



IIT ROORKEE



**NPTEL ONLINE
CERTIFICATION COURSE**

Computer aided power system analysis

Power system state estimation (Contd.)

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Measurement model.

$$z_1 = h_1(x_1, x_2, \dots, x_n) + e_1$$

$$z_2 = h_2(x_1, x_2, \dots, x_n) + e_2$$

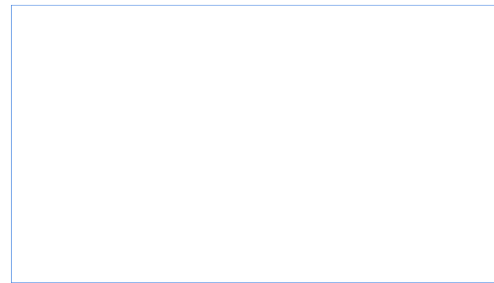
$$\vdots$$
$$z_m = h_m(x_1, x_2, \dots, x_n) + e_m$$

$m > n$
 $h_1(\cdot), h_2(\cdot), \dots, h_m(\cdot) \rightarrow \text{non-linear.}$

We take some initial guess

$$\bar{x}^{(0)} = [x_1^{(0)} \quad x_2^{(0)} \quad \dots \quad x_n^{(0)}]^T$$

Now, we apply Taylor Series expansion



$$Z_1 = h_1(x_1^{(0)}, x_2^{(0)} \dots x_n^{(0)}) + \frac{\partial h_1}{\partial x_1} \Delta x_1 + \frac{\partial h_1}{\partial x_2} \Delta x_2 + \dots + \frac{\partial h_1}{\partial x_n} \Delta x_n + e_1$$

$$Z_2 = h_2(x_1^{(0)}, x_2^{(0)} \dots x_n^{(0)}) + \frac{\partial h_2}{\partial x_1} \Delta x_1 + \frac{\partial h_2}{\partial x_2} \Delta x_2 + \dots + \frac{\partial h_2}{\partial x_n} \Delta x_n + e_2$$

$$\vdots$$

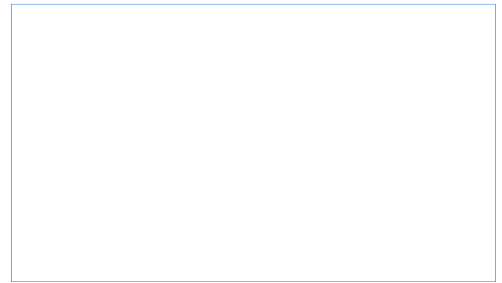
$$Z_m = h_m(x_1^{(0)}, x_2^{(0)} \dots x_n^{(0)}) + \frac{\partial h_m}{\partial x_1} \Delta x_1 + \frac{\partial h_m}{\partial x_2} \Delta x_2 + \dots + \frac{\partial h_m}{\partial x_n} \Delta x_n + e_m$$

$$\Rightarrow Z_1 - h_1(\bar{x}^{(0)}) = \frac{\partial h_1}{\partial x_1} \Delta x_1 + \frac{\partial h_1}{\partial x_2} \Delta x_2 + \dots + \frac{\partial h_1}{\partial x_n} \Delta x_n + e_1$$

$$Z_2 - h_2(\bar{x}^{(0)}) = \frac{\partial h_2}{\partial x_1} \Delta x_1 + \frac{\partial h_2}{\partial x_2} \Delta x_2 + \dots + \frac{\partial h_2}{\partial x_n} \Delta x_n + e_2$$

$$\vdots$$

$$Z_m - h_m(\bar{x}^{(0)}) = \frac{\partial h_m}{\partial x_1} \Delta x_1 + \frac{\partial h_m}{\partial x_2} \Delta x_2 + \dots + \frac{\partial h_m}{\partial x_n} \Delta x_n + e_m$$



$$\begin{bmatrix} z_1 - h_1(\bar{x}^{(0)}) \\ z_2 - h_2(\bar{x}^{(0)}) \\ \vdots \\ z_m - h_m(\bar{x}^{(0)}) \end{bmatrix} = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_m}{\partial x_1} & \frac{\partial h_m}{\partial x_2} & \cdots & \frac{\partial h_m}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix}$$

$H_{\bar{x}} \rightarrow$ Jacobian matrix

$H_{\bar{x}^{(0)}} \rightarrow$ Jacobian matrix evaluated with $\bar{x} = \bar{x}^{(0)}$

Denote $\Delta z_i = z_i - h_i(\bar{x}^{(0)}) \rightarrow \text{known}$ $H_{\bar{x}^{(0)}}$

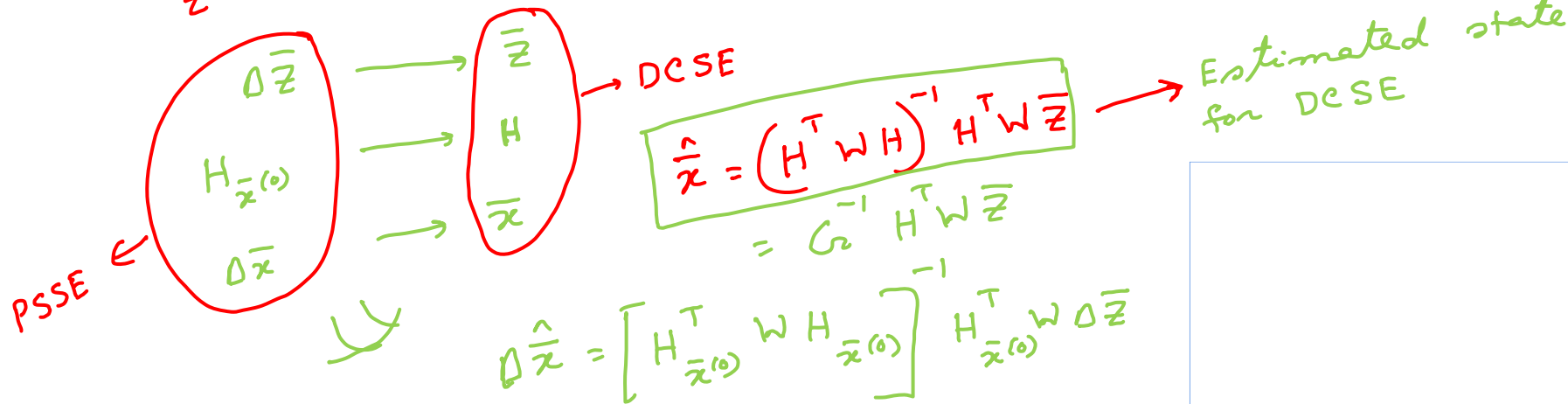
$$\begin{bmatrix} \Delta z_1 \\ \Delta z_2 \\ \vdots \\ \Delta z_n \end{bmatrix} = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_1}{\partial x_2} & \cdots & \frac{\partial h_1}{\partial x_n} \\ \frac{\partial h_2}{\partial x_1} & \frac{\partial h_2}{\partial x_2} & \cdots & \frac{\partial h_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial h_m}{\partial x_1} & \frac{\partial h_m}{\partial x_2} & \cdots & \frac{\partial h_m}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix}$$



$\Delta \bar{z} = H_{\bar{x}(0)} \cdot \Delta \bar{x} + \bar{e} \rightarrow \text{Linearized measurement model.}$

where $\Delta \bar{x} = [\Delta x_1 \quad \Delta x_2 \quad \dots \quad \Delta x_n]^T$
 $\bar{e} = [e_1 \quad e_2 \quad \dots \quad e_m]^T$

$\bar{z} = H \bar{x} + \bar{e} \rightarrow \text{Linear exact state estimation model.}$



$$W \rightarrow R^{-1}$$

Detailed algorithm

1. Take an initial guess $\bar{x}^{(0)}$; Calculate $\Delta \bar{z}$ vector.

2. $k = 0 \rightarrow$ iteration count.

$$3. \Delta \bar{x}^{(k)} = \left[H_{\bar{x}^{(k)}}^T W H_{\bar{x}^{(k)}} \right]^{-1} H_{\bar{x}^{(k)}} W \cdot \Delta \bar{z}$$

$$4. \bar{x}^{(k+1)} = \bar{x}^{(k)} + \Delta \bar{x}^{(k)}$$

5. Check $\max |\bar{x}^{(k+1)} - \bar{x}^{(k)}| < \epsilon$. If yes,

6. If not, Calculate $\Delta \bar{z} \Big|_{\bar{x}^{(k+1)}}$

7. $k = k+1$ and go to step 3.

$\hat{\bar{x}} = \bar{x}^{(k+1)}$ $\rightarrow$ Final result.

