



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

Power system state estimation (Contd.)

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Algorithm for PSSE

$m \rightarrow$ no. of measurement
 $n \rightarrow$ no. of states.

1. Take initial guess $\bar{x}^{(0)}$ and choose $c(k, d)$ for a given d (we choose d)
2. Set $k=0 \rightarrow$ iteration count. and set $W = R^{-1}$ matrix
3. Evaluate $h_i(\bar{x}^{(k)})$ and $\Delta z_i = z_i - h_i(\bar{x}^{(k)}) \quad \forall i=1, \dots, m$
3. Evaluate $H_{\bar{x}^{(k)}} \rightarrow$ Jacobian matrix $\rightarrow H_{\bar{x}^{(k)}}$ is the Jacobian matrix evaluated at $\bar{x} = \bar{x}^{(k)}$.
4. Calculate $\Delta \bar{x}$
5. Calculate $x_i^{(k+1)} = x_i^{(k)} + \Delta x_i \quad \forall i=1, \dots, n$
6. Calculate $e = \max_i |x_i^{(k+1)} - x_i^{(k)}|$
7. Check $e < \epsilon$ Yes $\rightarrow \bar{x}^{(k+1)}$ is the solution, and go to step 9.
No $\rightarrow k = k+1$ and go back to step 3.



Algorithm (contd)

9. Calculate $\hat{z}_i \quad \forall i=1, \dots, m$
10. Calculate $\hat{e}_i = z_i - \hat{z}_i \quad \forall i=1, \dots, m$
11. Calculate $\hat{f}$.
12. If $\hat{f} \leq C(k, \alpha)$ Yes
no no bad data and therefore final solution is obtained.
Go to step 13.
13. Calculate R' matrix
14. Calculate $\gamma_i = \frac{\hat{e}_i}{R'_{ii}} \quad \forall i=1, \dots, m$
15. Calculate $p = \max_{\forall i=1, \dots, m} |\gamma_i|$ and find the index of 'p' in the vector $\bar{\gamma}$. Let this index be 'i'. and hence, i^{th} measurement is bad.

$$\bar{\gamma} = [\gamma_1, \gamma_2, \dots, \gamma_m]^T$$



Algorithm

16. Remove the i^{th} measurement from the set of measurements.
17. Go back to step 1.

Structure of the Jacobian matrix

measurements are:

$$z_i = h_i(x_1, x_2, \dots, x_n)$$

$$\forall i=1, \dots, m$$

$$H = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \dots & \frac{\partial h_1}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_m}{\partial x_1} & \frac{\partial h_m}{\partial x_2} & \dots & \frac{\partial h_m}{\partial x_n} \end{bmatrix} \rightarrow (m \times n)$$

measurements can be
 $p_{ij}, q_{ij}, v_i, p_i, q_i, |I_{ij}|$



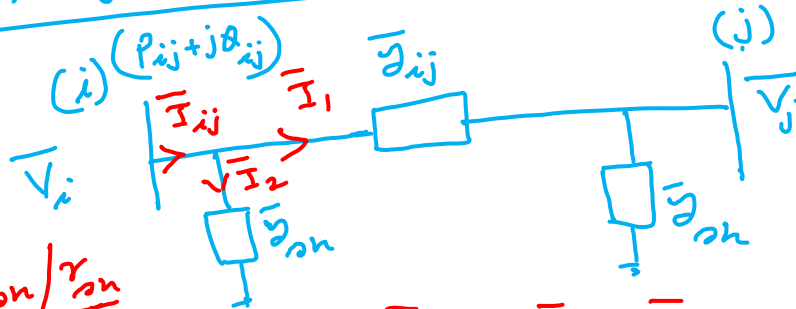
Expressions of measured quantities

$$P_i = \sum_{j=1}^N V_i V_j Y_{ij} \cos(\theta_i - \theta_j - \alpha_{ij})$$

$$Q_i = \sum_{j=1}^N V_i V_j Y_{ij} \sin(\theta_i - \theta_j - \alpha_{ij})$$

$N \rightarrow$ no. of bus in the system
 $n = 2N - 1$

$P_{ij}, Q_{ij}, |I_{ij}|$



$$\bar{Y}_{oh} = Y_{oh} \angle \gamma_{oh} = jY_{oh}$$

$$\bar{I}_{ij} = \bar{I}_1 + \bar{I}_2$$

$\bar{V}_i, \bar{V}_j \rightarrow$ complex voltages of bus i and j
 $\bar{Y}_{ij}, \bar{Y}_{oh} \rightarrow$ are complex admittances of the line
 $\bar{Y}_{oh} \rightarrow$ half-line charging susceptance of the line.

$$\bar{V}_i = V_i \angle \theta_i$$

$$\bar{V}_j = V_j \angle \theta_j$$

$$\bar{Y}_{ij} = Y_{ij} \angle \beta_{ij}$$



$$\begin{aligned}
 \bar{I}_{ij} &= (\bar{V}_i - \bar{V}_j) \mathcal{Y}_{ij} + \bar{V}_i \mathcal{Y}_{sh} \\
 &= \left(V_i \angle \theta_i - V_j \angle \theta_j \right) \mathcal{Y}_{ij} / \beta_{ij} + \left(V_i \angle \theta_i \right) (j \mathcal{Y}_{sh}) \\
 &= \left(V_i \angle \theta_i + \beta_{ij} - V_j \angle \theta_j + \beta_{ij} \right) \mathcal{Y}_{ij} + \mathcal{Y}_{sh} V_i \angle \theta_i + 90^\circ \\
 &= \left[V_i \cos(\theta_i + \beta_{ij}) - V_j \cos(\theta_j + \beta_{ij}) \right] \mathcal{Y}_{ij} + \\
 &= \left[V_i \sin(\theta_i + \beta_{ij}) - V_j \sin(\theta_j + \beta_{ij}) \right] \mathcal{Y}_{ij} - \mathcal{Y}_{sh} V_i \sin \theta_i \\
 &\quad + j \mathcal{Y}_{sh} V_i \cos \theta_i
 \end{aligned}$$

complex
operator

$\Rightarrow j = \sqrt{-1}$

$$\Rightarrow \bar{I}_{ij} = a_{ij} + j b_{ij}$$

bus index

complex operator

$$(P_{ij} + jQ_{ij}) = \bar{V}_i \bar{I}_{ij}^* = (V_i \cos \theta_i + j V_i \sin \theta_i) (a_{ij} - j b_{ij})$$



$$\Rightarrow P_{ij} = V_i (a_{ij} \cos \theta_i + b_{ij} \sin \theta_i)$$

$$\Rightarrow Q_{ij} = V_i (a_{ij} \sin \theta_i - b_{ij} \cos \theta_i)$$

$$\Rightarrow |I_{ij}| = \sqrt{a_{ij}^2 + b_{ij}^2}$$

$\Rightarrow$ For known line parameters,

$$P_{ij} = P_{ij}(V_i, \theta_i, V_j, \theta_j)$$

$$Q_{ij} = Q_{ij}(V_i, \theta_i, V_j, \theta_j)$$

$$|I_{ij}| = f(V_i, \theta_i, V_j, \theta_j)$$

