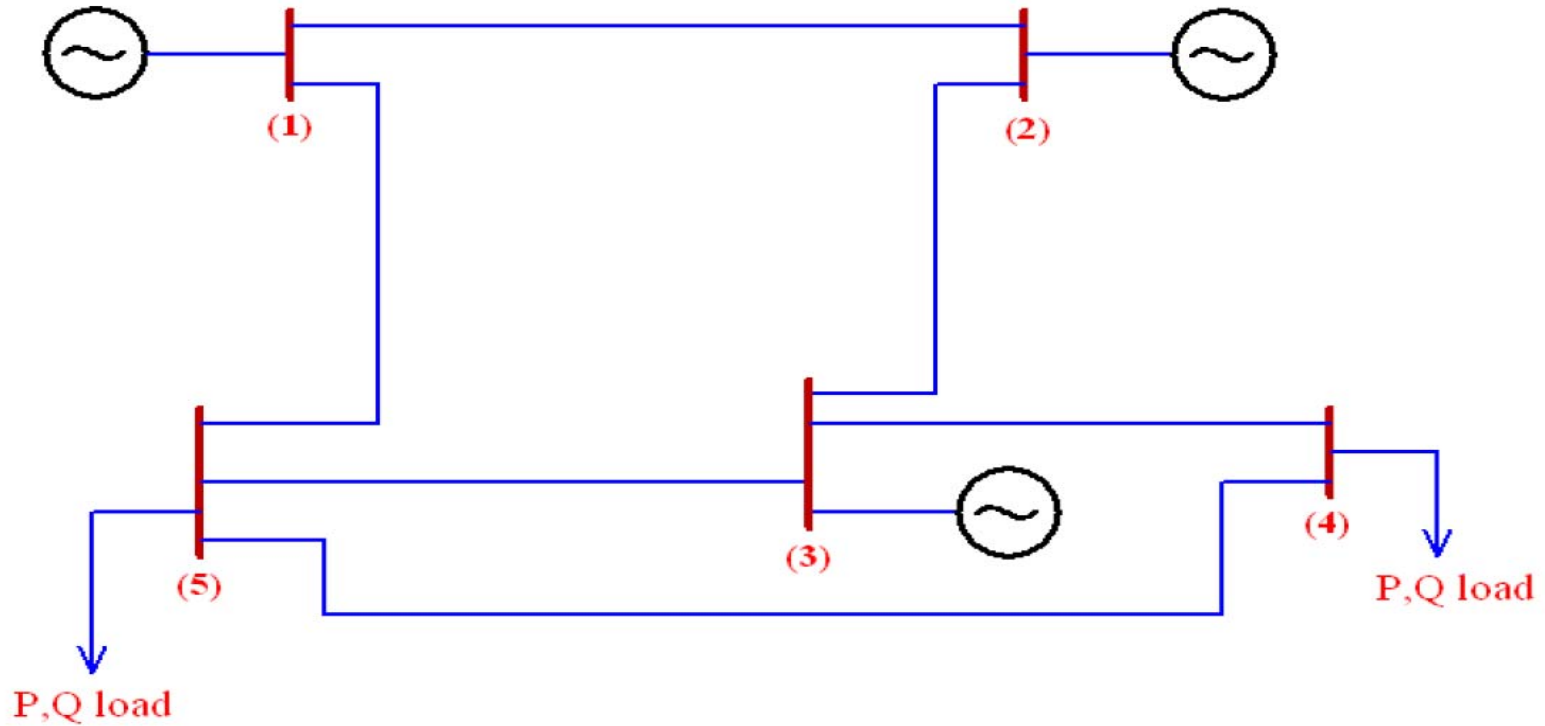
$$P_2 = f_2(v_1, v_2, v_3, \theta_2, \theta_3)$$

$\gamma_{24} = 0$

$$Q_3 = \sum_{k=1}^4 v_3 v_k \gamma_{3k} \sin(\theta_3 - \theta_k - d_{3k}) = f_5(v_2, v_3, v_4, \theta_2, \theta_3, \theta_4)$$



Power system state estimation



Power system state estimation

load flow result

Bus no.	vm	theta (rad)
1	1	0
2	1	0.0289
3	1	-0.0159
4	0.9059	-0.1458
5	0.944	-0.0877

SE result (no error)
($M = 12$)

vm	theta (rad)
1	0
1	0.0289
1	-0.0159
0.906	-0.1457
0.944	-0.0877

measurements ($M = 12$)

P12, Q12, P15, Q15, P23, Q23, P34, Q34, P35, Q35, P54, Q54

Power system state estimation

SE result ($\sigma=0.02$)
($M = 12$)

vm	theta
0.9832	0
0.9832	0.0297
0.9832	-0.016
0.8892	-0.1518
0.927	-0.0914

SE result ($\sigma=0.02$)
($M = 18$)

vm	theta
0.9955	0
0.9954	0.0289
0.9957	-0.0151
0.9022	-0.1473
0.9397	-0.0894

SE result ($\sigma=0.02$)
($M = 24$)

vm	theta
0.996	0
0.9959	0.0288
0.9962	-0.015
0.9029	-0.1483
0.9402	-0.0893

measurements ($M = 18$)

(all of $M = 12$) + P21, Q21, P51, Q51, P32, Q32

measurements ($M = 24$)

(all of $M = 18$) + P43, Q43, P53, Q53, P45, Q45



Power system state estimation

- $\alpha = 0.05$
- For $M = 24$ (with no bad data error), $C(k, \alpha) = 24.9958$; $\hat{f} = 1.4789$
- For $M = 24$ and $P_{54}^{bad} = 10.2193$ ($P_{54}^{meas} = 0.2193$) $\hat{f} = 196532.65$
- For $M = 23$ (with no bad data error), $C(k, \alpha) = 23.684$; $\hat{f} = 1.4721$

