



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

Computer aided power system analysis

Power system state estimation

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Bad data Detection

Calculate $\hat{f}$
if $\hat{f} > C(k, d) \rightarrow$ bad data exists

$$\hat{e}_i = (z_i - \hat{z}_i)$$

$\downarrow$
estimated error

$\frac{z_i - \hat{z}_i}{\sqrt{R'_{ii}}} \rightarrow$ standardised error.

$$E \left[\frac{(z_i - \hat{z}_i)^2}{R'_{ii}} \right] = 1 \Rightarrow E \left[\left(\frac{z_i - \hat{z}_i}{\sqrt{R'_{ii}}} \right)^2 \right] = 1$$

$$\frac{\hat{e}_i - 0}{\sqrt{R'_{ii}}} = \frac{z_i - \hat{z}_i}{\sqrt{R'_{ii}}}$$



$m \rightarrow$ no. of measurement
 $n \rightarrow$ no. of states

$$m = 4 \quad m = 3$$

$$n = 2$$

$$(m-1)$$

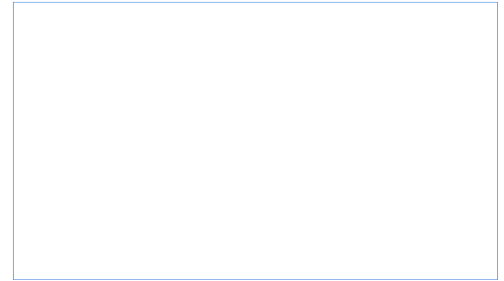
Let there are 'l' meters which have gone bad

$$(m-1)$$

$$(m-2)$$

$$m > n$$

$$(m-l) < n$$



Power System state estimation

Let There are N no. of buses.

$$V_i, \theta_i \quad \forall i=1, \dots, N$$

$$n = 2N$$

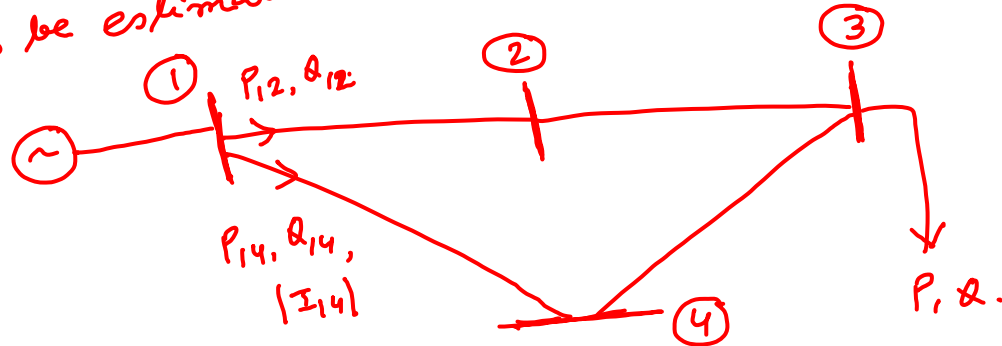
$$n = 2N - 1$$

$$\bar{\theta} = [\theta_2 \quad \theta_3 \quad \dots \quad \theta_N]^T \rightarrow (N-1) \times 1$$

$$\bar{V} = [V_1 \quad V_2 \quad \dots \quad V_N]^T \rightarrow (N \times 1)$$

$$\theta_1 = 0$$

↙
Total no. of states
to be estimated



Possible measurements:

$$P_{ij}, Q_{ij}, |I_{ij}|, V_i, P_i, Q_i$$

Real power
flow over
line 'i-j'

relative
power flow
over line
'i-j'

magnitude of
current flow
over line 'i-j'

$$P_i = P_i(\bar{V}, \bar{\theta})$$

$$Q_i = Q_i(\bar{V}, \bar{\theta})$$

$$\bar{E} = H\bar{x} + \bar{e}$$

Constant
matrix

$$P_i = \sum_{k=1}^N V_i V_k Y_{ik} \cos(\theta_i - \theta_k - \alpha_{ik})$$

$$Q_i = \sum_{k=1}^N V_i V_k Y_{ik} \sin(\theta_i - \theta_k - \alpha_{ik})$$

Non-linear
functions



In general, in power ^{system} state estimation, the measured quantities are non-linear functions of the state variables

Now, let $\bar{z} = [\bar{V}^T \bar{\theta}^T]^T \rightarrow \bar{z}$ is a column vector $(2N-1) \times 1$

$$\bar{V}^T \rightarrow [V_1 \ V_2 \ \dots \ V_N]$$

$$\bar{\theta}^T \Rightarrow [\theta_2 \ \theta_3 \ \dots \ \theta_N]$$

$$[\bar{V}^T \ \bar{\theta}^T] = [V_1 \ V_2 \ \dots \ V_N \ \theta_2 \ \theta_3 \ \dots \ \theta_N]^T$$

$$[\bar{V}^T \ \bar{\theta}^T]^T = [V_1 \ V_2 \ \dots \ V_N \ \theta_2 \ \theta_3 \ \dots \ \theta_N]^T$$

$$m > n$$

measurements are
 $z_i \quad \forall i = 1, 2, \dots, m$



$$\left\{ \begin{array}{l} z_1 = h_1(\bar{x}) + e_1 \\ z_2 = h_2(\bar{x}) + e_2 \\ \vdots \\ z_m = h_m(\bar{x}) + e_m \end{array} \right.$$

$h_1(\cdot), h_2(\cdot) \dots h_m(\cdot)$ are non-linear functions of $\bar{x}$
 Now, $\bar{x} = [x_1, x_2, \dots, x_n]^T$; $n = (2N-1)$



$$\left\{ \begin{array}{l} z_1 = h_1(x_1, x_2, \dots, x_n) + e_1 \\ z_2 = h_2(x_1, x_2, \dots, x_n) + e_2 \\ \vdots \\ z_m = h_m(x_1, x_2, \dots, x_m) + e_m \end{array} \right.$$

General measurement model for power system state estimation.

